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A tripartite evolutionary game for strategic decision-making in live-streaming e-commerce

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ABSTRACT

The rapid growth of live-streaming has transformed traditional e-commerce into an interactive and immersive experience, giving birth to live-streaming e-commerce. This paper investigates the strategic interactions between brands, social media influencers, and consumers under this mechanism. Using evolutionary game theory, we model decision-making dynamics across these three parties and analyze how their strategies develop over time. Our framework incorporates contractual penalties between brands and influencers, rewards for influencers, product returns, and subscription fees to capture realistic market behaviors. We derive replicator dynamics equations for each participant group and identify stable equilibrium strategies for the entire system. The application of replicator dynamics offers valuable perspectives on temporary states and strategies that achieve long-term equilibrium. We also present numerical simulations to validate the effectiveness of our model. In addition, we show how parameters, such as penalties and rewards, influence strategy selection and allow the system to achieve stability successfully. This research provides actionable recommendations for optimizing partnerships in live-streaming e-commerce supply chains.

1. Introduction

Live streaming e-commerce has developed into a revolutionary business model combining real-time interactivity and online shopping. This business model integrates live-streaming technology with online shopping, enabling real-time interaction between influencers, consumers, and brands. Allowing influencers to demonstrate and endorse products while responding to live queries, the model constructs trust, reduces uncertainty about product quality, and creates an immersive shopping experience; see [1]. According to Forbes, in 2021, live-streaming shopping was 327 billion in China, reflecting 108% growth, on top of 220% growth the year before.

In recent years, live streaming has become a trending research topic. In [2], the authors presented a three-party game composed of manufacturers, social media influencers, and consumers in the live-streaming e-commerce model. They provided evolutionary stable strategies via replicator dynamics equations and showed some numerical simulations. In [3], the authors investigated when a brand should use an influencer channel. They proposed a three-stage game composed of a manufacturer, an e-commerce channel, and an influencer channel. They obtained the equilibrium prices backward. In addition, the issue of misfits and product returns deserves attention. In [4], the authors addressed the problem of return-freight insurance on live-streaming shopping platforms using evolutionary game theory. They provided evolutionary stable strategies for the entire system.

Current studies on live-streaming e-commerce have explored various aspects of these interactions but lack an integrated approach to analyzing their interplay. Few studies have investigated the strategic interactions of the parties in this supply chain. This paper addresses this gap by developing a tripartite evolutionary game model that captures the decision-making dynamics among brands, influencers, and consumers. Unlike previous studies, such as [2], this paper introduces product evaluations by both consumers and influencers and product return costs, offering a more nuanced understanding of decision-making. We also analyze the role of penalties and rewards and their impact on brand and influencer strategies. Finally, we consider the subscription cost to join the influencer channel as a factor influencing consumer engagement. Incorporating these key factors, the model provides a realistic framework for analyzing strategy evolution over time.

The main contributions of our research are:

- developing a tripartite evolutionary game model composed of brands, influencers, and consumers to analyze their strategic interactions in live-streaming e-commerce.

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- incorporating practical elements such as contractual penalties, consumer rewards, product returns, and subscription fees into the analysis.
- identifying evolutionary stable strategies using replicator dynamics, providing insights into conditions for cooperation among the players.
- providing simulations to explore the sensitivity of key parameters (e.g., penalties, rewards, and potential losses) on player behavior and system outcomes.
- offering actionable recommendations for optimizing brand-influencer partnerships, consumer engagement, and overall supply chain efficiency in live-streaming e-commerce.

This paper is structured as follows: Section 2 reviews the relevant literature, Section 3 introduces the evolutionary game model, and Section 4 analyzes replicator dynamics. Stability analysis and numerical simulations are presented in Sections 5 and 6, respectively. Finally, Section 7 concludes the study with practical implications and future research directions.

2. Literature review

In this section, we review the most relevant literature to our study. From the methodological point of view, game theory has a fundamental role in supply chain management. In particular, classical game theory (non-cooperative and cooperative games with rational players) is applied to study the decisions of the members of the supply chain in several contexts [5,6], including social networks [7–9], and social media platforms profit analysis [10,11]. However, evolutionary game theory is becoming an essential tool to examine how the expected revenues and players' preferences change over time. Evolutionary game theory, see [12], describes the strategic interactions among several users under a dynamic process. It was first applied in biology to study the evolution of the behavior of animal populations and was then extended to other disciplines. The most important game dynamics is the replicator equation, introduced for a single species in [13]. The replicator dynamics equations are widely used in different fields, ranging from engineering and emergency management to logistics, see [14–16]. In [17], an evolutionary game decision model for network attack-defense based on the regret minimization algorithm is proposed to optimize the learning mechanism based on replication dynamics equations. In [18], the authors present a model of value co-creation behavior evolution of focal companies and non-focal subjects in the digital innovation ecosystem and construct a complex network evolutionary game model to study the dynamic decision-making process in the system. In [19], the authors model offloading with evolutionary game theory, using replicator dynamics to evolve user strategies to adapt to the fog/cloud environment.

Even if the advantages of e-commerce have been extensively investigated, fewer studies are devoted to live-streaming e-commerce models. In [20], the authors propose a tripartite game model including social media influencers, the live-streaming e-commerce platform, and consumers and analyze the dynamic evolution process of the strategy selection among subjects and influencing factors using evolutionary game theory. In [21], the authors explored the impact of various live-streaming factors on the number of sold users. In [22], the authors investigated online retailers' optimal strategies in a highly competitive market where they distribute products via regular selling or live-streaming selling with spillover effects. They examined the impacts of these strategies on consumer surplus and social welfare. In [23], the authors construct a two-period game model to study the effects of social influences on channel subscriptions and pricing decisions for a new luxury fashion product launched in a direct channel distribution system and a retail channel one, formulated as a Stackelberg game, where the manufacturer acts as the leader and the retailer, as the follower. In [24], the paper analyzed how live-streaming anchor behavior, supplier behavior, and streaming platform strategic decisions affect live streaming. This study presented a game model of value co-creation evolution in live-streaming ecosystems. In [25], the authors focused on exploring the problems and the behavioral strategies of stakeholders in the governance process and discussed how to guide behavioral decision-making to control the problem of live-streaming. In [26], the authors analyze the impact of top and ordinary streamers on the quality decision of live streaming e-commerce supply chains. A differential game model is constructed to optimize the trajectory of live-streaming product quality under the two types of streamers.

Unlike prior research, which typically focuses on brand-influencer partnerships, influencer-consumer interactions, or platform-influencer-consumer relations, this study develops an evolutionary game model that considers brands, influencers, and consumers simultaneously. In addition, this study incorporates factors that significantly influence strategy dynamics such as penalties for non-compliance between brands and influencers, consumer rewards to incentivize influencer engagement, product returns and associated costs, consumer loss, and subscription fees charged by influencers, adding a financial aspect to consumer decisions.

In conclusion, this study bridges critical gaps in the existing literature by providing a dynamic, comprehensive, and realistic analysis of live-streaming e-commerce supply chains. It advances both theoretical understanding and practical applications in this rapidly growing domain.

3. Evolutionary game model

This study examines a supply chain that includes brands, social media influencers, and consumers engaging through a live-streaming e-commerce platform. Brands supply products in response to orders made by influencers on the platform, allowing them to monitor customer preferences to optimize production and improve product quality. Influencers create a unique shopping experience for consumers by presenting product demonstrations and answering inquiries in real time [27]. Influencers need to personally test the products to provide authentic feedback and insights to their audience. Influencers provide exclusive promotional codes to engage consumers during a live shopping event that are available only to live viewers and have a limited duration. Influencers play a crucial role in improving brand perception and driving sales. In contrast, brands use influencers' visibility to connect with their audience and attract new customers. Consumers participate in the live stream and receive a limited-time promotional offer. Subsequently, they have the option to purchase products and return them if they are not satisfied. In addition, consumers can show appreciation for influencers and reward them.

Therefore, the game represents the strategic interactions among three groups: brands, influencers on social networks, and consumers. We assume that brands and influencers regulate the cooperation through a contractual agreement [2]. Violating the agreement incurs penalties that involve compensatory payments to the damaged party. Specifically, if the brand engages in a partnership with an influencer who fails to ensure adequate performance (for example, by posting late or misrepresenting the brand) the influencer has to compensate the brand with a penalty W. In contrast, if the influencer acts as a proactive ambassador of a brand that does not fulfill the influencer's expectation or requires strategic adjustments, the brand must pay a penalty K to the influencer.

The brand has two possible strategies: respecting the influencer partnership (RP) or non-respecting (NRP). In the first scenario, the brand benefits from the advantageous cooperation with the influencer. In contrast, in the second scenario, the brand does not comply with the requirements of the ambassador. Similarly, the influencer has two strategic options: one is to be an active brand ambassador (AA) the other is to be an inactive brand ambassador (NAA). On one hand, the influencer has the potential to enhance their follower count and increase profits. On the other hand, the influencer may breach the contract and

Table 1
Variables and parameters of the game model.

Symbol	Description	Value
x	Probability of B selecting to respect the contract	$0 \le x \le 1$
y	Probability of I being an active brand ambassador	$0 \le y \le 1$
z	Probability of C following influencers' recommendations	$0 \le z \le 1$
V	Product evaluation of C (uniformly distributed in $[0,1]$)	$V > 0$
P	Product price for C	$P > 0$
d	Discount that C gets during the live-streaming shopping	$d > 0$
S	Subscription paid by C to the channel of I	$s > 0$
v	Product evaluation according to I	$v > 0$
c	Production cost of B	$c > 0$
C_B	Investment cost of B	$C_B > 0$
C_I	Investment cost of I	$C_I > 0$
S^+	Subscription gain increase of I	$S^+ > 0$
W	Penalty paid by I to B	$W > 0$
K	Penalty paid by B to I	$K > 0$
R	Reward given by C to I	$R > 0$
E	Gain increase of B	$E > 0$
C_{BI}	Income paid by B to I	$C_{BI} > 0$
f	Loss of C if B is inactive	$f > 0$
r	Parameter = 0 if C returns the product, 1 otherwise	$r \in \{0,1\}$
C_r	Holding cost of B	$C_r > 0$
C_t	Transportation cost in case of product return	$C_t > 0$
C_s	Shipping cost of B	$C_s > 0$

fail to meet the brand's expectations. The consumer has two possible strategies: following the influencer's purchasing suggestions (FS) or opting not to follow these recommendations (NFS). In the first case, the consumer may perceive advantages in acquiring product information and receiving discounts. In addition, consumers can benefit from price comparisons that enable them to make informed purchasing decisions. In the second situation, the consumer can be negative to influencer marketing. We note that consumers can purchase products through alternative channels or online retailers; however, these associated costs are excluded from our analysis, as we focus only on live-streaming shopping.

The parameters and variables used in the model are shown in Table 1. Since the payoffs of each player group are affected by the strategic decisions of the other groups, there are eight possible combinations of strategies. The payoff matrices for the eight possible cases of interactions are shown in Tables 2 and 3. We consider a representative for each group (B is the brand, I is the influencer, and C is the consumer). The proportion of brands that respect the influencer partnerships is x, then the proportion of brands who choose the non-respecting strategy is $1-x$, $0 \le x \le 1$. The proportion of influencers who decide to be an active brand ambassador is y, then the proportion of influencers who select the inactive brand ambassador strategy is $1-y$, $0 \le y \le 1$. The proportion of consumers following the influencer's recommendations is z, then the proportion of consumers selecting the non-following strategy is $1-z$, $0 \le z \le 1$.

4. Evolutionary dynamics of the game

We develop our game assuming that the game groups have bounded rationality and cannot rationally make decisions to maximize their profit. This assumption indicates that players are not always fully informed and do not exhibit completely rational behavior. Consequently, they cannot rationally choose how to maximize their profit. However, players can learn to optimize their behavior. Under these circumstances, evolutionary game theory offers a solid basis for investigating how players can make decisions in such a complex environment. Specifically, by introducing the replicator dynamics, we develop a dynamic framework in which the distribution of different strategies within a population changes over time; see [13]. The replicator dynamics is formalized as a system of ordinary differential equations. Each replicator is associated with a single pure strategy passed down to all its descendants. In this context, individuals can adopt various strategies, and the proportion of the population employing a specific strategy will increase over time if it outperforms the current average of the population.

4.1. Replicator dynamics of brands

According to Tables 2 and 3, the expected revenue when brands respect the influencer partnership is:

$$\begin{aligned}\pi_{B_1} &= yz(rdP - c - C_B - C_{BI} - C_r - C_t - C_s)\\ &+ (1-y)z(rP - c - C_B + W - C_r - C_t - C_s)\\ &+ y(1-z)(rP - c - C_B - C_{BI} - C_r - C_s)\\ &+ (1-y)(1-z)(rP - c - C_B + W - C_r - C_s), \end{aligned} \tag{1}$$

and the expected revenue when brands do not respect the influencer partnerships is:

$$\begin{aligned}\pi_{B_2} &= yz(rdP - c - K + E - C_r - C_t - C_s)\\ &+ (1-y)z(rP - c - C_r - C_t - C_s)\\ &+ y(1-z)(rP - c - K + E)\\ &+ (1-y)(1-z)(rP - c). \end{aligned} \tag{2}$$

The average expected revenue for the total population of brands in the live-streaming e-commerce model is given by $\pi_B = x\pi_{B_1} + (1-x)\pi_{B_2}$. By the Malthusian equation, the growth rate of those brands who engage in influencer partnerships equals the difference between the expected revenue π_{B_1} and the average expected revenue π_B. Therefore, the replicator dynamics equation for the portion of brands that respect the influencer partnerships is given by:

$$\dot{x} = x(\pi_{B_1} - \pi_B) = x(1-x)(\pi_{B_1} - \pi_{B_2}). \tag{3}$$

By (1) and (2), the replicator dynamics equation of brands (3) becomes:

$$\dot{x} = F(x), \tag{4}$$

where

$$F(x) = x(1-x)\Big(-C_B - (1-z)C_r - C_S(1-z) + W - y(C_{BI} + E - K + W)\Big).$$

The equilibria or rest points of the dynamics of the brands are now computed. Letting $F(x) = 0$, we find three potential equilibrium solutions:

$$x = 0,\ x = 1,\ \bar{z} = \frac{C_B + C_r + C_S - W + y(C_{BI} + E - K + W)}{C_r + C_s}.$$

The following cases are possible:

1. If $z = \bar{z}$, then all values of x in $[0,1]$ lead to an equilibrium state. The system depends on the initial conditions.
2. If $z \neq \bar{z}$, then $F(x) = 0$ can be obtained for $x = 0$ or $x = 1$. To determine the equilibrium state, we evaluate $F'(x)$ and construct the auxiliary function $g(z) = -C_B - (1-z)C_r - C_S(1-z) + W - y(C_{BI} + E - K + W)$. Being $g'(z) = C_r + C_S$, we easily find that $g(z)$ is a linearly increasing function of z. Then, if $z < \bar{z}$, then $g(z) < 0$, $F'(0) < 0$ and $F'(1) > 0$. Thus, the only equilibrium state is $x = 0$ and the best strategy for brands is to breach the agreement. If $z > \bar{z}$, then $g(z) > 0$, $F'(0) > 0$ and $F'(1) < 0$. Therefore, the only rest point is $x = 1$ and the best strategy is to respect the contract.

4.2. Replicator dynamics of influencers

According to Tables 2 and 3, the expected revenue when influencers decide to be active brand ambassadors is:

$$\begin{aligned}\pi_{I_1} &= xz(S^+ - C_I + R + C_{BI})\\ &+ (1-x)z(S^+ - C_I + K + R)\end{aligned}$$

Table 2
The expected revenue matrix of brand, influencer, and consumer, if C follows $\mathcal{I}$'s buying recommendations.

$\mathcal{B}$	$\mathcal{C}$ **FS** (z)	
	$\mathcal{I}$ **AA** (y)	$\mathcal{I}$ **NAA** $(1-y)$
RP (x)	$rdP - c - C_B - C_{BI} - C_r - C_t - C_s$ $S^+ - C_I + R + C_{BI}$ $V - rdP - S + v - R$	$rP - c - C_B + W - C_r - C_t - C_s$ $S - W$ $V - rP - S + v$
NRP $(1-x)$	$rdP - c - K + E - C_r - C_t - C_s$ $S^+ - C_I + K + R$ $V - rdP - S + v - R - f$	$rP - c - C_r - C_t - C_s$ S $V - rP - S - f$

Table 3
The expected revenue matrix of brand, influencer, and consumer, if C does not follow I's buying recommendations.

$\mathcal{B}$	$\mathcal{C}$ **NFS** $(1-z)$	
	$\mathcal{I}$ **AA** (y)	$\mathcal{I}$ **NAA** $(1-y)$
RP (x)	$rP - c - C_B - C_{BI} - C_r - C_s$ $-C_I + C_{BI}$ $V - rP - C_t$	$rP - c - C_B + W - C_r - C_s$ $-W$ $V - rP - C_t$
NRP $(1-x)$	$rP - c - K + E$ $-C_I + K$ $V - rP - C_t$	$rP - c$ 0 $V - rP - C_t$

$$+\ x(1-z)(-C_I + C_{BI}) + (1-x)(1-z)(-C_I + K), \tag{5}$$

and the expected revenue when influencers are inactive brand ambassadors is:

$$\pi_{I_2} = xz(S - W) + (1-x)z(S) + x(1-z)(-W). \tag{6}$$

The average expected revenue for the total population of influencers in the live-streaming e-commerce model is given by $\pi_I = y\pi_{I_1} + (1-y)\pi_{I_2}$. The growth rate of those influencers who decide to be active brand ambassadors equals the difference between the expected revenue π_{I_1} and the average expected revenue π_I. Thus, the replicator dynamics equation for the portion of influencers who decide to be active brand ambassadors is given by:

$$\dot{y} = y(\pi_{I_1} - \pi_I) = y(1-y)(\pi_{I_1} - \pi_{I_2}). \tag{7}$$

By (5) and (6) the replicator dynamics equation of influencers (7) becomes:

$$\dot{y} = G(y), \tag{8}$$

where

$$G(y) = y(1-y)\Big(-C_I + K + z(R - S + S^+) + x(C_{BI} - K + W)\Big).$$

We now look for the equilibrium states of the dynamics of the influencers. Letting $G(y) = 0$ in (8), we find the potential equilibrium states:

$$y = 0,\ y = 1,\ \bar{\bar{z}} = \frac{C_I - K - x(C_{BI} - K + W)}{R - S + S^+}.$$

1. If $z = \bar{\bar{z}}$, then all values of y in $[0,1]$ are rest points, and the system depends on the initial conditions.
2. If $z \neq \bar{\bar{z}}$, then $G(y) = 0$ can be obtained for $y = 0$ and $y = 1$. To determine the equilibrium states, we construct the auxiliary function $\bar{g}(z) = -C_I + K + z(R - S + S^+) + x(C_{BI} - K + W)$. From $\bar{g}'(z) = R - S + S^+$ with $S^+ > S$, we easily find that $\bar{g}(z)$ is a linearly increasing function of z. If $z < \bar{\bar{z}}$, then $\bar{g}(z) < 0$, $G'(0) < 0$ and $G'(1) > 0$. Thus, the only rest point is $y = 0$ and the best strategy for influencers is to be inactive brand ambassadors. If $z > \bar{\bar{z}}$, then $\bar{g}(z) > 0$, $G'(0) > 0$ and $G'(1) < 0$. Therefore, the only rest point is $y = 1$ and the best strategy is to be active brand ambassadors.

4.3. Replicator dynamics of consumers

According to Tables 2 and 3, the expected revenue when consumers follow influencer I's buying recommendations on at least one social media platform is:

$$\begin{aligned}\pi_{C_1} &= xy(V - rdP - S + v - R)\\ &+ x(1-y)(V - rP - S + v)\\ &+ (1-x)y(V - rdP - S + v - R - f)\\ &+ (1-x)(1-y)(V - rP - S - f),\end{aligned} \tag{9}$$

and the expected revenue when consumers do not follow influencer I's buying recommendations on at least one social media platform is:

$$\begin{aligned}\pi_{C_2} &= xy(V - rP - C_t)\\ &+ x(1-y)(V - rP - C_t)\\ &+ (1-x)y(V - rP - C_t)\\ &+ (1-x)(1-y)(V - rP - C_t).\end{aligned} \tag{10}$$

The average expected revenue for the total population of consumers in the live-streaming e-commerce model is given by $\pi_C = z\pi_{C_1} + (1-z)\pi_{C_2}$. The growth rate of those consumers who decide to follow influencer I's buying recommendations equals the difference between the expected revenue π_{C_1} and the average expected revenue π_C. Thus, the replicator dynamics equation for the portion of consumers who follow influencer I's buying recommendations is given by:

$$\dot{z} = z(\pi_{C_1} - \pi_C) = z(1-z)(\pi_{C_1} - \pi_{C_2}). \tag{11}$$

As a consequence of (9) and (10), the replicator dynamics equation of consumers (11) becomes

$$\dot{z} = H(z), \tag{12}$$

where

$$H(z) = z(1-z)\Big(C_t + xv + yr(P - dP) + y(1-x)v - S - yR - f(1-x)\Big).$$

We now compute the equilibrium states of the dynamics of consumers. Letting $H(z) = 0$ in (12), we find three potential equilibrium solutions

$$z = 0,\ z = 1,\ \bar{y} = \frac{C_t - (1-x)f - S + xv}{R + rdP + rP - (1-x)v}.$$

1. If $y = \bar{y}$, then any value z in $[0,1]$ is a equilibrium state. The system depends on the initial conditions.
2. If $y \neq \bar{y}$, then $H(z) = 0$ is obtained for $z = 0$ or $z = 1$. To determine the equilibrium points we discuss the following cases.

   (a) When $C_t + xv + yr(P - dP) + y(1-x)v - S - yR - f(1-x) < 0$, then $H'(0) < 0$ and $z = 0$ is the unique equilibrium state. This means that when the total cost of following the influencers, given by $S + yR + f(1-x)$, is greater than the shopping experience cost, given by $C_t + xv + yr(P - dP) + y(1-x)v$, consumers are inclined to opt for a buying strategy that does not follow influencer recommendations.

Table 4
Eigenvalues of the Jacobian matrix.

X_i	λ_1	λ_2	λ_3
X_1	$-C_I + K$	$C_t - f - S$	$-C_B - C_r - C_s + W$
X_2	$C_t - S + v$	$C_B + C_r + C_s - W$	$-C_I + C_{BI} + W$
X_3	$C_I - K$	$-C_B - C_{BI} - C_r - C_s - E + K$	$C_t - f - R + rP(1-d) - S + v$
X_4	$-C_t + S + f$	$-C_I + R + K - S + S^+$	$-C_B + W$
X_5	$C_B + C_{BI} + C_r + C_s + E - K$	$C_t - R + rP(1-d) - S + v$	$C_I - C_{BI} - W$
X_6	$-C_t + S - v$	$C_B - W$	$-C_I + C_{BI} + R - S + S^+ + W$
X_7	$-C_B - C_{BI} - E + K$	$C_I - R - K + S - S^+$	$-C_t + f + R - rP(1-d) + S - v$
X_8	$C_B + C_{BI} + E - K$	$-C_t + R - rP(1-d) + S - v$	$C_I - C_{BI} - R + S - S^+ - W$

(b) When $C_t + xv + yr(P - dP) + y(1-x)v - S - yR - f(1-x) > 0$, namely when the total cost of following the influencers is less than the shopping experience cost, there are the following two situations.

i. When $\bar{y} < y < 1$, $H'(0) < 0$ and $z = 0$ is the unique equilibrium state.
ii. When $0 < y < \bar{y}$, $H'(1) < 0$ and $z = 1$ is the unique equilibrium state. This indicates that consumers are more likely to prefer the buying recommendations.

Table 5
Stability analysis of the equilibrium points.

Equilibrium point	λ_1	λ_2	λ_3	Stability
$(0,0,0)$	$-,+$	$-$	$-,+$	Stable, Unstable
$(1,0,0)$	$+$	$-,+$	$+$	Unstable
$(0,1,0)$	$-,+$	$-,+$	$-,+$	Stable, Unstable
$(0,0,1)$	$+$	$+$	$-$	Unstable
$(1,1,0)$	$-,+$	$+$	$-$	Unstable
$(1,0,1)$	$-$	$+$	$+$	Unstable
$(0,1,1)$	$-,+$	$-$	$-,+$	Stable, Unstable
$(1,1,1)$	$-,+$	$-$	$-$	Stable, Unstable

5. Stability analysis of equilibrium strategies

The previous section showed the evolutionary game analysis by separately examining the equilibria of players. However, the players' decisions are mutually influenced, and their interactions develop over time. For this reason, it is crucial to study comprehensively the dynamic evolution of the strategies of the three player groups. Therefore, we consider the system of ordinary differential equations:

$$\begin{cases} \dot{x} = F(x) \\ \dot{y} = G(y) \\ \dot{z} = H(z). \end{cases} \tag{13}$$

The trajectories of the dynamical system illustrate the potential developmental pathways of the e-commerce live-streaming mechanism according to our model. Taking into account the setting of our model, the state space for the dynamics is the unit cube $[0,1] \times [0,1] \times [0,1]$. The equilibria of the dynamical system (13) can be found by solving the set of equations $F(x) = 0$, $G(y) = 0$, and $H(z) = 0$. Eight equilibrium points correspond to the vertices of the cube $X_1 = (0,0,0)$, $X_2 = (1,0,0)$, $X_3 = (0,1,0)$, $X_4 = (0,0,1)$, $X_5 = (1,1,0)$, $X_6 = (1,0,1)$, $X_7 = (0,1,1)$, $X_8 = (1,1,1)$. The other equilibrium points are on the faces and edges of the cube, or in the interior of the cube.

The stability of the equilibria is essential to understanding the behavior of the groups of players. Specifically, the study of stability according to Lyapunov makes it possible to detect which equilibria will prevail among the various potential ones. We now focus on the vertices of the cube. The stability analysis of the equilibrium points at the vertices is made by studying the local stability of the corresponding linearized system. In particular, following [28,29], if all the eigenvalues of the Jacobian matrix of the corresponding linearized system are negative, then the equilibrium is a stable solution of the entire system; otherwise, the equilibrium is unstable. Therefore, the eight vertices are substituted into the Jacobian matrix to observe the sign of the eigenvalues.

The eigenvalues of the Jacobian matrix at the equilibrium point X_i, $i = 1, \dots 8$, are shown in Table 4, and the asymptotic stability is analyzed in Table 5.

To facilitate the analysis of the size of the eigenvalues and without loss of generality, we assume that $C_t < f + S$, $W < C_B$, and $C_{BI} > C_I$. The first condition refers to when the transportation costs in case of product return are less than the costs incurred by the consumer, that is, the sum of the subscription cost and the eventual loss if B is inactive. A typical situation is when the product is returned for free. The second condition $W < C_B$ implies that the penalty incurred by the influencer is less than the investment cost of the brand. Finally, we suppose that $C_{BI} > C_I$, that is, the income paid by the brand to the influencer is greater than the investment cost of the influencer.

From Table 5, it can be seen that $X_1 = (0,0,0)$, $X_3 = (0,1,0)$, $X_7 = (0,1,1)$ and $X_8 = (1,1,1)$ can become the stable equilibria of the game.

The strategies represented by $X_1 = (0,0,0)$ are that brands do not respect the influencer partnerships, influencers are inactive brand ambassadors and consumers do not follow influencers' shopping recommendations. The conditions for these player groups to reach asymptotic stability are: $-C_I + K < 0$ and $-C_B - C_r - C_s + W < 0$. $-C_I + K < 0$ indicates that the penalty incurred by the brand is less than the investment cost of the influencer. $-C_B - C_r - C_s + W < 0$, that is, $W < C_B + C_r + C_s$ suggests that the penalty incurred by the influencer is less than the brand's costs. This scenario can induce brands to evolve into a non-respecting strategy, influencers into an inactive strategy, and consumers into a non-following influencers strategy.

The strategies represented by $X_3 = (0,1,0)$ are that brands do not respect the influencer partnerships, influencers are active brand ambassadors and consumers do not follow influencers' shopping recommendations. The conditions for X_3 to become the stable equilibrium of the game are: $C_I - K < 0$, $-C_B - C_{BI} - C_r - C_s - E + K < 0$ and $C_t - f - R + rP(1-d) - S + v < 0$. $C_I - K < 0$ indicates that the penalty incurred by the brand is greater than the investment cost of the influencer. $-C_B - C_{BI} - C_r - C_s - E + K < 0$, that is $K < C_B + C_{BI} + C_r + C_s + E$, means that the penalty incurred by the brand is less than the sum of the investment cost of the brand and the gain increase of the influencer. $C_t - f - R + rP(1-d) - S + v < 0$ indicates that $C_t + rP(1-d) + v < f + R + S$, meaning the costs associated with purchasing the product are less than the costs of actively following the influencer's channel. This can induce brands to evolve into a non-respecting strategy, influencers into an active strategy, and consumers into a non-following influencers strategy.

The strategies represented by $X_7 = (0,1,1)$ are that brands do not respect the influencer partnerships, influencers are active brand ambassadors and consumers follow influencers' shopping recommendations. The conditions for X_7 to become the stable equilibrium of the game are: $-C_B - C_{BI} - E + K < 0$ and $-C_t + f + R - rP(1-d) + S - v < 0$. $-C_B - C_{BI} - E + K < 0$, that is $K < C_B + C_{BI} + E$, indicates that the penalty incurred by the brand is less than the sum of the brand's costs and the gain increase of the influencer. $-C_t + f + R - rP(1-d) + S - v < 0$ indicates that $C_t + rP(1-d) + v > f + R$, meaning the expenses related to

Table 6
Stability of the equilibrium points in different scenarios.

Equilibrium	Parameter scenario
$(0,0,0)$	$K < C_I,\ W < C_B + C_r + C_s$
$(0,1,0)$	$C_I < K < C_B + C_{BI} + C_r + C_s + E,\ C_t + rP(1-d) + v < f + R$
$(0,1,1)$	$K < C_B + C_{BI} + E,\ C_t + rP(1-d) + v > f + R$
$(1,1,1)$	$K > C_B + C_{BI} + E$

purchasing the product are greater than the costs of actively following the influencer's channel. This scenario can induce brands to evolve into a non-respecting strategy, influencers into an active ambassador strategy, and consumers into a following influencers strategy.

The strategies represented by $X_8 = (1,1,1)$ are that brands respect the influencer partnerships, influencers are active brand ambassadors and consumers follow influencers' shopping recommendations. The conditions for X_8 to become the stable equilibrium of the game is: $C_B + C_{BI} + E - K < 0$, that is $K > C_B + C_{BI} + E$. This indicates that the penalty incurred by the brand is greater than the sum of the brand's costs and the gain increase of the influencer. This scenario encourages brands to respect the agreement, influencers to be active ambassadors, and consumers to follow influencers' recommendations. Table 6 displays the stability analysis and the corresponding parameter scenarios.

We now analyze the stability of the equilibria in the interior of the cube. An equilibrium inside the cube is an equilibrium point (x^*, y^*, z^*) of the system (13) with $x^*, y^*, z^* \in (0,1)$. The analytical solution is too long to be displayed and will be omitted. The Jacobian matrix computed at the interior solution (x^*, y^*, z^*) has the form

$$\begin{pmatrix} 0 & \alpha & \beta \\ \gamma & 0 & \epsilon \\ \zeta & \eta & 0 \end{pmatrix},$$

where

$$\alpha = x^*(1-x^*)(-C_{BI} - E + K - W),$$
$$\beta = x^*(1-x^*)(C_r + C_s),$$
$$\gamma = y^*(1-y^*)(C_{BI} - K + W),$$
$$\epsilon = y^*(1-y^*)(R - S - S^+),$$
$$\zeta = z^*(1-z^*)(f + v - vy^*),$$
$$\eta = -z^*(1-z^*)(R - rdP - rP - v(1-x^*)).$$

The characteristic polynomial is given by:

$$-\lambda^3 + (\epsilon\eta + \alpha\gamma + \beta\zeta)\lambda + \beta\eta\gamma.$$

The cubic polynomial has at least one real root and by Viète's formulas, we know that the sum of the roots is equal to zero. Therefore, if $\beta\eta\gamma = 0$, one root equals zero and the other two are either two conjugate pure imaginary numbers or two real numbers of opposite sign. The point is not hyperbolic in both cases, and we cannot apply the Hartman–Grobman theorem to study stability. If $\beta\eta\gamma \neq 0$, the polynomial has a non-zero real solution. Again applying Viète's formulas, we conclude that the other two roots cannot be two conjugate pure imaginary numbers. Thus, all roots have non-zero real parts with different signs. According to the Hartman–Grobman theorem, the equilibrium point is unstable. Therefore, vertices $X_1 = (0,0,0)$, $X_3 = (0,1,0)$, $X_7 = (0,1,1)$, and $X_8 = (1,1,1)$ are the only asymptotically stable points. For a discussion on the dynamics along the edges and faces of the cube, we address the interested reader to [30].

6. Numerical simulations of the evolutionary game

In this section, we provide some numerical simulations to show the validity of our evolutionary game model. According to some parameters provided by other literature focusing on live-streaming e-commerce models, see [2,24], we analyze the various equilibrium solutions. We present the results obtained using Mathematica software to simulate the stable evolution process of the strategy selections of players, and the Python module *matplotlib.pyplot* to visualize them. For the conducted experiments, the execution time is approximately on the order of 0.1 s, making it negligible in practical applications. The experiments[1] were performed on a MacBook Air (M1, 2020) equipped with an Apple M1 chip (8-core CPU, 7-core GPU, and 16-core Neural Engine) and 8 GB of unified memory.

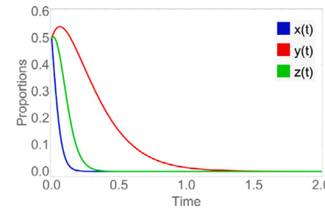

Fig. 1. Temporal evolution of $(0,0,0)$.

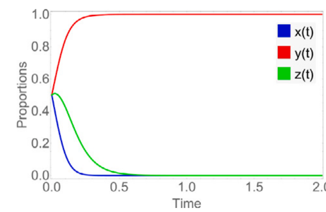

Fig. 2. Temporal evolution of $(0,1,0)$.

In the following numerical example, we choose the parameters corresponding to the scenarios in Table 6. The relevant parameters of income and cost are measured in thousands of dollars. For all scenarios we set: $C_I = 10$, $H = 3$, $S = 4$, $S^+ = 5$, $C_{BI} = 15$, $W = 6$, $C_t = 13$, $d = 0.8$, $rP = 30$, $v = 15$, $C_B = 15$, $C_s = 7$, $C_r = 4$, $E = 10$, $R = 5$. We choose different values of the penalty K according to the scenario. The penalty $K = 50$ will lead the system to the stable point $X_8 = (1,1,1)$, where brands, influencers, and consumers choose to participate actively in the live-streaming e-commerce mechanism. The penalty $K = 25$ may lead the system to $X_3 = (0,1,0)$ or $X_7 = (0,1,1)$, where influencers decide to become active brand ambassadors, but brands have scarce incentive to sponsor. In this case, the consumers' decision does not influence the strategy selection of the other players. Finally, when the penalty has the value $K = 5$, brands, influencers, and consumers are not active and the system will reach the point $X_1 = (0,0,0)$. Moreover, the loss f of the consumers is set as 7 or 35, depending on the strategy decision FS or NFS, respectively.

6.1. Simulations with neutral players

We now show the temporal evolution of the various equilibria and assume that in the initial game brands, influencers, and consumers all have a 50% probability of choosing different live-streaming e-commerce strategies. Figs. 1–4 illustrate the experimental results using Mathematica software with initial values $x_0 = 0.5$, $y_0 = 0.5$, $z_0 = 0.5$.

The plots confirm the stability analysis of the equilibrium points across the various scenarios outlined in Table 6. Fig. 1 shows that the system evolves towards the point $X_1 = (0,0,0)$. We observe that, initially, the probability of influencers selecting the active brand ambassador strategy slightly increases. Subsequently, the probability decreases, and influencers take longer than other players to stabilize the inactive ambassador strategy. Fig. 2 shows that the system reaches

[1] https://github.com/GeoFargy/TripartiteGame/upload/main

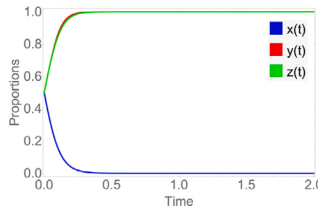

Fig. 3. Temporal evolution of $(0, 1, 1)$.

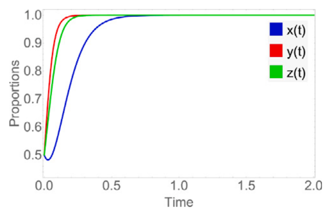

Fig. 4. Temporal evolution of $(1, 1, 1)$.

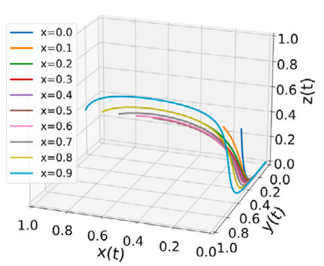

Fig. 5. Brands' evolving patterns across diverse response likelihoods for $(0, 0, 0)$.

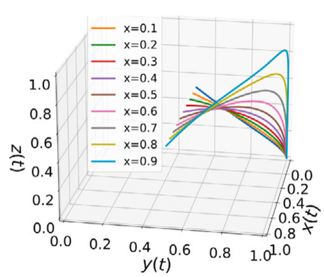

Fig. 6. Brands' evolving patterns across diverse response likelihoods for $(0, 1, 0)$.

the steady state $X_3 = (0, 1, 0)$. The brands and the influencers will choose their strategies almost simultaneously. The time to achieve stability becomes longer for the consumers, potentially influenced by the strategic decisions made by the influencers. Fig. 3 shows that the system reaches the steady state $X_7 = (0, 1, 1)$. Influencers and consumers take the same time to converge to the active strategy, while brands rapidly stabilize at the non-respecting strategy. Finally, Fig. 4 shows that the system reaches the steady state $X_8 = (1, 1, 1)$. All the players will choose the active strategy and, compared with the influencers and consumers, the brands will be the last to make the respecting strategy decision.

6.2. Simulations with varying initial probabilities

The second part of the numerical example evaluates the evolving patterns across diverse response likelihoods for the different equilibria using Python.

As shown in Figs. 5–8, 9-12 and 13–16, different initial values for the three player groups have no impact on the ultimate stable states.

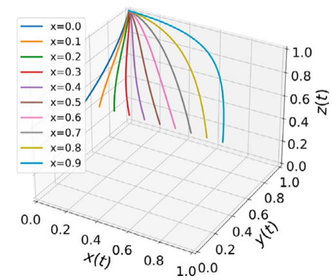

Fig. 7. Brands' evolving patterns across diverse response likelihoods for $(0, 1, 1)$.

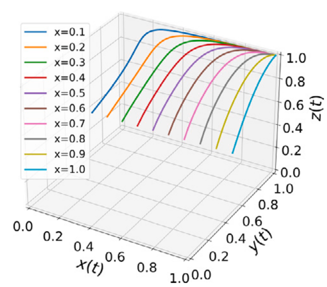

Fig. 8. Brands' evolving patterns across diverse response likelihoods for $(1, 1, 1)$.

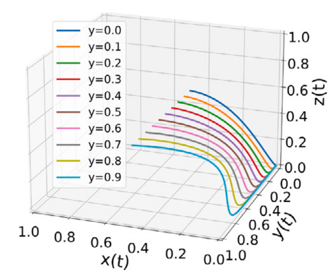

Fig. 9. Influencers' evolving patterns across diverse response likelihoods for $(0, 0, 0)$.

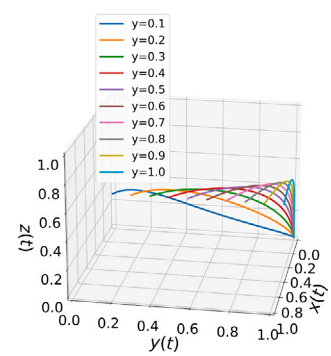

Fig. 10. Influencers' evolving patterns across diverse response likelihoods for $(0, 1, 0)$.

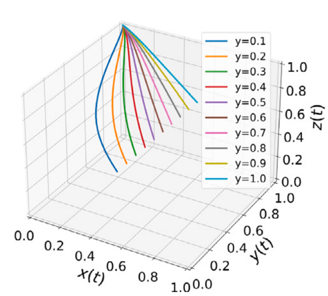

Fig. 11. Influencers' evolving patterns across diverse response likelihoods for $(0, 1, 1)$.

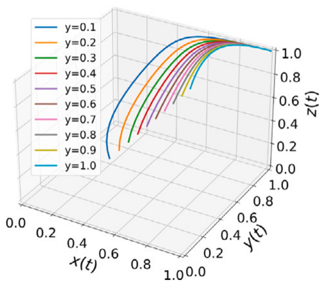

Fig. 12. Influencers' evolving patterns across diverse response likelihoods for $(1,1,1)$.

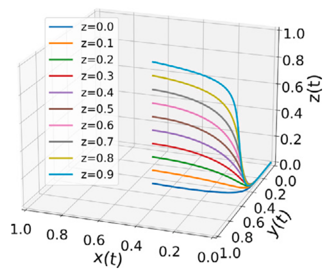

Fig. 13. Consumers' evolving patterns across diverse response likelihoods for $(0,0,0)$.

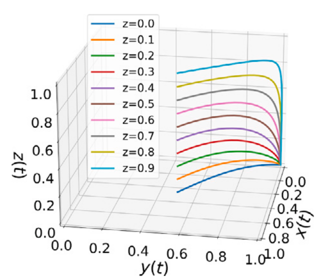

Fig. 14. Consumers' evolving patterns across diverse response likelihoods for $(0,1,0)$.

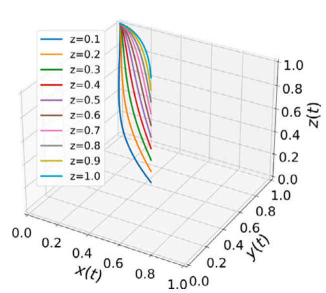

Fig. 15. Consumers' evolving patterns across diverse response likelihoods for $(0,1,1)$.

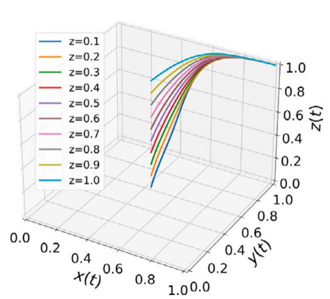

Fig. 16. Consumers' evolving patterns across diverse response likelihoods for $(1,1,1)$.

Figs. 5–8 present the results of the simulations where we fixed y_0 and z_0 equal to 0.5 and x varies from 0 to 0.9 or from 0.1 to 1 depending on the equilibrium considered. Fig. 5 displays that when the willingness of brands to be active gradually increases, the system will eventually evolve to $(0,0,0)$. This suggests that, given the scenario parameters, even if brands have a high value of initial probability, then they tend to choose the non-respecting strategy. The attitude of brands is not enough to encourage the other players to participate actively in the live-streaming e-commerce. In Fig. 6, the gradual increase in brands' probability makes the system evolve to $(0,1,0)$. This suggests that the influencers benefit from promoting products in their channels, even if the other players do not engage in the live-streaming e-commerce. In Fig. 7, when the probability of brands respecting the influencer partnerships increases, the final equilibrium point tends to be $X_7 = (0,1,1)$. In this case, even if brands initially follow an active strategy, they tend to deviate and reach the non-respecting strategy. Thus, the increase in the probability of brands does not impact influencers' and consumers' decisions to choose active behaviors. Finally, in Fig. 8, when the proportion of brands respecting the influencer partnerships increases, the final equilibrium point tends to be $X_8 = (1,1,1)$. Moreover, the more the value of x increases, the faster the game system evolves to X_8. All the player groups choose to carry out actively live-streaming e-commerce strategies. Thus, the increase in the probability of brands encourages influencers and consumers to select active behaviors. Figs. 9–12 present the results of the simulations, where we fixed x_0 and z_0 equal to 0.5 and let y vary. Figs. 13–16 presents the results of the simulations, where we fixed x_0 and y_0 equal to 0.5 and let x vary.

6.3. Sensitivity analysis

This section analyzes the correlation matrix (Fig. 17) to evaluate the intricate relationships between key parameters and the dynamic interactions of brands B, influencers I, and consumers C within the live-streaming e-commerce supply chain. A strong positive correlation is observed between the parameter R (e.g., rewards for influencers given by consumers) and all variables (x, y, z), suggesting that increasing rewards effectively encourages cooperative strategies among all participants. As R increases, influencers are more likely to promote products actively, which, in turn, strengthens engagement among brands, influencers, and consumers. Conversely, the parameter K, the penalty eventually paid by brands to influencers, shows a positive correlation with x and z. This indicates that reasonable penalties can encourage brands to respect agreements and consumers to approve collaborations. However, excessively high values of K might lead brands to disengage, as they perceive the cost of cooperation as too high, potentially destabilizing the system. These findings highlight the role of carefully balancing R and K to maintain cooperative strategies among the participants. On the other hand, the parameter W, the penalty that influencers have to pay to brands affects y and z differently. While W positively impacts y (influencers tend to cooperate when facing higher penalties), it has a weaker or even negative effect on x, as brands may reduce investments in partnerships if penalties are perceived as overly strict. Influencers, being private actors, are more inclined to move toward active strategies under moderate increases in W, as they seek to avoid penalties and maintain their credibility. Parameter f represents the loss incurred by consumers when brands are inactive in the collaborations with influencers. It negatively correlates with both y and z. This suggests that as consumers experience a loss, both influencers and consumers are less likely to engage. In contrast, x remains largely unaffected by f, reflecting that brands may continue sponsoring influencers regardless of consumer-side losses. Overall, these correlations emphasize the essential role of parameters like R, K, W, and f in shaping strategic decisions. These findings align with the evolutionary dynamics of the system, where parameter tuning can guide brands, influencers, and consumers toward optimal strategies, enhancing the efficiency and sustainability of the live-streaming e-commerce business model.

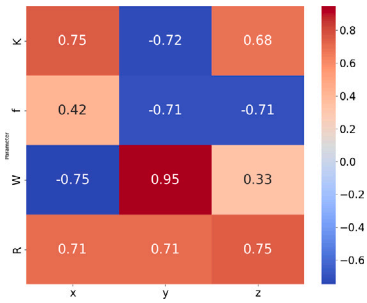

Fig. 17. Correlation matrix of parameters K, f, W and R in relation with all variables x, y, z.

6.4. Discussion

In our numerical simulations, we have identified four equilibria corresponding to four different parameter scenarios. The equilibrium point X_8 signifies that each player group decides to implement an active strategy. This leads to a stable equilibrium where all players decide to follow the trend of advertising through social networks. On the other hand, the point X_1 represents the opposite equilibrium, where no one decides to invest time and money in social networks. Given that only a minority of influencers do not rely on their earnings as brand ambassadors, creating the equilibrium discussed earlier X_1, the other two equilibria obtained are represented by the influencers' choice to participate actively as brand ambassadors. In this case, two conditions arise: the first when the consumers decide not to follow the influencers' advice on a product, and the other when they decide to purchase it. Specifically, in equilibria X_3 and X_4, the brands decide not to invest further in influencers as they have incurred losses from incorrectly sponsoring those influencers. Anything the influencers post affects the brand and, as a consequence, the choice of the wrong influencer can damage the brand's reputation. On the other hand, the influencers continue to use the strategy of sponsoring some products that are not tied to the brands, as their optimal strategy is to invest, hoping to gain followers in case they have not decided to follow the influencer's recommendations. Our simulations show that the initial assignment of parameters is the key factor in making the system evolve into the final strategic combination. The sensitivity analysis further reinforces this observation, as it highlights how small changes in critical parameters, such as rewards R, penalties K, W, and consumer losses f, can significantly alter the trajectory of the system toward different equilibria. By systematically analyzing the influence of these parameters, we were able to identify thresholds beyond which the system stabilizes into a cooperative state or diverges into inactive strategies. For example, excessively high values of K were shown to discourage brand investment, while moderate values promoted cooperative behavior across all players. Similarly, increasing R positively influenced the active participation of influencers and consumers, ensuring the stability of the system. The analysis of the parameters has managerial implications as it may guide the managers' decision-making and influence the business plans. By understanding the sensitivity of the system to these parameters, managers can design policies that encourage cooperation and optimize investments in the live-streaming e-commerce supply chain.

7. Conclusions

The live-streaming e-commerce has become a crucial tool for firms to achieve competitiveness in modern business. This paper aimed to investigate the interactions between brands, influencers, and consumers under a live-streaming mechanism. We presented an evolutionary game model to examine the dynamic evolution process of the strategies of each participant in the supply chain. We studied the replicator dynamics equations of each player group. Then, we looked for the evolutionary stable strategies of the complete system. We also showed that the parameters strongly affect the system so that the final equilibrium strategy can be changed by adjusting their value.

The contributions of our work are summarized in two aspects. First, our work contributed to the literature on evolutionary game models applied to live-streaming e-commerce platforms. Our work explored the behavior of the player groups and provided novel insights into optimal strategies in live-streaming selling. Second, we comprehensively examined the factors which mainly impact the participants' decisions in the supply chain. This study offers valuable recommendations aimed at enhancing the operational strategies of the live-streaming e-commerce supply chain. We have presented novel perspectives on the roles of brands, social media influencers, and consumer revenue, particularly in light of the recent increase in the advertising influence of social media influencers in the market.

Several directions for future research may be proposed. This study focused on a single brand-influencer-consumer interaction without accounting for competition among multiple brands or influencers, which is common in real-world live-streaming e-commerce. Another future research line is analyzing the role of the e-commerce platform which is essential in facilitating interactions and imposing platform-specific rules or fees. Moreover, the study does not explore consumer behavior after the purchase, such as sharing reviews or influencing other potential buyers. This aspect may be another research opportunity.

CRediT authorship contribution statement

Georgia Fargetta: Writing – review & editing, Writing – original draft, Software, Methodology, Data curation. **Laura R.M. Scrimali:** Writing – review & editing, Writing – original draft, Supervision, Methodology, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

I have shared the link to my codes in the article.

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