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## Exploiting Benefits of a Periodically-Forced Nonlinear Oscillator for Energy Harvesting from Ambient Vibrations

C. Trigona<sup>a\*</sup>, N. Dumas<sup>a</sup>, L. Latorre<sup>a</sup>, B. Andò<sup>b</sup>, S. Baglio<sup>b</sup>, P. Nouet<sup>a</sup>

<sup>a</sup>LIRMM, University Montpellier II, 161 rue Ada-34095 Montpellier Cedex 5, France

<sup>b</sup>University of Catania, v.le A. Doria 6-95125 Catania, Italy

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### Abstract

In this paper, we present an experimental and theoretical study of a nonlinear oscillator for energy harvesting applications. The collected energy comes from ambient vibrations as external energy sources. Specifically, we consider both periodic (e.g. 50Hz or 60Hz power line couplings) and stochastic sources (e.g. human-made devices, automobiles, train, etc.). The proposed mechanism is a double-well oscillator with a large amplitude response over a broad range of frequencies. Such system improves proposed scavengers based on linear mechanical principles, which only give appreciable response if the dominant ambient vibration frequency is close to the mechanical resonance. At the same time, it improves nonlinear harvesters (e.g. bi-stable) proposed in the literature by decreasing the vibration amplitude required to exceed the potential barrier of the double-well function through the benefit of the additive sinusoid. Modeling, numerical simulations and experiments are here reported to show a good accordance with expectations.

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*Keywords:* Energy harvesting; Wide-spectrum vibrations; Nonlinear dynamics

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### 1. Introduction

There has been much recent interest in the concept of self-powered devices, autonomous sensor node, Wireless Sensor Networks (WSNs) that are commonly powered using batteries. For applications where the system is expected to operate for long durations, energy becomes a very important system's requirement and many efforts have been conducted in order to efficiently use the battery energy [1].

\* Carlo Trigona. Tel.: +33-(0)4-67418665; fax: +33-(0)4-67418500.  
E-mail address: [carlo.trigona@lirmm.fr](mailto:carlo.trigona@lirmm.fr).

An approach is to sustain/replace the batteries by using an energy harvester that collect energy from various ambient sources, including solar power, thermal gradients and vibration [2]. However, it's unlikely that any single solution will satisfy all application spaces, as each method has its own constraints: solar methods require sufficient light energy, thermal gradients need sufficient temperature variation, and vibration-based systems need sufficient vibration sources. Vibration sources are generally more ubiquitous and can be readily found in the environment (induced oscillations, vehicle motion, etc.).

In this context, several vibration-powered energy harvesters have been proposed in literature and a lot of ways and mechanisms to optimize the amount of recovered energy have been explored such as the use of i) array of linear systems [3], ii) frequency-tunable resonators [4] and iii) nonlinear oscillators [5].

The common target is to obtain a uniform power distribution in order to efficiently collect energy from external vibrations typically characterized by a wide spectral distribution (from tens of Hz to few hundred of Hz [6]). In particular for the latter systems, the behaviour is strictly correlated with the potential energy function, with the amplitude of its barrier and the level of ambient vibrations that plays a key role: 1) if the vibration level is high enough to exceed the level of the potential barrier, the system will bi-stably oscillate, obtaining a wide spectral response, 2) if the magnitude of external noise is not enough to exceed the barrier, the system will oscillate in a single well and the potential can be assumed parabolic, as consequence a dominant resonance frequency will appear.

In this paper will be exploited the benefits of a periodically-forced nonlinear oscillator for energy harvesting from low-level ambient vibrations.

## 2. Working principle

A wide range of applications/solutions have been introduced for vibration-based energy harvesters. Focusing on nonlinear techniques, authors already proposed a device based on a nonlinear behaviour (bi-stable) to enhance device response to external vibrations [7]. It must be observed that a bi-stable oscillator has an energy barrier and its amplitude represents “the threshold” required to switch between one steady state to the other. In order to decrease the threshold, a possible solution is to tune the energy barrier which is unfortunately often fixed or at least difficult to change.

The approach that will be pursued in this paper is based on a time-periodic excitation mixed with external ambient noise to decrease the threshold level necessary to switch between stable states (Fig.1a). This approach allows adjusting the energy barrier similarly to approaches used in other domains such as i) electronic (bias point), ii) system theory (noise-added systems) or, iii) image processing (stochastic resonance) [8]. In particular here the noise effects on the device performances are considered in the opposite way with respect to already introduced principle of “noise activated devices” [8] where the noise is used to allow the information to be captured by the sensor. In fact the energy harvesting device is aimed here to capture the largest amount of energy available in the environment, in order to achieve this goal, we propose to consider the combination of one deterministic excitation and one stochastic excitation to allow the maximum energy transfer from the stochastic signal. The system has been modelled as a second order resonant system (a cantilever beam) with a quartic potential  $U(x)$ , as shown in Eq.1.

$$0.0014\ddot{x} + 0.013\dot{x} + 202x + \left. \frac{\partial U(x)}{\partial x} \right|_{U(x)=-101.33x^2+1e5 \cdot x^4} = n(t) + A \cdot \sin(2\pi ft)|_{f=50\text{Hz}} \quad (1)$$

Where  $x$  represents the beam displacement,  $n(t)$  the vibration noise (band limited white noise) and  $A$  the amplitude of the sinusoidal excitation. The mass (0.0014 kg) and the damping coefficient (0.013 kg/s) have been experimentally estimated, the elastic constant (202 kg/s<sup>2</sup>) has been evaluated through the single-beam model. Other coefficients have been estimated through a fitting procedure (Nelder–Mead nonlinear algorithm).

### 3. Results

The prototype used to validate the mechanical principle is an aluminum beam having a length of 53mm and a width of 8.5mm. Two permanent magnets with opposite polarities have been used to create the two stable states, the first one deposited on the cantilever tip and the second one fixed at 2mm of distance, as shown in Fig. 1b.

Fig. 2 shows the simulation results: the vibration level of 0.9g (standard deviation value) is not enough to start the switching mechanism (sub-threshold condition), so the beam oscillates inside a single well. In presence of an additive sinusoidal acceleration of 2g (rms value) at 50Hz, the excitation level exceeds the potential barrier and the beam displacement evolves between its two stable regions. Furthermore a periodic over-excitation produce a dominant condition. The value of  $\Delta$ , distance between the two magnets, has been fixed to 2mm. The simulations have been conducted by using Matlab simulink and Dormand-Prince solver.

In terms of spectral analysis (Fig. 3), a vibration level of 3.35g represents the threshold necessary to initiate bi-stable oscillations and to enlarge bandwidth. With a lower level of vibration noise (e.g. 0.9g), the system oscillates inside a single well (linear response). Optimal conditions are obtained with a 50Hz source (e.g. electric source, machines, etc.) considering a total acceleration ranging from 2.2g to about 4.7g. For combined sources higher than 4.79g, the sinusoid dominates the dynamic and the improvement in terms of large bandwidth response disappears. The acceleration values have been measured by using a built-in accelerometer on the shaker.

In other words, it is possible to collect energy from low amplitudes of vibrations by optimally positioning the system between a mechanical source (e.g. 50Hz) and vibrations (e.g. car motion, noisy source).

The two excitation signals considered here are almost ubiquitous, in fact several examples can be made where a periodic signal is present together with a noisy signal, moreover in the absence of the periodic excitation this can be produced internally to the energy harvesting device with the goal to greatly improve its scavenging capability still keeping the overall power budget positive.

In future works, validations of this principle applied to a MEMS prototype with PZT material for the energy conversion will be conducted.

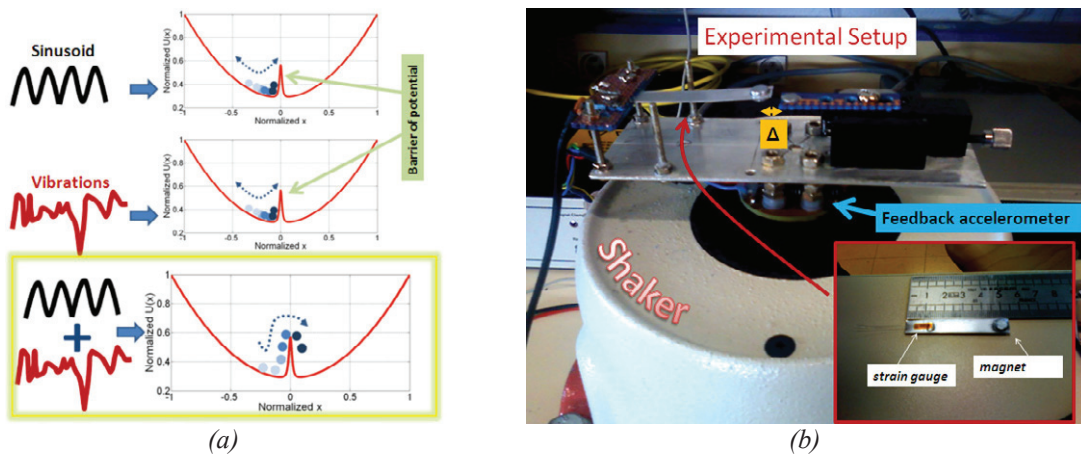


Fig. 1. a) Principle of a periodically-forced double-well oscillator to harvest energy from low level vibration sources. b) Experimental setup: the oscillator has been excited at the base by vertical mechanical vibrations generated by a shaker. A strain gauge has been fixed and conditioned through a Wheatstone bridge in order to monitor the mechanical displacement

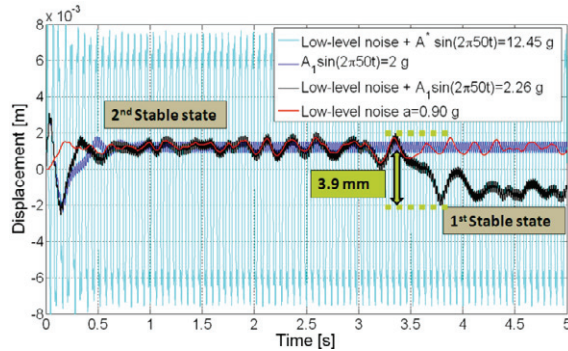


Fig. 2. Simulation results: in presence of noise level or sinusoidal bias of 0.9g (red) and 2g (blue) respectively, the system oscillates in a single-well (second stable state). The mixing of both allows switching between the two states (black). The last evolution represents the over-excited condition ( $A^* \gg A_1$ , cyan)

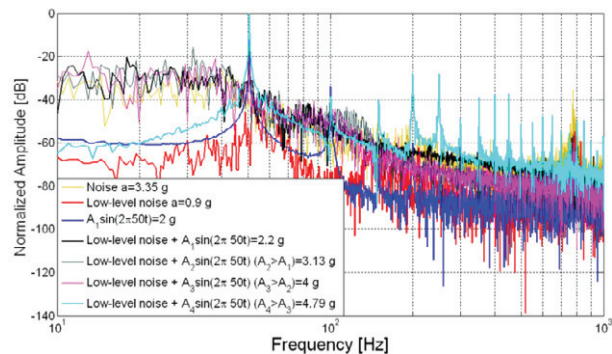


Fig. 3. Experimental results: in presence of low noise level, the system will oscillate inside a single well. By adding a sinusoid of amplitude  $A_1$ , the beam nonlinearly oscillates and a wider bandwidth appears. For periodic excitations with magnitudes higher than  $A_4$ , a sinusoidal prevalence appears, losing the bandwidth benefit

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