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Reagents and swab tests during the COVID-19 Pandemic: An optimized supply chain management with UAVs

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ABSTRACT

In this paper, we develop a supply chain optimization model for the preparation, provision, transportation, and execution of swab tests during COVID-19 pandemic. The proposed approach is based on a multi-tiered network consisting of manufacturing companies of reagents, processing laboratories (where the swab kits are prepared and some swab tests are analyzed), landing stations for UAVs and test centers. As innovations in the supply chain, the sharing of reagents between processing laboratories and the use of UAVs, using 5G technology, are contemplated in the management of the COVID-19 Pandemic. To obtain the optimal solutions of the underlying optimization problem, we provide a variational formulation problem for which results of existence and uniqueness will be provided. Finally, some numerical simulations are examined to validate the effectiveness of our approach.

1. Introduction

The COVID-19 pandemic, which broke out in China at the end of 2019, has certainly revolutionized our habits. The sudden and uncontrollable spread of infections has forced the entire population to periods of quarantines and restrictions and normal daily activities were closely monitored in order to ensure a slowdown in the spread of infections. Face-to-face education was suspended, giving rise to distance learning; work activities, when possible, was converted into work from home activities. On March 11, 2020, after assessing the levels of spread and severity of the SARS-CoV-2 infection, the World Health Organization (WHO) declared that the COVID-19 outbreak recorded over the past months can be characterized as a pandemic (see [1]), but, despite that, it can still be controlled. On 30 January 2020, following a second meeting of the Emergency Committee, the WHO Director-General had already declared the international SARS-CoV-2 outbreak a Public Health Emergency of International Concern (PHEIC), as defined in the International Health Regulations (IHR, 2005), [2]. Authors in [3] determined the criteria to be used in the decision of admission of COVID-19 patients to the intensive care units, while many researchers studied the future trend of the pandemic (see, for example, [4], where a hybrid reinforcement learning based algorithm is designed or [5], where a regression-based Robust Optimization (RO) approach to efficiently predict the number of patients with confirmed infection caused by the COVID-19 is developed).

Despite the initial uncertainties, the use of masks and protective devices has become one of the most important means in countering the spread of the virus [6]. The mandatory use of Personal Protective Equipment (PPE) was associated with an integrated surveillance system for the spread of COVID-19 [7], which continuously and systematically collected, compared and analyzed information on all cases of SARS-CoV-2 infection, confirmed by molecular diagnosis performed in public and private health facilities and in diagnostic and analysis laboratories (also the Computer Tomography scan images can be suitable for COVID-19 detection, see [8]).

It was, and still is, a necessary and useful observation tool both for informing citizens about the impact and evolution of the pandemic and for offering decision support for public health responses from health authorities. This surveillance system is based on the election of a sample of the respiratory tract. Generally, this sample is collected from the upper respiratory tract through a rhino-pharyngeal and oropharyngeal swab. According to the American Centers of Disease Control and Prevention (CDC), anterior nasal swab and turbinate nasal swab med can also be used. From the sample of biological material, transported to the laboratory at a controlled temperature, the extraction and purification of RNA is performed for the subsequent search for viral RNA using a rapid molecular method called Reverse Real-Time PCR (rRT-PCR) [9].

To perform the diagnostic tests, it is necessary to use specific reagents, which, in some phases, especially in the phases of peaks of infections, are difficult to produce, to find and store (see [10,11]). Moreover, also the production and provision of swab kits and swab tests encountered delays due to the huge demand. The difficulties

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in finding PPE, swab kits and tests and reagents, due to the large and not expected number of requests, dictate the need to provide suppliers and companies involved in the supply chain with optimization models, based on mathematical models, in order to avoid delays and/or shortages in supply (see [12] for a mathematical model devoted to design a sustainable mask Closed-Loop Supply Chain Network (CLSCN) during the COVID-19 outbreak in which the locational, supply, production, distribution, collection, quarantine, recycling, reuse, and disposal decisions are taken into account).

For these reasons, in this paper we provide a network-based supply chain optimization model for the execution of swab tests and the provision of reagent needed to execute a swab test (see [13] for a supply chain network with inclusion of labor, [14] for a reverse supply chain which consists of a pharmaceutical manufacturer and a retailer, [15] for a stochastic optimization of supply chain operations under ripple effect caused by pandemic and [16] for an integer programming formulation to maximize the number of swab tests during a pandemic). We assume that the manufacturing companies produce reagents and sell them to processing laboratories, where the swab kits are prepared, and some swab tests are performed on people and analyzed. As innovation, we suppose that some processing laboratories can self-produce some kinds of reagents and send them to other laboratories, to guarantee a fluidity in the management of a large number of requests for the execution of swabs.

An absolute novelty that for the first time is taken into consideration in an optimization model that concerns the provision and supply chain of reagents, and the execution of swabs is to consider, as a means of transport, in addition to the usual ones (cars, trucks, ...), the Unmanned Aerial Vehicles (UAVs), managed and orchestrated through innovative communications infrastructures, such as 5G Networks. UAVs, recently, have been proposed as an alternative in order to overcome the limitations and shortcomings of current practices (see [17] for an extensive and comprehensive review of papers dealing with the use of UAVs in engineering transportation and [18] for the use of fleets of Multi-access Edge Computing UAVs in 5G environments). The great potential of their use in transportation and delivering has been observed by the engineering community, although today some limitations are present.

In [19], the author emphasizes the multiple possibilities of using UAVs in improving and facilitating transport problems, in addition to the possibility to use them in the planning, designing and monitoring of transport and other infrastructure. Particularly, their use could be of fundamental importance in transportation of goods, in medical supply, in civil and transportation engineering and in traffic engineering problems. There are many examples of UAVs used in the delivering of medical items. For instance, in 2019, the government of Malawi set up drone corridor networks to serve the East African country of nearly 20 million people that has an inadequate health infrastructure. Starting with childhood vaccines for malaria, TB and rotavirus, the drone network quickly expanded to warehouse or 'beehive' pharmaceuticals for delivery on-demand. Similarly, thanks to their ability to fly over the country's dense rainforest, the Democratic Republic of the Congo decided in 2021 to active an Ebola vaccine project with the aim of vaccinating people in remote locations using bi-directional medical delivery drones, which drop-off the Ebola vaccine and then collect biometric data from vaccines, together with other health reports. Finally, since 2011, Zipline company delivers whole blood, platelets, frozen plasma, and cryoprecipitate along with medical products, including vaccines, infusions, and common medical commodities in Nigeria, Cote d'Ivoire and Kenya. Despite the obvious benefits and advantages of using UAVs in medical items delivering, however, it is clear that we must take into account the engineering limitations of UAVs, first of all the need to recharge their batteries.

Therefore, in this framework, we consider a network with five levels in which, at the highest level of the network, manufacturing companies produce reagents and sell them to processing laboratories, placed in the second level of the network. Processing laboratories purchase reagents from manufacturing companies, prepare the swab kits, analyze/process the swab tests and can execute swab tests directly on people. We also take into account the possibility for processing laboratories to self-produce some kinds of reagent and to exchange reagents with each other. The third level of the network consists of the landing stations that can be used (or not) by drones for the delivery of the swab kits to the test centers, that constitutes the fourth level of the network. Here, the test centers receive the swab kits from the processing laboratories (through the traditional transportation or via UAVs), execute the swab tests directly on people, representing the last level of the network, and analyze them or send them to the processing laboratories for the tests.

We emphasize that the use of UAVs as means of transport is supported by a 5G infrastructure through which all requests (for both kits to be sent and swabs to be collected), the geographical position of the people, the type of swab requested, the type of means of transport to be sent, the results of the swabs analyzed etc, are transmitted. It is known, in fact, that the recent development of such an innovative technology allows a secure and fast transmission of information which is therefore shared in real-time. On the other hand, assuming that geographically difficult to reach areas can be also considered, the combination of UAVs-5G infrastructure (which do not require physical connections with the ground) is considerably advantageous (see [20]).

In a system optimization point of view, this mathematical model aims to determine the optimal flow of reagents, swab kits and swab tests in order to maximize the quantity of executed swab tests and, simultaneously, to maximize the profits of laboratories. Moreover, a multitude of linear and non-linear constraints, in terms of capacity, execution times and conservation of flows, must be satisfied. To the obtained constrained optimization problem, we associate a variational formulation, for which results in terms of existence and uniqueness of the solution are provided. In addition, the duality theory is examined to guarantee an alternative variational formulation more tractable in computational terms.

The rest of the paper is organized as follows. In Section 2, we describe in a more detailed way the multi-tiered network on which our optimization model is based, that we describe in Section 3. Section 4 is devoted to the variational formulation of the model and, finally, in Section 7 we provide three numerical simulations to validate the effectiveness and the potentiality of our model. Particularly, we show how the sharing of reagents among laboratories and the use of UAVs through 5G technology as means of transport is of fundamental importance (and suitable in terms of objective function) in the supply chain of medical items, and, specifically, of reagents, kits and swab tests. We also show how our model is able to capture the most important objective, that is the maximization of the swabs analyzed, while maximizing the profit at the same time.

2. Network description

In this section we describe the topology of the network on which the model proposed in the following section is based (see [21] for another optimization model on the management of reagents and swabs, based on a multi-level network).

As already mentioned above, it is well known that a large number of people require to undergo COVID-19 swab tests for various reasons (such as to know if they are positive following contact with a positive person or before a particular event, for work reasons, to be able to travel, and so on).

There exist different types of tests that can be used for COVID-19 detection, including diagnostic tests that look for active coronavirus infection in mucus or saliva using a swab (which is inserted into the throat and the nose). Some swab-tests, the antigen tests, are generally quick and cheap and for this reason are called *rapid tests*; they look for a piece of the coating of the virus. Other types of swab-tests, the molecular tests, are often called *Real Time-Polymerase Chain Reaction*

(RT-PCR) *tests* for the lab technique used to detect the nucleic acid (such as RNA) belonging to the coronavirus, and they are considered the most sensitive and highly accurate swab-tests. Since there exist different types of swab-tests, in this paper we consider S types of swab-tests and denote by s the generic one.

Furthermore, preparing a swab-kit, running a swab-test and reading its results requires particular equipment but especially one or more reagents, that are specific chemicals without which it is not possible to obtain the necessary results and are often in short supply because of the high number of requests. Moreover, each type of swab-test and kit requires different quantities of each type of reagent. Therefore, we denote by $1, \dots, r, \dots, R$ the different types of reagents.

People can be grouped into G groups $(1, \dots, g, \dots, G)$, distinguished both by geographical area and by category: minors or adults (for which there are different prices established by the Government or sales policies), people who can swab for a fee or for free of charge (for the latter it is the National Health Service that pays, so the laboratory or the test center that execute the swab test get a different contribution than that received when a person pays directly), people who move or stay "at home" (for the latter the price is increased by the travel costs, also based on the geographical area of residence). Note that the model we present in the next section is generic, hence, different and/or other categories of people can be assumed.

People can get any type of swab-test by going (or contacting, for home execution) to one of the H test centers, such as hospitals, pharmacies, analysis laboratories, local health centers, dedicated hubs, etc. In the generic test center h, after swabbing people, the swab tests can be analyzed there or sent to a processing laboratory, depending on the type of swab test to be analyzed and the available equipment. The processing laboratories are specific laboratories, accredited by the National Health System, in which all types of swab tests can be analyzed. We denote such processing laboratories with $1, \dots, p, \dots, P$. Furthermore, we assume that these laboratories deal not only with analyzing the swab tests received from the test centers (or executed on people who decide to go directly to these processing laboratories), but also with the preparation of the swab kits to be sent to the test centers. Moreover, each processing laboratory buys the reagents from the manufacturing companies $(1, \dots, a, \dots, A)$ or could auto-produce one or more types of reagents. In this paper we also assume that each processing laboratory could buy or sell reagents from or to other laboratories.

One of the innovative aspects of this paper, as mentioned in the previous Section, concerns the possibility to use some UAVs as a means of transporting swab tests to be performed or analyzed. Therefore, in Fig. 1 the network topology is depicted, where continuous links and dashed links indicate traditional means of transport and through UAVs, respectively. Moreover, we suppose that the test center could be reached using the traditional means of transport or through UAVs. The UAVs can directly reach the test centers or stop at the landing stations $1, \dots, l, \dots, L$.

We now explain the network in detail. It consists of five levels. The highest level of the network is composed by the manufacturing companies $(1, \dots, a, \dots, A)$ which produce the reagents $(1, \dots, r, \dots, R)$. The reagents are purchased by the processing laboratories $(1, \dots, p, \dots, P)$, which could also self-produce some kinds of reagent and which are mainly concerned with preparing the swab kits and analyzing/processing the swab tests (if the test centers do not do it). In addition, processing laboratories can also swab the test directly on people. Therefore, the second layer of the network consists of the combination between the processing laboratory and the swab tests. Note that the links between the processing laboratories refer to the purchase and sale of reagents.

The intermediate level of the network consists of the landing stations $(1, \dots, l, \dots, L)$ that can be used (or not) by UAVs to transport the swab kits to the test centers.

The combination of test centers and swab tests constitutes the fourth level of the network. Each test center receives the swab kits from the processing labs (through the traditional transportation or via UAVs), gets the swab tests (on people) and then analyzes them or sends them to the processing laboratories for the test analysis.

The lowest level of the network consists of the groups of people (as previously described) requiring the swab tests.

We specify that:

- the transaction of reagents between companies and processing laboratories takes place only through the traditional method (black continuous links), while between laboratories the transaction can take place both through the traditional method and through UAVs (black continuous links and blue dashed links, respectively);
- the transport of swab kits and swab tests between the second (the processing laboratories) and the third layer (landing stations) takes place only via UAVs (blue dashed links);
- the transport of swab kits and swab tests between the third (landing stations) and the fourth layer (test centers) takes place only via the traditional transportation means (black continuous links);
- the transport of swab kits and swab tests between the second (the processing laboratories) and the fourth layer (test centers) can take place both through the traditional method and through UAVs (for simplicity we denoted them with gray dashed links, but each of them represents a pair of parallel links to indicate both modes of transport); moreover, note that these links connect nodes of the same swab-type;
- people could reach (or be reached by specialized personnel from) the test centers or directly the processing laboratories (black continuous links).

We underline that the main extension, respect to the paper in [21] (where a multi-period resource allocation model is analyzed, with the objective of simultaneously maximize the quantity of all performed swabs and minimize the time required to obtain the swabs result), consists in using some UAVs as a means of transporting. Therefore, also the intermediate level of the network, which consists of the landing stations (that can be used, or not, by UAVs to transport the swab kits to the test centers) represents a crucial innovation of this paper. Furthermore, in this paper we assume that the processing laboratories deal not only with analyzing the swab tests received from the test centers (or directly executed on people), but also with the preparation of the swab kits to be sent to the test centers. On the contrary, in [21] the swab kits are not taken into account. Moreover, here we are supposing that each laboratory could sell or buy reagents (not swab tests) to or from other laboratories and that the test centers are also able to analyze some types of swab test. Finally, the models deal with different aspects: both of them aim to determine the optimal flow of reagents and swab tests in order to maximize the quantity of executed swab tests, but in this paper the optimal flow of swab kits are also established and the laboratories' profits are maximized.

3. Optimization model

In this section we describe the optimization model which allows us to determine the optimal flow of reagents, swab kits and swab tests in order to maximize the quantity of executed swab tests while maximizing the labs' profits and ensuring that capacity, time and processing capacities constraints are met.

We firstly introduce the used notation (variables, parameters, cost and time functions) and then we propose a mathematical formulation.

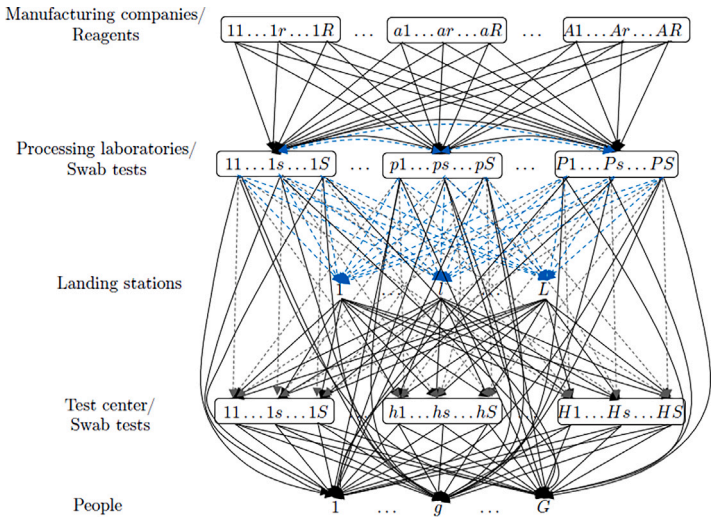

Fig. 1. Network Topology.

Table 1
Variables of the model.

Variable	Description
y_{arp}	Quantity of reagent r purchased by the processing laboratory p from the company a
$y_{pr}^{(AR)}$	Quantity of reagent r self-produced by the processing laboratory p
$y_{p\tilde{p}rm}^{(SR)}$	Quantity of reagent r sent by the processing laboratory p to $\tilde{p}$ in mode m, where $m=1$ means that the traditional transportation is chosen, while $m=2$ means that the transport by UAV is used
$x_{psl}^{(LS)}$	Quantity of s-type swab tests sent from the processing laboratory p to the landing station l (via UAV)
$x_{pshm}^{(TC)}$	Quantity of s-type swab tests sent from the processing laboratory p to the test center h through mode m
$x_{psg}^{(P)}$	Quantity of s-type swab tests requested by group g of people and processed directly by the processing laboratory p
z_{pslh}	Quantity of s-type swab tests (coming from the processing laboratory p) sent from the landing station l to the test center h (through the traditional transportation mode)
w_{hsg}	Quantity of s-type swab tests requested by group g of people at the test center h

3.1. Notation

The main purpose of the model is to maximize the number of swab tests obtained and analyzed, taking into account that particular reagents are required for each swab. The variables of the model are reported in Table 1, in which the used notation and the description of each variable are shown.

The mathematical model includes two types of functions: the cost functions, denoted by $c(\cdot)$, and the time functions, denoted by $t(\cdot)$. All the functions are defined as follows. Let:

- $c_{arp}^{(BR)}$ be the cost for the processing laboratory p to buy reagent r from company a and we assume that such a cost is a function of the amount of purchased reagent: $c_{arp}^{(BR)} = c_{arp}^{(BR)}(y_{arp})$, $\forall a = 1,\dots,A$, $\forall r = 1,\dots,R$, $\forall p = 1,\dots,P$;
- $c_{pr}^{(AR)}$ be the cost that the processing laboratory p has to pay to auto-produce reagent r and we assume that such a cost is a function of the amount of self-produced reagent: $c_{pr}^{(AR)} = c_{pr}^{(AR)}(y_{pr}^{(AR)})$, $\forall p = 1,\dots,P$, $\forall r = 1,\dots,R$;
- $c_{\tilde{p}prm}^{(SR)}$ be the cost that the processing laboratory p has to pay to receive reagent r by the processing laboratory $\tilde{p}$ in mode m and we assume that such a cost is a function of the amount of transported reagent: $c_{\tilde{p}prm}^{(SR)} = c_{\tilde{p}prm}^{(SR)}(y_{\tilde{p}prm}^{(SR)})$, $\forall p,\tilde{p} = 1,\dots,P$, $\forall r = 1,\dots,R$, $\forall m = 1,2$;
- $c_{ps}^{(K)}$ be the cost to prepare an s-type swab kit in laboratory p (to send at the test centers, directly or through the landing stations, or to get it on a person) and we assume that such a cost is a function of the amount of prepared kits: $c_{ps}^{(K)} = c_{ps}^{(K)}\left(\sum_{l=1}^{L} x_{psl}^{(LS)} + \sum_{m=1}^{2}\sum_{h=1}^{H} x_{pshm}^{(TC)} + \sum_{g=1}^{G} x_{psg}^{(P)}\right)$, $\forall p = 1,\dots,P$, $\forall s = 1,\dots,S$;
- $c_{ps}^{(Pr)}$ be the cost to process the swab test s in laboratory p and we assume that such a cost is a function of the amount of swab tests processed by p (we will define this quantity later);
- $c_{psl}^{(LS)}$ be the cost to send a UAV from p to the land station l, transporting the s-type swab kits and we assume that such a cost is a function of the amount of transported swab tests: $c_{psl}^{(LS)} = c_{psl}^{(LS)}(x_{psl}^{(LS)})$, $\forall p = 1,\dots,P$, $\forall s = 1,\dots,S$, $\forall l = 1,\dots,L$;
- $c_{pshm}^{(TC)}$ be the cost to transport the s-type swab kits from p to the test center h in mode m and we assume that such a cost is a function of the amount of transported swab tests: $c_{pshm}^{(TC)} = c_{pshm}^{(TC)}(x_{pshm}^{(TC)})$, $\forall p = 1,\dots,P$, $\forall s = 1,\dots,S$, $\forall h = 1,\dots,H$, $\forall m = 1,2$;
- $c_{lhs}^{(LH)}$ be the cost to transport the s-type swab kits from l to h and we assume that such a cost is a function of the amount of transported swab tests: $c_{lhs}^{(LH)} = c_{lhs}^{(LH)}(z_{pslh})$, $\forall l = 1,\dots,L$, $\forall h = 1,\dots,H$, $\forall s = 1,\dots,S$;
- $t_{p\tilde{p}m}^{(SR)}$ be the time to transport reagents from the processing laboratory p to $\tilde{p}$ and we suppose that such a time is a function of the total amount of reagents transported between p and $\tilde{p}$: $t_{p\tilde{p}m}^{(SR)} = t_{p\tilde{p}m}^{(SR)}\left(\sum_{r=1}^{R} y_{p\tilde{p}rm}^{(SR)}\right)$, $\forall p,\tilde{p} = 1,\dots,P$, $\forall m = 1,2$;
- $t_{phm}^{(TC)}$ be the time to transport swab kits from the processing laboratory p to the test center h in mode m and we suppose that such a time is a function of the total amount of swab kits transported between p and h: $t_{phm}^{(TC)} = t_{phm}^{(TC)}\left(\sum_{s=1}^{S} x_{pshm}^{(TC)}\right)$, $\forall p = 1,\dots,P$, $\forall h = 1,\dots,H$, $\forall m = 1,2$;
- $t_{pl}^{(LS)}$ be the time to transport swab kits from the processing laboratory p to the landing station l (via UAV) and we suppose that such a time is a function of the total amount of swab kits transported between p and l: $t_{pl}^{(LS)} = t_{pl}^{(LS)}\left(\sum_{s=1}^{S} x_{psl}^{(LS)}\right)$, $\forall p = 1,\dots,P$, $\forall l = 1,\dots,L$;
- t_{lh} be the time to transport swab kits from the landing station l to the test center h (through the traditional mode) and we suppose that such a time is a function of the total amount of swab kits transported between l and h: $t_{lh} = t_{lh}\left(\sum_{p=1}^{P}\sum_{s=1}^{S} z_{pslh}\right)$, $\forall l = 1,\dots,L$, $\forall h = 1,\dots,H$;

- $t_{pg}^{(P)}$ be the time to transport swab kits from the processing laboratory p to the group of people g and we suppose that such a time is a function of the total amount of swab kits transported between p and g: $t_{pg}^{(P)} = t_{pg}^{(P)} \left(\sum_{s=1}^{S} x_{psg}^{(P)} \right),\ \forall p = 1, \dots, P,\ \forall g = 1, \dots, G.$

We now describe all the parameters used for the formulation. Let:

- $\rho_{psg}^{(P)}$ be the revenue obtained by the processing laboratory p from the group g of people for the s-type swab test; it includes the kit preparation, swabbing the patient, the possible transport (which depends on the category of g) and the analysis of the swab test;
- $\rho_{psh}^{(TC)}$ be the revenue obtained by the processing laboratory p from the test center h for the s-type swab test and it refers only to the prepared kit;
- $\rho_{psh}^{(PrTC)}$ be the revenue obtained by the processing laboratory p from the test center h for the analysis of an s-type swab test (we underline that the processing laboratory p receives such a revenue only if the test center h is not able to analyze the s-type swab test and sends it to p);
- $\rho_{pslh}^{(LH)}$ be the revenue obtained by the processing laboratory p for a prepared s-type swab kit sent to the landing station l for the test center h (observe that $\rho_{psh}^{(TC)}$ and $\rho_{pslh}^{(LH)}$ are analogous but we are assuming that they could be different for sales strategies; clearly nothing prevents them from being equal);
- $\rho_{pslh}^{(PrLH)}$ be the revenue analogous of $\rho_{psh}^{(PrTC)}$ obtained using the landing station l;
- C_{ar} be the overall amount of reagent r produced by the company a;
- C_{pr} be the maximum quantity of reagent r that the processing laboratory p is able to self-produce;
- $f_{sr}^{(K)}$ be the quantity of reagent r needed to prepare an s-type swab test kit;
- $f_{sr}^{(Pr)}$ be the quantity of reagent r needed to analyze/process an s-type swab test;
- D_{sg} be the quantity of s-type swab tests required by the group g of people;
- $b_{ps}^{(K)}$ be the processing resources required by the processing laboratory p to prepare an s-type swab test;
- $b_{ps}^{(Pr)}$ be the processing resources required by the processing laboratory p to analyze an s-type swab test;
- $b_{ps}^{(E)}$ be the processing resources required by the processing laboratory p to get an s-type swab test;
- $b_{hs}^{(E)}$ be the processing resources required by the test center h to get an s-type swab test;
- $b_{hs}^{(Pr)}$ be the processing resources required by the test center h to analyze an s-type swab test;
- B_p be the maximum processing capacity available for the processing laboratory p, it represents the productivity efficiency and depends on machinery, on time, but above all on the workforce available at p;
- B_h be the maximum processing capacity available for the test center h;
- B_{ps} be the maximum quantity of s-type swab tests that the processing laboratory p is able to analyze (note that such a quantity could be zero if p cannot analyze s-type swab tests);
- B_{hs} be the maximum quantity of s-type swab tests that the test center h is able to analyze (note that such a quantity could be zero if h cannot analyze s-type swab tests);
- $\Theta_{p\bar{p}m}^{(R)}$ be the maximum reagents transportation capacity in the link between the processing laboratories p and $\bar{p}$ in mode m (this is the maximum capacity of the used transport mode);
- $\Theta_{phm}^{(S)}$ be the maximum swab tests transportation capacity in the link between the processing laboratory p and the test center h in mode m;
- $\Theta_{pl}^{(SL)}$ be the maximum swab tests transportation capacity in the link between the processing laboratory p and the landing station l (this is the UAV's maximum capacity);

Table 2
Parameters of the model.

Parameters	Description
$\rho_{psg}^{(P)}$, $\rho_{psh}^{(TC)}$, $\rho_{psh}^{(PrTC)}$, $\rho_{pslh}^{(LH)}$, $\rho_{pslh}^{(PrLH)}$	Revenues obtained by the processing laboratory p for the kit preparation and/or test analysis
C_{ar}, C_{pr}	Amount of reagent r produced
$f_{sr}^{(K)}$, $f_{sr}^{(Pr)}$	Quantity of reagent r needed to prepare a kit or analyze a test
D_{sg}	Quantity of s-type swab tests required by the group g of people
$b_{ps}^{(K)}$, $b_{ps}^{(Pr)}$, $b_{ps}^{(E)}$, $b_{hs}^{(E)}$, $b_{hs}^{(Pr)}$	Processing resources required to prepare a kit or analyze a test
B_p, B_h, B_{ps}, B_{hs}	Maximum processing capacity available at p or h and maximum quantity of s-type swab tests that p or h is able to analyze
$\Theta_{p\bar{p}m}^{(R)}$, $\Theta_{phm}^{(S)}$, $\Theta_{pl}^{(SL)}$	Maximum reagents or swab tests transportation capacity
$\overline{T}_r$, $\overline{T}_s$	Maximum allowed reagent or swab tests transport time
d_{hsm}	The roundtrip parameter

- $\overline{T}_r$ be the maximum allowed reagent transport time, so that the reagent r does not deteriorate;
- $\overline{T}_s$ be the maximum allowed swab kit and test transport time, so that the s-type swab test or kit does not deteriorate;
- d_{hsm} be the "roundtrip" parameter. Such a parameter is equal to 1 if the test center h is able to analyze the s-type swab tests, and, hence, the swab kits do not come back to the processing laboratory. On the contrary, if the test center h cannot analyze the s-type swab tests, the "roundtrip" parameter is equal to 2 because the swab kits have to come back to the processing laboratory to be analyzed. Note that the "roundtrip" parameter d_{hsm} also has the index m because in some cases the transportation means have to return to the laboratory. Indeed, in this paper we will assume that the drone always returns, however, the traditional vehicle (equipped cars, vans, helicopters, etc.) returns only if some swabs have to go back (to be processed in the laboratory).

For further clarity, all the parameters of the model are summarized in Table 2.

3.2. Mathematical formulation

The main aim of our model is to maximize the amount of the analyzed swab tests because, as previously discussed, it is very important to know who the positive (infected) people are, to prevent the contagion from increasing more and more. At the same time, the model is able to determine the optimal distribution flows that allow us to maximize the profit of each processing laboratory, that is maximize the revenues while minimizing the sum of costs.

Therefore, since the model involves more than one objective function which must be maximized, we have a multi-objective programming problem and we use the weighted sum method that combines and converts all the objective functions into a single-objective composite function. Let $\alpha,\ \beta \in [0, 1]$ two weights, then the problem becomes:

$$\max \left\{ \alpha \sum_{p=1}^{P} \sum_{s=1}^{S} \left[\sum_{l=1}^{L} x_{psl}^{(LS)} + \sum_{m=1}^{2} \sum_{h=1}^{H} x_{pshm}^{(TC)} + \sum_{g=1}^{G} x_{psg}^{(P)} \right] + \right.$$

$$\beta \sum_{p=1}^{P} \left[\sum_{s=1}^{S} \left[\sum_{g=1}^{G} \rho_{psg}^{(P)} x_{psg}^{(P)} + \sum_{h=1}^{H} \sum_{m=1}^{2} \left(\rho_{psh}^{(TC)} + (d_{hs1} - 1) \rho_{psh}^{(PrTC)} \right) x_{pshm}^{(TC)} + \right.\right.$$

$$\left.\left. \sum_{l=1}^{L} \sum_{h=1}^{H} \left(\rho_{pslh}^{(LH)} + (d_{hs1} - 1) \rho_{pslh}^{(PrLH)} \right) z_{pslh} \right] -$$

$$\left[\sum_{a=1}^{A} \sum_{r=1}^{R} c_{arp}^{(BR)} (y_{arp}) + \sum_{r=1}^{R} c_{pr}^{(AR)} (y_{pr}^{(AR)}) + \sum_{\substack{\bar{p}=1 \\ \bar{p} \neq p}}^{P} \sum_{r=1}^{R} \sum_{m=1}^{2} c_{\bar{p}prm}^{(SR)} (y_{\bar{p}prm}^{(SR)}) + \right.$$

$$\sum_{s=1}^{S}\left(c_{ps}^{(K)}\left(\sum_{l=1}^{L}x_{psl}^{(LS)}+\sum_{m=1}^{2}\sum_{h=1}^{H}x_{pshm}^{(TC)}+\sum_{g=1}^{G}x_{psg}^{(P)}\right)+ c_{ps}^{(Pr)}\left(\sum_{l=1}^{L}\sum_{h=1}^{H}(d_{hs1}-1)z_{pslh}+\sum_{m=1}^{2}\sum_{h=1}^{H}(d_{hs1}-1)x_{pshm}^{(TC)}+\sum_{g=1}^{G}x_{psg}^{(P)}\right)+ \sum_{h=1}^{H}\sum_{m=1}^{2}d_{hsm}c_{pshm}^{(TC)}(x_{pshm}^{(TC)})+2\sum_{l=1}^{L}c_{psl}^{(LS)}(x_{psl}^{(LS)})+ \sum_{l=1}^{L}\sum_{h=1}^{H}d_{hs1}c_{lhs}^{(LH)}(z_{pslh})\right)\Bigg]\Bigg]\Bigg\} \tag{1}$$

subject to:

$$\sum_{p=1}^{P}y_{arp}\le C_{ar},\quad \forall a=1,\dots,A,\ \forall r=1,\dots,R \tag{2}$$

$$y_{pr}^{(AR)}\le C_{pr},\quad \forall p=1,\dots,P,\ \forall r=1,\dots,R \tag{3}$$

$$\sum_{\substack{\tilde{p}=1\\ \tilde{p}\neq p}}^{P}\sum_{m=1}^{2}y_{p\tilde{p}rm}^{(SR)}\le\sum_{a=1}^{A}y_{arp}+y_{pr}^{(AR)}+\sum_{\substack{\tilde{p}=1\\ \tilde{p}\neq p}}^{P}\sum_{m=1}^{2}y_{\tilde{p}prm}^{(SR)},\quad \forall p=1,\dots,P,\ \forall r=1,\dots,R \tag{4}$$

$$\sum_{s=1}^{S}f_{sr}^{(K)}\left(\sum_{l=1}^{L}x_{psl}^{(LS)}+\sum_{h=1}^{H}\sum_{m=1}^{2}x_{pshm}^{(TC)}+\sum_{g=1}^{G}x_{psg}^{(P)}\right)+ \sum_{s=1}^{S}f_{sr}^{(Pr)}\left(\sum_{l=1}^{L}\sum_{h=1}^{H}(d_{hs1}-1)z_{pslh}+\sum_{m=1}^{2}\sum_{h=1}^{H}(d_{hs1-1})x_{pshm}^{(TC)}+ \sum_{g=1}^{G}x_{psg}^{(P)}\right)\le\sum_{a=1}^{A}y_{arp}+y_{pr}^{(AR)}-\sum_{\substack{\tilde{p}=1\\ \tilde{p}\neq p}}^{P}\sum_{m=1}^{2}y_{p\tilde{p}rm}^{(SR)}+\sum_{\substack{\tilde{p}=1\\ \tilde{p}\neq p}}^{P}\sum_{m=1}^{2}y_{\tilde{p}prm}^{(SR)},\quad \forall r=1,\dots,R,\ \forall p=1,\dots,P \tag{5}$$

$$\sum_{h=1}^{H}z_{pslh}=x_{psl}^{(LS)},\quad \forall p=1,\dots,P,\ \forall s=1,\dots,S,\ \forall l=1,\dots,L \tag{6}$$

$$\sum_{g=1}^{G}w_{hsg}=\sum_{p=1}^{P}\sum_{m=1}^{2}x_{pshm}^{(TC)}+\sum_{p=1}^{P}\sum_{l=1}^{L}z_{pslh},\quad \forall h=1,\dots,H,\ \forall s=1,\dots,S \tag{7}$$

$$\sum_{h=1}^{H}w_{hsg}+\sum_{p=1}^{P}x_{psg}^{(P)}\le D_{sg},\quad \forall s=1,\dots,S,\ \forall g=1,\dots,G \tag{8}$$

$$\sum_{s=1}^{S}b_{ps}^{(K)}\left(\sum_{l=1}^{L}x_{psl}^{(LS)}+\sum_{h=1}^{H}\sum_{m=1}^{2}x_{pshm}^{(TC)}+\sum_{g=1}^{G}x_{psg}^{(P)}\right)+ \sum_{s=1}^{S}b_{ps}^{(Pr)}\left(\sum_{l=1}^{L}\sum_{h=1}^{H}(d_{hs1}-1)z_{pslh}+\sum_{m=1}^{2}\sum_{h=1}^{H}(d_{hs1-1})x_{pshm}^{(TC)}+\sum_{g=1}^{G}x_{psg}^{(P)}\right)+ \sum_{s=1}^{S}\sum_{g=1}^{G}b_{ps}^{(E)}x_{psg}^{(P)}\le B_{p},\quad \forall p=1,\dots,P \tag{9}$$

$$\sum_{l=1}^{L}x_{psl}^{(LS)}+\sum_{h=1}^{H}\sum_{m=1}^{2}x_{pshm}^{(TC)}+\sum_{g=1}^{G}x_{psg}^{(P)}\le B_{ps},\quad \forall p=1,\dots,P,\ \forall s=1,\dots,S \tag{10}$$

$$\sum_{s=1}^{S}\sum_{g=1}^{G}[b_{hs}^{(E)}+(2-d_{hs1})b_{hs}^{(Pr)}]w_{hsg}\le B_{h},\quad \forall h=1,\dots,H, \tag{11}$$

$$\sum_{g=1}^{G}w_{hsg}\le B_{hs},\quad \forall h=1,\dots,H,\ \forall s=1,\dots,S, \tag{12}$$

$$\sum_{r=1}^{R}x_{p\tilde{p}rm}\le\Theta_{p\tilde{p}m}^{(R)},\quad \forall p,\tilde{p}=1,\dots,P,\ \forall m=1,2 \tag{13}$$

$$\sum_{s=1}^{S}x_{pshm}^{(TC)}\le\Theta_{phm}^{(S)},\quad \forall p=1,\dots,P,\ \forall h=1,\dots,H,\ \forall m=1,2 \tag{14}$$

$$\sum_{s=1}^{S}x_{psl}^{(LS)}\le\Theta_{pl}^{(SL)},\quad \forall p=1,\dots,P,\ \forall l=1,\dots,L \tag{15}$$

$$t_{p\tilde{p}m}^{(SR)}\left(\sum_{r=1}^{R}y_{p\tilde{p}rm}^{(SR)}\right)\le\overline{T}_{r},\quad \forall p,\tilde{p}=1,\dots,P,\ \forall r=1,\dots,R,\ \forall m=1,2 \tag{16}$$

$$t_{phm}^{(TC)}\left(\sum_{s=1}^{S}x_{pshm}^{(TC)}\right)\le\overline{T}_{s},\quad \forall p=1,\dots,P,\ \forall s=1,\dots,S,\ \forall h=1,\dots,H,\ \forall m=1,2 \tag{17}$$

$$t_{pl}^{(LS)}\left(\sum_{s=1}^{S}x_{psl}^{(LS)}\right)+t_{lh}\left(\sum_{p=1}^{P}\sum_{s=1}^{S}z_{pslh}\right)\le\overline{T}_{s},\quad \forall p=1,\dots,P,\ \forall s=1,\dots,S,\ \forall l=1,\dots,L,\ \forall h=1,\dots,H \tag{18}$$

$$t_{pg}^{(P)}\left(\sum_{s=1}^{S}x_{psg}^{(P)}\right)\le\overline{T}_{s},\quad \forall p=1,\dots,P,\ \forall s=1,\dots,S,\ \forall g=1,\dots,G, \tag{19}$$

$$y_{arp},y_{pr}^{(AR)},y_{p\tilde{p}rm}^{(SR)},x_{psl}^{(LS)},x_{pshm}^{(TC)},x_{psg}^{(P)},z_{pslh},w_{hsg}\ge 0,\quad \forall a=1,\dots,A,\ \forall r=1,\dots,R,\ \forall p,\tilde{p}=1,\dots,P,\ \forall m=1,2,\ \forall s=1,\dots,S,\ \forall l=1,\dots,L,\ \forall h=1,\dots,H,\ \forall g=1,\dots,G \tag{20}$$

The objective function (1) consists of two main terms: the total amount of analyzed swab tests, multiplied by the weight α, and the total profit, multiplied by β. The first term, that is the total amount of analyzed swab tests is given by the sum of (all types of) swab test kits sent by all processing laboratories to all the test centers (using or not the landing stations: $\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{l=1}^{L}x_{psl}^{(LS)}$ and $\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{m=1}^{2}\sum_{h=1}^{H}x_{pshm}^{(TC)}$, respectively) and the tests performed directly on people ($\sum_{g=1}^{G}x_{psg}^{(P)}$). Observe that all the swab test kits sent by the processing labs are executed and analyzed (at the test centers or return at the labs). The latter sentence is justified from realty, since the number of requests of swab tests from people is very high and the case of swab test kits prepared, sent and ready but not used is excluded. On the contrary, it may happen that customer requests cannot be met due to a lack of swab tests (or rather, reagents needed for the analysis of the swab tests).

The second term of the objective function (1), multiplied by the weight β, represents the total profit, given by the difference between the revenues and the costs. Specifically, the overall revenue is obtained summing the following terms:

- $\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{g=1}^{G}\rho_{psg}^{(P)}x_{psg}^{(P)}$, the total revenue obtained by the processing laboratories from the people for the executed swab tests (including the kit preparation, swabbing the patient, the possible transport and the analysis of the swab test);
- $\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{h=1}^{H}\sum_{m=1}^{2}\left(\rho_{psh}^{(TC)}+(d_{hs1}-1)\rho_{psh}^{(PrTC)}\right)x_{pshm}^{(TC)}$, the total revenue obtained by the processing laboratories from the test centers for both the prepared and sent swab test kits and the possible (if done by the processing labs) analysis of the swab tests; we remind that $d_{hs1}=2$ ($\Rightarrow\ (d_{hs1}-1)=1$) if the swab test has to come back to the processing laboratory where it is analyzed, while $d_{hs1}=1$ ($\Rightarrow\ (d_{hs1}-1)=0$) if the swab test is analyzed at the test center (hence, in the latter case, the processing laboratory does not obtain any revenue for the swab analysis);
- $\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{l=1}^{L}\sum_{h=1}^{H}\left(\rho_{pslh}^{(LH)}+(d_{hs1}-1)\rho_{pslh}^{(PrLH)}\right)z_{pslh}$, the total revenue obtained by the processing laboratories from the test centers, using the landing stations, for both the prepared and sent swab test kits and the possible (if done by the processing labs) analysis of the swab tests.

The overall cost is obtained summing the following terms:

- $\sum_{p=1}^{P}\sum_{a=1}^{A}\sum_{r=1}^{R}c_{arp}^{(BR)}(y_{arp})$, the cost for the processing laboratories to buy reagents from companies;
- $\sum_{p=1}^{P}\sum_{r=1}^{R}c_{pr}^{(AR)}(y_{pr}^{(AR)})$, the cost for the processing laboratories to self-produce reagents;
- $\sum_{p=1}^{P}\sum_{\substack{\tilde{p}=1\\ \tilde{p}\neq p}}^{P}\sum_{r=1}^{R}\sum_{m=1}^{2}c_{\tilde{p}prm}^{(SR)}(y_{\tilde{p}prm}^{(SR)})$, the cost for the processing laboratories to receive reagents from other laboratories; note that we are assuming that only the laboratory that receives the reagents pays, moreover, this cost refers to the cost of transport (since the purchase cost vanishes in the objective function);

- $\sum_{p=1}^{P}\sum_{s=1}^{S} c_{ps}^{(K)}\left(\sum_{l=1}^{L} x_{psl}^{(LS)} + \sum_{m=1}^{2}\sum_{h=1}^{H} x_{pshm}^{(TC)} + \sum_{g=1}^{G} x_{psg}^{(P)}\right)$, the cost for the processing laboratories to prepare all the swab test kits;
- $\sum_{p=1}^{P}\sum_{s=1}^{S} c_{ps}^{(Pr)}\left(\sum_{l=1}^{L}\sum_{h=1}^{H}(d_{hs1}-1)z_{pslh} + \sum_{m=1}^{2}\sum_{h=1}^{H}(d_{hs1}-1)\right.$ $\left. x_{pshm}^{(TC)} + \sum_{g=1}^{G} x_{psg}^{(P)}\right)$, the cost for the processing laboratories to analyze all the swab tests; note that these costs are functions of the amount of swab tests processed by each lab (hence, the total quantity of the swab tests coming back from the test centers to the labs summed to the swab tests performed on people);
- $\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{h=1}^{H}\sum_{m=1}^{2} d_{hsm} c_{pshm}^{(TC)}(x_{pshm}^{(TC)})$, the cost to transport all the swab tests from the processing laboratories to the test centers; we underline that these costs are multiplied by the round trip parameter d_{hsm} that equals 2 or 1 if the transportation means come back or not to the laboratory, respectively;
- $2\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{l=1}^{L} c_{psl}^{(LS)}(x_{psl}^{(LS)})$, the cost to transport all the swab tests from the processing laboratories to the landing stations; we underline that these costs are multiplied by 2 because these routes are traveled only by UAVs, which must always return to the starting point (with or without the swab tests to be analyzed);
- $\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{l=1}^{L}\sum_{h=1}^{H} d_{hs1} c_{lhs}^{(LH)}(z_{pslh})$, the cost to transport all the swab tests from the landing stations (and coming from the labs) to the test centers; clearly these costs are multiplied by the round trip parameter d_{hs1}, where $m = 1$ because the used transportation means for these routes (from the landing stations to the test centers) are the traditional ones which come back only if any swab tests have to be analyzed at the processing laboratories.

We now explain all the constraints.

Constraints (2) and (3) represent the capacity constraints, according to which, each company cannot send more reagent than produced (see constraint (2)) and, analogously, at each processing laboratory the maximum quantity of each reagent that it is possible to self-produce must not be exceed (see constraint (3)).

Constraint (4) refers to the sharing of reagents among the processing laboratories. Indeed, it establishes that the amount of each reagent that each laboratory could share with (that is, send to) other laboratories (through the traditional mean of transport and through UAVs) must not exceed the total amount of reagent possessed, that is the sum of the quantity of reagent purchased by all the manufacturing companies, the quantity of reagent self-produced and the quantity of reagent received from other laboratories.

The relation between the swab tests and the amount of reagents needed to prepare and analyze them is expressed by constraint (5). It guarantees that, for each processing laboratory, the amount of each reagent needed to:

- prepare the swab test kits to send at the landing stations, at the test centers and to use directly on people and
- analyze the swab tests received from the test centers (using or not the landing station) and performed on people

must be less than or equal to the total amount of such a reagent owned by the laboratory. Particularly, the owned reagent is determined by the sum of the amount purchased by all the manufacturing companies, the quantity of reagent self-produced and the quantity of reagent received from other laboratories, minus the quantity of reagent sent to other laboratories.

Constraints (6) and (7) represent the conservation laws, according to which in each landing station and each test center, respectively, the inflows and outflows must be equal. Specifically, the quantity of each swab test type sent by a processing laboratory to a landing station must be equal to the sum of swab tests sent by the landing station to all the test centers; analogously, the quantity of each swab test type sent by all the processing laboratories and landing stations to a test center must be equal to the swab tests executed by the test center to all groups of people.

Clearly, the number of requests from people cannot be exceeded, as ensured by constraint (8).

Each processing laboratory has a maximum processing capacity, depending on the machinery, the time and the workforce, as determined by constraint (9). Moreover, constraint (10) establishes whether a processing laboratory is able to prepare (and analyze) a swab test type: if the parameter $B_{ps} = 0$, then the processing laboratory p is not able to create the s-type swab tests, otherwise p can create and send them.

Constraints (11) and (12) are the analogous constraints of (9) and (10) for each test center. We underline that in constraint (11) the multiplication of $b_{hs}^{(Pr)}$ by $(2 - d_{hs1})$ allows us to add such processing resources usage only if the (s-type) swab tests are analyzed in the test center h (and they are not sent back to the laboratories). Furthermore, in realty constraint (12) is fundamental since it allows us to formulate the case in which a test center is able or not to get some type of swab tests.

The space capacities of each transport mean used for the links between the laboratories, between laboratories and test centers, and between laboratories and landing stations are guaranteed by constraints (13), (14) and (15).

It is well known that each reagent, each swab test kit and swabbed test have a shelf-life, that is the maximum allowed transport time. Therefore, constraints (16)–(19) ensure that the shelf-life of each reagent and each swab test is not exceeded.

Finally, constraint (20) represents the domain of the variables of the model.

4. Variational formulation

In this section, we provide a variational formulation of the proposed model presented in Section 3. This alternative formulation allows us to use the well-known variational inequality theory, which provides, under appropriate hypotheses, results of existence and uniqueness of the solution. Moreover, the variational inequality formulation allows also the rigorous computation of the optimal solutions, as described in the next section.

First, we assume that all the cost and time functions involved in the objective function (1) are continuously differentiable and convex.

Theorem 4.1. *The optimal solutions to the maximization problem (1); (3)–(20) are equivalent to the solutions to the variational inequality problem given by: determine* $(x^{(LS)*}, x^{(TC)*}, x^{(P)*}, y^{*}, y^{(AR)*}, y^{(SR)*}, w^{*}, z^{*}) \in \mathbb{K}$, *satisfying:*

$$\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{l=1}^{L}\left[\beta\frac{\partial c_{ps}^{(K)}\left(x^{(LS)*}, x^{(TC)*}, x^{(P)*}\right)}{\partial x_{psl}^{(LS)}} + 2\beta\frac{\partial c_{psl}^{(LS)}\left(x_{psl}^{(LS)*}\right)}{\partial x_{psl}^{(LS)}} - \alpha\right] \times \left(x_{psl}^{(LS)} - x_{psl}^{(LS)*}\right)$$

$$+\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\beta\frac{\partial c_{ps}^{(K)}\left(x^{(LS)*}, x^{(TC)*}, x^{(P)*}\right)}{\partial x_{pshm}^{(TC)}} + \beta\frac{\partial c_{ps}^{Pr}\left(x^{(TC)*}, x^{(P)*}, z^{*}\right)}{\partial x_{pshm}^{(TC)}}\right.$$

$$\left.+\beta d_{hsm}\frac{\partial c_{pshm}^{(TC)}\left(x_{pshm}^{(TC)*}\right)}{\partial x_{pshm}^{(TC)}} - \alpha - \beta\left(\rho_{psh}^{(TC)} + (d_{hs1}-1)\rho_{psh}^{(PrTC)}\right)\right] \times \left(x_{pshm}^{(TC)} - x_{pshm}^{(TC)*}\right)$$

$$+\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{g=1}^{G}\left[\beta\frac{\partial c_{ps}^{(K)}\left(x^{(LS)*}, x^{(TC)*}, x^{(P)*}\right)}{\partial x_{psg}^{(P)}} - \alpha - \beta\rho_{psg}^{(P)}\right.$$

$$\left.+\beta\frac{\partial c_{ps}^{(Pr)}\left(x^{(TC)*}, x^{(P)*}, z^{*}\right)}{\partial x_{psg}^{(P)}}\right] \times \left(x_{psg}^{(P)} - x_{psg}^{(P)*}\right)$$

$$+\beta\sum_{a=1}^{A}\sum_{r=1}^{R}\sum_{p=1}^{P}\left[\frac{\partial c_{arp}^{(BR)}\left(y_{arp}^{*}\right)}{\partial y_{arp}}\right] \times \left(y_{arp} - y_{arp}^{*}\right)$$

$$+\beta\sum_{p=1}^{P}\sum_{r=1}^{R}\left[\frac{\partial c_{pr}^{(AR)}\left(y_{pr}^{(AR)*}\right)}{\partial y_{pr}^{(AR)}}\right] \times \left(y_{pr}^{(AR)} - y_{pr}^{(AR)*}\right)$$

$$+\beta\sum_{\substack{\tilde{p}=1\\ \tilde{p}\neq p}}^{P}\sum_{p=1}^{P}\sum_{m=1}^{2}\left[\frac{\partial c_{\tilde{p}prm}^{(SR)}\left(y_{\tilde{p}prm}^{(SR)*}\right)}{\partial y_{\tilde{p}prm}^{(SR)}}\right]\times\left(y_{\tilde{p}prm}^{(SR)}-y_{\tilde{p}prm}^{(SR)*}\right)$$
$$+\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{l=1}^{L}\sum_{h=1}^{H}\left[\beta\frac{\partial c_{ps}^{(Pr)}\left(x^{(TC)*},x^{(P)*},z^{*}\right)}{\partial z_{pslh}}-\beta\left(\rho_{pslh}^{(LH)}+(d_{hs1}-1)\rho_{pslh}^{(PrLH)}\right)\right.$$
$$\left.+\beta d_{hs1}\frac{\partial c_{lhs}^{(LH)}(z_{pslh}^{*})}{\partial z_{pslh}}\right]\times\left(z_{pslh}-z_{pslh}^{*}\right)$$

$$\forall(x^{(LS)},x^{(TC)},x^{(P)},y,y^{(AR)},y^{(SR)},w,z)\in\mathbb{K},\tag{21}$$

where

$$\mathbb{K}:=\left\{(x^{(LS)},x^{(TC)},x^{(P)},y,y^{(AR)},y^{(SR)},w,z)\in\mathbb{R}_{+}^{N}\ :\ (3)\text{–}(20)\ hold\right\},\tag{22}$$

and where $N=PLS+2PSH+PSG+ARP+RP+(P-1)PRM+PSLH$.

Proof. It follows by results presented in [22]. □

For easy reference in the subsequent discussions, we put variational inequality (21) into standard form (see [22–25]), that is: determine $X^{*}\in\mathcal{K}$ satisfying:

$$\langle F(X^{*}),X-X^{*}\rangle\geq 0,\quad\forall X\in\mathcal{X}.\tag{23}$$

We set $X\equiv(x^{(LS)},x^{(TC)},x^{(P)},y^{(SR)},z)$, $F(X)=(F^{i}(X))_{i=1,\dots,7}$, with the generic (p,l,s)-th component of $F^{1}(X)$ given by:

$$F_{pls}^{1}(X)\equiv\left[\beta\frac{\partial c_{ps}^{(K)}\left(x^{(LS)},x^{(TC)},x^{(P)}\right)}{\partial x_{psl}^{(LS)}}-2\beta\frac{\partial c_{pls}^{(LS)}\left(x_{pls}^{(LS)}\right)}{\partial x_{pls}^{(LS)}}-\alpha\right],\quad\forall p,l,s\tag{24}$$

the generic (p,s,h,m)-th component of $F^{2}(X)$ given by:

$$F_{pshm}^{2}(X)\equiv\left[\beta\frac{\partial c_{ps}^{(K)}\left(x^{(LS)},x^{(TC)},x^{(P)}\right)}{\partial x_{pshm}^{(TC)}}+\beta\frac{\partial c_{ps}^{Pr}\left(x^{(TC)},x^{(P)},z\right)}{\partial x_{pshm}^{(TC)}}\right.$$
$$\left.+\beta d_{hsm}\frac{\partial c_{pshm}^{(TC)}\left(x_{pshm}^{(TC)}\right)}{\partial x_{pshm}^{(TC)}}-\alpha-\beta\left(\rho_{psh}^{(TC)}+(d_{hs1}-1)\rho_{psh}^{(PrTC)}\right)\right],$$
$$\forall p,s,h,m,\tag{25}$$

the generic (p,s,g)-th component of $F^{3}(X)$ given by:

$$F_{psg}^{3}(X)\equiv\left[\beta\frac{\partial c_{ps}^{(K)}\left(x^{(LS)},x^{(TC)},x^{(P)}\right)}{\partial x_{psg}^{(P)}}+\beta\frac{\partial c_{ps}^{Pr}\left(x^{(TC)},x^{(P)},z\right)}{\partial x_{psg}^{(P)}}\right.$$
$$\left.-\alpha-\beta\rho_{psg}^{(P)}\right],\quad\forall p,s,g,\tag{26}$$

the generic (a,r,p)-th component of $F^{4}(X)$ given by:

$$F^{4}(X)=\left[\beta\frac{\partial c_{arp}^{(BR)}\left(y_{arp}\right)}{\partial y_{arp}}\right],\quad\forall a,r,p,\tag{27}$$

the generic (p,r)-th component of $F^{5}(X)$ given by:

$$F^{5}(X)=\left[\beta\frac{\partial c_{pr}^{(AR)}\left(y_{pr}^{(AR)}\right)}{\partial y_{pr}^{(AR)}}\right],\quad\forall p,r,\tag{28}$$

the generic $(\tilde{p},p,r,m)$-th component of $F^{6}(X)$ given by:

$$F_{\tilde{p}prm}^{6}(X)\equiv\left[\beta\frac{\partial c_{\tilde{p}prm}^{(SR)}\left(y_{\tilde{p}prm}^{(SR)}\right)}{\partial y_{\tilde{p}prm}^{(SR)}}\right],\quad\forall\tilde{p},p,r,m,\tag{29}$$

and the generic (p,s,l,h)-th component of $F^{7}(X)$ given by:

$$F_{pslh}^{7}(X)\equiv\left[\beta\frac{\partial c_{ps}^{(Pr)}\left(x^{(TC)*},x^{(P)*},z^{*}\right)}{\partial z_{pslh}}+\beta d_{hs1}\frac{\partial c_{lhs}^{(LH)}(z_{pslh}^{*})}{\partial z_{pslh}}\right.$$
$$\left.-\beta\left(\rho_{pslh}^{(LH)}+(d_{hs1}-1)\rho_{pslh}^{(PrLH)}\right)\right],\quad\forall p,s,l,h,\tag{30}$$

and $\mathcal{K}\equiv\mathbb{K}$. Therefore, variational inequality (21) can be rewritten into standard form (23). We observe that in the variational inequality problem in standard form (23), the feasible set $\mathcal{K}$ is closed and convex, properties guaranteed by the nature of constraints (3)–(20) and the assumption of continuously differentiability of all the time functions, and that the function that enters the variational inequality, F, is continuous, property that follows from the continuously differentiability of the all cost functions involved in the previous formulation. Hence, the existence of a solution to variational inequality (23), or equivalently, (21), is guaranteed from the classical variational inequality theory (see [26]).

Finally, following [22,26], we can state the following uniqueness result.

Theorem 4.2. *The uniqueness of the solution to the variational inequality (21) or, equivalently, to the variational inequality (23) is guaranteed if the function $F(X)$ is strictly monotone on $\mathcal{K}$, that is:*

$$\langle F(X^{1})-F(X^{2}),X^{1}-X^{2}\rangle>0,\quad\forall X^{1},X^{2}\in\mathcal{K},\ X^{1}\neq X^{2}.\tag{31}$$

The following result ensures a sufficient condition to the strictly monotonicity of the function F.

Proposition 4.3. *If the cost functions $c_{arp}^{(BR)}(\cdot)$, $c_{pr}^{(AR)}(\cdot)$, $c_{\tilde{p}prm}^{(SR)}$, $c_{ps}^{(K)}(\cdot)$, $c_{ps}^{(Pr)}(\cdot)$, $c_{pshm}^{(TC)}(\cdot)$, $c_{psl}^{(LS)}(\cdot)$, and $c_{lhs}^{(LH)}(\cdot)$ are strictly convex with respect to their own variables, $c_{ps}^{(K)}(\cdot)$ are additive with respect to $x^{(LS)}$, $x^{(TC)}$ and $x^{(P)}$ and $c_{ps}^{Pr}(\cdot)$ are additive with respect to $x^{(TC)}$, $x^{(P)}$ and z, then $F(X)$ is a strictly monotone function according to (31).*

Proof. Let $X^{1},X^{2}\in\mathcal{K}$ be two feasible vectors such that $X^{1}\neq X^{2}$. We evaluate the following quantity:

$$\langle F(X^{1})-F(X^{2})\rangle=$$
$$\beta\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{l=1}^{L}\left[\frac{\partial c_{ps}^{(K)}\left(x^{(LS),1},x^{(TC),1},x^{(P),1}\right)}{\partial x_{psl}^{(LS)}}-\frac{\partial c_{ps}^{(K)}\left(x^{(LS),2},x^{(TC),2},x^{(P),2}\right)}{\partial x_{psl}^{(LS)}}\right]$$
$$\times\left(x_{pls}^{(LS),1}-x_{pls}^{(LS),2}\right)$$
$$+2\beta\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{l=1}^{L}\left[\frac{\partial c_{pls}^{(LS)}\left(x_{pls}^{(LS),1}\right)}{\partial x_{pls}^{(LS)}}-\frac{\partial c_{pls}^{(LS)}\left(x_{pls}^{(LS),2}\right)}{\partial x_{pls}^{(LS)}}\right]\times\left(x_{pls}^{(LS),1}-x_{pls}^{(LS),2}\right)$$
$$+\beta\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial c_{ps}^{(K)}\left(x^{(LS),1},x^{(TC),1},x^{(P),1}\right)}{\partial x_{pshm}^{(TC)}}-\frac{\partial c_{ps}^{(K)}\left(x^{(LS),2},x^{(TC),2},x^{(P),2}\right)}{\partial x_{pshm}^{(TC)}}\right]$$
$$\times\left(x_{pshm}^{(TC),1}-x_{pshm}^{(TC),2}\right)$$
$$+\beta\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial c_{ps}^{Pr}\left(x^{(TC),1},x^{(P),1},z^{1}\right)}{\partial x_{pshm}^{(TC)}}-\frac{\partial c_{ps}^{Pr}\left(x^{(TC),2},x^{(P),2},z^{2}\right)}{\partial x_{pshm}^{(TC)}}\right]$$
$$\times\left(x_{pshm}^{(TC),1}-x_{pshm}^{(TC),2}\right)$$
$$+\beta\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{h=1}^{H}\sum_{m=1}^{2}d_{hsm}\left[\frac{\partial c_{pshm}^{(TC)}\left(x_{pshm}^{(TC),1}\right)}{\partial x_{pshm}^{(TC)}}-\frac{\partial c_{pshm}^{(TC)}\left(x_{pshm}^{(TC),2}\right)}{\partial x_{pshm}^{(TC)}}\right]$$
$$\times\left(x_{pshm}^{(TC),1}-x_{pshm}^{(TC),2}\right)$$
$$+\beta\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{g=1}^{G}\left[\frac{\partial c_{ps}^{(K)}\left(x^{(LS),1},x^{(TC),1},x^{(P),1}\right)}{\partial x_{psg}^{(P)}}-\frac{\partial c_{ps}^{(K)}\left(x^{(LS),2},x^{(TC),2},x^{(P),2}\right)}{\partial x_{psg}^{(P)}}\right]$$
$$\times\left(x_{psg}^{(P),1}-x_{psg}^{(P),2}\right)$$

$$+\beta\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{g=1}^{G}\left[\frac{\partial c_{ps}^{(Pr)}\left(x^{(TC),1},x^{(P),1},z^{1}\right)}{\partial x_{psg}^{(P)}}-\frac{\partial c_{ps}^{(Pr)}\left(x^{(TC),2},x^{(P),2},z^{2}\right)}{\partial x_{psg}^{(P)}}\right]\times\left(x_{psg}^{(P),1}-x_{psg}^{(P),2}\right)$$

$$+\beta\sum_{a=1}^{A}\sum_{r=1}^{R}\sum_{p=1}^{P}\left[\frac{\partial c_{arp}^{(BR)}\left(y_{arp}^{1}\right)}{\partial y_{arp}}-\frac{\partial c_{arp}^{(BR)}\left(y_{arp}^{2}\right)}{\partial y_{arp}}\right]\times\left(y_{arp}^{1}-y_{arp}^{2}\right)$$

$$+\beta\sum_{p=1}^{P}\sum_{r=1}^{R}\left[\frac{\partial c_{pr}^{(AR)}\left(y_{pr}^{(AR),1}\right)}{\partial y_{pr}^{(AR)}}-\frac{\partial c_{pr}^{(AR)}\left(y_{pr}^{(AR),2}\right)}{\partial y_{pr}^{(AR)}}\right]\times\left(y_{pr}^{(AR),1}-y_{pr}^{(AR),2}\right)$$

$$+\beta\sum_{\substack{\bar{p}=1\\ \bar{p}\neq p}}^{P}\sum_{p=1}^{P}\sum_{m=1}^{2}\left[\frac{\partial c_{\bar{p}prm}^{(SR)}\left(y_{\bar{p}prm}^{(SR),1}\right)}{\partial y_{\bar{p}prm}^{(SR)}}-\frac{\partial c_{\bar{p}prm}^{(SR)}\left(y_{\bar{p}prm}^{(SR),2}\right)}{\partial y_{\bar{p}prm}^{(SR)}}\right]\times\left(y_{\bar{p}prm}^{(SR),1}-y_{\bar{p}prm}^{(SR),2}\right)$$

$$+\beta\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{l=1}^{L}\sum_{h=1}^{H}\left[\frac{\partial c_{ps}^{(Pr)}\left(x^{(TC),1},x^{(P),1},z^{2}\right)}{\partial z_{pslh}}-\frac{\partial c_{ps}^{(Pr)}\left(x^{(TC),2},x^{(P),2},z^{2}\right)}{\partial z_{pslh}}\right]\times\left(z_{pslh}^{1}-z_{pslh}^{2}\right)$$

$$+\beta\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{l=1}^{L}\sum_{h=1}^{H}d_{hs1}\left[\frac{\partial c_{lhs}^{(LH)}(z_{pslh}^{1})}{\partial z_{pslh}}-\frac{\partial c_{lhs}^{(LH)}(z_{pslh}^{2})}{\partial z_{pslh}}\right]\times\left(z_{pslh}^{1}-z_{pslh}^{2}\right).$$

We observe that each of the addends of the previous expression is strictly positive since, for assumption, each of the cost functions $c_{arp}^{(BR)}(\cdot)$, $c_{pr}^{(AR)}(\cdot)$, $c_{\bar{p}prm}^{(SR)}$, $c_{ps}^{(K)}(\cdot)$, $c_{ps}^{(Pr)}(\cdot)$, $c_{pshm}^{(TC)}(\cdot)$, $c_{psl}^{(LS)}(\cdot)$, and $c_{lhs}^{(LH)}(\cdot)$ is strictly convex and the cost functions $c_{ps}^{(K)}$ and $c_{ps}^{(Pr)}$ are additive. Therefore, function F is strictly monotone according to Definition (31). □

5. Duality theory and an equivalent formulation of the variational inequality

In this Section, we want to derive an alternative formulation to variational inequality (23) by means of the Lagrange multipliers associated with the constraints defining the feasible set $\mathcal{K}$ (see [25,27,28]). First of all, we observe that constraints (3)–(20) can be rewritten as follows (we will write only the constraint (3) for simplicity of notation; for all constraints with a minor sign we will perform the same operation):

$$\sum_{p=1}^{P}y_{arp}-C_{ar}\leq 0,\quad \forall a=1,\dots,A,\ \forall r=1,\dots,R \tag{32}$$

Moreover, variational inequality (23) can be equivalently rewritten as a minimization problem. Indeed, by setting

$$V(X)=\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{l=1}^{L}\left[\beta\frac{\partial c_{ps}^{(K)}\left(x^{(LS)*},x^{(TC)*},x^{(P)*}\right)}{\partial x_{psl}^{(LS)}}+2\beta\frac{\partial c_{pls}^{(LS)}\left(x_{pls}^{(LS)*}\right)}{\partial x_{pls}^{(LS)}}-\alpha\right]\times\left(x_{pls}^{(LS)}-x_{pls}^{(LS)*}\right)$$

$$+\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\beta\frac{\partial c_{ps}^{(K)}\left(x^{(LS)*},x^{(TC)*},x^{(P)*}\right)}{\partial x_{pshm}^{(TC)}}+\beta\frac{\partial c_{ps}^{Pr}\left(x^{(TC)*},x^{(P)*},z^{*}\right)}{\partial x_{pshm}^{(TC)}}+\beta d_{hsm}\frac{\partial c_{pshm}^{(TC)}\left(x_{pshm}^{(TC)*}\right)}{\partial x_{pshm}^{(TC)}}-\alpha-\beta\left(\rho_{psh}^{(TC)}+(d_{hs1}-1)\rho_{psh}^{(PrTC)}\right)\right]\times\left(x_{pshm}^{(TC)}-x_{pshm}^{(TC)*}\right)$$

$$+\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{g=1}^{G}\left[\beta\frac{\partial c_{ps}^{(K)}\left(x^{(LS)*},x^{(TC)*},x^{(P)*}\right)}{\partial x_{psg}^{(P)}}-\alpha-\beta\rho_{psg}^{(P)}+\beta\frac{\partial c_{ps}^{(Pr)}\left(x^{(TC)*},x^{(P)*},z^{*}\right)}{\partial x_{psg}^{(P)}}\right]\times\left(x_{psg}^{(P)}-x_{psg}^{(P)*}\right)$$

$$+\sum_{a=1}^{A}\sum_{r=1}^{R}\sum_{p=1}^{P}\left[\beta\frac{\partial c_{arp}^{(BR)}\left(y_{arp}^{*}\right)}{\partial y_{arp}}\right]\times\left(y_{arp}-y_{arp}^{*}\right)$$

$$+\sum_{p=1}^{P}\sum_{r=1}^{R}\left[\beta\frac{\partial c_{pr}^{(AR)}\left(y_{pr}^{(AR)*}\right)}{\partial y_{pr}^{(AR)}}\right]\times\left(y_{pr}^{(AR)}-y_{pr}^{(AR)*}\right)$$

$$+\sum_{\substack{\bar{p}=1\\ \bar{p}\neq p}}^{P}\sum_{p=1}^{P}\sum_{m=1}^{2}\left[\beta\frac{\partial c_{\bar{p}prm}^{(SR)}\left(y_{\bar{p}prm}^{(SR)*}\right)}{\partial y_{\bar{p}prm}^{(SR)}}\right]\times\left(y_{\bar{p}prm}^{(SR)}-y_{\bar{p}prm}^{(SR)*}\right)$$

$$+\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{l=1}^{L}\sum_{h=1}^{H}\left[\beta\frac{\partial c_{ps}^{(Pr)}\left(x^{(TC)*},x^{(P)*},z^{*}\right)}{\partial z_{pslh}}-\beta\left(\rho_{pslh}^{(LH)}+(d_{hs1}-1)\rho_{pslh}^{(PrLH)}\right)+\beta d_{hs1}\frac{\partial c_{lhs}^{(LH)}(z_{pslh}^{*})}{\partial z_{pslh}}\right]\times\left(z_{pslh}-z_{pslh}^{*}\right)$$

we have:

$$V(X)\geq 0,\ \forall X\in\mathcal{K}\ \text{and}\ \min_{X\in\mathcal{K}}V(X)=V(X^{*})=0.$$

Hence, we can consider the following Lagrange function:

$$\mathcal{L}(X,\lambda,\mu)=V(X)+\sum_{a=1}^{A}\sum_{r=1}^{R}\lambda_{ar}^{1}\left(\sum_{p=1}^{P}y_{arp}-C_{ar}\right)+\sum_{p=1}^{P}\sum_{r=1}^{R}\lambda_{pr}^{2}\left(y_{pr}^{(AR)}-C_{pr}\right)$$

$$+\sum_{p=1}^{P}\sum_{r=1}^{R}\lambda_{pr}^{3}\left(\sum_{\substack{\bar{p}=1\\ \bar{p}\neq p}}^{P}\sum_{m=1}^{2}y_{\bar{p}prm}^{(SR)}-\sum_{a=1}^{A}y_{arp}+y_{pr}^{(AR)}-\sum_{\substack{\bar{p}=1\\ \bar{p}\neq p}}^{P}\sum_{m=1}^{2}y_{\bar{p}prm}^{(SR)}\right)$$

$$+\sum_{r=1}^{R}\sum_{p=1}^{P}\lambda_{rp}^{4}\left[\sum_{s=1}^{S}f_{sr}^{(K)}\left(\sum_{l=1}^{L}x_{psl}^{(LS)}+\sum_{h=1}^{H}\sum_{m=1}^{2}x_{pshm}^{(TC)}+\sum_{g=1}^{G}x_{psg}^{(P)}\right)+\sum_{s=1}^{S}f_{sr}^{(Pr)}\left(\sum_{l=1}^{L}\sum_{h=1}^{H}(d_{hs1}-1)z_{pslh}+\sum_{m=1}^{2}\sum_{h=1}^{H}(d_{hs1-1})x_{pshm}^{(TC)}+\sum_{g=1}^{G}x_{psg}^{(P)}\right)\right.$$

$$\left.-\sum_{a=1}^{A}y_{arp}-y_{pr}^{(AR)}+\sum_{\substack{\bar{p}=1\\ \bar{p}\neq p}}^{P}\sum_{m=1}^{2}y_{\bar{p}prm}^{(SR)}-\sum_{\substack{\bar{p}=1\\ \bar{p}\neq p}}^{P}\sum_{m=1}^{2}y_{\bar{p}prm}^{(SR)}\right]+\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{l=1}^{L}\mu_{psl}^{1}\left(\sum_{h=1}^{H}z_{pslh}-x_{pls}^{(LS)}\right)$$

$$+\sum_{h=1}^{H}\sum_{s=1}^{S}\mu_{hs}^{2}\left(\sum_{g=1}^{G}w_{hsg}-\sum_{p=1}^{P}\sum_{m=1}^{2}x_{pshm}^{(TC)}-\sum_{p=1}^{P}\sum_{l=1}^{L}z_{pslh}\right)$$

$$+\sum_{s=1}^{S}\sum_{g=1}^{G}\lambda_{sg}^{5}\left(\sum_{h=1}^{H}w_{hsg}+\sum_{p=1}^{P}x_{psg}^{(P)}-D_{sg}\right)$$

$$+\sum_{p=1}^{P}\lambda_{p}^{6}\left[\sum_{s=1}^{S}b_{ps}^{(K)}\left(\sum_{l=1}^{L}x_{psl}^{(LS)}+\sum_{h=1}^{H}\sum_{m=1}^{2}x_{pshm}^{(TC)}+\sum_{g=1}^{G}x_{psg}^{(P)}\right)+\sum_{s=1}^{S}b_{ps}^{(Pr)}\left(\sum_{l=1}^{L}\sum_{h=1}^{H}(d_{hs1}-1)z_{pslh}+\sum_{m=1}^{2}\sum_{h=1}^{H}(d_{hs1-1})x_{pshm}^{(TC)}+\sum_{g=1}^{G}x_{psg}^{(P)}\right)\right.$$

$$\left.+\sum_{s=1}^{S}\sum_{g=1}^{G}b_{ps}^{(E)}x_{psg}^{(P)}-B_{p}\right]+\sum_{p=1}^{P}\sum_{s=1}^{S}\lambda_{ps}^{7}\left(\sum_{l=1}^{L}x_{psl}^{(LS)}+\sum_{h=1}^{H}\sum_{m=1}^{2}x_{pshm}^{(TC)}+\sum_{g=1}^{G}x_{psg}^{(P)}-B_{ps}\right)$$

$$+\sum_{h=1}^{H}\lambda_{h}^{8}\left(\sum_{s=1}^{S}\sum_{g=1}^{G}[b_{hs}^{(E)}+(2-d_{hs1})b_{hs}^{(Pr)}]w_{hsg}-B_{h}\right)+\sum_{h=1}^{H}\sum_{s=1}^{S}\lambda_{hs}^{9}\left(\sum_{g=1}^{G}w_{hsg}-B_{hs}\right)$$

$$+\sum_{p=1}^{P}\sum_{\bar{p}=1}^{P}\sum_{m=1}^{2}\lambda_{\bar{p}\bar{p}m}^{10}\left(\sum_{r=1}^{R}x_{p\bar{p}rm}-\Theta_{p\bar{p}m}^{(R)}\right)+\sum_{p=1}^{P}\sum_{h=1}^{H}\sum_{m=1}^{2}\lambda_{phm}^{11}\left(\sum_{s=1}^{S}x_{pshm}^{(TC)}-\Theta_{phm}^{(S)}\right)$$

$$+\sum_{p=1}^{P}\sum_{l=1}^{L}\lambda_{pl}^{12}\left(\sum_{s=1}^{S}x_{psl}^{(LS)}-\Theta_{pl}^{(SL)}\right)+\sum_{p=1}^{P}\sum_{\bar{p}=1}^{P}\sum_{s=1}^{S}\sum_{g=1}^{G}\lambda_{p\bar{p}sg}^{13}\left[t_{p\bar{p}m}^{(SR)}\left(\sum_{r=1}^{R}y_{\bar{p}prm}^{(SR)}\right)-\overline{T}_{r}\right]$$

$$+\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{h=1}^{H}\sum_{m=1}^{2}\lambda_{pshm}^{14}\left[t_{phm}^{(TC)}\left(\sum_{s=1}^{S}x_{pshm}^{(TC)}\right)-\overline{T}_{s}\right]$$

$$+\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{l=1}^{L}\sum_{h=1}^{H}\lambda_{pslh}^{15}\left[t_{pl}^{(LS)}\left(\sum_{s=1}^{S}x_{psl}^{(LS)}\right)+t_{lh}\left(\sum_{p=1}^{P}\sum_{s=1}^{S}z_{pslh}\right)-\overline{T}_{s}\right]$$

$$+\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{g=1}^{G}\lambda_{psg}^{16}\left[t_{pg}^{(P)}\left(\sum_{s=1}^{S}x_{psg}^{(P)}\right)-\overline{T}_{s}\right]+\sum_{a=1}^{A}\sum_{r=1}^{R}\sum_{p=1}^{P}\lambda_{arp}^{17}\left(-y_{arp}\right)$$

$$+\sum_{p=1}^{P}\sum_{\bar{p}=1}^{P}\sum_{r=1}^{R}\sum_{m=1}^{2}\lambda_{p\bar{p}rm}^{18}\left(-y_{\bar{p}prm}^{(SR)}\right)+\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{l=1}^{L}\lambda_{psl}^{19}\left(-x_{psl}^{(LS)}\right)$$

$$+\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{h=1}^{H}\sum_{m=1}^{2}\lambda_{pshm}^{20}\left(-x_{pshm}^{(TC)}\right)$$

$$+\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{g=1}^{G}\lambda_{psg}^{21}\left(-x_{psg}^{(P)}\right)+\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{l=1}^{L}\sum_{h=1}^{H}\lambda_{pslh}^{22}\left(-z_{pslh}\right)$$
$$+\sum_{h=1}^{H}\sum_{s=1}^{S}\sum_{g=1}^{G}\lambda_{hsg}^{23}\left(-w_{hsg}\right)+\sum_{p=1}^{P}\sum_{r=1}^{R}\lambda_{pr}^{24}\left(-y_{pr}^{(AR)}\right), \tag{33}$$

where $X \in \mathbb{R}^N$, $\lambda = \left(\lambda^j\right)_{j=1,\dots,24} \in \mathbb{R}_+^M$ and $\mu = (\mu^k)_{k=1,2} \in \mathbb{R}^{PSL+HS}$ and

$$\begin{aligned}\mathcal{M} = {} & AR + 3PR + SG + P + PS + H + HS + P^2M + PHM + PL \\ & + P^2SG + PSHM + PSLH + PSG + ARP + P^2RM + PSL \\ & + PSHM + PSG + PSLH + HSG + PR.\end{aligned}$$

We want to prove that the Lagrange multipliers that appear in the Lagrange function (33) exist and that the system of KKT conditions (34) is equivalent to the variational inequality (23). More specifically, we will demonstrate that if X^* is a solution to variational inequality (23), then KKT conditions (34) hold and vice versa, that is from KKT conditions (34) variational inequality (23) follows. To this aim, we will prove that the following theorem holds true.

Theorem 5.1. *The Lagrange multipliers that appear in Lagrange function (33) exist and, for all $a = 1,\dots,A$, $r = 1,\dots,R$, $p,\tilde{p} = 1 = \dots,P$, $s = 1,\dots,S$, $h = 1,\dots,H$, $m = 1,2$, $l = 1,\dots,L$, the following KKT conditions hold:*

$$\overline{\lambda}_{ar}^{1}\left(\sum_{p=1}^{P} y_{arp}^{*} - C_{ar}\right) = 0, \quad \overline{\lambda}_{pr}^{2}\left(y_{pr}^{(AR)*} - C_{pr}\right) = 0$$

$$\overline{\lambda}_{pr}^{3}\left(\sum_{\substack{\tilde{p}=1\\ \tilde{p}\neq p}}^{P}\sum_{m=1}^{2} y_{p\tilde{p}rm}^{(SR)*} - \sum_{a=1}^{A} y_{arp}^{*} + y_{pr}^{(AR)*} - \sum_{\substack{\tilde{p}=1\\ \tilde{p}\neq p}}^{P}\sum_{m=1}^{2} y_{\tilde{p}prm}^{(SR)*}\right) = 0,$$

$$\begin{aligned}&\overline{\lambda}_{rp}^{4}\left[\sum_{s=1}^{S} f_{sr}^{(K)}\left(\sum_{l=1}^{L} x_{psl}^{(LS)*} + \sum_{h=1}^{H}\sum_{m=1}^{2} x_{pshm}^{(TC)*} + \sum_{g=1}^{G} x_{psg}^{(P)*}\right)\right.\\ &\quad + \sum_{s=1}^{S} f_{sr}^{(Pr)}\left(\sum_{l=1}^{L}\sum_{h=1}^{H}(d_{hs1}-1)z_{pslh}^{*} + \sum_{m=1}^{2}\sum_{h=1}^{H}(d_{hs1-1})x_{pshm}^{(TC)*} + \sum_{g=1}^{G} x_{psg}^{(P)*}\right)\\ &\quad \left. - \sum_{a=1}^{A} y_{arp} - y_{pr}^{(AR)*} + \sum_{\substack{\tilde{p}=1\\ \tilde{p}\neq p}}^{P}\sum_{m=1}^{2} y_{p\tilde{p}rm}^{(SR)*} - \sum_{\substack{\tilde{p}=1\\ \tilde{p}\neq p}}^{P}\sum_{m=1}^{2} y_{\tilde{p}prm}^{(SR)*}\right] = 0,\end{aligned}$$

$$\overline{\lambda}_{sg}^{5}\left(\sum_{h=1}^{H} w_{hsg}^{*} + \sum_{p=1}^{P} x_{psg}^{(P)*} - D_{sg}\right) = 0,$$

$$\begin{aligned}&\overline{\lambda}_{p}^{6}\left[\sum_{s=1}^{S} b_{ps}^{(K)}\left(\sum_{l=1}^{L} x_{psl}^{(LS)*} + \sum_{h=1}^{H}\sum_{m=1}^{2} x_{pshm}^{(TC)*} + \sum_{g=1}^{G} x_{psg}^{(P)*}\right)\right.\\ &\quad + \sum_{s=1}^{S} b_{ps}^{(Pr)}\left(\sum_{l=1}^{L}\sum_{h=1}^{H}(d_{hs1}-1)z_{pslh}^{*} + \sum_{m=1}^{2}\sum_{h=1}^{H}(d_{hs1-1})x_{pshm}^{(TC)*} + \sum_{g=1}^{G} x_{psg}^{(P)*}\right)\\ &\quad \left. + \sum_{s=1}^{S}\sum_{g=1}^{G} b_{ps}^{(E)} x_{psg}^{(P)*} - B_{p}\right] = 0,\end{aligned}$$

$$\overline{\lambda}_{ps}^{7}\left(\sum_{l=1}^{L} x_{psl}^{(LS)*} + \sum_{h=1}^{H}\sum_{m=1}^{2} x_{pshm}^{(TC)*} + \sum_{g=1}^{G} x_{psg}^{(P)*} - B_{ps}\right) = 0,$$

$$\overline{\lambda}_{h}^{8}\left(\sum_{s=1}^{S}\sum_{g=1}^{G}[b_{hs}^{(E)} + (2-d_{hs1})b_{hs}^{(Pr)}]w_{hsg}^{*} - B_{h}\right) = 0, \quad \overline{\lambda}_{hs}^{9}\left(\sum_{g=1}^{G} w_{hsg}^{*} - B_{hs}\right) = 0,$$

$$\overline{\lambda}_{p\tilde{p}m}^{10}\left(\sum_{r=1}^{R} x_{p\tilde{p}rm}^{*} - \Theta_{p\tilde{p}m}^{(R)}\right) = 0, \quad \overline{\lambda}_{phm}^{11}\left(\sum_{s=1}^{S} x_{pshm}^{(TC)*} - \Theta_{phm}^{(S)}\right) = 0,$$

$$\overline{\lambda}_{pl}^{12}\left(\sum_{s=1}^{S} x_{psl}^{(LS)*} - \Theta_{pl}^{(SL)}\right) = 0, \quad \overline{\lambda}_{p\tilde{p}sg}^{13}\left[t_{p\tilde{p}m}^{(SR)}\left(\sum_{r=1}^{R} y_{p\tilde{p}rm}^{(SR)*}\right) - \overline{T}_{r}\right] = 0,$$

$$\overline{\lambda}_{pshm}^{14}\left[t_{phm}^{(TC)}\left(\sum_{s=1}^{S} x_{pshm}^{(TC)*}\right) - \overline{T}_{s}\right] = 0,$$

$$\begin{aligned}&\overline{\lambda}_{pslh}^{15}\left[t_{pl}^{(LS)}\left(\sum_{s=1}^{S} x_{psl}^{(LS)*}\right) + t_{lh}\left(\sum_{p=1}^{P}\sum_{s=1}^{S} z_{pslh}^{*}\right) - \overline{T}_{s}\right] = 0,\\ &\overline{\lambda}_{psg}^{16}\left[t_{pg}^{(P)}\left(\sum_{s=1}^{S} x_{psg}^{(P)*}\right) - \overline{T}_{s}\right] = 0\\ &\overline{\lambda}_{arp}^{17}\left(-y_{arp}^{*}\right) = 0, \quad \overline{\lambda}_{p\tilde{p}rm}^{18}\left(-y_{p\tilde{p}rm}^{(SR)*}\right) = 0, \quad \overline{\lambda}_{psl}^{19}\left(-x_{psl}^{(LS)*}\right) = 0,\\ &\overline{\lambda}_{pshm}^{20}\left(-x_{pshm}^{(TC)*}\right) = 0, \overline{\lambda}_{psg}^{21}\left(-x_{psg}^{(P)*}\right) = 0, \quad \overline{\lambda}_{pslh}^{22}\left(-z_{pslh}^{*}\right) = 0,\\ &\overline{\lambda}_{hsg}^{23}\left(-w_{hsg}^{*}\right) = 0, \quad \overline{\lambda}_{pr}^{24}\left(-y_{pr}^{(AR)}\right) = 0.\end{aligned} \tag{34}$$

Moreover, the following conditions are satisfied:

$$\begin{aligned}&\beta\frac{\partial c_{ps}^{(K)}\left(x^{(LS)*}, x^{(TC)*}, x^{(P)*}\right)}{\partial x_{psl}^{(LS)}} + 2\beta\frac{\partial c_{pls}^{(LS)}\left(x_{pls}^{(LS)*}\right)}{\partial x_{pls}^{(LS)}}\\ &-\alpha + \sum_{r=1}^{R}\overline{\lambda}_{rp}^{4} f_{sr}^{(K)} - \mu_{psl}^{1} + \overline{\lambda}_{p}^{6} b_{ps}^{(K)} + \overline{\lambda}_{ps}^{7}\\ &+\overline{\lambda}_{pl}^{12} + \sum_{h=1}^{H}\overline{\lambda}_{pslh}^{15}\frac{\partial t_{pl}^{(LS)}\left(\sum_{s=1}^{S} x_{psl}^{(LS)*}\right)}{\partial x_{psl}^{(LS)}} = 0,\end{aligned} \tag{35}$$

$$\begin{aligned}&\beta\frac{\partial c_{ps}^{(K)}\left(x^{(LS)*}, x^{(TC)*}, x^{(P)*}\right)}{\partial x_{pshm}^{(TC)}} + \beta\frac{\partial c_{ps}^{Pr}\left(x^{(TC)*}, x^{(P)*}, z^{*}\right)}{\partial x_{pshm}^{(TC)}} + \beta d_{hsm}\frac{\partial c_{pshm}^{(TC)}\left(x_{pshm}^{(TC)*}\right)}{\partial x_{pshm}^{(TC)}}\\ &-\alpha - \beta\left(\rho_{psh}^{(TC)} + (d_{hs1}-1)\rho_{psh}^{(PrTC)}\right) + \sum_{r=1}^{R}\overline{\lambda}_{rp}^{4}\left(f_{sr}^{(K)} + f_{sr}^{(Pr)}(d_{sh1}-1)\right) - \overline{\mu}_{hs}^{2}\\ &+\overline{\lambda}_{p}^{6}\left(b_{ps}^{(K)} + b_{ps}^{(Pr)}(d_{sh1}-1)\right) + \overline{\lambda}_{ps}^{7} + \overline{\lambda}_{phm}^{11} + \overline{\lambda}_{pshm}^{14}\frac{\partial t_{phm}^{(TC)}\left(\sum_{s=1}^{S} x_{pshm}^{(TC)*}\right)}{\partial x_{pshm}^{(TC)}} - \overline{\lambda}_{pshm}^{20} = 0,\end{aligned} \tag{36}$$

$$\begin{aligned}&\beta\frac{\partial c_{ps}^{(K)}\left(x^{(LS)*}, x^{(TC)*}, x^{(P)*}\right)}{\partial x_{psg}^{(P)}} - \alpha - \beta\rho_{psg}^{(P)} + \beta\frac{\partial c_{ps}^{(Pr)}\left(x^{(TC)*}, x^{(P)*}, z^{*}\right)}{\partial x_{psg}^{(P)}}\\ &+\sum_{r=1}^{R}\overline{\lambda}_{rp}^{4}\left(f_{sr}^{(K)} + f_{sr}^{(Pr)}\right) + \overline{\lambda}_{sg}^{5} + \overline{\lambda}_{p}^{6}\left(b_{ps}^{(K)} + b_{ps}^{(Pr)} + b_{ps}^{(E)}\right) + \overline{\lambda}_{ps}^{7}\\ &+\overline{\lambda}_{psg}^{16}\frac{\partial t_{pg}^{(P)}\left(\sum_{s=1}^{S} x_{psg}^{(P)*}\right)}{\partial x_{psg}^{(P)}} - \lambda_{psg}^{21} = 0,\end{aligned} \tag{37}$$

$$\beta\frac{\partial c_{arp}^{(BR)}\left(y_{arp}^{*}\right)}{\partial y_{arp}} + \overline{\lambda}_{ar}^{1} - \overline{\lambda}_{pr}^{3} - \overline{\lambda}_{rp}^{4} - \overline{\lambda}_{arp}^{17} = 0, \tag{38}$$

$$\beta\frac{\partial c_{pr}^{(AR)}\left(y_{pr}^{(AR)*}\right)}{\partial y_{pr}^{(AR)}} + \overline{\lambda}_{pr}^{2} + \overline{\lambda}_{pr}^{3} - \overline{\lambda}_{rp}^{4} - \overline{\lambda}_{pr}^{24} = 0, \tag{39}$$

$$\begin{aligned}&\beta\frac{\partial c_{\tilde{p}prm}^{(SR)}\left(y_{\tilde{p}prm}^{(SR)*}\right)}{\partial y_{\tilde{p}prm}^{(SR)}} - \overline{\lambda}_{pr}^{3} + \overline{\lambda}_{\tilde{p}r}^{3} - \overline{\lambda}_{rp}^{4} + \overline{\lambda}_{r\tilde{p}}^{4} + \overline{\lambda}_{\tilde{p}pm}^{13}\frac{\partial t_{\tilde{p}pm}\left(\sum_{r=1}^{R} y_{\tilde{p}prm}^{(SR)*}\right)}{\partial y_{\tilde{p}prm}^{(SR)}} - \overline{\lambda}_{\tilde{p}prm}^{18} = 0,\\ &\beta\frac{\partial c_{ps}^{(Pr)}\left(x^{(TC)*}, x^{(P)*}, z^{*}\right)}{\partial z_{pslh}} - \beta\left(\rho_{pslh}^{(LH)} + (d_{hs1}-1)\rho_{pslh}^{(PrLH)}\right) + \beta d_{hs1}\frac{\partial c_{lhs}^{(LH)}(z_{pslh}^{*})}{\partial z_{pslh}}\\ &+\sum_{r=1}^{R}\overline{\lambda}_{rp}^{4} f_{sr}^{(Pr)}(d_{hs1}-1) + \overline{\mu}_{psl}^{1} - \overline{\mu}_{hs}^{2} + \overline{\lambda}_{p}^{6} b_{ps}^{(Pr)}(d_{sh1}-1)\\ &+\overline{\lambda}_{pslh}^{15}\frac{\partial t_{lh}\left(\sum_{p=1}^{P}\sum_{s=1}^{S} z_{pslh}^{*}\right)}{\partial z_{pslh}} - \lambda_{pslh}^{22} = 0,\end{aligned} \tag{40, 41}$$

$$\overline{\mu}_{hs}^{2} + \overline{\lambda}_{sg}^{5} + [b_{hs}^{(E)} + (2-d_{hs1})b_{hs}^{(Pr)}] + \overline{\lambda}_{hs}^{9} - \overline{\lambda}_{hsg}^{23} = 0. \tag{42}$$

Finally, the strong duality also holds true, namely,

$$V(X^*) = \min_{\mathcal{K}} V(X) = \max_{(\lambda,\mu)\in\mathbb{R}_+^M\times\mathbb{R}^{PLS+HS}}\left(\min_{X\in\mathbb{R}_+^N}\mathcal{L}(X,\lambda,\mu)\right).$$

Proof. See [28] or [25] for the proof. □

Through KKT conditions (34), together with conditions (35)–(42), it is possible to derive an alternative variational formulation of the previous one provided in Section 4 (see [29–31] for a similar procedure). The advantage of using this new formulation lies in relaxing all the constraints present in the feasible set $\mathcal{K}$ within the objective function by using the associated Lagrange multipliers. Particularly, this advantage is essential to better manage the non-linear time constraints (16)–(19), which, otherwise, could create difficulties in the computation of the optimal solution. Thereby, it is legitimate for us to use the computational procedure described in [32].

6. Numerical examples

In this section we provide two numerical simulations in order to clearly express the potentiality of our model. Particularly, we show a first simulation where we suppose that no sharing of reagents is allowed and also where UAVs are not used. Then, we compare the results of this first simulation with those obtained from a second simulation, in which we assume that sharing of reagents and use of UAVs are allowed.

For both the numerical examples we used the same contest, where the network, depicted in Fig. 2, consists of:

- $A = 2$ manufacturing companies, each of which produces one type of reagent (so we have $R = 2$ types of reagents);
- $S = 2$ types of swab-tests (each of which needs one type of reagent for preparing and processing), rapid tests and RT-PCR tests;
- $P = 2$ processing laboratories and we assume that both could prepare and analyze both types of swab tests; moreover, we assume that the second processing laboratory can self-produce the second type of reagent;
- $L = 1$ landing station (because we are assuming to simulate only one geographical zone);
- $H = 3$ test centers (the first one represents a small laboratory that processes only swabs of the second type; the second is a COVID-19 hub, that is a local center usually managed by the Local or National Health Service; the last represents a pharmacy, where we assume that only swab tests of the first type could be analyzed);
- $G = 4$ groups of people (the first group represents the people who do not directly pay for the swabbing, as in reality, the National Health Service pays the hub center for them; the second and third groups are composed by adults and minors, respectively. We divided them because the prices for swab tests are different; the last group includes people who stay at home, hence they pay also the travel costs).

Note that in Fig. 2 we indicated with a light color the links of the network which are absent in our considered context (such as the output links from the first company refereed to the second reagent, or the links connecting the second swab-type test of the third test center).

We assume that the quantities D_{sg} of swab test types required by the groups of people are as follows:

$$D_{11} = 0,\ D_{12} = 45,\ D_{13} = 25,\ D_{14} = 0,\ D_{21} = 10,$$
$$D_{22} = 15,\ D_{23} = 20,\ D_{24} = 15.$$

Therefore, the total amount of required swab tests is 130.

As previously described, each group of people pays different prices for swab tests (based not only on the different type of swab test, but also on the type of people constituting the group):

$$\rho^{(P)}_{psg}:\ \rho^{(P)}_{111} = 4,\ \rho^{(P)}_{121} = 10,\ \rho^{(P)}_{211} = 4,\ \rho^{(P)}_{221} = 10,$$
$$\rho^{(P)}_{112} = 12,\ \rho^{(P)}_{122} = 35,\ \rho^{(P)}_{212} = 12,\ \rho^{(P)}_{222} = 35,$$
$$\rho^{(P)}_{113} = 6,\ \rho^{(P)}_{123} = 17,\ \rho^{(P)}_{213} = 6,\ \rho^{(P)}_{223} = 17,$$
$$\rho^{(P)}_{114} = 17,\ \rho^{(P)}_{124} = 50,\ \rho^{(P)}_{214} = 17,\ \rho^{(P)}_{224} = 50.$$

The other revenues obtained by the processing laboratories from the test centers (for the provision of swab kits and/or the processing of swab tests) are the following:

$$\rho^{(TC)}_{psh}:\ \rho^{(TC)}_{p1h} = 2\,\rho^{(TC)}_{p2h} = 10,\ \forall p = 1,2,\ \forall h = 1,2,3;$$

$$\rho^{(PrTC)}_{psh}:\ \rho^{(PrTC)}_{p1h} = 8,\ \rho^{(PrTC)}_{p2h} = 15,\ \forall p = 1,2,\ \forall h = 1,2,3;$$

$$\rho^{(LH)}_{pslh}:\ \rho^{(LH)}_{p1lh} = 2;\rho^{(LH)}_{p2lh} = 10,\ \forall p = 1,2,\ l = 1,\ \forall h = 1,2,3;$$

$$\rho^{(PrLH)}_{pslh}:\ \rho^{(PrLH)}_{p1lh} = 8,\ \rho^{(PrLH)}_{p2lh} = 15,\ \forall p = 1,2,\ l = 1,\ \forall h = 1,2,3.$$

We also suppose that the overall amount of reagents produced by the manufacturing companies, C_{ar}, are $C_{11} = 150$ and $C_{22} = 100$, while $C_{12} = C_{21} = 0$ since we assumed that each company produces only one type of reagent. Furthermore, the reagent $r = 1$ is not self-produced by any laboratory (it is produced only by the first company) and the processing $p = 2$ does not self-produce any type of reagent, while the maximum quantity of reagent $r = 2$ that the processing laboratory $p = 2$ is able to self-produce is $C_{pr} = C_{22} = 50$.

The quantities of reagents needed to prepare the swab kits ($f^{(K)}_{sr}$) and to analyze/process the swab tests ($f^{(Pr)}_{sr}$) are: $f^{(K)}_{11} = f^{(K)}_{22} = f^{(Pr)}_{11} = f^{(Pr)}_{22} = 1$, $f^{(K)}_{12} = f^{(K)}_{21} = f^{(Pr)}_{12} = f^{(Pr)}_{21} = 0$, indeed, we assumed that each type of swab-test needs only one type of reagent for preparing and processing ($s = 1$ needs $r = 1$, while $s = 2$ needs $r = 2$).

The processing resources required to prepare the kits or get or analyze the tests are the following:

$$b^{(K)}_{ps} = 1\ \forall p, s = 1,2;$$

$$b^{(Pr)}_{ps}:\ b^{(Pr)}_{11} = 1,\ b^{(Pr)}_{12} = 3,\ b^{(Pr)}_{21} = 1,\ b^{(Pr)}_{22} = 2;$$

$$b^{(E)}_{ps} = b^{(E)}_{hs} = 1,\ \forall p, s = 1,2,\ \forall h = 1,2,3;$$

$$b^{(Pr)}_{hs} = 1,\ \forall h = 1,2,3,\ s = 1,\ b^{(Pr)}_{hs} = 2,\ \forall h = 1,2,3,\ s = 2.$$

Hence, to analyze the second type of swab requires more resources than preparing or getting any type of swab kit or swab tests and analyzing the second type of swab test. Moreover, to analyze the second type of swab at the first processing laboratory requires more resources than somewhere else. However, in the simulations here proposed we assume that the available maximum processing capacities are large enough to process all swabs.

Furthermore, in these simulations we assume that each test center h is able to analyze each type of swab test s, except for $h = 2$ and $s = 2$, hence, we impose that $d_{hsm} = 1,\ \forall h = 1,3,\ \forall s = 1,2,\ d_{21m} = 1,\ d_{22m} = 2,\ \forall m = 1,2$, according to which the second type of swab kits at the second test center (the COVID-19 hub) have to come back to the processing laboratory to be analyzed.

Finally, we assume that all the cost functions have a generic quadratic expression with a null constant term. It is clear that all these assumptions could be modified according to the specific pandemic situation.

The optimal solutions of the proposed numerical simulations are computed via the Euler Method (see [32] for a detailed description) using the Matlab program on an HP laptop with an AMD compute cores 2C+3G processor, 8 GB RAM.

6.1. Simulation 1: without sharing and UAVs

As previously mentioned, the first simulation consists in determining the optimal flows, imposing that both the sharing of reagents and the use of UAVs for the transport of reagents or swab tests are forbidden (or, better, that they are not contemplated): $y^{(SR)}_{p\tilde{p}rm} = 0,\ \forall p, \tilde{p}, r, m = 1,2$, $x^{(LS)}_{psl} = 0,\ \forall p, s = 1,2,\ l = 1,\ x^{(TC)}_{psh1} = 0,\ \forall p, s = 1,2,\ \forall h = 1,2,3$ and $z_{pslh} = 0,\ \forall p, s = 1,2, l = 1, \forall h = 1,2,3$.

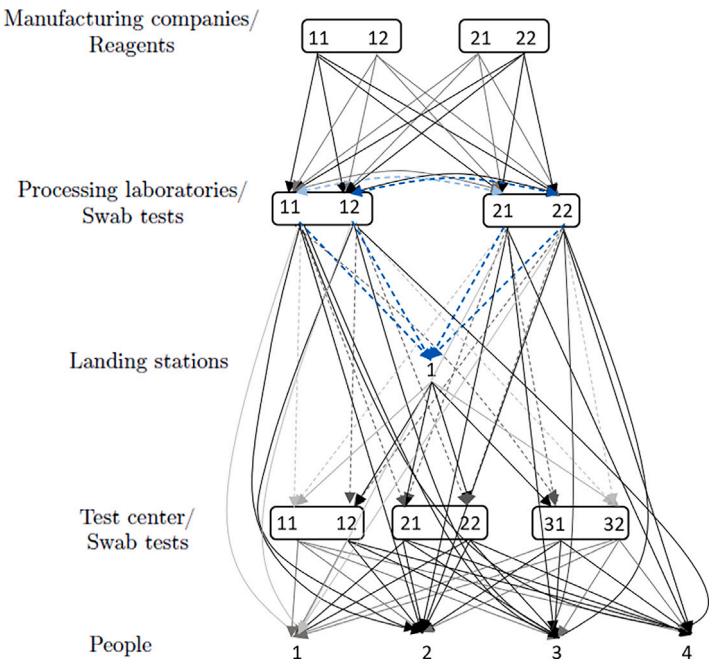

Fig. 2. Network Topology for simulation.

Table 3
Optimal solutions: reagent flows from companies to processing laboratories (Simulation 1).

y^*_{arp}	$p=1$	$p=2$
$a=1, r=1$	42.90	42.89
$a=2, r=2$	21.93	28.07

Table 4
Optimal solutions: swab test flows from processing laboratories to test centers through the traditional transportation mode (Simulation 1).

$x^{(TC)*}_{psh1}$	$h=1$	$h=2$	$h=3$
$p=1, s=1$	0	10.04	10.04
$p=1, s=2$	3.76	4.08	0
$p=2, s=1$	0	17.07	17.07
$p=2, s=2$	16.24	5.92	0

Table 5
Optimal solutions: swab test flows from processing laboratories to people groups (Simulation 1).

$x^{(P)*}_{psg}$	$g=1$	$g=2$	$g=3$	$g=4$
$p=1, s=1$	0	11.41	0	0
$p=1, s=2$	0	15	0	15
$p=2, s=1$	0	4.38	0	0
$p=2, s=2$	0	0	0	0

Observe that, since no UAVs can be used, all the input and output flows to and from the landing station are null.

The optimal solutions are reported in Tables 3–6. Furthermore, the quantity of reagent $r=2$ self-produced by the processing laboratory $p=2$ is equal to the maximum quantity it is able to self-produce: $y^{(AR)*}_{22}=C_{22}=50$.

6.2. Simulation 2: with sharing and UAVs

The second example allows us to explore what occurs when reagents sharing and UAVs could be used. The optimal solutions are reported in Tables 7–13. Also in this case, the quantity of reagent $r=2$ self-produced by the processing laboratory $p=2$ is equal to the maximum quantity it is able to self-produce: $y^{(AR)*}_{22}=C_{22}=50$, while the quantity of reagent $r=2$ sent by the processing laboratory $p=1$ to $p=2$ in mode $m=1$ (that means that the traditional transportation is chosen) is $y^{(SR)*}_{1221}=1.42$, in mode $m=2$ (that is, when the transport by UAV is used) is $y^{(SR)*}_{1222}=5.27$.

Table 6
Optimal solutions: swab test flows from test centers to people groups (Simulation 1).

w^*_{hsg}	$g=1$	$g=2$	$g=3$	$g=4$
$h=1, s=2$	0	0	20	0
$h=2, s=1$	0	14.61	12.5	0
$h=2, s=2$	10	0	0	0
$h=3, s=1$	0	14.61	12.5	0

Table 7
Optimal solutions: reagent flows from companies to processing laboratories (Simulation 2).

y^*_{arp}	$p=1$	$p=2$
$a=1, r=1$	36.56	36.79
$a=2, r=2$	24.42	25.58

Table 8
Optimal solutions: swab test flows from processing laboratories to the landing station $l=1$ (Simulation 2).

$y^{(LS)*}_{ps1}$	$s=1$	$s=2$
$p=1$	9.31	1.65
$p=2$	6.52	4.88

Table 9
Optimal solutions: swab test flows from processing laboratories to test centers through the traditional transportation mode (Simulation 2).

$x^{(TC)*}_{psh1}$	$h=1$	$h=2$	$h=3$
$p=1, s=1$	0	3.26	3.26
$p=1, s=2$	0	0.57	0
$p=2, s=1$	0	4.88	4.88
$p=2, s=2$	4.88	1.88	0

Table 10
Optimal solutions: swab test flows from processing laboratories to test centers through the UAVs (Simulation 2).

$x^{(TC)*}_{psh2}$	$h=1$	$h=2$	$h=3$
$p=1, s=1$	0	7.02	7.02
$p=1, s=2$	0	1.64	0
$p=2, s=1$	0	10.25	10.25
$p=2, s=2$	10.24	4.25	0

Table 11
Optimal solutions: swab test flows from processing laboratories to people groups (Simulation 2).

$x^{(P)*}_{psg}$	$g=1$	$g=2$	$g=3$	$g=4$
$p=1, s=1$	0	3.35	0	0
$p=1, s=2$	0	15	0	15
$p=2, s=1$	0	0	0	0
$p=2, s=2$	0	0	0	0

Table 12
Optimal solutions: swab test flows from the landing station (by processing laboratories) to test centers (Simulation 2).

z^{*}_{pslh}	$h=1$	$h=2$	$h=3$
$p=1, s=1$	0	4.66	4.66
$p=1, s=2$	0	1.65	0
$p=2, s=1$	0	3.26	3.26
$p=2, s=2$	4.88	0	0

Table 13
Optimal solutions: swab test flows from test centers to people groups (Simulation 2).

w^{*}_{hsg}	$g=1$	$g=2$	$g=3$	$g=4$
$h=1, s=2$	0	0	20	0
$h=2, s=1$	0	20.83	12.5	0
$h=2, s=2$	10	0	0	0
$h=3, s=1$	0	20.83	12.5	0

Table 14
Optimal solutions: reagent flows from companies to processing laboratories (Simulation 3).

y^{*}_{arp}	$p=1$	$p=2$
$a=1, r=1$	36.65	36.65
$a=2, r=2$	24.32	25.24

Table 15
Optimal solutions: swab test flows from processing laboratories to the landing station $l=1$ (Simulation 3).

$y^{(LS)*}_{ps1}$	$s=1$	$s=2$
$p=1$	5.48	5.62
$p=2$	5.48	4.98

6.3. Simulation 3: with only the maximization of the analyzed swab tests

The third example aims at exploring the solutions obtained with only the first objective function considered. Therefore, in this simulation we take into account only the objective of maximizing the number of the analyzed swab tests. We, hence, ignore the objective of maximizing the profit of each processing laboratory. The optimal solutions are reported in Tables 14–20. In this case, unlike the previous simulations, the quantity of reagent $r=2$ self-produced by the processing laboratory $p=2$ is not equal to the maximum quantity it is able to self-produce, but it is about half: $y^{(AR)*}_{22} = C_{22} = 24.32$. Moreover, in this simulation, we obtain no difference on the used transportation mode, $m=1$ (that means that the traditional transportation is chosen) or $m=2$ (that is, when the transport by UAV is used), to send reagent $r=2$ by the processing laboratory $p=1$ to $p=2$: $y^{(SR)*}_{1221} = y^{(SR)*}_{1222} = 1.92$.

Table 16
Optimal solutions: swab test flows from processing laboratories to test centers through the traditional transportation mode (Simulation 3).

$x^{(TC)*}_{psh1}$	$h=1$	$h=2$	$h=3$
$p=1, s=1$	0	7.22	7.22
$p=1, s=2$	9.58	3.18	0
$p=2, s=1$	0	7.22	7.22
$p=2, s=2$	8.66	1.34	0

Table 17
Optimal solutions: swab test flows from processing laboratories to test centers through the UAVs (Simulation 3).

$x^{(TC)*}_{psh2}$	$h=1$	$h=2$	$h=3$
$p=1, s=1$	0	7.22	7.22
$p=1, s=2$	9.58	3.18	0
$p=2, s=1$	0	7.22	7.22
$p=2, s=2$	8.66	1.34	0

Table 18
Optimal solutions: swab test flows from processing laboratories to people groups (Simulation 3).

$x^{(P)*}_{psg}$	$g=1$	$g=2$	$g=3$	$g=4$
$p=1, s=1$	0	0.65	0	0
$p=1, s=2$	0	0.37	2.59	0.37
$p=2, s=1$	0	0.65	0	0
$p=2, s=2$	0	0	0.55	0

Table 19
Optimal solutions: swab test flows from the landing station (by processing laboratories) to test centers (Simulation 3).

z^{*}_{pslh}	$h=1$	$h=2$	$h=3$
$p=1, s=1$	4.96	2.74	2.74
$p=1, s=2$	0	0.66	0
$p=2, s=1$	0	2.74	2.74
$p=2, s=2$	4.68	0.30	0

Table 20
Optimal solutions: swab test flows from test centers to people groups (Simulation 3).

w^{*}_{hsg}	$g=1$	$g=2$	$g=3$	$g=4$
$h=1, s=2$	0	14.63	16.85	14.63
$h=2, s=1$	0	21.85	12.5	0
$h=2, s=2$	10	0	0	0
$h=3, s=1$	0	21.85	12.5	0

6.4. Analysis and comparison of the results

From the obtained results we can observe that in all the numerical examples (with and without sharing and UAVs, with a single objective function or multi-objective), all requests are satisfied. Indeed, since in emergency situations (such as a pandemic situation), having the results of the swabs is necessary and urgent, satisfying the greatest number of requests from end users is the main objective of this work. Therefore, in our model we have chosen a value of the weight α greater than β.

Furthermore, we can note that with sharing, the second processing laboratory $p=2$ buys fewer reagents from the manufacturing companies (see Tables 3 and 7), because it can receive (and, actually, it receives) the reagents also from the first processing laboratory $p=1$. We also underline that, if present, both landing stations and UAVs are used for the transport of reagents and swab kits and tests. Indeed, it is noted that it is more convenient to use UAVs more than the traditional method of transport (see Tables 9 and 10). In Simulation 2 (with sharing and UAVs), the direct flows of swab tests from processing laboratories to people groups are smaller than those of Simulation 1 (see Tables 5 and 11); this is motivated by observing that in Simulation 2 the people groups receive more swab tests from the test centers

Table 21
Comparison between the terms of the objective function.

	Simulation 1	Simulation 2	Simulation 3
T1	130	130	130
T2	396.19	651.39	−1097.6

(where the swab tests arrive also using the landing station and the UAVs; see Tables 6, 13 and 12). Comparing the optimal solutions obtained by Simulation 2 and Simulation 3, we can observe how the total quantity of swab tests send from processing laboratories to test centers through the traditional transportation mode in Simulation 3 is much higher than in Simulation 2 (see Tables 9 and 16). On the contrary, the total swab test flows from processing laboratories to people groups in Simulation 3 is less than that in Simulation 2 (see Tables 11 and 18).

We now analyze and compare the terms of the objective function (1). Table 21 shows the optimal values that the two terms of the objective function assume in the numerical examples (without or with sharing and UAVs and with only the first objective function), where $T1$ is the total amount of analyzed swab test and $T2$ is the total profit (given by the difference between the revenues and the costs):

$$T1 = \sum_{p=1}^{P}\sum_{s=1}^{S}\left[\sum_{l=1}^{L} x_{psl}^{(LS)} + \sum_{m=1}^{2}\sum_{h=1}^{H} x_{pshm}^{(TC)} + \sum_{g=1}^{G} x_{psg}^{(P)}\right],$$

$$\begin{aligned} T2 = \sum_{p=1}^{P}\Bigg[\sum_{s=1}^{S}\Bigg[\sum_{g=1}^{G}\rho_{psg}^{(P)}x_{psg}^{(P)} + \sum_{h=1}^{H}\sum_{m=1}^{2}\left(\rho_{psh}^{(TC)} + (d_{hs1}-1)\rho_{psh}^{(PrTC)}\right)x_{pshm}^{(TC)} \\ \sum_{l=1}^{L}\sum_{h=1}^{H}\left(\rho_{pslh}^{(LH)} + (d_{hs1}-1)\rho_{pslh}^{(PrLH)}\right)z_{pslh}\Bigg] - \\ \Bigg[\sum_{a=1}^{A}\sum_{r=1}^{R}c_{arp}^{(BR)}(y_{arp}) + \sum_{r=1}^{R}c_{pr}^{(AR)}(y_{pr}^{(AR)}) + \sum_{\substack{\bar{p}=1\\ \bar{p}\neq p}}^{P}\sum_{r=1}^{R}\sum_{m=1}^{2}c_{\bar{p}prm}^{(SR)}(y_{\bar{p}prm}^{(SR)}) + \\ \sum_{s=1}^{S}\left(c_{ps}^{(K)}\left(\sum_{l=1}^{L}x_{psl}^{(LS)} + \sum_{m=1}^{2}\sum_{h=1}^{H}x_{pshm}^{(TC)} + \sum_{g=1}^{G}x_{psg}^{(P)}\right)\right. \\ c_{ps}^{(Pr)}\left(\sum_{l=1}^{L}\sum_{h=1}^{H}(d_{hs1}-1)z_{pslh} + \sum_{m=1}^{2}\sum_{h=1}^{H}(d_{hs1}-1)x_{pshm}^{(TC)} + \sum_{g=1}^{G}x_{psg}^{(P)}\right) \\ \left.+\sum_{h=1}^{H}\sum_{m=1}^{2}d_{hsm}c_{pshm}^{(TC)}(x_{pshm}^{(TC)}) + 2\sum_{l=1}^{L}c_{psl}^{(LS)}(x_{psl}^{(LS)}) + \sum_{l=1}^{L}\sum_{h=1}^{H}d_{hs1}c_{lhs}^{(LH)}(z_{pslh})\right)\Bigg]\Bigg]. \end{aligned}$$

We highlight that, as previously observed, since all the requested swab tests are satisfied, the first term $T1$ of the objective function (the total amount of analyzed swab tests) are the same for all the simulations. Therefore, the difference between the three simulations refers only to the second term $T2$. Particularly, when we aim to maximize only the number of executed swab tests (Simulation 3), we obtain the same $T1$ value (that is, the same number of analyzed swab tests of the other simulations), and a negative $T2$ value. The latter means that the total cost is higher than the total revenue. Indeed, in Simulation 3, the total revenue is 1898.1, while the total cost is 2995.8 (we remind that the total profit $T2$ is given by revenues minus the costs). Furthermore, it is important to underline that using our extended model (Simulation 2) we obtain a considerable benefit, that is a percentage increase, with respect of both, Simulation 1 and Simulation 3: $PI_1 = \left(\frac{T2_2 - T2_1}{T2_1}\cdot 100\right)\% = 64.41\%$ and $PI_3 = \left(\frac{T2_2 - T2_3}{T2_3}\cdot 100\right)\% = 159,35\%$.

7. Conclusions

The severity of a pandemic, such as that of COVID-19, which emerged at the end of 2019 and is still ongoing today, can lead to devastating consequences. In recent years, unfortunately, we have witnessed millions of deaths and infected people around the world (6,405,080 deaths and 578,142,444 confirmed infected people as of August 4, 2022, see the WHO Health Emergency Dashboard, [33]) and these numbers continue to grow. Therefore, it is clear that a correct management of all means to limit the infection as much as possible is of fundamental importance. One of the means that the population has to limit the infection consists in the detection of the infected, through swabs. Indeed, if, due to the positivity to COVID-19, a person (even asymptomatic) is in contact with other people, could infect them. Instead, knowing if a person is infected with COVID-19 or not, leads to a monitoring that guarantees greater safety to the whole population: a person who, through the swab, certifies (or discovers) his/her positivity is isolated, avoiding infecting other people with direct contact.

One of the biggest issues in performing swab tests on people is the lack of reagents needed for the preparation of swab kits and the analysis of swab tests. For these reasons, in this paper we focus on the management of reagents, swab kits and swab tests during the COVID-19 pandemic. Therefore, we propose a supply chain optimization model, based on a multi-level network, that allows us to establish the optimal flow of reagents, swab kits and swab tests in order to maximize the quantity of performed swab tests and, simultaneously, to maximize the profits of laboratories (note that the maximization of profits allows a greater number of swabs performed, indeed, if, due to an incorrect management of the flows, the costs exceed the revenues, no laboratory could perform the swabs).

The main innovative aspects (in addition to the use of an optimization model for the management of reagents, kits and swabs) refer to the sharing of reagents among laboratories and to the possibility to use the UAVs through 5G technology for transporting reagents, kits and swabs. We provide the associated variational formulation with existence and uniqueness results and examine the duality theory. Finally, we validate the effectiveness of our model through three numerical simulations that show how the use of UAVs and the sharing of reagents is much more convenient.

The proposed model could be used for different situations or contexts, such as the provision of medical items, vaccines, medications, medicines, organs, blood bags, but also perishable products, and so on.

For future works we intend to take into account the deterioration of the reagents, tending to minimize it also by minimizing transport times. Moreover, considering that during the COVID-19 pandemic there was a strong competition between suppliers for the supply of reagents or, more generally, of PPE, it may be interesting to take into account this feature through a Nash Equilibrium framework.

CRediT authorship contribution statement

Gabriella Colajanni: Conceptualization, Methodology, Software, Formal analysis, Data curation, Writing. **Patrizia Daniele:** Conceptualization, Methodology, Supervision, Project administration. **Daniele Sciacca:** Conceptualization, Methodology, Formal analysis, Writing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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