

University of Catania
Department of Civil Engineering and Architecture
University of Perugia
Department of Civil and Environmental Engineering

NATIONAL DOCTORAL PROGRAM

**IN DEFENSE AGAINST NATURAL RISKS AND
ECOLOGICAL TRANSITION OF
BUILT ENVIRONMENT**

PhD thesis
XXXVIII cycle

Experimental and numerical investigations on the seismic dynamic response of masonry barrel vaults

PhD Candidate
Roberta Scungio

Tutor
Massimiliano Giofrè

International Tutor
Paulo B. Lourenço

Co - Tutor
Giacomo Navarra



**Università
di Catania**



unipg



**Universidade
do Minho**

International Faculty Board

PhD Coordinator

Massimo Cuomo

Faculty Board members

Assante Dario	Uninettuno University
Barabino Benedetto	University of Brescia
Benassai Guido	"Parthenope" University of Naples
Biondi Daniela	University of Calabria
Caliò Ivo	University of Catania
Callisto Luigi	"La Sapienza" University of Rome
Cancelliere Antonino	University of Catania
Capparelli Giovanna	University of Calabria
Castelli Francesco	Kore University
Caterino Nicola	"Parthenope" University of Naples
Cavaleri Liborio	University of Palermo
Cazzani Antonio Maria	University of Cagliari
Ceroni Francesca	"Parthenope" University of Naples
Cesarano Clemente	Uninettuno University
Claps Pierluigi	Polytechnic University of Turin
Contrafatto Loredana	University of Catania
Cuomo Massimo	University of Catania
D'Annibale Francesco	University of L'Aquila
De Bartolo Samuele	University of Salento
De Bellis Maria Laura	University of Chieti-Pescara
De Biagi Valerio	Polytechnic University of Turin
De Felice Gianmarco	Roma Tre University
De Luca Davide Luciano	University of Calabria
Dell'Isola Francesco	University of L'Aquila
Di Laora Raffaele	University of Campania - "L. Vanvitelli"
Di Matteo Alberto	University of Palermo
Dimitri Rossana	University of Salento
Fatone Francesco	Polytechnic University of Marche
Feo Luciano	University of Salerno
Ferrari Alessio	University of Palermo
Fidelibus Corrado	University of Salento
Fiori Aldo	Roma Tre University
Fiorino Donatella Rita	University of Cagliari
Formica Giovanni	Roma Tre University
Foti Enrico	University of Catania

Fraternali Fernando	University of Salerno
Frattolillo Andrea	University of Cagliari
Freni Gabriele	Kore University
Gabriele Stefano	Roma Tre University
Gambarotta Luigi	University of Genoa
Garcea Giovanni	University of Calabria
Gioffrè Massimiliano	University of Perugia
Giorgio Ivan	University of L'Aquila
Iezzì Gianluca	University of Chieti-Pescara
Imposa Sebastiano	University of Catania
Lacarbonara Walter	"La Sapienza" University of Rome
Lagomarsino Sergio	University of Genoa
Laio Francesco	Polytechnic University of Turin
Lanzoni Stefano	University of Padova
Lenci Stefano	Polytechnic University of Marche
Leone Antonio	University of Salento
Lepidi Marco	University of Genoa
Luongo Angelo	University of L'Aquila
Maiorana Carmelo	University of Padova
Maiorano Rosa Maria Stefania	"Parthenope" University of Naples
Marfia Sonia	Roma Tre University
Marincioni Fausto	Polytechnic University of Marche
Massabò Roberta	University of Genoa
Micelli Francesco	University of Salento
Minutolo Vincenzo	University of Campania - "L. Vanvitelli"
Mollaioli Fabrizio	"La Sapienza" University of Rome
Monaco Paola	University of L'Aquila
Montelpore Sergio	University of Chieti-Pescara
Musmarra Dino	University of Campania - "L. Vanvitelli"
Musumeci Rosaria Ester	University of Catania
Navarra Giacomo Camillo	Kore University
Nocera Francesco	University of Catania
Pace Bruno	University of Chieti-Pescara
Pagliaroli Alessandro	University of Chieti-Pescara
Panteghini Andrea	University of Brescia
Paolacci Fabrizio	Roma Tre University
Piccardo Giuseppe	University of Genoa
Pilotti Marco	University of Brescia
Placidi Luca	Uninettuno University
Porcu Maria Cristina	University of Cagliari
Preti Marco	University of Brescia
Quagliarini Enrico	Polytechnic University of Marche
Quaranta Giuseppe	"La Sapienza" University of Rome

Romano Elpidio
Salerno Ginevra
Simonini Paolo
Spacone Enrico
Tomasicchio Giuseppe
Ubertini Filippo
Vaccari Mentore
Varano Valerio
Ventura Giulio
Versaci Antonella
Xotta Giovanna
Zambianchi Enrico
Zulli Daniele

Uninettuno University
Roma Tre University
University of Padova
University of Chieti-Pescara
University of Salento
University of Perugia
University of Brescia
Roma Tre University
Polytechnic University of Turin
Kore University
University of Padova
"Parthenope" University of Naples
University of L'Aquila

International Review Board:

- Professor Daniel V. Oliveira, Univesity of Minho (Portugal);
- Professor Luca Pelà, Polytechnic University of Catalonia (Spain).

Submitted for review: September 2025

Defense of the thesis: January 2026.

Funding: This research received no external funding and was fully supported by the University of Perugia through the XXXVIII Cycle funds allocated to the National Doctoral Program in Defense Against Natural Risks and Ecological Transition of the Built Environment.

Data Availability Statement: Data will be made available on request.

Design/research activity in national or international programs: The experimental tests from the first and second experimental campaigns were supported by Regione Umbria, Italy (Project Code: P.S.R. Umbria 2014/2020, Misura 16.2.2–84250132283; Scientific coordinator: Professor Gioffrè) and by Regione Umbria, Italy (Project Code: P.S.R. Umbria 2014/2020, Misura 16.2.1 – CUP I69H22000030006; Scientific coordinator: Professor Gioffrè), respectively.

Acknowledgments:

Approaching the end of this long research journey, I feel the need to express my deepest gratitude to all those who have supported and guided me throughout this period. My first and heartfelt thanks go to my supervisors. Professor Massimiliano Giofrè was the first to introduce me to the world of seismic dynamics and masonry structures. From him I learned the importance of grounding theories in data, avoiding the risk of bending facts to fit preconceived ideas. My sincerest thanks also go to Professor Paulo B. Lourenço, who was always available to discuss the progress of my thesis and who welcomed me into the research group, making possible a truly enriching research experience abroad. I am also grateful to Professor Giacomo Navarra, whose leadership of the L.E.D.A. laboratory and its state-of-the-art shaking table facilities provided the invaluable opportunity to perform seismic tests under multiple load configurations.

The list of people I should acknowledge is endless, but I would particularly like to thank Dr. Chiara Pepi, Professor Vittorio Gusella, and Professor Nicola Cavalagli for their willingness to listen and for their valuable discussions on results and numerical approaches during the most challenging moments of this work.

I am deeply grateful to my colleagues Maria Eleonora Pipistrelli, Pier Francesco Greco, Mattia Schiantella, Andrea Belli, and Mehran Shahpari, who created a supportive work environment and always managed to lighten the mood when needed. I would also like to acknowledge all the wonderful colleagues of the International Doctoral Program in Civil and Environmental Engineering at the University of Perugia, as well as those of the National Doctoral Program in Defense against Natural Disasters and Ecological Transition of the Built Environment at the University of Catania.

Finally, I cannot forget to thank my family and my partner Hayden, whose trust in my abilities never ceases to amaze me, and who have always been there to listen and support me. A special thank you also goes to all my close friends in Umbria and around the world.

“Perfer et obdura! Dolor hic tibi proderit olim”

— Ovidio, *Amores*

Abstract

Masonry is one of the oldest and most widespread construction materials, adopted across civilizations worldwide. Despite its historical relevance, research on the seismic behaviour of masonry structures remains limited, with most investigations focusing on static capacity rather than dynamic response. Among masonry typologies, vaults are particularly underrepresented in literature studies, even though many are located in seismic-prone regions, making their assessment an open research challenge. The aim of this thesis is to investigate the dynamic seismic response of masonry barrel vaults subjected to increasing levels of seismic excitation, combining experimental and numerical approaches. The research follows three complementary directions. On the experimental side, the work builds upon a previous campaign in which two full-scale vault specimens with identical geometry were tested on shaking tables: (i) the Unreinforced Vault (UV), representative of existing structures, and (ii) the Reinforced Vault (RV), built *ex-novo* and strengthened with an innovative hemp-rope-based bio-composite reinforcement. A new campaign was then performed on (iii) the Repaired-Damaged Unreinforced Vault (R-DUV), corresponding to the UV specimen after damage and subsequently repaired using the same reinforcement system. Comparative analyses are carried out to systematically evaluate the seismic performance of unreinforced, reinforced, and repaired specimens. Special attention is given to the onset of damage, failure mechanisms, and both qualitative and quantitative indicators, thus addressing several aspects on masonry vault behaviour. On the numerical side, finite element-based simulations are developed, calibrated, and validated against the experimental results. Particular focus is placed on modelling rocking mechanisms as damage-induced phenomena and capturing their evolution under dynamic input, providing a benchmark for future applications.

This dual experimental–numerical investigation contributes to a deeper understanding of the seismic behaviour of masonry vaults and evaluates the effectiveness of innovative bio-composite reinforcement techniques for structural risk mitigation.

Contents

1	Introduction	1
1.1	Motivations	1
1.2	Research objectives	3
1.3	Thesis outline and main contributions	5
2	A literature review on masonry barrel vaults: historical evolution and experimental investigations	9
2.1	Masonry arches and barrel vaults	10
2.2	Experimental investigation on masonry arches and barrel vaults . .	14
2.2.1	Main reinforcement systems and experimental static investigations	15
2.2.2	Experimental investigation of the seismic response in the dynamic field	20
2.2.3	Conclusive remarks	25
3	A literature review on masonry barrel vaults: analysis methods and numerical investigations	27
3.1	From historical approaches to limit analysis	28
3.1.1	Historical approaches	28
3.1.2	Limit analysis of masonry arches	29
3.2	Numerical methods	32
3.2.1	Finite Element Method	32
3.2.1.1	Micro-modelling	33
3.2.1.2	Macro-modelling	38
3.2.2	Discrete Element Method	42
3.2.3	Discrete Macro Element Method	46
3.2.4	Conclusive remarks	49
4	Experimental campaign test setups	51
4.1	First experimental shaking table test campaign	52
4.1.1	Specimens' definition and constructions	52
4.1.1.1	Unreinforced Vault	54
4.1.1.2	Reinforced Vault	54
4.1.2	Characterization of the bio-composite material	57

4.1.3	Shaking table testing setup definition	61
4.1.3.1	Seismic tests input	63
4.1.3.2	Setup and instrumentation	67
4.2	Second experimental shaking table test campaign	71
4.2.1	Specimen definition and construction	73
4.2.2	Setup and instrumentation	74
5	Experimental campaign test results	79
5.1	Damage and failure identification through visual inspections	80
5.1.1	Methodology	80
5.1.2	Unreinforced Vault	80
5.1.3	Reinforced Vault	82
5.1.4	Repaired-Damaged Unreinforced Vault	84
5.1.5	Key findings	87
5.2	Damage and failure identification through variation of the modal properties	89
5.2.1	Methodology	89
5.2.2	Unreinforced Vault	93
5.2.3	Reinforced Vault	94
5.2.4	Repaired-Damaged Unreinforced Vault	99
5.2.5	Key findings	102
5.3	Rapid identification of sensitive locations as a basis for numerical comparison	106
5.3.1	Methodology	106
5.3.1.1	Relative displacements	107
5.3.1.2	Angular variation indicator	108
5.3.2	Unreinforced Vault	108
5.3.3	Reinforced Vault	113
5.3.3.1	Results at 2.1RE	113
5.3.3.2	Results at 2.2RE	116
5.3.3.3	Results at 2.3RE	118
5.3.4	Repaired-Damaged Unreinforced Vault	127
5.3.4.1	Results at 1.9RE	131
5.3.4.2	Results at 2.0RE	133
5.3.4.3	Results at 2.1RE	138
5.3.4.4	Results at 2.2RE	144
5.4	Conclusive remarks	151
6	Influence of non-linear interface parameters in modelling rock- ing failure mechanisms in curved unreinforced masonry structures	153
6.1	Modelling strategies for rocking mechanism	154
6.1.1	Solution strategies for dynamic problems	154
6.1.2	Material and geometric non-linearities	156

6.1.3	Modelling methodology adopted in this framework	158
6.2	Validation of the methodology though the Unreinforced Vault case study	159
6.2.1	Model description	159
6.2.2	Methodology of validation of the model	162
6.2.3	Model updating of the elastic parameters	163
6.3	Parametric study for strategy validation	166
6.3.1	Results using literature-based non-linear interface parameters . . .	168
6.3.2	Results using literature-based tensile strength and fracture energy of 1×10^{-3} N/mm	175
6.3.3	Results assuming a more brittle behaviour at the target interfaces	181
6.3.4	Results assuming a pre-damaged condition	187
6.3.5	Results considering pre-damaged condition and brittle behaviour of the target interfaces (fracture energy set to 1×10^{-5} N/mm) .	194
6.3.6	Results considering pre-damaged condition and brittle behaviour of the target interfaces (fracture energy set to 1×10^{-6} N/mm) .	199
6.4	Conclusive remarks	205
7	Conclusion	209
7.1	Summary of contributions and findings	210
7.2	Future research directions	213

List of Figures

1.1	Number of publications since 2000 on concrete structure, steel structure and masonry structure detected from SCOPUS	2
1.2	Number of publications since 2000	3
1.3	Number of publications on masonry vaults since 2000	4
2.1	Arch formation: (a) Landscape Arch in Utah (USA) [CNHA, 2025], (b) natural formation process [Great Runs, 2025] and (c) comparison between a corbel and a “true” arch [Standage, 2025]	11
2.2	Arch components	12
2.3	Vault system evolution: (a) introduction of ribbed arches in pozzolanic-concrete vaults [Lancaster, 2015]; (b) structural system description in Hagia Sophia [Mainstone, 1997].	13
2.4	Brick arrangements (adapted from Boni et al. [2022]): (a) radial, (b) transversal, (c) diagonal and (d) herringbone.	14
2.5	Different brick arrangements in vaults [Lancaster, 2015]: (a) vertical and radial bricks, and (b) inclined bricks.	15
2.6	Seismic tests on masonry barrel vaults (adapted from Baratta et al. [2008]): (a) geometrical configuration, (b) collapse of the second specimen	21
2.7	Seismic tests on GFRP reinforced masonry vault (adaped from Taranu et al. [2010]): (a) testing setup conditions, (b) collapse phases	22
2.8	Seismic investigation on unreinforced and retrofitted vault specimen (adapted from Giamundo et al. [2015]): (a) testing setup of the unreinforced specimen, (b) testing setup of the reinforced one. . . .	23
2.9	Seismic investigation on two different specimens (adapted from Ramaglia et al. [2016]): (a) layout of the first specimen, (b) failure mechanism of the first specimen, (c) layout of the second specimen, (d) failure of the second specimen	24
3.1	Limit analysis: (a) graphical formulation of the theorems, (b) comparison of results obtained using limit analysis and other numerical formulations for an arch subjected to a point load at one-quarter of the span [Lourenço, 2002].	30

3.2	Masonry modelling approached (adapted from Lourenço [1996]): (a) masonry apparatus, (b) detailed micro-modelling, (c) simplified micro-modelling and (d) macro-modelling	34
3.3	Analysis of seismic dynamic aspects from several contributions: (a- b) deformed shapes for the unreinforced and reinforced models (adapted from Giamundo [2014]), (c-d) deformed shapes under cyclic quasi- static loads and seismic inputs (adapted from Plavsic et al. [2021]), and (e-f) FE model and corresponding deformed shape (adapted from Marini et al. [2008])	36
3.4	Analysis of different units arrangements from several contributions: (a) deformed shapes under vertical settlements simulations (adapted from Alforno [2021]), and (b) principal stress distributions (adapted from Hejazi and Sadeghi [2022])	37
3.5	Reinforced vault models from previous contributions: (a) general FE model and details (adapted from Castellano et al. [2020]), and (b- c) modelling approach and deformed shapes with highlighted hinges (adapted from Castellano et al. [2023])	39
3.6	Macro-modelling of barrel vaults from previous contributions: (a-b) models used and model of the proposed reinforced system (adapted from Alejo et al. [2021]), and (c-d) mesh and macroelements of the church (adapted from Martínez et al. [2025])	40
3.7	Macro-modelling of barrel vaults from previous contributions: (a) principal strains (adapted from Creazza et al. [2000]), and (b-c) stress distributions (adapted from Bartoli and Betti [2013])	42
3.8	Previous studies on barrel vaults adopting a macro-modelling ap- proach: (a) FE model (adapted from Reccia et al. [2014]), and (b) FE model with different load application points (adapted from Tarhan and Uysal [2023])	43
3.9	Overview of previous studies on barrel vaults and arches using DEM: (a) Failure of arches [Lemos, 2007], (b) examples of some of the DEM models adopted (adapted from Alexakis and Makris [2017]), and (c) stress distribution at failure (adapted from Girardello and Valluzzi [2013])	45
3.10	DE models form literature: (a) comparison between TLA and DEM results (adapted from Fang et al. [2019]), (b) model with and with- out infill (adapted from Coisson et al. [2020]), and (c) different units pattern analysed (adapted from Boni et al. [2022])	47
3.11	DMEM models form literature: (a-b) model and damage pattern (adapted from Caddemi et al. [2019]), and (c) conceptual scheme of discretisation (adapted from Pantò et al. [2021])	48
4.1	Barrel vault specimen's layout and dimensions in meter: (a) general 3-D view, and (b) frontal view	53

4.2	3-D view of the foundation frame and centering system: (a) focus on the centering elements, and (b) details of the steel foundation frame.	55
4.3	Main phases of construction of the UV (a-f)	56
4.4	Construction of the hemp-rope net (a-c)	57
4.5	Main phases of the strengthening application on the RV (a-f)	58
4.6	Hemp-ropes net constituents [Pepi et al., 2024]: (a) hemp rope, (b) from the top down: hemp rope, strand, yarn and fiber, and (c) scheme of the hierarchical structure	59
4.7	Results from Giofrè and Pepi [2024] : (a) schematic representation of the typical mechanical behavior of hemp ropes, (b) stress strain curves obtained from the three points bending tests, and (c) stress strain curves obtained from the uniaxial compression tests	60
4.8	Cocciopesto matrix [Giofrè and Pepi, 2024] : (a) preparation, (b) three point bending test sample, and (c) uniaxial compression test sample	61
4.9	L.E.D.A. laboratory (adapted from Fossetti et al. [2017]): (a) dual system of 4 m by 4 m shaking tables, (b) linked single 10 m by 4 m shaking tables, (c) reaction mass plane view, and (d) transversal section.	62
4.10	Reinforced and Unreinforced Vaults: (a-d) General view on the shaking tables facility,(e) focus on the Unreinforced specimen, and (f) focus on the reinforced specimen	64
4.11	Reference Earthquake: (a) acceleration time histories, (b) displacement time histories, and (c) pseudo-acceleration response spectra . .	66
4.12	Acceleration time histories, intense phases and Arias Intensity of the RE	67
4.13	Testing acquisition Setup #1	69
4.14	Testing acquisition Setup #2	70
4.15	UV and RV specimens on the shaking table: (a) frontal view, (b) focus on the RV specimen, and (c-e) close-ups of the setup schemes	71
4.16	Damaged Unreinforced Vault damaged pattern	73
4.17	Main phase of the strengthening solutions for the R-DUV (a-e)	75
4.18	Acquisition setup for R-DUV tests	76
4.19	R-DUV specimen on the shaking table: (a) frontal view and (b) back view, (c-f) close-ups for the setups schemes	77
5.1	Damage pattern of the UV specimen: (a) schematised view, (b) 3-D frontal view, (c) 3-D lateral view from the left abutment, and (d) 3-D lateral view from the right abutment.	81
5.2	Damage pattern of the RV specimen from section A-A: (a) schematised view, and (b) 3-D view and details of some cracks	83

5.3	Damage pattern of the RV specimen from section B-B: (a) schematised view, (b) 3-D view and details of some cracks	85
5.4	Damage pattern of the R-DUV specimen up to test 2.1RE	86
5.5	Damage pattern evolution during the last seismic test for the R-DUV specimen from section A-A: (a) rocking mechanism around the hinges in the abutments, (b) expulsion of part of the abutment, (c) complete rotation of the specimen, and (d) final damage pattern.	86
5.6	Damage pattern of the R-DUV specimen from section B-B: (a) schematised view, and (b) 3-D view and details of some cracks	88
5.7	Damage pattern comparison among the three specimens, considering the UV as reference: (a) damage pattern in the UV, (b) damage of the RV in similar locations, and (c) damage of the R-DUV in similar locations.	89
5.8	Damage pattern comparison among the strengthened specimens: (a) damage of the RV in similar locations, and (b) damage of the R-DUV in similar locations	90
5.9	Singular values curves assessed for UV: (a) undamaged condition, and (b) after test 0.4RE	93
5.10	First three vibration modes of the UV specimen	94
5.11	UV damage indicators: (a) mode 1, (b) mode 2 and (c) mode 3.	95
5.12	Singular values curves for RV:(a) undamaged condition, (b) after test 0.4RE, (c) after test 1.0RE, (d) after test 2.0RE, (e) after test 2.2RE, and (f) after test 2.3RE	96
5.13	First three mode shapes of the RV specimen	97
5.14	RV damage indicators: (a) mode 1, (b) mode 2, and (c) mode 3	98
5.15	Singular values curves for R-DUV:(a) undamaged condition; (b) after test 0.4RE, (c) after test 1.0RE, (d) after test 1.8RE, (e) after test 2.0RE; (f) after test 2.1RE	100
5.16	First three mode shapes of the RDV specimen	100
5.17	R-DUV damage indicators: (a) mode 1, (b) mode 2, and (c) mode 3	102
5.18	Compatibility assessments of the proposed strengthening through comparison of modal parameters	104
5.19	Comparison of the modal properties between RV and R-DUV	105
5.20	Structural response of the UV specimen during test 0.4RE at 14.2 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.	109
5.21	Structural response of the UV specimen during test 0.4RE at 14.6 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.	111
5.22	Relative displacement time histories of selected monitored locations of the UV specimen during test 0.4RE: (a) Section A-A for reference, (b) location M9, (c) location M12, and (d) location M16.	112

5.23	Structural response of the RV specimen during test 2.1RE at 12.4 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.	114
5.24	Relative displacement time histories of selected monitored locations of the RV specimen during test 2.1RE: (a) Section A-A for reference, (b) location M7, (c) location M9, and (d) location M12.	115
5.25	Structural response of the RV specimen during test 2.2RE at 12.8 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.	116
5.26	Structural response of the RV specimen during test 2.2RE at 14.0 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.	117
5.27	Relative displacement time histories of selected monitored locations of the RV specimen during test 2.2RE: (a) Section A-A for reference, (b) location M12, (c) location M13, and (d) location M16.	119
5.28	Relative displacement time histories of selected monitored locations of the RV specimen during test 2.2RE: (a) Section A-A for reference, (b) location M7, and (c) location M9.	120
5.29	Structural response of the RV specimen during test 2.3RE at 12.3 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.	121
5.30	Structural response of the RV specimen during test 2.3RE at 12.4 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.	122
5.31	Structural response of the RV specimen during test 2.3RE at 12.8 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.	123
5.32	Structural response of the RV specimen during test 2.3RE at 12.9 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.	124
5.33	Structural response of the RV specimen during test 2.3RE at 14.2 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.	125
5.34	Structural response of the RV specimen during test 2.3RE at 19.4 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.	126
5.35	Relative displacement time histories of selected monitored locations of the RV specimen during test 2.2RE: (a) Section A-A for reference, (b) location M5, (c) location M6, and (d) location M7.	128
5.36	Relative displacement time histories of selected monitored locations of the RV specimen during test 2.2RE: (a) Section A-A for reference, (b) location M8, (c) location M12, and (d) location M13.	129

5.37	Relative displacement time histories of selected monitored locations of the RV specimen during test 2.2RE: (a) Section A-A for reference, (b) location M16, (c) location M17, and (d) location M21.	130
5.38	Structural response of the R-DUV specimen during test 1.9RE at 12.8 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern. .	131
5.39	Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 1.9RE: (a) Section A-A for reference, (b) location M44, and (c) location M46	132
5.40	Structural response of the R-DUV specimen during test 2.0RE at 12.8 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern. .	134
5.41	Structural response of the R-DUV specimen during test 2.0RE at 17.4 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern. .	135
5.42	Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 2.0RE: (a) Section A-A for reference, (b) location M16, (c) location M18, and (d) location M30	136
5.43	Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 2.0RE: (a) Section A-A for reference, (b) location M30, (c) location M44, and (d) location M46	137
5.44	Structural response of the R-DUV specimen during test 2.1RE at 13.3 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern. .	138
5.45	Structural response of the R-DUV specimen during test 2.1RE at 14.5 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern. .	140
5.46	Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 2.1RE: (a) Section A-A for reference, (b) location M1, (c) location M14, and (d) location M16	141
5.47	Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 2.1RE: (a) Section A-A for reference, (b) location M18, (c) location M30, and (d) location M32	142
5.48	Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 2.1RE: (a) Section A-A for reference, (b) location M44, (c) location M46, and (d) location M60	143
5.49	Structural response of the R-DUV specimen during test 2.2RE at 12.9 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern. .	144
5.50	Structural response of the R-DUV specimen during test 2.2RE at 13.5 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern. .	146

5.51	Structural response of the R-DUV specimen during test 2.2RE at 14.0 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern. .	147
5.52	Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 2.2RE: (a) Section A-A for reference, (b) location M14, (c) location M16, and (d) location M18	148
5.53	Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 2.2RE: (a) Section A-A for reference, (b) location M28, (c) location M30, and (d) location M32	149
5.54	Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 2.2RE: (a) Section A-A for reference, (b) location M44, (c) location M46, and (d) location M50	150
5.55	Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 2.2RE: (a) Section A-A for reference, (b) location M2, and (c) location M60.	151
6.1	Typical formulations of compressive and tensile behaviours	156
6.2	Typical constitutive laws for tensile materials	157
6.3	Element types in the numerical model [DIANA FEA, 2025]	160
6.4	Interfaces constitutive law in the normal direction: (a) ideal elastic behaviour in compression and linear softening Mode I in tension, and (b) detail of the tensile softening and secant unloading and reloading	161
6.5	Adopted elements to validated the models: target interfaces (C1, C3, C4 and C5), and significant locations	163
6.6	Impact of interface stiffness parameters on the frequencies: (a–b) influence of C_n , (c) influence of C_s	165
6.7	Graphical comparison of the experimental and numerical mode shapes and MAC values: (a) first mode, and (b) second numerical mode and third experimental mode	166
6.8	Calibration of the modal parameters: (a) comparison between numerical and experimental frequency values, and (b) cross matrix of the MAC, noting that the second numerical modes matched the third experimental one.	167
6.9	Analysis for the significant duration of the acceleration time histories	168
6.10	Comparison of the experimental inputs and the numerical adopted ones.	169
6.11	Results using literature-based interface N-L parameters: traction time histories at the extreme points of the target interfaces	170
6.12	Results using literature-based interface N-L parameter: numerical vs theoretical behaviour at the extreme points of the target interfaces	171

6.13	Results using literature-based interface N-L parameters: (a) deformed shape at 5.1 s, and (b) comparison of the RDTHs at the keystone (left panels) and corresponding numerical response (right panels)	172
6.14	Results using literature-based interface N-L parameter: (a) deformed shape at 9.1 s, and (b) comparison of the RDTHs at location M16 (left panels) and corresponding numerical response (right panels) . .	173
6.15	Results using literature-based non-linear interface parameters at 6.7 s: (a) input signals, (b) comparison between numerical and experimental results, and (c) numerical results	174
6.16	Results assuming $f_t = 0.1$ MPa and $G_f^I = 1 \times 10^{-3}$ N/mm: numerical vs theoretical behaviour at the extreme points of the target interfaces	176
6.17	Results assuming $f_t = 0.1$ MPa and $G_f^I = 1 \times 10^{-3}$ N/mm: numerical vs theoretical behaviour at the extreme points of the target interfaces	177
6.18	Results assuming $f_t = 0.1$ MPa and $G_f^I = 1 \times 10^{-3}$ N/mm: (a) deformed shape at 6.7 s, and (b) comparison of RDTHs at the keystone (left panels) and corresponding numerical response (right panels)	178
6.19	Results assuming $f_t = 0.1$ MPa and $G_f^I = 1 \times 10^{-3}$ N/mm: (a) deformed shape at 7.1 s, and (b) comparison of RDTHs at location M16 (left panels) and corresponding numerical response (right panels)	179
6.20	Results assuming $f_t = 0.1$ MPa and $G_f^I = 1 \times 10^{-3}$ N/mm: at 6.7 s: (a) input signals, (b) comparison between numerical and experimental results, and (c) numerical results	180
6.21	Results assuming a more brittle behaviour of the target interfaces: traction time histories at the extreme points of the target interfaces	182
6.22	Results assuming a more brittle behaviour of the target interfaces: numerical vs theoretical behaviour at the extreme points of the target interfaces	183
6.23	Results assuming a more brittle behaviour of the target interfaces: (a) deformed shape at 7.3 s, and (b) comparison of RDTHs at the keystone (left panels) and corresponding numerical response (right panels)	184
6.24	Results assuming a more brittle behaviour of the target interfaces: (a) Deformed shape at 8.9 s; (b) comparison of RDTHs at location M16 (left panels) and corresponding numerical response (right panels)	185
6.25	Results assuming a more brittle behaviour of the target interfaces at 6.7 s: (a) input signals, (b) comparison between numerical and experimental results, and (c) numerical results	186

6.26	Results assuming a pre-existing damaged condition ($ft = 5$ kPa and $G_f^I = 0.01$ N/mm): traction time histories at the extreme points of the target interfaces	189
6.27	Results assuming a pre-existing damaged condition ($ft = 5$ kPa and $G_f^I = 0.01$ N/mm): numerical vs theoretical behaviour at the extreme points of the target interfaces	190
6.28	Results assuming a pre-existing damaged condition ($ft = 5$ kPa and $G_f^I = 0.01$ N/mm): (a) deformed shape at 6.7 s, and (b) comparison of RDTHs at the keystone (left panels) and corresponding numerical response (right panels)	191
6.29	Results assuming a pre-existing damaged condition ($ft = 5$ kPa and $G_f^I = 0.01$ N/mm): (a) deformed shape at 7.1 s, and (b) comparison of RDTHs at location M16 (left panels) and corresponding numerical response (right panels)	192
6.30	Results assuming a pre-existing damaged condition ($ft = 5$ kPa and $G_f^I = 0.01$ N/mm) at 6.7 s: (a) input signals, and (b) comparison between numerical and experimental results, and (c) numerical results	193
6.31	Results considering pre-damaged condition and fracture energy set to 1×10^{-5} N/mm: traction time histories at the extreme points of the target interfaces	195
6.32	Results considering pre-damaged condition and fracture energy set to 1×10^{-5} N/mm: numerical vs theoretical behaviour at the extreme points of the target interfaces	196
6.33	Results considering pre-damaged condition and fracture energy set to 1×10^{-5} N/mm: (a) deformed shape at 7.0 s, and (b) comparison of RDTHs at the keystone (left panels) and corresponding numerical response (right panels)	197
6.34	Results considering pre-damaged condition and fracture energy set to 1×10^{-5} N/mm: (a) deformed shape at 7.2 s, and (b) comparison of RDTHs at location M16 (left panels) and corresponding numerical response (right panels)	198
6.35	Results considering pre-damaged condition and fracture energy set to 1×10^{-5} N/mm at 6.7 s: (a) input signals, and (b) comparison between numerical and experimental results, (c) numerical results	200
6.36	Results considering pre-damaged condition and fracture energy set to 1×10^{-6} N/mm: traction time histories at the extreme points of the target interfaces	201
6.37	Results considering pre-damaged condition and fracture energy set to 1×10^{-6} N/mm: numerical vs theoretical behaviour at the extreme points of the target interfaces	202

6.38	Results considering pre-damaged condition and fracture energy set to 1×10^{-6} N/mm: (a) deformed shape at 7.0 s, and (b) comparison of RDTHs at the keystone (left panels) and corresponding numerical response (right panels)	203
6.39	Results considering pre-damaged condition and fracture energy set to 1×10^{-6} N/mm: (a) deformed shape at 7.2 s, and (b) comparison of RDTHs at location M16 (left panels) and corresponding numerical response (right panels)	204
6.40	Results considering pre-damaged condition and fracture energy set to 1×10^{-6} N/mm at 6.7 s: (a) input signals, and (b) comparison between numerical and experimental results, (c) numerical results	206
6.41	Results considering pre-damaged condition and fracture energy set to 1×10^{-6} N/mm at 5.8 seconds: (a) input signals, and (b) comparison between numerical and experimental results, (c) numerical results	207

List of Tables

4.1	Material mechanical properties	52
4.2	Mean values and CoV from direct tensile tests on hemp ropes . . .	60
4.3	Mean values and CoV from three-point bending and uniaxial compression tests from Gioffrè and Pepi [2024]	61
4.4	Shaking table test sequence of the first experimental campaign . . .	72
4.5	Shaking table test sequence for the R-DUV specimen	78
5.1	Frequency results for the UV specimen	93
5.2	Frequency results for RV specimen	97
5.3	Frequency results for R-DUV specimen	101
5.4	Percent frequency errors with reference to the UV condition	103
5.5	Percent frequency errors with reference to the RV condition	105
6.1	Material properties adopted in the UV numerical model in first instance	162
6.2	Sensitivity analysis for tuning the interface stiffness parameters . . .	165

1. Introduction

This chapter begins by discussing the motivations underlying the research, with particular emphasis on the gaps in the existing literature that form the foundation of the present study. The second part introduces the objectives and the innovative aspects of the work. Finally, the chapter concludes with an outline of the thesis, highlighting its main contributions to the scientific discussion on masonry structures through both experimental and numerical investigations.

1.1. Motivations

Masonry is one of the most widely used construction materials worldwide, owing to its versatility in both geometry and constituent materials. It is a heterogeneous composite formed by the assemblage of units, such as stone, brick, or adobe, typically bonded with mortar layers. Mortar itself can vary significantly in composition, ranging from lime-based to cement-based mixtures and beyond. In addition, the arrangement of the units, the presence or absence of mortar joints, and the shape and size of the blocks contribute to a wide variety of possible configurations [Foraboschi, 2019; Galano and Betti, 2024; Lourenço, 2021]. This variability represents one of the main challenges in developing comprehensive and universally valid conclusions about the structural behaviour of masonry systems.

It is also important to note that, despite the growing interest in masonry-based construction, research on this topic remains relatively limited. In particular, an analysis of the SCOPUS database¹, summarised in Figure 1.1, highlights a marked gap in the number of publications: 210,596 results are available for steel structures, 477,752 for concrete structures, and only 16,208 for masonry

¹Data retrieved on 31 July 2025

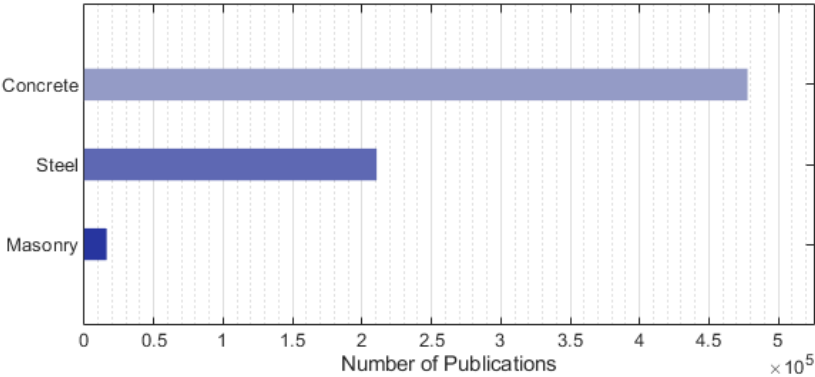


Fig. 1.1: Number of publications since 2000 on concrete structure, steel structure and masonry structure detected from SCOPUS

structures. This discrepancy is particularly striking, especially considering that masonry structures are estimated to represent approximately 50–70% of the global building stock [Marthys and Noland, 1989], with significant differences across countries.

Among masonry structures, arches and vaults represent one of the most historically significant typologies. These elements optimized the use of available materials to achieve large spans, relying primarily on geometry to resist vertical loads through compression. Masonry vaults are key components of built cultural heritage, commonly found in churches, monasteries, and residential buildings. Their widespread presence reflects both cultural identity and socio-economic conditions, as their development was closely tied to material constraints and availability.

Structurally, an arch is a curved, thrust-bearing element that enables wide spans with minimal material consumption, as it primarily carries compressive forces. Notable examples of masonry vaults and arches are found across Europe, including the Pantheon and Domus Aurea in Rome (Italy), the Etruscan Arch in Perugia (Italy), Sainte-Chapelle in Paris (France), Jerónimos Monastery in Lisbon (Portugal), and Hagia Sophia in Istanbul (Turkey).

Although masonry vaults hold significant historical and cultural value, the number of publications dedicated to this structural typology remains limited compared to the broader body of research on masonry structures. This disparity is clearly illustrated by a literature review conducted using the SCOPUS database, which quantified the number of publications on these topics² (Figure 1.2). Both trends show a growing interest over the past twenty-five years; however, a substantial discrepancy persists. For instance, in 2024, more than 1,200 papers addressed masonry structures in general, whereas fewer than 100

²Data accessed on 31 July 2025

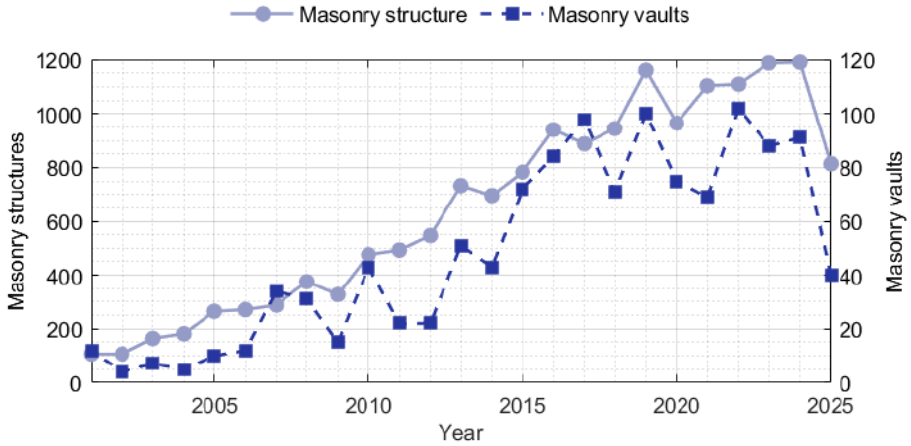


Fig. 1.2: Number of publications since 2000

focused specifically on masonry vaults. This pronounced gap in research output highlights that masonry vaults remain an underexplored topic within the current scientific discourse.

The discrepancy becomes even more pronounced when the focus is narrowed to publications specifically addressing masonry vaults in the seismic field. Figure 1.3 presents the results of a refined search, restricted to studies explicitly focused on masonry vaults, based on the presence of relevant terms in the title, keywords, or introduction. The figure shows these publications as a percentage of the total number of papers on masonry vaults. As previously noted, masonry vaults, like most masonry components, are primarily designed to carry vertical loads, which makes their vulnerability to lateral actions, such as earthquakes, particularly critical. However, only 27% of the publications on masonry vaults include the term *seismic*, despite the well-recognized vulnerability of these elements to such loading conditions [Alforno et al., 2022; Bianchini et al., 2023; Gaetani et al., 2017; Scacco et al., 2021; Yuan and Milani, 2023]. The percentage decreases further when considering only studies that focus on seismic dynamic response: just seventy contributions were published between 2000 and 2025. When restricted to barrel vaults, the number of studies addressing both seismic and dynamic aspects falls to only twelve over the same period.

1.2. Research objectives

In light of the presented background, this thesis aims to explore and address several research gaps and emerging topics. The seismic dynamic response of masonry vaults remains an open field of investigation, particularly in the case of barrel vaults. In recent years, research has increasingly focused on cross vaults (e.g., Alforno et al. [2022]; Bianchini [2023]; Gaetani et al. [2017]; Milani et al. [2016]; Rossi et al. [2016]), while studies on barrel vaults remain limited, and

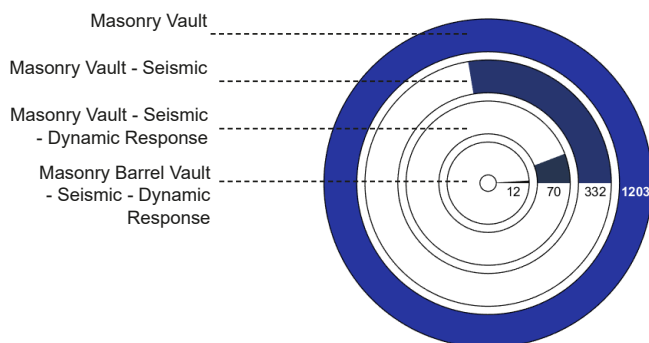


Fig. 1.3: Number of publications on masonry vaults since 2000

even fewer have examined reinforced configurations. Given the intrinsic vulnerability of historic masonry structures to extreme events, the scientific community has shown strong interest in improving their load-bearing capacity, not only to ensure public safety, but also to preserve the cultural and historical value of heritage buildings. Since the establishment of the first international council on architecture in the 20th century, various reinforcement techniques have been proposed and applied. More recently, attention has shifted toward bio-based solutions, which exhibit mechanical properties compatible with masonry, thereby avoiding compatibility issues and offering a wide range of potential applications. This thesis therefore aims to examine this underaddressed topic, which has not been exhaustively investigated through either experimental or numerical approaches. The main focus of the work is to study the seismic behaviour of full-scale specimens subjected to shaking table tests, while simultaneously evaluating the effectiveness of novel strengthening solutions, and to provide significant insights and modelling approaches for the seismic dynamic behaviour of unreinforced masonry barrel vaults.

This thesis builds upon the initial experimental investigation of two full-scale masonry barrel vaults carried out by Gioffré et al. [2023] through shaking table tests. That study examined damage patterns, ultimate load capacity, and variations in vibration frequencies and modes to identify the onset of damage in two twin barrel vaults, one unreinforced and the other reinforced, under progressively increasing seismic excitations. The reinforcement consisted of a novel bio-based strengthening system made of a hemp-rope net embedded in a cocciopesto matrix, the latter composed of an organic binder mixed with finely ground clay bricks. Although this study provided significant insights, the results cannot be considered exhaustive, as several aspects remain unexplored. This work continues the cited contribution by addressing the seismic dynamic behaviour of masonry barrel vaults through a three-directional approach, with

interconnected aims:

- **Experimental campaign:** Building on the unreinforced specimen tested in Gioffré et al. [2023], which sustained damage during seismic tests, a new campaign was designed to assess the effectiveness of the proposed reinforcement system in repairing the previously damaged vault. The specimen was repaired with the same bio-composite system and subjected to a new sequence of shaking table tests following the same loading protocol. Given the novelty of the reinforcement, the campaign investigates its ability to recover dynamic properties, delay the onset of damage, and enhance ultimate load-bearing capacity, while also assessing its effectiveness in a more realistic structural scenario.
- **Comparative analysis:** The datasets from both campaigns enable a comprehensive evaluation of the seismic behaviour of masonry barrel vaults in unreinforced, reinforced, and repaired conditions. This analysis combines qualitative and quantitative approaches. Special attention is given to identifying the onset of damage, describing failure mechanisms, and assessing the vaults' response through the extensive dataset acquired. This discussion aims to clarify critical aspects and contribute to filling current research gaps.
- **Numerical modelling:** Numerical investigations are carried out to develop and validate modelling strategies capable of reproducing the seismic dynamic response of unreinforced masonry structures. The proposed strategy is calibrated and validated against the shaking table test results, thus providing a reliable benchmark case study for the modelling approach.

1.3. Thesis outline and main contributions

The present work is structured into seven chapters, including the introduction and the conclusions. The introductory chapter outlines the motivation for this research, identifies the main open research questions, defines the objectives, and describes both the framework and the principal contributions of this thesis. Chapters 2 and 3 provide an extensive literature review, addressing experimental and numerical investigations on the seismic dynamic behaviour of masonry barrel vaults. Chapter 4 describes the setup of the two experimental campaigns addressed in this work. Chapters 5 and 6 present the results of the experimental and numerical investigations, respectively. Finally, Chapter 7 summarises the conclusions of the study and outlines future research perspectives and potential developments. In detail:

- Introduction (**Chapter 1**) presents the motivations of this work, with particular emphasis on existing gaps in the literature concerning masonry

barrel vaults. The research objectives are defined, highlighting their significance and novelty. The chapter concludes with an outline of the thesis and a summary of the main contributions of each chapter.

- **Chapter 2** provides a literature review of the current state of the art, focusing first on the historical evolution and use of masonry barrel vaults. It highlights the most relevant developments from the Roman to the Renaissance era and introduces the constitutive elements of arches. The second part reviews contributions addressing reinforcement methods and the seismic dynamic behaviour of masonry barrel vaults, particularly those investigated by shaking table tests. The chapter concludes with remarks that set the basis for the experimental campaign presented in this thesis.
- **Chapter 3** reviews the main analysis methods applied to masonry barrel vaults. The first section discusses analytical approaches, starting from purely geometric formulations and progressively incorporating simplified material assumptions, culminating in the three theorems of limit analysis. The second section reviews widely adopted numerical methods; namely the Finite Element Method (FEM), the Discrete Element Method (DEM), and the Discrete Macro-Element Method (DMEM). For each, the basic modelling formulations are outlined, and selected applications from the literature are discussed. The chapter concludes with a synthesis of the main findings.
- **Chapter 4** presents the two experimental campaigns, describing the specimens in detail, their construction phases, and their main differences. The novel hemp-rope-based bio-composite material is also introduced, together with its mechanical properties. Three vault specimens are considered under three conditions: an Unreinforced Vault (UV), a Reinforced Vault (RV), and a Repaired-Damaged Unreinforced Vault (R-DUV), all tested under the same excitation levels.
An in-depth description of the shaking table facilities is provided, along with the selection criteria for the input ground motions. The instrumentation layout used to record the dynamic response is presented, and the main objectives of the two campaigns are outlined.
- **Chapter 5** presents a comparative analysis of the shaking table tests performed on the three vault specimens, examining their seismic performance from different perspectives. The chapter begins by outlining the methodology adopted to assess seismic behaviour, which combines qualitative and quantitative approaches and relies on the extensive dataset collected during the campaigns. A qualitative description of the observed damage evolution is first provided for each specimen, followed by a comparative overview highlighting the main similarities among the three configurations. The quantitative assessment focuses on the variation of modal parameters obtained from Ambient Vibration Tests (AVTs) after significant seismic inputs. This analysis enables tracking the evolution of damage

through changes in vibration frequencies and modes, while also comparing the compatibility of modal properties across the unreinforced, reinforced, and repaired configurations. The seismic response is further investigated through the analysis of deformed shapes, relative displacement patterns, and a proposed angular variation indicator. Together, they allow the identification of critical locations, working as a base for numerical model comparison. Finally, the effectiveness of the bio-composite strengthening system is evaluated in terms of enhanced seismic performance, delayed damage onset, and recovery of dynamic properties, thus providing a comprehensive overview of the strengths and limitations of the different vault configurations.

- **Chapter 6** introduces a modelling strategy to capture the rocking mechanism of unreinforced structures within a FEM framework. In this thesis, a formulation for unreinforced masonry structures subjected to pure rocking, i.e., without sliding or crushing, is proposed. The failure phenomenon is modelled through a simplified two-dimensional micro-modelling approach, where units remain elastic and all inelastic properties are concentrated at zero-thickness interfaces. An implicit time step solution scheme is adopted, as it controls numerical damping and updates the stiffness matrix at each increment. The non-linear interface behaviour, described by a discrete cracking law, governs the response through tensile strength, linear softening, and fracture energy in tension. The formulation is applied to a representative case study: the Unreinforced Vault, which failed predominantly through rocking. A numerical model is developed and calibrated in two stages: (i) validation of the elastic response against experimental vibration modes and frequencies, and (ii) an extensive parametric analysis investigating the role of tensile strength and fracture energy in tension. In this second phase, the model performance is assessed by comparing displacement time histories at two key locations, and by examining hinge formation and global displacements at representative time instants. The results confirm the key role of fracture energy in governing the rocking mechanism, as it controls the rate of strength degradation beyond the tensile threshold and the progression to a zero-strength, zero-stiffness state, enabling hinge formation and crack opening. Tensile strength mainly influences the activation time of rocking. Overall, the findings demonstrate that structural capacity depends not solely on tensile strength, but on the combined contribution of non-linear parameters.
- **Conclusions (Chapter 7)** synthesise the main outcomes of the research, reviewing the key contributions and limitations. The chapter concludes with perspectives for future research, building on the insights gained from both the experimental and numerical investigations.

2. A literature review on masonry barrel vaults: historical evolution and experimental investigations

Barrel vaults represent the simplest evolution of masonry arches, as they can be seen as side-by-side arches [Como, 2016]. In this configuration, the arch is referred to as the directrix, while the generatrix lines are parallel to the springing lines. This straightforward geometry was already known to the Sumerians and Babylonians (5th-4th millennium BCE), and can be found across many civilizations worldwide, as highlighted by numerous scientific contributions. For instance, Martínez et al. [2025] investigated the linear and non-linear behaviour of a church in Mexico, identifying the barrel vaults as the most vulnerable structural component. Similarly, Hejazi and Sadeghi [2022] analysed Persian brick masonry vaults, while Acikgoz et al. [2017] studied the response of masonry barrel vaults under settlement-induced movements, validating the observed crack patterns in the viaduct at London Bridge Station. These recent studies confirm the ongoing global interest in masonry barrel vaults.

This chapter will begin with a brief geometrical and historical overview, proceed with an outline of the main strengthening techniques, and conclude with an in-depth review of the seismic behaviour of masonry barrel vaults, as documented in dynamic studies reported in the literature. It is worth noting that masonry arches and vaults have fascinated generations of researchers. Although the body of work on this topic is extensive, this chapter focuses only on a limited

selection of contributions, emphasizing those most relevant to the scope of this thesis and primarily produced in the last thirty years.

2.1. Masonry arches and barrel vaults

The origin of arched forms can be traced back to natural structures shaped over time by erosion and similar processes (Figure 2.1(a–b)). A comparable principle is observed in damaged masonry structures, which tend to develop arch-shaped failure mechanisms as they seek an equilibrium form capable of self-support. Primitive examples of arches and vaults date back to the third millennium BCE in Egypt and Mesopotamia [Alforno, 2021], where the forms more closely resembled corbel arches rather than true arches. Figure 2.1(c) illustrates the difference between a corbel (false) arch and a true arch. In a corbel arch, stones are arranged in horizontal courses forming small cantilevers that span spaces while reducing material use. For similar reasons, corbel arches and vaults were among the first to be adopted, as they do not require centering and are self-supporting structures. According to Roca et al. [2019], one of the earliest known examples of a corbel vault is found in the Treasury of Atreus (also known as the Tomb of Agamemnon), built in the 13th century BCE in Mycenae, Greece. Examples of this technique are found worldwide, with similar structures documented in Iran (3rd century CE) and in Mexico (6th–7th centuries CE). Before moving on to the historical evolution of these structures, it is necessary to introduce the main components pertaining to arches and vaults, as illustrated in Figure 2.2:

- Intrados: The inner curve or surface of the arch.
- Extrados: The outer curve or surface of the arch, opposite the intrados.
- Span: The horizontal distance between the two supports (abutments) of the arch.
- Springing line: The horizontal surface at the top of the abutments where the arch begins to curve.
- Rise: The vertical distance from the springing line to the highest point of the arch.
- Abutments (Buttresses): Supporting structures on which the arch rests.
- Voussoirs: The individual wedge-shaped stones or blocks forming the arch, each contributing to its shape and structural integrity.
- Keystone: The central, topmost stone of the arch that locks the other voussoirs in place.
- Joint: The separation between consecutive voussoirs.
- Infill: The material placed on the sides or upper parts of the arch.

The Romans significantly advanced arch and vault construction techniques by adopting pozzolan-cement, wedge-shaped stones and bricks, sometimes even pre-shaped in curved forms [Lancaster, 2015]. One of the most remarkable examples of the use of the opus caementicium is the Pantheon, where the

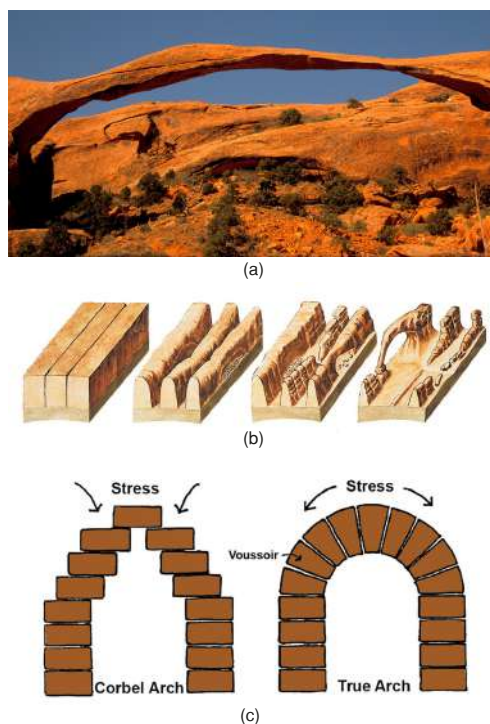


Fig. 2.1: Arch formation: (a) Landscape Arch in Utah (USA) [CNHA, 2025], (b) natural formation process [Great Runs, 2025] and (c) comparison between a corbel and a “true” arch [Standage, 2025]

dome was built using pozzolanic-concrete with a thickness that varies from the oculus to the base to reduce weight and optimise structural efficiency. In the external parts of the dome, several relieving arches are visible, i.e., bricks arranged in circular patterns embedded in the concrete, which help reduce radial thrust, the force exerted by the arch as a result of the applied loads. In other cases, particularly in barrel vault configurations, the low tensile strength of the materials often led to cracking at the keystone, resulting in the formation of a three-hinged mechanism [Alforno, 2021]. To address this, the Romans began incorporating ribbed arches into the structural system, as shown in Figure 2.3(a) [De Matteis et al., 2019; Lancaster, 2015]. Lancaster [2015] dedicated an entire volume to the vaulting systems of the Roman era. While this topic is beyond the scope of the present work, the interested reader is referred to this comprehensive resource.

After the division of the Roman Empire, vault constructions in the Eastern Roman Empire were significantly influenced by Egyptian and Persian traditions [Alforno, 2021]. One innovative solution developed in this context involved counteracting the thrust of an arch or vault by introducing consecutive curved

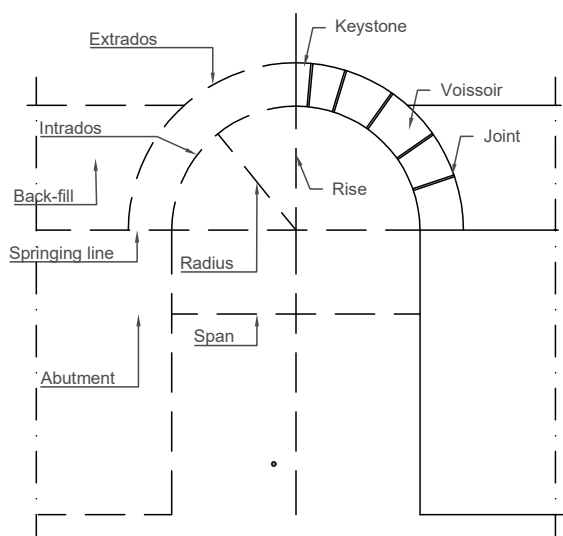
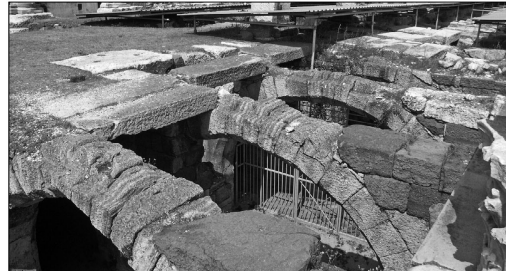


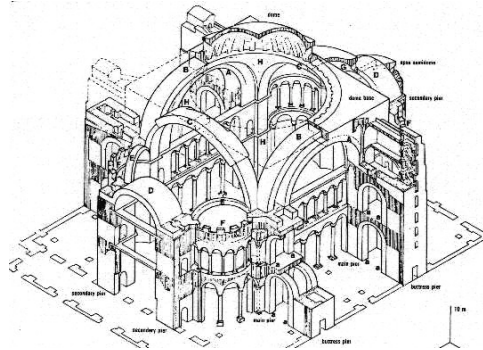
Fig. 2.2: Arch components

structures. A prominent example is the Hagia Sophia (Figure 2.3(b)), where this structural approach was implemented. However, due to the complexity and the scale of the design, the building experienced multiple partial collapses over time.

During the medieval period, particularly between the 10th and 12th centuries, construction primarily employed monolithic barrel vaults and massive load-bearing walls, a choice influenced by the economic downturn of the era. These structures, inspired by Roman architecture, required large quantities of material to span relatively short distances. However, their robust and continuous form provided sufficient ductility to withstand considerable deformations, such as those caused by soil settlements, without leading to collapse [Roca et al., 2019]. A significant architectural shift occurred between the 13th and 14th centuries, moving away from monolithic barrel vaults towards more efficient geometries such as ribbed and cross vaults. These new vault systems relied mainly on support from columns or piers positioned at the four corners and were often combined with flying buttresses. Additionally, increasing the rise and adopting more pointed arches allowed for a reduction in material usage and led to a more balanced distribution of structural forces. During the Renaissance era, there was a revival of classical architectural forms; builders drew inspiration from ancient Roman and Greek traditions, while incorporating lessons learned from Gothic builders. Among these forms, domes gained particular significance in Christian religious architecture. Notable examples include the dome of Santa Maria del Fiore in Florence, Italy, and the later dome of St. Peter's Basilica in Rome. Additionally, during this period, increasing attention was paid to the arrange-



(a)



(b)

Fig. 2.3: Vault system evolution: (a) introduction of ribbed arches in pozzolanic-concrete vaults [Lancaster, 2015]; (b) structural system description in Hagia Sophia [Mainstone, 1997].

ment of masonry units. This subject was thoroughly examined in the doctoral thesis by Alforno [2021], whose principal findings are briefly summarised in the following section.

Masonry barrel vaults were commonly built using one of four primary brick arrangements, whose static response was studied by Boni et al. [2022]:

- Radial or longitudinal: bed joints are aligned parallel to the springing lines (Figure 2.4(a)).
- Transversal: bed joints are perpendicular to the springing lines (Figure 2.4(b)).
- Oblique or diagonal: bed joints are oriented towards the center of the vault (Figure 2.4(c)).
- Herringbone: bed joints are arranged diagonally in alternating directions (Figure 2.4(d)).

The radial arrangement was the most widely adopted construction technique [Alforno, 2021], following the same principles as arch construction. Typically,

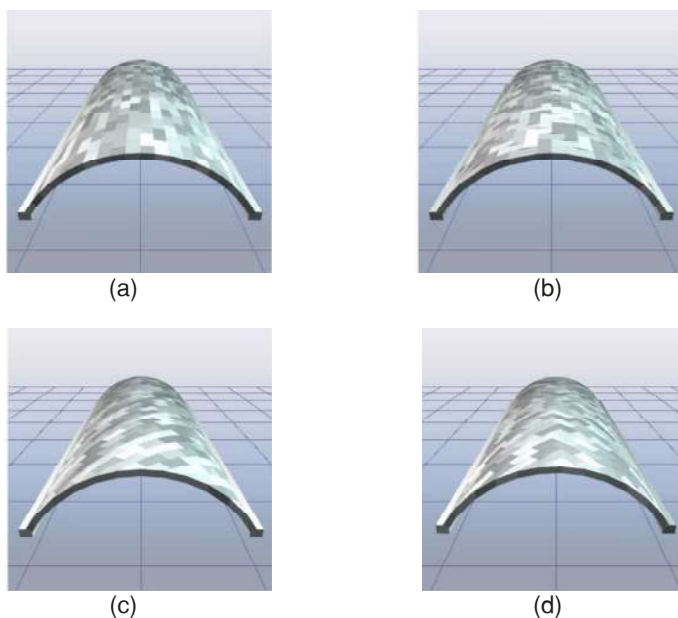


Fig. 2.4: Brick arrangements (adapted from Boni et al. [2022]): (a) radial, (b) transversal, (c) diagonal and (d) herringbone.

a wooden centering was used to support the vault during construction, while bricks or wedge-shaped stones were symmetrically placed relative to the key-stone's location. The keystone was the final element to be placed, locking the entire structure into its final form. In certain cases, to avoid the need for centering, bricks were laid at an inclined angle instead of vertically. This technique allowed the vault to be self-supporting during construction, relying on mortar bonding until the ring was completed. Such configurations are often referred to as pitched transversal vaults [Roca et al., 2019; Lancaster, 2015]. Figure 2.5 illustrates vaults featuring different brick arrangements.

This section thus highlights the widespread use and adoption of masonry barrel vaults across various historical periods, passed down as architectural heritage to the present day. Although this section has mainly presented well-known examples, it is worth mentioning that these structures were also widely adopted in common construction. Indeed, a survey conducted by Galli et al. [2017] following the Umbria-Marche seismic events revealed that a significant number of buildings featured masonry barrel vaults on their first floors.

2.2.Experimental investigation on masonry arches and barrel vaults

Masonry barrel vaults are frequently encountered in built heritage constructions, prompting numerous investigations into their static and dynamic behaviour. Among these, several studies have focused on the effects of differential settle-

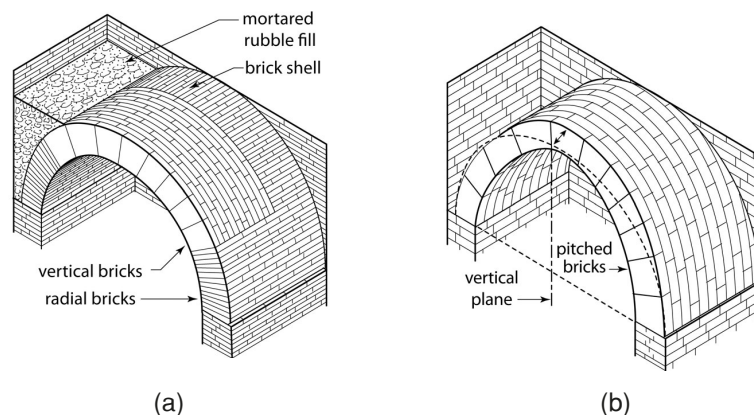


Fig. 2.5: Different brick arrangements in vaults [Lancaster, 2015]: (a) vertical and radial bricks, and (b) inclined bricks.

ments, both vertical [Bove et al., 2019; Castellano et al., 2021; Hudson and Theodossopoulos, 2017] and horizontal [Cardinali et al., 2022; Roselli et al., 2023], which are commonly simulated by introducing movable supports at the edges or corners of the structure. Barrel vaults are particularly sensitive to such displacements, as their stability primarily relies on their geometric configuration. Additionally, other studies have explored the influence of backfill on the structural performance and capacity of barrel vaults. Notably, Krajewski and Hojdys [2015; 2024] investigated how different types of backfill affect the load-bearing capacity of masonry barrel vaults, while Sarhosis et al. [2019] examined an existing structure incorporating backfill.

Since additional numerical investigations will be discussed in Chapter 3, this section focuses specifically on experimental studies aimed at improving the response of masonry arches and barrel vaults, as reported in the literature. The first part addresses the main reinforcement techniques and experimental investigations under static conditions, while the second part concentrates exclusively on experimental campaigns conducted on masonry barrel vaults to assess their seismic dynamic response.

2.2.1. Main reinforcement systems and experimental static investigations

Historical masonry structures are intrinsically vulnerable to extreme events due to the properties of the materials, such as low tensile strength, high density, and a lack of effective connection details. The scientific community has demonstrated significant interest in improving the load-bearing capacity of these structures to ensure public safety while preserving the cultural and historical value and authenticity of heritage buildings.

Over the years, a variety of reinforcement strategies have been investigated,

encompassing both traditional and composite-based methods. The most commonly employed include Fiber Reinforced Polymers (FRP), Textile Reinforced Mortars (TRM) and Fiber Reinforced Cementitious Matrix (FRCM). Concurrently, international organizations dedicated to the protection of built heritage have increasingly been established. For instance, the International Scientific Committee on the Analysis and Restoration of Structures of Architectural Heritage (ISCARSAH), a technical committee of the International Council on Monuments and Sites (ICOMOS), has issued several principles and guidelines for interventions on built heritage. Among these guidelines, though not exhaustive, key principles are that interventions should remain reversible and compatible with the original materials, design principles, construction techniques, and cultural values of the structure [ICOMOS/ISCARSAH, 2024]. These guidelines aim to limit the invasiveness of reinforcement and strengthening applications. As a result, more intrusive approaches, such as reinforced concrete jacketing or the insertion of steel bars, should be carefully considered. However, these traditional conservation approaches were also implicitly promoted by foundational documents such as the Athens Charter (1931) and the Venice Charter [ICOMOS, 1964], which continue to shape conservation philosophy. This legacy has posed significant challenges in shifting the general mindset towards more compatible strengthening solutions.

The following section presents a concise overview of the most common traditional reinforcement methods in masonry arches and vaults, emphasising their limitations.

- Increase of buttress sections: This method typically involves enlarging the buttresses from top to bottom to maintain the thrust line within the masonry geometry. A prominent example of this technique is Stern's intervention at the Colosseum, which effectively "froze" several arches in time. However, this approach significantly alters the original design principles and has a substantial visual impact.
- Addition of steel ties: Steel ties are typically inserted at the level of the springing line to counteract the horizontal component of the thrust. While effective, this intervention can have a significant visual impact and may obstruct the view, particularly when the springing line is at a lower height. Moreover, steel is known to be incompatible with masonry in terms of mechanical, thermal, and long-term behaviour, which introduces additional challenges.
- Reinforced concrete jacketing: This technique involves creating a secondary reinforced concrete layer at the extrados of the vault. However, it presents several drawbacks that severely affect the structural behaviour of heritage masonry.

Collectively, these strengthening methods substantially alter the structural identity of curved structures by increasing mass and stiffness, which in turn modifies the overall system behaviour. Furthermore, differences in thermal, mechanical, and rheological properties between original and new materials frequently give rise to compatibility issues.

Consequently, several innovative reinforcement systems have been investigated. For example, numerous arch and vault structures have been studied applying FRP (e.g. Di Primio et al. [2021]; Tarhan and Uysal [2023]; Witzany et al. [2020]) or TRM/FRCM techniques (e.g., Boem and Gattesco [2021]; Bove et al. [2019]; Gattesco and Boem [2019]; Sadeghi et al. [2021]). Below, a brief review is provided of notable case studies identified by Zampieri et al. [2018] for each of these reinforcement approaches.

FRP applications have been actively studied for their properties, including low weight, high strength, minimal thickness, and easy adaptation to different surfaces [Babatunde, 2017; Bertolesi et al., 2021; Reboul et al., 2023; Scacco et al., 2021]. These materials are composed of fibers embedded in an epoxy matrix, where the fiber is the main load-carrying component and the matrix helps transfer stress among the elements. Fibers are usually made of Glass, Carbon, or Aramid, thus referred to as GFRP, CFRP, and AFRP, respectively. Significant improvements have been achieved by applying FRP to curved masonry structures, which can withstand peak loads several times higher than their unreinforced counterparts and reach higher ultimate displacements prior to failure.

Valluzzi et al. [2001] investigated the response of masonry barrel vaults subjected to a point load applied at one-quarter of the span and reinforced with either CFRP or GFRP strips. The specimens reinforced discontinuously at the extrados with CFRP strips exhibited significant deformation, failing due to the sliding of bricks along the mortar joints. Similar behaviour was observed for specimens reinforced with discontinuous GFRP strips at the extrados. Conversely, reinforcement with continuous GFRP strips applied at the intrados effectively prevented collapse. This study highlights the critical influence of reinforcement location, extrados versus intrados, on the structural performance of masonry vaults. Building upon these findings, Oliveira et al. [2010a] experimentally evaluated the load capacity and displacement response up to failure of twelve masonry segmental arches scaled at 1:2. Two specimens were tested in unreinforced condition as baseline references. The tests employed displacement-controlled loading applied at one-quarter of the arch span. The unreinforced arches failed without developing a full collapse mechanism, exhibiting a four-hinge failure pattern. Using these results as benchmarks, several reinforcement techniques were investigated and compared in terms of load capacity enhancement. Arches reinforced with continuous GFRP strips at the intrados exhibited a damage pattern characterised by the formation of three hinges and partial debonding of the strips, including the detachment of a thin brick layer. Al-

though the initial stiffness remained largely unaffected, the load capacity significantly increased by approximately 165% compared to unreinforced specimens, alongside notable changes in displacement response. In contrast, specimens reinforced with continuous GFRP strips at the extrados failed by forming one hinge beneath the applied load and two additional hinges at the supports, achieving an 89% increase in load capacity relative to unreinforced arches. Finally, the originally unreinforced specimens, after experiencing damage, were locally reinforced at the hinge locations and subsequently retested. These specimens developed a new four-hinge failure mechanism in different locations than before, while also demonstrating a further increase in peak load capacity.

Although these materials have demonstrated high performance, their use in heritage structures is often limited due to the presence of epoxy resins, which compromise the reversibility of the intervention. At the same time, there is growing interest in more compatible reinforcement systems, such as Fiber-Reinforced Mortars (FRM) or Fiber-Reinforced Cementitious Matrices (FRCM), which consist of a fiber grid embedded between two layers of mortar. In the composite, the matrix protects the fibers and is responsible for transferring stress from the reinforcement to the substrate [D'Antino et al., 2020; Garcia-Ramonda et al., 2022]. Gattesco and Boem [2019] investigated the effectiveness of FRM with glass fiber mesh applied either at the intrados or extrados of masonry barrel vaults. The experimental study was conducted on a full-scale masonry vault with reduced depth, tested under quasi-static conditions. Loads were applied in eight different positions to simulate the inertial forces generated during earthquakes. The unreinforced specimen developed the expected four-hinge mechanism. In contrast, in the reinforced specimen, the formation of the fourth hinge was inhibited by the tensile strength provided by the reinforcement. The study specifically focused on the connection between the reinforcement and the substrate, particularly in the springer area, which could lead to premature failure due to sliding or uplift of the cracked zone, thereby highlighting the need for further investigation in this area. Carozzi et al. [2018] compared the response of in-situ masonry arches and vaults reinforced with Steel Reinforced Grout (SRG), TRM, and FRP systems. The SRG system consisted of two mortar layers separated by a steel textile, the TRM used a glass fiber mesh, and the FRP consisted of a carbon fiber textile applied using epoxy resin. The specimens were tested using a concentrated load applied at one-quarter of their spans. The load was introduced through a series of loading and unloading cycles. The unreinforced specimens consistently exhibited the formation of four alternating hinges. The FRP reinforced vault showed a crack distribution similar to that observed by Oliveira et al. [2010a], with the first hinge forming directly beneath the load application and detachment of the reinforcement occurring near the springing line, reaching a peak load four times higher than that of the unreinforced counterpart. The TRM reinforced specimen exhibited its first crack at the load application point, a second near three-quarters of the span, and two

additional cracks at the abutments. These led to detachment of the reinforcement and rupture of the fibers, although the vault reached a maximum load 5.5 times higher than the unreinforced specimen. The SRG reinforcement was applied to a tested arch, which achieved a peak load four times higher than that of the unreinforced specimen. However, it also resulted in a significant increase in the stiffness of the system.

More recently, as a result of growing global awareness regarding environmental preservation, the use of bio-based fiber applications has gained increasing attention, owing to their low cost and favorable mechanical properties [Barbhuiya et al., 2025; Elfaleh et al., 2023; Pipistrelli et al., 2025]. In particular, Barbhuiya et al. [2025] provides a comparative analysis of various properties of bio-based materials relative to conventional construction materials such as steel and concrete. The density of these materials ranges from very high for steel, to moderate for concrete, and significantly lower for natural fibers. Steel is well known for its high tensile strength, which makes it suitable for reinforcement applications, whereas concrete exhibits low tensile resistance. Natural fibers, on the other hand, exhibit moderate to high tensile strength depending on the species. Among the various types investigated, applications tend to favor plant-based fibers, which offer several opportunities: fibers can be added to the mortar mix to improve the flexural capacity, textile elements can be used in FRM or FRCM and lastly, it is also possible to embed ropes in several matrices [Abbass et al., 2024; de Carvalho Bello et al., 2019; Greco et al., 2024; Illampas et al., 2023; Majumder et al., 2025; Pipistrelli et al., 2025]. It is worth pointing out that such natural fibers may exhibit ageing and long-term effect issues, which several authors have attempted to address (e.g., Azwa et al. [2013]; Bui et al. [2021]; Giofrè and Pepi [2024]; Zuccarello et al. [2023]), although these aspects remain an active topic of investigation.

Bio-based reinforcement systems have been tested on masonry elements in several studies [Davino et al., 2025; Menna et al., 2015; de Carvalho Bello et al., 2021]. However, their application to curved masonry structures remains extremely limited. In this regard, Giofrè et al. [2021] reported preliminary experimental tests on masonry arches, both unreinforced and reinforced with a novel hemp-rope-based biocomposite. In this system, the hemp rope was fully embedded in a cocciopesto matrix, i.e., a mix of finely ground clay bricks and an organic resin. The load-bearing capacity was investigated by means of a vertical point load applied at the keystone using a hydraulic jack, while the seismic response was assessed through tilt tests, where the base was incrementally inclined to simulate an equivalent monotonically increasing horizontal static force, replicating the effects of ground motion acceleration. The reinforced specimens exhibited a significant improvement, achieving up to a tenfold increase in load capacity compared to their unreinforced counterparts under vertically increasing loading, and reaching significantly higher displacement levels. Tilt test results further confirmed the effectiveness of the reinforcement, as the strengthened

structures did not reach collapse when subjected to the same collapse multiplier as the unreinforced specimens.

2.2.2. Experimental investigation of the seismic response in the dynamic field

While the contributions cited in the previous section have significantly advanced the understanding of barrel vault behaviour under various conditions, they provide limited insight into the seismic dynamic performance of these systems. Notably, most of the referenced studies have employed simplified loading schemes, with only a few addressing the full dynamic response of masonry structures. The following section reviews the available studies that specifically focus on this topic.

Baratta et al. [2008] investigated the seismic response of a barrel vault based on tuff masonry. The vault had a depth of 100 cm, an inner span of 200 cm, and a radius of 100 cm, forming a segmental arch resting on two masonry walls measuring 50 cm by 70 cm in height (see Figure 2.6(a)). The specimen was provided with an overload and was tested on a 6 Degrees of Freedom (DoFs) shaking table, with dimensions of 4 m by 4 m, and frequency range varying from 0 to 50 Hz. The first specimen, with a constant overload configuration, was tested in two phases. In the first phase, the input was based on a ground motion recorded during the Campania–Lucania earthquake (23 November 1980), scaled by increasing factors ranging the Peak Ground Acceleration (PGA) from 0.3 g to 1.5 g to simulate progressively increasing seismic excitations. Thereafter, a vertical input with a PGA of 1.5 g was added to the unidirectional horizontal motion, until a horizontal PGA of 1.8 g was reached, at which point collapse occurred. Since failure was reached only at exceptionally high PGA levels, which is uncommon for unreinforced masonry structures, the authors suggest that filtering applied to the seismic input prior to testing may have significantly influenced its effective frequency range. The second masonry specimen was instead subjected to an increasing level of seismic excitation sequence, added to a significant variation of the overload configuration. This investigation was carried out in three phases: a first phase with an overload of approximately 8 kN, and the specimen was subjected to two levels of excitation reaching PGAs of 0.1 g and 0.2 g. In the second phase, the overload was increased to 14 kN, matching that of the first specimen, and the same levels of excitation (0.1 g and 0.2 g) were applied. This phase was interrupted due to the detachment of voussoirs from the intrados. After this occurrence and subsequent to the restoration of the structure, the third phase began, with seismic excitation increased up to 0.7 g, which led to collapse. This study emphasized the uncertainties inherent in the dynamic behaviour of masonry barrel vaults and highlighted the influence of varying overloads, which, in real applications, may result from the collapse of adjacent structural elements. It is noteworthy that although both specimens were identically constructed and made of the same materials, they failed at dif-

ferent PGA levels. The authors attributed this discrepancy to variations in crack development between the specimens, underscoring the inherent unpredictability of such structures under seismic dynamic loading.

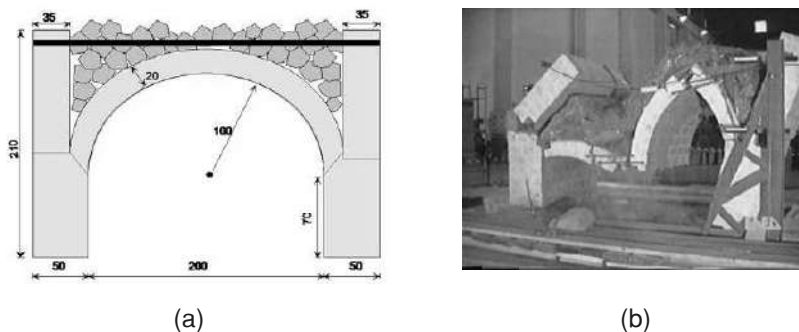


Fig. 2.6: Seismic tests on masonry barrel vaults (adapted from Baratta et al. [2008]): (a) geometrical configuration, (b) collapse of the second specimen

Taranu et al. [2010] conducted an experimental investigation on a 1:3 scaled masonry vault built atop supporting walls. The specimen had an inner span of approximately 140 cm, wall height of 80 cm, and a vault rise of 37 cm. The vault was constructed using clay bricks and mortar joints. It was strengthened with GFRP strips, each 20 cm wide and spaced at 25 cm intervals. The specimen was tested on a shaking table facility operating within a frequency range of 0.5–50 Hz. It was subjected to tri-axial input, simulating real earthquake conditions by including all seismic components. Three seismic inputs were applied: the El Centro earthquake (USA, 1940), with accelerations ranging from 0.2 g to 0.5 g, and the Vrancea earthquake (Romania, 1986), with accelerations from 0.1 g to 0.4 g. After these two seismic tests, the structure exhibited minimal damage. A sine-sweep excitation was subsequently applied, maintaining the PGA at 0.5 g while varying the frequency content to induce resonance phenomena. This test led to the formation of five cracks, with the most prominent ones characterised by sliding along the mortar bed joints at the impost level (see Figure 2.7). This study highlights how the modal properties of the structure, such as vibration frequencies and mode shapes, can significantly influence the response when the seismic input contains frequency components close to those of the investigated structure.

Giamundo et al. [2015] tested a full-scale unreinforced masonry barrel vault with dimensions of 298 cm, 114 cm and 220 cm for span, rise and depth of the vault respectively. The in-plane geometry was based on a segmental arch profile. The vault was constructed using solid clay bricks and a pozzolanic-based mortar. The unreinforced specimen underwent a sequence of shaking table tests



Fig. 2.7: Seismic tests on GFRP reinforced masonry vault (adaped from Taranu et al. [2010]): (a) testing setup conditions, (b) collapse phases

using an artificial seismic input with a frequency content of 0–30 Hz and a PGA of 0.46 g, applied in the horizontal direction, as shown in Figure 2.8(a). The seismic sequence was performed at progressively increasing intensities, starting with 10% of the selected seismic input and incrementally reaching 100% of its full intensity. At this stage, the vault exhibited an opening-closing pattern of multiple cracks, symmetrically distributed around the keystone, indicating the onset of a hinge mechanism. The specimen was then retrofitted by first sealing the cracks and applying an Inorganic Matrix composite Grid (IMG) system, which consisted of a mortar layer, followed by the placement of a basalt fiber grid, and a second mortar layer. The reinforcement was applied at the extrados, covering the entire surface of the vault. The strengthened specimen was then subjected to the same seismic input sequence as the unreinforced counterpart, up to 100% intensity of the original input, and did not exhibit any visually detectable damage, confirming the effectiveness of the proposed intervention. Subsequently, the vault was tested up to 275% of the original seismic intensity. At this level, an opening-closing hinge mechanism emerged from the intrados but was effectively restrained by the extrados reinforcement.

Further investigations were carried out by Giamundo et al. [2016] on an unreinforced masonry barrel vault with the same geometry as the specimen previously described. The structure was tested on a shaking table under two different seismic inputs. The first input was derived from a natural ground motion recorded during the Irpinia earthquake on November 23, 1980, with a PGA of 0.18 g. The input was scaled from 25% to 150% of the original intensity and had a dominant frequency content up to 15 Hz. A second seismic input was adopted, consisting of an artificial accelerograms with frequency content up to 30 Hz and a PGA of 0.46 g. Additionally, random excitations were employed to extract the modal properties of the vault following selected seismic tests. The structural response was evaluated through visual inspection and by analysing the dynamic

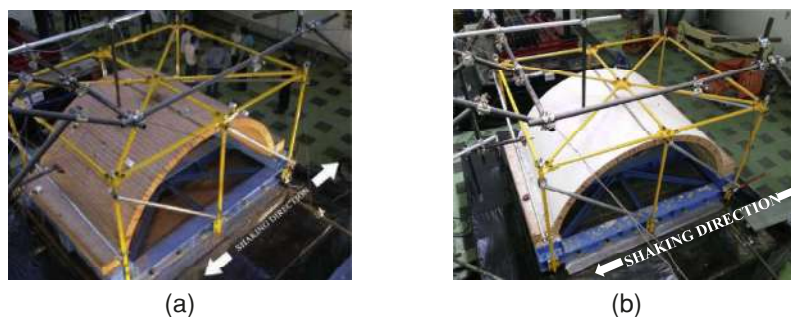


Fig. 2.8: Seismic investigation on unreinforced and retrofitted vault specimen (adapted from Giamundo et al. [2015]): (a) testing setup of the unreinforced specimen, (b) testing setup of the reinforced one.

amplification and peak acceleration values recorded at various locations on the specimen. Under the Irpinia-based input, the vault exhibited no detectable damage and showed consistent results between dynamic amplification factors and peak response accelerations across monitoring points. Subsequent seismic tests led to the formation of cracks at the extrados, followed by the development of additional cracks indicative of the hinge mechanism. However, full collapse was not reached during this stage. Final failure occurred when the vault was excited with 100% of the artificial intensity input. The vault was then retrofitted using the same IMG technique described in Giamundo et al. [2015], and retested with the artificial seismic input. Testing continued until a PGA of 1.2 g was reached, at which point the experiment was stopped to avoid collapse, as the opening–closing crack mechanism was clearly evident. This study not only confirmed the effectiveness of the proposed reinforcement technique, but also provided valuable insights into the variation of modal parameters before and after retrofitting. It highlighted that the modal properties were only marginally affected, thus demonstrating the compatibility of the intervention in terms of stiffness and mass distribution between the unreinforced and strengthened configurations.

In the study published by Ramaglia et al. [2016], the response of two masonry barrel vaults under unidirectional seismic input is compared. The specimens differed in geometry and structural configuration: the first consisted of a full-scale, single-brick-layer masonry barrel vault, while the second included a vault constructed atop two masonry walls, raising the impost line by approximately 1 m from the base (see Figure 2.9(a-b) respectively, for reference). The first specimen was subjected to increasing seismic input levels based on an artificial accelerogram, reaching near-collapse at a PGA of 0.48 g. The second specimen underwent a sequence of tests using incremental scaled natural ground motion,

with collapse occurring at a PGA of 0.22 g. Both specimens exhibited a typical hinge-mechanism failure, a collapse mode that is difficult to reproduce in static tests due to the prevalence of sliding. In the first specimen, which consisted solely of a masonry vault with reduced abutment dimensions, cracks developed along the arch-shaped profile (Figure 2.9(b)). In the second specimen, cracks also extended into the abutments (Figure 2.9(d)).

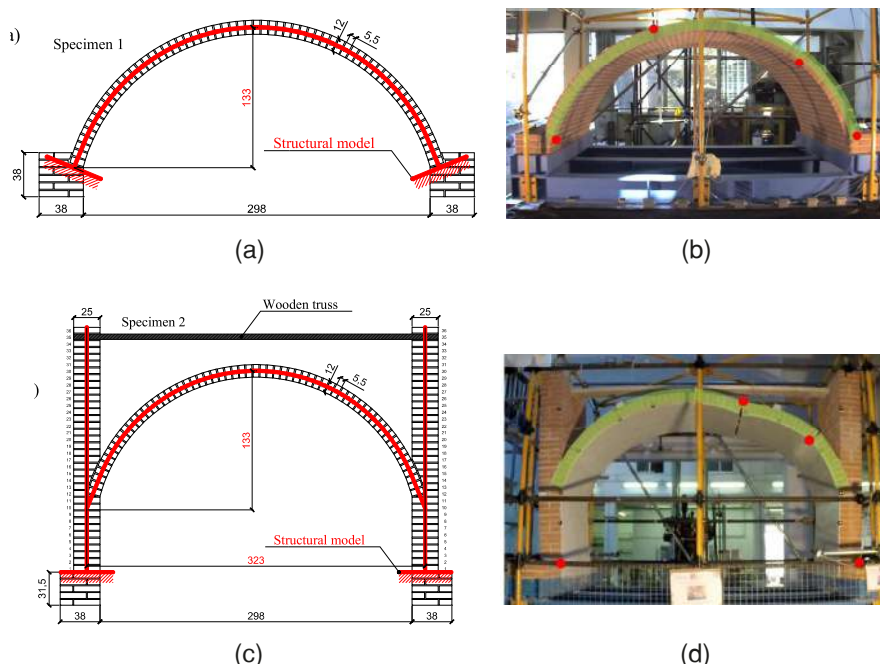


Fig. 2.9: Seismic investigation on two different specimens (adapted from Ramaglia et al. [2016]): (a) layout of the first specimen, (b) failure mechanism of the first specimen, (c) layout of the second specimen, (d) failure of the second specimen

Sequentially, Ramaglia et al. [2017] reported tests on the same vault with abutments discussed above, strengthened using a TRM system equivalent to the previously discussed IMG technique. Additional retrofitting measures included backfilling, the construction of a masonry rib, and the installation of a stainless steel tie placed atop the rib. The retrofitted specimen was retested using the same increasing amplitude sequence applied to the unreinforced configuration, reaching up to 250% of the Irpinia-based input. Subsequently, another seismic input was applied, leading to collapse at a PGA of 0.52 g. In addition to comparing seismic performance, damage onset was evaluated through the analysis of frequency variations and the monitoring of acceleration responses at various instrumented locations, giving significant insights on the masonry unreinforced and reinforced behaviour.

2.2.3. Conclusive remarks

All the experimental studies reviewed to date have significantly contributed to the understanding of the behaviour masonry barrel vaults and arches. They have also demonstrated the effectiveness of various reinforcement techniques when applied to curved structural elements.

The key findings from this section can be summarised as follows:

- Most studies focused on full-scale specimens of masonry barrel vaults and arches, with scaled models used in only a limited number of cases, predominantly for arches.
- The barrel vaults reviewed are generally built with radial brick arrangements, and the arch profiles are typically segmental.
- The load is typically applied at significant locations along the arch geometry, e.g., at the keystone, at one-quarter of the span, or at the base using shaking table tests.
- The behaviour of unreinforced specimens is systematically compared with that of reinforced counterparts.
- Unreinforced masonry elements consistently develop hinge-based failure mechanisms, which, in some cases, result in complete collapse.
- The seismic dynamic behaviour of vaults has mainly been studied under unidirectional input using both natural and artificial ground motions on shaking table platforms.
- Several reinforcement systems have been investigated, including FRP, FRCM, and natural FRCM, applied at both the intrados and extrados, significantly enhancing the peak-load capacity.
- All reinforcement strategies tested on shaking tables were applied exclusively at the extrados of the vaults, demonstrating their effectiveness in enhancing seismic capacity and delaying the onset of damage.
- The seismic response is typically assessed through visual inspection to identify damage, as well as by analysing changes in dynamic properties such as frequency and acceleration during sequential seismic tests of increasing intensity.

Building on current state of the art, both regarding the potential of reinforcement systems and the limited knowledge of seismic dynamic behaviour, several investigations have been conducted by the research group led by Gioffré et al. [2023]. In this research, the seismic behaviour of a full-scale vault reinforced at the extrados with a novel hemp-rope net fully embedded in a cocchiopesto matrix was investigated, highlighting enhanced performance compared to its unreinforced twin. The two vault specimens were tested simultaneously on the shaking table facilities of the Laboratory of Earthquake engineering and Dynamic Analysis (L.E.D.A.) at the University of Enna Kore (Italy), under increasing levels of seismic excitation. The reinforced vault withstood seismic loads up to six times higher than the unreinforced specimen before collapsing.

The study also examined damage both qualitatively, through visual inspection and quantitatively, by introducing damage indicators based on variations in vibration properties, which were monitored through significant seismic tests. The results provided valuable insights into the behaviour of both unreinforced and reinforced specimens, confirming the effectiveness of the reinforcement in enhancing seismic capacity. Moreover, the intervention was deemed compatible, as it did not significantly alter the modal properties of the structure in undamaged conditions. This work forms the foundation of the present thesis; for this reason, a more detailed description of the experimental campaign will be provided in the Chapter 4.

3. A literature review on masonry barrel vaults: analysis methods and numerical investigations

The form of arches and vaults has fascinated engineers and builders throughout all epochs. The optimal way of building, developed through accumulated experience and learning from past failures, was traditionally transmitted across generations, often in oral form.

The need to assess the structural capacity and performance of vulnerable masonry structures, particularly barrel vaults and arches, has become increasingly relevant in the current era. This growing interest arises from multiple factors. On one hand, there is a continuing effort to safeguard historical constructions, motivated not only by their cultural and historical significance but also by their economic value. On the other hand, the safety of people who visit or work in such structures must be ensured under both normal service conditions and extreme loading scenarios. This chapter provides a concise review of the analytical methods developed for masonry arches, starting from the earliest approaches based solely on geometry and progressing to plastic theory applied to arches and vaults. In the second part, numerical methods are presented, with emphasis on fundamental formulations and selected contributions from the literature that are most relevant to the scope of this work. Finally, the conclusive remarks summarise the key findings of this chapter, leading to the choice of the most suitable method to be adopted for the purposes of this thesis, which will be

further discussed in Chapter 6.

3.1.From historical approaches to limit analysis

Until the end of the 18th century, building rules for the construction of arches and vaults were based on simple geometrical definitions, likely developed through a process of trial and error, without a defined quantitative approach for assessing the stability of arches. These empirical methods gave rise to the so-called “rules of thumb,” which formed the basis for construction practice before the introduction of analytical and numerical models. It is important to note that these rules, which ignored material strength and considered only static conditions, have nonetheless proven to be reliable [Di Pasquale, 1996]. Comprehensive discussions on the historical development of structural analysis and design methods for masonry arches and vaults are available in Como [2016]; Heyman [1982]; Huerta [2001; 2008]; Di Pasquale [1996], and a brief overview is provided below.

3.1.1.Historical approaches

In his ten books on architecture, Vitruvius (1st century BCE) acknowledged the thrusting action of arches, although he still relied on geometric rules to describe their stability [Buttarazzi, 2013]. No formal written treatises appeared again until the Renaissance and Baroque periods, as the empirical “rules of thumb” were primarily transmitted orally during the Gothic period (12th–16th centuries) [Gaetani, 2020; Roca et al., 2019]. Among the most notable rules from the Gothic period were those later attributed to Blondel and Fr. Derand (17th century), which prescribed that the width of the abutment should equal one-third of the intrados length. Hernán Ruiz the Younger (16th century), in his notebook, correlated the width of the abutment to the thickness of the arch, representing a first simplified attempt to account for the weight of the structure [Gaetani, 2020; Roca et al., 2019].

In his treatise *De Re Aedificatoria*, Leon Battista Alberti reviewed the construction principles known up to the 15th century and established proportional relationships among the height, thickness, and abutment width of arched bridges [Buttarazzi, 2013; Roca et al., 2019]. An important exception of the period is represented by the studies of Leonardo da Vinci on arch behavior. He can be considered the first to approach arch stability and strength as a problem influenced not solely by geometric dimensions. Notably, he defined the arch as “*nothing but a strength caused by two weaknesses*” and stated, “*If the arch, in its attempt to break, exerts a force of 200 libra, provide an opposing force or weight of 200 libra, and it will hold well*”, offering one of the earliest scientific insights into arch behavior. For more details on da Vinci’s reflections on arch mechanics, the reader is referred to Buttarazzi [2013].

During the 17th century, new approaches for analyzing the form and stability of arches emerged: (i) methods based on internal force distribution, and (ii) kinematic approaches addressing the mechanisms of arch failure.

Hooke's catenary principle represented the first scientific contribution to understanding arch behaviour in terms of internal force distribution, expressed through an analogy: as hangs the flexible cable, so inverted will stand the arch [Como, 2016; Huerta, 2008; Roca et al., 2010]. Building on this, Gregory in 1690 refined the concept by stating that an arch, regardless of its shape, can be considered stable only if an inverted catenary, also known as an anti-funicular curve, is fully contained within its thickness.

In the same period, Philippe de La Hire investigated arch stability from a kinematic perspective, providing a scientific foundation for the empirical rules of thumb. He validated these traditional rules for the specific case of a semi-circular arch, macro-modelling the structure as a rigid block system in which cracks form at a 45° angle from the springing line. This crack pattern results in three rigid segments, assumed to interact through sliding between voussoirs, thus neglecting friction, and allowing rotation only at the boundaries. Couplet (1729–1730) refined this theory by linking arch stability to the maximum number of hinges that can form before the structure becomes a mechanism, with the voussoirs rotating around hinges positioned at either the intrados or the extrados. His observations were based on three assumptions that remain the foundation of modern limit analysis [Gaetani, 2020; Roca et al., 2019]: (1) sliding between voussoirs is possible but not predominant, (2) the material has zero tensile strength, and (3) the compressive strength is considered infinite. In 1773, Coulomb introduced the method of maxima and minima, representing an early application of the lower and upper-bound theorems [Buzzetti et al., 2024]. According to his approach, collapse in arches is caused by relative rotation between rigid blocks, with cracks potentially forming at any joint, while sliding between blocks is possible but generally prevented by frictional forces.

3.1.2. Limit analysis of masonry arches

Limit or plastic analysis was first applied to masonry structures, particularly arches, by Heyman [1982], although earlier applications to masonry were provided by Kooharian [1952]; Prager [1959] [Huerta, 2008; Como, 2016]. Limit analysis, as introduced by professor Heyman was carried out under certain assumptions, listed below as initially introduced:

- "Sliding failure cannot occur"
- "Masonry has no tensile strength"
- "Masonry has an infinite compressive strength"

These assumptions are generally conservative, as sliding failures are unlikely to occur. The hypothesis of infinite friction is more restrictive than reality, since typical friction coefficients for masonry range from 0.60 to 0.75 [Roca et al., 2019]. A similar reasoning applies to tensile strength, as masonry typically has low tensile capacity. Failure generally occurs at unit–mortar interfaces, where tensile strength ranges from 0 MPa for dry joints to approximately 0.2 MPa in

historical mortar joints. The third assumption could be considered unsafe; however, due to the overabundance and redundancy of material, masonry structures are typically stressed to values two orders of magnitude below their compressive strength [Huerta, 2008; Roca et al., 2019].

According to these three main assumption, three theorems were formulated [Roca et al., 2010]:

1. Lower bound theorem (also known as safe or static theorem) : the structure is considered safe, i.e., collapse will not occur, if a statically admissible state of equilibrium can be found;
2. Upper bound theorem (also known as kinematic theorem): collapse will occur if a kinematically admissible mechanism can be found for which the work produced by the external forces is null or positive ;
3. Uniqueness theorem: if both a statically and a kinematically admissible mechanism can be found, a condition of analysis is reached

A graphical representation of the limit analysis theorems is shown in Figure 3.1(a). The collapse load for a structure is indicated as F_u , while any load solution obtained using the lower bound theorem (1), denoted F_{sa} , is smaller than the collapse load and is therefore considered safe; conversely, any load exceeding F_u , denoted F_{ka} , is obtained using the upper bound theorem.

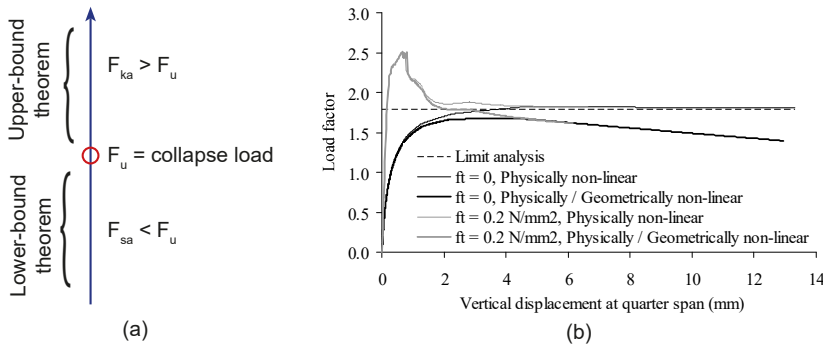


Fig. 3.1: Limit analysis: (a) graphical formulation of the theorems, (b) comparison of results obtained using limit analysis and other numerical formulations for an arch subjected to a point load at one-quarter of the span [Lourenço, 2002].

The hypotheses of theorem 1 are implicitly satisfied if the thrust line, i.e., the geometrical locus of points where sectional forces act, constituted only by compressive stresses [Roca et al., 2019], lies entirely within the geometry of the structure. Since an arch is a three time indeterminate structure, multiple solutions can be obtained by varying the three unknowns, typically defined as the direction, point of application, and magnitude of the external force.

This theorem is mainly solved using the graphical static approach for two-dimensional structures, but it was later extended to vaulted structures through the slicing technique, which involves analysing elementary arches within the geometry of vaults and domes. This approach allowed the analysis of complex geometries using a simple tool, particularly when studying domes or cross vaults. A comprehensive discussion can be found in Huerta [2001; 2008], to which the reader is referred.

Theorem 2 is based on the identification of a collapse mechanism, which in the case of arches requires a minimum of four hinges. The mechanism is analysed through the determination of instantaneous centers of rotation and the calculation of the corresponding displacements. Given the forces and the associated displacements, the virtual work equation can then be formulated, allowing the determination of the load.

Finally, Theorem 3 requires that a kinematic mechanism satisfies both the upper and lower-bound theorems. If this condition is verified, the resulting load corresponds exactly to the true collapse load.

The effectiveness of this tool has been demonstrated in several publications. A clear understanding is provided by Figure 3.1(b), which shows the results of analyses performed by Lourenço [2002] of an arch under a point load applied at one-quarter of the span. It is worth noting that when the tensile strength of the material is neglected, the load capacity obtained from limit analysis using the kinematic approach coincides with that derived from Finite Element Method simulations with the same assumptions. However, when geometric non-linearities are considered, a significant discrepancy arises because limit analysis cannot account for the load history. Despite this limitation, limit analysis remains the subject of ongoing research, with numerous contributions focusing on masonry structures, e.g. see the publications by D'Altri et al. [2020]; Galassi et al. [2018]; Oliveira et al. [2010b]; Portioli et al. [2014]; Schiantella et al. [2024]. Modern approaches are generally formulated on the basis of either the kinematic theorem or the static theorem. Methods implementing the upper-bound theorem incorporate more realistic material properties and were initially developed using rigid-block formulations to assess both in-plane and out-of-plane collapse loads. In the latter case, simplified formulations have been proposed to estimate collapse loads under seismic actions. For more details, the reader is referred to Gilbert and Melbourne [1994]; Gilbert et al. [2006]; Orduña and Lourenço [2003; 2005a;b]; Roca et al. [2010]. Furthermore, Milani [2008] proposed a six-node curved element to model masonry vaults within a simplified Finite Element (FE) environment. The material was represented through a homogeneous approach, and kinematic limit analysis was subsequently applied. A subsequent approach was developed by Chiozzi et al. [2017] and Grillanda et al. [2019], in which the mesh is composed of rigid bodies whose geometry is defined by a parametric surface. This model reduces computational effort and increases accuracy by combining homogenization with a limited number of

mesh elements [Bianchini, 2023].

On the other hand, an optimization of the safe theorem for masonry vaults and domes in a three-dimensional environment was proposed by O'Dwyer [1999] and further developed by Block and Ochsendorf [2007] and Block [2009]. This approach, introduced as the Thrust Network Method (TNM), generates surfaces defined exclusively by compression forces, thereby determining the corresponding thrust lines. The initial formulation was limited to equilibrium states under vertical loads but was later extended to account for horizontal actions in the work of Marmo and Rosati [2017].

As previously mentioned, despite the limitations of the limit analysis method, plastic analysis offers a reliable framework for assessing the ultimate load capacity of masonry structures. Moreover, it remains computationally efficient, making it a valuable tool for both research and practical applications.

3.2.Numerical methods

The following sections aim to present and review the numerical methods most commonly applied to the analysis of masonry structures, with particular attention to masonry barrel vaults. Specifically, the discussion will address the Finite Element Method, the Discrete Element Method, and the Discrete Macro-Element Method, highlighting their potential and limitations. These aspects will be examined through a review of selected contributions from the literature, mainly developed over the past 20–30 years. The studies considered in this section primarily focus on unreinforced and reinforced specimens, with an emphasis on seismic capacity assessment and the modelling strategies that are most relevant to the scope of this work. This review does not aim to be exhaustive, but rather to provide a targeted synthesis aligned with the objectives of the thesis. Finally, it should be noted that the term barrel vault is sometimes concealed in the literature under alternative denominations, particularly in studies on arch bridges, which can essentially be regarded as barrel vaults (e.g., Pelà et al. [2009]; Reccia et al. [2014]; Tecchio et al. [2023]; Pantò et al. [2024]).

3.2.1.Finite Element Method

The Finite Element Method (FEM) provides a wide range of possibilities for modelling masonry, ensuring accurate geometrical representation and multiple levels of detail in describing the material. Unlike limit analysis, FEM not only evaluates load capacity but also captures the evolution of the damage state by incorporating constitutive laws that describe the stress–strain response at every point and throughout the entire analysis process [Bianchini, 2023; Gaetani, 2020; Roca et al., 2010]. Masonry can be represented through several modelling strategies, each offering different levels of detail and accuracy. A masonry panel, composed of units and joints, can mainly be described according to two modelling categories (see Figure 3.2):

- Micro-modelling: In this approach, all the constituent elements of the

masonry assembly are explicitly represented under more or less realistic assumptions. This technique allows for detailed monitoring of the structural response and can be further divided into two variants:

- Detailed micro-modelling
- Simplified micro-modelling

The main difference between these two approaches lies in the representation of mortar joints and mortar–brick interfaces. In the simplified case, they are modelled as a single zero-thickness interface, with the units expanded to compensate for the absence of joint thickness. In both variants, the units and mortar joints are represented as continuum elements, with elastic properties (and inelastic ones if required) assigned accordingly. The interfaces, instead, primarily govern cracking and sliding phenomena. This modelling strategy is typically adopted in research applications, where detailed results are required. Simplified micro-modelling, however, may also be applied to entire buildings by introducing an elementary equivalent cell that represents the assemblage of multiple masonry units.

- Macro-modelling: In this approach, the masonry assembly is represented as a continuum by means of homogenization techniques, and a constitutive law is uniformly assigned to the entire structure. The material is modelled as anisotropic, and the constitutive law may be derived either phenomenologically or from micro-mechanical considerations. In the phenomenological case, masonry properties are obtained from tests on sufficiently large specimens subjected to homogeneous stress–strain states. In the micro-mechanical case, the material parameters are computed as average stress–strain responses through mathematical formulations that account for the properties of individual components and their geometry. This macro-modelling approach is particularly suitable for professional practice, as it requires less computational effort and fewer input parameters.

For further details on this topic, the reader is referred to Lourenço and Gaetani [2022].

As expected, each of these approaches entails different levels of accuracy and computational cost. Their reliability is strongly dependent on the availability of experimental data, which are required not only to calibrate material properties but also to identify the failure mechanisms governing the overall structural response.

3.2.1.1. Micro-modelling

The following section presents a review of recent contributions that adopt a micro-modelling approach. The discussion first considers unreinforced specimens, with particular focus on their seismic response investigated through non-

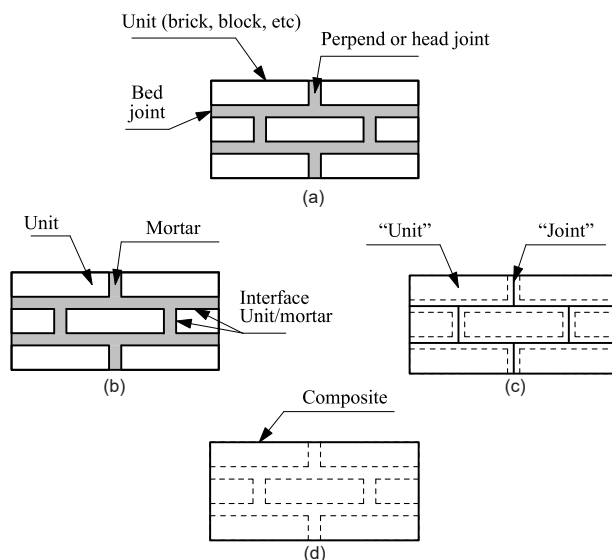


Fig. 3.2: Masonry modelling approached (adapted from Lourenço [1996]): (a) masonry apparatus, (b) detailed micro-modelling, (c) simplified micro-modelling and (d) macro-modelling

linear dynamic analysis. Subsequently, studies aimed at improving the understanding of masonry barrel vault behaviour are reviewed, addressing aspects such as different load configurations, the effects of unit arrangement, and other influencing factors. Finally, contributions that investigate the effectiveness of reinforcement systems are discussed.

To the author's knowledge, only two scientific studies have been published on the seismic dynamic behaviour of masonry vaults using micro-modelling approaches, which are briefly reviewed below.

In his Ph.D. thesis, Giamundo [2014] modelled a full-scale masonry barrel vault using a detailed micro-modelling strategy. The modelling was intended to validate the FE model against experimental results obtained from shaking-table seismic tests, previously reviewed in Section 2.2.2. For the unreinforced specimens, all non-linearities were concentrated at the interfaces, which were governed by a Mohr–Coulomb plasticity model. The retrofitted specimen was analysed with a micro-approach where the mortar matrix and reinforcing grid were represented separately using elastic truss elements. The mortar matrix was assumed to exhibit ideal plasticity in compression. In contrast, the basalt grid was modelled with brittle tensile failure and no compressive capacity, described through a combined Rankine/Von Mises yield criterion. The numerical models were used to perform various analyses aimed at validating the FE strategy and

reproducing the experimental shaking-table results. Modal analyses provided the vault's fundamental modal properties, including frequencies and participating masses in both horizontal and vertical directions. Static non-linear analyses were validated against load–displacement curves, deformed shapes, crack patterns, and stress contour maps (see Figure 3.3(a-b)). For the retrofitted vault, reinforcement at the extrados effectively prevented crack opening in that region, confining the damage to the intrados. Dynamic non-linear analyses further validated the numerical models against shaking-table tests. The results produced force–displacement hysteresis cycles and maximum acceleration profiles, showing good agreement with experiments for undamaged vaults. For damaged vaults, however, the analyses highlighted the strong influence of damping parameters. This finding underlined the need to update the models with appropriate damping ratios to capture the progressive damage observed under increasing seismic input levels.

In a subsequent study, Plavsic et al. [2021] simulated the behaviour of unreinforced masonry vaults under cyclic quasi-static and dynamic earthquake loading. Their objective was to model these structures accurately to assess their non-linear response and ultimate strength, with particular emphasis on seismic vulnerability. The authors adopted a simplified 3-D micro–modelling approach in which masonry units were represented as expanded blocks. The interfaces were defined as zero-thickness elements with non-linear properties governed by a damage–plasticity constitutive law capable of capturing tensile, compressive, and shear failure. Cyclic quasi-static tests were simulated by applying an acceleration time history at a master node, successfully reproducing the formation of four hinges and the associated rocking mechanism of three distinct blocks (see Figure 3.3(c)). The model was then subjected to seismic simulations replicating the 1980 Irpinia earthquake in Italy, with both horizontal components applied to the 3-D structure. Three seismic scenarios were analysed by scaling the earthquake amplitude. In all cases, a four-hinge mechanism developed, with crack locations consistent with those observed under quasi-static cyclic loading. At higher intensity levels, an additional fifth hinge was also detected (Figure 3.3(d)).

Marini et al. [2008] carried out an experimental and numerical study to investigate the seismic behaviour of barrel-vaulted structures, with particular focus on their rocking mechanisms under horizontal loading. In the experimental campaign, a horizontal load was applied at the springing level. Different numerical models were implemented to simulate the structural response. The non-linear behaviour of the vaults was modelled using interface elements positioned between units, along the vault–backfill interface, between backfill and abutments, and at the abutment bases, adopting a simplified micro-modelling approach (Figure 3.3(e)). The models accounted for dead loads, vertical point loads at the abutment tops, and horizontal point loads applied at the springing level. Numerical results showed good agreement with experimental data in terms of

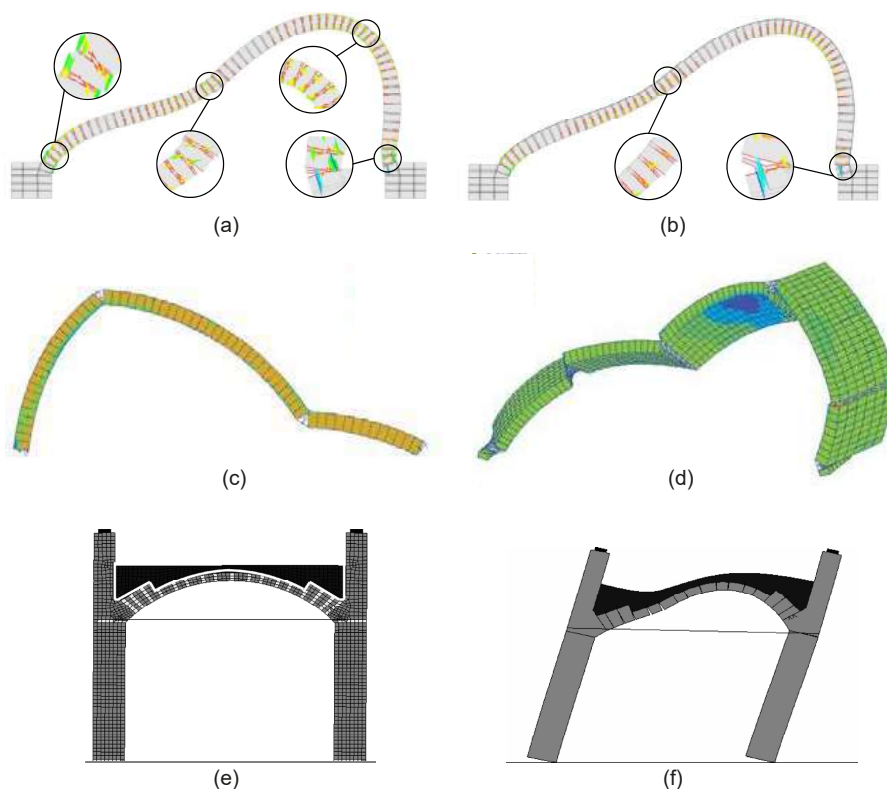


Fig. 3.3: Analysis of seismic dynamic aspects from several contributions: (a-b) deformed shapes for the unreinforced and reinforced models (adapted from Giamundo [2014]), (c-d) deformed shapes under cyclic quasi-static loads and seismic inputs (adapted from Plavsic et al. [2021]), and (e-f) FE model and corresponding deformed shape (adapted from Marini et al. [2008])

crack patterns (see Figure 3.3(f)), load–displacement curves, and vertical displacements along the vault crown. This comparison validated the accuracy of the implemented numerical models.

Other studies have also adopted a micro-modelling approach to examine the response of masonry barrel vaults under different load configurations and to investigate the influence of unit arrangement.

In his Ph.D. thesis, Alforno [2021] adopted a simplified micro-modelling approach to investigate the arrangement of bricks in a masonry barrel vault while maintaining the same overall geometry. The study highlighted the effectiveness of different brick arrangements under various load scenarios, including self-weight, abutment settlements, and both vertical and horizontal shear failure mechanisms at the boundaries. The analysis primarily consisted of a paramet-

ric study of brick arrangements, mainly. radial, diagonal, and vertical patterns (see Figure 3.4(a)). The material parameters for the study were selected based on previous literature contributions. Overall, the results demonstrated that no single brick pattern performed best across all load scenarios.

Hejazi and Sadeghi [2022] subsequently carried out a similar investigation on masonry barrel vaults, comparing Roman and Persian brick arrangements using a detailed micro-modelling approach. The interfaces between brick units and mortar were explicitly modelled using contact and target elements, with mechanical properties drawn from the literature. The Willam–Warnke failure criterion was adopted to describe the inelastic behaviour of the components. The geometrical models included three arch profiles prevalent in Persian architecture and two primary brick arrangements were considered: Roman, with bricks laid normal to the arch plane, and Persian, with bricks laid in annular courses perpendicular to the arch length (see Figure 3.4(b)). The investigation considered vaults of varying depths and arch profiles. The results highlighted how different combinations of arch shapes and brick arrangements are better suited to specific load patterns.

Other studies have focused on the micro-modelling not only of the masonry apparatus but also of reinforcement systems, which were typically applied at the extrados.

Castellano et al. [2020] modelled a masonry barrel vault including all constituent parts and FRCM strengthening at the extrados (see Figure 3.5(a)). The fibers of the reinforcement system were idealised as equivalent elasto-damaging trusses,

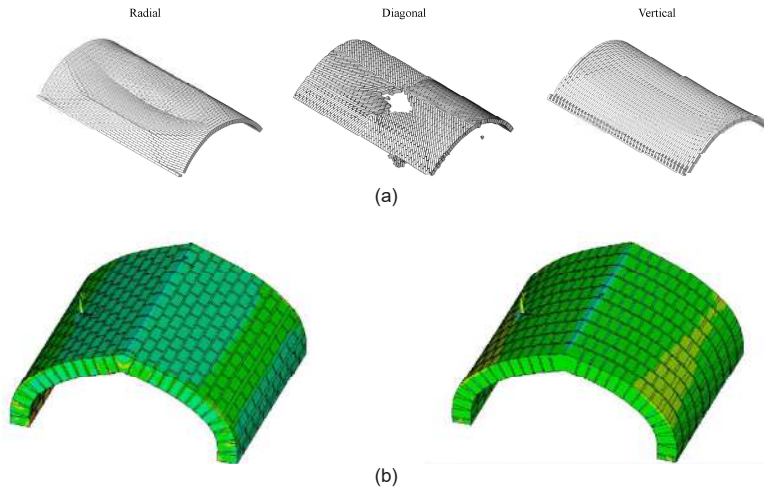


Fig. 3.4: Analysis of different units arrangements from several contributions: (a) deformed shapes under vertical settlements simulations (adapted from Alforno [2021]), and (b) principal stress distributions (adapted from Hejazi and Sadeghi [2022])

exhibiting brittle behaviour with negligible compressive strength. A Concrete Damage Plasticity (CDP) model was assigned to the stones, mortar joints, and cementitious matrix. This formulation incorporates stiffness degradation governed by damage variables ranging from 0 (undamaged) to 1 (complete stiffness loss), with distinct behaviours in tension (exponential softening) and compression (parabolic softening). To reduce computational effort, only a 100 mm portion of the vault's thickness was modelled. A non-linear analysis was performed under an incremental load applied at approximately one-third of the span. The model successfully reproduced the global stiffness of the system, although the simulated damage pattern did not fully match the experimental observations used for validation.

Futhermore, Castellano et al. [2023] proposed a method to model masonry barrel vaults reinforced at the extrados. The masonry apparatus was represented using a simplified micro-modelling approach, while the reinforcement system was modelled as a continuum with embedded bars (Figure 3.5(b)). The masonry units were represented using a Total Strain Rotating Crack model, while the zero-thickness interfaces followed a discrete cracking law. The reinforcement matrix was also governed by a Total Strain Rotating Crack model, with the reinforcement–substrate interfaces again described using the discrete cracking model. The numerical analyses successfully reproduced the global behaviour of the reinforced vault, accurately capturing the collapse load and displacement at peak load. The model also simulated the formation of hinges and the subsequent progressive debonding and cracking patterns, demonstrating its capability to represent stress-transfer mechanisms within the reinforced structure (see Figure 3.5(c)). Minor discrepancies in the results of the unreinforced vault were observed. The model was also employed for pushover analyses to assess seismic capacity, which demonstrated a significantly increased horizontal load multiplier for the reinforced vault compared to the unreinforced one.

3.2.1.2. Macro-modelling

Similarly to the previous section, the following review focuses on studies investigating the behaviour of masonry barrel vaults using a homogeneous approach. Contributions addressing seismic capacity are presented first, followed by studies examining other load configurations and additional aspects of interest for the aim of this work.

Alejo et al. [2021] investigated the seismic behaviour of a church in Moreira, Mexico, focusing on its nave barrel vault and evaluating a proposed strengthening technique for this structural element. Several numerical models were developed to study the vault, ranging from a 2-D sectional model to a comprehensive 3-D model of the entire church, each with a different level of detail (Figure 3.6(a)). Non-linear pushover analyses were performed on all models, revealing that the lowest load capacity of the church occurred when analysing the vault. A strengthening technique was then proposed for the nave's barrel

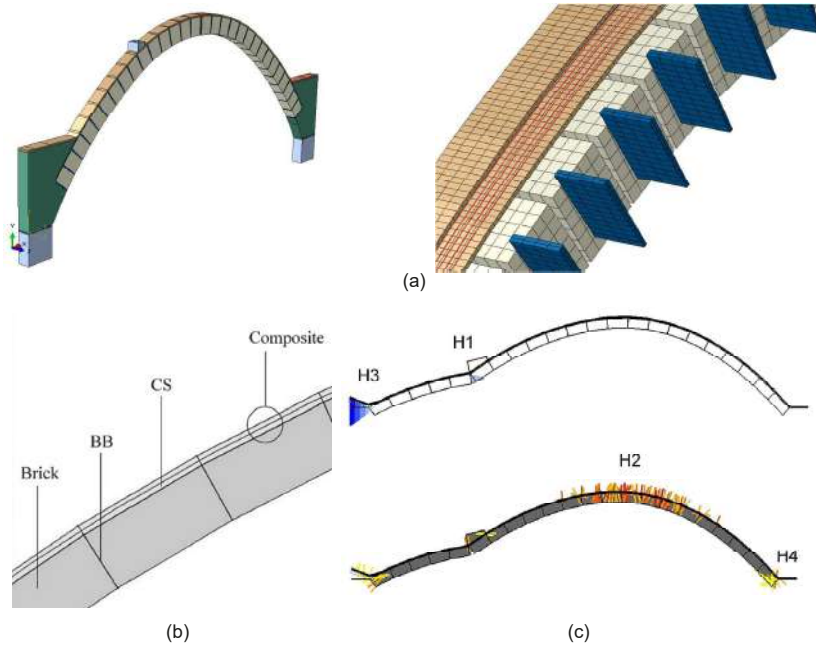


Fig. 3.5: Reinforced vault models from previous contributions: (a) general FE model and details (adapted from Castellano et al. [2020]), and (b-c) modelling approach and deformed shapes with highlighted hinges (adapted from Castellano et al. [2023])

vault, consisting of a masonry stiffening element at the extrados and a carbon-FRCM layer applied on top, modelled with a perfect bond (Figure 3.6(b)). This intervention significantly increased the horizontal load capacity of the vault. In the unstrengthened configuration, hinges formed in four locations, while in the strengthened vault, hinges appeared at the same locations, but the damage distribution shifted toward the connection between the lateral stiffener and the vault.

Martínez et al. [2025] continued the analysis of the previously studied church to evaluate the expected linear and non-linear dynamic behaviour under three-component seismic excitations. The complete church model (Figure 3.6(c)) was subdivided into six independent macro elements to facilitate calibration of dynamic properties and allow more detailed analysis. One of these macro elements corresponded to a barrel vault, selected based on zones typically most seismically vulnerable in churches of this type (Figure 3.6(d)). The CDP model was employed to represent the compressive and tensile behaviour of masonry. Model calibration was performed using vibration frequencies and mode shapes obtained from an experimental campaign. For seismic loading, four accelerograms from the 2017 Mexico City earthquake, two accelerograms from other

recorded ground motion events, and two synthetic accelerograms were used to broaden the seismic demand scenarios. The analysis results confirmed existing crack patterns in the barrel vault, particularly along the keystone and its connection with the bell tower. A longitudinal crack along the intrados of the barrel vault was also observed.

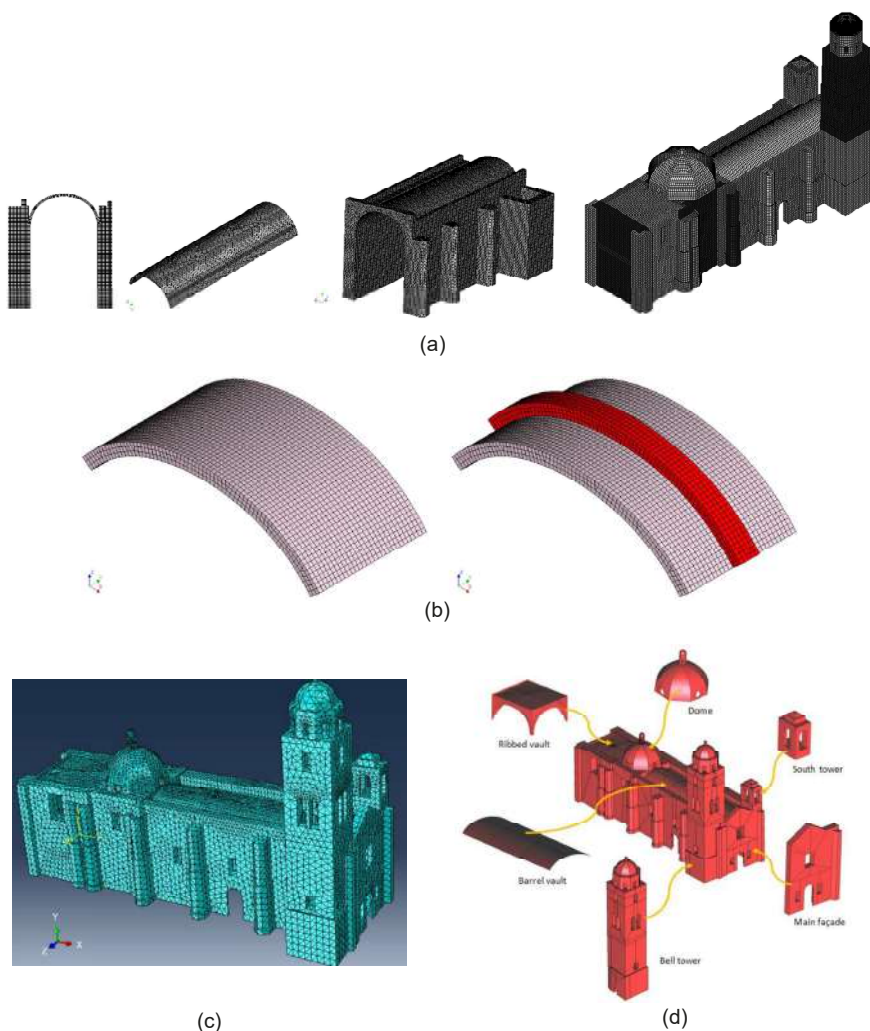


Fig. 3.6: Macro-modelling of barrel vaults from previous contributions: (a-b) models used and model of the proposed reinforced system (adapted from Alejo et al. [2021]), and (c-d) mesh and macroelements of the church (adapted from Martínez et al. [2025])

Subsequently, several studies addressed the behaviour of masonry barrel vaults under self-weight or service loads, also focusing on real-case applications. Creazza et al. [2000] proposed a numerical model using a homogenization ap-

proach to describe both the damage patterns and the ultimate load capacity of a barrel vault subjected to self-weight and a progressively applied load near one-quarter of the span. The model employed a two-parameter damage formulation, treating the composite material (mortar and brick) as an equivalent isotropic, homogeneous continuum. This damage formulation accounted damage for both tensile and compressive stresses. Experimental results confirmed the formation of four hinges, which the 3-D numerical model successfully replicated (Figure 3.7(a)).

Bartoli and Betti [2013] conducted a detailed study to investigate the sudden collapse of the keystone of a barrel vault in the Cappella dei Principi, Florence, Italy. The vault, tambour, and main structural elements were modelled in both 2-D and 3-D using a macro-modelling approach (see Figure 3.7(b-c)), with material properties derived from in-situ testing. Both linear and non-linear analyses were carried out to validate the numerical models and identify structural problems leading to the local collapse. The results showed that vertical loads and the relative stiffness of structural elements, rather than the dome's horizontal thrust, were the primary causes of tensile stresses and subsequent keystone failure.

Reccia et al. [2014] adopted a homogenization approach based on a Representative Elementary Volume (REV) to model the Venice trans-lagoon railway bridge, which featured a masonry barrel vault (Figure 3.8(a)). The vault was represented as an orthotropic homogenized material exhibiting softening behavior, derived from a 3-D finite element discretization of the REV. The latter was defined based on the actual arrangement of bricks and governed by a Mohr–Coulomb failure criterion. Numerical analyses of the railway bridge provided insight into its non-linear structural response under various loading conditions, mainly train loads and pier settlements. Additionally, the analyses investigated the contributions of backfill and cohesion to the overall structural response through pushover simulations.

Coisson et al. [2020] investigated a pavilion barrel vault at the Pilotta Palace, Parma, Italy. The vault geometry was accurately represented through 2-D-beam finite elements, with dead and variable loads applied as concentrated forces at centroids. The CDP model was adopted to describe the mechanical behaviour of the masonry. Analyses were conducted in two main phases: first, the vault was modelled with constraints at the springings to compare directly with classical analytical estimates of maximum compressive stress and horizontal thrust, assuming a three-hinge mechanism; second, the model incorporated supporting walls and metal tie-rods represented by beam elements. Collapse mechanisms observed in the FEM analyses were typically driven by compressive failure of the masonry, while the tie-rods remained within the elastic range throughout the simulations.

Lastly, Tarhan and Uysal [2023] investigated the effect of optimal reinforcement arrangements on masonry vaults through a FEM-based study. Numerical

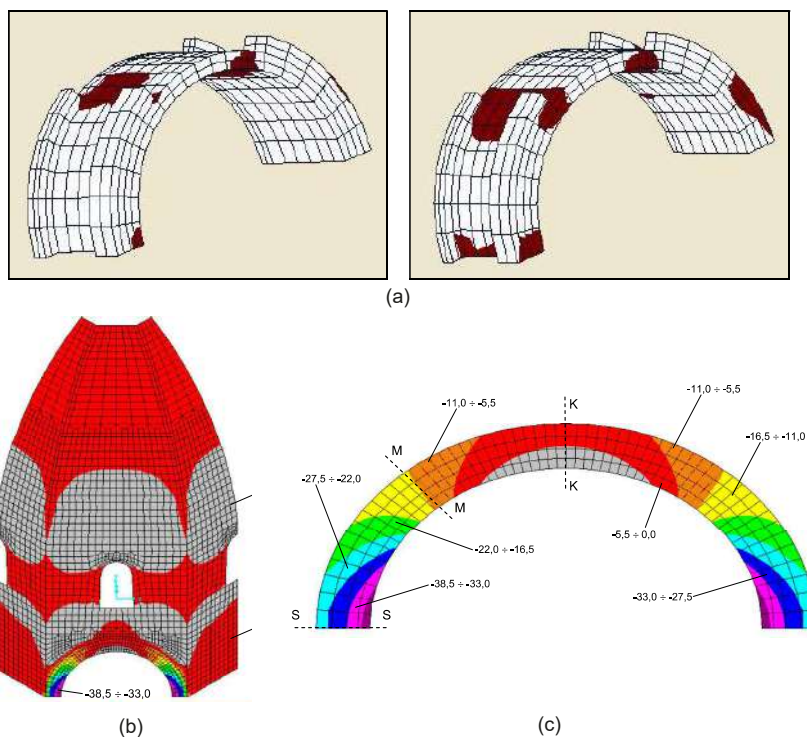


Fig. 3.7: Macro-modelling of barrel vaults from previous contributions: (a) principal strains (adapted from Creazza et al. [2000]), and (b-c) stress distributions (adapted from Bartoli and Betti [2013])

specimens were reinforced at the extrados, intrados, or both using FRP materials under two loading conditions (see Figure 3.8(b)), and a 3-D macro-model with homogeneous material configurations was developed to simulate their behaviour. Mechanical properties were derived from experimental tests and considered exclusively in the linear range. Contact between the reinforcement layer and the masonry substrate was assumed to be a perfect bond, and validation showed good agreement between the numerical model and experimental results from previous studies available in literature. Reinforcement at the intrados was found to be the most effective in terms of material efficiency and overall enhancement of structural performance.

3.2.2. Discrete Element Method

Although the Discrete Element Method (DEM) was originally developed for analysing slope mechanisms in hard rock, where separation paths are already known, it has found extensive applications in the field of masonry structures (e.g., Bianchini [2023]; de Felice et al. [2017]; Giamundo et al. [2014]; Petracca et al. [2017]; Pulatsu et al. [2016; 2020a]).

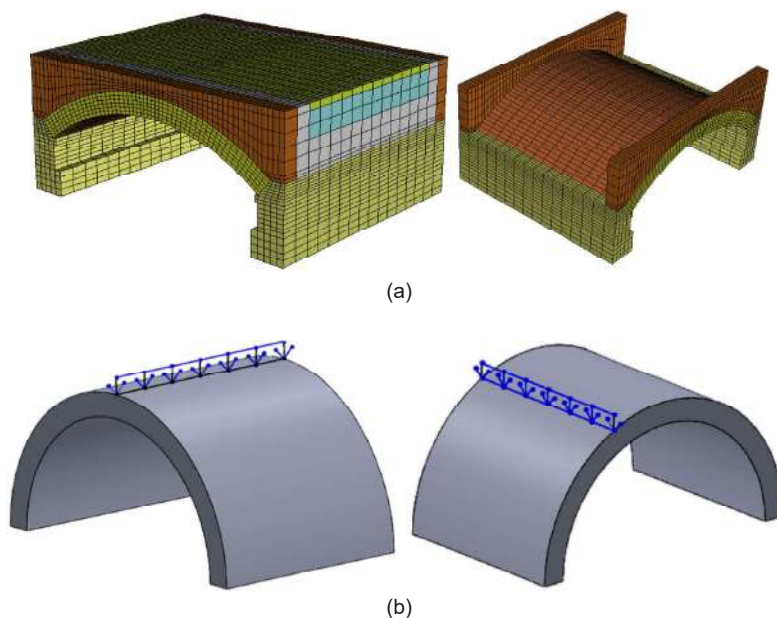


Fig. 3.8: Previous studies on barrel vaults adopting a macro-modelling approach: (a) FE model (adapted from Reccia et al. [2014]), and (b) FE model with different load application points (adapted from Tarhan and Uysal [2023])

The method relies on a discontinuous approach, in which masonry is modelled as units, generally assumed as rigid bodies, separated by joints represented by a set of contact points or edges.

According to its pioneers Cundall and Hart [1971] and subsequent studies [Roca et al., 2010; Lemos, 2007], DEM differs from the FEM in several ways:

- Blocks are rigid or deformable, but the system's overall deformability is primarily controlled by the joints;
- No restrictions are imposed on the shape of the blocks;
- Interactions between blocks are governed by contact points or edges, and the stress distribution across the contact surface can not be directly obtained;
- Contacts can be soft, described by normal and tangential stiffness, or hard.
- Each block can be discretised independently of adjacent blocks, and full separation between blocks is allowed without restrictions on the magnitude of displacements.
- Changes in forces result from changes in displacements, and these incremental forces are added to the pre-existing forces.

- DEM models typically adopt a time-stepping solution algorithm.
- Displacements and velocities are derived from accelerations, which are computed from forces and moments acting on each block.

DEM has been widely applied to the dynamic analysis of masonry structures subjected to seismic actions (e.g., AlShawa et al. [2012]; Baggio and Trovalusci [1995]; Gobbin [2021a]; Pagnoni [1994]; Schiavoni et al. [2023]). Masonry arches and vaults, in particular, are key elements for DEM applications [Alexakis and Makris, 2017; Lemos, 2007; Pulatsu et al., 2018].

A review of notable studies on masonry barrel vaults using DEM is presented below. Contributions are first grouped according to investigations of seismic behaviour, followed by studies addressing other structural aspects.

Lemos [1998] employed a 3-D Discrete Element (DE) model to simulate the seismic behaviour and failure mechanisms of arch structures. The solution was obtained by numerically integrating the equations of motion over time using an explicit algorithm. Material properties of the joints were modelled using a Mohr–Coulomb constitutive law. Loading involved first applying self-weight, followed by dynamic loading prescribed as motion at the support blocks, with the same earthquake input applied to both supports and scaled until structural failure occurred. The DE model successfully captured failure processes, including the typical four-hinge mechanism observed in circular arches (see Figure 3.9(a)).

Alexakis and Makris [2017] used a DE model primarily to validate a new formulation methodology to determine the limit states of single-nave masonry barrel vaults. Blocks were treated as rigid bodies, friction angles along joints were set to prevent sliding, and the tensile strength of mortar was neglected. Gravitational and horizontal inertial loads were applied at the centroids of each block as static forces. Two hinging mechanisms were evaluated: hinges developing only in the arch profile, or hinges extending into the buttresses (Figure 3.9(b)). The last configuration was found to be more critical, as it activated at a lower seismic input.

Girardello and Valluzzi [2013] adopted a DEM approach to model and compare numerical and experimental results for both unreinforced and strengthened masonry vaults. The vaults were tested under vertical cyclic loads applied either at one-quarter or at three-quarters of the span. Blocks were assigned elastic isotropic material properties, while contact areas were governed by a Coulomb yielding criterion. Reinforcement at the extrados was simulated using deformable blocks connected by two-node cable elements. The unreinforced model was analysed through a force-based time-history procedure. Given the high computational cost, the strengthened vault was instead loaded with the maximum force recorded during the corresponding experimental test. Numerical results for the unreinforced specimen showed good agreement with the experimental response, successfully reproducing hinge-opening behaviour. In

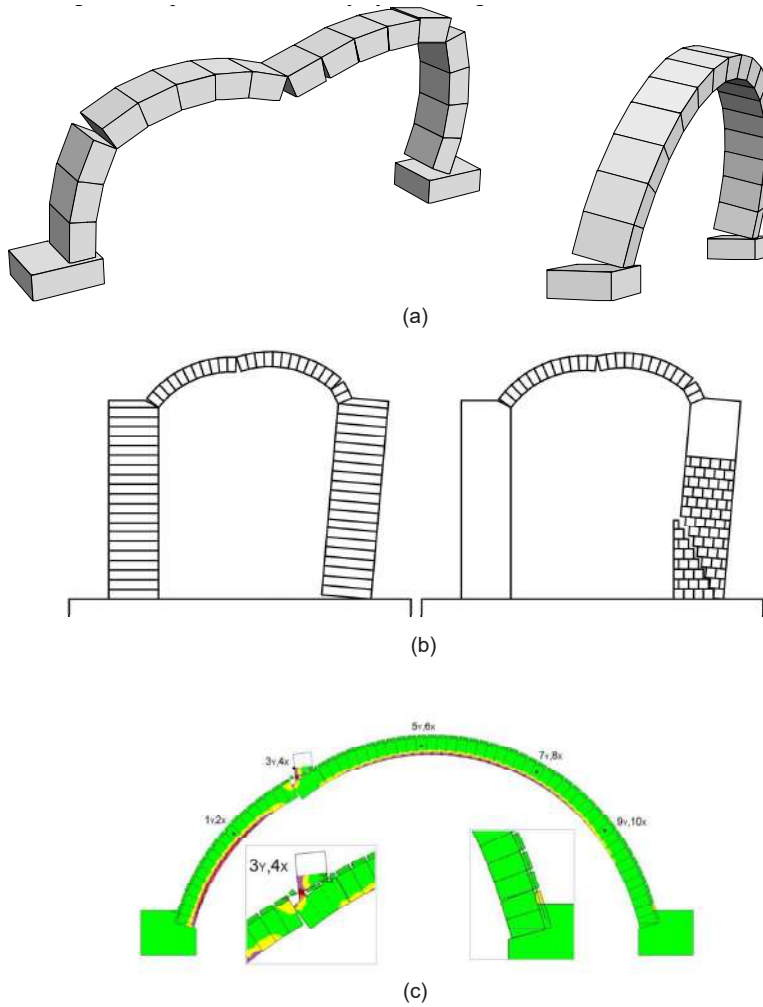


Fig. 3.9: Overview of previous studies on barrel vaults and arches using DEM: (a) Failure of arches [Lemos, 2007], (b) examples of some of the DEM models adopted (adapted from Alexakis and Makris [2017]), and (c) stress distribution at failure (adapted from Girardello and Valluzzi [2013])

the strengthened vault, the model replicated the sliding failure mechanism and detachment of reinforcement around the loading point (Figure 3.9(c)).

Fang et al. [2019] adopted DEM to overcome the limitations of Thrust Line Analysis (TLA), an application of the lower-bound theorem, by providing a more detailed stability assessment. A barrel vault was modelled as the extrusion of a semicircular arch, and the results showed that stability predictions and hinge locations were consistent with those of the arch. Specifically, the study

determined the minimum thickness-radius ratio required for stability under self-weight. This threshold for barrel vaults closely matched that predicted by TLA (see Figure 3.10(a)).

In their study on the Pilotta Palace vault, Coïsson et al. [2020] employed a DE model where masonry units were idealized as large macro-blocks to enhance computational efficiency. The model simplified the structure to a representative 1-meter section. Non-structural elements were considered as concentrated forces applied at the centroids of the respective elements. A sensitivity to parameter calibration was found, especially when dealing with stiffness between blocks. This issue led to the conclusion that DEM is predominantly a powerful research tool for research case studies, but its application in standard professional practice remains challenging.

The effects of brick pattern and infill on the stability of masonry vaults under support settlements were investigated by Boni et al. [2022]. In the DE model, the blocks were treated as rigid elements, while the infill was represented by rigid spheres. The study considered a barrel vault as a reference case and analysed six different brick arrangements (see Figure 3.10(c)). Vaults were subjected to three types of quasi-static support settlement, applied either vertically or horizontally on one or two supports. The results showed a typical three-hinge failure mechanism, with brick pattern significantly influencing vault stability. The presence of infill generally enhanced stability by increasing the displacement capacity at collapse.

3.2.3. Discrete Macro Element Method

The Discrete Macro-Element Method (DMEM), first introduced by Calìo et al. [2005] and Calìo and Pantò [2008], approximates the non-linear response of masonry structures using discrete macro-elements. Each element is conceptualized as an articulated quadrilateral with rigid edges and hinged vertices, interconnected by two non-linear diagonal springs that capture in-plane shear behavior. Interactions with adjacent elements are modeled through zero-thickness interfaces consisting of discrete non-linear springs, which simulate typical failure mechanisms such as flexural rocking, diagonal cracking, and sliding. Originally limited to 2-D in-plane analysis, the method was later extended to 3-D to incorporate out-of-plane motions and rigid body movements, maintaining a balance between accuracy and computational efficiency.

A formulation for masonry curved structures was proposed by Calìo et al. [2010] and Cannizzaro [2011]. In this extension, the macro-element is represented as an articulated irregular quadrilateral with rigid edges, where in-plane deformation is controlled by non-linear diagonal links. Interaction with adjacent elements or external supports is managed by spatial interfaces, generally inclined relative to the quadrilateral's plane. These contain discrete non-linear links calibrated to reproduce membrane, flexural, torsional, and sliding behaviour, thereby decoupling distinct failure mechanisms.

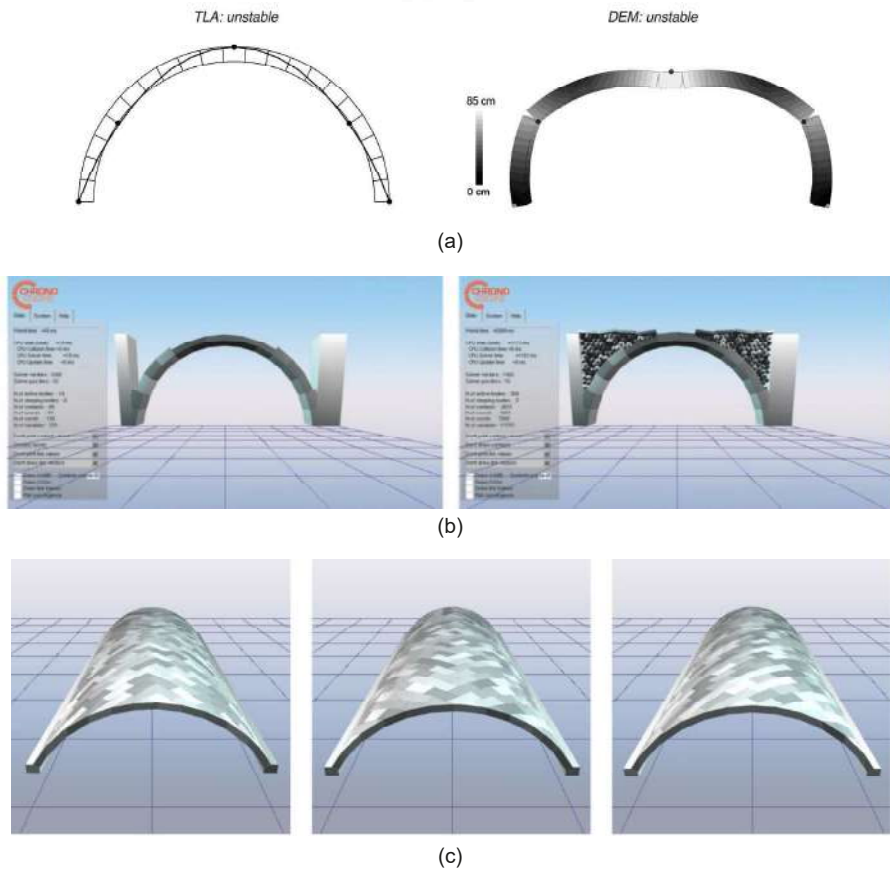


Fig. 3.10: DE models from literature: (a) comparison between TLA and DEM results (adapted from Fang et al. [2019]), (b) model with and without infill (adapted from Coïsson et al. [2020]), and (c) different units pattern analysed (adapted from Boni et al. [2022])

Many contributions have been provided in the last 20 years, especially focusing on the seismic behaviour of masonry structures (e.g., Calì et al. [2012]; Cannizzaro and Lourenço [2016]; Castellazzi et al. [2022]). Although, as previously stated, a formulation on curved structures has been provided, to this author's knowledge, only two studies address specifically masonry barrel vaults, of which a brief review will below be provided.

Caddemi et al. [2019] applied DMEM to investigate the non-linear behaviour of a five-arch masonry railway bridge in northern Italy. The interaction between adjacent macro-elements was concentrated in zero-thickness cohesive interfaces, which incorporate the non-linear behaviour of masonry through a homogenisation technique based on calibration along two principal directions.

The interfaces were discretised with transversal non-linear links. Additional sliding links, including one aligned with the interface axis and two orthogonal to it, governed in-plane and out-of-plane sliding as well as torsional behaviour. Diagonal shear was represented using a Mohr–Coulomb law with linear softening in both tension and compression. Validation was carried out by comparing the first three vibration modes and frequencies, showing excellent agreement with linear-elastic behaviour. A pushdown analysis, simulating an anti-symmetric loading condition induced by a train convoy, was also performed (see Figure 3.11(a-b)). The model accurately predicted the deformed shape and collapse damage pattern, which showed good agreement with observed behaviour.

Pantò et al. [2021] proposed a DMEM-based strategy to simulate masonry arches strengthened with Steel Reinforced Grout (SRG) composites. Both masonry and reinforcement were modelled with rigid elements interacting through one-dimensional non-linear links (see Figure 3.11(c)). Masonry blocks were represented by hinged panels with rigid edges, where flexural behaviour was governed by an elasto-plastic law, shear response was elastic, and joint sliding followed a rigid-plastic Mohr–Coulomb criterion. The reinforcement was de-

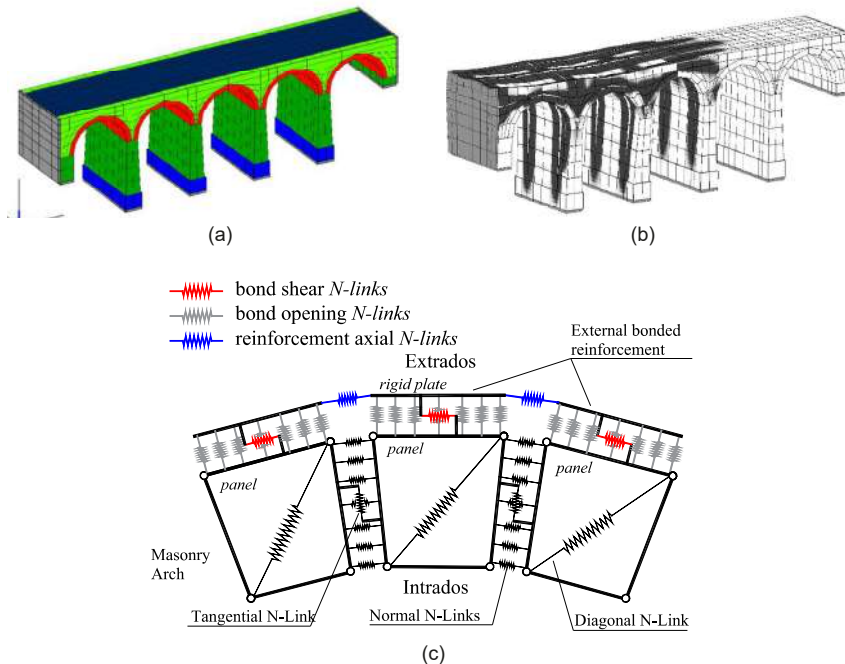


Fig. 3.11: DMEM models from literature: (a-b) model and damage pattern (adapted from Caddemi et al. [2019]), and (c) conceptual scheme of discretisation (adapted from Pantò et al. [2021])

scribed as rigid plates connected by axial links reproducing tensile behaviour up to rupture, bonded to the masonry arch through zero-thickness interfaces. The non-linear response of the arches was simulated under asymmetric vertical loading, specifically using one concentrated monotonic force. The model captured the primary failure mechanisms of both SRG and masonry, including fabric rupture, debonding, and masonry cracking, crushing, shear, or sliding. The solution procedure adopted an iterative Newton–Raphson scheme combined with the arc-length method. Substantial agreement was observed with experimental results, particularly regarding ultimate load capacity and failure modes involving flexural plastic hinges.

3.2.4. Conclusive remarks

In conclusion, the main findings emerging from the review of numerical models applied to masonry barrel vaults can be summarised as follows:

- Numerical models have been developed to address both unreinforced and reinforced masonry barrel vaults.
- Within FEM formulations, studies focusing exclusively on masonry barrel vaults through acceleration time histories are mainly motivated by scientific and research purposes rather than practical applications.
- Micro-modelling approaches are better suited to capture the dynamic behaviour of these structural elements throughout the analysis.
- Macro-modelling is more appropriate for case studies of historical constructions, where the complexity of the structures and their connections with adjacent elements would be too demanding to address with more detailed approaches.
- Numerical models are employed both to assess the current state of damage and to evaluate the effectiveness of strengthening interventions.
- Applications of the DEM are almost exclusively limited to research case studies.
- DEM requires a clear separation between units and interface elements, which reduces its applicability to well-defined elements whose material properties are known.
- In several DEM applications, non-structural elements and loads have been applied at the centroid of masonry units, creating challenges in determining the exact number of elements to be modelled.
- Most analyses evaluate the vault capacity under a point load applied either at one-quarter of the span or at the keystone.
- Assessments of seismic capacity are usually conducted by means of pushover analysis, with only limited applications of non-linear dynamic analysis.

4. Experimental campaign test setups

This chapter provides a comprehensive overview of the tested specimens, experimental setups, and testing methodologies adopted in this research. Two experimental campaigns were carried out, during which a total of three masonry barrel vault specimens were investigated. The chapter is organized into two parts. The first covers the initial experimental campaign, assessing the Unreinforced Vault (UV) and the Reinforced Vault (RV) and whose initial results are provided in Gioffré et al. [2023]. The second describes the testing of the Damaged Unreinforced Vault (DUV), i.e., the original UV specimen in its damaged state after the first campaign, which was subsequently retrofitted and retested. This strengthened specimen is referred to as the Reinforced-Damaged Unreinforced Vault (R-DUV) throughout this study. Both sections provide detailed descriptions of the geometric configurations, layouts, and construction phases of each tested specimen. Additionally, the experimental setups are described, including the criteria for selecting seismic inputs, the adopted instrumentation layouts, and the specifics of the two experimental campaigns. This chapter also introduces a novel hemp-rope-based bio-composite material proposed as a reinforcement system for masonry vaults, accompanied by an overview of its mechanical characterization based on existing test data.

The campaigns aimed to investigate the dynamic and seismic responses of similar vault systems under increasing seismic intensity levels, using identical input signals to ensure comparable excitation across specimens. Specifically, the first campaign sought to test and compare the UV and RV specimens, both newly constructed, to assess the reinforcement system's contribution to seismic

behavior, load capacity, and dynamic response variations. The second experimental campaign focused on the R-DUV specimen, aiming to evaluate the bio-composite material's effect on the seismic response of a damaged vault. This setup simulates a more realistic historical configuration, considering both the structural layout and decay state.

4.1.First experimental shaking table test campaign

This section provides detailed information on the experimental tests initially presented by Gioffré et al. [2023]. The geometry and layout of the tested specimens were designed to be representative of typical full-scale barrel vaults commonly found in historic masonry structures across Italy. The dimensional configuration was selected to ensure comparability with both the shaking table facilities and previous experimental investigations available in the literature (e.g., see Giamundo et al. [2015; 2016]; Ramaglia et al. [2016]). As illustrated in Figure 4.1, the specimen featured a segmental barrel vault geometry with a inner span of 2.78 m and a depth of 2.07 m, corresponding to an arch radius of 1.50 m. The total rise of the vault was 1.58 m, with a uniform thickness of 0.12 m consisting of a single brick layer. The vault rested on two abutments with base dimensions of approximately 0.35 m by 0.50 m. The masonry units employed were solid clay bricks, measuring 5.5 cm × 12 cm × 25 cm and joints filled with a commercial premixed mortar, characterized by an average compressive strength of approximately 5 MPa with an average thickness ranging between 10 mm and 15 mm. The mechanical properties of the constituent materials, as provided by the manufacturers, are summarized in Table 4.1. It is worth noting that despite the fact that the influence of backfill on the load-capacity of masonry vaults is well studied in literature [Cavicchi and Gambarotta, 2005; Tralli et al., 2014; Zizi et al., 2022], the specimens were built without any backfill, which is commonly found in historic roofing systems such as those of churches in Italy [Cardinali et al., 2022; Giamundo et al., 2015; Ramaglia et al., 2016].

Table 4.1: Material mechanical properties

Material	Compressive Strength (MPa)	Elastic Modulus (GPa)
Brick	29.3	5.7
Mortar	≥ 5	2.6

4.1.1.Specimens' definition and constructions

The specimens were built following traditional masonry techniques on a custom-designed foundation frame and centering system, specifically developed for the experimental campaign. Figure 4.2 illustrates the main components of both the foundation and the centering. These supporting systems were fabricated entirely from structural steel with a nominal yield strength of 235 MPa (S235JR, in accordance with CEN [2005]). The foundation frame (Figure 4.2(b)) con-

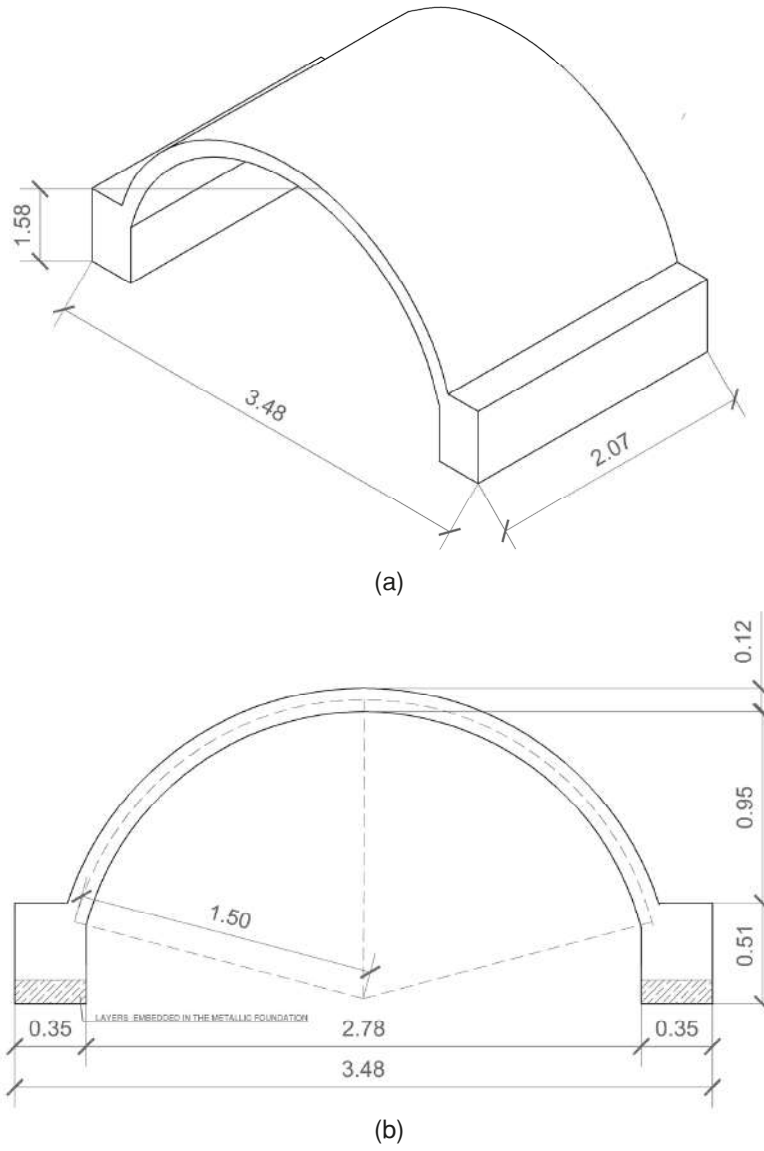


Fig. 4.1: Barrel vault specimen's layout and dimensions in meter: (a) general 3-D view, and (b) frontal view

sisted of a rigid steel structure composed, from bottom to top, of three HEA240 wide flange profiles, stiffened by a bracing system made of four 40 mm square hollow sections. The two outermost HEA beams were designed to host the lifting connections, and to anchor the specimen to the shaking tables via 30 mm diameters bolts. For the construction of the vault base, two 400 mm U-shaped profiles were employed to support the first two courses of the abutments. Consequently, the foundation system fulfilled a dual function: it provided a rigid base for transportation and handling of the specimen, and ensured secure attachment to the shaking table system. The centering was built using a system of three primary steel arches, transversely braced and covered with corrugated steel sheets (see Figure 4.2(a)). The structural elements consisted of 40 mm square hollow profiles, while the corrugated sheets were selected based on the brick unit dimensions and bearing capacity. In addition to serve as temporary framework, the centering was also designed to protect the shaking table and associated instrumentation during the seismic tests. Particular attention was given to the connections between the centering and the foundation frame. To enable controlled lowering of the system, six pantograph jacks were employed. Once removed, the centering could be firmly secured to the HEA beams to ensure safety and stability during testing.

4.1.1.1. Unreinforced Vault

The UV unreinforced specimen was built following traditional methods, ensuring that the bricks were adequately soaked in water prior to the construction process. This step prevented the bricks from absorbing the water from the mortar layers, which could otherwise compromise the mortar quality. The construction began symmetrically from the abutments. Figure 4.3 illustrates the main phases of the building process. It started with the construction of the base (Figure 4.3(a–b)), where the first two layers of bricks were embedded in the U-shaped frame using only a layer of mortar for bonding. The construction progressed symmetrically, alternating from one side to the other (Figure 4.3(c–d)), while ensuring proper alignment at the top to allow for the correct placement of the keystone unit (Figure 4.3(e–f)).

Two specimens were built following this procedure; however, while one remained unaltered, aiming to represent a traditional masonry barrel vault, the second was reinforced with a strengthening layer.

4.1.1.2. Reinforced Vault

The Reinforced Vault featured the same unreinforced masonry as the UV, but with an additional layer of reinforcement at the extrados. This reinforcement consisted of a novel hemp-rope-based material, featuring a hemp-rope net embedded in a ground-brick-based *cocciopesto* matrix. The hemp-rope net was manually fabricated using a wooden loom, starting from individual ropes with a nominal diameter of 4 mm, and arranged in a grid with 20 mm by 70 mm

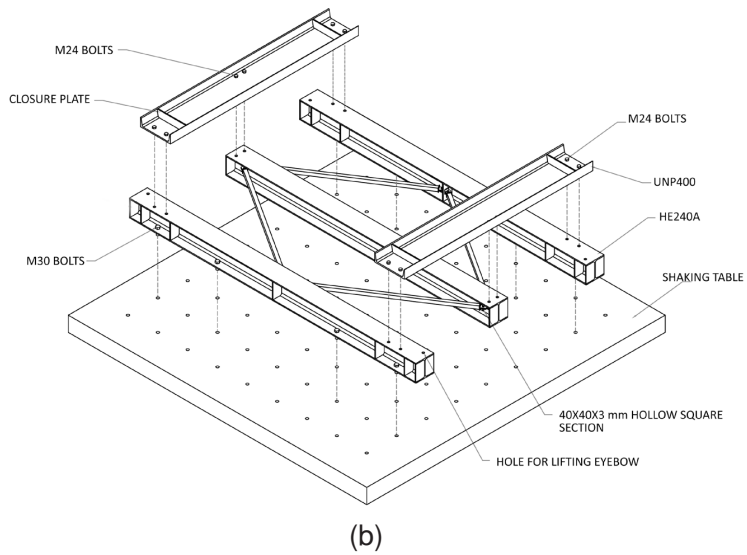
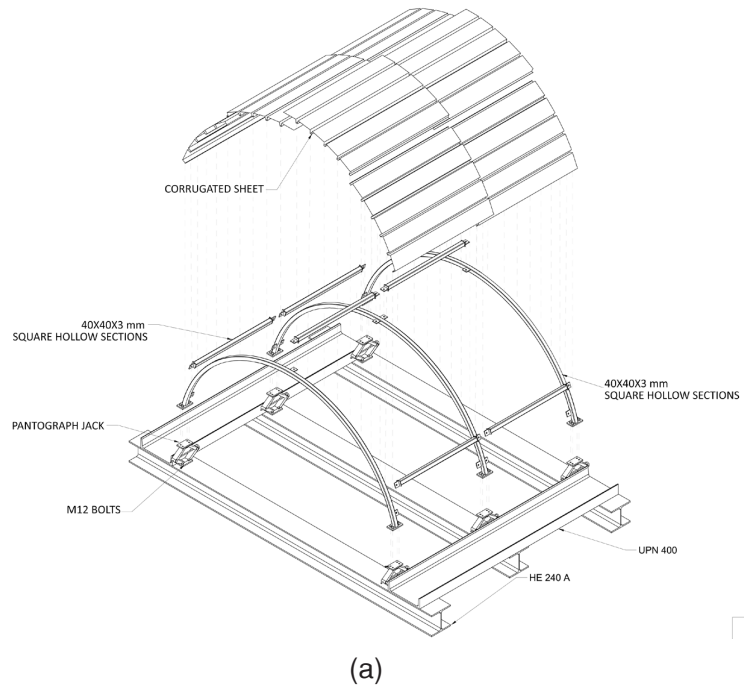


Fig. 4.2: 3-D view of the foundation frame and centering system: (a) focus on the centering elements, and (b) details of the steel foundation frame.

spacing, as shown in Figure 4.4.

The main phases of the reinforcement application are shown in Figure 4.5. Panel (a) shows the initial cleaning of the specimen to remove dust and impurities. Next, a layer of organic resin was sprayed onto the surface (Figure 4.5(b)) to improve bonding between the reinforcement and the substrate. The hemp-rope net was then positioned atop the resin layer, extending beyond the vault's curvature, with a portion embedded between the top two layers of the abutment

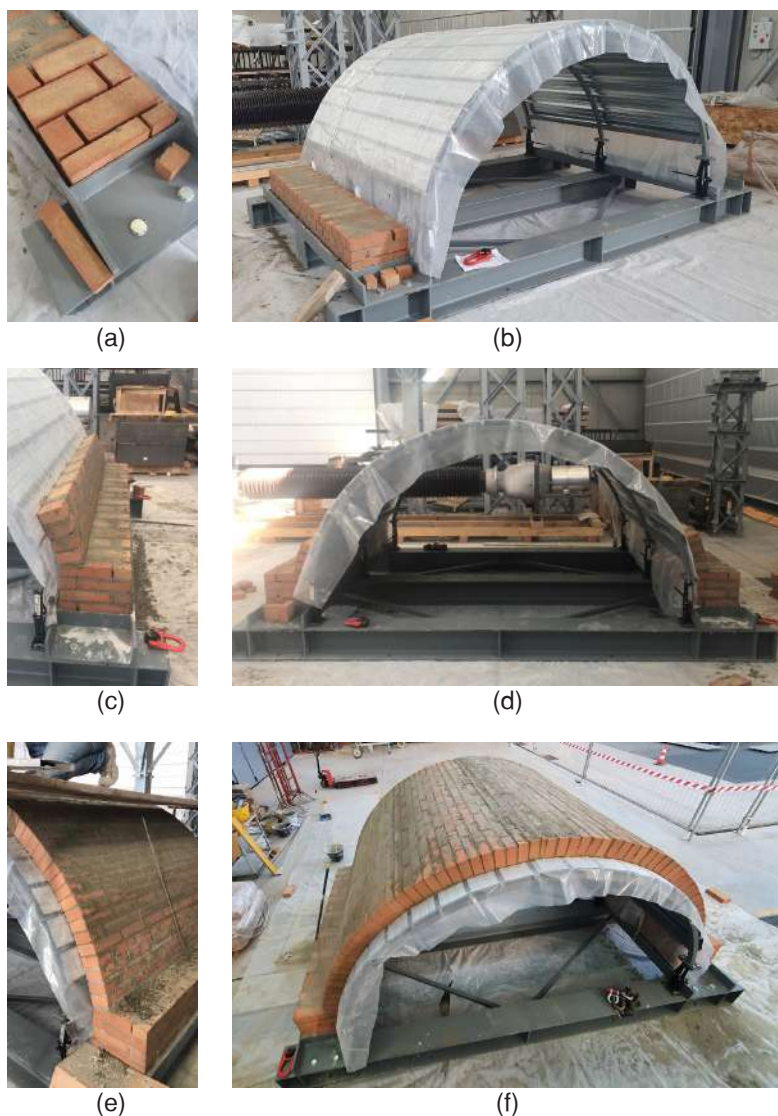


Fig. 4.3: Main phases of construction of the UV (a-f)

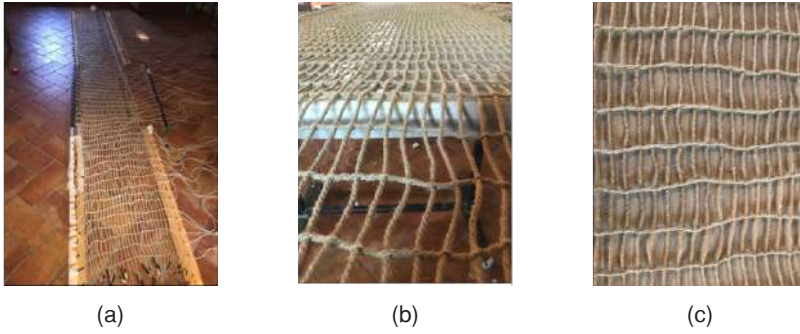


Fig. 4.4: Construction of the hemp-rope net (a-c)

units, as illustrated in panel (c). This detail was carefully executed to simulate traditional reinforcement methods and to improve the connection between the strengthening layer and the masonry. Sequentially, wooden formworks were used during the casting process to ensure a uniform matrix thickness of 30 mm. The matrix was poured and compacted to minimize the presence of voids that could lead to weaker sections (Figure 4.5(d-f)).

After allowing for appropriate curing periods, i.e., 28 days for the mortar and 7 days for the strengthening matrix, the specimens were transported to the shaking table. They were then weighed, and secured in place. The centering was lowered; however, for the unreinforced vault specimen, it was not completely removed but was instead fixed to the foundation system to serve as a safety measure in case of the sudden collapse of the specimen. The total weights of the specimens were approximately 42 kN and 45 kN, for the Unreinforced and Reinforced vaults respectively, including the steel foundation and centering elements. These metallic components were estimated to be about 12.5 kN. The reinforcement contributed to an approximately 9% increase in total weight, with a material density stated at 1.2 tons/m³.

4.1.2.Characterization of the bio-composite material

The mechanical properties of the materials employed in the novel strengthening system were evaluated through dedicated experimental campaigns. The hemp ropes were tested under direct tensile loading to determine their tensile strength, elastic modulus, and post-peak response. The cociopesto matrix was characterised through three-point bending and uniaxial compression tests. These tests allowed for the assessment of its flexural and compressive strengths, elastic modulus, and behavior beyond peak load.

Hemp rope samples were obtained from commercially available ropes composed of untreated hemp fibers, harvested in Romania. Each rope consisted of three strands, each formed by eight yarns made up of an uncountable number of

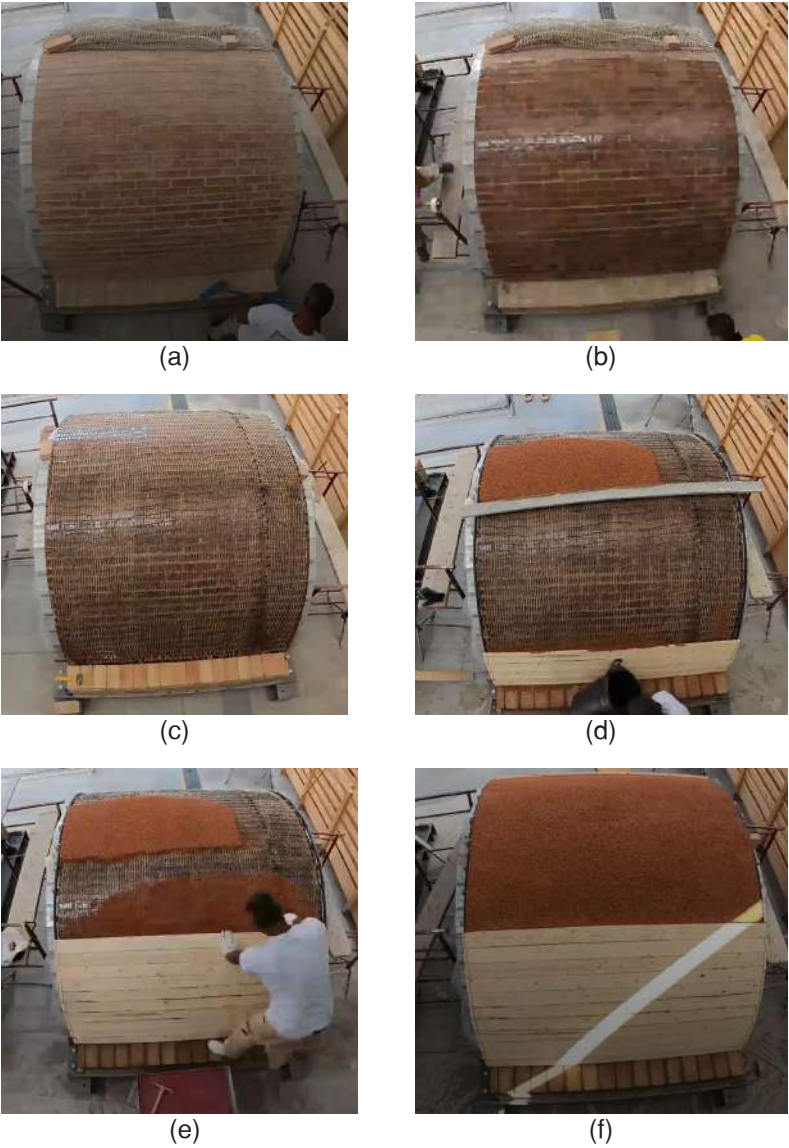


Fig. 4.5: Main phases of the strengthening application on the RV (a-f)

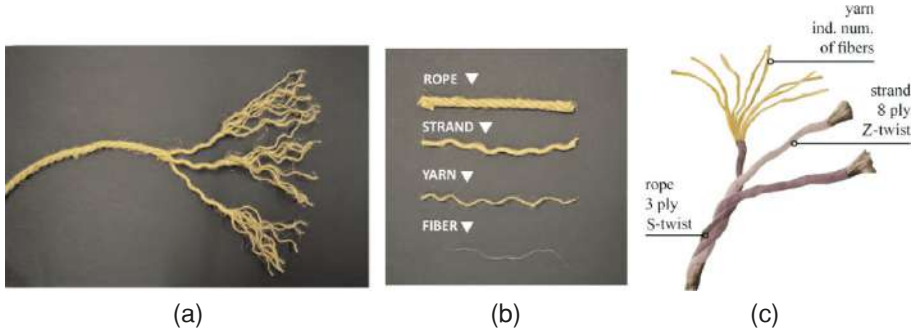


Fig. 4.6: Hemp-ropes net constituents [Pepi et al., 2024]: (a) hemp rope, (b) from the top down: hemp rope, strand, yarn and fiber, and (c) scheme of the hierarchical structure

individual hemp fibers, as illustrated in Figure 4.6 [Pepi et al., 2024].

Pepi et al. [2024] and Giofrè and Pepi [2024] carried out several experimental campaigns to investigate the mechanical properties of hemp ropes and their constituents under controlled-displacement conditions, applying a loading rate of 0.17 mm/s. The tests were performed as direct tensile tests, following the guidelines provided in ASTM [1999]. The testing machine used (Lloyd Instrument LR30) was equipped with a 30 kN load cell. The rope samples had an effective diameter of approximately 4.00–4.10 mm, while the strands exhibited an average diameter of 2.04 mm. The yarns, assuming a circular cross-section, had an average apparent diameter of 0.73 mm. Given the scope of this thesis, only the mechanical properties of the hemp ropes will be discussed.

Giofrè and Pepi [2024] conducted experimental tests on 10 rope samples with a length of 150 mm each, while Pepi et al. [2024] tested 64 samples, each 200 mm long. The typical stress-strain curve behaviour of natural material ropes is presented in Figure 4.7(a) and can be divided in three main sections as shown. The behaviour in Region I, also known as Crimp in literature, is characterised by a re-alignment of the fibers constituting the ropes along the loading axis. This is followed by Region II, where the response can be described by a linear trend up to the tensile strength, while Region III is characterised by a progressive failure of the rope constituents. The mechanical properties investigated include the tensile strength ($\sigma_{\max}$) and the corresponding maximum strain ($\epsilon_{\max}$). Given that the stress-strain response of the hemp rope is bi-linear, two elastic moduli were defined. The first modulus, E_{in} , refers to Region I and is calculated over the strain interval from 7% to 30% of the maximum strain. The second modulus, E , is defined as the secant modulus between 75% and 95% of the maximum strain, corresponding to Region II. It is also worth noting that the tensile strength corresponds to the peak stress, and the maximum strain refers

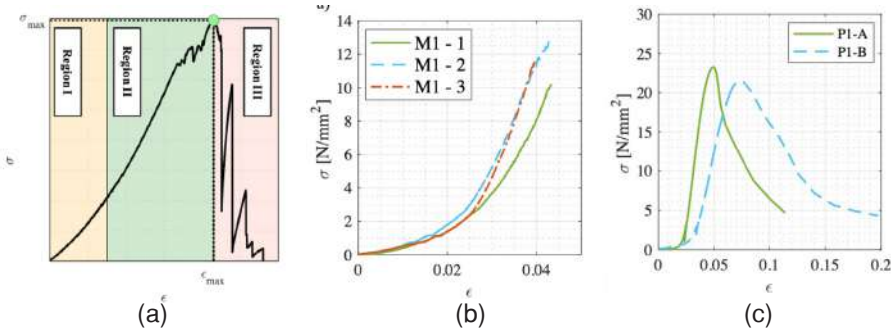


Fig. 4.7: Results from Giofrè and Pepi [2024] : (a) schematic representation of the typical mechanical behavior of hemp ropes, (b) stress strain curves obtained from the three points bending tests, and (c) stress strain curves obtained from the uniaxial compression tests

to the strain value at which this peak stress is reached. Table 4.2 reports the results in terms of mean values and Coefficients of Variation (CoV). For the two different sample lengths, the average tensile strength values were 75.40 MPa and 77.23 MPa, respectively. The corresponding maximum strain values ranged from 0.48 to 0.34. The first elastic modulus (E_{in}) varied approximately between 139 MPa and 155 MPa, while the second elastic modulus (E) ranged from 227.8 MPa to 379.2 MPa.

Table 4.2: Mean values and CoV from direct tensile tests on hemp ropes

	σ_{max} (MPa)		ϵ_{max}		E_{in} (MPa)		E (MPa)	
Reference	Mean	CoV	Mean	CoV	Mean	CoV	Mean	CoV
Giofrè and Pepi [2024]	75.40	0.12	0.48	0.35	139.04	0.11	227.78	0.15
Pepi et al. [2024]	77.23	0.10	0.34	0.11	155.16	0.23	379.18	0.10

Giofrè and Pepi [2024] characterized the matrix response through three-point bending tests. Compression tests were then conducted on the resulting halves of the prismatic specimens, in accordance with the UNI EN 1015-11:2019 standard [CEN, 2019], as shown in Figure 4.8. The matrix consisted of a mixture of aggregates and an organic bi-component binder, reaching the aimed mechanical properties within seven days.

Three prismatic specimens, each measuring 40 mm × 40 mm × 160 mm, were cast and cured under controlled temperature and humidity conditions. The bending tests were carried out in displacement-controlled mode at a rate of 1.5 mm/min, aiming for failure between 60 and 90 seconds. The two halves resulting from the bending tests were then subjected to compression tests, with a load application rate of 30 N/mm. The load was applied over

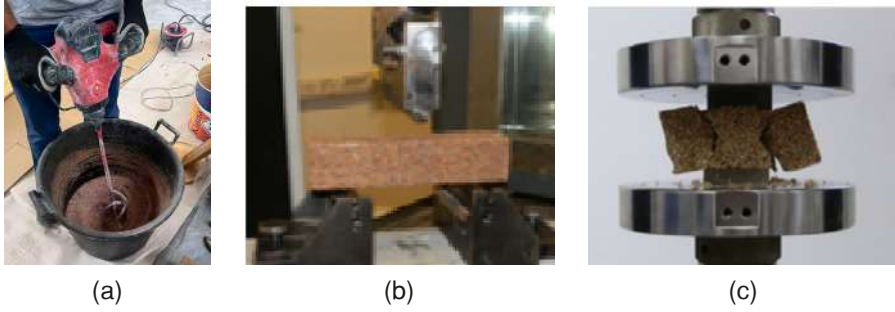


Fig. 4.8: Cocciopesto matrix [Giofrè and Pepi, 2024] : (a) preparation, (b) three point bending test sample, and (c) uniaxial compression test sample

a 40 mm $\times$ 40 mm surface until failure. Both tests were performed using a testing machine equipped with a 30 kN load cell. The results are summarized in Table 4.3, while typical stress–strain curves are shown in Figure 4.7(b-c). The flexural response of the matrix exhibited a bi-linear behavior, with a linear increase up to the peak stress, followed by a brittle fracture initiating from the bottom face. The maximum stress was evaluated at the peak of the curve, and the corresponding strain was taken as the maximum strain. The elastic modulus was calculated as the secant modulus from the origin to the peak. The average flexural strength and maximum strain were 11.53 MPa and 0.04, respectively, with an estimated elastic modulus of approximately 275 MPa. The compressive stress–strain response followed a typical parabolic curve. The average peak compressive strength was 25.87 MPa, with a corresponding strain of 0.13, resulting in a secant elastic modulus of approximately 1100 MPa.

Table 4.3: Mean values and CoV from three-point bending and uniaxial compression tests from Giofrè and Pepi [2024]

	Three-Point Bending Tests		Uniaxial Compression Tests	
	Mean	CoV	Mean	CoV
$\sigma_{\max}$ (MPa)	11.53	0.09	25.87	0.10
$\epsilon_{\max}$	0.04	0.04	0.13	0.26
E (MPa)	275.33	0.10	1091.5	0.24

4.1.3. Shaking table testing setup definition

The experimental campaign was carried out at L.E.D.A. research institute of the University of Enna Kore (Italy), which is equipped with a shaking table facility. It consists of two identical 6-DoFs shaking tables, with dimensions of 4 m $\times$ 4 m. It is possible to test specimens either on a single shaking table or on both tables using synchronous or asynchronous motions. Moreover,

the tables can be linked together to form a single 6-DoFs shaking table of 10 m $\times$ 4 m, allowing for testing either larger specimens or two specimens at the same time (Figure 4.9). This configuration increases the load-bearing capacity up to 100 tons and can operate in a frequency range between 0.01 and 60 Hz. The stroke limits reach 400 mm for horizontal movements (X and Y axes) and 250 mm for vertical movements. The system's acceleration capacity can achieve 1.05 g for horizontal and 0.7 g for vertical movements at full load. Each table is equipped with eight servo-hydraulic actuators, with two 500 kN actuators per direction, to apply dynamic forces effectively. To absorb and dampen the forces and vibrations generated by the actuators during shaking table tests, the laboratory is equipped with a large reinforced concrete block Reaction Mass (RM) with a volume of 1100 m³ and a mass of approximately 3000 tons. To isolate the testing system from the surrounding environment, the reaction mass is dynamically isolated from the remaining laboratory structure using an air-spring system. The isolation system is composed of 96 air springs placed at the bottom of the reaction mass. The springs are characterised by a low lateral stiffness, allowing effective vibration isolation. For more details concerning the laboratory and the shaking tables facility, the reader is referred to Fossetti et al. [2017]; Lo Iacono et al. [2017].

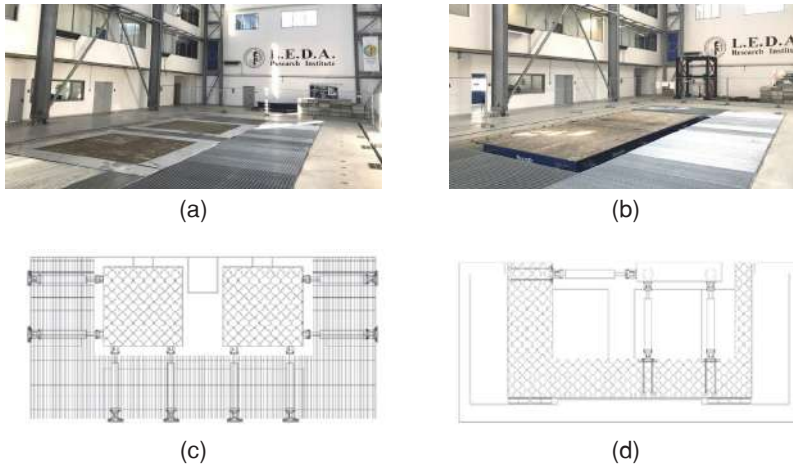


Fig. 4.9: L.E.D.A. laboratory (adapted from Fossetti et al. [2017]): (a) dual system of 4 m by 4 m shaking tables, (b) linked single 10 m by 4 m shaking tables, (c) reaction mass plane view, and (d) transversal section.

The shaking tables experimental campaign featured two kinds of tests: Seismic Tests (STs) and Ambient Vibration Tests (AVTs). The first were performed in sequence by selecting a Reference Earthquake (RE) time history and carrying out progressive tests at increasing amplitudes (i.e., a coefficient was used as a

scale factor for the intensity of the earthquake). The goal was to subject the vaults to an increasing level of excitation until the occurrence of severe damage. AVTs were performed without any active input from the shaking tables in a low-level environmental noise condition. The choice of not having a stronger noise level was made to avoid the occurrence of additional damage not due to the seismic tests. The objective was to subject the vaults to progressively increasing seismic load and to evaluate, through AVTs, the occurrence of damage by variation in the vibration frequencies and correlated vibration shapes, which can be attributed to a change in the stiffness of the structure [Chopra, 2014; Rytter, 1993].

Figure 4.10 presents multiple views of the two specimens after being transported to the shaking tables. Panels (e–f) focus on the UV and the RV, respectively.

4.1.3.1. Seismic tests input

A seismic input can be chosen among recorded earthquakes, artificial and simulated ones as detailed in CEN [2004]. A recorded earthquake refers to a seismic event that occurred at a specific site and for which acceleration time histories, and the derived response data, were instrumentally captured. It is also possible to use simulated earthquakes, based on site-specific studies and conditions, or artificial earthquakes, which are time histories generated to match a target elastic response spectrum featuring a minimum stationary duration of 10 seconds. Although the code specifies that an input in three directions needs to be analysed, for common building the vertical component has been an aspect often disregarded in past studies [Camata et al., 2023; Carydis et al., 2012; de Nardis et al., 2014]. However, recent research emphasizes the critical role of this vertical component in influencing structural behaviour, cracking patterns, and failure mechanisms. Several field data from significant past earthquakes have shown how the vertical motion contributed to various types of structural failure during the Kalamata (Greece, 1986), Northridge (USA, 1994), and Kobe (Japan, 1995) seismic events [Dailamani and Croll, 2017; Papazoglou and El-nashai, 1996]. In Italy, during the earthquakes that have occurred over the last twenty years, several studies have highlighted the impact of the vertical component on the response of masonry structures, which can significantly affect the cracking pattern and collapse of masonry elements [Di Michele et al., 2020; Liberatore et al., 2019]. A significant confirmation of the influence of the vertical component of a seismic action is provided by the collapse of the clock tower in Emilia Romagna during the 2012 seismic sequence [Acito et al., 2014]. The failure and damage crack pattern were validated by numerical studies taking into account all the seismic input directions and proving their effects on structures with low material strengths. It has been demonstrated that the impact of the vertical component on structural response varies depending on the orientation of the structural elements, vertical (e.g., walls and towers) or horizontal (e.g., roofs and floors). Horizontal elements are primarily affected by

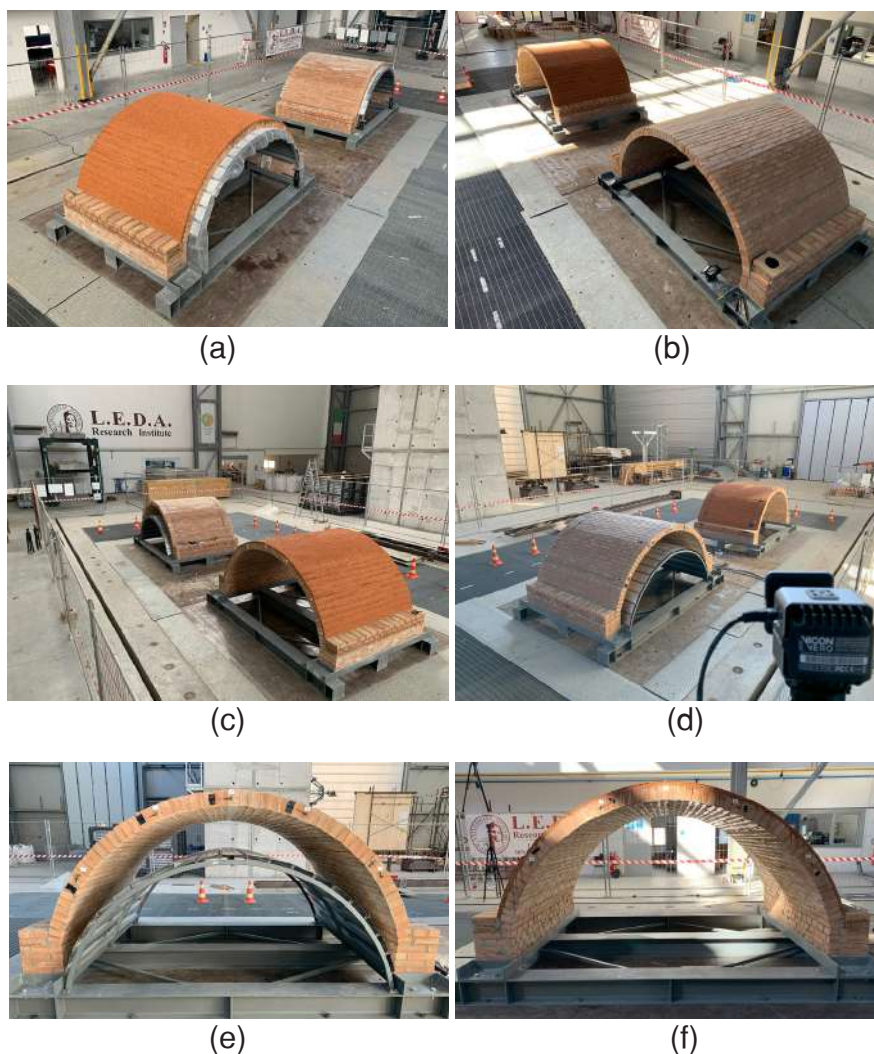


Fig. 4.10: Reinforced and Unreinforced Vaults: (a-d) General view on the shaking tables facility,(e) focus on the Unreinforced specimen, and (f) focus on the reinforced specimen

vibration-type mechanisms, which excite the structure in the vertical direction. Vertical elements may be more susceptible to impact-type mechanisms [Carydis et al., 2012]. During seismic excitation, the vertical seismic component can cause repeated cycles of compression and decompression in masonry structures, potentially leading to irreversible damage and separation of mortar joints. This effect is more pronounced when structures present low quality mortar.

This mechanism may decrease both bending and shear resistance, particularly if a sliding shear mechanism may occur [Pugi et al., 2019]. It has also been shown that under reduced gravitational loads, the vertical component of seismic action can introduce tensile stresses in usually compressed elements [Di Michele et al., 2020]. Several experimental tests also highlight how the vertical component may reduce the axial and shear capacity and induce damage (such as sliding and cracking) in structures with low interface cohesion, while its effect is less pronounced on structures with higher interface cohesion [Chieffo et al., 2021; Liberatore et al., 2019]. Although these findings underscore the key role of the seismic vertical input components, further detailed studies are needed to draw general conclusions because several factors may influence the structural response, e.g., the loads and the structural typology [Camata et al., 2023; Kallioras et al., 2022].

Masonry vaults, which rely heavily on self-weight for stability, are particularly vulnerable to the distinct stresses and deformations introduced by the vertical seismic component. However, the current literature offers limited insight into its specific effects on such structures, highlighting the need for more targeted research.

Considering this background, the earthquake that occurred on October 30th, 2016, in the center of Italy (Norcia) was chosen as the Reference Earthquake (RE). This seismic event was among the strongest in the 2016 Umbria-Marche seismic sequence and was the most intense earthquake in Italy after the Irpinia Earthquake in 1980 [Civico et al., 2018; Delorme et al., 2020; Galli et al., 2017]. Notably, the Peak Ground Velocity (PGV) was around two to three times higher than that expected for a 475 year return period [Liberatore et al., 2019].

The severity and intensity of an earthquake can be described using various strong-motion parameters, including the Peak Ground Acceleration (PGA), the Peak Ground Displacement (PGD), the PGV, pseudo-acceleration response spectra, and the Moment Magnitude (M_w). The recorded time histories in terms of acceleration and displacement, along with the pseudo-acceleration response spectra for all recorded directions of the seismic event, are presented in Figure 4.11. This strong motion event was characterized by a M_w equal to 6.6. The pseudo-acceleration response spectra values exceeded 0.5 times the gravitational acceleration (g) in all directions for stiff structures with a fundamental period below 0.5 seconds (Figure 4.11(c)), while the values ranged between one to two times g for typical period of two-to-three-story unreinforced masonry buildings [Liberatore et al., 2019]. As previously noted, for standard buildings, the vertical component of seismic events is often neglected. However, in this case, the pseudo-acceleration values in the vertical direction are of the same order of magnitude as those in the horizontal directions. The acceleration and displacement time histories are shown in Figure 4.11(a-b). In the X direction, the earthquake recorded a PGA of approximately 0.6 g and a PGD of 30 mm, while the Y direction exhibited a PGA of 0.5 g and a PGD of

14 mm. The notably significant vertical component exhibited a PGA of 0.4 g, comparable in magnitude to the two horizontal components.

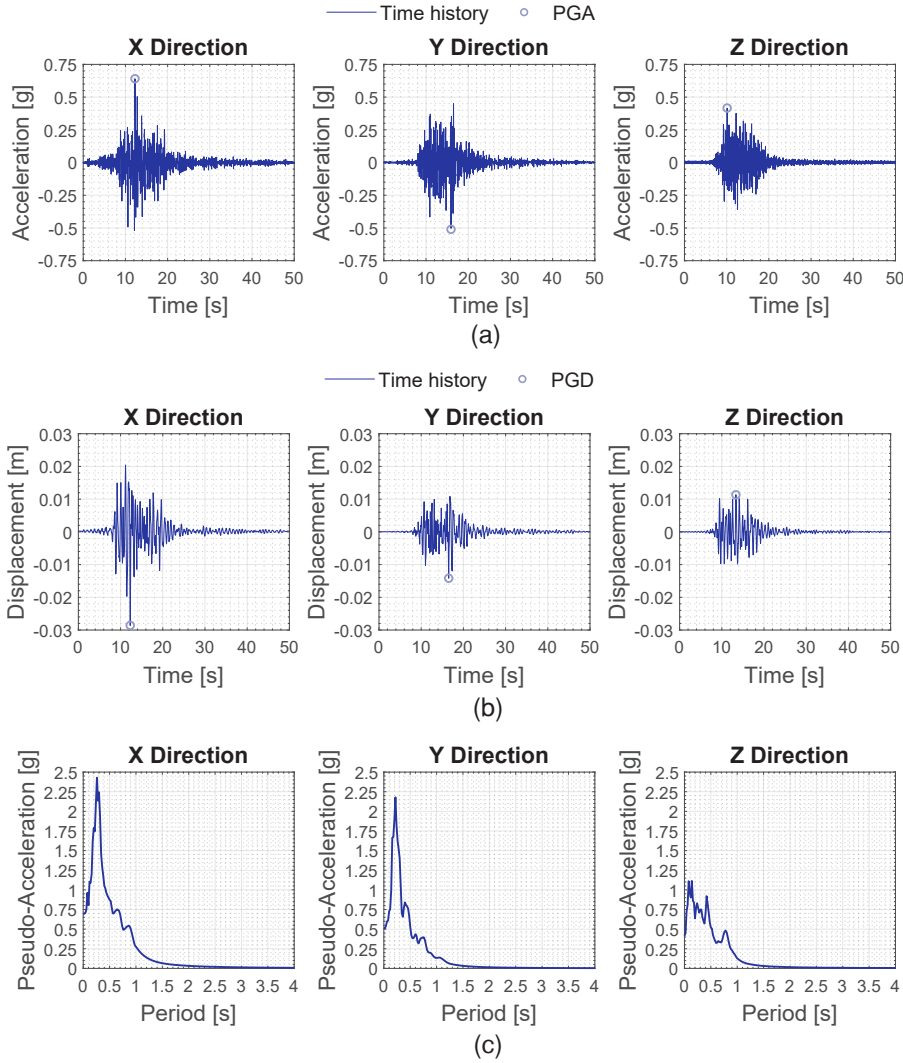


Fig. 4.11: Reference Earthquake: (a) acceleration time histories, (b) displacement time histories, and (c) pseudo-acceleration response spectra

It is also noteworthy that the earthquake had an intense phase between 10 and 20 seconds across all directions, approximately. All PGA values are obtained within this time frame. The intense phase is defined, in this work, through the Arias Intensity (I_A) [Arias, 1970], often used to capture the potential damage of a strong motion (for more insights on this topic, see Travararou et

al. [2003]). The intense phase is defined as the portion of the accelerograms within the significant duration [Aşıkoğlu et al., 2024]. As it is possible to note from Figure 4.12(right-panels), the significant duration is obtained as the time frame between 5% and 95% of I_A , and the correspondent parts of the accelerograms are highlighted in the left-panels.

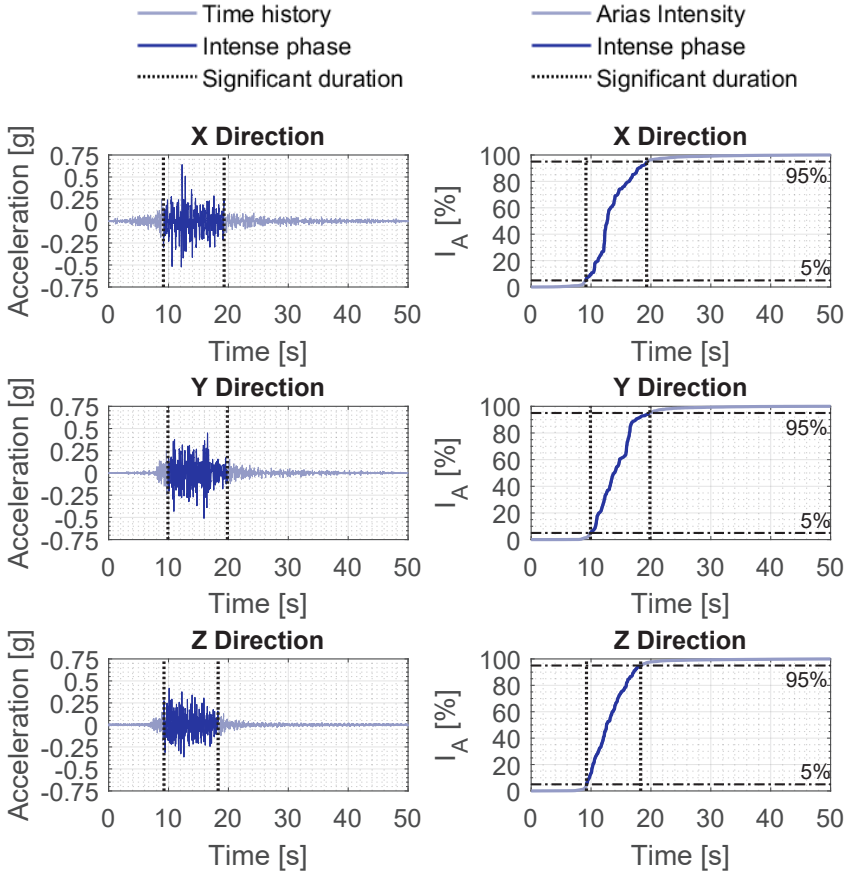


Fig. 4.12: Acceleration time histories, intense phases and Arias Intensity of the RE

4.1.3.2. Setup and instrumentation

The aim of this experimental campaign was to compare the dynamic and seismic behavior of the two vault specimens (UV and RV, as described in Section 4.1.1) under identical seismic input, with particular attention to their failure mechanisms, seismic capacity, and dynamic response. To this end, the single 10 m × 4 m shaking table configuration was selected, allowing both specimens to be tested simultaneously.

Given that the dynamic behavior of a barrel vault is often represented as an envelope of several arch ribs, two sections, labeled Section A-A and Section B-B in Figures 4.13 and 4.14, were instrumented. The acceleration responses of the vault specimens were recorded using piezoelectric accelerometers, installed at several locations along the facades of the vaults to measure acceleration time histories, with a sensitivity of 1 V/g. The displacement time histories at multiple locations were monitored through reflective markers glued to the specimens. This was done using eight high-speed, high-resolution infrared cameras, which tracked the motion of the reflective markers in real time. These cameras enabled accurate tracking of the marker positions, allowing to measure the displacement during the shaking table tests with a sensitivity error of 0.2 mm. This system is based on the creation of a network of three markers, which work as the reference system. To capture the acceleration and displacement data in three primary directions, two distinct sensor setups were used: Setup #1 (see Figure 4.13) and Setup #2 (see Figure 4.14). Each setup employed a different number and arrangement of sensors to capture a comprehensive picture of the vaults' dynamic responses and maximize the number of acquisition channels.

The test acquisition Setup #1 was characterized by a combination of tri-axial and bi-axial accelerometers distributed on Section A-A. A tri-axial accelerometer (labelled as A3 in Figure 4.13) was placed at the keystone of Section A-A to measure accelerations in all three directions: radial, tangential, and longitudinal. Four bi-axial accelerometers were positioned on both the left and right sides of the keystone at 22° and 52° from the keystone (labelled as A1, A2, A4 and A5 from left to right, respectively). These sensors measured the radial and tangential acceleration components, capturing the acceleration time histories of the specimens in both directions. On the same facade, the displacement time histories were recorded through 23 reflective markers glued every 2 units (labelled as M1, M2, ..., M23, from left to right). It is worth noting that only two markers were glued on the abutments, one for each side. These peculiar setups allowed for the monitoring of the structure's movements in two horizontal directions (X, Y) and one vertical (Z). For Section B-B, five uni-axial accelerometers recorded only radial acceleration components (marked in Figure 4.13 as A6, A7, A8, A9, A10). These accelerometers were placed at corresponding locations to those in Section A-A. The displacement time histories in three directions (X, Y, Z) on this facade were recorded by 9 markers placed one every 4 units, and they are labelled as the correspondent ones in Section A-A. This setup allowed monitoring both vaults using the same acquisition setup in both acceleration and displacement.

In Setup #2, the instrumentation for Section A-A was slightly modified to focus on capturing more detailed dynamic response data in the acceleration field in all directions using tri-axial accelerometers. Five accelerometers were placed on Section A-A (labelled as A1, A2, A3, A4 and A5). These sensors measured the accelerations in the radial, tangential, and longitudinal directions. Controver-

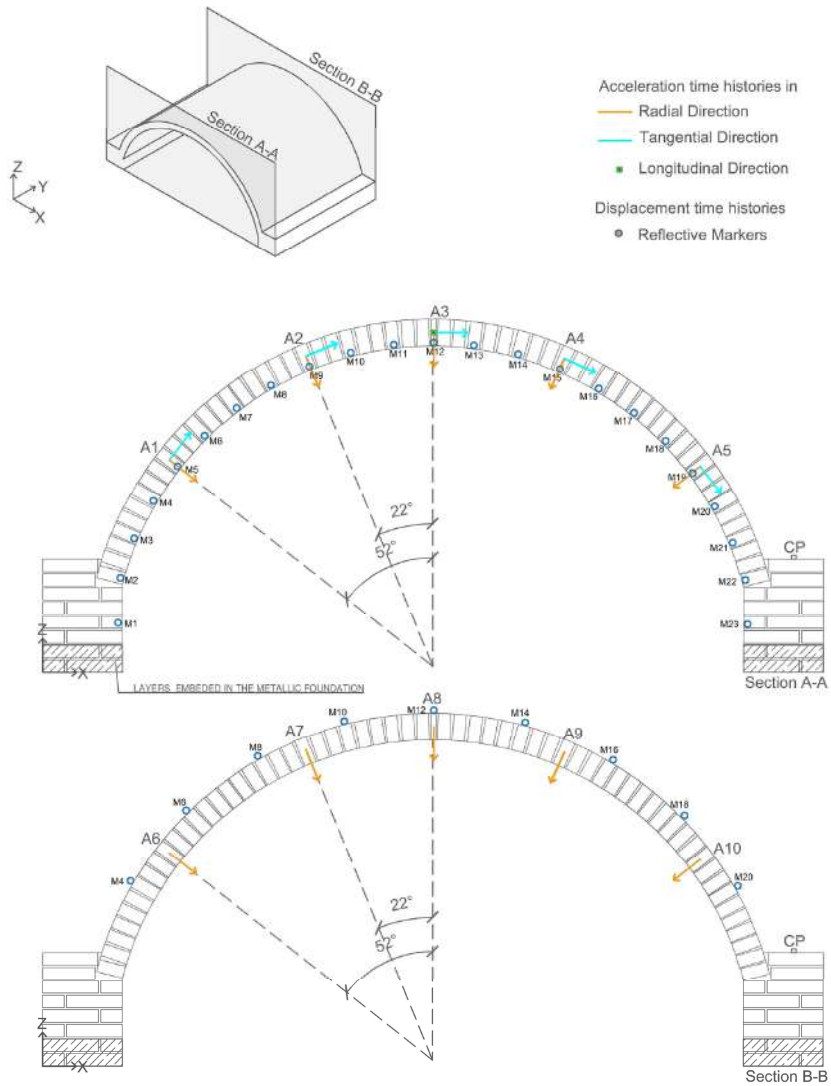


Fig. 4.13: Testing acquisition Setup #1

sially, no significant changes were made to the positions of the markers on the same facade. Five bi-axial accelerometers were positioned along Section B-B (labelled as A6, A7, A8, A9 and A10) to record the response in the radial and tangential directions. The number of reflective markers was increased to 21 on this facade (labelled M3, M4,..., M21), allowing for a more detailed monitoring of the displacement time histories. In this second setup, Section B-B was instrumented similarly to Section A-A, except for the omission of two markers

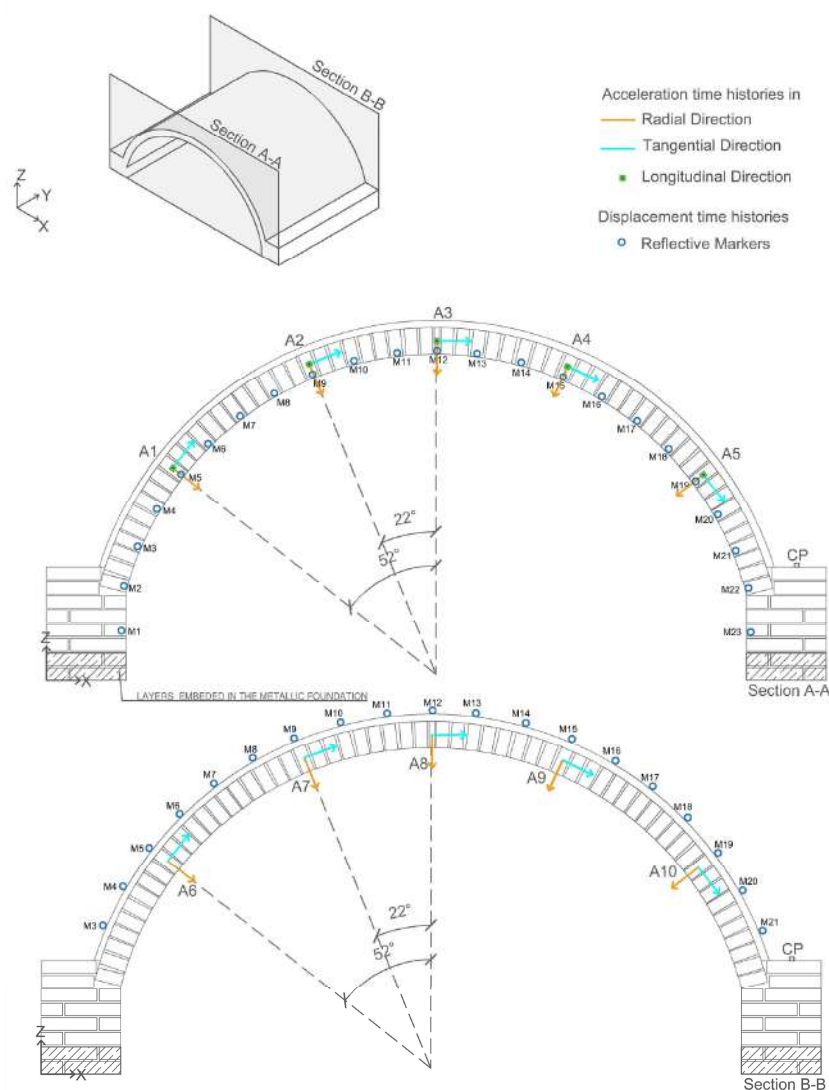


Fig. 4.14: Testing acquisition Setup #2

positioned on the abutments. Additionally, a tri-axial accelerometer was placed on the right abutments of the vault to measure the effective acceleration transmitted to the vault in both setup configurations (labelled as CP in both the setups). Figure 4.15 shows that the UV specimen was still equipped with the centering, which had been fully lowered to protect the shaking table in the event of a sudden collapse. This precaution was not deemed necessary for the Reinforced Vault, based on the results of previous capacity tests performed on

reinforced arches [Gioffré et al., 2021].

Table 4.4 summarizes the shaking table test sequence, indicating the setup used and the various types of tests performed. A total of 31 tests were carried out, including 21 seismic tests with intensities ranging from 20% to 230% of RE. The AVTs were performed without any active input from the shaking-table facilities to avoid introducing additional damage unrelated to seismic loading. The first two seismic tests were performed on both specimens (up to 40% scale of the RE), while the subsequent seismic tests were carried out only on the RV. Following the seismic test at 0.4RE and its following AVT, the instrumentation configuration was changed from Setup #1 to Setup #2. From that point onward, AVTs were no longer performed after every seismic test, but were instead recorded after reaching significant input level.

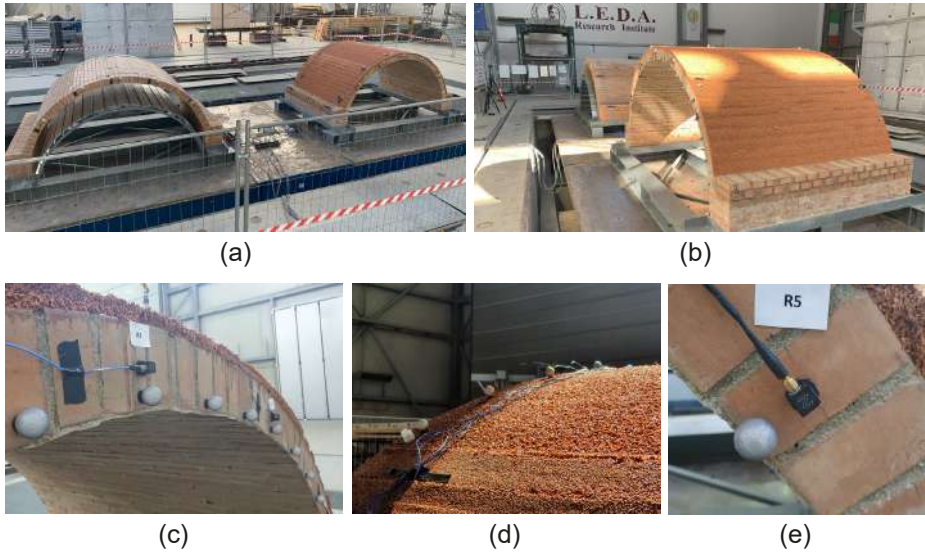


Fig. 4.15: UV and RV specimens on the shaking table: (a) frontal view, (b) focus on the RV specimen, and (c-e) close-ups of the setup schemes

4.2. Second experimental shaking table test campaign

Building on the previous experimental campaign described in the previous section and in Gioffré et al. [2023], a second experimental shaking table test campaign was carried out. The primary objective was to evaluate the effectiveness of a hemp-rope-based reinforcement in improving the seismic performance of a specimen already in a damaged state prior to the application of the strengthening system. This campaign focused on simulating a more realistic structural scenario, as historical vaults are rarely found in undamaged states. Such structures often exhibit cracks in the joints or unit sliding caused by various factors,

Table 4.4: Shaking table test sequence of the first experimental campaign

Test Code	Test Typology	Duration/Intensity	PGA [g]			Setup
			X	Y	Z	
AVT0	AVT	12 min				#1
0.2RE	ST	20%	0.1	0.1	0.1	#1
AVT_0.2RE	AVT	12 min				#1
0.4RE	ST	40%	0.3	0.2	0.2	#1
AVT_0.4RE	AVT	12 min				#1
0.5RE	ST	50%	0.3	0.3	0.2	#2
AVT_0.5RE	AVT	12 min				#2
0.6RE	ST	60%	0.4	0.3	0.2	#2
AVT_0.6RE	AVT	12 min				#2
0.7RE	ST	70%	0.4	0.4	0.3	#2
AVT_0.7RE	AVT	12 min				#2
0.8RE	ST	80%	0.5	0.4	0.3	#2
0.9RE	ST	90%	0.6	0.5	0.4	#2
1.0RE	ST	100%	0.6	0.5	0.4	#2
AVT_1.0RE	AVT	12 min				#2
1.1RE	ST	110%	0.7	0.6	0.5	#2
1.2RE	ST	120%	0.8	0.6	0.5	#2
1.3RE	ST	130%	0.9	0.7	0.5	#2
1.4RE	ST	140%	1.0	0.7	0.6	#2
1.5RE	ST	150%	1.0	0.8	0.6	#2
1.6RE	ST	160%	1.1	0.8	0.6	#2
1.7RE	ST	170%	1.2	0.9	0.7	#2
1.8RE	ST	180%	1.3	0.9	0.7	#2
1.9RE	ST	190%	1.3	1.0	0.8	#2
2.0RE	ST	200%	1.4	1.0	0.8	#2
AVT_2.0RE	AVT	12 min				#2
2.1RE	ST	210%	1.5	1.1	0.8	#2
AVT_2.1RE	AVT	12 min				#2
2.2RE	ST	220%	1.6	1.1	0.9	#2
AVT_2.2RE	AVT	12 min				#2
2.3RE	ST	230%	1.7	1.3	1.1	#2
AVT_2.3RE	AVT	12 min				#2

including past extreme loading events, changes in load configuration, environmental effects, and other damage-inducing causes. Testing a specimen in a realistic damaged configuration allows for a more accurate assessment of the novel system's contribution to load-bearing capacity and dynamic seismic behavior, which may otherwise be incorrectly estimated if evaluated on an undamaged specimen.

4.2.1. Specimen definition and construction

The previously tested UV specimen was employed again for a second experimental campaign after having sustained significant damage during the initial shaking table tests. This Damaged Unreinforced Vault (DUV) exhibited multiple cracks and partial sliding of individual units, as shown in Figure 4.16. In particular, the units located between cracks pC1-pC2 and pC3 and pC4 experienced sliding. The damage pattern appears almost symmetrical with respect to the keystone position, as also highlighted in the figure. More details about the damage pattern of the DUV and its tracking will be provided in Chapter 5.

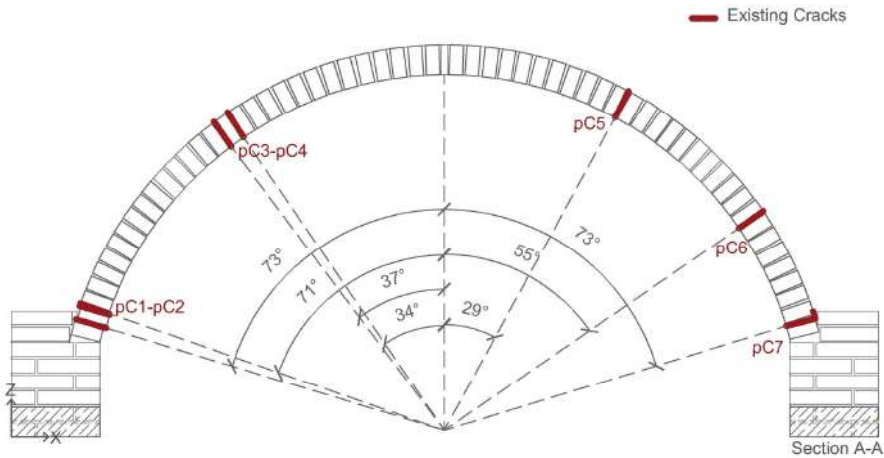


Fig. 4.16: Damaged Unreinforced Vault damaged pattern

The specimen was retrofitted using the same novel strengthening system investigated in the previous campaign, namely, a hemp-rope net embedded in a 30 mm-thick cocchiopesto matrix. The reinforcement process followed the procedure described in Section 4.1.1.2, which included surface cleaning of the extrados, application of the bonding resin, and placement of the hemp-rope net. These phases are illustrated in Figure 4.17. Panel (b) shows the placement of the hemp-rope net, arranged in a 2 cm × 7 cm grid, which was applied only after careful surface cleaning and bonding resin application. Panel (c) highlights the net positioned on the extrados of the DUV, with free hemp ropes extending over the abutments. Panel (d) shows the casting of the cocchiopesto

matrix using wooden formworks supported by a rubber-based tubular platform that acted as a guide during pouring, ensuring a uniform 30 mm thickness. However, one key modification was introduced: in the previous campaign, the reinforcement matrix was cast only after completing the abutments, embedding the net between the final two brick layers. In this case, the matrix was poured prior to brick placement (Figure 4.17(d–e)) to improve the bond between the strengthening layer and the masonry substrate, as this interface had previously proven to be the system’s most vulnerable point, consistently with other extrados applications [Zampieri et al., 2018]. No additional strengthening or repair interventions were carried out, i.e., no mortar repointing, crack filling, or unit realignment was performed. This specimen will be therefore referred to as Repaired-Unreinforced Damaged Vault (R-DUV) in this study.

4.2.2. Setup and instrumentation

The second experimental campaign was conducted exclusively on the R-DUV, positioned on one of the two shaking tables of the L.E.D.A. laboratory. The campaign had two primary objectives: (i) to assess the effectiveness of the bio-composite strengthening applied to the damaged specimen in terms of recovering its initial stiffness and modal properties, and (ii) to compare its seismic capacity, failure behavior, and dynamic response with those of the UV and RV specimens, respectively. This specimen was instrumented with a layout similar to that adopted in the previous campaign, but with an increased number of recording devices. Figure 4.18 highlights the extensive instrumentation on Sections A–A and B–B, corresponding to the front and back facades, respectively. Each section was instrumented with five accelerometers, at angular positions of 22° and 52° from the keystone in the same locations described in Section 4.1.3.2, recording acceleration components in the radial, tangential, and longitudinal directions. Section A–A was equipped with accelerometers, recording all three components, radial, tangential, and longitudinal, corresponding to sensors A1, A2, A3, and A4. These devices recorded the complete acceleration time histories for all three components. In contrast, accelerometer A5 on Section A–A and accelerometers A6 to A10 on Section B–B recorded only the radial and tangential components. Additionally, a tri-axial accelerometer was installed on the abutment (labelled CP). All accelerometers were piezoelectric devices with a sensitivity of 1 V/g. Additionally, as in the previous campaign, reflective markers were glued to the specimen, but solely on Section A–A. The displacement time histories of several locations were tracked by means of 8 high speed and high resolution infrared cameras. This setup enabled detailed displacement monitoring using 74 reflective markers, distributed as follows: 61 markers tracked the displacement time histories of individual units along the arch profile (labeled M1 to M61), while each abutment was equipped with six markers to assess boundary conditions and the connection between the masonry and the reinforcement system. Of these, four markers were glued to the ma-

sonry and two were positioned on the metallic foundation. Markers on the left abutment of Section A-A were labeled L1, L2, ..., L6, while those on the right abutment were labeled R1, R2, ..., R6. The markers were alternately placed on the intrados and extrados surfaces to capture possible crack openings on both sides. Additionally, Figure 4.19 shows that, in this configuration, the centering was lowered and anchored to the metallic foundation, as in the UV setup, with the aim of protecting the instrumentation. The figure also provides close-up

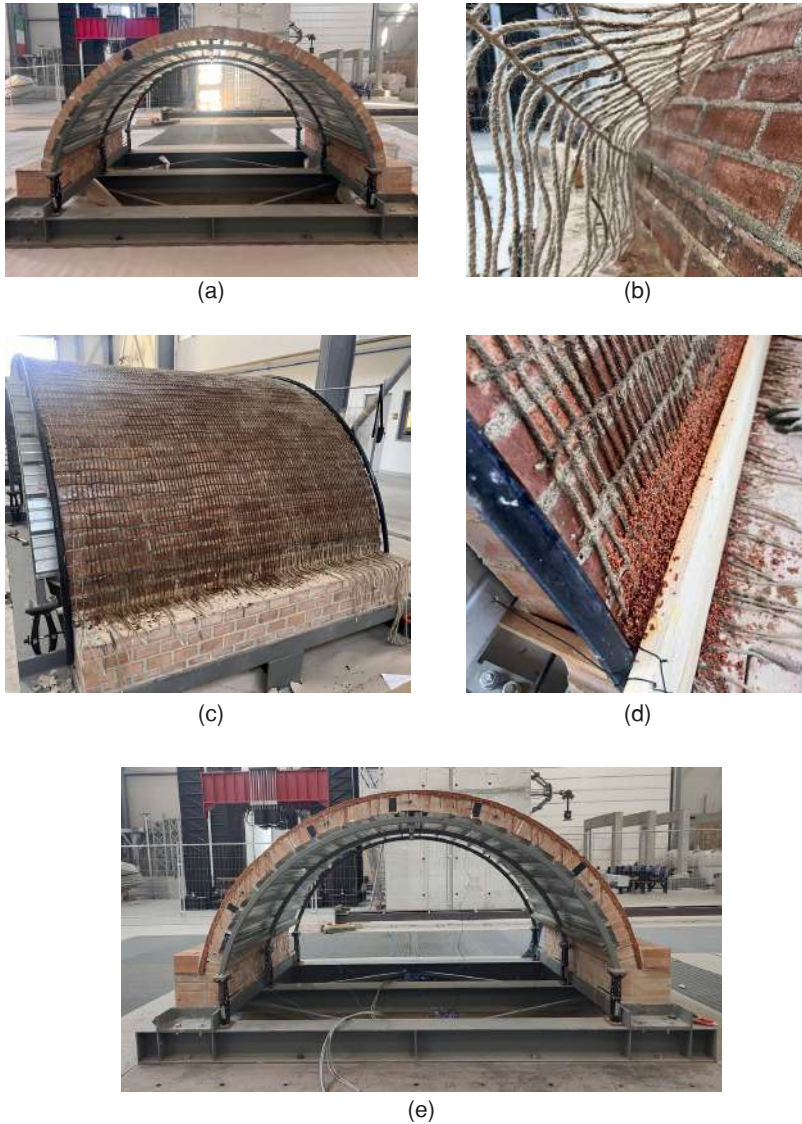


Fig. 4.17: Main phase of the strengthening solutions for the R-DUV (a-e)

views of the installed sensors.

A test plan similar to that of the previous campaign was adopted, as summarized in Table 4.5. As shown, the campaign comprised both STs and AVTs. The seismic tests were conducted at progressively increasing intensities, applying scale factors to the RE, which was again set as the Norcia earthquake of October

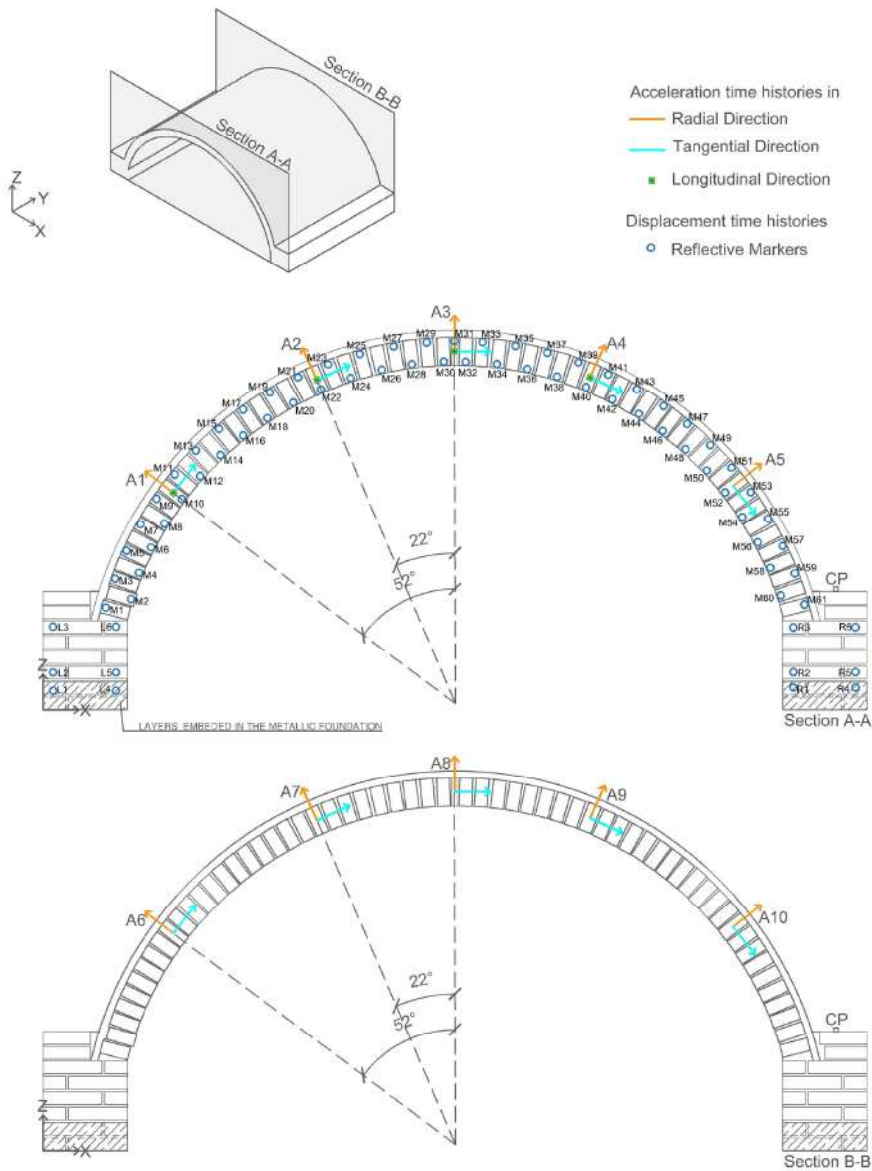


Fig. 4.18: Acquisition setup for R-DUV tests

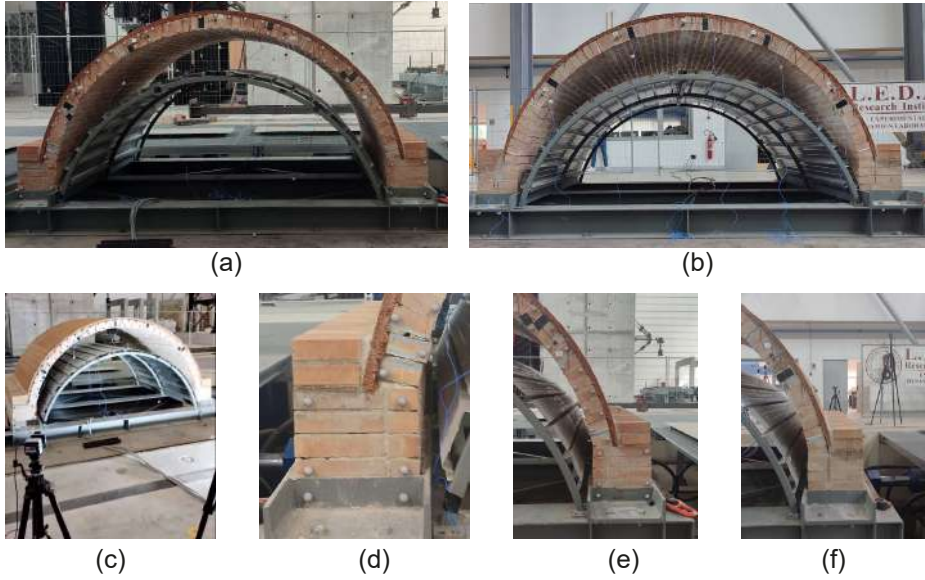


Fig. 4.19: R-DUV specimen on the shaking table: (a) frontal view and (b) back view, (c-f) close-ups for the setups schemes

30, 2016 (see Section 4.1.3.1 for further details). This procedure ensured that all specimens, UV, RV, and R-DUV, were subjected to identical input conditions, thereby minimizing uncertainties related to the excitation. AVTs were carried out without any additional inputs from the shaking tables.

A total of 40 tests were carried out, including both AVTs and STs, until the specimen could no longer be tested due to severe damage. The amplitude of the RE was scaled by factors ranging from 0.4 to 2.2, and AVTs of 15 minute duration were performed between successive seismic tests.

Table 4.5: Shaking table test sequence for the R-DUV specimen

Test Code	Test Typology	Duration/Intensity
AVT0	Ambient Vibration Test	15 min
0.2RE	Seismic Test	20%
AVT_0.2RE	Ambient Vibration Test	15 min
0.4RE	Seismic Test	40%
AVT_0.4RE	Ambient Vibration Test	15 min
0.5RE	Seismic Test	50%
AVT_0.5RE	Ambient Vibration Test	15 min
0.6RE	Seismic Test	60%
AVT_0.6RE	Ambient Vibration Test	15 min
0.7RE	Seismic Test	70%
AVT_0.7RE	Ambient Vibration Test	15 min
0.8RE	Seismic Test	80%
0.9RE	Seismic Test	90%
AVT_1.0RE	Ambient Vibration Test	15 min
1.1RE	Seismic Test	110%
AVT_1.1RE	Ambient Vibration Test	15 min
1.2RE	Seismic Test	120%
AVT_1.2RE	Ambient Vibration Test	15 min
1.3RE	Seismic Test	130%
AVT_1.3RE	Ambient Vibration Test	15 min
1.4RE	Seismic Test	140%
AVT_1.4RE	Ambient Vibration Test	15 min
1.5RE	Seismic Test	150%
AVT_1.5RE	Ambient Vibration Test	15 min
1.6RE	Seismic Test	160%
AVT_1.6RE	Ambient Vibration Test	15 min
1.7RE	Seismic Test	170%
AVT_1.7RE	Ambient Vibration Test	15 min
1.8RE	Seismic Test	180%
AVT_1.8RE	Ambient Vibration Test	15 min
1.9RE	Seismic Test	190%
AVT_1.9RE	Ambient Vibration Test	15 min
2.0RE	Seismic Test	200%
AVT_2.0RE	Ambient Vibration Test	15 min
2.1RE	Seismic Test	210%
AVT_2.1RE	Ambient Vibration Test	15 min
2.2RE	Seismic Test	220%

5. Experimental campaign test results

The experimental test campaigns pursued multiple objectives. The primary goal was to evaluate the seismic performance of masonry barrel vaults in their unreinforced, reinforced, and repaired conditions. Because investigations on full-scale vaults remain limited, difficulties arise in fully understanding their behaviour, as scale effects cannot be neglected. The proposed reinforcement and repair technique was based on a novel hemp-rope composite material, described in the previous chapter. In the first campaign, the tests were designed to highlight the effects of this reinforcement by directly comparing the behaviour of an unreinforced vault with that of a newly built, reinforced specimen. This ensured that any observed differences in performance could be attributed solely to the reinforcement. The second campaign was carried out to assess the effectiveness of the reinforcement when applied to a damaged specimen, thereby better replicating the condition of existing structures.

The comparative analysis of the experimental results is based on consistent parameters across both campaigns, with the unreinforced vault serving as the primary reference in most cases. The following chapters present the experimental results, beginning with the evolution of the damage pattern and the assessment of seismic capacity. In this study, seismic capacity is defined as the factor by which the Reference Earthquake (RE) was multiplied at the onset of structural failure. As highlighted in the literature, damage evolution can be correlated with stiffness degradation in specimens, which in turn affects their modal parameters. For this reason, a dedicated section reports the results obtained from Ambient Vibration Tests (AVTs), expressed in terms of variations

in modal parameters. Finally, the tracking of displacement time histories during the seismic tests enabled the analysis of deformed shapes and the monitoring of several locations' behaviour, based solely on the structural displacements, i.e., excluding the displacement contribution from the input. In this context, the observation of the displacement-based data is essential to individuate the most significant and critical locations working as base for numerical result comparisons. The chapter concludes with some conclusive remarks assessing the compatibility of the studied reinforcement systems and their effectiveness in enhancing seismic capacity. Additionally, each section concludes with the key findings derived from the comparison of results among the three specimens.

5.1. Damage and failure identification through visual inspections

5.1.1. Methodology

This section presents the damage patterns observed in the three tested specimens: the Unreinforced Vault (UV), the Reinforced Vault (RV), and the Repaired–Damaged Unreinforced Vault (R-DUV). The description addresses key aspects such as the location and distribution of cracks, the formation of hinges around which the blocks rotated, the debonding of reinforcement, and other critical features that developed during testing. Particular attention is given to the evolution of damage over time, including the sequence of crack formation throughout the seismic tests and the specific forms of damage to which reinforced vaults are especially vulnerable. The purpose of this section is to document the observed damage and provide a comprehensive interpretation of the structural response under seismic loading. By examining crack propagation, deformation patterns, and residual load-bearing capacity, the study evaluates whether cracks tended to develop in similar locations across the different vaults. Overall, the analysis contributes to a deeper understanding of failure mechanisms in reinforced masonry structures, highlighting the influence of reinforcement on both the initiation and progression of damage.

5.1.2. Unreinforced Vault

The UV specimen did not exhibit any visually detectable damage until it was subjected to the 0.4RE test (i.e., when the input intensity reached 40% of the RE). The resulting damage pattern, illustrated in Figure 5.1, was severe enough to warrant interruption of further seismic testing in order to prevent collapse. The vault sustained five cracks at the interfaces between the mortar joints and the bricks, extending through the entire depth of the specimen. The crack pattern was nearly identical on both facades, as schematized in Figure 5.1(a), while panels (b–d) provide three-dimensional views of the observed damage. The damage pattern is characterized by five cracks (labelled from C1 to C5) positioned with respect to the keystone. The pattern is almost symmetrical with respect to the keystone, with the exception of C2. Cracks C1 and C5 formed near the abutments at angles of approximately 71° and 73°, respectively. Two

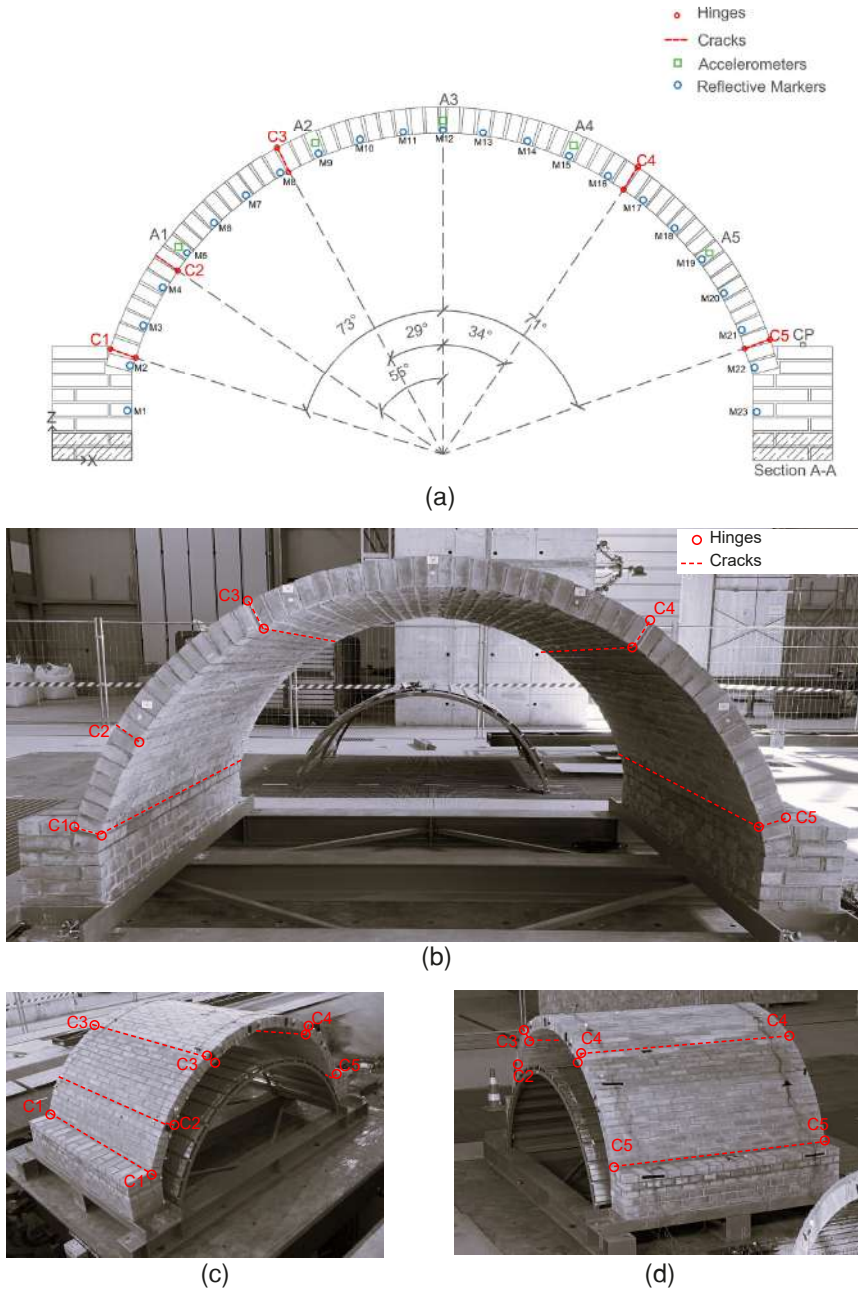


Fig. 5.1: Damage pattern of the UV specimen: (a) schematised view, (b) 3-D frontal view, (c) 3-D lateral view from the left abutment, and (d) 3-D lateral view from the right abutment.

intermediate cracks, C3 and C4, developed between bricks 19–20 (at about one-third of the span) and between bricks 17–18 (at about three-quarters of the span), forming angles of approximately 29° and 34° , respectively. The non-symmetrical crack C2 opened on the left side of the vault at an angle of 55° . In Figure 5.1, red circles mark the hinges, located at the opposite extremities of the cracks. As established in the literature, the formation of four hinges is sufficient to activate a kinematic mechanism, implying that the system becomes statically indeterminate [Heyman, 1982; Galano and Betti, 2024; Como, 2016]. In this case, the vault exhibited a clear rocking mechanism, with rotations occurring primarily around cracks C1, C3, C4, and C5. These cracks induced the rotation of the upper body portion of the vault (between C3 and C4) around alternating hinges at the intrados and extrados, thus forming a visible mechanism. Following this test, the centering was raised to support the vault, which was subsequently moved out of the shaking table facility. In the following discussion, the Unreinforced Vault in this damaged state will be referred to as the Damaged Unreinforced Vault (DUV).

5.1.3. Reinforced Vault

The RV specimen did not exhibit any visually detectable damage until the 2.2RE test, which led to the detachment of mortar pieces from the intrados and the initial formation of cracks at the mortar-brick interface. These cracks, labeled C1, C2, C3, and C4 in Figure 5.2, formed at 34° on both the left and right sides of the keystone, and at 44° and 71° on the right side, respectively. Notably, C1 and C2 were perfectly symmetrical with respect to the keystone, while C3 and C4 appeared only on the right side of the vault. It is worth noting that the crack openings could be detected only after reviewing the recorded video and were not clearly visible to the naked eye during the test. This highlights the starting of a damage pattern mechanism, made noticeable primarily due to the mortar detachment at the intrados. Moreover, these cracks opened up to the masonry-reinforcement interfaces and were therefore visible with openings at the intrados, highlighting a typical behaviour of masonry vault reinforced at the extrados.

Figures 5.2 and 5.3 highlight the critical damage observed at the end of the last seismic test (when 2.3 was used as a multiplying factor for RE) in sections A-A and B-B, respectively. As it is possible to note, especially on the right side of the structure (with reference to the keystone), a significant residual displacement is detectable. This seismic input caused severe structural damage, with multiple cracks opening across the vault. Most of these cracks extended through the depth of the specimen, but only a few propagated through the entire thickness, affecting even the reinforcement. Cracks C1, C3, C9, and C11 fully propagated through the extrados, forming hinges at the reinforcement layer. Meanwhile, cracks C2, C6, C7, C8, C10, and C12 extended up to the composite-masonry interface. Cracks C4 and C5 appeared near the abutments, at the connection

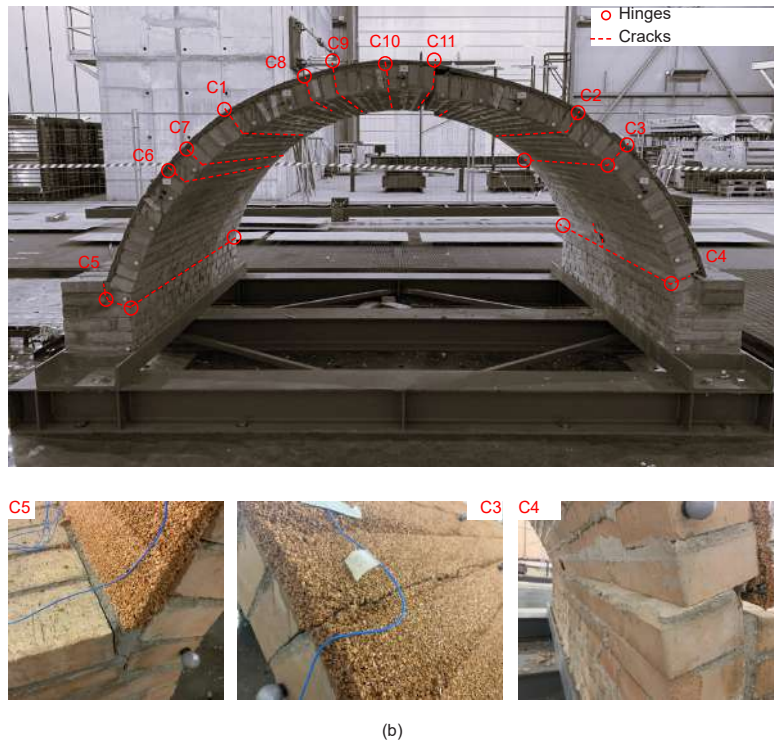
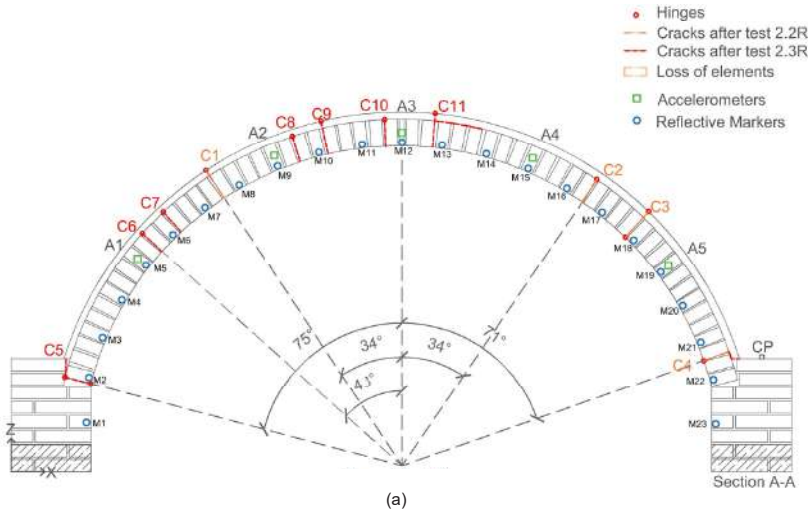


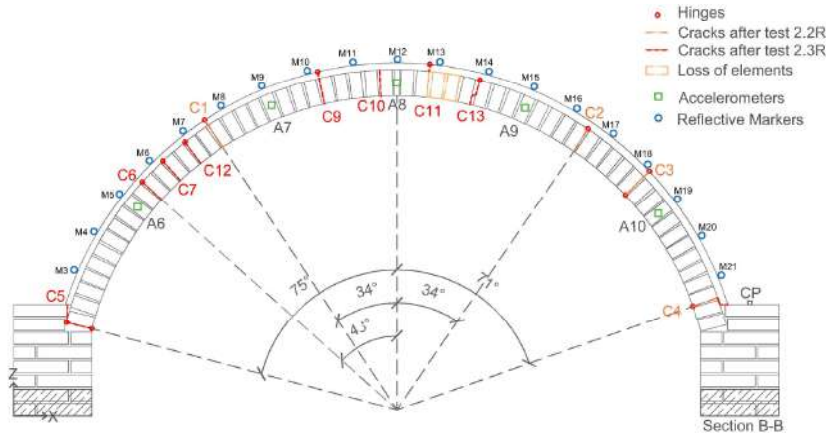
Fig. 5.2: Damage pattern of the RV specimen from section A-A: (a) schematised view, and (b) 3-D view and details of some cracks

between the reinforcement and the masonry, a common weak point in vaults reinforced at the extrados [Zampieri et al., 2018; Oliveira et al., 2010a; Alecci et al., 2016]. An additional debonding section created close to the keystone location and characterised by crack C11. Furthermore, section B–B exhibited substantial damage, with seven bricks detaching. In an unreinforced specimen, such a configuration would likely have resulted in total collapse, as the loss of these elements would have prevented the arch from maintaining its shape, which is essential for structural stability. The presence of the reinforcement, particularly the hemp rope net, prevented complete collapse. This effect is clearly observed in crack C4, which developed along a shear plane, causing debonding between the composite and the masonry, placing the hemp net in tension (as shown in Figure 5.3(b)) and thereby maintaining structural integrity.

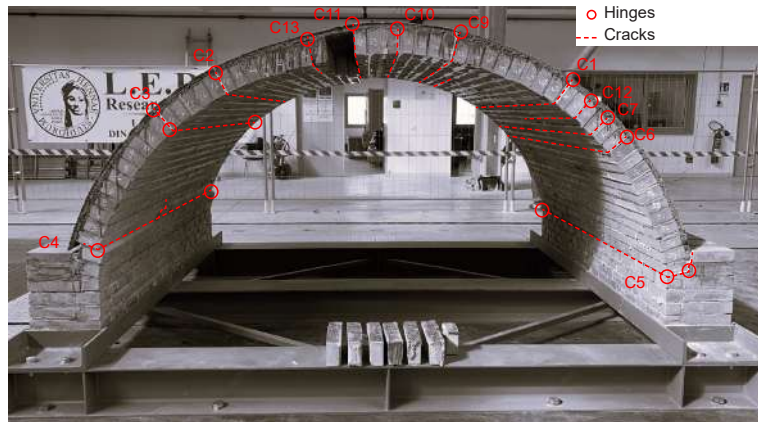
5.1.4. Repaired-Damaged Unreinforced Vault

As detailed in Section 4.2.2, the specimen already exhibited several cracks (labelled here as pC1, pC2, pC3, pC4, pC5, pC6, and pC7) and some misalignment of bricks. However, no visually detectable damage was observed until reaching test 2.0RE, i.e., when an input level twice that of the RE was applied. This test led to the formation of cracks labelled C1, C2, and C3 in Figure 5.4, which were primarily characterised by the detachment of mortar pieces. The latter two formed on the left side of the keystone at angles of 49° and 11° , respectively. As expected for masonry vaults and arches reinforced at the extrados, the cracks initially developed from the intrados of the specimen, without reaching the reinforcement layer. The subsequent seismic test (2.1RE) generated several new cracks, labelled C4, C5, C6, and C7. Notably, cracks C5 and C6 opened at positions almost symmetrical to cracks C1 and C2, but on the right side, with the keystone serving as reference. The most significant cracks formed, symmetrically, at the connection between the abutments and the steel foundation, leading to the activation of a rocking mechanism of the specimen around cracks C4 and C7 (Figure 5.4). These hinges ultimately led to the collapse of the specimen, together with the complete loss of part of the right abutment during the final seismic test, with an input equal to 2.2 times the RE (Figure 5.5).

The progression of damage during the final seismic test is illustrated in Figure 5.5. Panel (a) highlights the rocking mechanism around the cracks at the bases of the abutments, while panel (b) captures the expulsion of a portion of the right abutment, revealing a well-defined shear plane that coincided with the interface between the masonry substrate and the bio-composite reinforcement layer. The rocking of the abutment and the resulting loss of connection caused the complete rotation of the specimen, leading to the formation of multiple hinges and the detachment of the reinforcement layer from the bricks (panel (c)). This mechanism induced shear forces on the hemp ropes, causing their failure and resulting in the complete loss of a section of the vault (panel (d)). The hinges formed during seismic test 2.2RE are highlighted in



(a)



(b)

Fig. 5.3: Damage pattern of the RV specimen from section B-B: (a) schematised view, (b) 3-D view and details of some cracks

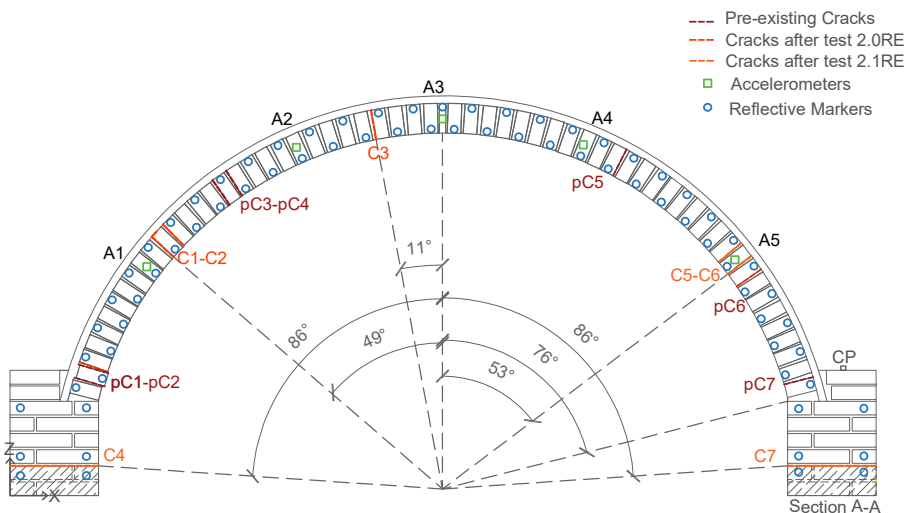


Fig. 5.4: Damage pattern of the R-DUV specimen up to test 2.1RE

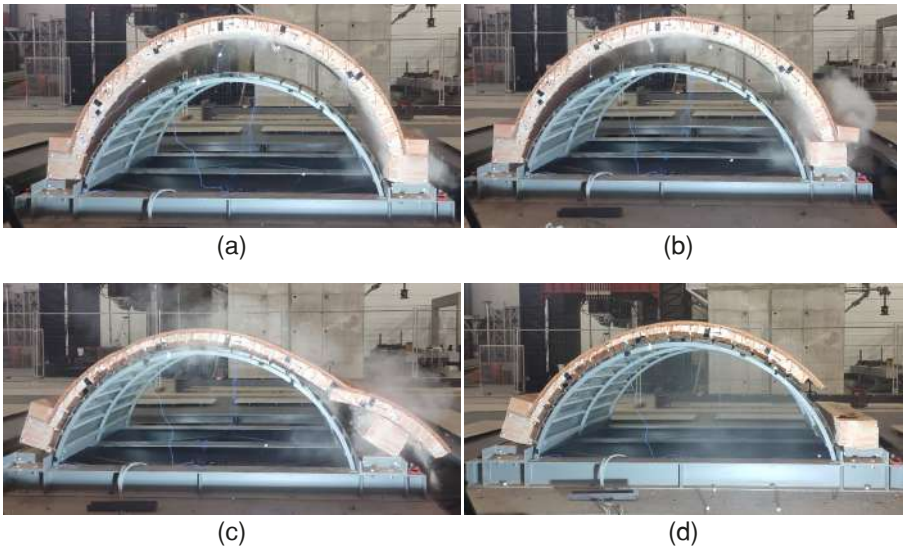


Fig. 5.5: Damage pattern evolution during the last seismic test for the R-DUV specimen from section A-A: (a) rocking mechanism around the hinges in the abutments, (b) expulsion of part of the abutment, (c) complete rotation of the specimen, and (d) final damage pattern.

Figure 5.6 by radial lines connecting the cracks to the geometric centre of the vault. These cracks, labelled C8, C9, C10, and C11, fully propagated through the thickness of the vault, with hinges forming at the extrados of the reinforcement, as evident in Figure 5.6(b). As previously noted, cracks C4 and C7 are symmetrical, forming angles of 86° with respect to the keystone, while cracks in similar positions, namely C8 and C10, opened at 39° (approximately at one-sixth of the span) and 34° (approximately at four-fifths of the span), respectively. Cracks C9 and C11 opened near the keystone and in correspondence with the shear plane.

5.1.5.Key findings

As a general conclusion, it is worth noting that the damage patterns between the several specimens developed in similar locations. In particular, the first cracks opened in comparable positions across the three configurations, located very close to those of the unreinforced specimen. As highlighted in Figure 5.7, both strengthened specimens, namely the reinforced and the repaired ones, exhibited crack formation in positions close to those where failure occurred in the UV specimen. In the figure, cracks in specimens RV and R-DUV that share similar positions with those of the UV are marked in red, with their location indicated by radial lines and angles measured with respect to the keystone. The RV specimen, in particular, exhibited a very close crack distribution, especially regarding the first cracks (labelled C1 to C6 in panel (b), and marked in red). This confirms that the initial damage developed in the same way for both specimens, although at different excitation levels. Specifically, the RV specimen failed under an input almost six times higher than that which caused the observed damage pattern in the UV specimen. Similar conclusions can be drawn for the R-DUV specimen, whose failure pattern was also close to that of the UV specimen, although the cracks farthest from the keystone developed in different locations (i.e., cracks C1 and C5 in the UV versus cracks C4 and C7 in the R-DUV). Additionally, collapse of the R-DUV occurred only when subjected to an excitation level 5.5 times that of the UV. This finding highlights that the initial responses of the three specimens were very similar. This is a significant observation, since the reinforced and repaired structures, despite having additional mass and stiffness, still developed cracks in the most stressed zones, which remained unaltered by the strengthening layer. After this initial damage, the effect of the reinforcement became evident, as it enabled the cracks to spread much more extensively.

In contrast, Figure 5.8 compares the damage pattern of the RV specimen with that of the R-DUV. Once again, several cracks appeared in similar positions, highlighted by red lines and angle values. Of particular interest are those that most affected the structural response. In both specimens, failure was characterised by debonding of the reinforcement at two locations: cracks C11 and C4 in the RV, and cracks C10 and C11 in the R-DUV. Moreover, crack C4

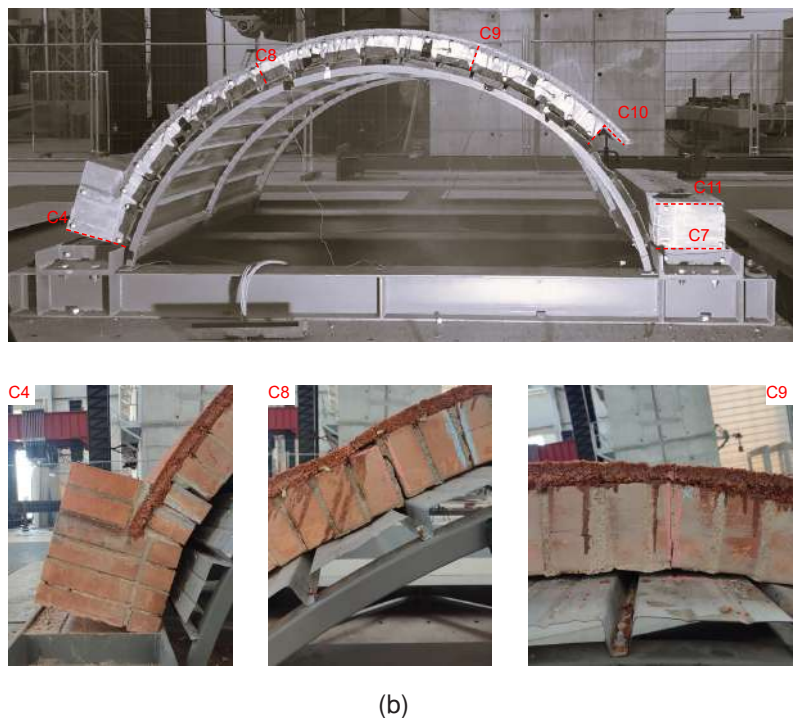
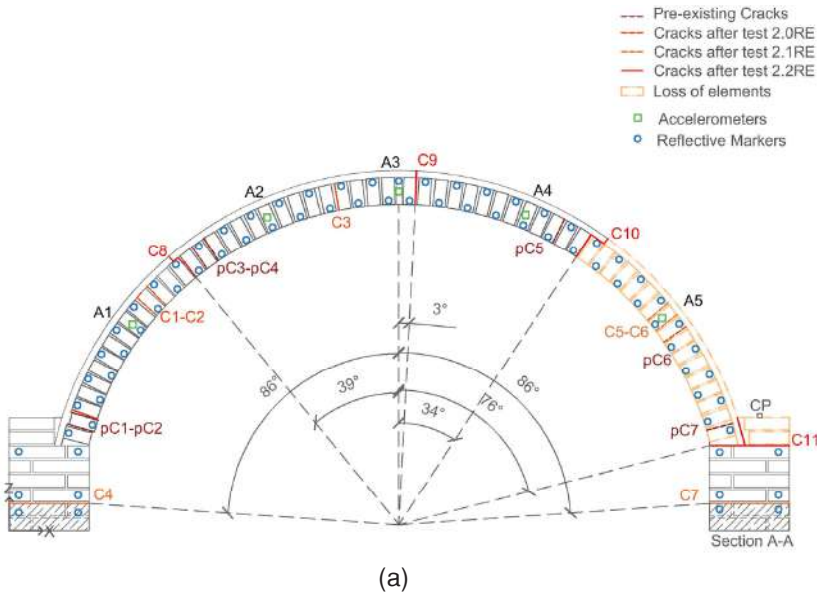


Fig. 5.6: Damage pattern of the R-DUV specimen from section B-B: (a) schematised view, and (b) 3-D view and details of some cracks

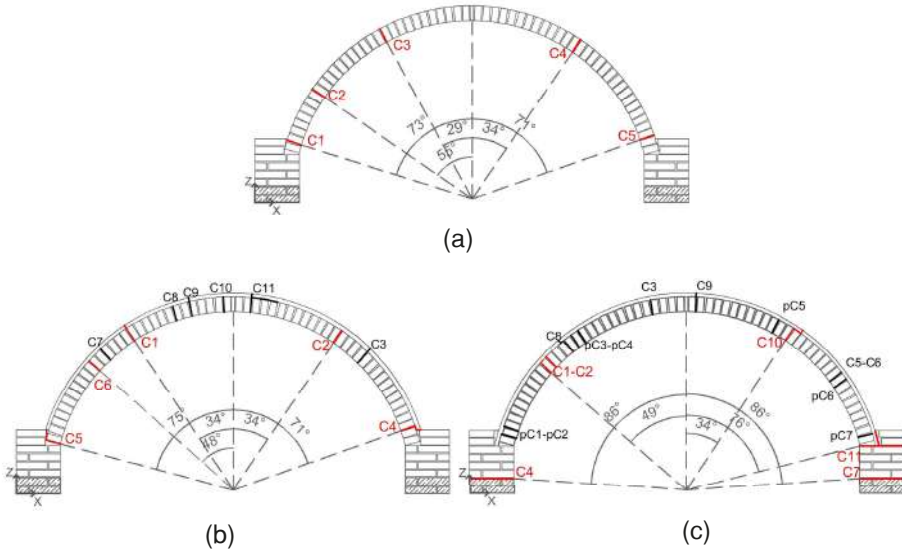


Fig. 5.7: Damage pattern comparison among the three specimens, considering the UV as reference: (a) damage pattern in the UV, (b) damage of the RV in similar locations, and (c) damage of the R-DUV in similar locations.

in panel (a) for the RV can be compared with crack C11 in panel (b) for the R-DUV. Both cracks developed at the interface between the strengthening layer and the masonry in proximity of the abutments, highlighting the criticality of this location, where shear failure occurred. Additionally, crack C11 in the RV and crack C9 in the R-DUV opened close to the keystone, another critical position. This finding confirms that the behaviour of the RV and R-DUV specimens was similar, even though the first reproduced a newly built structure, while the second was obtained from the DUV, i.e., the UV in its damaged condition. The latter better aimed to represent real existing structures, which usually present some pre-existing damages due to several factors (e.g., past extreme events, ageing). From the perspective of load capacity, both strengthened vaults exhibited almost the same performance under seismic loading, although with a complete different collapse mechanism, which was highlight influence by the connection detail between the reinforcement layer and the abutments, as described in Chapter 4.

5.2. Damage and failure identification through variation of the modal properties

5.2.1. Methodology

Literature provides several vibration-based methods for carrying out the dynamic identification of a system, which is of utmost importance in the civil

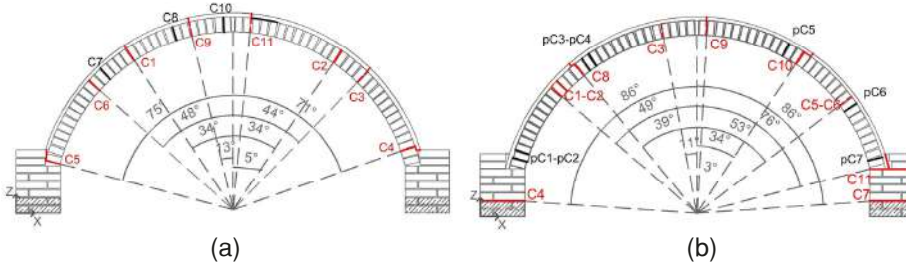


Fig. 5.8: Damage pattern comparison among the strengthened specimens: (a) damage of the RV in similar locations, and (b) damage of the R-DUV in similar locations

engineering field since it allows for the determination of vibration frequencies and corresponding mode shapes. These modal parameters are correlated with the mass and stiffness of the system.

The eigenvalue problem in the free vibration equation (Eq. 5.1) can be written in terms of the mass matrix $\mathbf{M}$ and the stiffness matrix $\mathbf{K}$.

$$(\mathbf{K} - \omega^2 \mathbf{M})\phi = 0 \quad (5.1)$$

where the vibration frequency f is obtained as ω (i.e., ω^2 is the eigenvalue) divided by 2π , while ϕ is the eigenvector, which represents the mode shape. Dynamic identification methods are essential for characterizing the behaviour of a structure and indirectly provide information about the $\mathbf{K}$ matrix, since there are usually fewer uncertainties about the mass matrix $\mathbf{M}$. Moreover, it has been proven that a change in the modal parameters can indicate a change in the dynamics of the system and, consequently, in the stiffness matrix $\mathbf{K}$, which is related to the evolution of damage [Rytter, 1993; Chopra, 2014] or the changes in the boundary conditions. It is possible to relate the evolution of damage to a change in the modal parameters if a pre-damaged condition is available.

Given this background, two damage indicators are commonly used, both based on the variation of modal parameters ([Gioffré et al., 2023; Mendes et al., 2014; Pepi et al., 2021]):

1. Damage indicator based on the variation of the mode shapes (d^m):

$$d_i^m = 1 - \text{MAC}_{u,c}^i \quad (5.2)$$

where the subscript i refers to the i -th mode considered, and the Modal Assurance Criterion (MAC) (defined as in Eq. 5.3) is calculate between the current condition kept into account (c) and the undamaged conditions (u):

$$MAC(\varphi_u, \varphi_c) = \frac{|\varphi_u^* \varphi_c|^2}{\|\varphi_u\|^2 \|\varphi_c\|^2} \quad (5.3)$$

where:

- φ_u and φ_c are two mode shape vectors.
- $|\varphi_u^* \varphi_c|^2$ is the squared absolute value of the Hermitian inner product.
- The superscript * indicates the conjugate transpose of a complex vector.
- $\|\varphi_u\|^2 \|\varphi_c\|^2$ is the product of the squared norms of the two vectors.

The MAC value ranges between 1 (perfect correlation between modes) and 0 (absence of correlation). Generally, MAC values higher than 0.7 indicate a good agreement between the mode shapes [Pepi et al., 2021; Gioffré et al., 2023]. Therefore, it can be assumed that if the damage indicator d_i^m is less than 0.2, the damage does not significantly affect the i -th mode shape.

2. Damage indicator based on the decrease in vibration frequencies (d_i^f):

$$d_i^f = 1 - \frac{f_{i,c}}{f_{i,u}} \quad (5.4)$$

where $f_{i,c}$ is the vibration frequency of the system in the current condition (c) and $f_{i,u}$ is the vibration frequency in the undamaged condition (u). The index i denotes the i -th mode considered. The assumption to evaluate the change in the i -th frequency is based on the correlation of the MAC for the i -th mode, indicating a good match between the compared modes in the undamaged and current conditions (i.e., MAC higher than 0.7).

The vibration frequencies and the correlated mode shapes were obtained through the Enhanced Frequency Domain Decomposition (EFDD), which is a widely used techniques for modal identification in the frequency domain. Following, a brief summary of this method is provide. For a more detailed discussion, the reader is encouraged to refer to Au [2017], Brincker and Ventura [2015] and Pepi [2019].

The EFDD method builds upon the classical Frequency Domain Decomposition (FDD) approach, improving its robustness and accuracy, particularly in the presence of noise and closely spaced modes. The core of EFDD lies in transforming the time-domain response signals (such as acceleration, velocity, and displacement) into the frequency domain using the Fourier transform. This allows to compute Power Spectral Density (PSD) matrix, which captures the

frequency content and cross-correlation of the structural system response across multiple monitored Degrees of Freedom (DoFs). The PSD matrix is then decomposed using Singular Value Decomposition (SVD) at each frequency line. Mathematically, this process can be expressed as follows (Eq. 5.5):

$$G_{yy}(f) = U(f)S(f)U(f)^H \quad (5.5)$$

where:

- $G_{yy}(f)$ is the PSD matrix at frequency f ,
- $U(f)$ contains the singular vectors (mode shapes),
- $S(f)$ contains the singular values,
- The superscript H denotes the Hermitian transpose.

Mathematically, the PSD matrix $G_{yy}(f)$ is expressed as (Eq. 5.6):

$$G_{yy}(f) = H(f)G_{xx}(f)H(f)^H \quad (5.6)$$

where $G_{xx}(f)$ is the input PSD matrix and $H(f)$ is the frequency response function.

In FDD, the singular values and singular vectors approximate the modal parameters, with peaks in the singular value plot indicating vibration frequencies and the corresponding singular vectors representing vibration mode shapes. EFDD enhances this process by identifying peaks in the frequency domain and converting the single-degree-of-freedom (SDOF) PSD functions back into the time domain via the inverse Fourier transform. This produces autocorrelation functions for each mode, from which damping ratios and refined vibration frequencies can be estimated accurately.

For output-only identification (such as ambient vibration tests), the input is unknown, and the method relies solely on $G_{yy}(f)$. EFDD refines the modal identification by fitting SDOF bell functions around the peaks and analyzing their time-domain counterparts. This method assumes that the system is lightly damped and that the modes are reasonably separated. However, it can handle moderate noise and coupling effects through careful peak selection and averaging techniques.

Damage identification based on the modal parameters was carried out by Giofr  et al. [2023] for the UV and RV specimens, using all the acquisition channels, as detailed in Section 4.1.3.2, which involved a significant amount of data. However, the vaults were investigated with different configurations. In order to establish a common basis for further analyses, the following investigations will therefore be carried out using only the acceleration time histories recorded in the radial direction for all three specimens, thus ensuring an equivalent number of channels for each case.

5.2.2. Unreinforced Vault

The EFDD method was applied to perform the modal identification, using the acceleration time histories recorded during tests AVT0, AVT_0.2RE, and AVT_0.4RE, i.e., before testing, after applying a seismic input equal to 20% of RE, and after 40% of RE, sampled at 1000 Hz. The analyses were conducted over the frequency range between 0 and 50 Hz, with subsequent decimation of the signals. The first three Singular Value Curves (SVCs) for the initial and final tests are shown in Figure 5.9. It is worth noting that harmonic excitations at multiples of 5 Hz were observed in all the tests, which could potentially affect the identification of the vibration modes.

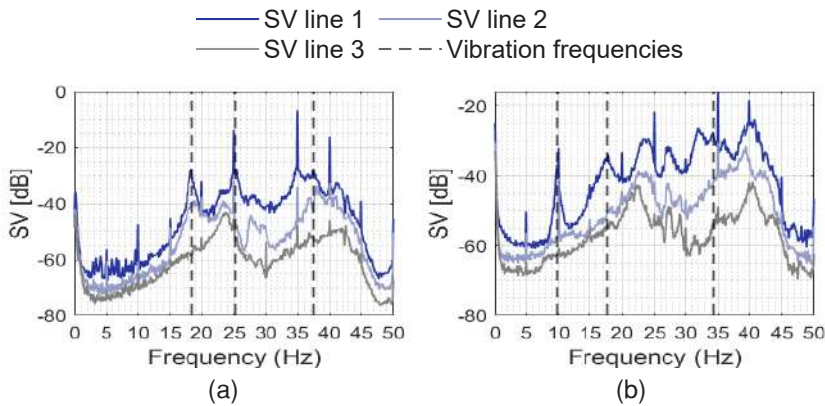


Fig. 5.9: Singular values curves assessed for UV: (a) undamaged condition, and (b) after test 0.4RE

This procedure allowed for the identification of three vibration frequencies and their corresponding mode shapes, which are summarised in Table 5.1. In the undamaged condition (AVT0), the first vibration frequency was identified at 18.4 Hz, with the corresponding mode shape shown in Figure 5.10(a), described as an in-plane mode. The second frequency was determined to be at 25.2 Hz, with its mode shape depicted in Figure 5.10(b). This mode exhibited an asymmetric movement of the two facades, which were asynchronous, particularly in the vertical components. Finally, the third frequency was identified at 37.5 Hz, with the corresponding mode shape presented in Figure 5.10(c), characterised by a predominant vertical motion.

Table 5.1: Frequency results for the UV specimen

	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]
AVT0	18.4	25.2	37.5
AVT_0.2RE	17.6	24.8	36.7
AVT_0.4RE	9.8	17.7	34.3

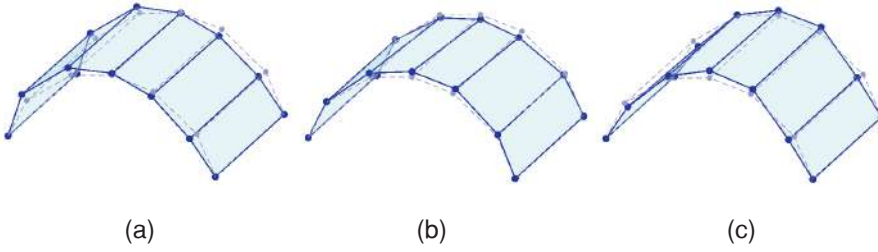


Fig. 5.10: First three vibration modes of the UV specimen

Significant insight into the variation of vibration frequencies is already provided in Table 5.1, but a clearer understanding of the effects of damage on the modal parameters is presented in Figure 5.11. Initially, it is worth noting that after seismic test 0.2RE, no substantial variations were observed in the modal properties, as the damage indicators (d) remained below 0.05 for all parameters except for the third vibration mode, where d^m reached approximately 0.5. Conversely, following the final seismic test, the presence of visible damage was reflected in a significant variation in the modal properties. The frequency-based damage indicators (d_f) reached values of approximately 0.5, 0.3, and 0.1 for the first, second, and third modes, respectively, indicating a marked variation in the frequency content, which is also evident in the singular value curves. The final dynamic identification, carried out using the inputs recorded during test AVT_0.4RE, showed that the first frequency decreased from 18.4 Hz to 9.8 Hz, the second frequency from 25.2 Hz to 17.7 Hz, and the third frequency only slightly, from 37.5 Hz to 34.3 Hz. However, no substantial variations were detected in the first two mode shapes, where d^m remained close to 0.05, while the third mode shape exhibited a considerable variation, with a damage indicator of approximately 0.6.

These findings highlight that damage affects each modal parameter differently, since changes in the stiffness matrix may influence the results in distinct ways: the first and second modes were more affected in terms of frequency variation than in mode shape, while the third mode was strongly affected in terms of mode shape but only marginally in frequency.

5.2.3. Reinforced Vault

Similarly, for the RV specimen, the EFDD method was adopted to perform the dynamic identifications, following the same procedure outlined in the previous section. The modal properties were identified for the following AVTs: AVT0, AVT_0.2NRC, AVT_0.4NRC, AVT_1.0NRC, AVT_2.0NRC, AVT_2.2NRC, and AVT_2.3NRC, i.e., before testing and after significant seismic excitation levels. The SVCs are reported in Figure 5.12 for selected tests: panel (a) shows the

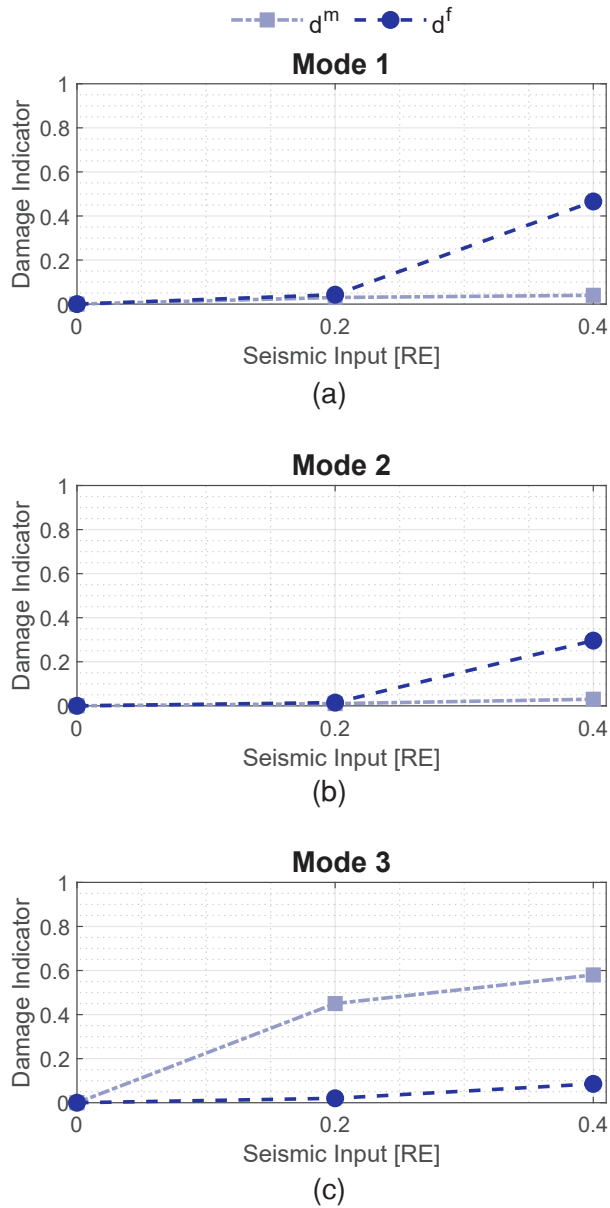


Fig. 5.11: UV damage indicators: (a) mode 1, (b) mode 2 and (c) mode 3.

curves in the undamaged condition, while panels (b–c) present the results after tests carried out at 40% and 100% of RE. The last three panels (d–f) highlight the decline in frequencies after the final seismic tests (with factors equal to 2, 2.2, and 2.3). As in the previous analyses, the curves also show the presence of harmonic excitations at frequency values corresponding to multiples of 5 Hz.

The analyses enabled the identification of the first three frequencies and their associated vibration mode shapes within the frequency range of 0–50 Hz, with results presented in Table 5.2.

For the undamaged condition (AVT0), the three vibration frequencies were identified along with their corresponding mode shapes, shown in Figure 5.13. The first frequency was found at 19.4 Hz, the second at 27.6 Hz, and the third

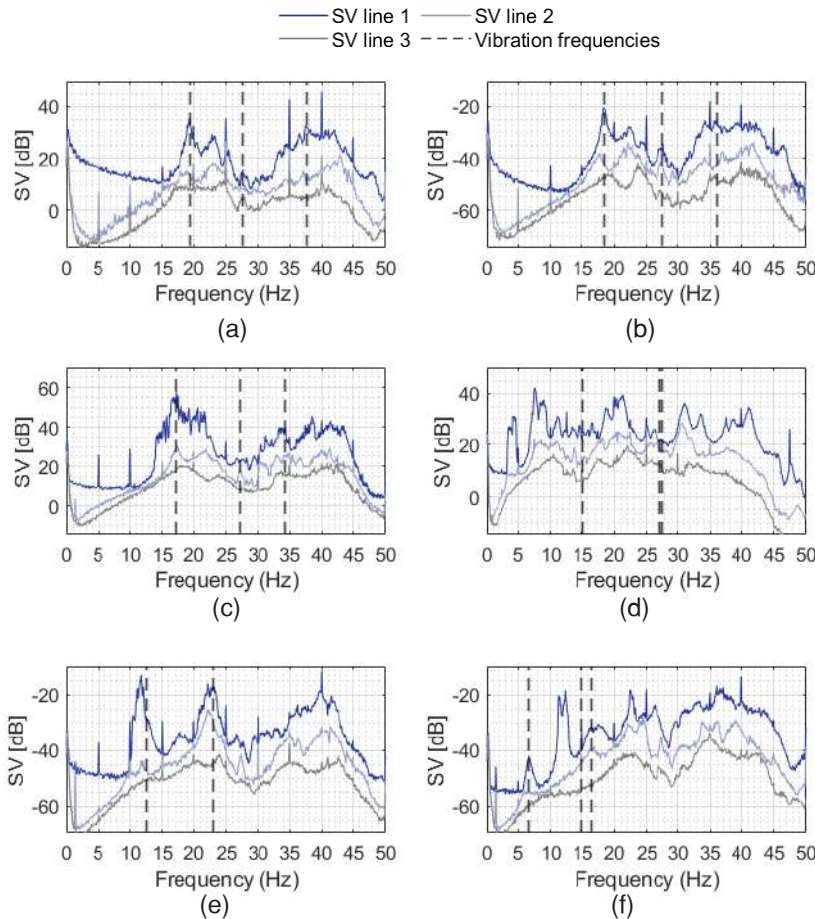
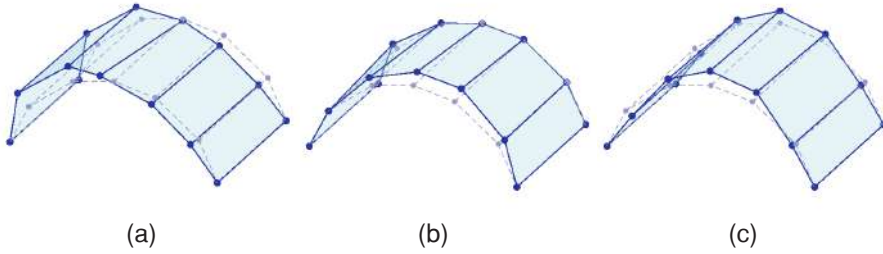


Fig. 5.12: Singular values curves for RV:(a) undamaged condition, (b) after test 0.4RE, (c) after test 1.0RE, (d) after test 2.0RE, (e) after test 2.2RE, and (f) after test 2.3RE

Table 5.2: Frequency results for RV specimen

Test Code	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]
AVT0	19.4	27.6	37.6
AVT_0.2RE	18.7	27.5	37.4
AVT_0.4RE	18.5	27.5	36.0
AVT_1.0RE	17.1	25	34.2
AVT_2.0RE	15.2	25	27.5
AVT_2.2RE	13.3	24.3	23.1
AVT_2.3RE	6.7	15.0	16.5

at 37.6 Hz. The first vibration mode can be described by an in-plane mode (Figure 5.13(a)), typical of curved structures, while the second mode can be described by asynchronous motion between the two facades (Figure 5.13(b)) and the third mode is described in the vertical direction (Figure 5.13(c)).

**Fig. 5.13:** First three mode shapes of the RV specimen

The damage indicator results based on the modal parameter variations are presented in Figure 5.14. Although no visible damage was observed before test 2.2RE, the adopted indicators provided information on the underlying variations. In particular, up to the test following 40% of the RE input, the only indicator with $d > 0.05$ was the damage indicator based on the vibration mode variation of mode 2, which reached a value of approximately 0.2. It is worth reminding that the damage indicators based on mode shape variation up to 0.2 did not reveal significant changes in the mode shapes. Subsequently, the results obtained after the seismic test with an input equal to RE yielded increasing values of the damage indicators for all the frequencies and modes analysed, although they still remained below 0.2. At test AVT_2.0RE, i.e., when the input amplitude was twice that of the RE, the damage indicator values for the frequency variation (d^f) were approximately 0.2, 0.1, and 0.25, while the corresponding values for the mode shape variation (d^m) were about 0.1, 0.3, and 0.15. These results indicate that significant changes in mode shapes were detected only for the second mode, since variation of ($d^m < 0.2$) are not considered as signif-

icant variation in the vibration modes. In particular, the frequencies at this stage decreased to 15.2 Hz for the first mode, 25.0 Hz for the second mode, and 27.5 Hz for the third mode, compared to their initial values of 19.4 Hz, 27.6 Hz, and 37.6 Hz, respectively.

A significant change in the modal properties occurred after seismic test 2.2RE, where the d^f factors exceeded 0.2 for all frequencies, although no significant variations were observed in modes 1 and 3, while d^m reached 0.3 for mode 2. This increase in the damage indicators was followed by a subsequent shift in

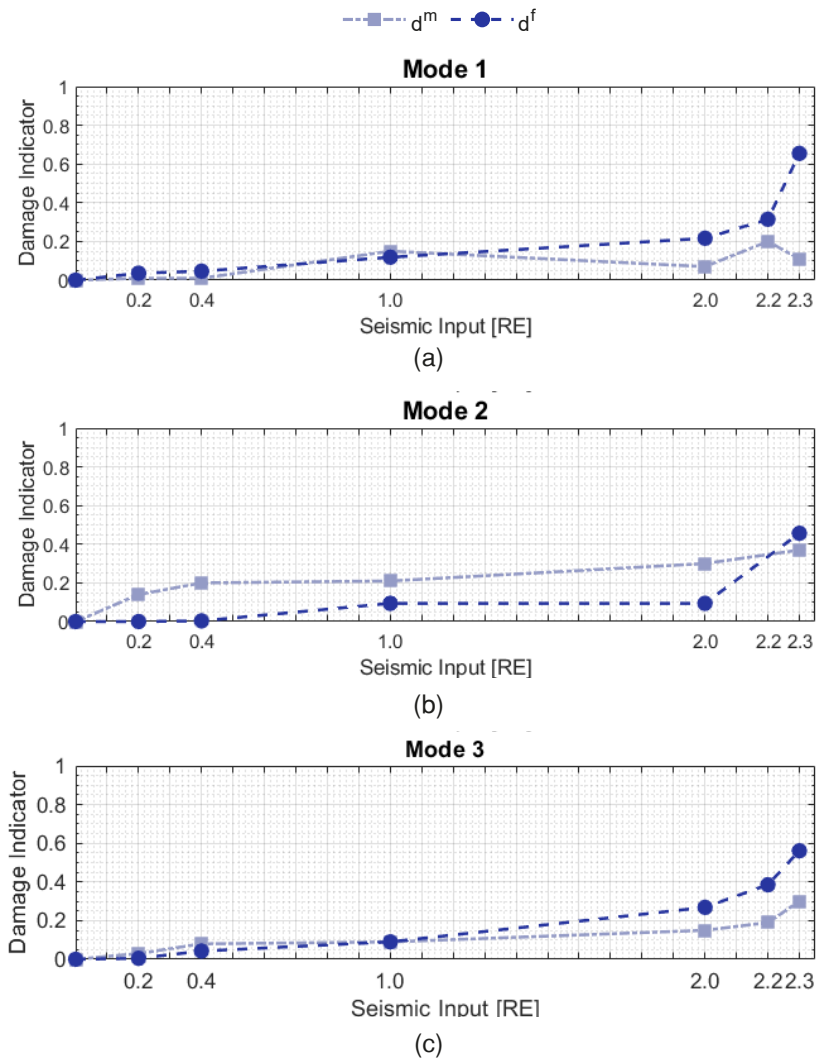


Fig. 5.14: RV damage indicators: (a) mode 1, (b) mode 2, and (c) mode 3

the trend after the final seismic test (2.3RE). At the end of the last test, the first frequency had decreased to 6.7 Hz (approximately one-third of its initial undamaged value), the second frequency was identified at 15.0 Hz, and the third at 16.5 Hz. The final damage configuration had a considerable impact on the first mode in terms of frequency reduction, with $d^f = 0.65$; however, the d^m value for the corresponding mode shape remained below 0.2, indicating a good correlation with the undamaged mode shape. Conversely, the second mode was affected more evenly in both frequency and mode shape, with d^f and d^m ranging between 0.4 and 0.5. The third mode was primarily affected in its frequency content, reaching $d^f \approx 0.6$, while the variation in mode shape was less pronounced, with $d^m = 0.3$.

These findings are consistent with the results obtained for the UV specimen, namely that stiffness degradation does not induce uniform effects across all modal parameters.

5.2.4. Repaired-Damaged Unreinforced Vault

A more detailed tracking of the variation of the modal parameters was carried out for the R-DUV specimen, with an AVT performed after each seismic test. This configuration allowed for the analysis of the effect of each seismic event on the damage evolution and, implicitly, on the stiffness variation. For this analysis as well, the same setup was adopted, i.e., carrying out the EFDD method using only the acceleration time histories recorded in the radial direction on both monitored sections, as highlighted in the setup (Figure 4.18). Figure 5.12(a) highlights the first three SVCs obtained from the response in the undamaged condition (AVT0), while panels (b–f) show the SVCs after seismic tests 0.4RE, 1.0RE, 1.8RE, 2.0RE, and 2.1RE, respectively.

Three vibration frequencies were identified below 50 Hz in the undamaged condition, corresponding to 17.0 Hz, 27.2 Hz, and 34.7 Hz. The associated mode shapes are reported in Figure 5.16. Panel (a) shows a clear in-plane motion, while panel (b) highlights the anti-phase motion between the monitored sections. Finally, panel (c) depicts a predominant vertical motion.

As shown in Figures 5.15(d) and 5.17, no significant changes in the frequencies or the corresponding mode shapes were observed up to an input level equal to the RE. All indicators remained below 0.2 until the seismic test with an input scaled by a factor of 1.7 times the RE, as reported in Table 5.3, which yielded values of 14.0 Hz, 24.8 Hz, and 29.3 Hz for the first three investigated frequencies. It is worth noting that, although the levels of the damage indicators are low, a visible progression of increasing values is still observed for all the modal parameters-based indicators considered.

After this test, the second mode exhibited indicator values exceeding 0.2 for both frequency variation and mode shape. In contrast, only the first mode d^f indicator was affected at this stage (seismic input 1.8RE), and no values exceeded 0.1 for the third mode or frequency. From this point onward, the damage

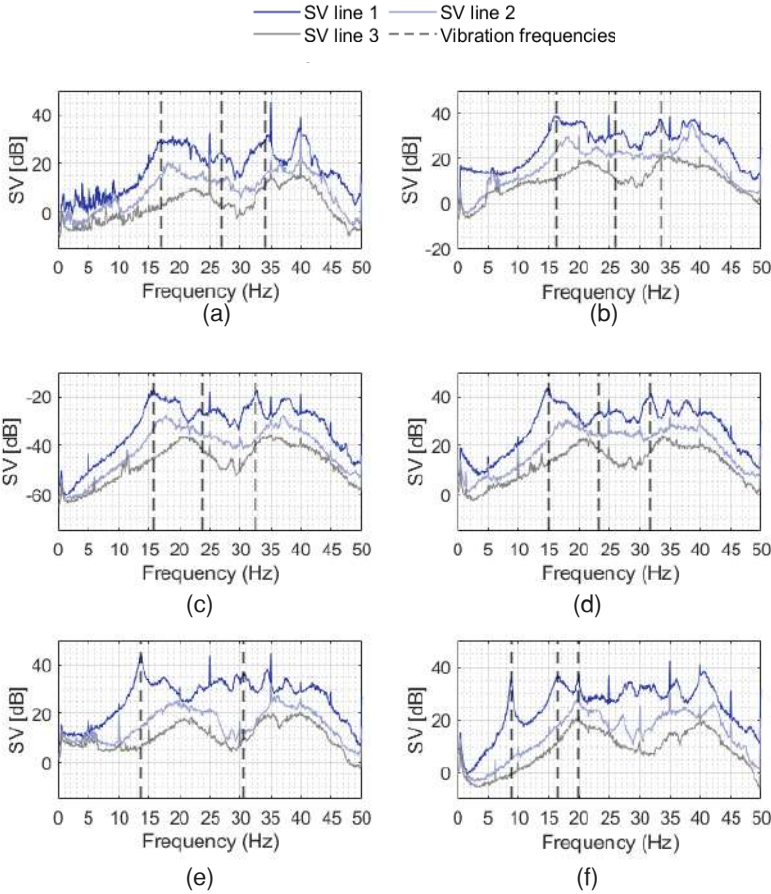


Fig. 5.15: Singular values curves for R-DUV:(a) undamaged condition; (b) after test 0.4RE, (c) after test 1.0RE, (d) after test 1.8RE, (e) after test 2.0RE; (f) after test 2.1RE

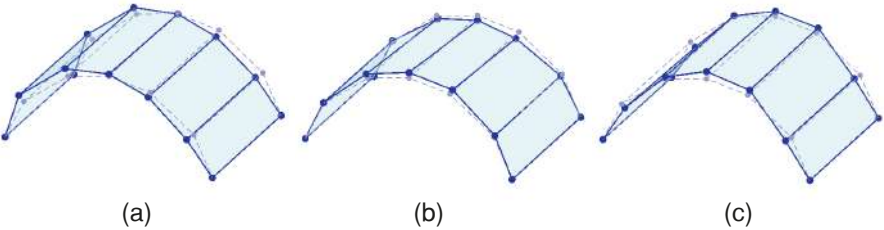


Fig. 5.16: First three mode shapes of the RDV specimen

Table 5.3: Frequency results for R-DUV specimen

Test Code	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]
AVT0	17.0	27.2	33.9
AVT_0.2RE	16.7	26.9	33.4
AVT_0.4RE	16.5	26.6	33.2
AVT_0.5RE	16.2	26.6	32.9
AVT_0.6RE	16.1	25.0	32.4
AVT_0.7RE	16.0	25.2	32.3
AVT_0.8RE	15.9	25.1	32.2
AVT_0.9RE	15.7	25.0	31.8
AVT_1.0RE	15.7	25.0	31.6
AVT_1.1RE	15.6	25.0	31.4
AVT_1.2RE	15.4	25.0	31.5
AVT_1.3RE	15.1	25.0	31.3
AVT_1.4RE	14.6	24.9	31.3
AVT_1.5RE	14.3	24.9	31.2
AVT_1.6RE	14.3	25.0	30.8
AVT_1.7RE	14.0	24.8	29.3
AVT_1.8RE	13.6	23.4	29.2
AVT_1.9RE	12.7	20.6	27.6
AVT_2.0RE	11.6	19.6	25.0
AVT_2.1RE	8.7	16.6	20.2

significantly impacted the first frequency, with a noticeable change in trend after test 2.1RE, resulting in a final $d^f \approx 0.5$ and a corresponding frequency of 8.7 Hz. However, the d^m value for mode 1 remained below 0.2, indicating that the mode shape did not experience any significant change. Conversely, mode 2 was similarly affected by the damage in terms of both frequency decay and mode shape variation, with slightly higher values generally recorded for d^m . Final values reached 0.5 and 0.4 for d^m and d^f , respectively, with the frequency assessed at 16.6 Hz. In contrast, the third mode exhibited a nearly identical evolution in both frequency and mode shape, with the curves almost perfectly overlapping and reaching a final value of 0.4 (frequency at 20.2 Hz).

Tracking the damage indicators throughout all seismic tests allowed the monitoring of modal property variations, which became significant (i.e., values higher than 0.2) only after an input equal to 180% of the RE was applied. It is worth noting that the progression of damage manifests differently across the modes. In particular, the first mode was more affected in terms of frequency, while its mode shape remained essentially unchanged. By contrast, the second and third modes and their corresponding frequencies were affected more evenly, with the progression of d^m and d^f almost overlapping for the third mode, whereas the second mode exhibited different progression values for the frequency varia-

tion and the mode shape. The assessment of damage is further supported by the pronounced shift in d value trends observed between tests following input excitations of 2.0RE and 2.1RE.

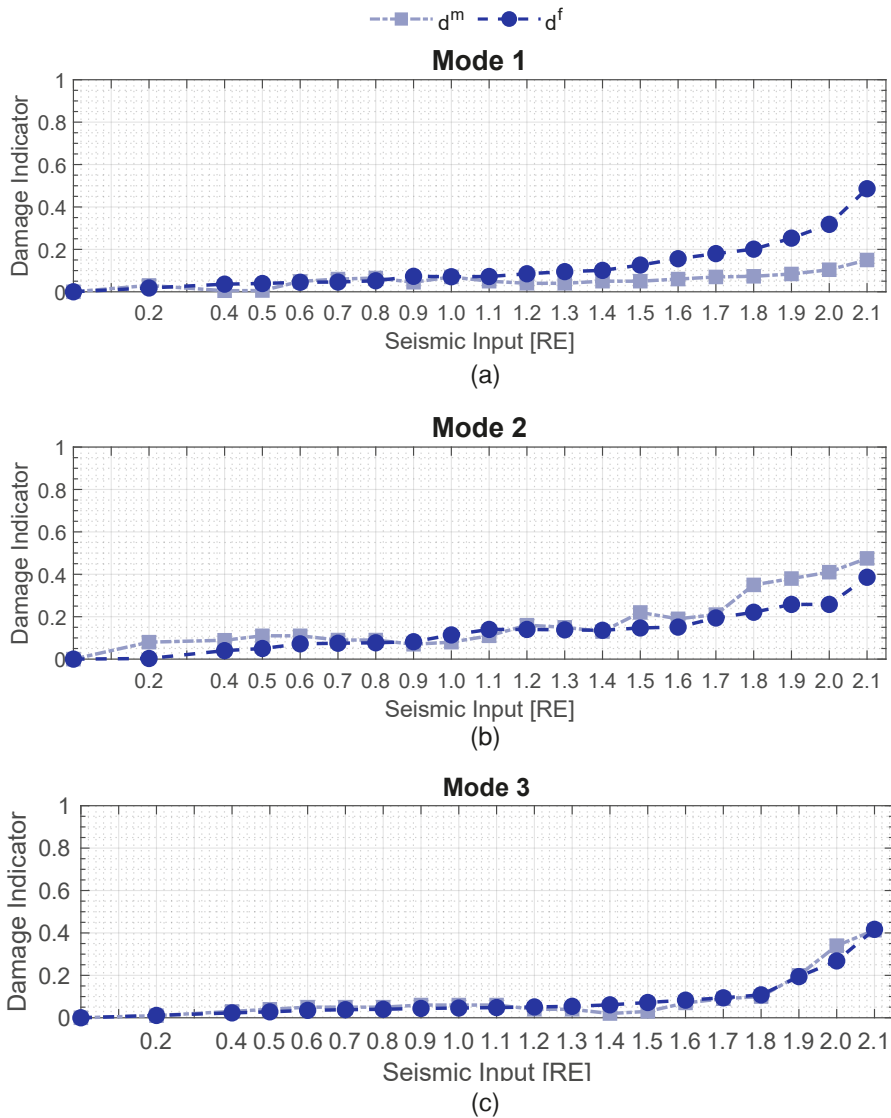


Fig. 5.17: R-DUV damage indicators: (a) mode 1, (b) mode 2, and (c) mode 3

5.2.5.Key findings

One of the main criteria to evaluate the compatibility of a reinforcement or repair technique is its influence on the modal properties of the structure, as these significantly affect the overall dynamic behaviour. In this context, the

effects of the proposed strengthening are assessed in terms of percentage errors in the vibration frequencies, with reference to the UV specimen, which represents a typical existing structure. These frequencies are compared with those of the Reinforced Vault and the Repaired-Damaged Unreinforced Vault. The comparison is carried out under the assumption that the i -th vibration modes are in good agreement. In this regard, MAC values close to 0.8 are generally considered to indicate a satisfactory correspondence between mode shapes.

The comparison between the frequencies and the corresponding MAC values for the same i mode is reported in Figure 5.18(a), while the frequency errors are summarised in Table 5.4, row (a). The RV specimen shows an increase in all the frequencies, although the percentage error remains below 9.5% for all modes, as highlighted by the grey hatched area in the figure. The first and second frequencies show the largest differences, with the second frequency exhibiting the highest variation, whereas the third one is only minimally affected, yielding a value very close to that of the unreinforced reference. Furthermore, all modes are associated with very good MAC values, equal to 0.95, 0.94, and 0.88 for the first, second, and third modes, respectively.

Table 5.4: Percent frequency errors with reference to the UV condition

Row	Specimen	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]
(a)	RV	+5.5%	+9.5%	+0.3%
(b)	DUV	-46%	-29.8%	-8.5%
(c)	R-DUV	-7.6%	+8.4%	-9.6%

Panel (b) of Figure 5.18 and row (c) of Table 5.4 present the comparison between the UV specimen and the R-DUV in the undamaged condition. In this case, not all frequencies increased compared to the UV specimen; rather, different effects were observed across the modes, highlighting how the strengthening layer, by introducing additional mass and stiffness, influenced the modes differently. The percent errors for the first three frequencies were -7.6%, +8.4%, and -9.6%, respectively. It is important to note, however, that the intervention was applied to a damaged specimen (DUV), i.e., the UV in its damaged condition. As previously recalled, the damage significantly reduced the frequencies by 46%, 29.8%, and 8.5% for the first, second, and third frequency, respectively, thus indicating a substantial change in the frequency content (see row (b) of Table 5.4). The proposed strengthening did not fully recover the initial undamaged configuration, but the percent errors remained within $\pm 10\%$. Moreover, the MAC values for all analysed modes exceeded 0.78, with this minimum value corresponding to the third vibration mode, which was the most affected by the damage in the DUV configuration, while the first and second modes underscored good correspondence with MAC results of 0.91 and 0.84, respectively.

Additionally, a comparison of the frequency content and vibration modes was carried out between the RV and R-DUV specimens, with results shown in Fig-

ure 5.19. A good correlation between the i -th modes was confirmed, with MAC values higher than 0.83; the lowest value was obtained for the second mode,

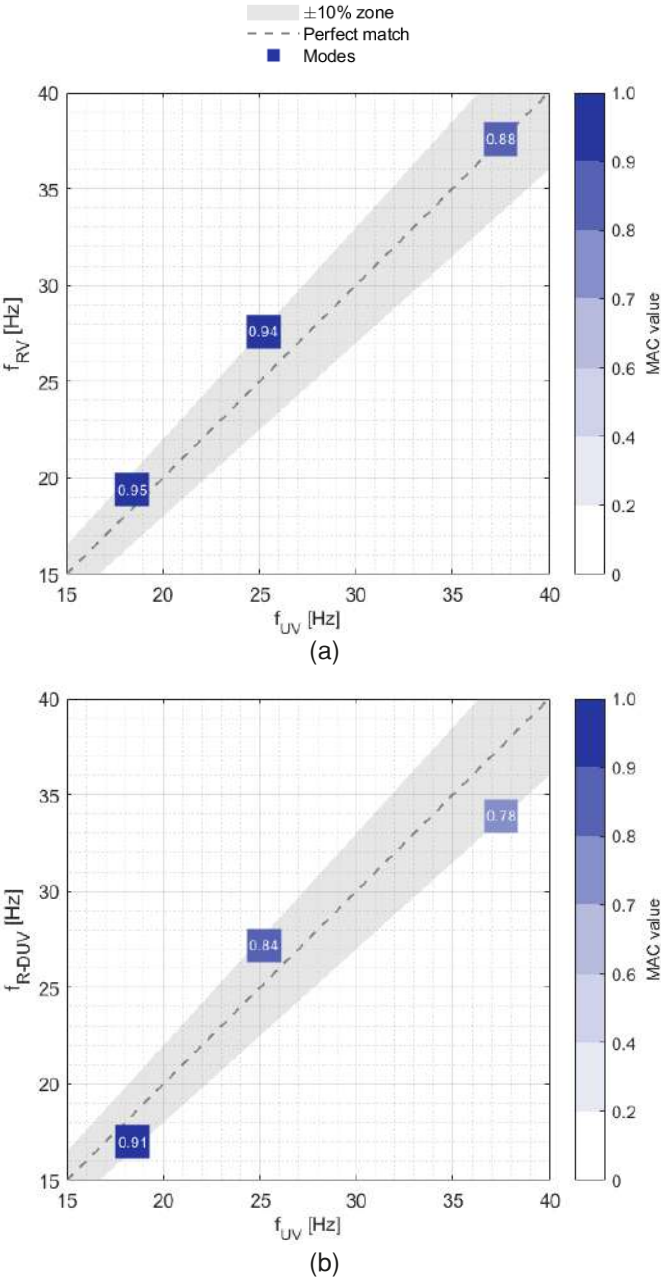


Fig. 5.18: Compatibility assessments of the proposed strengthening through comparison of modal parameters

while excellent agreement was found for the first ($\text{MAC} = 0.94$) and third mode ($\text{MAC} = 0.90$). As shown in the figure and in Table 5.5, the second and third frequencies presented percent errors lower than 10% between the RV and R-DUV conditions, while the first frequency exhibited a larger error of approximately 12.4%. These findings are consistent with the expected behaviour, since the RV specimen was constructed in an ex-novo condition, whereas the R-DUV specimen was derived from an element with degraded stiffness. The two specimens have the same mass, so the only parameter influencing the comparison is the effect of the additional stiffness. Although the results do not indicate a perfect match between the frequency contents of the specimens, it is worth noting that the second frequency is almost perfectly matched, while the errors recorded for the other two frequencies remain below 13%.

Table 5.5: Percent frequency errors with reference to the RV condition

Specimen	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]
R-DUV	-12.4%	-1.5%	-9.8%

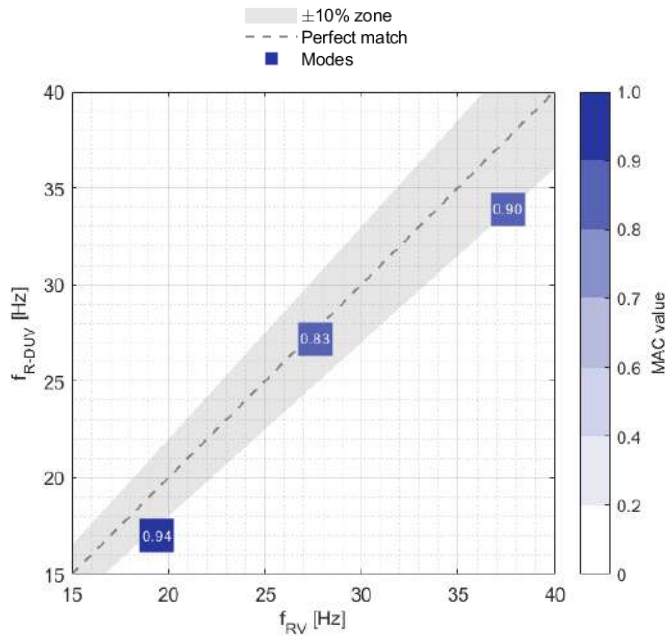


Fig. 5.19: Comparison of the modal properties between RV and R-DUV

Lastly, it is worth discussing the modal parameter-based damage indicators. Some of these are routinely used in Structural Health Monitoring (SHM) worldwide. Their reliability in tracking the evolution of damage has been demon-

strated in all tested configurations, and particular attention should be paid when variations exceed 20%. In addition, several considerations can be drawn regarding the damage evolution in masonry barrel vaults:

- The variation of the first frequency provides a valuable tool for monitoring the evolution and progression of damage, with reductions greater than 40% being particularly critical.
- The second and third frequencies, together with their associated mode shapes, are affected more evenly, with comparable influence from both frequency decay and mode shape variation.
- A significant evolution of damage can also be identified through changes in the damage indicator trend, i.e., when a marked shift occurs that is not captured by the previously considered indicators.

5.3. Rapid identification of sensitive locations as a basis for numerical comparison

5.3.1. Methodology

Displacement monitoring was primarily adopted during the Seismic Tests (STs) as it provides direct insight into the vault behaviour under seismic excitation. The purpose of the following analysis is not to derive displacement–capacity curves or to deliver a complete assessment of structural performance. Rather, the objective is to implement a rapid screening strategy able to identify the monitored locations that exhibit the most significant response, either in terms of displacement demand or relative angular variation. These locations are interpreted as sensitive points of the vault, i.e. areas where the response concentrates and which can therefore be considered as representative monitoring locations for subsequent numerical comparisons. To this end, only the most significant seismic tests are considered, corresponding to the cases in which the AVT-based modal damage indicators exceeded the threshold value of 0.2.

Given the large amount of data typically collected during dynamic tests (e.g., in the UV specimen, 37 monitored locations sampled at 100 Hz, corresponding to approximately 5000 instants per location), a detailed point-by-point analysis over the entire test would not be practical. The following adopted strategy is therefore conceived as an expeditious tool for reducing and interpreting the dataset, aimed at extracting the most sensitive locations with limited computational effort.

The procedure consists of three main steps:

- **Relative displacement patterns:** For each reliable monitored location, the maximum absolute displacement is identified, and the corresponding instants are used to reconstruct the relative displacement patterns of the vault. These provide a simplified picture of how deformation is distributed.

- **Angular variation indicator:** Based on the time histories of relative displacements, the angle between consecutive monitored points is computed. Variations with respect to the initial configuration are recorded as a qualitative indicator of potential changes in the locations' distribution. The analysis focuses only on the instants associated with maxima variations.
- **Combined assessment:** Displacement maxima and angular-variation maxima are compared to highlight common significant instants, providing a synthetic description of the response in terms of deformed shape, relative displacement patterns and angular variation indicator patterns.

This approach does not aim at delivering an exhaustive picture of damage evolution. Rather, it serves as a first-level analysis to rapidly identify the regions of the vault most affected by seismic input. Only reliable locations, i.e., monitored points whose measurements were free from drift, spikes, or other measurement errors, were considered in the analysis. When similar relative displacement patterns and angular variation indicator patterns were observed at multiple time instants, only those corresponding to the largest values were selected as representative of the structural response. This analysis allowed the identification of the most sensitive locations, whose Relative Displacement Time Histories (RDTHs) are examined in detail. Importantly, this procedure also provides a rational basis for selecting which monitored points should be prioritised in subsequent comparisons with numerical models, thereby supporting a more targeted and meaningful validation process.

5.3.1.1. Relative displacements

As detailed in Chapter 4, several locations on the vault specimens were instrumented and monitored throughout all the seismic tests, adopting different marker layouts for the UV, RV, and R-DUV specimens (see Figures 4.13, 4.14, and 4.18). The displacement time histories recorded by the markers were processed to compute what is hereafter referred to as the relative (re.) displacements, i.e., the displacements of the vault specimens relative to the imposed input motion. The relative displacements were calculated according to Eq. 5.7:

$$u_d^n(t) = \bar{u}_d^n(t) - \bar{u}_d^i(t) \quad (5.7)$$

where:

- $u_d^n(t)$ is the relative displacements time history;
- $\bar{u}_d^n(t)$ is the displacements time history recorded by the markers on the specimen;
- $\bar{u}_d^i(t)$ is the displacements time history applied as input;
- the superscript n refers to the location of the marker;
- the subscript d refers to the direction, i.e., X, Y, or Z.

For each reliable marker (i.e., excluding those affected by measurement errors or detachment during the tests), the maximum absolute relative displacement was determined together with the corresponding time instant. Several maxima occurred simultaneously across different markers; in these cases, only unique time instants were retained to avoid redundancy.

5.3.1.2. Angular variation indicator

In parallel, the variation of the angle between consecutive monitored locations was considered. Vectors were defined by connecting each marker to the centre of the arch geometry, and the angle between adjacent vectors defined a Section (S). The variation in these angles, compared to their initial values, was used as a qualitative indicator of potential significant changes in the configuration. If n markers were considered reliable, $n - 1$ angles were computed, one for each section, and significant deviations from the initial configuration were considered. At this stage, an angular variation indicator was proposed, labelled as d^θ , and defined as (Eq.5.8):

$$d_s^\theta(t) = 1 - \frac{\theta_s(t)}{\theta_s(0)} \quad (5.8)$$

in which:

- $\theta_s(t)$ is the angle evaluated at each time frame;
- $\theta_s(0)$ is the angle evaluated before the beginning of each seismic test;
- the subscript s refers to the section considered (e.g., the one evaluated between marker 2 and 3).

This indicator equals zero when no angular variation occurs, while significant increases or decreases may signal changes in the displacement in two consecutive locations. The maximum absolute value of d_s^θ was determined for each section, and the corresponding time instants were compared with those associated with peak relative displacements. Based on these significant instants, the deformed configuration of the vault was reconstructed, including the relative displacement of each location along the measured directions and the associated d_s^θ values.

This combined evaluation of relative displacements and angular variations provided a concise yet informative description of the vault response during seismic tests, enabling the rapid identification of sensitive locations.

5.3.2. Unreinforced Vault

As illustrated in Figure 4.13, the Unreinforced Vault was equipped with 23 markers along Section A–A, which recorded displacement time histories during two seismic tests, namely 0.2RE and 0.4RE.

It should be emphasized that, prior to the 0.4RE test, no reliable relative displacement records were obtained, as the detected amplitudes were at or below

the sensitivity threshold of the computed relative displacement (0.5 mm). Consequently, the discussion presented herein focused exclusively on the final seismic test of the UV specimen, with the monitored locations considered reliable from M2 to M22. Furthermore, the Y-direction response was disregarded, since its magnitude was comparable to the measurement uncertainties.

The procedure described in the previous section enabled the identification of several critical locations, considered representative of the structural response.

Figure 5.20(b) depicts the deformed configuration of the vault, highlighting significant changes in the upper portion of the structure at 14.2 s. The deformed shape captures the considerable displacement in the upper part, while minimal or no visible changes are observed in the lower locations. The clear displacement in the positive X direction emphasizes the different behaviour of the two portions of the vault, which is better explained by the relative displacement pattern.

As reported in panel (c), the relative displacements ranged between 20 mm and 30 mm for locations from M9 to M18, corresponding to the portion of the struc-

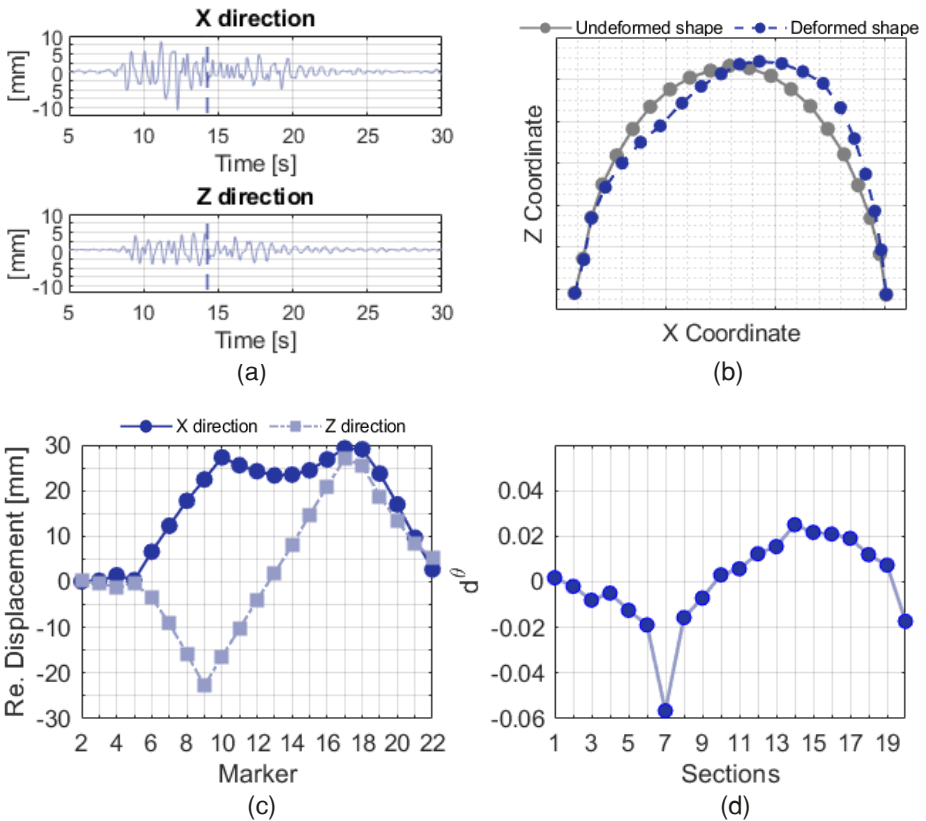


Fig. 5.20: Structural response of the UV specimen during test 0.4RE at 14.2 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.

ture bounded by cracks C3 and C4, which developed between locations M8–M9 and M16–M17, respectively. Controversially, the response gradually decreases toward the abutments. Location M16 registered the largest relative displacement in both monitored directions, with amplitudes of 30 mm (X direction) and 28 mm (Z direction). Another critical position is M9, where significant displacements were also recorded, namely 26 mm and 23 mm in the horizontal and vertical directions, respectively. All X direction records are positive, whereas the Z direction response exhibits an almost anti-symmetric distribution with respect to the keystone (see panels (b–c)). Panel (d) presents the evolution of the angular variation indicator, which reveals angular changes whose peaks correspond to crack formation associated with a rocking mechanism. The maximum absolute variation was observed in Section S7, between locations M8–M9, corresponding to the initiation of crack C3, around which rotation developed. It is also worth noting that, to identify significant changes derived from the angular variation, the analysis does not need to focus exclusively on deviations from zero, but rather on any deviation from an identified pattern, as in this case. This occurs because the center of the geometry is always defined with respect to the original configuration; therefore, when larger rotations of blocks take place, deviations from zero may also appear in sections where no significant changes between the relative positions of two consecutive monitored points are evident.

In contrast, the effects of the rocking mechanism in the opposite direction is captured in the deformed shape shown in panel (b) of Figure 5.21, which corresponds to the response at 14.6 seconds. Compared to the previous configuration, the deformed shape exhibits negative horizontal displacements, still reflecting the different behaviour of the left and right portions of the specimen, while panel (c) reports the relative displacements in the X and Z directions at this time instant. Locations between M6 and M20 recorded absolute value displacements ranging from 10 mm to 25 mm in both directions. The displacement pattern in the X direction appears as a mirrored response with respect to that observed in Figure 5.20, while the anti-symmetric distribution in the Z direction remains evident. The largest amplitudes occurred at M9, near crack C3, where displacements reached 22 mm (X) and 20 mm (Z), and at M16, which shows a comparable horizontal amplitude to M9 but a negative vertical response of approximately 10 mm. It should be emphasized that, similar to the previous time instant, the maximum angular variation indicator d^θ occurs on the opposite side of the maximum relative displacement. In this case, the peak value is detected at Section S15, corresponding to the angle between locations M16–M17, where crack C4 developed.

The analyses carried out allowed the identification of critical locations, which, when compared with the observed damage, were confirmed to be associated with structural damage. In particular, locations M9 and M16 were identified as the most sensitive and critical, while M12 was also included, as it marked

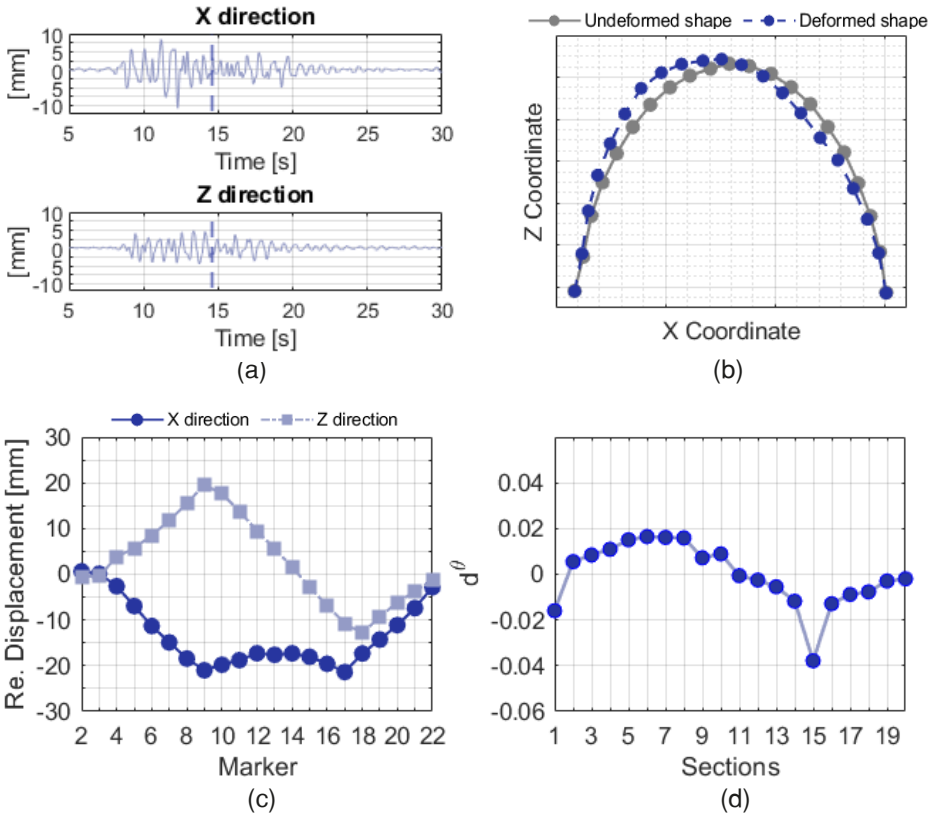


Fig. 5.21: Structural response of the UV specimen during test 0.4RE at 14.6 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.

a significant point for the reversal of the response. The relative displacement time histories of these locations are presented in Figure 5.22.

As can be observed, the responses are reported within the time window between 5 and 30 s, as this interval captures the transition from the elastic to the non-linear phase and the subsequent decreasing response at the end of the test. All RDTHs exhibit a clear transition from the elastic to the non-linear regime starting at approximately 12 s. During the initial stage, the specimen behaves elastically, and no appreciable relative displacements are recorded. The responses begin to diverge from near-zero relative displacement at approximately 12.3 s, corresponding to the instant when the PGD of the RE in the X direction is reached. Beyond this point, the system transitions to the non-linear regime, with increasing displacement values between 12 and 16 s. During this interval, locations M9, M12, and M16 recorded horizontal absolute displacement values ranging from 10 to 30 mm, while vertical values range between -25 mm and 28 mm. It is worth emphasizing that, while no appreciable differences are ob-

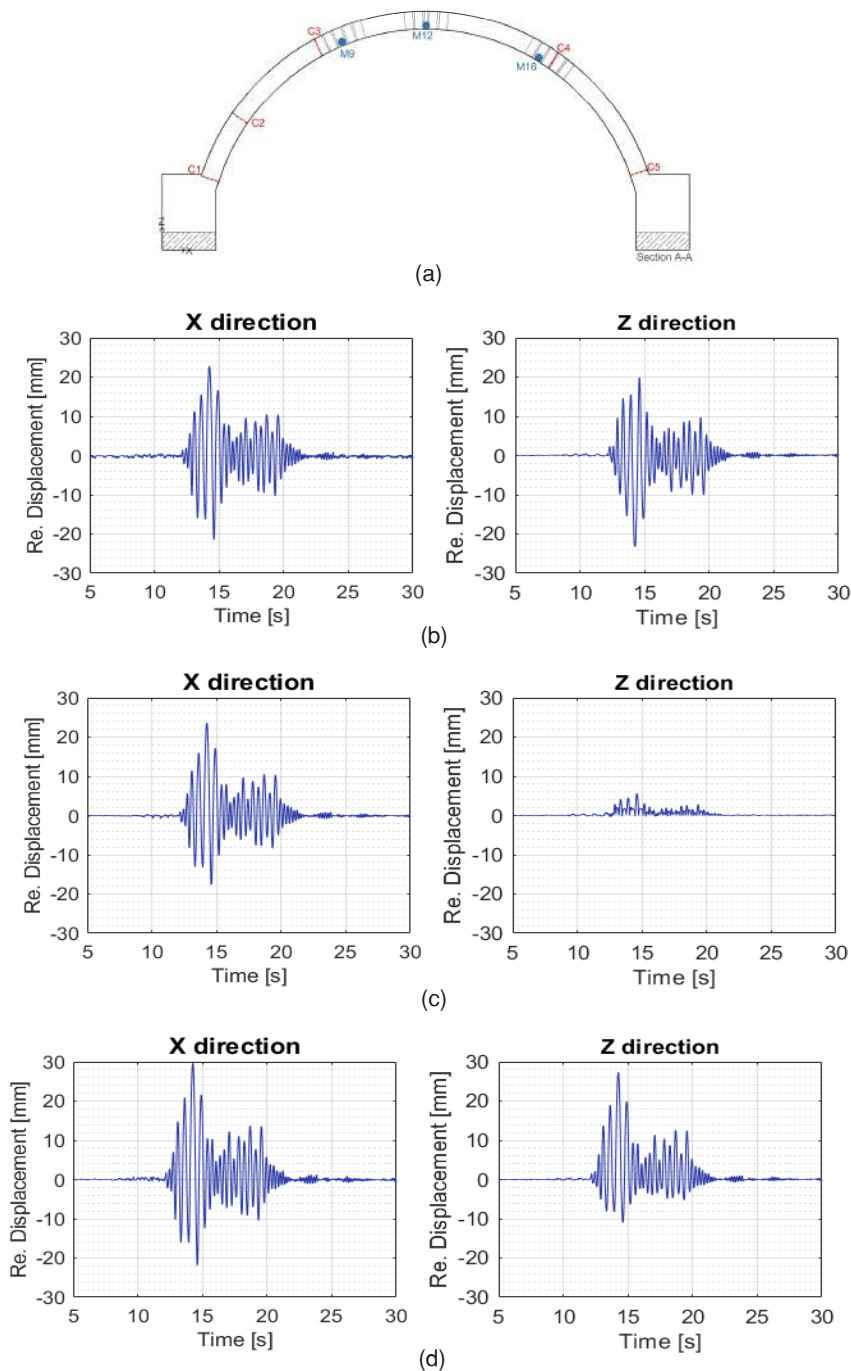


Fig. 5.22: Relative displacement time histories of selected monitored locations of the UV specimen during test 0.4RE: (a) Section A-A for reference, (b) location M9, (c) location M12, and (d) location M16.

served among the RDTHs in the horizontal direction, the Z response exhibits a mirrored pattern between panel (b) and panel (d), representing locations M9 and M16, respectively. This behavior clearly indicates the rocking mechanism, characterized by alternating vertical displacements in almost-symmetric portions of the specimen. Moreover, only positive vertical values are recorded at location M12, corresponding to the keystone. The intense phase observed in the RDTHs is directly associated with the development of the rocking mechanism, followed by a temporary reduction in amplitude and a subsequent secondary increase of lower magnitude. The divergence from zero displacement indicates the initiation of damage and the activation of rocking hinges, particularly those related to cracks C3 and C4, as illustrated in Figure 5.1 and confirmed by the angular variation indicator discussed previously.

In conclusion, the combined use of relative displacement and angular variation indicator data enabled a first assessment of the seismic dynamic response of the structure, allowing the identification of the most critical locations. The specimen experienced significant relative displacements in both the X and Z directions, primarily induced by rocking mechanisms around cracks C3 and C4. A Comparison of displacement patterns across directions and markers revealed anti-symmetric behavior in the vertical direction and mirrored responses in the horizontal direction during successive rocking phases. Relative displacement data, together with the angular variation indicator, confirmed the development of localized rotations at specific sections, coinciding with visible crack openings. These findings provide valuable insight into the most vulnerable regions of the structure, although the analysis cannot be considered exhaustive, as it represents only a rapid screening of the available data.

5.3.3.Reinforced Vault

The RV specimen was subjected to 21 seismic tests, with the displacement time histories recorded at several points, as highlighted in Setup #1 and #2 (see Figures 4.13 and 4.14). The following analyses are carried out on the monitored location of the arch profile on Section A-A, whose layout did not change between the different configurations. The analyses are carried out under three seismic input, equal to 2.1, 2.2 and 2.3 times the RE. The procedure highlighted in methodological section is applied for each of this seismic tests

5.3.3.1.Results at 2.1RE

The results of the procedure when the RV specimen was subjected to an input equal to 2.1 times the RE are presented in the following section. The only unreliable location in this analysis was M10. No significant changes were detected through the angular variation indicator, and the maxima in both directions occurred at the same time instant. For this reason, only Figure 5.23 is analysed to identify the most sensitive locations in this section. Panel (b) shows the deformed configuration at 12.4 seconds, which deviates only mini-

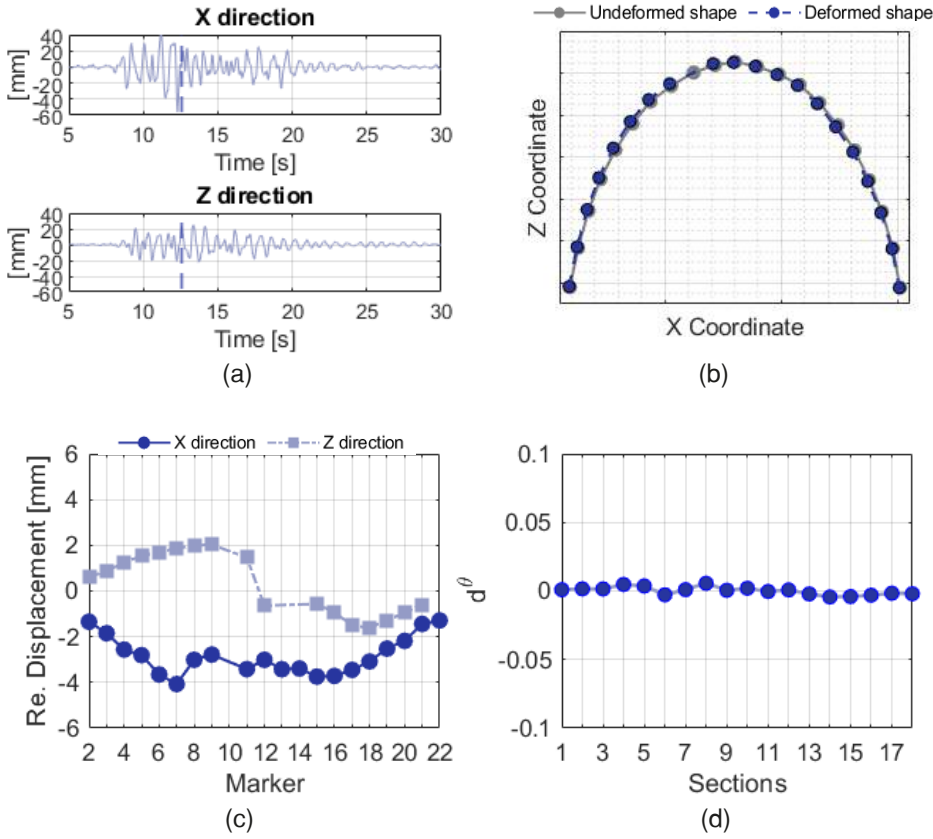


Fig. 5.23: Structural response of the RV specimen during test 2.1RE at 12.4 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.

mally from the undeformed state. Panel (c) depicts the relative displacement pattern, which reflects the typical structural response, with horizontal displacements sharing the same sign and an almost mirrored distribution in the vertical direction. Maximum amplitudes were attained at location M7, near crack C1 (Figure 5.24), reaching 4 mm in X and 1.8 mm in Z, and at M9, near crack C8, where the vertical displacement reached 2 mm and the horizontal displacement 3 mm. As anticipated, no significant deviations were detected when analysing the angular variation indicator (panel (d)).

The RDTHs were examined at three locations, M7, M9, and M12, whose responses are presented in the time window between 5 and 30 s (Figure 5.24). These locations are situated close to cracks C1, C8, and between C10 and C11, respectively. At all analysed points, the peak portions of the responses were recorded between 9 and 15 s, showing similar displacement patterns throughout

the observed period. In all cases, the response in the X direction reached higher values compared to the vertical direction.

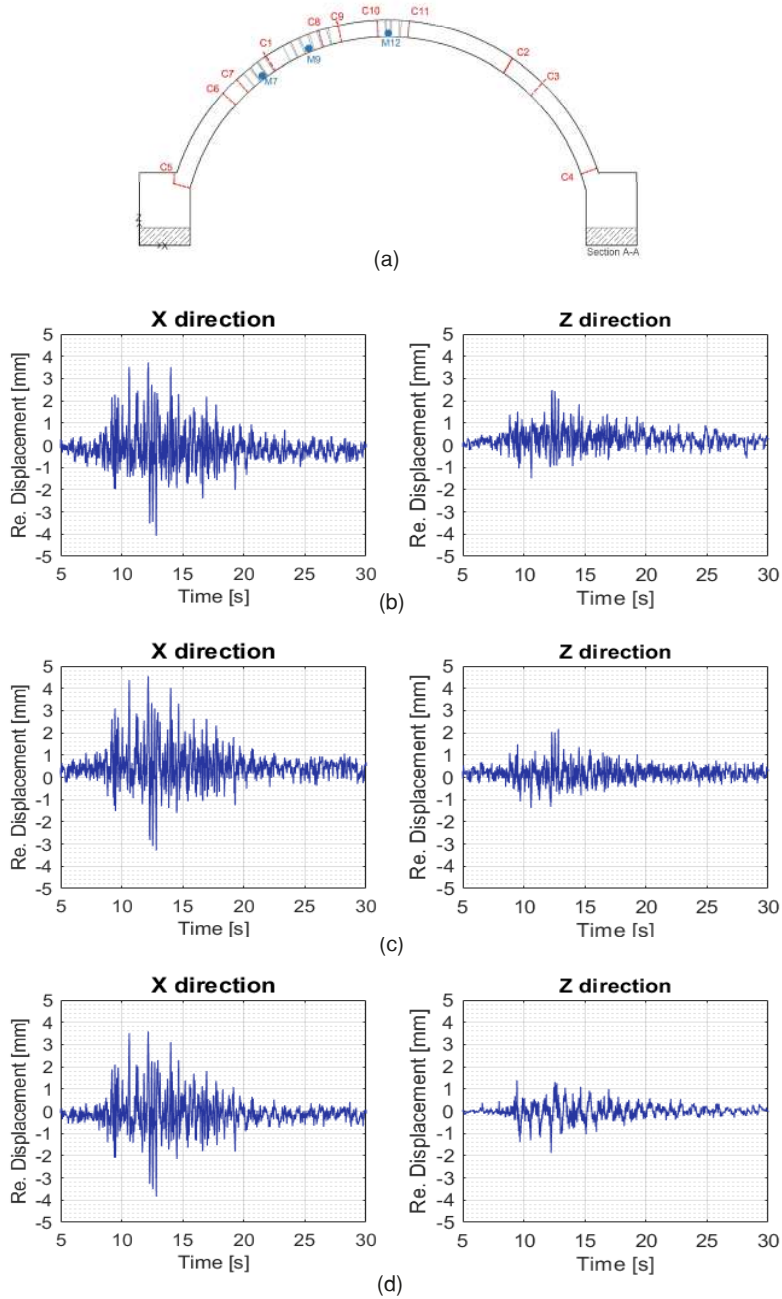


Fig. 5.24: Relative displacement time histories of selected monitored locations of the RV specimen during test 2.1RE: (a) Section A-A for reference, (b) location M7, (c) location M9, and (d) location M12.

5.3.3.2. Results at 2.2RE

The results of the penultimate test were carried out with a reduced amount of monitored locations, as mos were excluded from the analysis, as they were deemed unreliable. The monitored locations considered in the analysis were M2, M3, M4, M6, M7, M9, from M11 to M16, and from M19 to M22, resulting in a total of 14 sections evaluated using the angular variation indicator. In this analysis as well, no significant trends were detected from the angular variation indicator, so the discussion will focus primarily on the relative displacement patterns. The deformed shape (Figure 5.25(b)) differed only slightly from the original undeformed configuration. The deformed profile is highlighted by blue markers, while the grey ones represent the undeformed shape.

The input is displayed in panel (a), and the corresponding time instant was identified at 12.8 s, whereas the relative displacement patterns in the X and

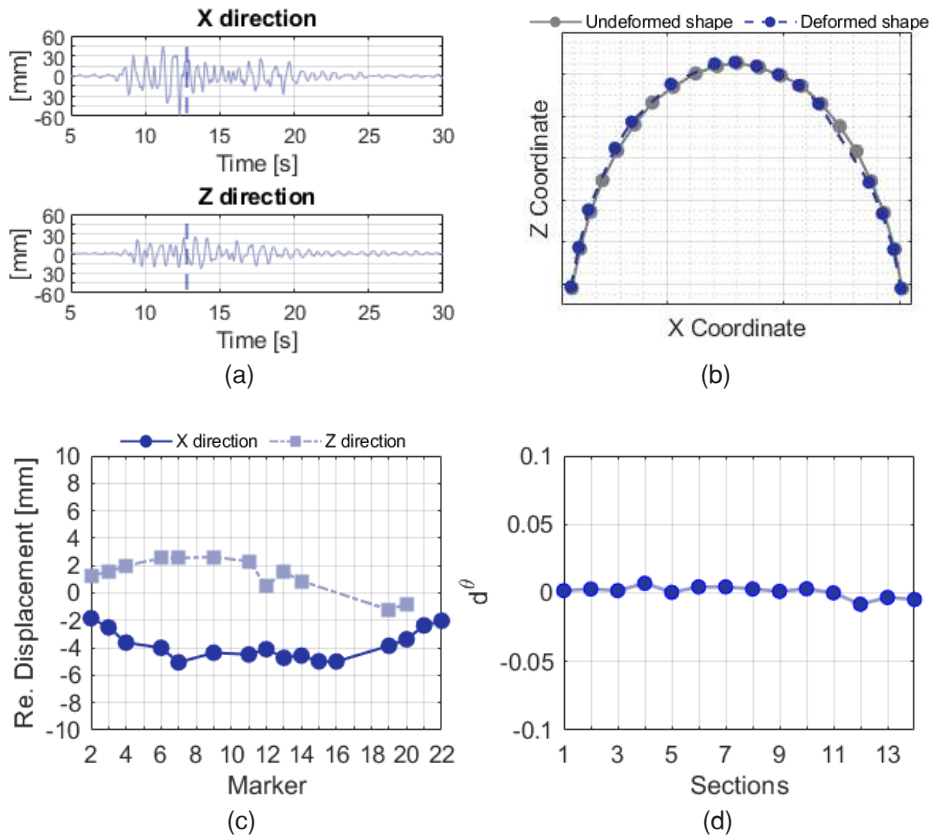


Fig. 5.25: Structural response of the RV specimen during test 2.2RE at 12.8 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.

Z directions are reported in panel (c). The location that recorded the highest absolute relative displacement was M7, near crack C1, which recorded nearly 5 mm in the horizontal (X) direction and 2.5 mm in the vertical (Z) direction. In contrast, all other reliable monitored locations recorded values between 2 mm and 5 mm in the horizontal direction and between 1 mm and 2.5 mm in the vertical direction. The angular variation indicator exhibited no significant peaks in panel (d).

Panel (b) of Figure 5.26 shows the deformed shape contrasted with the undeformed configuration, while panel (c) reports the relative displacements recorded at all reliable monitored points in both the X and Z directions at the time instant 14.05 s. Higher values were still observed in the horizontal direction, ranging between 1 mm and 8 mm. A clear mirrored pattern with respect to the keystone is also notable, although not perfectly symmetric.

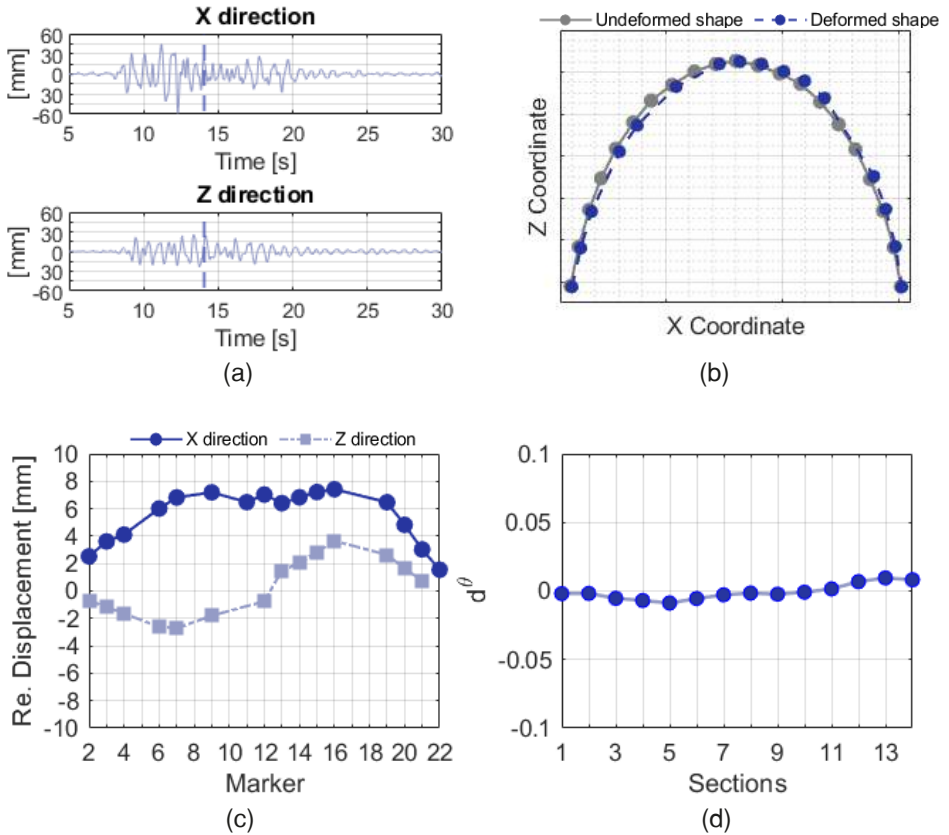


Fig. 5.26: Structural response of the RV specimen during test 2.2RE at 14.0 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.

The previously discussed anti-symmetric pattern was evident in the vertical direction, with higher values recorded in the right portion of the specimen, from location M13 onward. In particular, values smaller than 2.5 mm were obtained in the left portion, contrasted with results ranging from 2 mm to 4 mm on the right. Additionally, a small difference between the displacements at locations M12 and M13 is discernible. The highest displacement in the X direction was recorded at location M16, which also exhibited the largest vertical displacement. These relative displacements were 7.5 mm in X and 3.5 mm in Z, respectively. Location M9 (close to crack C8) recorded a similar value in the horizontal direction. No significant variation was observed in the d^0 index for this time instant.

The analyses carried out so far allowed the identification of five critical locations: M12, M13, and M16, as shown in Figure 5.27, and M7 and M9, significant points on the left portion of the structure as detailed in Figure 5.28. As is notable, all these locations are very close to where several cracks were detected during the tests. The RDTHs are presented over a time frame of 5–30 s, with the highest values observed between approximately 9 s and 18 s, and relative displacements at the end of the test returning near zero, highlighting the absence of permanent residual displacements. As previously discussed, the horizontal responses were similar across the monitored locations, while more insight can be gained from the vertical RDTHs. Specifically, locations closer to the keystone (M12) exhibited relatively small negative displacement values, whereas locations farther from the keystone displayed significant vertical responses, indicating movements in both directions.

5.3.3.3. Results at 2.3RE

The structural response of the RV specimen under seismic input 2.3RE is presented in this section. As previously discussed in Section 5.1.3, the specimen had reached an advanced damage state, with collapse being prevented solely by the presence of the extrados reinforcement. In the following analyses, only two locations were considered unreliable during the test, namely M10 and M22. Given the complex structural behaviour during this test, the analysis of the deformed shape and of the two proposed patterns aimed to identify the most critical locations, although the whole specimen recorded significant displacement values. It must be recalled that, especially in this analysis, the aim of the procedure was not to provide an exhaustive description of the structural behaviour, but rather to identify the most critical areas as a basis for comparison with numerical models.

The deformed shape (panel (b), Figure 5.29) clearly shows substantial displacements consistent with the observed damage, particularly corresponding to the shear failure of the reinforcement–masonry connection at the right abutment at this time instant (12.3 s). This behaviour is evidenced by an almost uniform shift in the right portion of the structure (from location M11 to M21, as shown

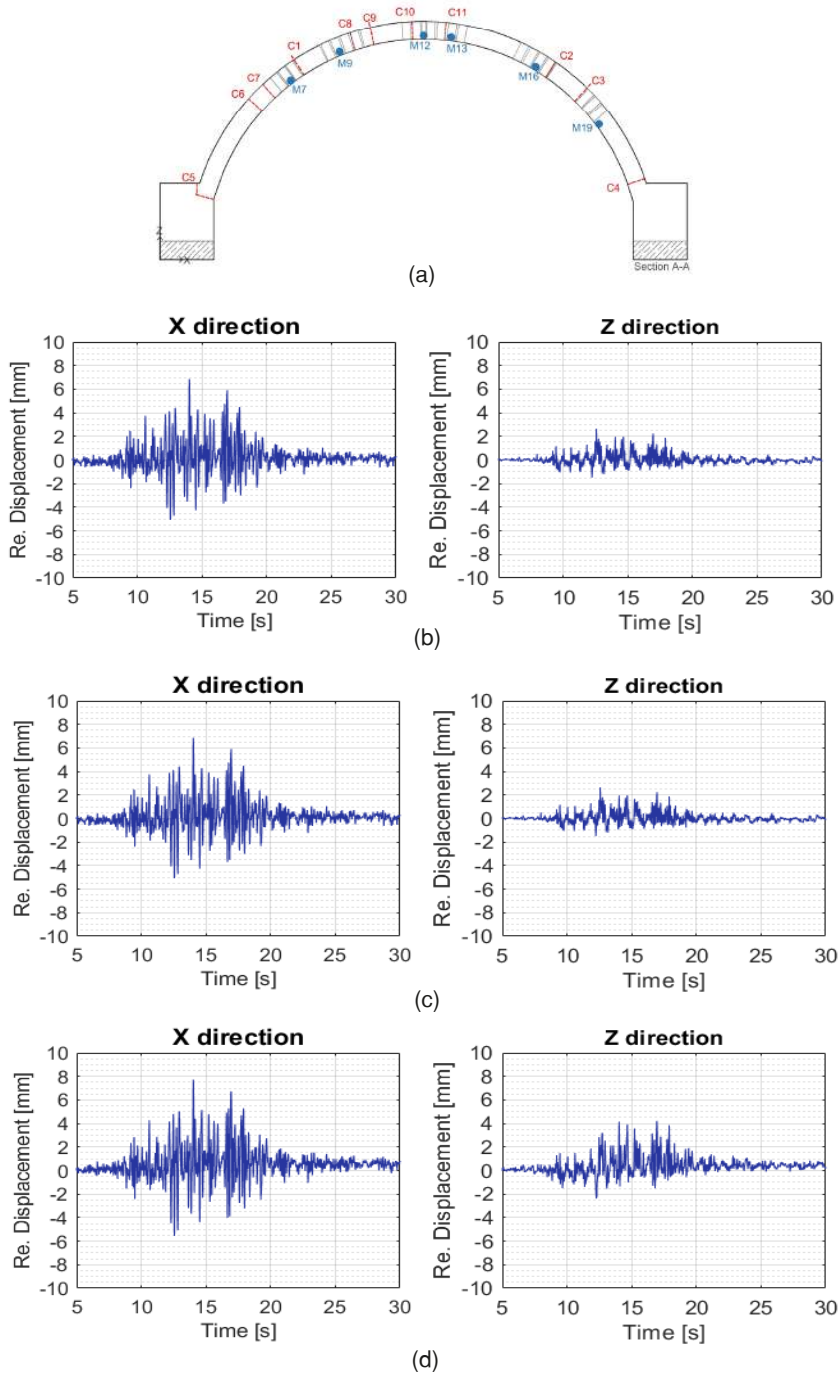


Fig. 5.27: Relative displacement time histories of selected monitored locations of the RV specimen during test 2.2RE: (a) Section A-A for reference, (b) location M12, (c) location M13, and (d) location M16.

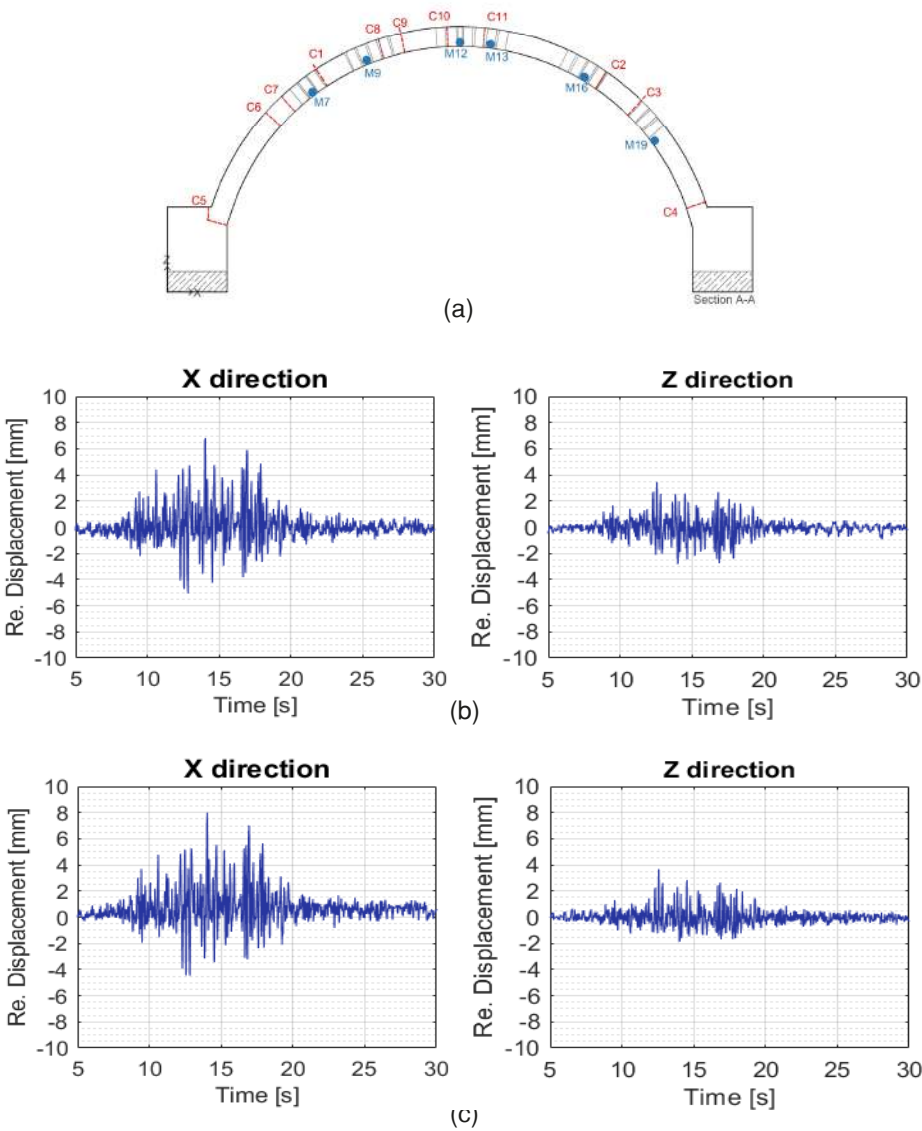


Fig. 5.28: Relative displacement time histories of selected monitored locations of the RV specimen during test 2.2RE: (a) Section A-A for reference, (b) location M7, and (c) location M9.

in panel (c)). All monitored locations recorded positive relative displacements exceeding 25 mm in the horizontal direction, with the exception of M2, M3, and M21. Conversely, an anti-symmetric displacement pattern was observed in the vertical direction with respect to the keystone, with relative displacements ranging between -25 mm and 27 mm. The maximum horizontal displacement was recorded at location M8, which reached 51 mm, while the same point at-

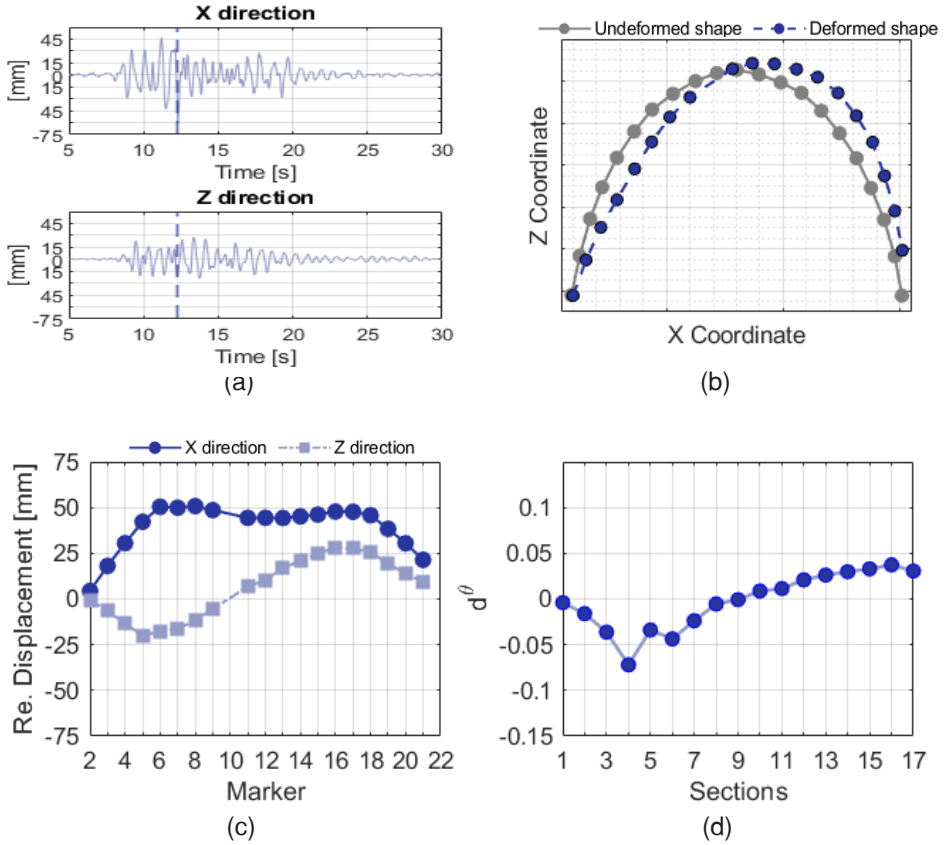


Fig. 5.29: Structural response of the RV specimen during test 2.3RE at 12.3 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.

tained a negative Z displacement of -11 mm. The largest vertical displacement was observed at location M16, which also exhibited a horizontal displacement of 48 mm. The d^θ (panel (d)) shows a distinct peak at Section S4, defined between locations M5-M6, corresponding to the position of crack C5 and highlighting its contribution to the overall deformation pattern in panel (b).

Another significant frame of the structural response is presented in Figure 5.30, corresponding to 12.4 s. The deformed shape in panel (b) illustrates a movement in the negative direction of the X axis, with the left portion of the vault shifting horizontally while simultaneously moving upward in the Z direction. In contrast, the right portion shows an almost linear trend from the abutment to the keystone. The relative displacement values presented in panel (c) confirmed this response, with the maximum displacement in the X direction recorded at location M16, reaching -61 mm. Most other markers registered values between

10 mm and 60 mm, except for M2 and M21. Notably, a progressive increase in the absolute X displacement was observed from M21 to M17, consistent with the visible deformed pattern. In contrast, the Z-displacement pattern could be described by a parabolic trend from M2 to M18, followed by a nearly linear correlation between M18 and M20. The maximum vertical displacement was recorded at location M8, equal to 42 mm, which corresponded to a horizontal displacement of nearly -50 mm. It is worth noting that the maxima absolute displacements in the X and Z directions occurred at positions symmetric with respect to the keystone. Finally, panel (d) shows that the angular variation indicator d^θ exhibited a peak at Section S15, deviating from the general trend and highlighting the formation of a rotational hinge between M17–M18, corresponding to the opening of crack C3.

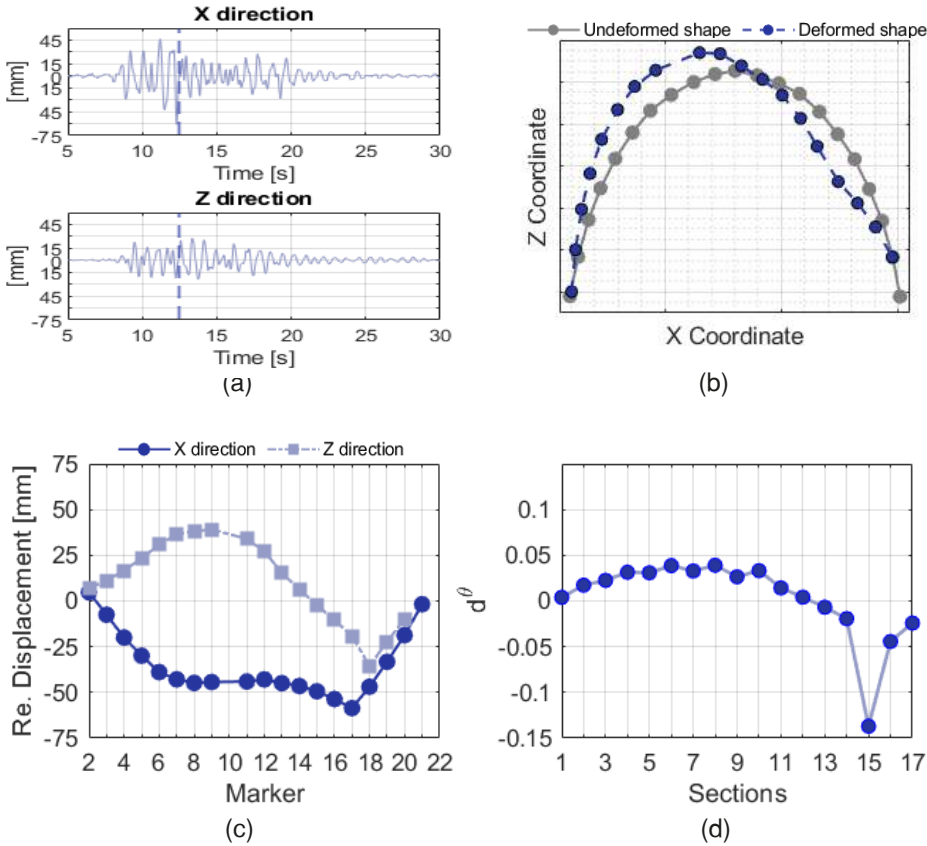


Fig. 5.30: Structural response of the RV specimen during test 2.3RE at 12.4 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.

Figure 5.31(b) shows the deformed shape: the specimen was affected by shear failure at the right abutment, causing an overall movement toward the positive side of the X axis at 12.8 s. The locations on the left side of the keystone followed an approximately linear trend, whereas a significant shift occurred at the keystone itself. This description is consistent with the relative displacement patterns in both X and Z directions (panel (c)). The Z displacement exhibited an anti-symmetric behavior, with the maximum value recorded at M16 (33 mm), while the other locations recorded values ranging approximately between -30 mm and 33 mm. The X displacements were uniformly positive, ranging between 30 mm and 65 mm, excluding locations M2 and M3, with the peak response observed at M8. Additionally, section S6, defined between locations M7–M8, recorded the maximum absolute values in the angular variation indicator pattern (panel (d)). This section corresponds to crack C1 observed in the damage pattern.

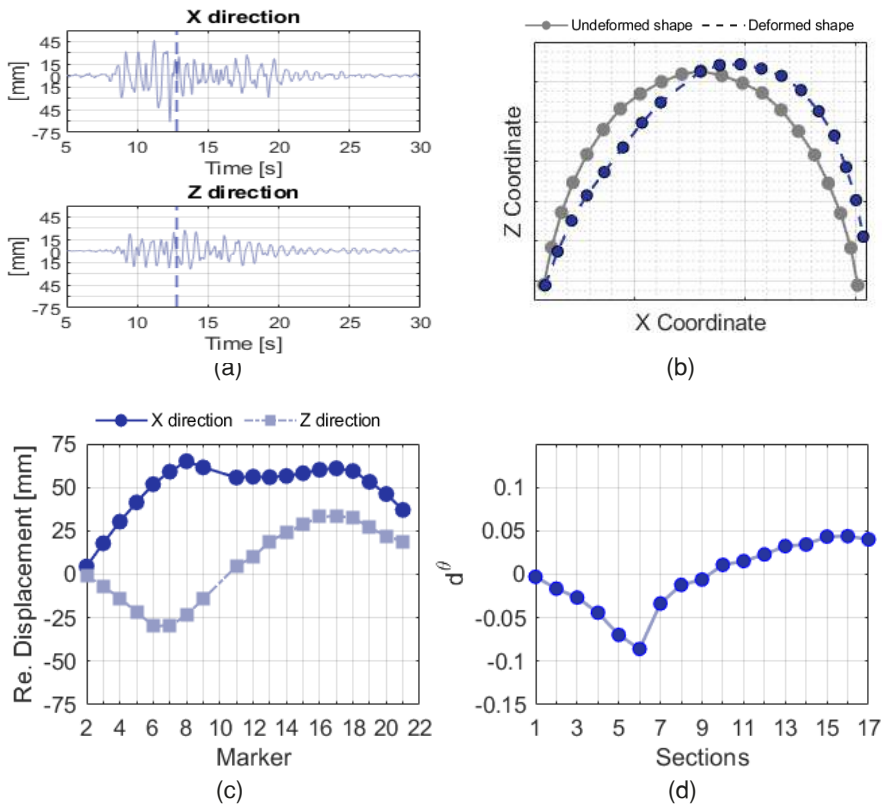


Fig. 5.31: Structural response of the RV specimen during test 2.3RE at 12.8 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.

The debonding mechanism identified through visual inspection is clearly distinguishable in the keystone area at 12.9 s, as highlighted by the deformed shape in panel (b) of Figure 5.32. The deformed configuration shows a horizontal shift in the right portion of the arch profile, accompanied by a downward movement in the Z direction, while this trend is not observed in the left portion. The difference in vertical displacement is well captured by locations M12 and M13 in panel (c), where the minimum value was recorded at M13, equal to -44 mm. A significant difference of approximately 20 mm was measured between M12 and M13. Unlike previous cases, the relative displacements no longer exhibit a fully anti-symmetric behavior across the keystone in the Z direction. Moreover, for the first time, X displacements include both positive and negative values, with the transition occurring at M13. The recorded values range from -25 mm to 30 mm, particularly concentrated near the right abutment. Finally, the angular variation indicator d^θ shows a pronounced peak at section S10, between M12–M13, corresponding to crack C11 and confirming the debonding mechanism. A smaller peak was also observed at Section S14, between M16–M17, associated with crack C2.

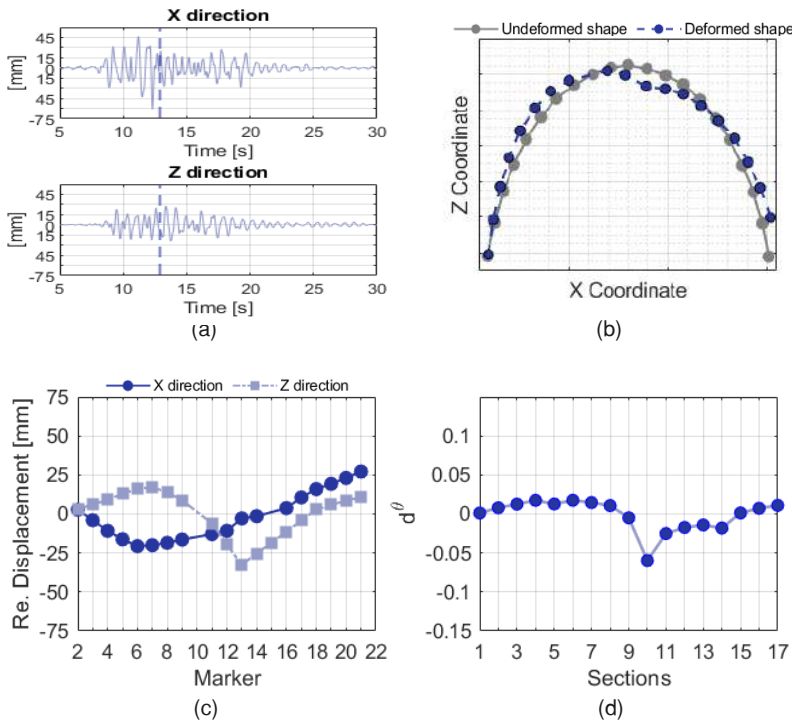


Fig. 5.32: Structural response of the RV specimen during test 2.3RE at 12.9 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.

The effect of debonding at the keystone is more evident in Figure 5.33(b), showing the response at 14.2 s. The whole specimen moved toward the positive X axis, but a clear change in the configuration was visible in the keystone area (M12). Additionally, panel (c) depicted a clear distinction between the keystone, which recorded the highest vertical displacement of 24 mm, and the adjacent location M13, which attained a negative vertical displacement of - 7.5 mm. This resulted in a difference of approximately 32 mm between the two adjacent markers, as shown in panels (b–c). At the same time, all monitored points recorded positive X displacements, with maximum values approaching 50 mm. Panel (d) clearly showed a pronounced peak in the angular variation indicator at section S4, which, as previously noted, corresponded to crack C6. The highest relative displacement between locations M12 and M13 was recorded at 19.4 s. Panel (b) of Figure 5.34 clearly illustrates how, simultaneously, the

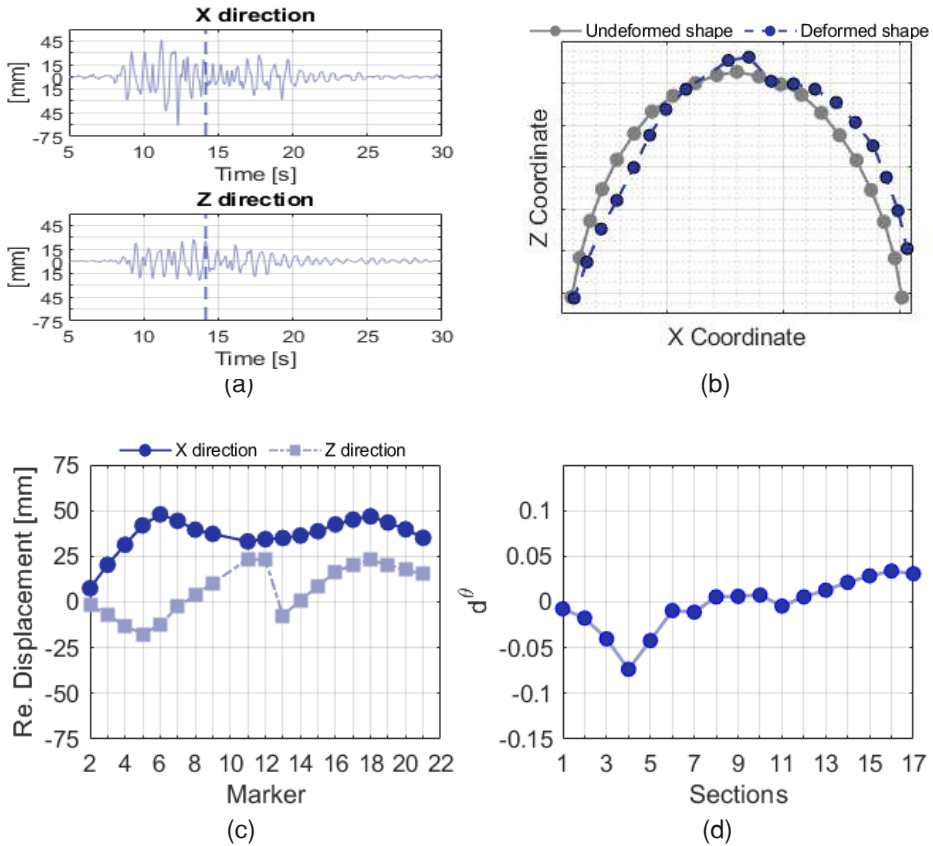


Fig. 5.33: Structural response of the RV specimen during test 2.3RE at 14.2 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.

right portion of the specimen shifted to the right due to shear failure and debonding at the right abutment, while a significant negative vertical shift occurred immediately after the keystone. These results highlight a difference of approximately 50 mm between locations M12 and M13. The latter recorded the largest vertical relative displacement, equal to -53 mm, whereas the corresponding X displacement was significantly smaller, reaching only -17 mm. The horizontal relative displacements ranged between -28 mm and 27 mm, while the vertical displacements varied from -55 mm to 25 mm. The angular variation indicator showed slight peaks at sections S11 and S15, located between M13-M14 and M16-M17.

The critical location obtained from the analysis are presented in the following figures. In particular, in Figure 5.35 the RDTHs show the response in the locations marked as M5, M6 and M7, posed on the left portion of the structure

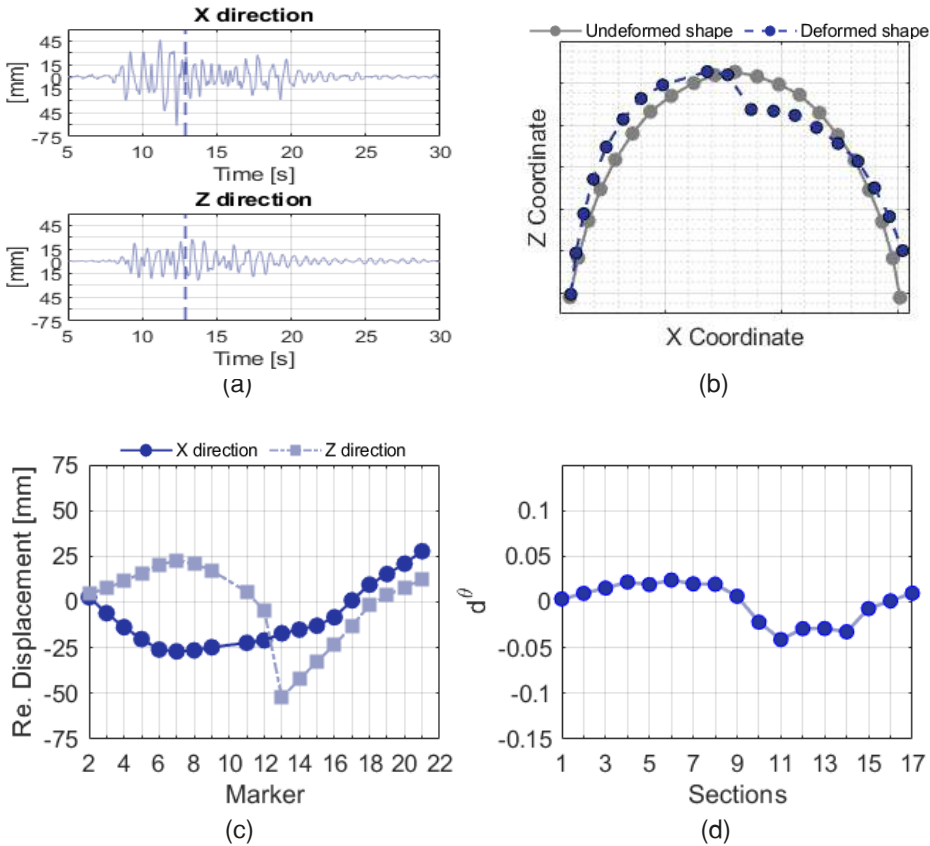


Fig. 5.34: Structural response of the RV specimen during test 2.3RE at 19.4 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.

and close to cracks C6, C7 and C1, respectively. The response is presented in the time frame between 5-30 seconds, which clearly highlights that relative displacements between 15 mm and 60 mm were recorded between 10 s and 20 s of the test. In detail, all these locations exhibited greater displacements in the X direction, compared with lower values in the Z direction, which varied approximately between -35 mm and 40 mm. Additionally, it is worth noting that, close to the end of the test, no significant shift from the original position was observed.

Subsequently, the RDTHs of locations M8, M12, and M13 are shown in Figure 5.36. The first highlighted location is situated close to crack C1, while M12 corresponded to the keystone and M13 is located next to crack C11. The responses are presented in the same time frame, which allowed the identification of a significant duration of the response, between 12-20 seconds. It is worth noting that, in panels (c-d) after 20 seconds, a shift from the zero value is clearly observed, indicating a permanent change of the monitored location. In particular, in the X direction, both M12 and M13 recorded positive values, indicating a movement to the right, while the effect of the previously discussed debonding phenomenon was evident in the downward shift of the Z displacements. Specifically, by the end of the test, location M13 exhibited a clear change in the vertical position of approximately 35 mm.

The last three identified critical locations are analysed in Figure 5.37, i.e., locations M16, M18 (close to cracks C2 and C3), and M21 (close to crack C4). In all the detailed responses, a clear peak is observed at 12.4 seconds. After this time instant, all the monitored locations showed a shift in the response, which was no longer centered around zero, but shifted towards positive values in the X direction, with increasing magnitude from the top to the bottom part of the right portion (location M21 exhibited a final deviation of 30 mm from the original position). In contrast, the final Z displacement depended on the location, with negative values recorded at M16 and M18 and a positive final value at M21.

5.3.4. Repaired-Damaged Unreinforced Vault

Given the extensive displacement acquisition dataset obtained during the second experimental campaign and previously described in Section 4.2.2, a total of 61 monitoring locations were considered illustrated in Figure 4.18 on the frontal facade. The analysis presented in this section aims to identify the critical location to be adopted as the reference for comparison with the numerical model results. In order to maintain consistency with the procedure followed in the previous sections, only the monitoring points located closer to the intrados are discussed (i.e., locations M2, M4, M6, ..., M60). The seismic tests selected as significant for this analysis correspond to 1.9RE, 2.0RE, 2.1RE, and 2.2RE.

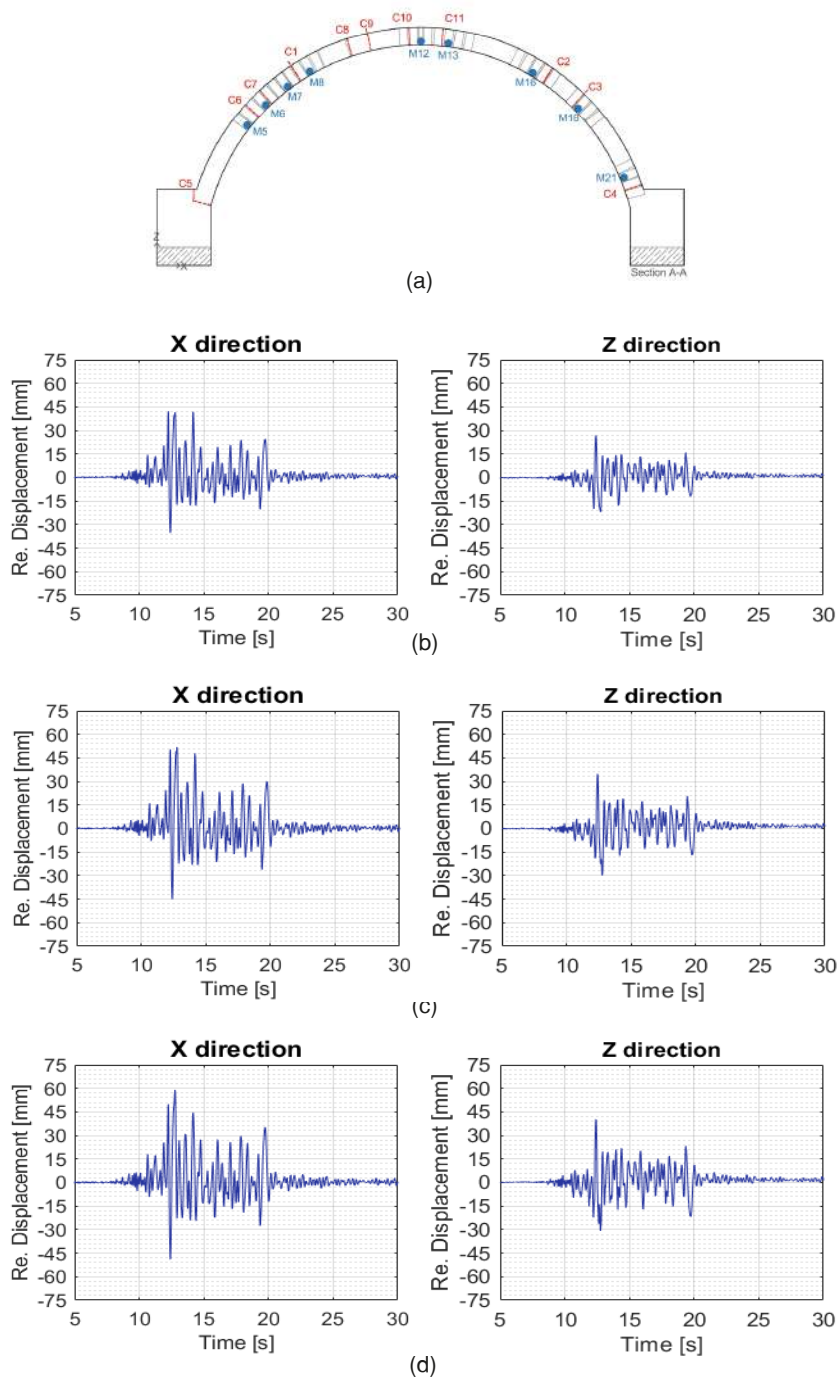


Fig. 5.35: Relative displacement time histories of selected monitored locations of the RV specimen during test 2.2RE: (a) Section A-A for reference, (b) location M5, (c) location M6, and (d) location M7.

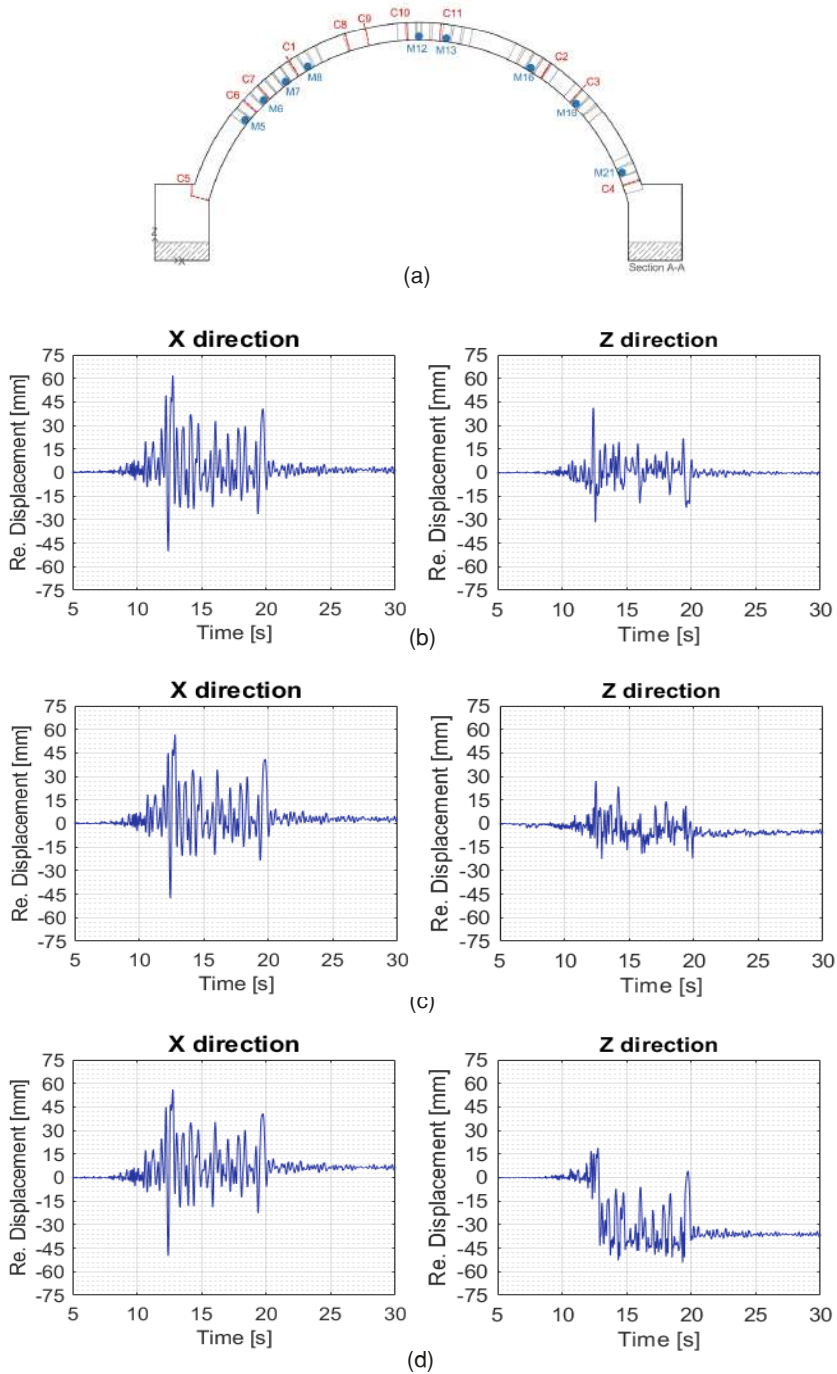


Fig. 5.36: Relative displacement time histories of selected monitored locations of the RV specimen during test 2.2RE: (a) Section A-A for reference, (b) location M8, (c) location M12, and (d) location M13.

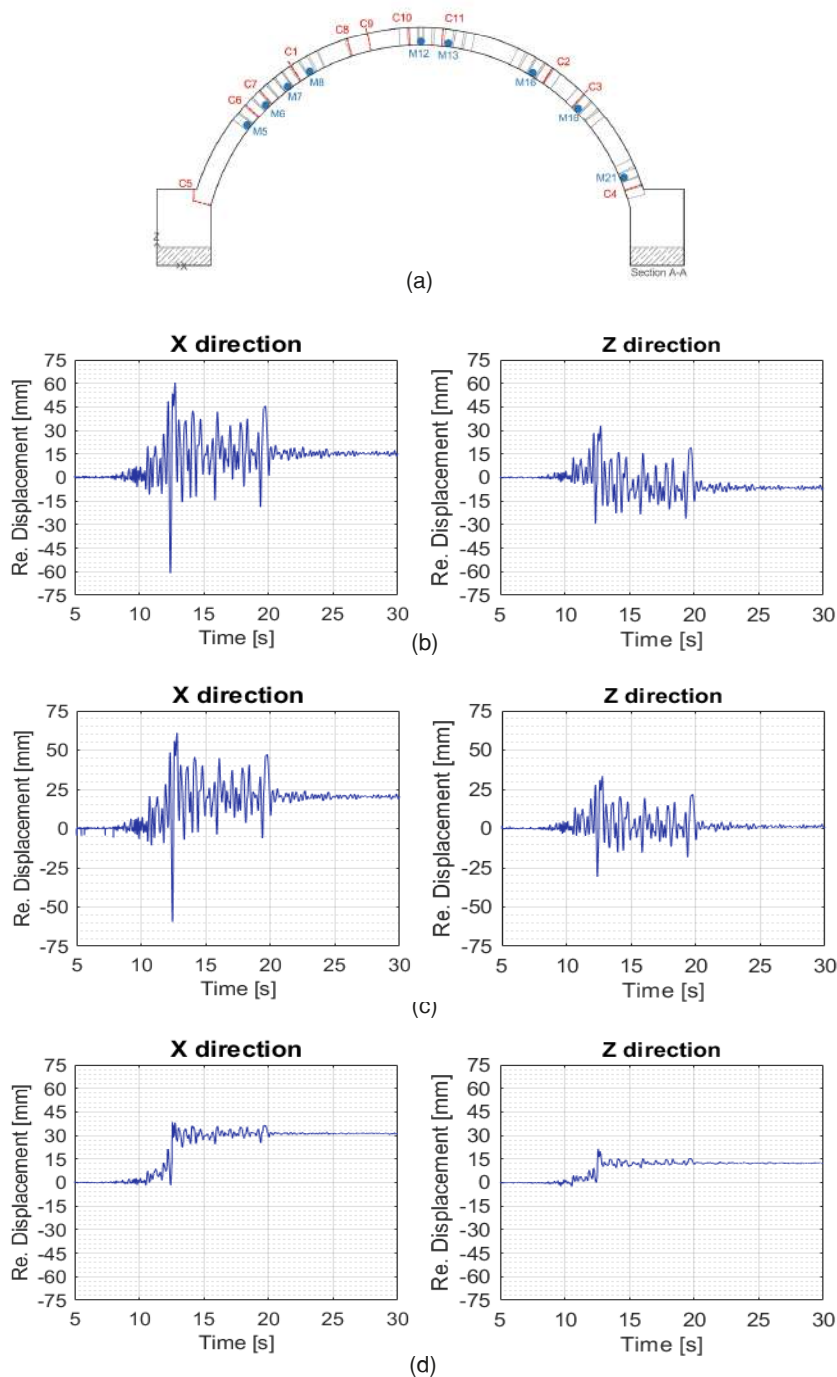


Fig. 5.37: Relative displacement time histories of selected monitored locations of the RV specimen during test 2.2RE: (a) Section A-A for reference, (b) location M16, (c) location M17, and (d) location M21.

5.3.4.1. Results at 1.9RE

The results obtained from test 1.9RE are discussed below. The analysis individuate the peaks in both relative locations and in the angular variation in the same time instant.

In particular, Figure 5.38(b) illustrates the deformed shape at 12.8 seconds, revealing only minor deviations from the undeformed configuration, more visible in the upper portion of the specimen, with a consistent displacement towards the negative X-axis.

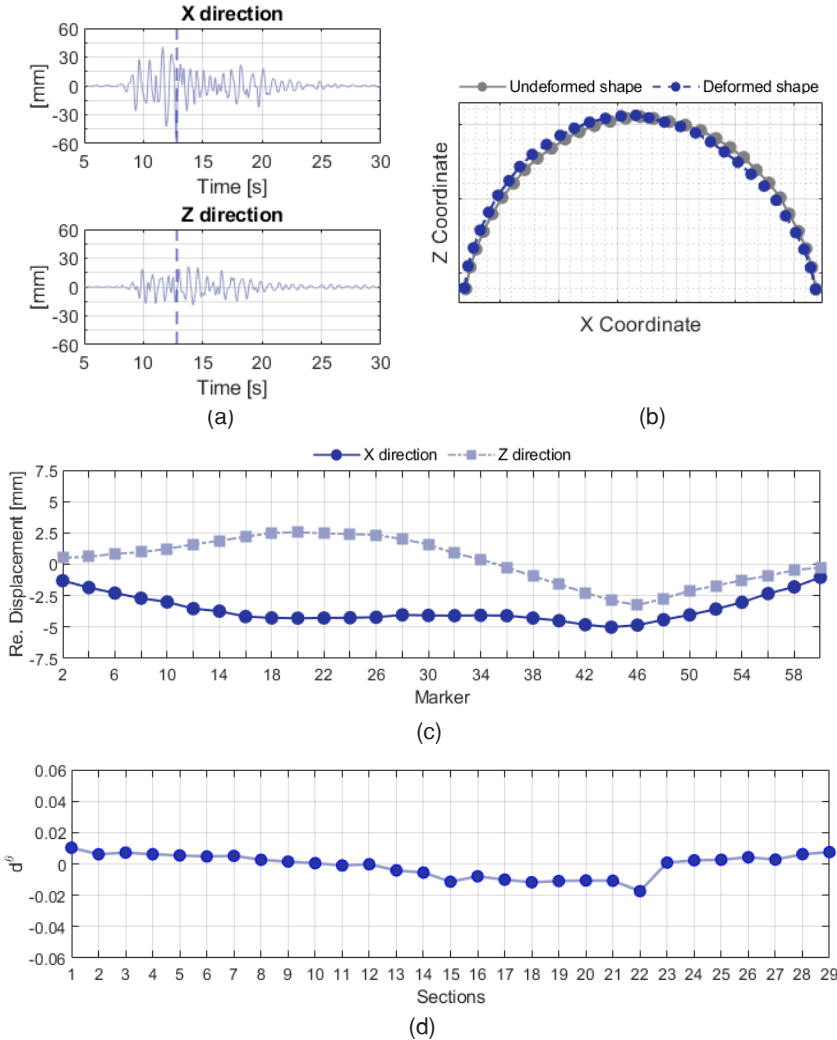


Fig. 5.38: Structural response of the R-DUV specimen during test 1.9RE at 12.8 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.

The expected anti-symmetric pattern is observed in the vertical displacements, whereas the X direction exhibits only negative values, almost symmetric with reference to the keystone. Consistently, the displacement response reaches higher absolute values at location M44 (panel (c)), with a maximum peak of approximately -5 mm in the X direction, while a pronounced peak is recorded at location M46 in the Z direction (-3 mm). The results of the angular variation indicators are shown in panel (d): the two most relevant sections are S22 and S23, defined between locations M42–M44 and M44–M6, respectively.

The analysis highlighted locations and sections which, compared with the damage pattern are in the surroundings of of crack C10 in the damage pattern (see Figure 5.6) and as represented in Figure 5.39(a).

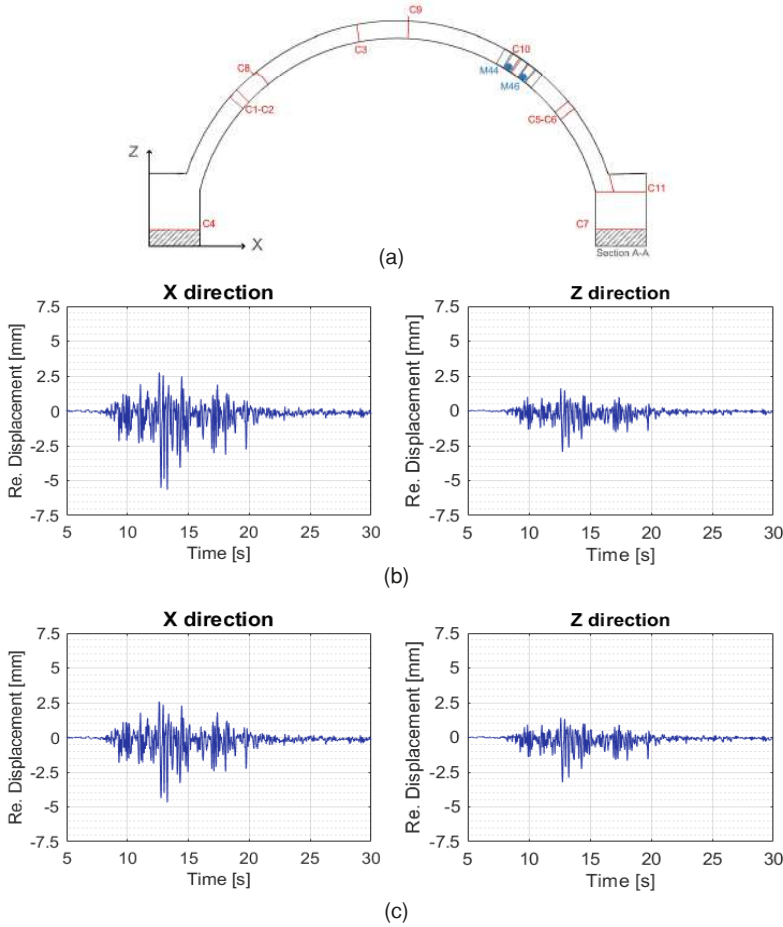


Fig. 5.39: Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 1.9RE: (a) Section A-A for reference, (b) location M44, and (c) location M46

The monitored locations M44 and M46 exhibited the highest displacement values and also characterized the response when adopting the angular variation indicator. The RDTHs of these locations are presented in panels (b–c) within the time frame 5–30 s. In particular, it is worth noting that the highest values in the response are obtained between 12 s and 15 s, with larger amplitudes in the X direction (ranging between –5 mm and 3 mm) than in the Z direction (between –3 mm and 1.8 mm). All the peaks in this time interval are negative for both axes, and no significant deviation from the original position is observed at the end of the test.

5.3.4.2. Results at 2.0RE

Subsequently, the results of the third-to-last seismic test to which the R-DUV specimen was subjected are presented in this section.

The deformed shape obtained at 12.8 s, shown in Figure 5.40(b), highlights that the upper portion is still the most affected. However, as visible in panel (c) of Figure 5.40, the locations closer to the abutments (M2 and M60) also record displacements of about 2–2.5 mm in the horizontal direction, while reduced values are observed in the vertical direction. Additionally, the highest displacement in the X direction is recorded at location M44, with 7.5 mm, whereas 4.4 mm is obtained at the same location in the Z direction. Conversely, the largest vertical displacement is found at location M18, reaching 4.8 mm, combined with a negative displacement of –7 mm in the X direction. The d^θ results for this time instant are detailed in panel (d) of Figure 5.40, which again emphasises the change in trend between sections S22 and S23, as already observed in the previous seismic test.

Figure 5.41(b) presents a deformed shape that appears mirrored with respect to the one previously discussed. The relative displacement at 17.4 s highlights exclusively positive values greater than 1.0 mm at all monitored locations, both in the extrados and intrados configurations. In contrast, displacements ranging between –1 mm and 4 mm are recorded in the vertical direction (panel (c)). Moreover, the pattern identified by the Z displacements is not anti-symmetric with respect to the keystone (located between M30–M32) but exhibits a peak at location M16, while the maximum value is observed at location M30, reaching approximately 2 mm. The largest horizontal displacements are instead obtained at location M16, which recorded values of about 6 mm. Consistent with the analysis of the relative displacement values, panel (d) highlights the response in terms of the angular variation indicator, showing a significant peak at section S8. It is worth noting that section S8 is contained between M14–M16 and corresponds to the location of crack C8.

Figure 5.42 presents the response of locations M16, M18, and M30. The relative displacements over time are shown in panels (b–d), while their positions are highlighted in panel (a). The response exhibits pronounced peaks between 10 s and 20 s, with larger amplitudes in the X direction compared to the

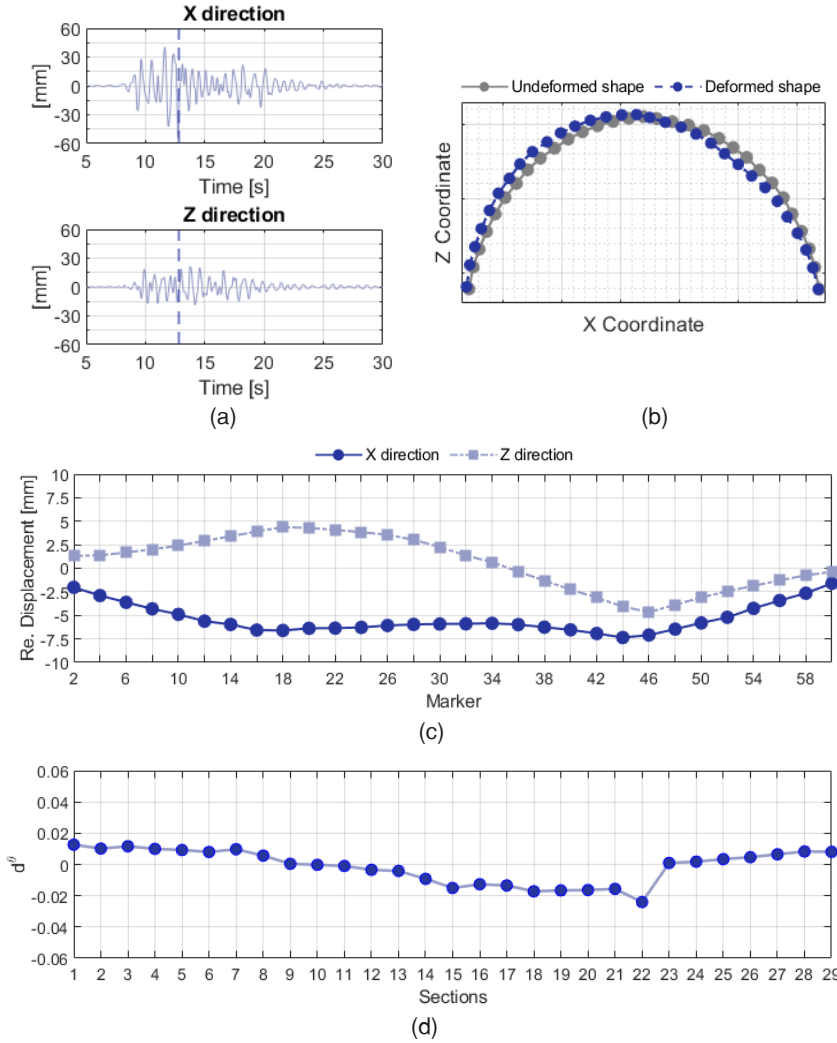


Fig. 5.40: Structural response of the R-DUV specimen during test 2.0RE at 12.8 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.

Z direction. It is also worth noting that higher vertical displacements were recorded at the locations farthest from the keystone. Figure 5.43 presents the RDTHs of locations positioned almost symmetrically to those discussed above. The same conclusions drawn for the previously mentioned locations remain valid, and it is important to note that none of the monitored locations exhibit residual permanent displacements by the end of the considered time frame (5–30 s).

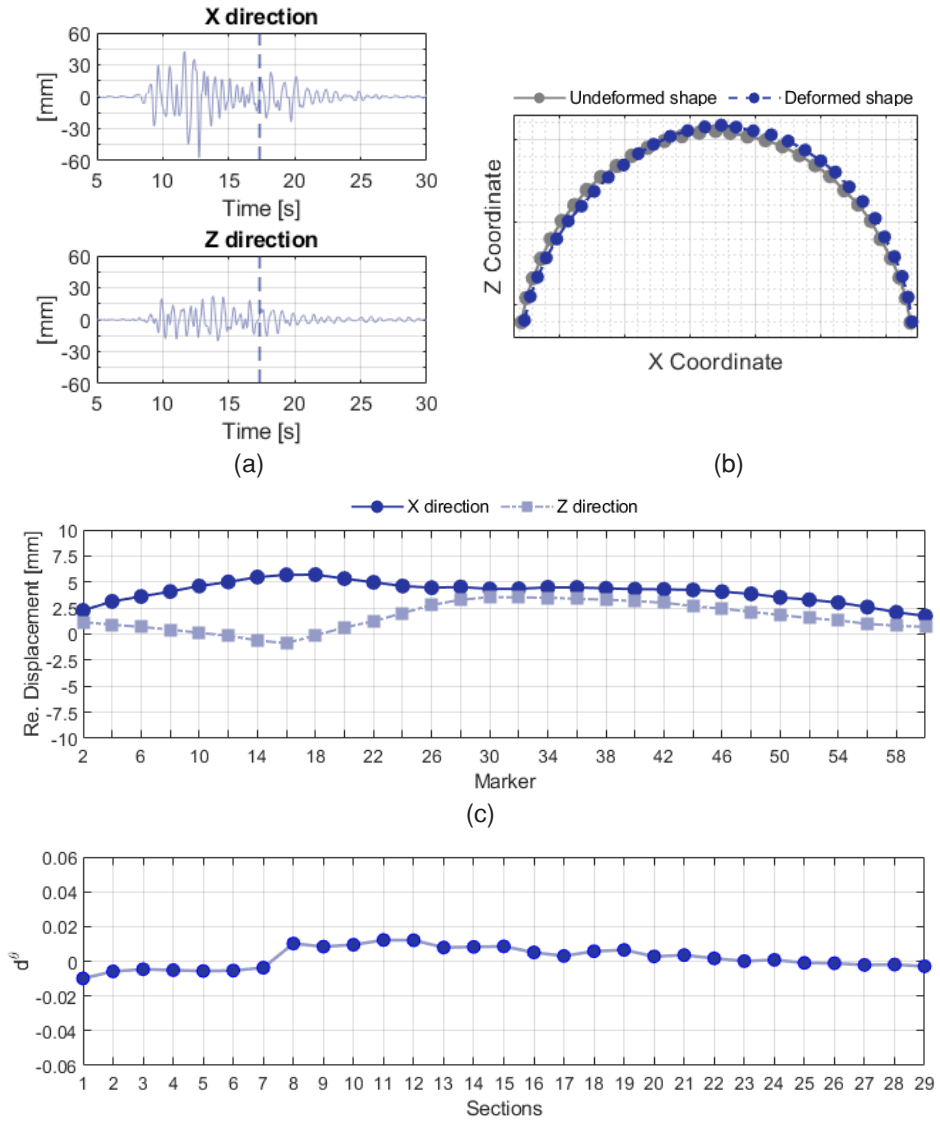


Fig. 5.41: Structural response of the R-DUV specimen during test 2.0RE at 17.4 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.

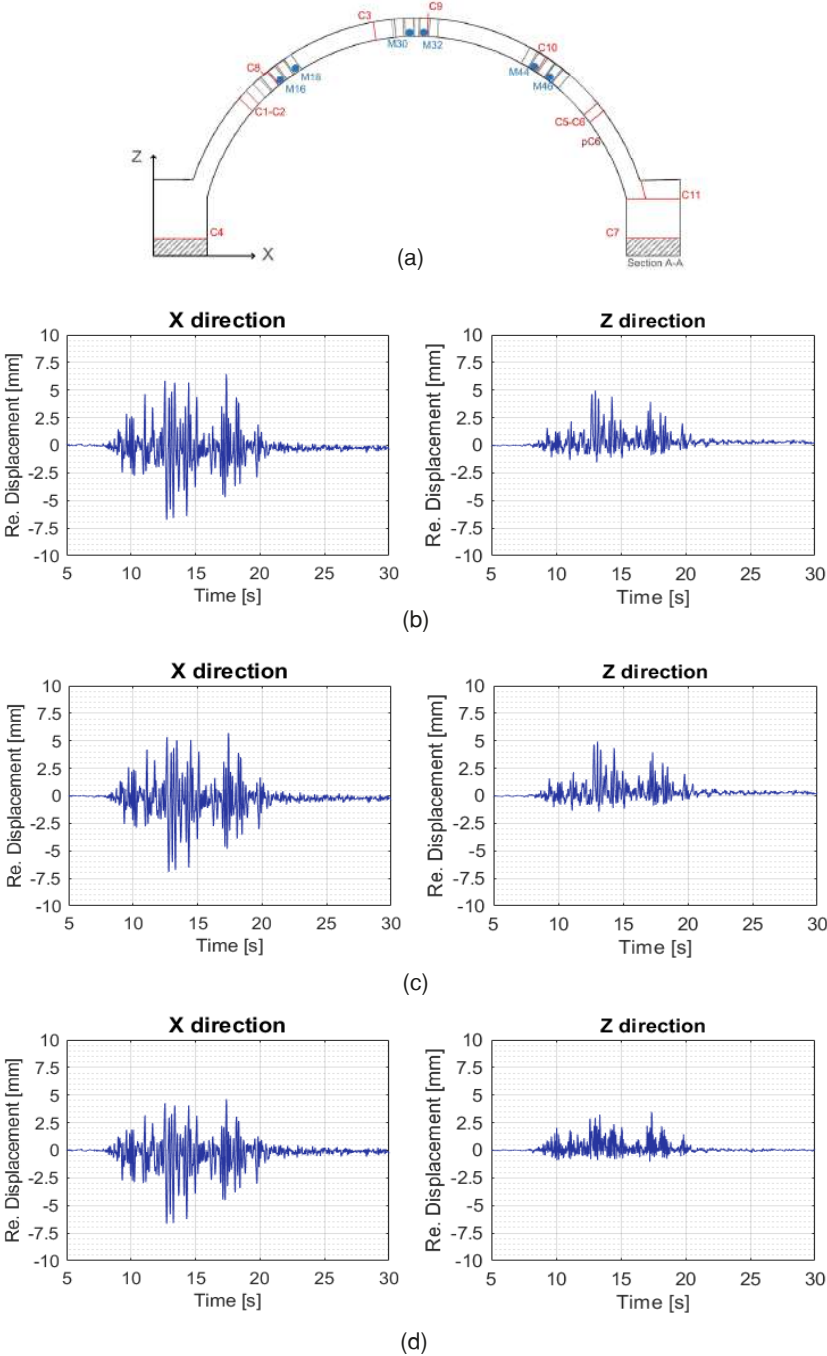


Fig. 5.42: Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 2.0RE: (a) Section A-A for reference, (b) location M16, (c) location M18, and (d) location M30

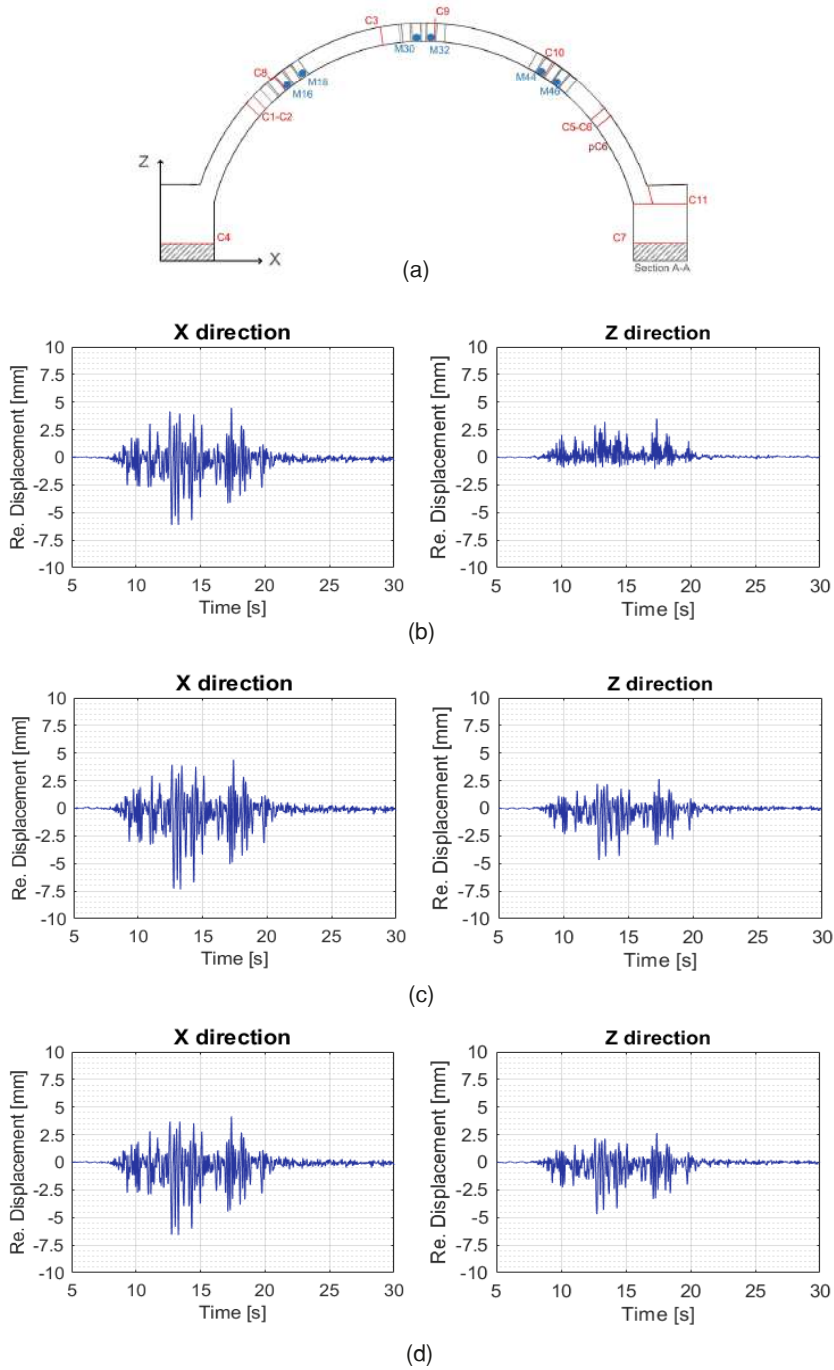


Fig. 5.43: Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 2.0RE: (a) Section A-A for reference, (b) location M30, (c) location M44, and (d) location M46

5.3.4.3. Results at 2.1RE

The response during the penultimate seismic test (i.e., when subjected to an input 2.1 times the RE), highlighted in terms of deformed shape, relative displacement pattern, and the angular variation indicator (d^θ), is presented and discussed in this section.

The deformed shape at 13.3 seconds effectively captures the significant displacements to which the specimen was subjected (Figure 5.44(b)).

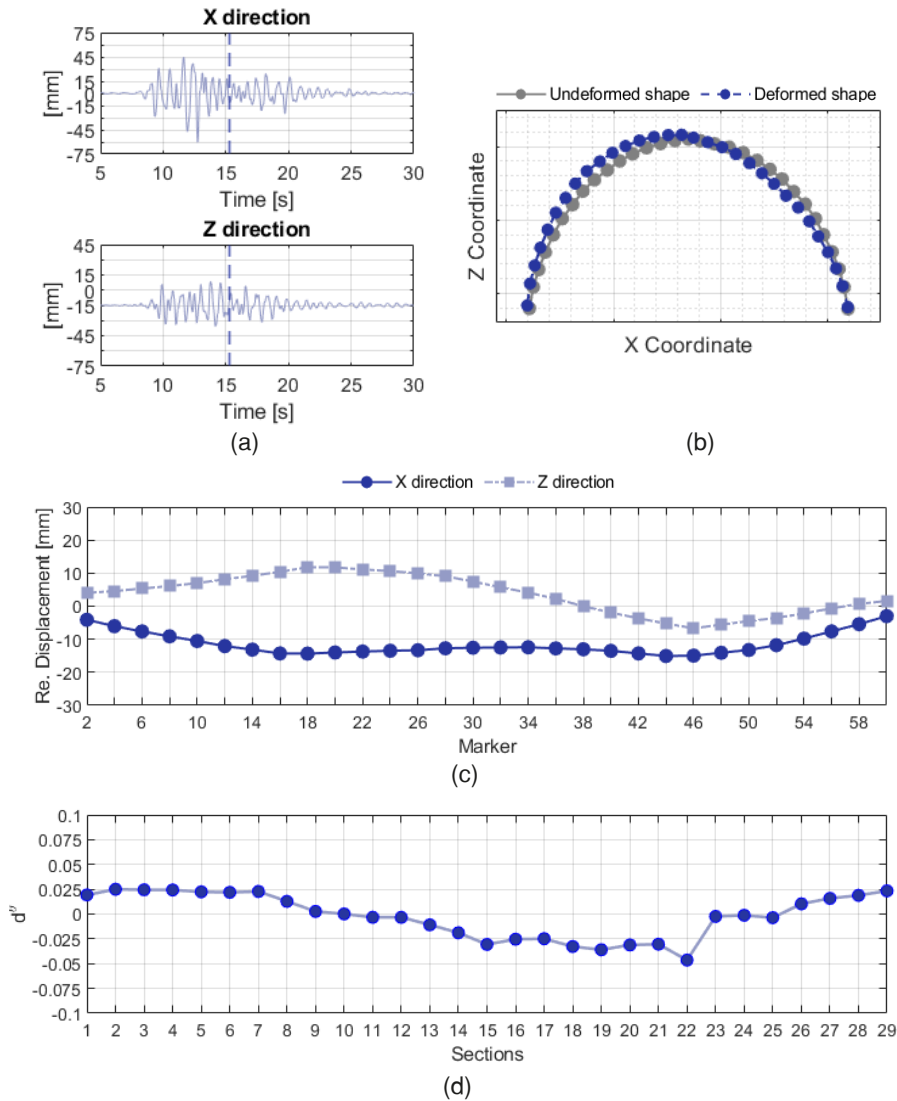


Fig. 5.44: Structural response of the R-DUV specimen during test 2.1RE at 13.3 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.

The re. displacements in the X direction exhibit exclusively negative values ranging approximately from -5 mm to -20 mm, with a response that is symmetrical with respect to the keystone (M30–M32). It must be noted that at the locations closest to the abutments (M2 and M60), horizontal displacements greater than 5 mm were recorded. These results are consistent with the observed rocking phenomenon, described by alternating rotations around the bases. The vertical displacement pattern still displays the usual anti-symmetry with respect to the keystone, although in this case the keystone itself records positive values of 5–10 mm. The locations experiencing the largest response values are M46, with -21 mm in the horizontal direction and -6 mm in the vertical direction, and M18, which records the highest vertical displacement of 13 mm together with -18.5 mm in the X direction (Figure 5.44(c)). Consequently, panel (d) of Figure 5.44 identifies sections S7, S15, S22, and S23 as the critical ones to be assessed, since they diverge from a clearly recognisable pattern. In particular, section S7 is bounded by locations M14–M16 and is consistent with crack C8. Section S15 is defined between M30 and M32, two locations close to crack C9. Finally, sections S22 and S23 are located between M42–M44 and M44–M46, respectively, and are consistent with the observed crack C10.

The deformed shape at 14.5 s is presented in panel (b) of Figure 5.45. A clear movement to the right is visible, while an upward displacement is also detected. This translation is also reflected in the horizontal displacement pattern in the X direction in panel (c). All monitored locations recorded displacements greater than 10 mm; the lowest values were obtained at the locations closest to the abutments (M2 and M60), while the peak response occurred at M16, with 26 mm of horizontal displacement and -3.5 mm of vertical displacement. The loss of the symmetrical pattern in the X direction and of the anti-symmetric one in the Z direction is clearly evident. Figure 5.45(d) presents the results of the angular variation indicator. A change in trend is clearly observed in sections S8 and S9: the first bounded by M14–M16, and the second between M16–M18, both still associated with crack C8.

The analysis conducted so far provided insight into the locations most sensitive to the response data. In particular, observing the RDTHs presented in Figure 5.46 for positions M2, M14, and M16, it is possible to note that the highest response values are recorded in the time frame between 12–20 s, ranging from -20 mm to 26 mm in the X direction. Specifically, location M2, the closest to the left abutment, recorded a peak value of 10 mm in the X direction and 5 mm in the Z direction. The X response is characterised by predominantly positive values, with peaks exceeding 10 mm of displacement.

Consequently, the analysis of the RDTHs is essential, as it provides more complete information about the response by highlighting permanent shifts. In particular, panels (c–d) of Figure 5.47, corresponding to locations M30 and M32, not only show the peaks in the response but also reveal a residual shift of the monitored locations at the end of the analysis. Specifically, a positive shift of

1 mm was obtained in the X direction, contrasted by a residual negative displacement of -2 mm in the vertical direction. This residual shift becomes more evident closer to the right abutment. In panels (b) and (c) of Figure 5.48, a positive shift of 2 mm is observed in the X direction, accompanied by a smaller translation in the vertical direction. Finally, panel (d) highlights that, closer to the abutment, the specimen experienced a final displacement of 4 mm in the X direction and 2 mm in the Z direction.

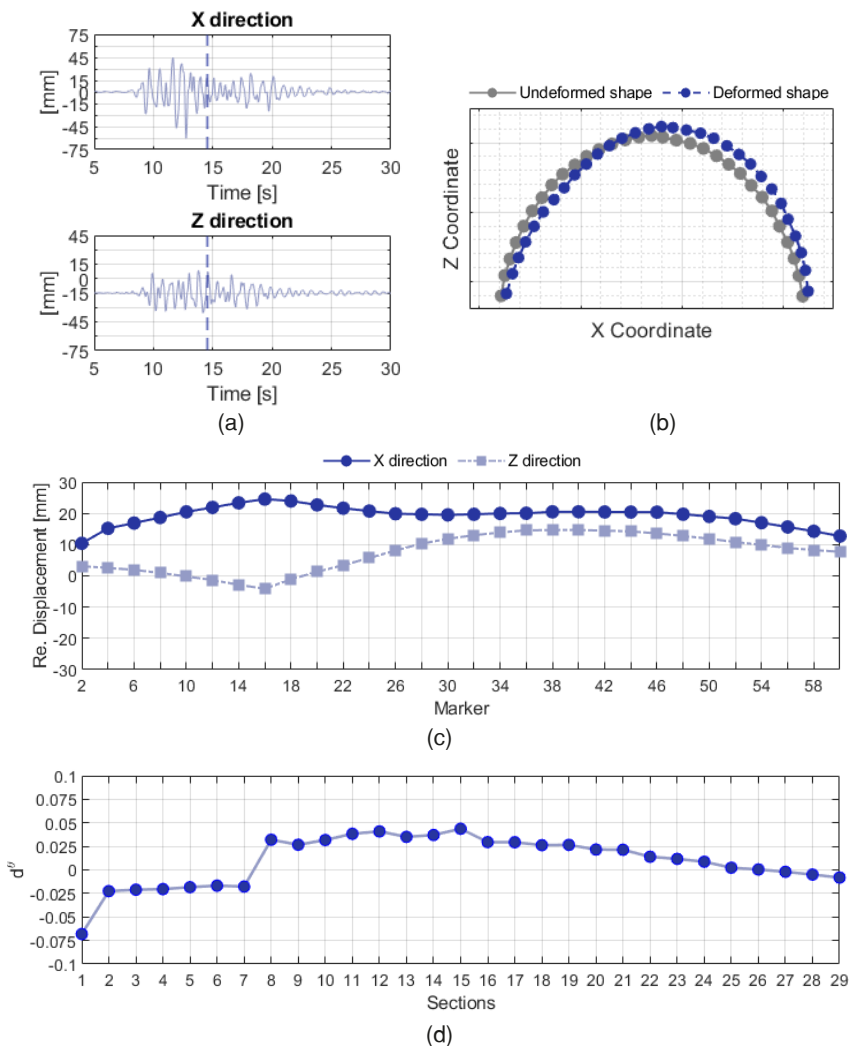


Fig. 5.45: Structural response of the R-DUV specimen during test 2.1RE at 14.5 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.

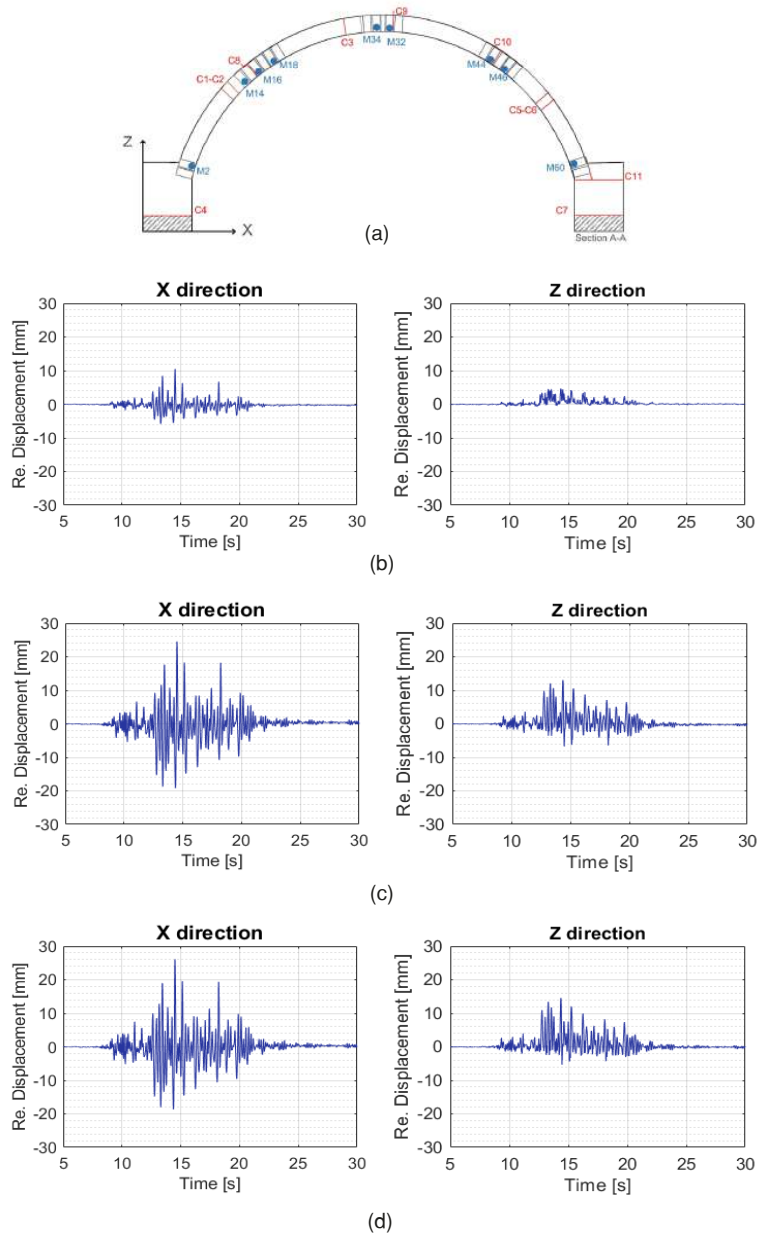


Fig. 5.46: Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 2.1RE: (a) Section A-A for reference, (b) location M1, (c) location M14, and (d) location M16

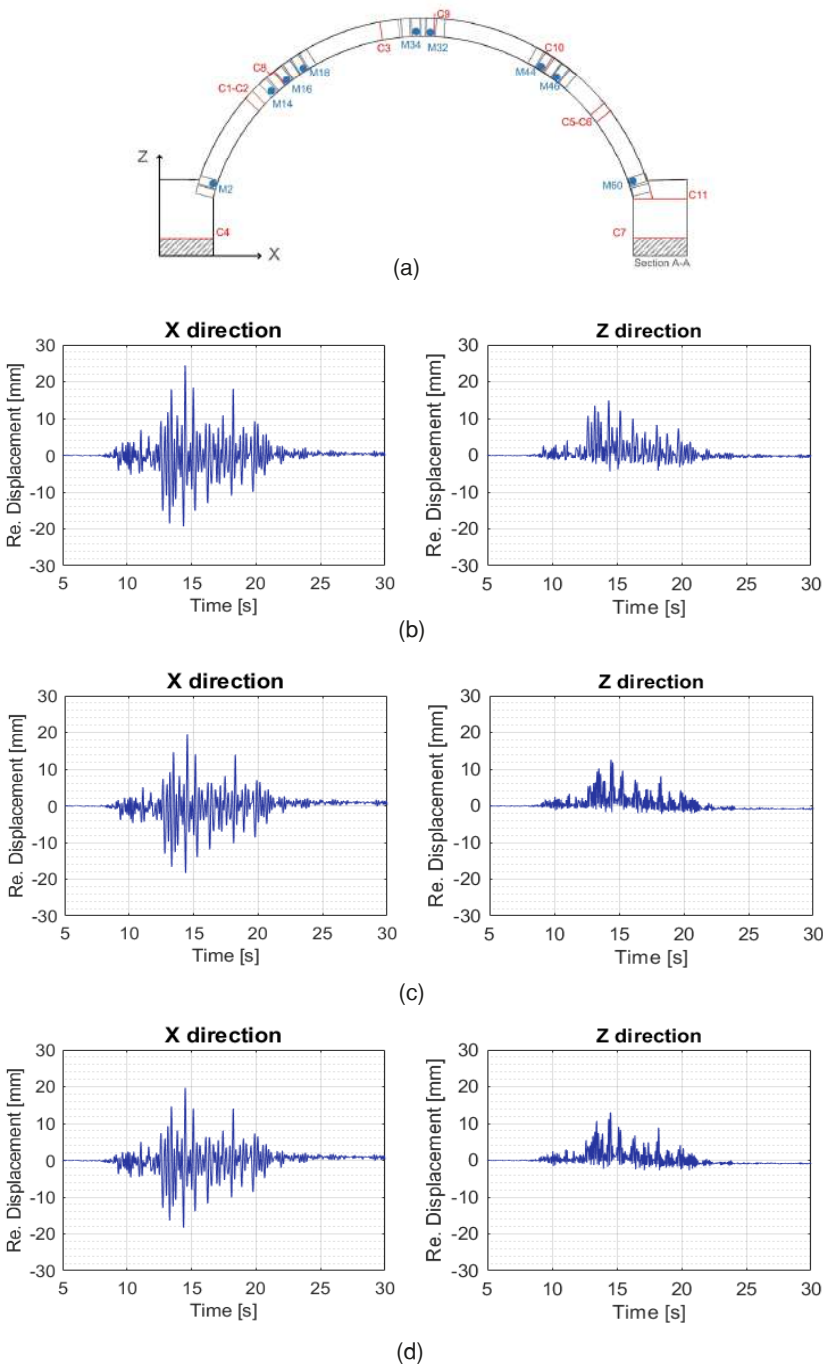


Fig. 5.47: Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 2.1RE: (a) Section A-A for reference, (b) location M18, (c) location M30, and (d) location M32

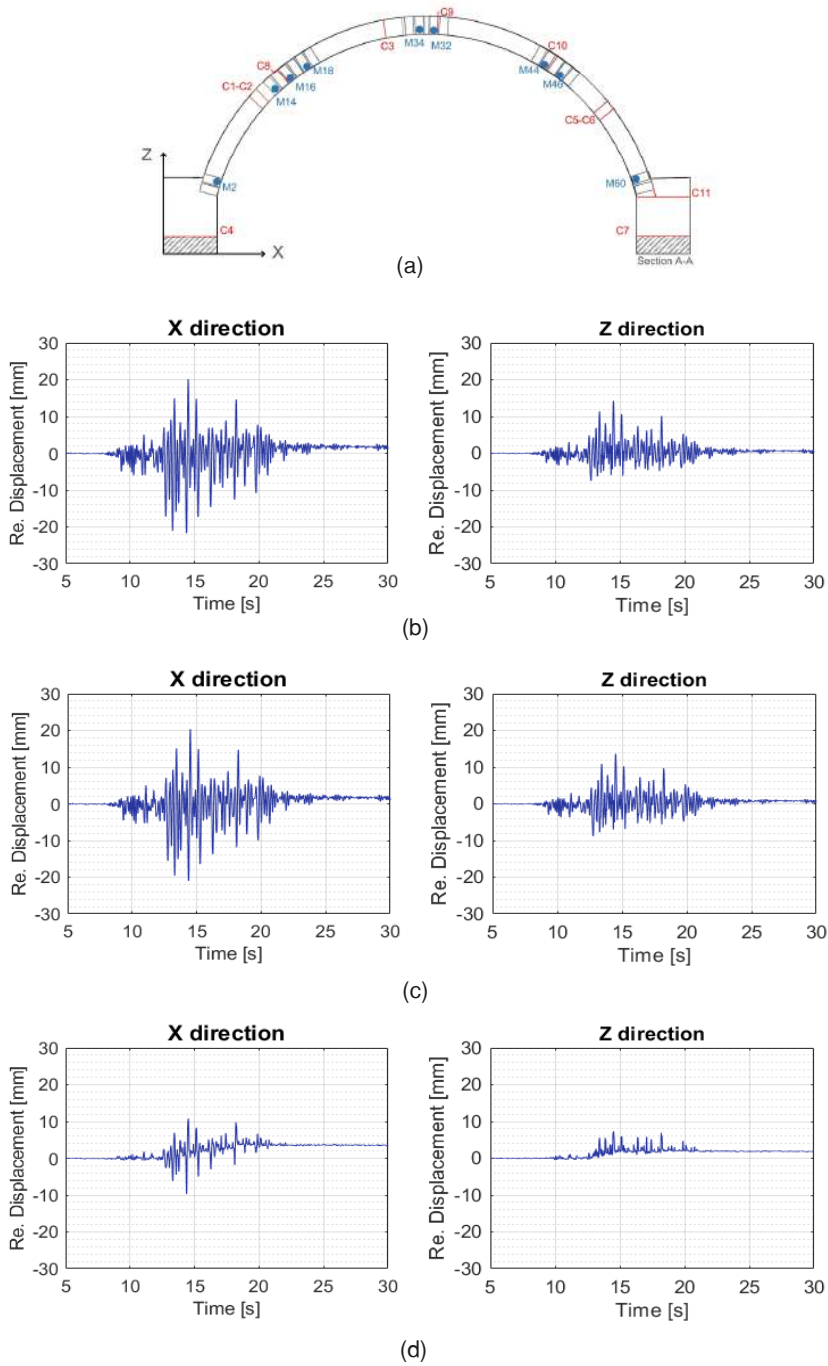


Fig. 5.48: Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 2.1RE: (a) Section A-A for reference, (b) location M44, (c) location M46, and (d) location M60

5.3.4.4. Results at 2.2RE

Given the damage pattern and its evolution during the last seismic test of the R-DUV specimen, the following analysis was carried out by restricting the time frame of investigation, which was interrupted at 14.0 s. This choice was necessary to highlight the significant variations in the vault behaviour during the test, while avoiding the sole identification of peaks corresponding to the collapse phase.

In particular, a representative frame of the global behaviour was detected at 12.9 s, as presented in Figure 5.49.

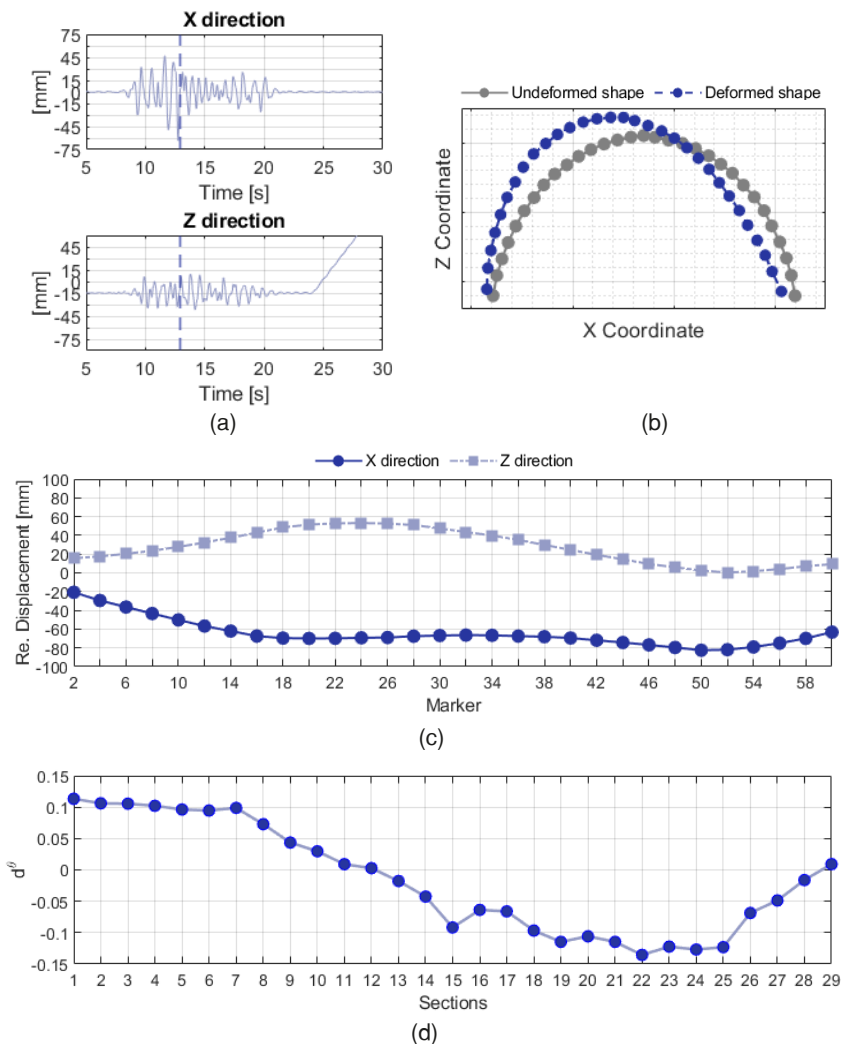


Fig. 5.49: Structural response of the R-DUV specimen during test 2.2RE at 12.9 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.

The significance of the deformed shape is illustrated in panel (b), showing a negative translation of the whole specimen, particularly at the lower monitored locations, while an upward movement is detected in the vertical direction. These results are further highlighted by the relative displacement values recorded in both the X and Z directions in panel (c). At locations M2 and M60, the first experienced a negative displacement of -20 mm in X and a positive displacement of nearly the same magnitude in Z, while the second recorded almost 30 mm in Z but twice this value in X (-60 mm). Additionally, all monitored points recorded displacements ranging approximately from -20 to -85 mm in the X direction, while smaller values were obtained in the vertical direction, ranging from 20 to 55 mm. The peak in the horizontal direction occurred at location M50, with 84 mm in X and -7 mm in Z, whereas the peak in the vertical direction was observed at location M22, with 53 mm in Z and 70 mm in X. Panel (d) emphasises the significant angular variations compared to the original configuration, indicating substantial changes in the arch geometry. Section S15, bounded by locations M30–M32, corresponds to crack C9, while sections S25 and S26, bounded by M48–M50 and M50–M52, respectively, are associated with cracks C5 and C6.

Another significant frame of the response was identified at 13.5 s, which highlighted the complete rotation caused by rocking around the left abutment (panel (b), Figure 5.50). Indeed, the vertical and horizontal displacements obtained in the right portion of the specimen (from M30 onwards) exhibit almost equal values of approximately 30 mm in both the X and Z directions. The peaks of the two displacement patterns were both recorded at location M16 (panel (c)), with approximately 37 mm in the X direction and -10 mm in the Z direction. Additionally, the angular variation indicator highlights a change in the trend in sections S7–S8, between M14–M16–M18, and in section S15, between M30–M32. Both of these locations identified by the d^θ analysis are correlated with the presence of visible damage.

Moreover, the deformed shape shown in Figure 5.51(b) highlights the formation of the shear plane at the reinforcement–masonry interface, which resulted in the expulsion of the upper part of the right abutment. This phenomenon is clearly detectable from the significant displacements recorded in the right portion of the vault. Panel (c) further illustrates how the displacements in the X direction increase progressively towards the right abutment, with M60 being the closest monitoring point. Values ranging from approximately 20 mm to 85 mm were recorded in the X direction, while values between -30 mm and 10 mm were obtained in the Z direction. Sections S13 and S15 were identified as the most significant in terms of the angular variation indicator, with the first located between M28–M30 and the second between M30–M32, confirming the critical condition of the area near the keystone.

As can be observed from the RDTHs of the locations presented in Figures 5.52, 5.53, 5.54, and 5.55, the analysis covers a time frame between 5 and 14 s.

It is worth noting that displacements exceeding 5 mm begin to appear from 9 s onwards. Negative peaks are recorded in the horizontal direction at all monitored points, while the vertical displacements exhibit exclusively positive peaks, emphasizing the global movement to which the specimen was subjected.

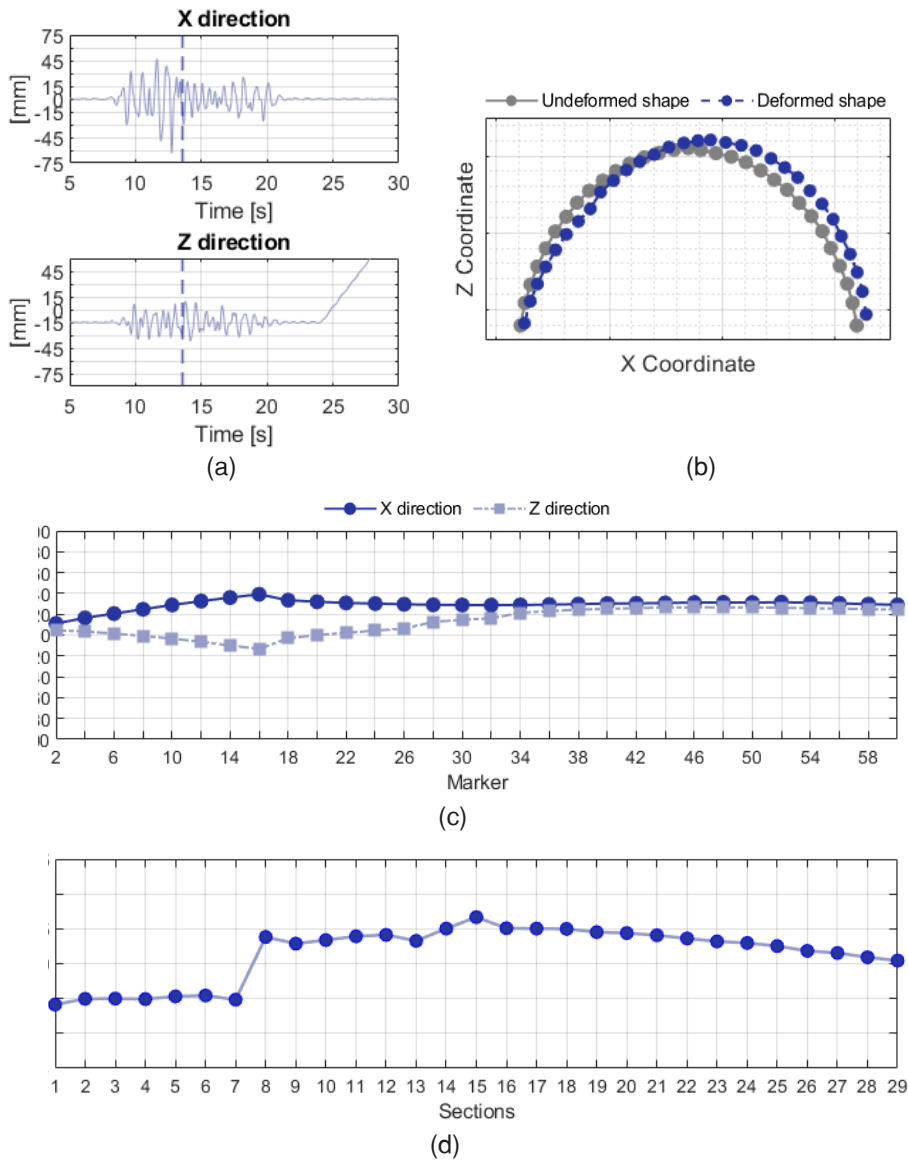


Fig. 5.50: Structural response of the R-DUV specimen during test 2.2RE at 13.5 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.

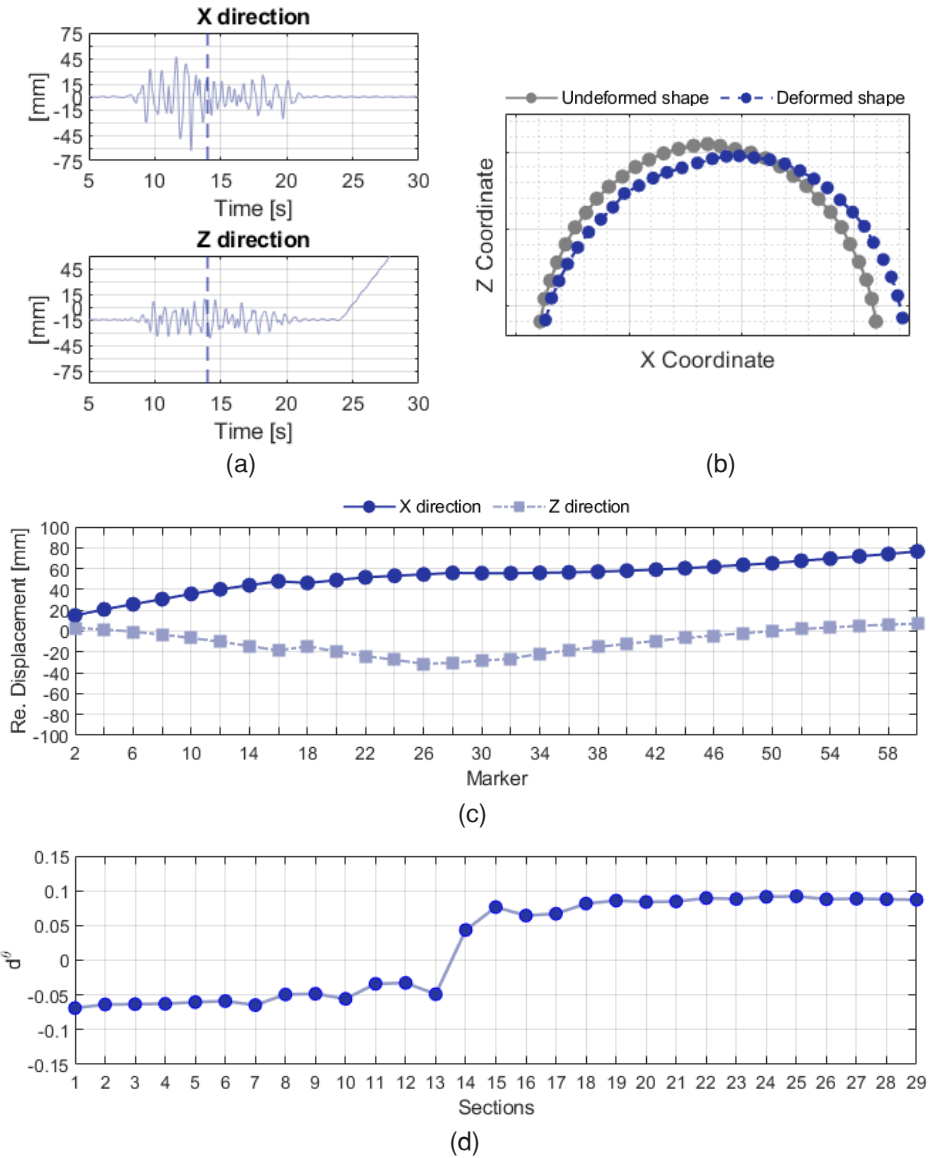


Fig. 5.51: Structural response of the R-DUV specimen during test 2.2RE at 14.0 s: (a) input, (b) deformed and undeformed shapes, (c) relative displacement patterns, and (d) angular variation indicator pattern.

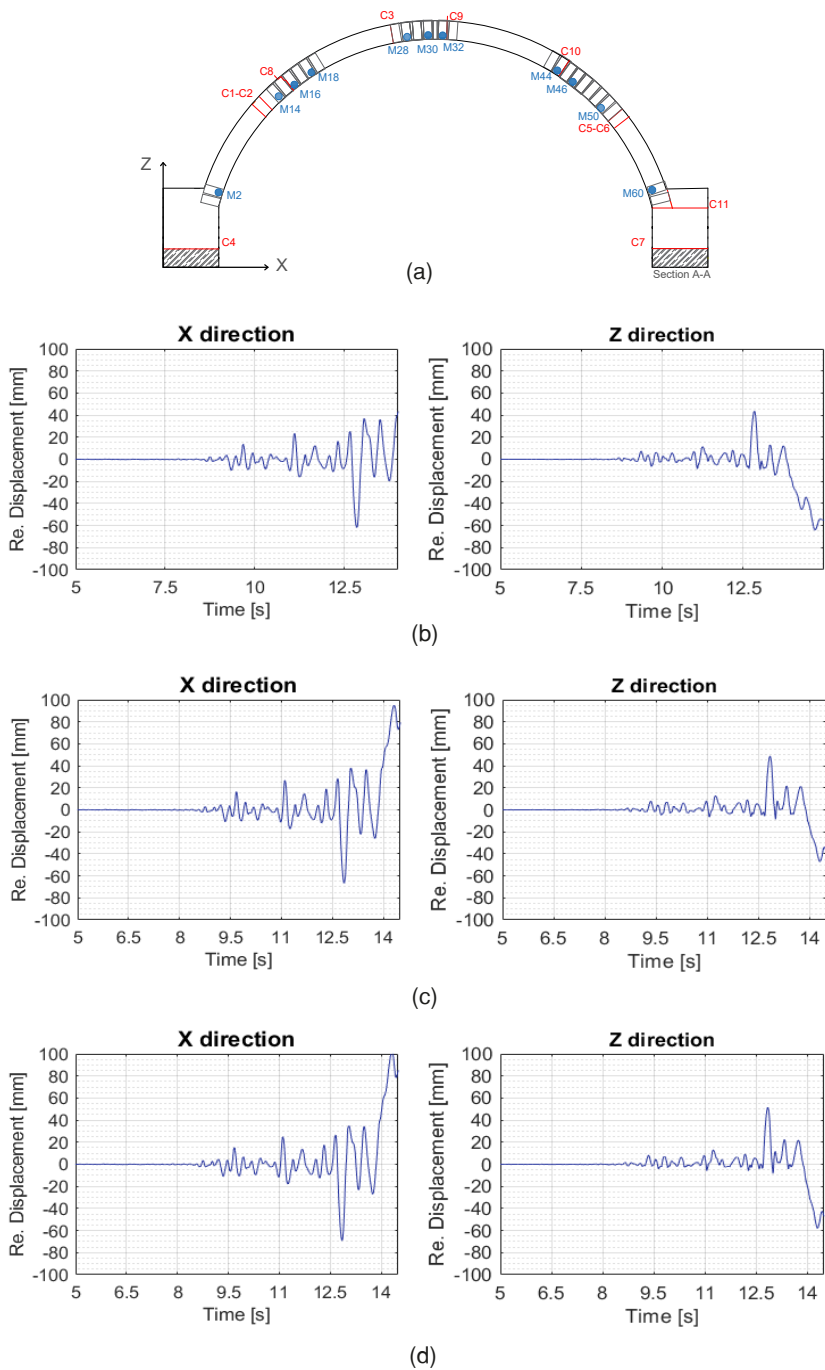


Fig. 5.52: Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 2.2RE: (a) Section A-A for reference, (b) location M14, (c) location M16, and (d) location M18

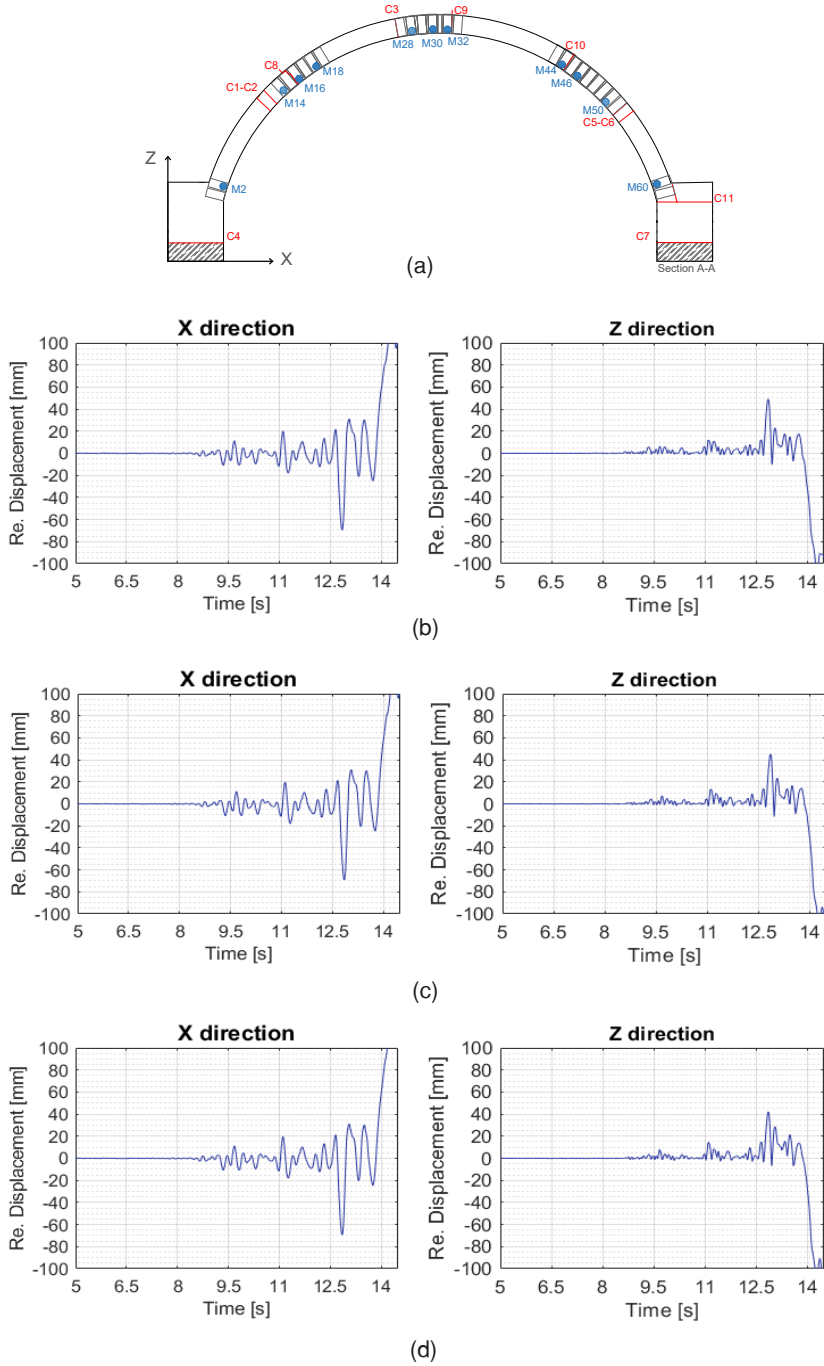


Fig. 5.53: Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 2.2RE: (a) Section A-A for reference, (b) location M28, (c) location M30, and (d) location M32

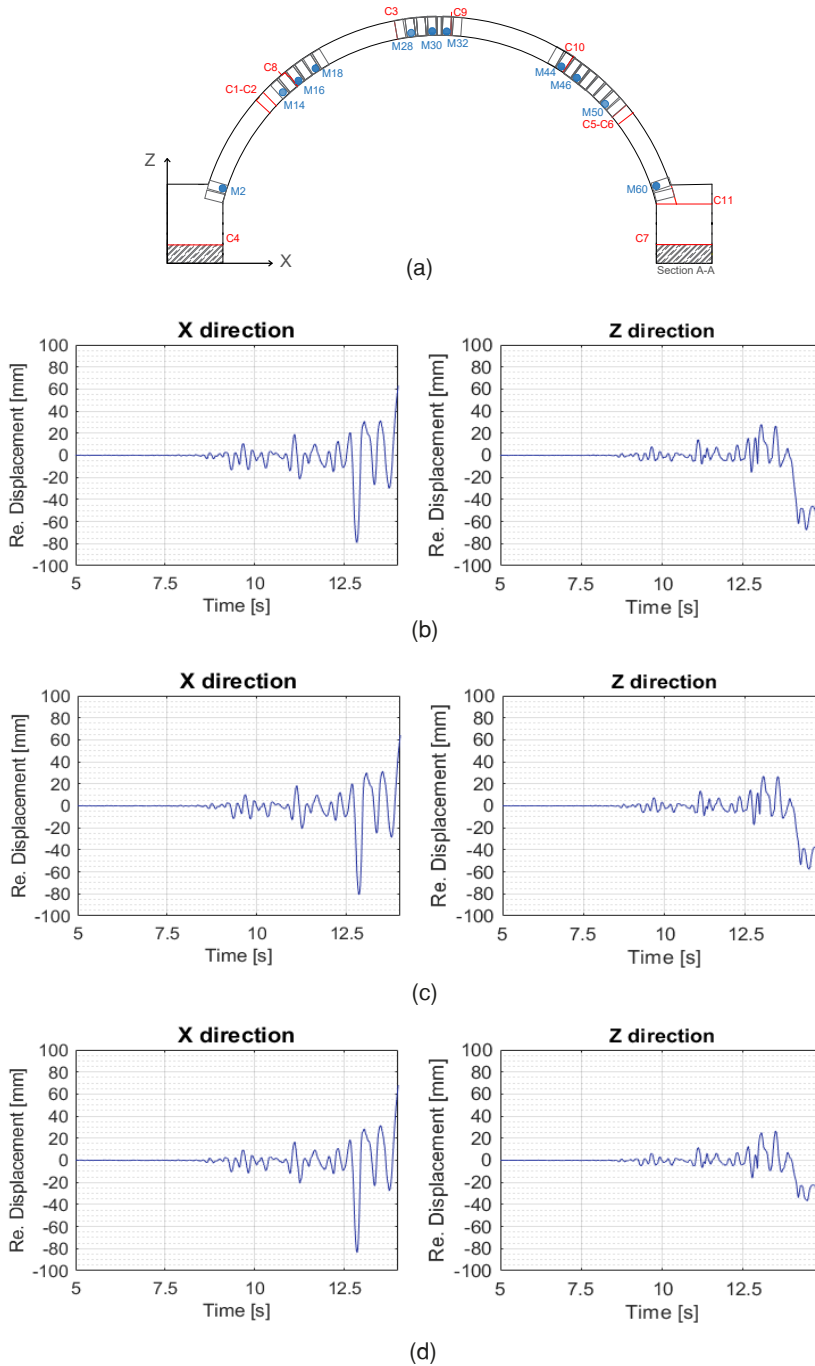


Fig. 5.54: Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 2.2RE: (a) Section A-A for reference, (b) location M44, (c) location M46, and (d) location M50

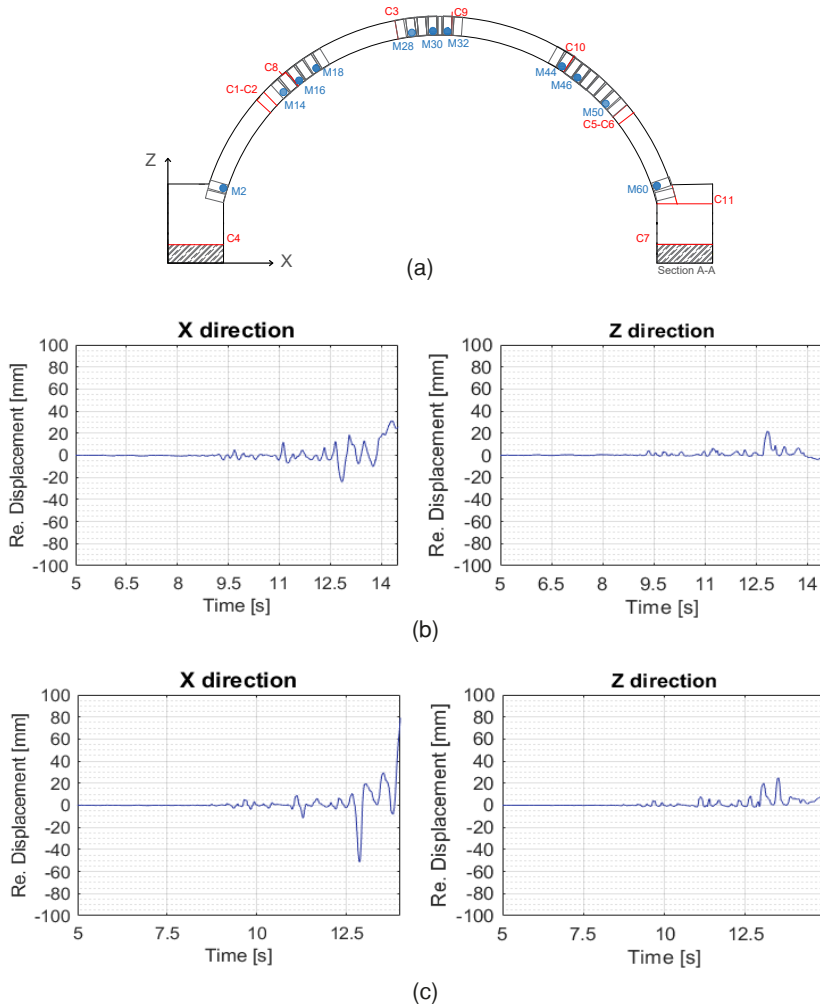


Fig. 5.55: Relative displacement time histories of selected monitored locations of the R-DUV specimen during test 2.2RE: (a) Section A-A for reference, (b) location M2, and (c) location M60.

5.4. Conclusive remarks

This chapter provided significant insights into the seismic capacity of masonry barrel vaults, in both unreinforced and strengthened conditions, subjected to dynamic seismic inputs of increasing intensity. In particular, the dynamic behaviour was assessed through a combined qualitative–quantitative approach. From this analysis, several final considerations were drawn:

- The effectiveness of the interventions in enhancing the load-bearing capacity of the specimens was assessed in both the repaired and reinforced

configurations, considering the multiplying factor applied to the input. The reinforced and repaired vaults reached ultimate conditions under input values approximately 5.5 times higher than those of the UV counterpart. Indeed, the unreinforced vault failed when a factor equal to 0.4 was applied to the RE, while the RV and R-DUV specimens collapsed at 2.3 and 2.2 times the RE, respectively.

- Under the applied inputs, the UV failed through the development of a rocking mechanism, while the repaired and reinforced vaults exhibited more widespread crack propagation, increasing the number of weak areas before reaching collapse.
- From the different failure mechanisms, it was possible to detect areas similarly affected by crack propagation, providing useful insights into the behaviour under comparable seismic inputs.
- The connection between the reinforcement layer and the abutments was identified as one of the most critical zones, together with the area around the keystone in the strengthened solutions, which played a significant role in the failure of both configurations.
- The details of the connection in the two strengthened conditions influenced the failure mechanism: while the connection was identified as a critical location in both cases, its position affected the capacity of the hemp ropes to act in tension, either ensuring a final equilibrium configuration that prevented collapse (as in RV) or failing to do so (as in R-DUV).
- The progression of damage was evaluated both visually and through modal property-based indicators, showing that damage and the associated stiffness degradation affected the assessed properties differently.
- The compatibility of the novel bio-composite strengthening technique was assessed by comparing the modal properties of the three specimens in the undamaged conditions:
 - Between the UV and RV specimens, percentage changes in frequency remained below 10%, while MAC values higher than 0.8 were obtained for all corresponding modes.
 - The strengthening applied to the DUV specimen allowed the recovery of the modal properties of the R-DUV to values close to those of the UV specimen, with percentage errors lower than 10% and MAC values higher than 0.78.
- The displacement-based analysis aimed at identifying sensitive locations to be used as a reference for comparison with the numerical model. This was carried out only for significant seismic tests, providing insights into the deformed shapes, the locations of maximum response in terms of relative displacements, and the variations in angular indicators.

6. Influence of non-linear interface parameters in modelling rocking failure mechanisms in curved unreinforced masonry structures

Several authors have studied the rocking behaviour of masonry structures, which *"represents the principal strength resource of masonry structures subjected to horizontal forces"* [Como et al., 2019]. Both analytical and numerical formulations have been proposed, and the mechanisms induced by rocking, as well as their analyses, remain an active subject of investigations (e.g., AlShawa et al. [2012]; Elgawady et al. [2011]; Housner [1963]; Ishiyama [1982]; Lagomarsino [2015]; Vecchio et al. [2025]; Yim et al. [1980]). In the specific case of masonry arches and barrel vaults, the first analytical and numerical investigation in the dynamics field was carried out by Oppenheim [1992], whose work focused on the onset of the rocking mechanism. Building on this foundation, further studies have been undertaken by several authors (e.g., Como et al. [2019]; Clemente [1998]; De Lorenzis et al. [2007]; Dejong and Ochsendorf [2010]).

The aim of this chapter is to provide new insights into modelling the rocking mechanism within a numerical framework. To this end, the chapter is structured into three main sections. The first section reviews the most common numerical strategies used for modelling rocking and, based on this review, proposes a

numerical approach to assess the response of unreinforced structures subjected to pure rocking. The second section applies the proposed modelling strategy to a reliable case study: the response of the unreinforced masonry vault. Finally, the third section presents a parametric analysis to evaluate the influence of the selected non-linear parameters.

6.1. Modelling strategies for rocking mechanism

In this work, an implicit time-stepping solution method is adopted within a Finite Element Method (FEM) framework. The rocking phenomenon is primarily modelled through the inelastic properties assigned to the interfaces, expressed in terms of tensile strength and, more importantly, Mode I fracture energy values. No additional damping formulation is introduced, while geometric non-linearities are explicitly considered.

To introduce the key aspects of this methodology, it is worth recalling that the rocking failure mechanism is inherently a dynamic and non-linear problem, involving both geometric and material non-linearities.

The following section presents a detailed discussion of each of these aspects.

6.1.1. Solution strategies for dynamic problems

A dynamic problem in a numerical model is solved by computing the equations of motion for a Multi-Degree-of-Freedom (MDoF) system, expressed as follows (Equation 6.1):

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}_{\text{ext}}(t) \quad (6.1)$$

Here, $\mathbf{M}$ denotes the mass matrix, $\mathbf{C}$ the viscous damping matrix, and $\mathbf{K}$ the stiffness matrix. The vectors $\ddot{\mathbf{u}}(t)$, $\dot{\mathbf{u}}(t)$, and $\mathbf{u}(t)$ represent acceleration, velocity, and displacement, respectively, while $\mathbf{f}_{\text{ext}}(t)$ is the external excitation vector. The system is considered non-linear if the matrices $\mathbf{K}$, $\mathbf{C}$, or $\mathbf{M}$ are time dependent; otherwise, it is regarded as linear.

When addressing rocking problems, several authors have adopted an explicit solution method, particularly within the framework of Discrete Element Method (DEM) approaches (e.g., Dimitri et al. [2011]; Galvez et al. [2022]; Gobbin et al. [2021b]; Malomo et al. [2021]; Peña et al. [2007]). A time integration scheme is defined as explicit if the displacement at time step $t + \Delta t$ is computed directly from the equilibrium condition at the previous time step t . In contrast, implicit time-stepping methods require equilibrium at $t + \Delta t$ to determine the corresponding displacement [Bathe, 1982; Chopra, 2014; Dokainish and Subbaraj, 1989; Gavin, 2020; Modak and Sotelin, 2000; Trcala et al., 2022]. Another key difference is that, in implicit methods, the stiffness matrix $\mathbf{K}$ must be updated at each time step, whereas explicit methods require its construction only once at the start of the time step [Chung and Lee, 1994; Kim and Lee, 2018]. This poses significant challenges for non-linear structural problems, where large changes in stiffness occur, as in rocking mechanisms. To overcome

this issue, several authors adopt simplified formulations (e.g., null-tension material conditions [Dimitri et al., 2011; Peña et al., 2007; De Lorenzis et al., 2007]), focusing primarily on capturing the rocking phenomenon rather than providing a full description of its progressive development. Moreover, explicit methods require an explicit formulation of the damping matrix, which is particularly challenging in structures failing through rocking mechanisms [Galvez et al., 2022; D’Altri et al., 2024; Vassiliou et al., 2021]. Implicit methods may instead rely on numerical damping, with the response depending on the chosen time step. Nevertheless, when the time step is adequately selected, this approach can simplify the problem by avoiding additional damping formulations.

Among implicit methods, the Newmark family of integration schemes is one of the most widely adopted approaches to relate displacement, velocity, and acceleration at time $t + \Delta t$, as expressed in Equations 6.2 and 6.3:

$$\mathbf{u}^{t+\Delta t} = \mathbf{u}^t + \dot{\mathbf{u}}^t \Delta t + \left[\left(\frac{1}{2} - \beta \right) \ddot{\mathbf{u}}^t + \beta \ddot{\mathbf{u}}^{t+\Delta t} \right] \Delta t^2 \quad (6.2)$$

$$\dot{\mathbf{u}}^{t+\Delta t} = \dot{\mathbf{u}}^t + \left[(1 - \gamma) \ddot{\mathbf{u}}^t + \gamma \ddot{\mathbf{u}}^{t+\Delta t} \right] \Delta t \quad (6.3)$$

The Newmark method is derived from a second-order Taylor expansion, where the parameters $\beta = 0.25$ and $\gamma = 0.5$ guarantee unconditional stability and second-order accuracy of the integration scheme. However, the method may introduce inaccuracies in the short-period (high-frequency) range when the associated response is smaller than the adopted time step Δt . These effects can be controlled by introducing artificial numerical damping.

The Hilber–Hughes–Taylor (HHT) method extends the Newmark formulation by introducing a numerical dissipation parameter α . The Newmark coefficients β and γ are then expressed as functions of α , as shown in Equations 6.4 and 6.5:

$$\beta = \frac{1}{4}(1 - \alpha)^2 \quad (6.4)$$

$$\gamma = \frac{1}{2}(1 - 2\alpha) \quad (6.5)$$

The parameter α is commonly set to -0.1 , but can assume values in the range $-0.3 \leq \alpha \leq 0.5$. When $\alpha = 0$, the HHT method reduces to the classical Newmark formulation. Lower values of α increase numerical damping, primarily affecting low-period modes while leaving high-period modes nearly unaffected. Importantly, this dissipation does not compromise the unconditional stability and second-order accuracy of the scheme [Gavin, 2020; DIANA FEA, 2025].

Finally, the choice of the time step Δt is critical to ensure that all frequencies of interest are adequately captured. A commonly adopted criterion is expressed in Equation 6.6, where f denotes the highest frequency considered:

$$\Delta t = \frac{1}{20} \cdot \frac{1}{f} \quad (6.6)$$

Building upon this review, this thesis aims to propose an approach using the HHT method, without relying on additional damping formulation, to describe the onset and progression of the rocking mechanism.

6.1.2. Material and geometric non-linearities

In numerical methods, particularly in Finite Element (FE) environments, the formulation of a rocking mechanism in MDoF problems can be obtained through a simplified micro-modelling approach, which assumes that all material non-linearity is concentrated in the interfaces. In this approach, where zero-thickness elements ideally represent the mortar–brick interfaces and the mortar joints themselves, the behaviour can be characterized by crushing, cracking, and sliding. In the following formulation, neither sliding behaviour nor crushing failure are considered, since a simple rocking mechanism, defined as rotation around hinges alternating between intrados and extrados, can be modelled through stiffness and strength loss in the tensile branch of the materials. To illustrate the strategy, a typical traction (σ)–interface relative normal displacement (Δ_u) curve in compression and tension is provided in Figure 6.1. In this formulation, crack width is used to describe crack opening, while crushing phenomena are not considered, as they are assumed to remain within the elastic range.

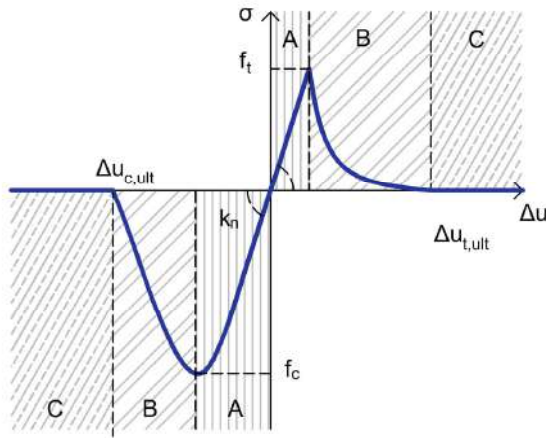


Fig. 6.1: Typical formulations of compressive and tensile behaviours

In the figure, three distinct regions can be identified:

- A. the elastic phase;
- B. the softening branch;
- C. the total loss of resistance.

The first region (A) is defined by elastic properties; in the case of interfaces, these are represented by the normal stiffness (k_n) and bounded by the tensile and compressive strengths (f_t and f_c , respectively). The softening branch can be defined in several ways, such as using fracture energy, ultimate strain, or a multi-linear approach. At the end of the softening branch, once the ultimate interface relative normal displacement ($\Delta_{u,ult}$) is reached, the material has lost its resistance properties, resulting in a loss of stiffness and increasing displacements without stress.

In this work, a focus on the parameters governing tensile constitutive laws is provided. Different formulations are available in the literature, but the most common softening rules for Mode I failure are based on exponential (Figure 6.2(a)), linear (Figure 6.2(b)), or fully brittle (Figure 6.2(c)) constitutive laws, which are primarily governed by four parameters:

- the initial stiffness (k_n);
- the tensile strength (f_t);
- the fracture energy (G_f^I);
- the softening curve shape.

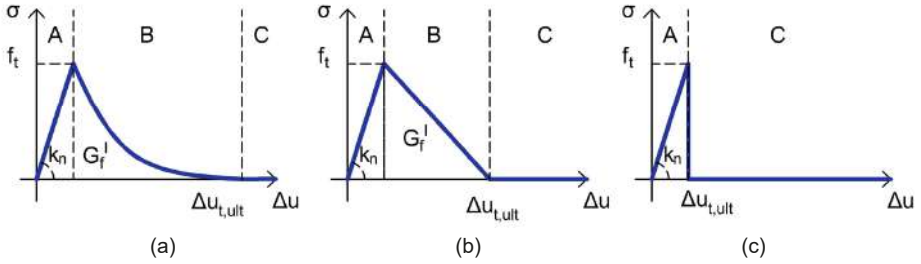


Fig. 6.2: Typical constitutive laws for tensile materials

The combination of the initial stiffness (k_n) and tensile strength (f_t) defines the elastic phase (A), while the softening branch (B) is governed by the tensile strength, the softening curve shape, and the fracture energy (G_f^I). It is worth noting that the ultimate interface relative normal displacement, referred to here in the tensile case, as the ultimate crack width opening ($\Delta_{u,ult}$), is determined by a combination of these three parameters. Different combinations lead to different ultimate values. Finally, once $\Delta_{u,ult}$ is reached, the material experiences a total loss of tensile resistance (region C).

On one hand, most numerical formulations of rocking available in the literature analyse the influence of stiffness parameters and tensile strength (e.g., Galvez et al. [2022]; Dimitri et al. [2011]; AlShawa et al. [2012]; Zhong and Christopoulos [2021]; Chatzis and Smyth [2012]). In many publications, tensile strength

is assumed to be negligible when modelling rocking between masonry blocks and is therefore set to zero, as in the case of dry joints, effectively neglecting the low tensile strength. However, this approach is difficult to implement in implicit time-step solution methods, as the solution may easily diverge. For this reason, a finite value of tensile strength must be introduced, along with a corresponding value of fracture energy. Using an ideally brittle law, such as that shown in Figure 6.2(c), often causes severe convergence problems in implicit solvers. Moreover, assigning a null tensile strength prevents the representation of the rocking phenomenon as a progressive damage process and only allows for capturing its onset. On the other hand, only a limited number of studies investigate the role of fracture energy, which is generally considered not to influence the overall capacity of the structure [Parisi et al., 2013; Pulatsu et al., 2020a;b]. Nevertheless, it is straightforward to understand that the value attributed to fracture energy strongly affects how quickly the material loses its resistance. A numerical example can clarify this aspect. Consider a linear softening branch with a tensile strength of 0.2 MPa and a corresponding fracture energy of 0.01 N/mm. In this case, the ultimate crack width ($\Delta_{u,ult}$) is reached at 0.1 mm. By keeping the same tensile strength but increasing the fracture energy to 0.1 N/mm, $\Delta_{u,ult}$ becomes 1 mm before total strength loss, resulting in a difference of one order of magnitude. Additionally, since the loading in a rocking mechanism is not monotonically increasing, this aspect may create difficulties in non-linear dynamic analyses. Interfaces are subjected to alternating compression and tension states, which complicates the numerical representation of the rocking mechanism.

Additionally, geometric non-linearity must be considered when dealing with rocking failure mechanisms, since it involves significant displacements and rotations, and very low strain values. In this context, a relevant formulation is the Total Lagrangian description, which is particularly suitable when displacements and rotations are large, while strains remain small. In this formulation, the reference configuration is the initial one (i.e., at time instant $t = 0$), and a Green–Lagrange strain formulation is adopted. The inclusion of this aspect is crucial in rocking analysis, which would otherwise be constrained to small displacements and rotations.

6.1.3. Modelling methodology adopted in this framework

Given this background, the proposed strategy is applied and subsequently validated in the following work. In a FEM environment, a simplified micro-modelling approach is adopted, concentrating all inelastic material properties in the zero-thickness interfaces. The constitutive law is defined by the normal stiffness and the softening law shown in Figure 6.2(b), where tensile strength and fracture energy are specified to assess the significant influence of each parameter in effectively representing the rocking mechanism as a progressive damage

process. The HHT implicit solution strategy is employed, and no additional damping formulation is introduced. The model nonetheless accounts for both geometrical and material non-linear effects.

6.2. Validation of the methodology through the Unreinforced Vault case study

The proposed modelling strategy was applied to describe and reproduce the rocking mechanism in unreinforced arch and vault specimens. Since the Unreinforced Vault (UV) failed primarily through this phenomenon and provided a large amount of experimental data, it was selected as an opportunity to validate the strategy within a real case study. This section and the following one (6.3) will therefore be devoted to validating the strategy, while conclusive remarks will be presented in Section 6.4.

6.2.1. Model description

A numerical model of the UV was developed in DIANA FEA¹ (Finite Element Analysis) with the aim of reproducing, in detail, the rocking mechanism extensively observed during the experimental tests and described in Chapter 5. The model was based on the FEM and adopted a micro-simplified 2-D approach. The geometry of the specimen was defined according to the layout in Section A–A (Figure 4.13), consistent with the adoption of an in-plane modelling approach. Due to the large number of elements required to explicitly represent bricks, mortar joints, and brick–mortar interfaces, a simplified micro-modelling strategy was employed. In this approach, bricks were geometrically expanded on each side to account for the thickness of mortar joints, which were not explicitly modelled. The abutments were represented as homogeneous continuum elements, with their material properties defined as homogeneous masonry. Interface elements were introduced between bricks to represent both mortar and brick–mortar joints. These interfaces were the only elements that were assigned Non-Linear (NL) mechanical properties, since no damage was experimentally observed in the bricks or abutments. A hexa/quad mesh was adopted, with an element size of 20 mm, ensuring six elements across the vault thickness in a 2-D model, as 3-D is not needed since there were no torsional modes identified. The element types adopted in the model were as follows:

- Quadrilateral plane-stress element with eight nodes (element CQ16M, Figure 6.3);
- Triangular plane-stress element with six nodes (element CT12M, Figure 6.3);
- Interface element defined between two lines in a 2-D configuration (element CL12I, Figure 6.3).

¹DIANA FEA, Release 10.9

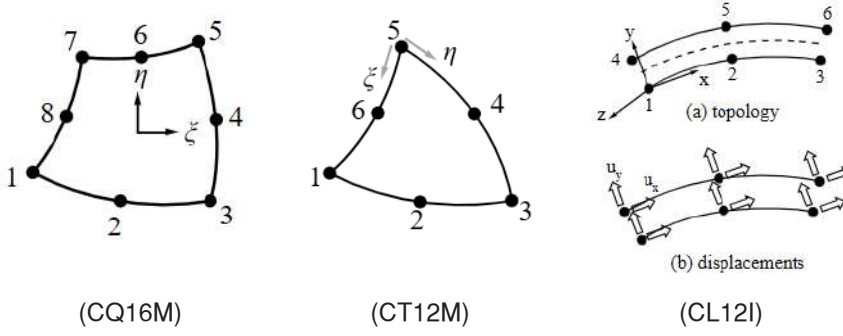


Fig. 6.3: Element types in the numerical model [DIANA FEA, 2025]

The model was constrained with translation-fixed boundary conditions at the base. It was comprised by 1945 quadrilateral elements, 25 triangular elements, and 381 interface elements, resulting in a total of 2351 degrees of freedom. As previously noted, the model was reproduced in a two-dimensional environment; therefore, the geometry of the elements accounted for the vault depth (i.e., 2.07 m). The expanded bricks and abutments were assumed to behave linearly, with their response governed exclusively by the elastic modulus and Poisson's ratio. This modelling choice was supported by the experimental results, which showed no damage in elements other than the brick–mortar interfaces. For the latter, a discrete cracking model was adopted to capture the N-L behaviour, while the elastic response was defined in terms of normal stiffness (k_n) and tangential stiffness (k_s), as expressed in Equations 6.7 and 6.8.

$$k_n = \frac{E_b \cdot E_M}{(h_m + h_b) \cdot (E_b - E_M)} \quad (6.7)$$

$$k_s = \frac{G_b \cdot G_M}{(h_m + h_b) \cdot (G_b - G_M)} = C_s \cdot k_n \quad (6.8)$$

in which:

- E_b : elastic modulus of bricks (MPa);
- E_M : elastic modulus of masonry (MPa);
- h_m : thickness of the mortar joints (mm);
- h_b : height of the brick (mm).

The N-L response was modelled using the discrete cracking approach implemented in DIANA FEA [2025]. This constitutive law describes the relationship between the normal traction (t_n) and the crack width (Δu_n), i.e., the total relative displacement across the joint (see Equation 6.9).

$$\frac{\sigma(\Delta u_n)}{f_t} = \begin{cases} 1 - \frac{\Delta u_n}{\Delta u_{n,ult}} & \text{if } 0 < \Delta u_n < \Delta u_{n,ult} \\ 0 & \text{if } \Delta u_{n,ult} < \Delta u_n < \infty \end{cases} \quad (6.9)$$

where f_t is the tensile strength and $\Delta u_{n,ult}$ is the ultimate crack width. A linear softening relation was set, so that the ultimate crack width ($\Delta u_{n,ult}$) was defined as two times the fracture energy (G_f^I) divided by the tensile strength (f_t). The behaviour of the interfaces in the normal direction is illustrated in Figure 6.4. Panel (a) presents the parameters governing the interface response, while panel (b) graphically depicts the unloading and reloading model defined with the secant approach. In this formulation, once the tensile strength is exceeded, the interface unloads along a secant line back to the origin. Upon reloading, the response follows the same path until it rejoins the softening branch, which continues until the tensile strength of the material is fully degraded.

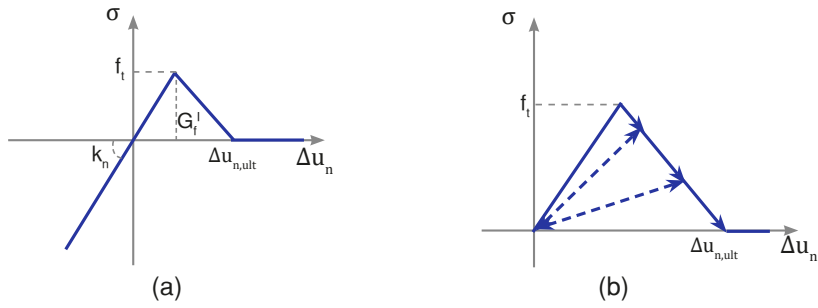


Fig. 6.4: Interfaces constitutive law in the normal direction: (a) ideal elastic behaviour in compression and linear softening Mode I in tension, and (b) detail of the tensile softening and secant unloading and reloading

The mechanical parameters adopted initially in the model are reported in Table 6.1. It is worth noting that the Elastic modulus (E) of the bricks was provided by the manufacturer [Gioffré et al., 2023], while that of the masonry was derived from Table C.8.5.I in Circolare [2019]. The Poisson's ratio (ν) was set to 0.2. The bricks' elastic modulus was assumed to be 5.7 GPa, while that of the masonry was 1.8 GPa. In the Equations 6.7 and 6.8, the mortar joint thickness was set to 10 mm, and the height of the brick was 120 mm. By adopting a Poisson's ratio of $\nu = 0.2$, the coefficient C_s was set to 0.4. Based on these assumptions, the expected values of the normal stiffness (k_n) and tangential stiffness (k_s) were 20 N/mm³ and 8 N/mm³, respectively.

All the materials were defined with a density of 1.8 Tons/m³ and the numerical specimen had a weight of 29.5 kN, consistent with the measured weight of the specimen.

Table 6.1: Material properties adopted in the UV numerical model in first instance

Material	E[GPa]	ν
Bricks	5.7	0.2
Masonry	1.8	0.2
	k_n [N/mm ³]	k_s [N/mm ³]
Interface	20	8

6.2.2. Methodology of validation of the model

The validation procedure of the numerical model followed a two-step approach that considers both the elastic and non-linear responses:

- The model was first assessed based on its ability to reproduce the vibration frequencies and corresponding modes identified experimentally through dynamic testing. These modal parameters provided a reliable representation of the mass and stiffness distribution governing the elastic behaviour of the system. A close agreement between numerical and experimental modal responses is considered a prerequisite for ensuring the reliability of the model before addressing the more complex non-linear dynamic behaviour.
- The model was subsequently subjected to seismic loading, and its dynamic response is compared with experimental data to characterise the rocking mechanism:
 - A critical aspect of the structural response concerns the behaviour of the interfaces, especially those expected to initiate cracking. These so-called *target interfaces* are analysed in detail due to their predominant influence on the overall rocking mechanism (see Figure 6.5). Their response is evaluated by tracking stress evolution over time and examining the tensile stress–crack width relationships. These results shed light on the transition from elastic to post-peak softening behaviour, the progressive loss of tensile capacity, and the formation of complete failure mechanisms.
 - The comparison focuses on the relative displacements recorded at selected monitoring locations along Section A–A, and individuated in the analysis presented in Section 5.3.2. Two reference locations are examined in detail: the keystone, representing a structurally significant position, and location M16, located near crack C4, which exhibited the largest displacement during the tests (see Figure 6.5). These two locations are representative of the global and local structural responses, and their displacement time histories are compared to assess the predictive capability of the model in terms of rocking onset, magnitude, and phase of response.

- In addition to time-history validation, a global comparison is performed at the instant of peak experimental response, i.e., at 6.7 s. At this time step, the relative displacements of all monitoring points along Section A–A are analysed in both the horizontal and vertical directions (X and Z). This assessment allows evaluation of the model's capacity to reproduce the deformed pattern of the structure and the potential initiation and development of hinge mechanisms. Numerical angular variation indicators (defined in Eq. 5.8 for the experimental results) are computed and compared with their experimental counterparts at the same instant, providing insights into the failure of the interfaces and the activation of rocking.

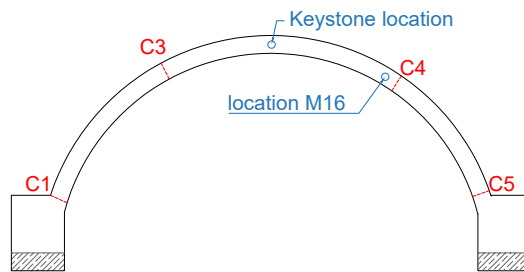


Fig. 6.5: Adopted elements to validated the models: target interfaces (C1, C3, C4 and C5), and significant locations

Through this multi-level validation strategy, combining frequency-domain analysis, time-history comparisons, and interface-level evaluations, the robustness of the numerical model is critically assessed. This integrated approach ensures that the model is capable not only of reproducing the global dynamic response but also of capturing the local non-linear phenomena that govern damage evolution and failure in masonry vaults under seismic loading.

6.2.3. Model updating of the elastic parameters

The model was calibrated based on the vibration frequencies and vibration modes obtained from the acceleration time histories recorded in ambient vibration conditions, prior to any seismic testing. The dynamic identification, performed through the Enhanced Frequency Domain Decomposition method, yielded the frequencies and mode shapes summarised in Section 5.2.2. The adopted calibration methodology is outlined as follows. The model was calibrated by ensuring that the numerical vibration modes were correlated with the experimental ones. The Modal Assurance Criterion (MAC), defined in Eq. 5.3, is the most commonly used metric in the literature for validating the correlation between two mode shapes, expressed as vectors [Buzzetti et al., 2024;

Luger et al., 2025; Pantò et al., 2024]. Since the MAC ranges between 0 and 1, indicating no correlation to perfect correlation, it is generally accepted that values greater than 0.8 indicate good agreement, whereas values below 0.3 imply poor correlation [Gioffré et al., 2023]. In addition to the MAC, the percent errors between the experimental and numerical frequencies was evaluated, with a target threshold set to 2%. This dual criterion, i.e., MAC for vibration modes and percent errors for frequencies, ensured a reliable calibration of the model. A well-calibrated model is essential to guarantee that the mass and stiffness matrices accurately represent those of the physical specimen.

A sensitivity analysis was carried out to investigate the effect of the interface stiffness values on the modal parameters. No significant variation in the vibration modes was observed with different values of k_n , as the MAC between experimental and numerical modes was always higher than 0.8 for corresponding modes. However, since a 2-D model was adopted, the second experimental mode, which is asymmetric between the two façades of the specimen, could not be reproduced. Therefore, in the following, the second numerical frequency and mode shape correspond to the third mode and frequency observed experimentally. The results of the sensitivity analysis are summarised in Table 6.2 and illustrated in Figure 6.6. In the latter, errors for the first frequency are represented with circular markers, and errors for the second frequency with square markers. In panels (a–b), results with errors $\leq 5\%$ are highlighted with a black border, while in panel (c), those with errors $< 2\%$ are similarly emphasised.

A multiplying coefficient C_n was applied to the normal stiffness k_n . An initial analysis was performed with C_n values equal to 1, 0.1, and 10, based on the stiffness calculated via Equation 6.8, while keeping C_s constant at 0.4. The value $C_n = 10$ yielded the lowest percent error for the first frequency (approximately 23%) and for the second frequency (about 28%). Conversely, the original k_n and k_s values ($C_n = 1$) led to percent errors of 31% and 26% for the first and second frequencies, respectively. When $C_n = 0.1$, the errors increased to approximately three times those obtained with the original stiffness values (Figure 6.6(a)). Among the additional tested values of C_n (2.5, 3, and 5), the best results were achieved for $C_n = 3$, with relative errors of 1.1% and 2.4% for the first and second frequencies, respectively (see Figure 6.6(b)). The influence of C_s was subsequently investigated, and the best agreement with experimental data was found for a ratio $k_s/k_n = 0.6$ (Figure 6.6(c)). A final calibration of C_s enabled fine-tuning of both frequencies to achieve percent errors below the target threshold of 2%. This configuration resulted in interface stiffness values of 60 N/mm³ and 35 N/mm³, corresponding to errors below 1.9% and calibrated frequencies of 18.2 Hz and 36.8 Hz, respectively.

For the final calibrated frequencies, a more detailed MAC analysis was performed and the results are reported in Figure 6.7. The latter shows visual comparisons between the vibration mode shape obtained from the experimental data and those resulting from the numerical model. As it is possible to note, both the

Table 6.2: Sensitivity analysis for tuning the interface stiffness parameters

k_n [N/mm ³]	k_s [N/mm ³]	C_n	C_s	Percent errors [%]	
				1 st frequency	2 nd frequency
20	8	1	0.4	30.4	25.9
2	0.8	0.1	0.4	75	75.2
200	80	10	0.4	23.4	27.2
100	40	5	0.4	10.9	9.3
50	20	2.5	0.4	5.9	7.2
60	25	3	0.4	1.1	2.4
60	12.5	3	0.2	2.2	3.7
60	35	3	0.6	1.1	1.9

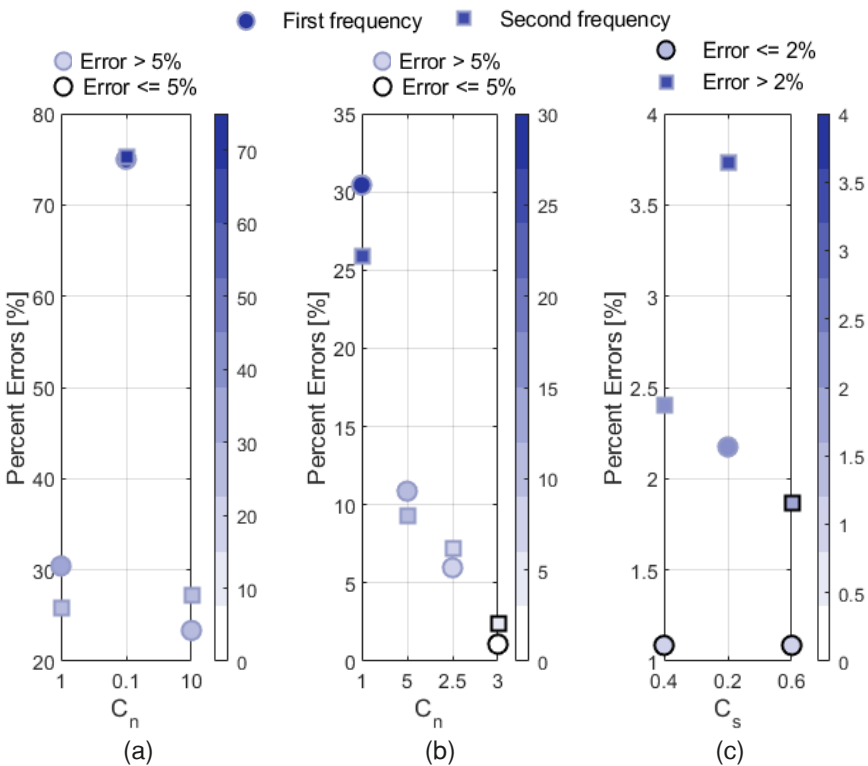


Fig. 6.6: Impact of interface stiffness parameters on the frequencies: (a–b) influence of C_n , (c) influence of C_s

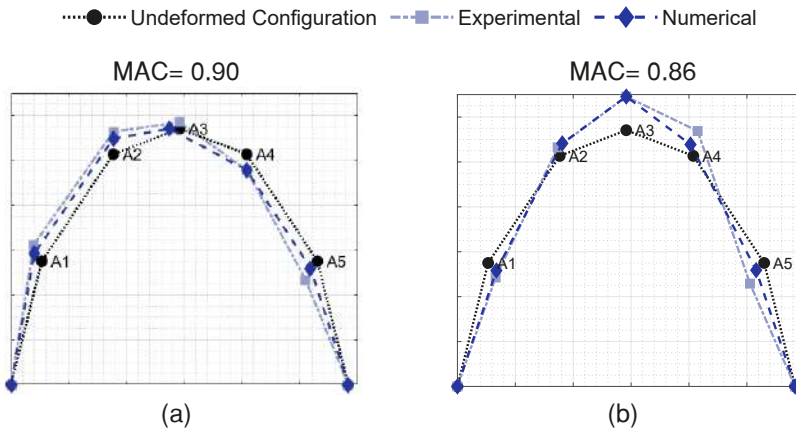


Fig. 6.7: Graphical comparison of the experimental and numerical mode shapes and MAC values: (a) first mode, and (b) second numerical mode and third experimental mode

same modes show a good correlation with a MAC higher than 0.86. It is worth noting that these values are calculated, using 8 out of 10 components of the eigenvectors and they are calibrated with reference to the vectors obtained from Section A-A (as in Figure 4.13), excluding the horizontal component of nodes A2 and A4. A final consideration regarding the calibration of the model is presented in Figure 6.8. In panel (a), it can be observed that the calibrated frequencies lie within the target error range of $\pm 2\%$, and are close to the configuration of a perfect match (i.e., when the numerical model reproduces the experimental data exactly). Panel (b) displays the cross-MAC matrix, which confirms the strong correlation between corresponding modes and the negligible correlation between non-corresponding ones. As previously noted, the calibration was performed for the first two numerical frequencies and their associated modes, which correspond to the first and third experimental modes.

6.3. Parametric study for strategy validation

Considering the parameters defined so far, the only remaining unknowns were the interface N-L parameters; namely, the tensile strength (f_t) and the Mode I fracture energy (G_f^I). Lourenço and Gaetani [2022] suggests typical values of $f_t = 0.1$ MPa and $G_f^I = 0.01$ N/mm, which will therefore be defined as literature based N-L parameters.

As described in Section 4.1.3.1, the seismic input consisted of three components with a total duration of 50 s, resulting in substantial computational demand. To reduce this, only the most intense phase of the acceleration time histories was retained (see Section 4.1.3.1). The reduced interval was defined by selecting the time corresponding to 5% and 95% of the Arias Intensity for each component,

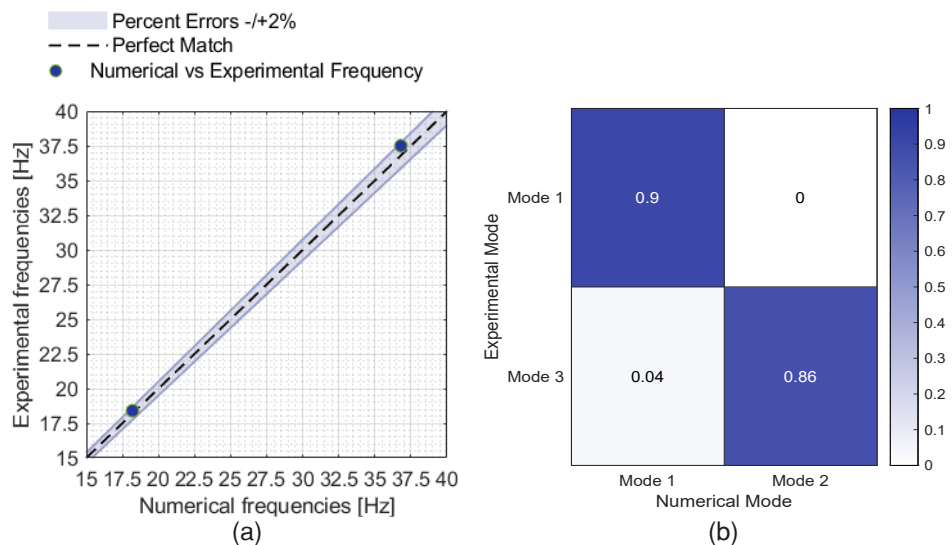


Fig. 6.8: Calibration of the modal parameters: (a) comparison between numerical and experimental frequency values, and (b) cross matrix of the MAC, noting that the second numerical modes matched the third experimental one.

with a slight extension to capture the early and late portions of the motion. Figure 6.9 illustrates the results of this analysis for the horizontal (X) and vertical (Z) components; the Y component is omitted because the model is limited to in-plane analysis, and its displacement values in the response were comparable to measurement noise.

The earliest start time across components was approximately 9 s (Z direction), and the latest end time was 20 s (X direction). To better capture the initial linear response and the late-stage behaviour, the interval was extended by 1.5 s at both ends, resulting in a final duration from 7.5 s to 21.5 s. Prior to use in the numerical model, the acceleration signals underwent baseline correction and high-pass filtering (cutoff 0.1 Hz) to remove low-frequency trends and non-physical drifts often introduced during integration to velocity and displacement [Boore and Bommer, 2005; Chen et al., 2021; Rupakhety and Sigbjörnsson, 2010]. Figure 6.10 compares the processed numerical inputs with the experimental signals, showing close reproduction of the original input accelerations in both directions.

Time History Analyses (THA) were carried out using the HHT method for time integration, with α set to -0.1 . Equilibrium at each time step was evaluated using the Secant (quasi-Newton) method, in which the stiffness matrix is computed based on the last two iterations. This method is particularly useful for THA involving a large number of time steps or when the stiffness matrix is non-symmetric [Bathe, 1982; Lourenço and Gaetani, 2022]. Convergence was

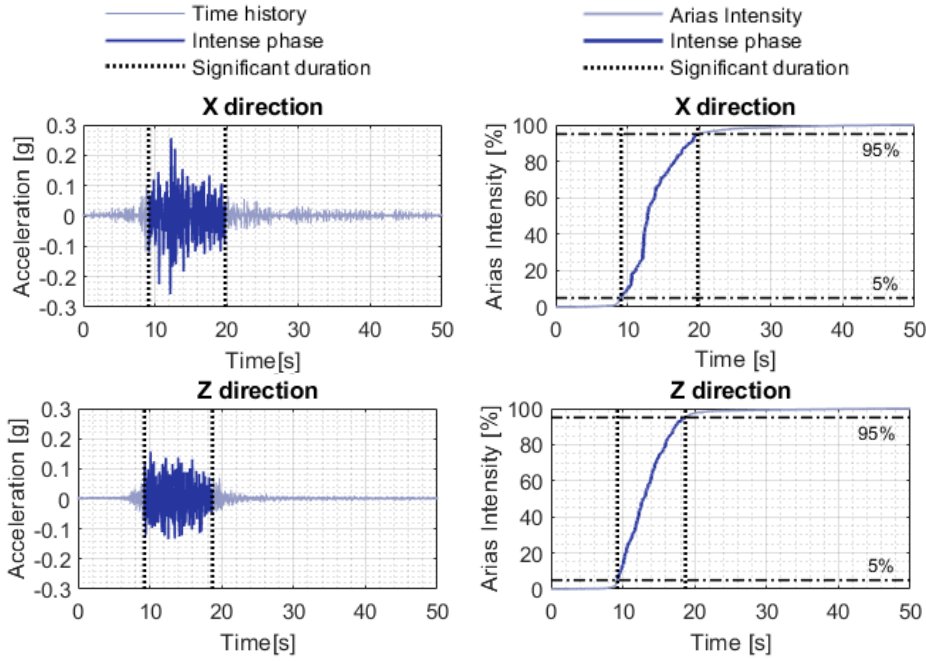


Fig. 6.9: Analysis for the significant duration of the acceleration time histories

evaluated through the variation of internal energy, with a tolerance threshold of 10^{-3} . A constant time increment of $\Delta t = 0.001$ s was adopted to satisfy the requirement in Eq. 6.6, in which the highest frequency of interest was set to the second calibrated frequency, i.e. 36.8 Hz, and to ensure that the time step is significantly smaller than the duration of the time history [Mendes, 2012]. The results were saved with a time step of $\Delta t = 0.01$ s, consistent with the sampling rate adopted for recording the displacement time histories. In some cases, particularly when commenting the interface responses, this may cause slight misinterpretations, making the behaviour appear different from that expected according to the imposed softening law, since not all the computed steps were saved.

6.3.1. Results using literature-based non-linear interface parameters

As stated, the N-L interface properties were defined as suggested by Lourenço and Gaetani [2022], with $f_t = 0.1$ MPa and $G_f^I = 0.01$ N/mm.

Since the global response is highly dependent on interface behaviour, the stress evolution over time was examined to verify whether the tensile strength was reached. To investigate the N-L behaviour of the interfaces governing the rocking mechanism, their response in terms of normal traction and crack width openings was also evaluated. Figure 6.11 shows the normal traction time histories for the target interfaces, with reference to the extreme points located

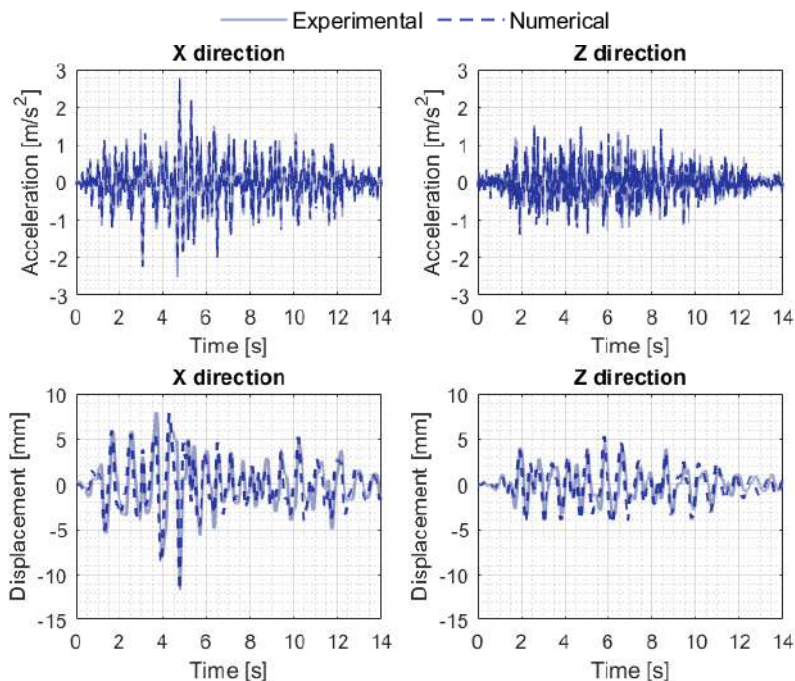


Fig. 6.10: Comparison of the experimental inputs and the numerical adopted ones.

at the intrados and extrados. At the beginning of the analysis, all monitored points at the extrados were in compression, while the points at the intrados of cracks C1 and C5 were slightly in tension. The extremes at the extrados of cracks C3 and C4 were also initially in compression. Most monitored points did not reach the tensile strength ($f_t = 0.1$ MPa), and therefore remained within the elastic range. Those that reached the tensile strength did not experience a complete loss of resistance, instead evolving along the softening branch. These observations indicate that the target interfaces did not fully open, preventing the activation of the rocking mechanism. A more detailed insight into the interface behaviour is provided in Figure 6.12, where normal traction is plotted against crack width opening (Δ_u) for the same monitored points. The results confirm the previous observations: the response either remained within the softening branch of the constitutive law or did not reach the tensile strength. This demonstrates that the interfaces never fully opened, as observed for cracks C1 and C5 at the intrados. At the other extremities, the tensile strength was not reached, confirming that the global behaviour remained nearly elastic.

The discussion on interface behaviour is consistent with the relative displacement response (i.e., the response excluding the base input), shown in Figures 6.13, 6.14, and 6.15. Figures 6.13 and 6.14 displayed the deformed shapes

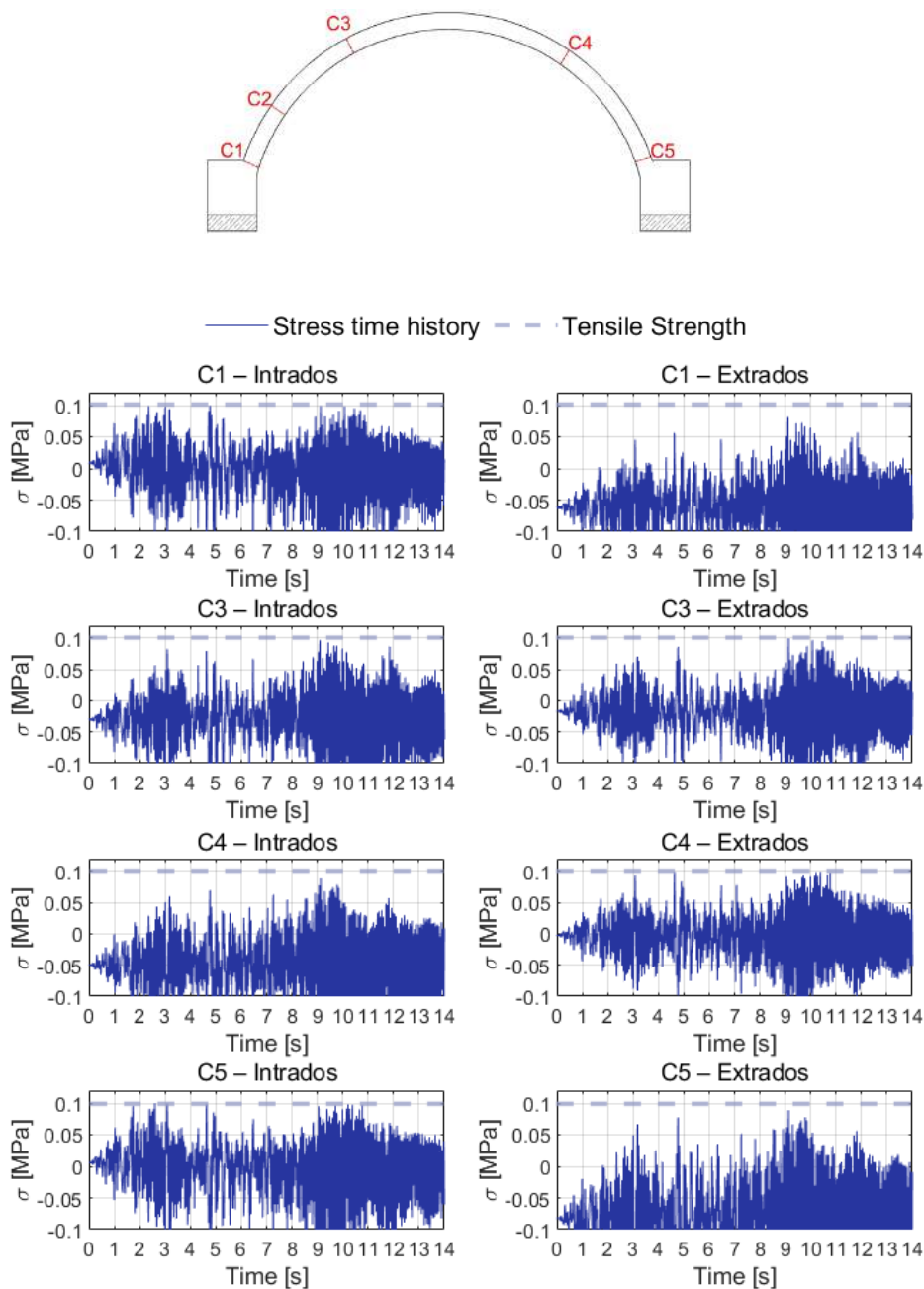


Fig. 6.11: Results using literature-based interface N-L parameters: traction time histories at the extreme points of the target interfaces

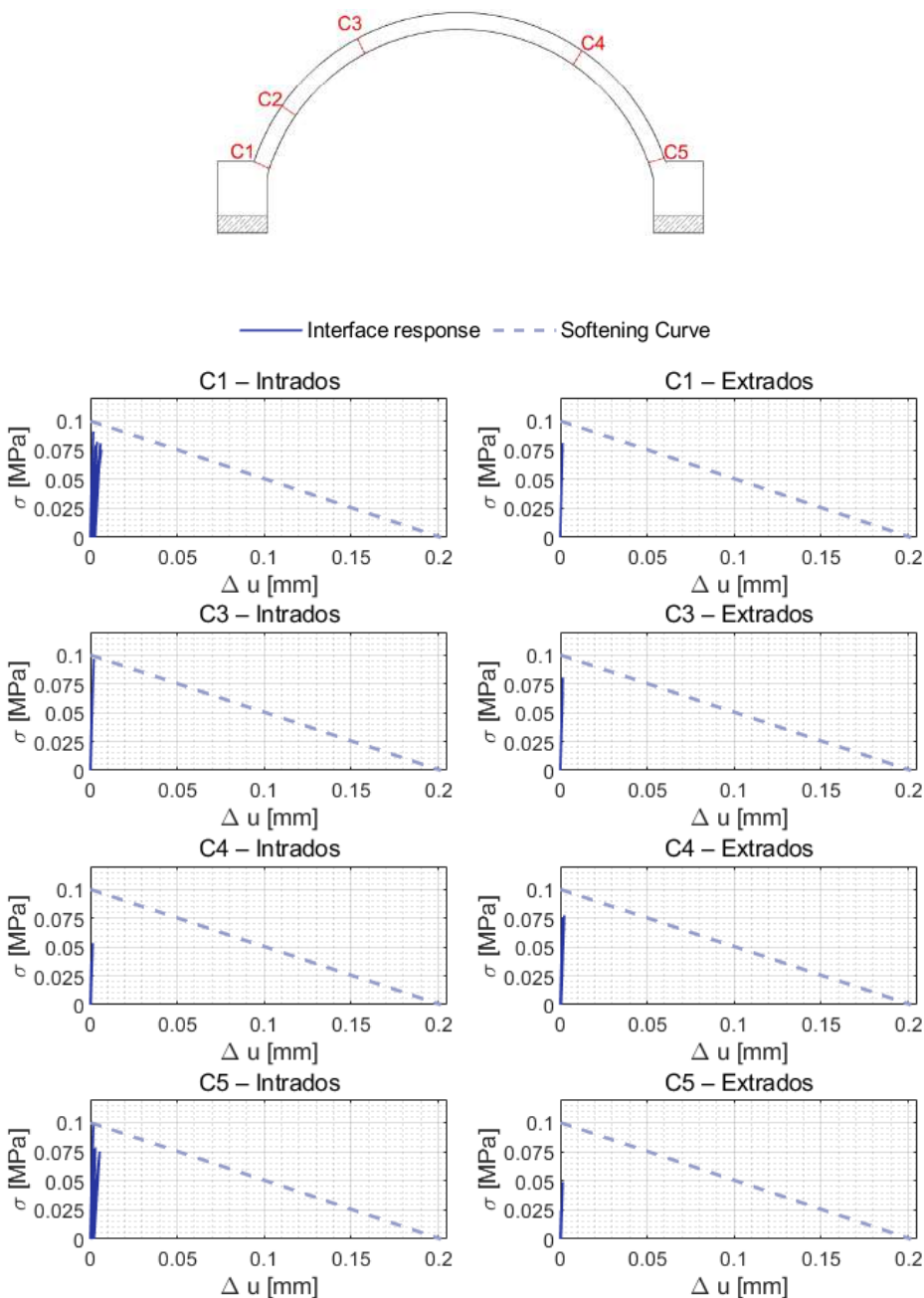


Fig. 6.12: Results using literature-based interface N-L parameter: numerical vs theoretical behaviour at the extreme points of the target interfaces

at two different time instants, 5.1 s and 9.1 s, corresponding to two significant time steps in the numerical response, respectively. In panels (a), no visible hinge openings were observed. Panel (b) of Figure 6.13 compared the experimental and numerical Relative Displacement Time Histories (RDTHs) at the keystone. The numerical results differed by approximately two orders of magnitude throughout the analysis in both the horizontal and vertical directions, with a maximum relative displacement of about 0.1 mm in both directions occurring around 10.5 s.

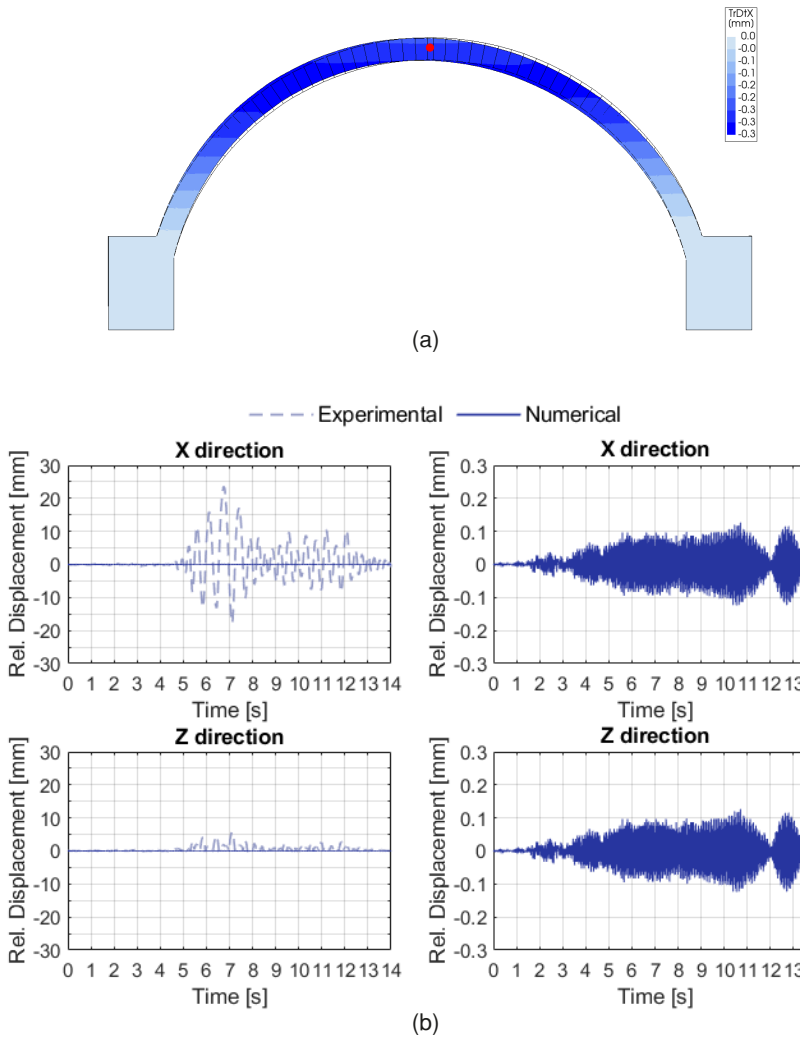


Fig. 6.13: Results using literature-based interface N-L parameters: (a) deformed shape at 5.1 s, and (b) comparison of the RDTHs at the keystone (left panels) and corresponding numerical response (right panels)

A similar trend was observed for the location that recorded the maximum relative displacement in the experimental campaign (location M16), as shown in panel (b) of Figure 6.14. In this case, the numerical simulation reached a peak displacement of approximately 0.2 mm in the X direction at around 9 s. Figure 6.15 shows the relative displacements of all monitored locations at 6.7 s, corresponding to the peak response instant observed in the experimental data. This figure is intended to provide a global validation of the model during the intense phase of the rocking. Panel (b) compares the experimental and numerical results, while panel (c) shows only the numerical response. The relative displacements along Section A–A of the unreinforced specimen are compared

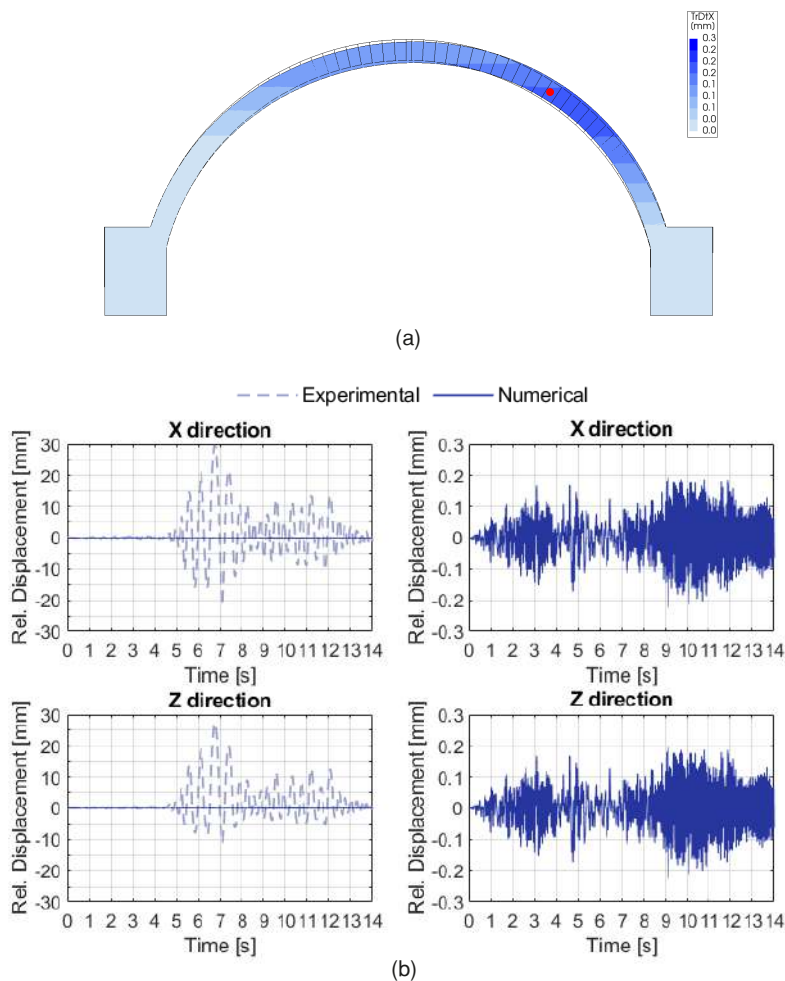


Fig. 6.14: Results using literature-based interface N-L parameter: (a) deformed shape at 9.1 s, and (b) comparison of the RDTHs at location M16 (left panels) and corresponding numerical response (right panels)

with the corresponding numerical results. As observed, the numerical response was approximately two orders of magnitude smaller than the experimental one, and the variation in angles, indicated by d^θ , was nearly zero. No hinges developed in the model, resulting in displacement profiles that differ significantly from the experimental observations, with very similar trends in both directions considered.

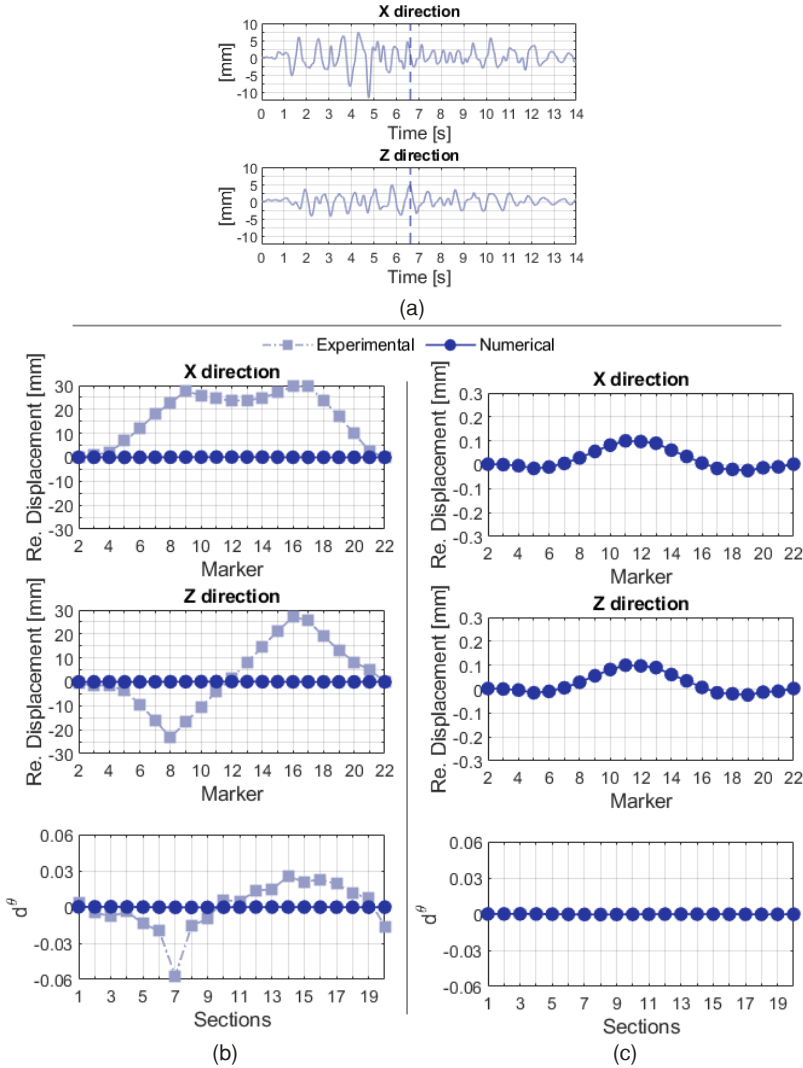


Fig. 6.15: Results using literature-based non-linear interface parameters at 6.7 s: (a) input signals, (b) comparison between numerical and experimental results, and (c) numerical results

6.3.2. Results using literature-based tensile strength and fracture energy of 1×10^{-3} N/mm

Given that the extreme points of the target interfaces reaching the tensile stress value always remained within the softening branch, the following analysis aimed to investigate the influence of the fracture energy while keeping the same tensile strength. All interfaces were defined with identical mechanical parameters: the maximum admissible principal stress was set to 0.1 MPa, while the fracture energy in tension was reduced by one order of magnitude, to 1×10^{-3} N/mm. The results of this analysis, focused on the behaviour of the target interfaces, are presented in Figure 6.16 in terms of normal traction versus time, and in Figure 6.17 as crack width opening versus normal traction.

As noted in Section 6.3.1, only the points at the extrados of cracks C1 and C5 reached the tensile strength and subsequently followed the descending branch of the constitutive law, whereas all other monitored points remained within the elastic range. In the current analysis, as shown in Figure 6.16, the intrados point of crack C1 experienced a complete loss of tensile strength after 10 seconds, while the other monitored intrados points still retained tensile resistance, thus remaining either in the elastic or the softening phase. By contrast, all extrados points lay either in the elastic or the softening domain. A clearer understanding of the interface behaviour is provided in Figure 6.17. The full evolution of crack opening is visible for the intrados point of crack C1: once the tensile strength was reached, the response exhibited unloading–reloading branches consistent with the softening law, with progressive loss of stiffness and resistance. After complete dissipation of the fracture energy, the interface degraded to zero strength and stiffness. In contrast, crack C5 at the intrados also started dissipating fracture energy along the softening curve, although without reaching full loss of resistance, while the intrados points of cracks C3 and C4 only began to enter the softening branch. Similar conclusions hold for the extrados points of cracks C3, C4, and C5, whereas the extrados point of crack C1 remained elastic throughout the analysis.

As expected, since only one monitored point reached full degradation, no significant changes were observed in the displacement response. Indeed, no significant modifications were observed in the time-history response at the monitored points (keystone and marker M16) or in the deformed shapes (Figures 6.18 and 6.19). Figures 6.18(a) and 6.19(a) illustrate the deformed configurations at 6.7 s and 9.1 s, respectively, i.e., the same instants considered in Section 6.3.1, confirming the absence of visible cracks. In Figure 6.18(b), the left panels compare experimental and numerical results at the keystone, while the right panels show the numerical response alone. Only a minor effect on the RDTHs is observed: in the horizontal direction the time history appears less noisy, and the peak displacement increases slightly, from 0.1 mm to 0.25 mm. No significant variation was observed in the vertical direction. Similarly, the response of location M16 share common findings, with responses shown in Figure 6.19(b).

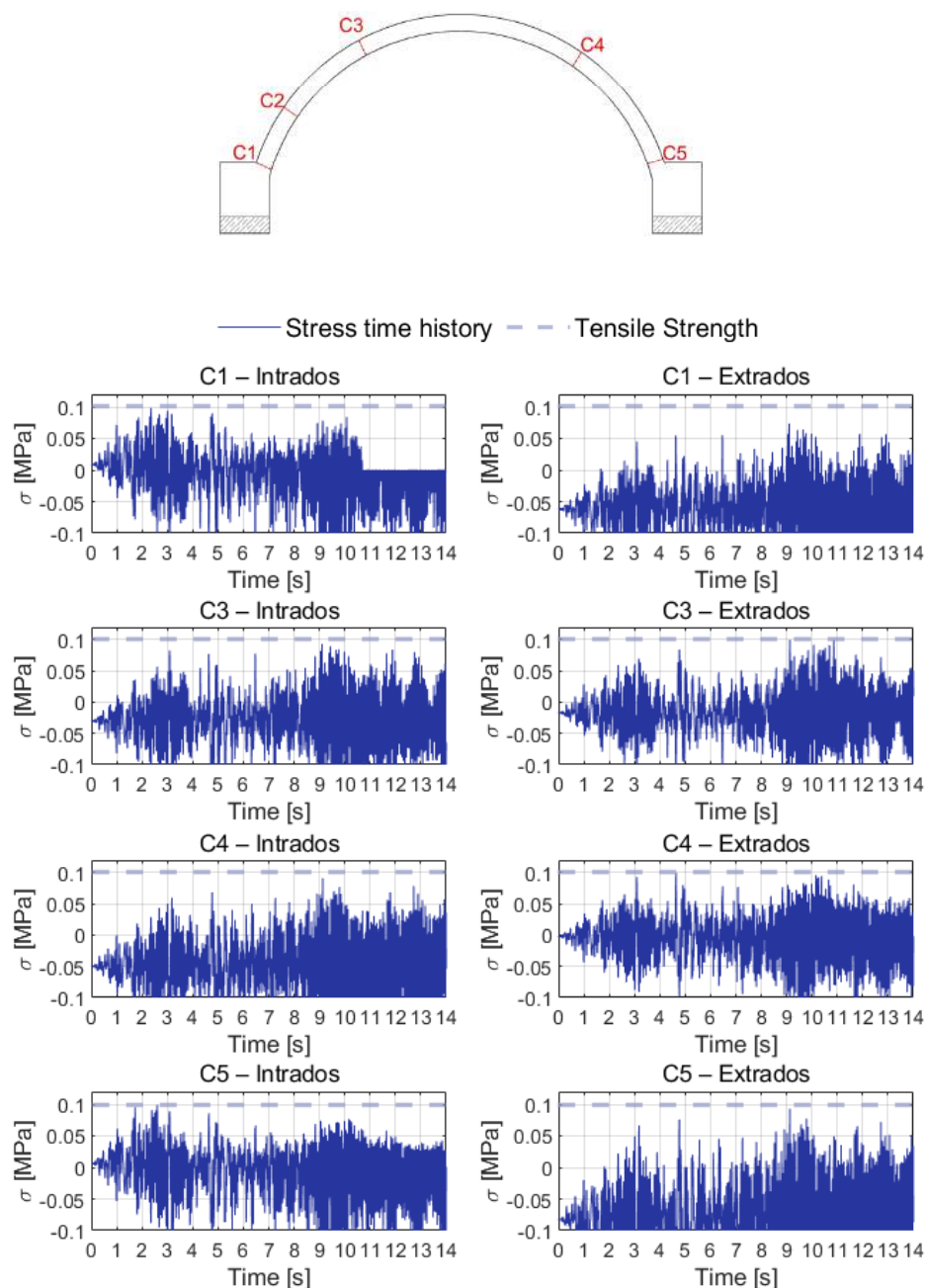


Fig. 6.16: Results assuming $f_t = 0.1$ MPa and $G_f^I = 1 \times 10^{-3}$ N/mm: numerical vs theoretical behaviour at the extreme points of the target interfaces

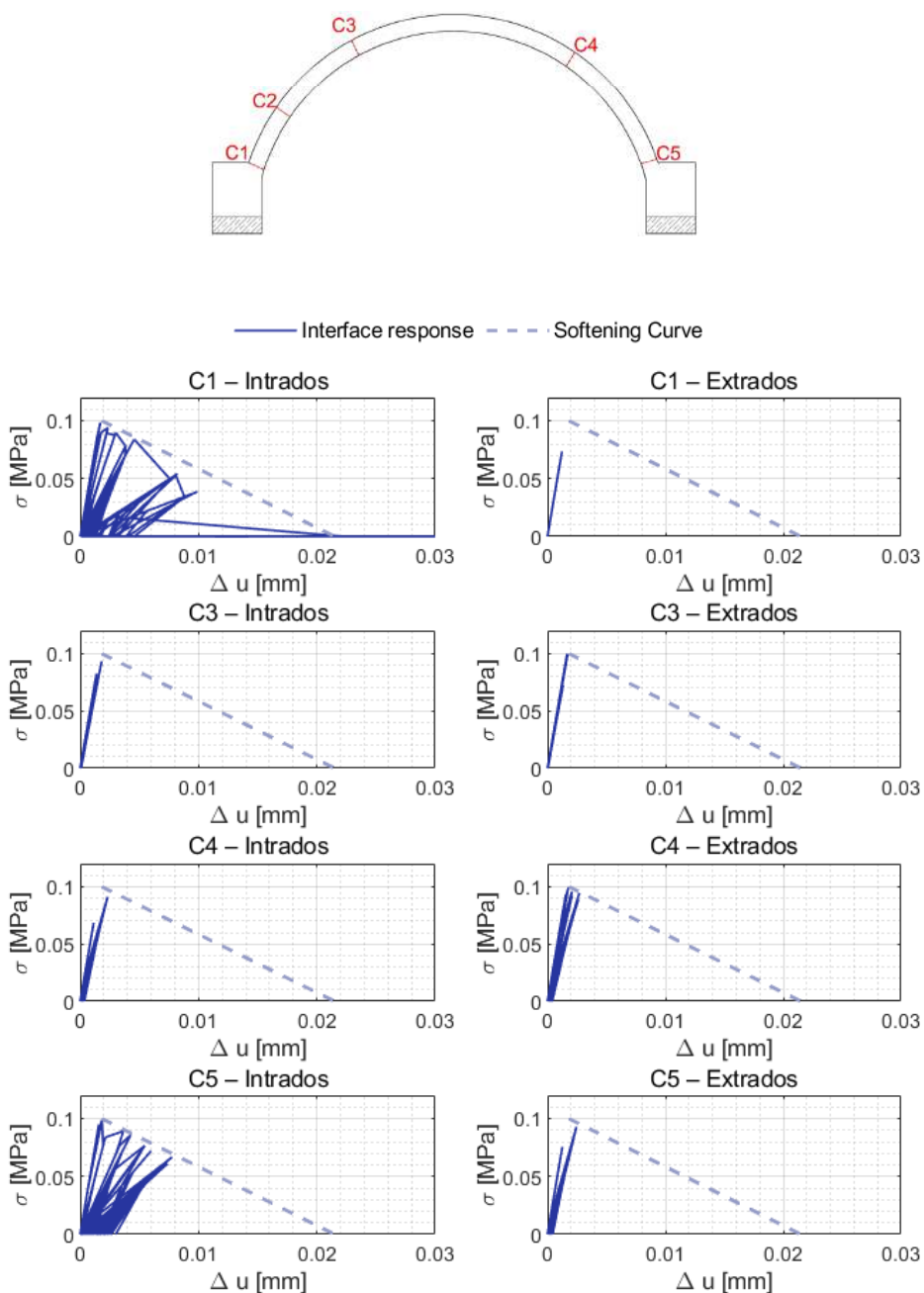


Fig. 6.17: Results assuming $f_t = 0.1$ MPa and $G_f^I = 1 \times 10^{-3}$ N/mm: numerical vs theoretical behaviour at the extreme points of the target interfaces

The left panels compare experimental and numerical RDTHs, while the right one show numerical results only. A peak displacement of approximately 0.3 mm was reached around 9 s, i.e., two orders of magnitude smaller than the experimental reference value, but still approximately 35% higher than in the previous analysis.

Figure 6.20, which provides an overview of the displacement field, shows no appreciable differences with respect to Figure 6.15. The order of magnitude of displacements remained about two times smaller than the experimental target

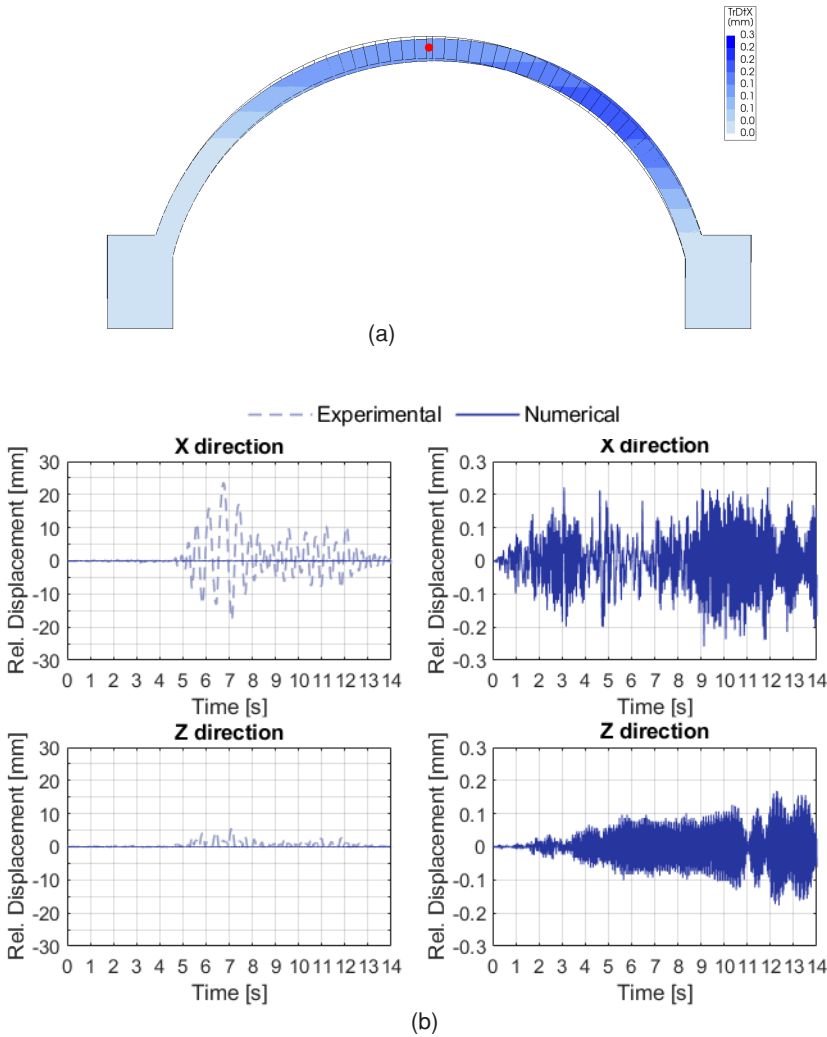


Fig. 6.18: Results assuming $f_t = 0.1$ MPa and $G_f^I = 1 \times 10^{-3}$ N/mm: (a) deformed shape at 6.7 s, and (b) comparison of RDTHs at the keystone (left panels) and corresponding numerical response (right panels)

values (panel c). Similarly, the angular variation indicator (d^θ) showed negligible variation from zero, indicating no relevant change in the overall configuration. This partial opening of only one end of the four cracks is not sufficient to significantly increase the displacement response, which highlights why the response is not so different from the literature-based values one. A further analysis was carried out with an even lower tensile fracture energy, set to 1×10^{-4} N/mm. However, this simulation encountered multiple convergence issues and ultimately failed due to divergence.

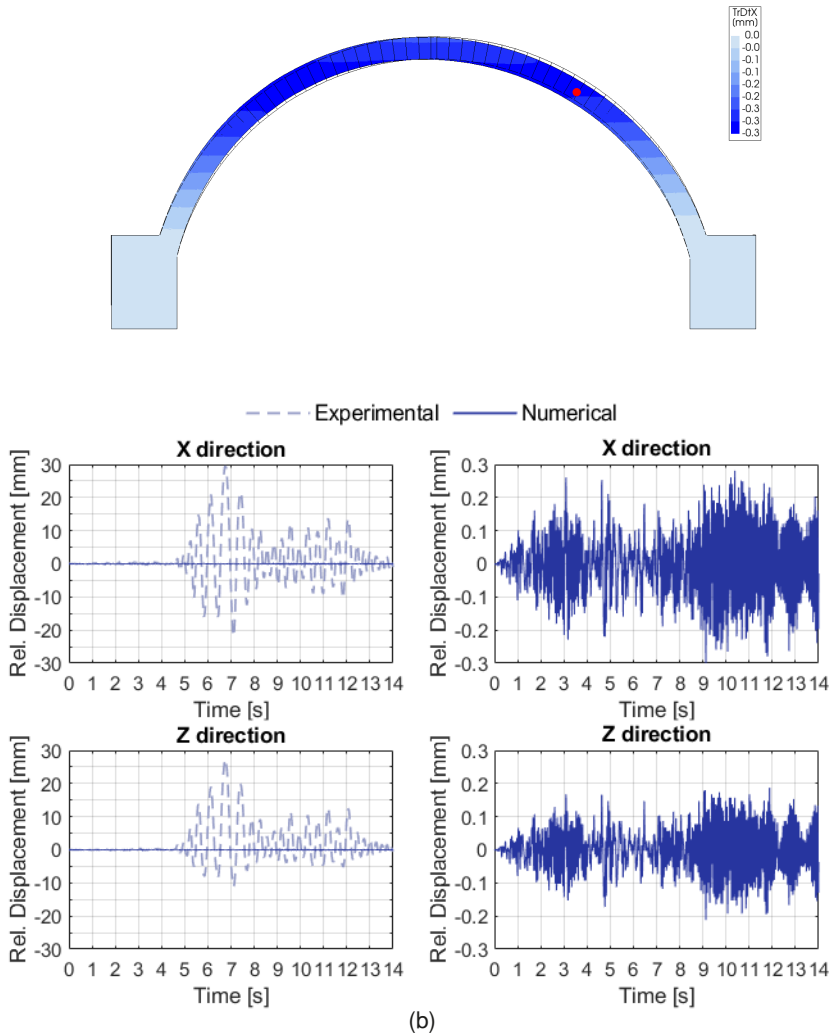


Fig. 6.19: Results assuming $f_t = 0.1$ MPa and $G_f^I = 1 \times 10^{-3}$ N/mm: (a) deformed shape at 7.1 s, and (b) comparison of RDTHs at location M16 (left panels) and corresponding numerical response (right panels)

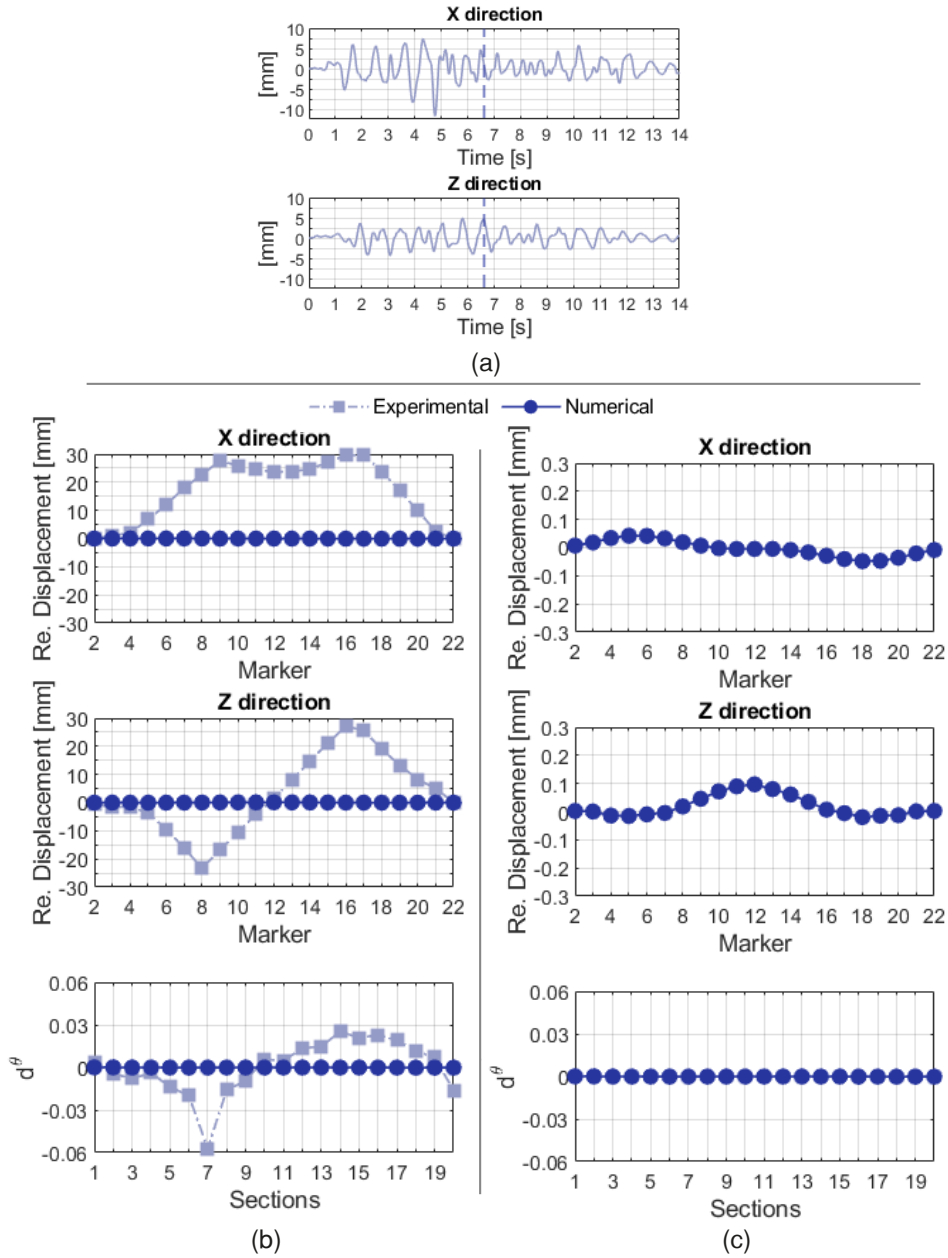


Fig. 6.20: Results assuming $f_t = 0.1$ MPa and $G_f^I = 1 \times 10^{-3}$ N/mm: at 6.7 s: (a) input signals, (b) comparison between numerical and experimental results, and (c) numerical results

6.3.3. Results assuming a more brittle behaviour at the target interfaces

One of the fundamental assumptions in any numerical model is that it represents an idealised version of the real specimen, neglecting local imperfections or defects that could significantly influence the actual response. To explore the potential effect of a more brittle behaviour, an additional analysis was carried out in which only the target interfaces were assumed to exhibit increased brittleness, i.e., a reduced Mode I fracture energy in tension. In this model, literature-based values were adopted for all interfaces except for the four target ones, where cracks were expected to develop. For these specific interfaces, the tensile strength was fixed at 0.1 MPa, while the fracture energy was reduced to 1×10^{-4} N/mm.

The evolution of normal traction in the target interfaces is presented in Figure 6.21. As shown, all monitored points at the extrados and intrados moved outside the softening branch, reaching zero tensile stress from 4.7 s onward. These findings are consistent with the crack width versus normal traction response shown in Figure 6.22, where all cracks developed displacements outside the softening branch, corresponding to zero normal traction. It should be clarified that small deviations from the theoretical softening curve are likely due to numerical approximations and the limited number of saved time steps during the analysis. A complete opening of the interfaces was made possible by the reduced tensile fracture energy, as highlighted by the deformed shapes in Figures 6.23 and 6.24, panels (a). In the first case, corresponding to 7.3 s of analysis, the opening of the cracks is evident, with hinges forming at crack C3 (left side) at the intrados, and an alternate hinge at crack C4 (right side), together with a partial opening of cracks C1 and C5 near the springing line. In Figure 6.24, the hinges at cracks C3 and C4 appear on opposite sides, providing the first useful indication of the successful initiation of the rocking mechanism. The RDTHs at the keystone is shown in Figure 6.23 for both monitored directions (X and Z). Although the numerical response does not fully match the experimental results, the strong influence of the loss of tensile resistance in the target interfaces is evident. First, the numerical response is of the same order of magnitude as the experimental one, although with lower peak values. In the horizontal direction, a maximum displacement of 14 mm is reached at 8.9 s, compared to the experimental peak of 23 mm at 6.7 s. In the vertical direction, the numerical maximum is -7 mm at 12.5 s, while the experimental maximum is 6 mm at 7.1 s. Moreover, the timing of the rocking mechanism differs between the numerical and experimental results. During the experimental test, rocking began immediately after the vertical PGA threshold was exceeded, at 4.7 seconds, whereas in the numerical analysis, it occurred at 7 seconds under the same input conditions.

The RDTHs for location M16 are shown in Figure 6.24, underlining similar conclusions. Experimentally, the maximum displacements were 29 mm and 27 mm in the X and Z directions, respectively, occurring at 6.7 seconds. Numerically,

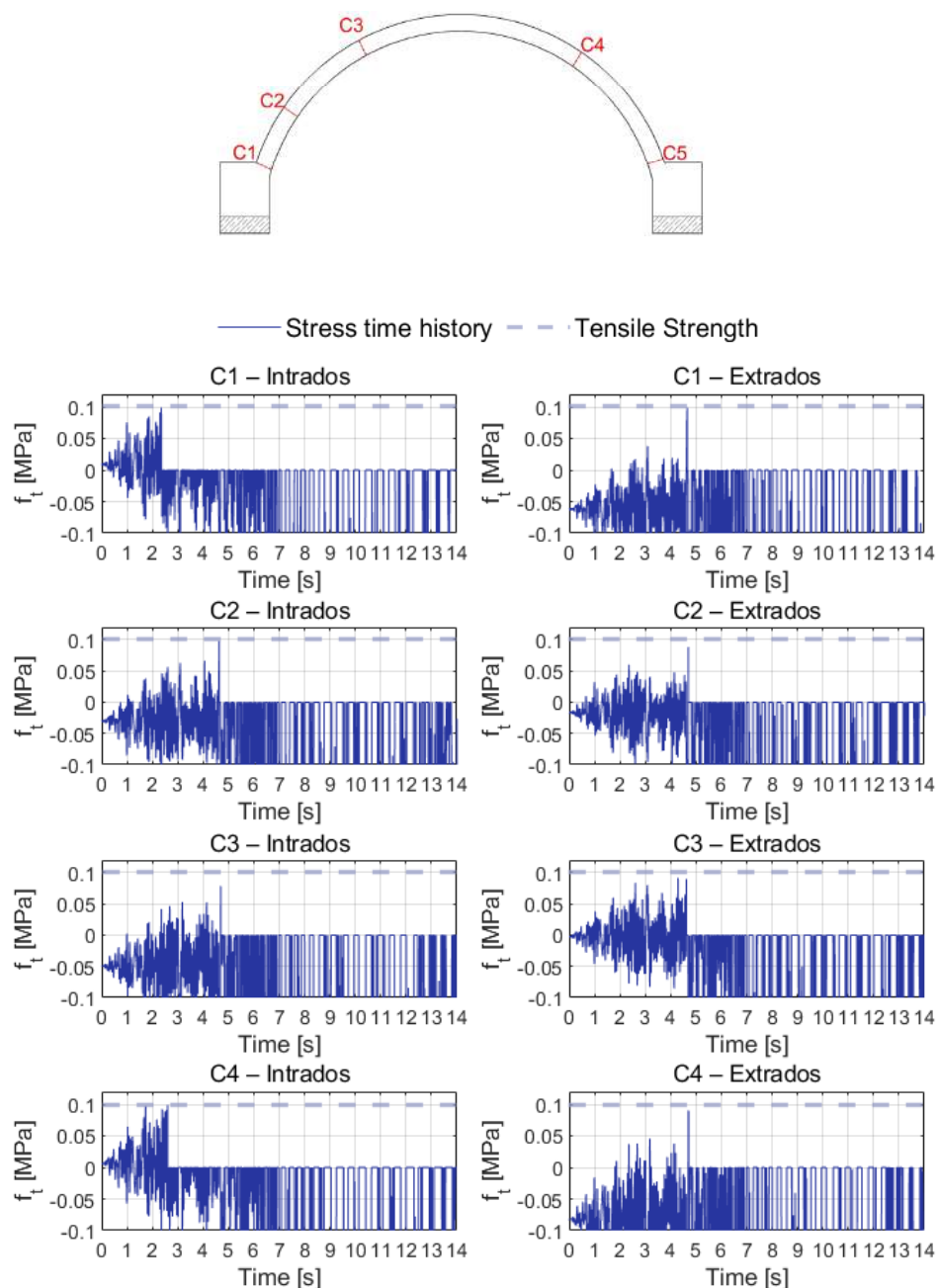


Fig. 6.21: Results assuming a more brittle behaviour of the target interfaces: traction time histories at the extreme points of the target interfaces

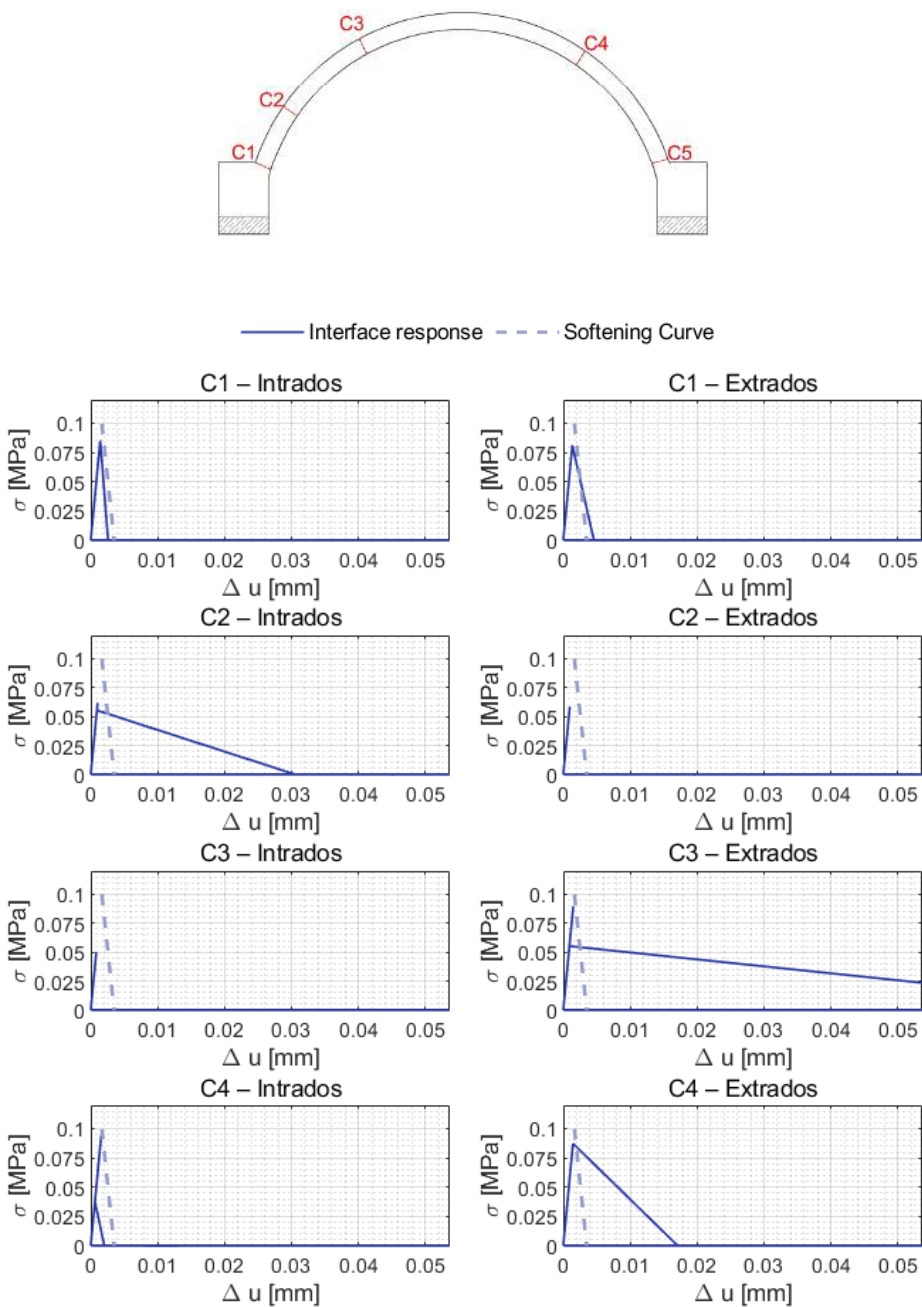


Fig. 6.22: Results assuming a more brittle behaviour of the target interfaces: numerical vs theoretical behaviour at the extreme points of the target interfaces

the corresponding maximum relative displacements were 18 mm in the horizontal direction and 16 mm in the vertical direction, occurring at 8.9 seconds.

Finally, a global comparison of numerical and experimental results at 6.7 s is presented in Figure 6.25, showing relative displacements. No significant changes are observed in the numerical displacement at this time, particularly when comparing X and Z directions and the angular variation δ^θ in panels (b). This is consistent with the rocking mechanism not yet being activated, as it only begins at 7 seconds. The overall response magnitude remains low; however, the

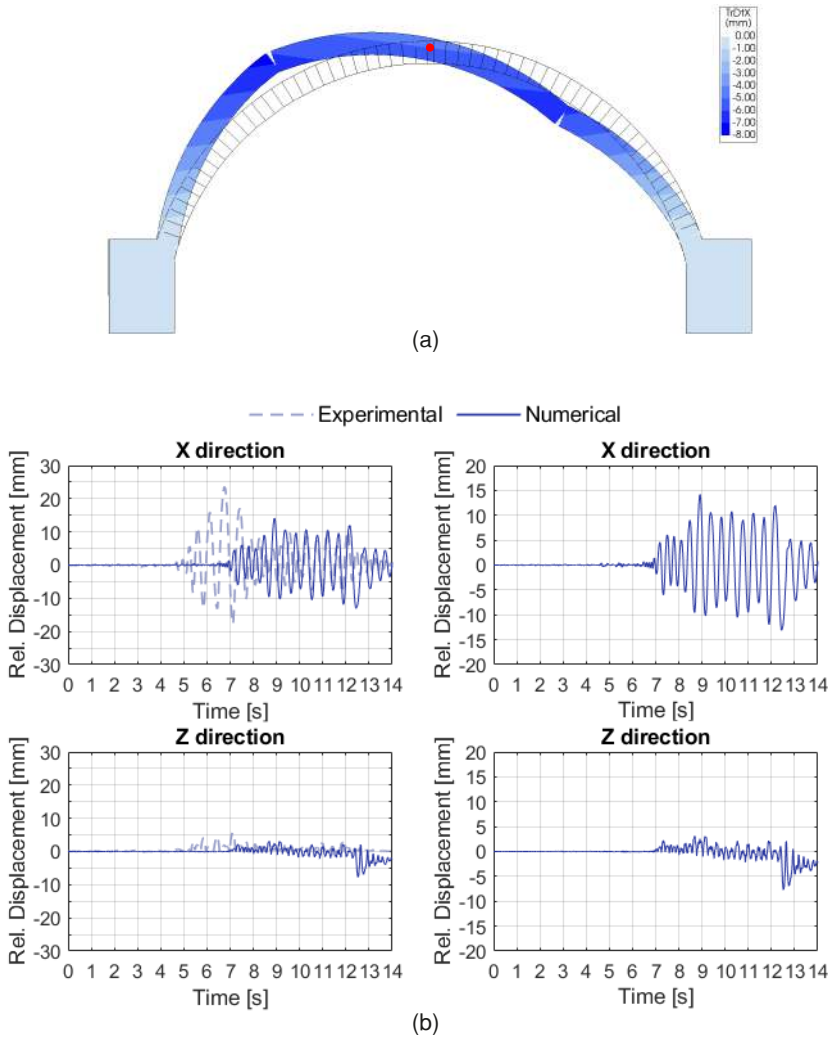


Fig. 6.23: Results assuming a more brittle behaviour of the target interfaces: (a) deformed shape at 7.3 s, and (b) comparison of RDTHs at the keystone (left panels) and corresponding numerical response (right panels)

deformed shape, although scaled down, qualitatively reflects the experimental deformation pattern, as shown in panels (c), where only numerical results are plotted.

Although the analysis does not fully reproduce the experimental data, the reduction in fracture energy had a remarkable effect on the response. In contrast to the previous analyses discussed in Sections 6.3.1 and 6.3.2, this model achieves displacements of the same order of magnitude as the experimental results, despite the tensile strength remaining unchanged. These results em-

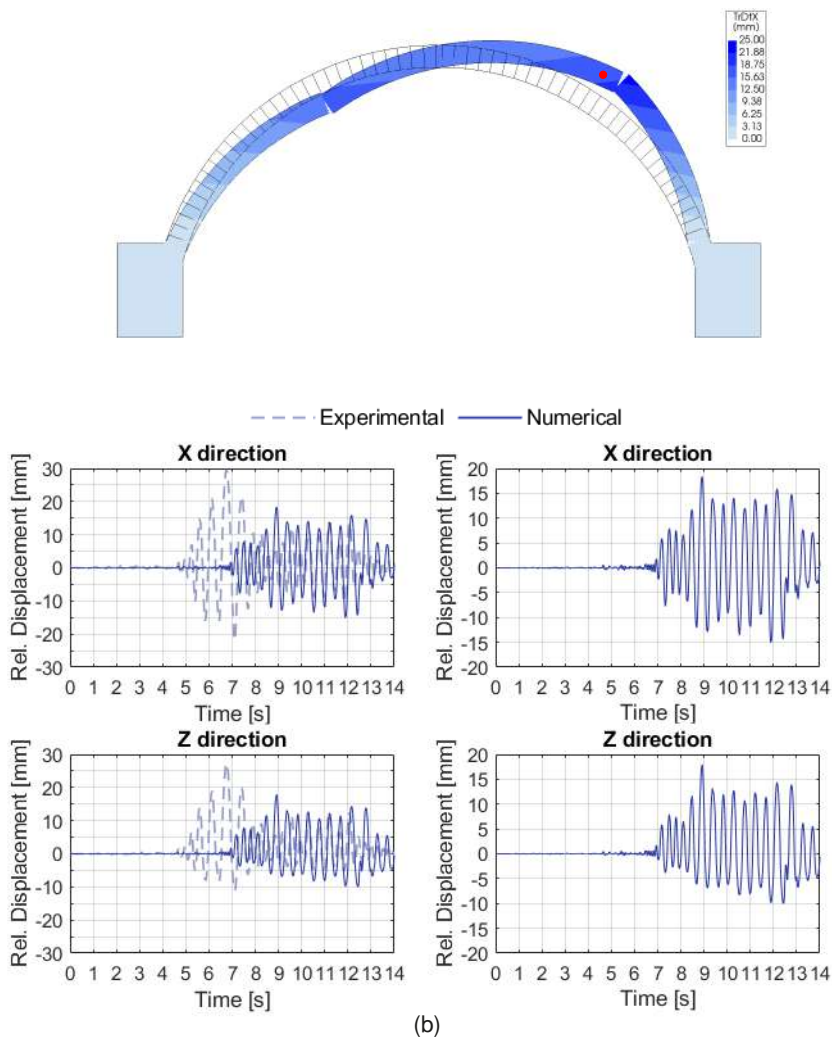


Fig. 6.24: Results assuming a more brittle behaviour of the target interfaces: (a) Deformed shape at 8.9 s; (b) comparison of RDTHs at location M16 (left panels) and corresponding numerical response (right panels)

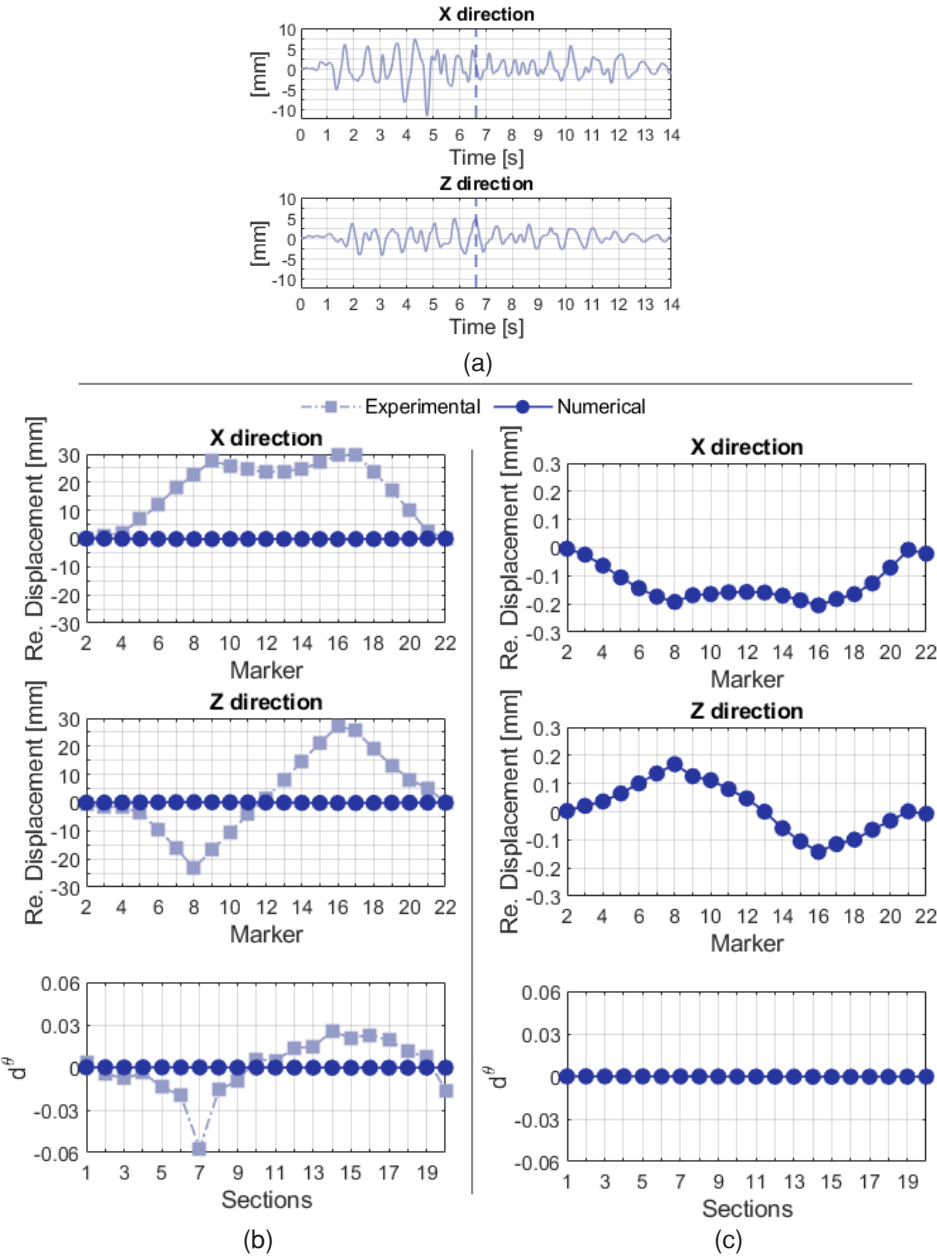


Fig. 6.25: Results assuming a more brittle behaviour of the target interfaces at 6.7 s: (a) input signals, (b) comparison between numerical and experimental results, and (c) numerical results

phasise the significant influence of post-peak behaviour under dynamic, non-monotonic loading. While the numerical displacements do not fully replicate the experimental trends, particularly in terms of the timing of rocking initiation, all monitored cracks exhibited a complete loss of tensile strength within the expected time-frame, i.e., prior to the onset of the rocking mechanism. These findings support the possibility of an initial pre-damaged condition, potentially induced by undetected micro-cracking during previous seismic events, which will following be addressed.

6.3.4. Results assuming a pre-damaged condition

A combination of literature-based tensile strength ($f_t = 0.1$ MPa) and a more brittle condition (lowering the fracture energy) at the target interfaces successfully highlighted the activation and progression of the rocking mechanism, although with reduced magnitude and delayed onset.

Building on this background, additional analyses were performed by reducing the tensile strength parameter, while keeping the fracture energy at 0.01 N/mm. However, all such cases resulted in divergent simulations. Specifically, analyses assuming a reduced tensile strength of 0.01 MPa or a no-tension material model failed to converge under the adopted time-step and convergence settings.

An assumption of a pre-damaged condition was also evaluated, considering that minor cracks may have developed during the first seismic test. These cracks could have remained undetected during the AVT due to their closure under the self-weight of the structure. The analysis was conducted using literature-based values for all interfaces, except for those where cracking was assumed. In these regions, a material with no tensile resistance was adopted, setting the tensile strength to 5 kPa, the lowest value that still ensured numerical convergence. The fracture energy was kept constant for all interfaces at 0.01 N/mm, while all other parameters remained as described in Section 6.2. Since this analysis assumed a pre-damaged model with null tensile strength at the target interfaces and a fracture energy of 0.01 N/mm, it implied an ultimate crack width openings $\Delta_{u,ult} = 4$ mm. This represents a relatively large value before complete loss of resistance, which does not physically reflect the weak and brittle interfaces described so far. Nevertheless, the purpose of this analysis was to investigate the effects of reducing only the tensile strength while retaining the literature-based fracture energy.

The results in terms of normal traction evolution over time are shown in Figure 6.26. A reduced scale was adopted for better visualization, and the tensile threshold was reached by all monitored points at both the intrados and extrados of the target interfaces. Although the tensile strength was significantly reduced (to 5 kPa, solely for numerical stability), it is worth noting that none of the monitored points exhibited a complete loss of tensile resistance and stiffness. As expected, this outcome reflects the combined effect of the two N-L parameters: tensile strength and fracture energy. The response in terms of normal

traction–crack width relationship is presented in Figure 6.27. Although all monitored interfaces exceeded the initial tensile strength, the definition of the set fracture energy combined with a linear softening law resulted in a residual tensile capacity, placing the interface in the softening branch and preventing full crack opening. These results highlight the unrealistic quasi-ductile behaviour induced by the softening model, which does not represent the actual brittle nature of brick–mortar interfaces. Moreover, they emphasize the critical role of post-peak behaviour in dynamic analyses, particularly under non-monotonic loading conditions. Since the applied loads are not strictly increasing, interfaces may alternate between tension and compression at different time steps. Consequently, a monitored point along a crack may experience successive cycles of tension and compression, where the input energy is not fully dissipated in the tensile state. In this context, the post-peak response, specifically the tensile fracture energy, plays a decisive role in determining whether full crack opening occurs.

The deformed shapes and RDTHs results are presented for the keystone and location M16 in the following Figures 6.28 and 6.29. In both cases, panels (a) illustrate the deformed configurations at two distinct time steps: 6.7 s and 7.1 s. A clear alternating opening of hinges is visible in the upper part of the structure. Although such results could be misleading if only the crack openings were considered, evaluation of the interface response clarifies that the interfaces behave like springs, allowing significant crack openings, without reaching failure. Panel (b) of Figure 6.28 compares the experimental and numerical RDTHs at the keystone. In both directions, the numerical response is consistently lower than the experimental one. However, when considering only the numerical trend, the shape of the response resembles the experimental behaviour in scaled form, giving useful insights in proving a pre-damaged state in the structure. At the keystone, the maximum numerical displacements were 3.1 mm in the X direction and 1.2 mm in the vertical direction, compared to 24 mm and 5 mm in the experimental results, respectively. Similarly, panel (b) of Figure 6.29 compares the experimental and numerical RDTHs at the point of maximum response (location M16). The same considerations made for the keystone apply here. The numerical time history captures the experimental trend with a magnitude approximately one order lower. The expected peak displacements were 29 mm (X direction) and 27 mm (Z direction), while the numerical model yielded 4.3 mm and 4.2 mm, respectively.

Lastly, Figure 6.30 presents a global comparison of the relative displacements of all monitored points at 6.7 seconds. This simulation exhibits higher displacement values and begins to highlight a potential damage pattern. Although the displacement magnitudes are lower than experimental values, the trend is consistent with the expected behaviour in both monitored directions. However, less reliable results are observed when examining d^θ , where angular variation is less pronounced than in the experimental outcomes. These observations are

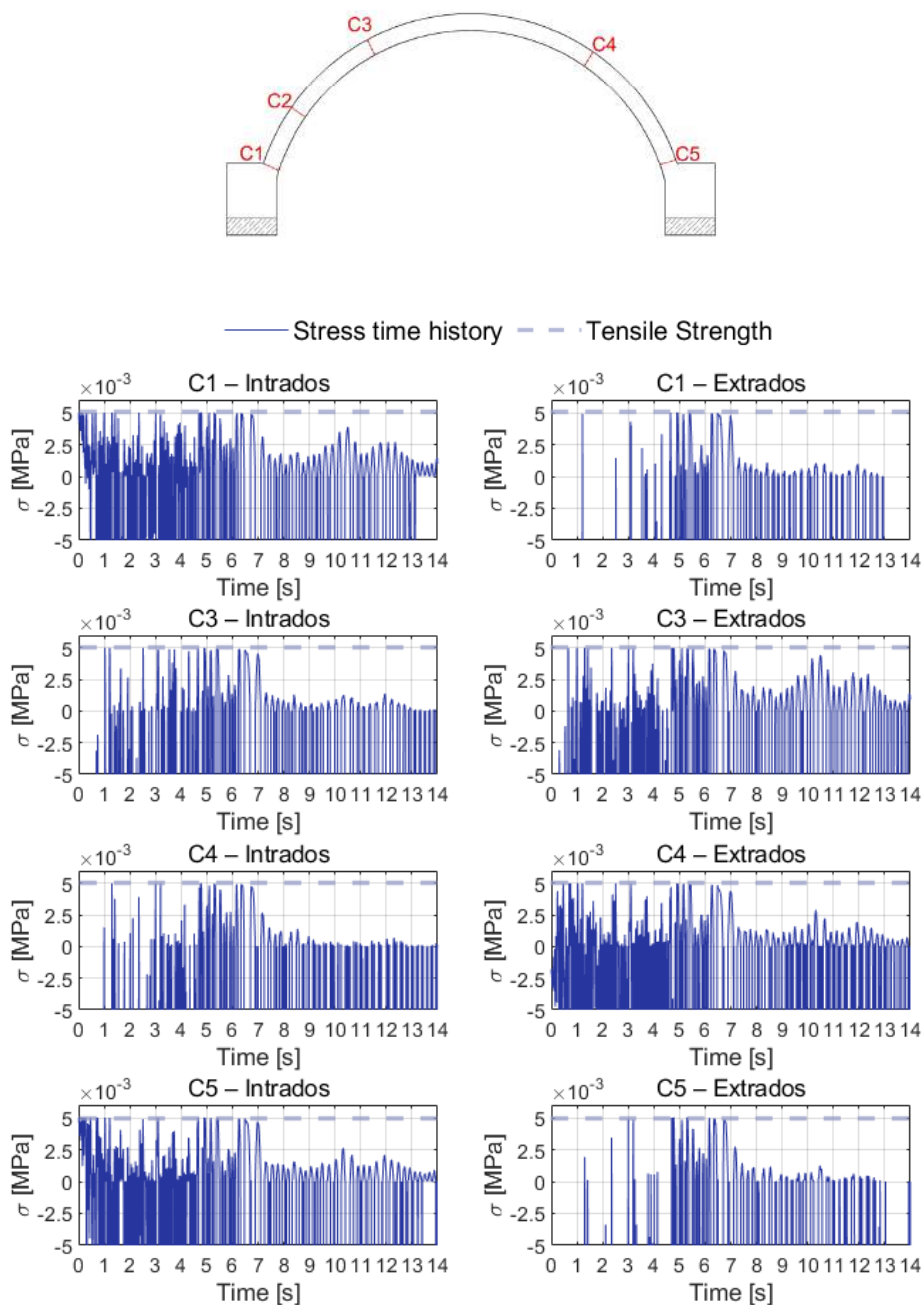


Fig. 6.26: Results assuming a pre-existing damaged condition ($f_t = 5$ kPa and $G_f^I = 0.01$ N/mm): traction time histories at the extreme points of the target interfaces

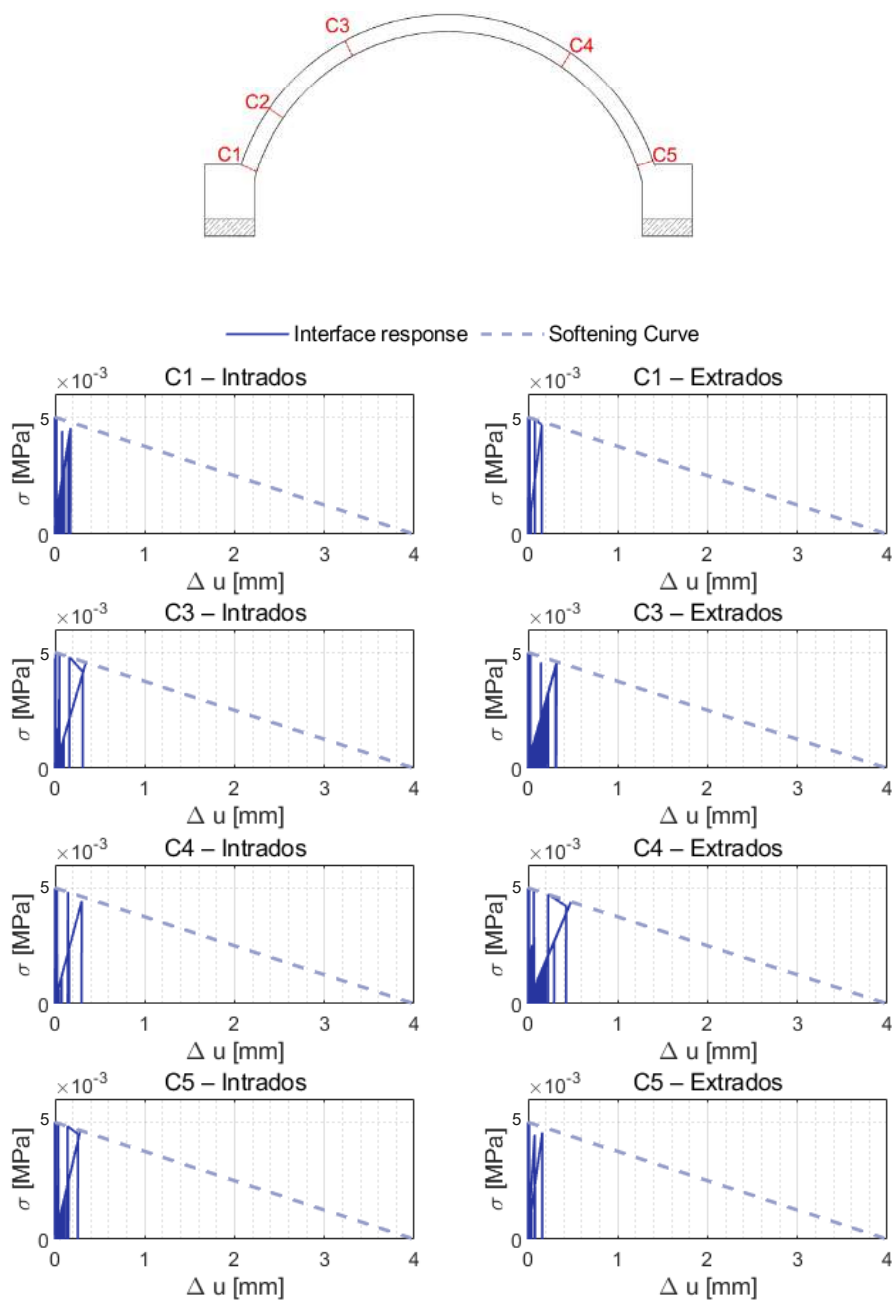


Fig. 6.27: Results assuming a pre-existing damaged condition ($ft = 5$ kPa and $G_f^I = 0.01$ N/mm): numerical vs theoretical behaviour at the extreme points of the target interfaces

consistent with the time-history displacements and with the findings regarding interfaces' behaviour.

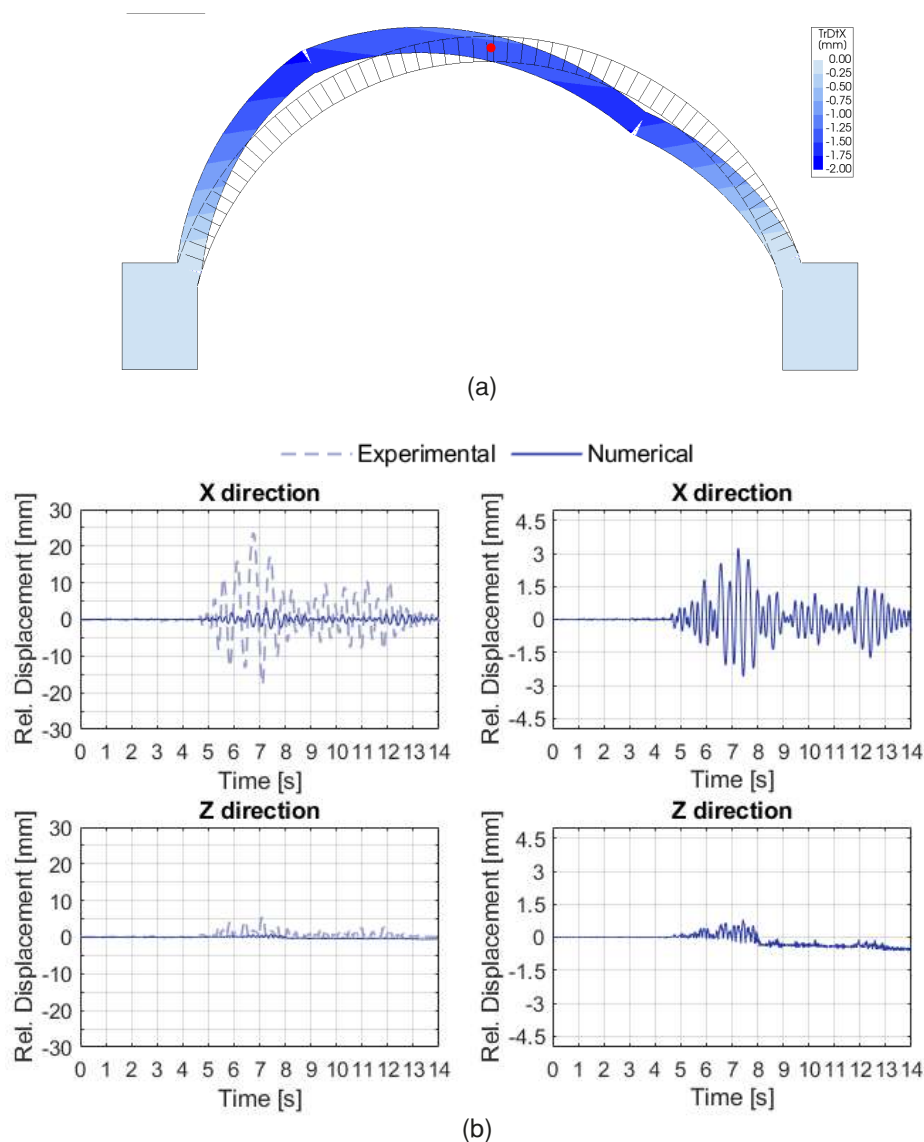


Fig. 6.28: Results assuming a pre-existing damaged condition ($f_t = 5$ kPa and $G_f^I = 0.01$ N/mm): (a) deformed shape at 6.7 s, and (b) comparison of RDTHs at the keystone (left panels) and corresponding numerical response (right panels)

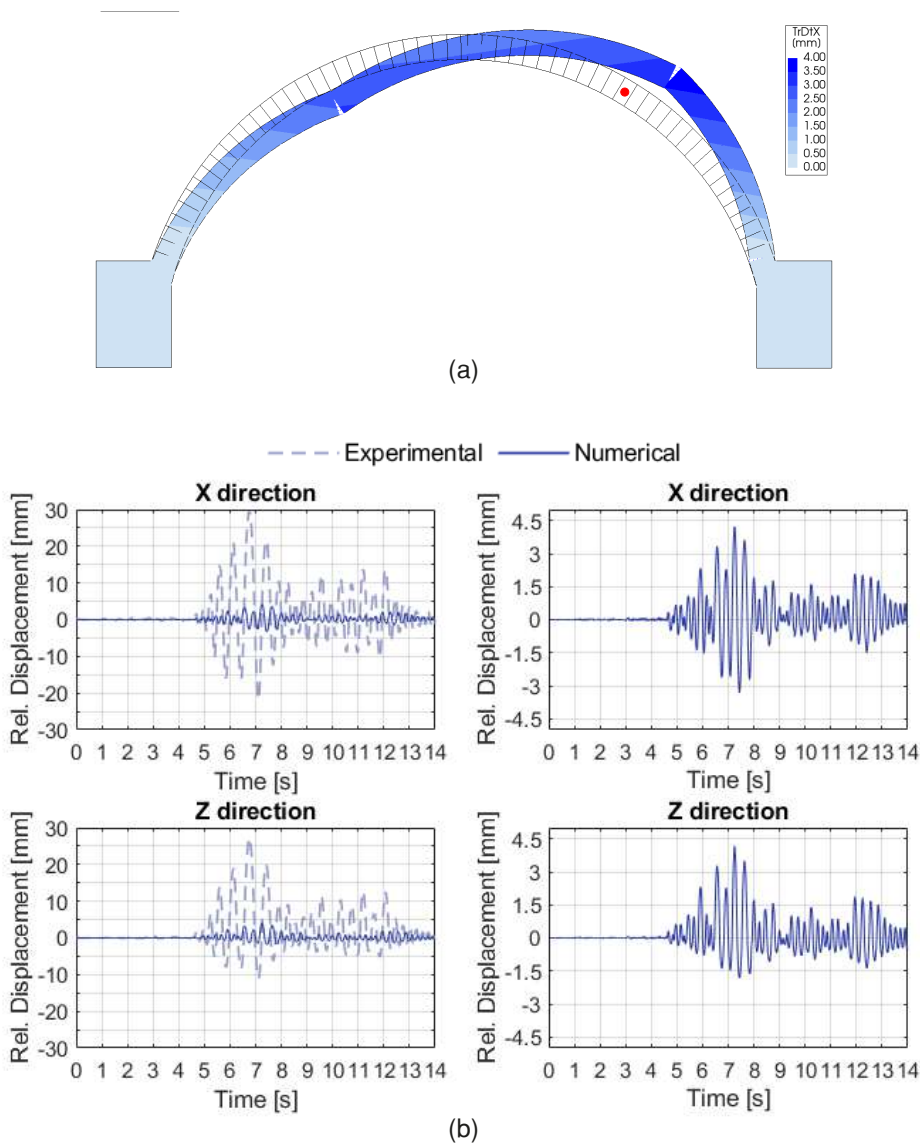


Fig. 6.29: Results assuming a pre-existing damaged condition ($f_t = 5$ kPa and $G_f^I = 0.01$ N/mm): (a) deformed shape at 7.1 s, and (b) comparison of RDTHs at location M16 (left panels) and corresponding numerical response (right panels)

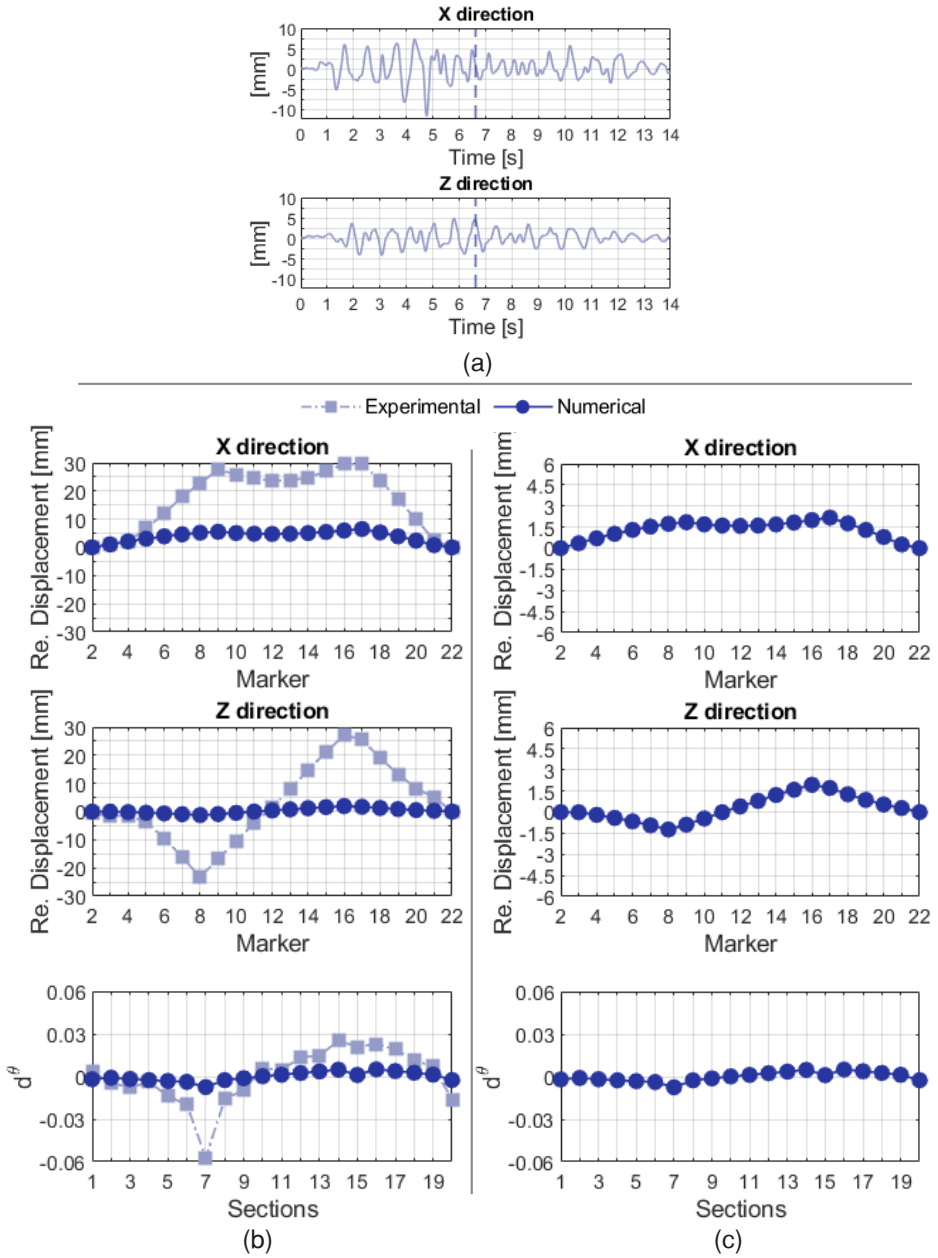


Fig. 6.30: Results assuming a pre-existing damaged condition ($f_t = 5$ kPa and $G_f^I = 0.01$ N/mm) at 6.7 s: (a) input signals, and (b) comparison between numerical and experimental results, and (c) numerical results

6.3.5. Results considering pre-damaged condition and brittle behaviour of the target interfaces (fracture energy set to 1×10^{-5} N/mm)

The previous analysis highlighted a consistent rocking response pattern at the investigated location under seismic loading, but it failed to reproduce the complete opening of the hinges, thus preventing the full activation of the mechanism. In contrast, the analysis in Section 6.3.3 emphasized the significant role of fracture energy in governing the activation of the rocking mechanism. Although all target interfaces exhibited a complete loss of tensile strength before 5 seconds of simulation time, the onset of rocking was nonetheless delayed until approximately 9 seconds. Based on these findings, an additional analysis was performed. This model retained the assumption of a highly brittle behaviour at the target interfaces (with Mode I fracture energy set to 1×10^{-5} N/mm), while also enforcing an almost-null tensile strength at the same locations. In particular, $G_f^I = 1 \times 10^{-5}$ N/mm was adopted to obtain an ultimate crack width opening comparable to the analysis in Section 6.3.3, and a tensile strength of $f_t = 5$ kPa was prescribed for numerical stability.

The behaviour of the target interfaces in terms of stress evolution over time is shown in Figure 6.31 in a scaled visualization. As it can be observed, the influence of reducing the fracture energy is immediately detectable. First, the intrados of cracks C1 and C5 experienced slight tensile stresses under self-weight only, leading to a complete loss of resistance within the first second of analysis. A similar observation applies to the extrados of cracks C3 and C4, where tensile resistance was fully lost within the same time frame. For the remaining monitored points, both intrados and extrados, the tensile strength was exceeded and reduced to zero during the first two seconds of analysis. From that point onward, the interfaces were characterized by zero tensile strength and stiffness. The normal traction versus crack- Δ_u relationship is illustrated in Figure 6.32. All monitored points, both intrados and extrados, exhibit a response extending beyond the linear elastic regime, indicating progression along the softening branch. In several cases, a gradual reduction in tensile stress is evident, consistent with the softening law. In others, the stress response drops rapidly, suggesting that the interface transitioned almost immediately to the zero strength regime within the saved time steps. It should be noted, however, that not all simulation steps were recorded. Consequently, the depicted responses may not fully capture the continuous evolution of interface behaviour, particularly in the post-peak phase. This limitation may partially explain the apparent abrupt transitions and should be considered when interpreting the results.

The time steps corresponding to 7.0 s and 7.2 s were selected to display the deformed shape, which clearly highlighted crack openings with hinges alternating between intrados and extrados along the same crack. The deformed shapes also revealed that the regions most affected by the rocking mechanism were

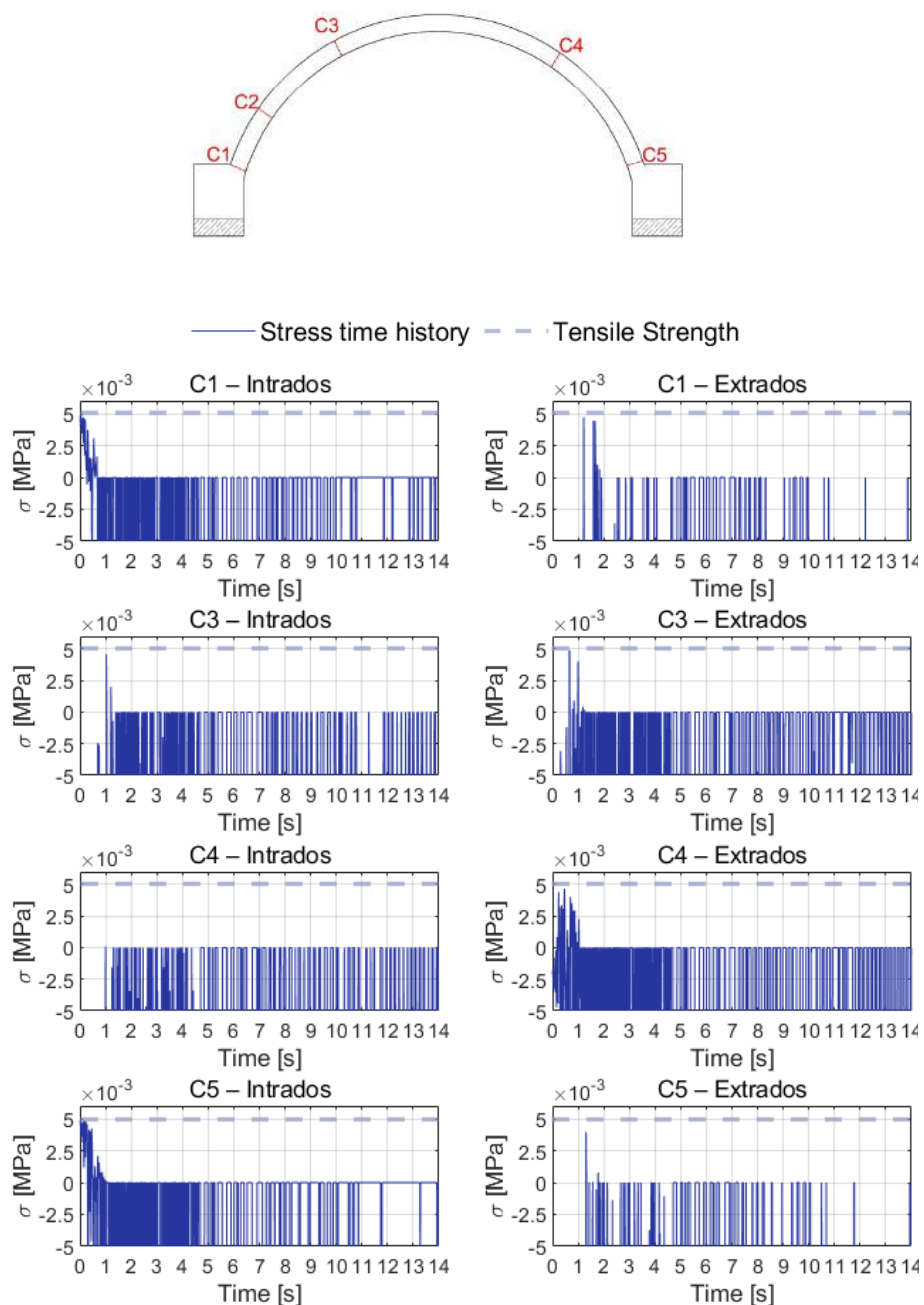


Fig. 6.31: Results considering pre-damaged condition and fracture energy set to 1×10^{-5} N/mm: traction time histories at the extreme points of the target interfaces

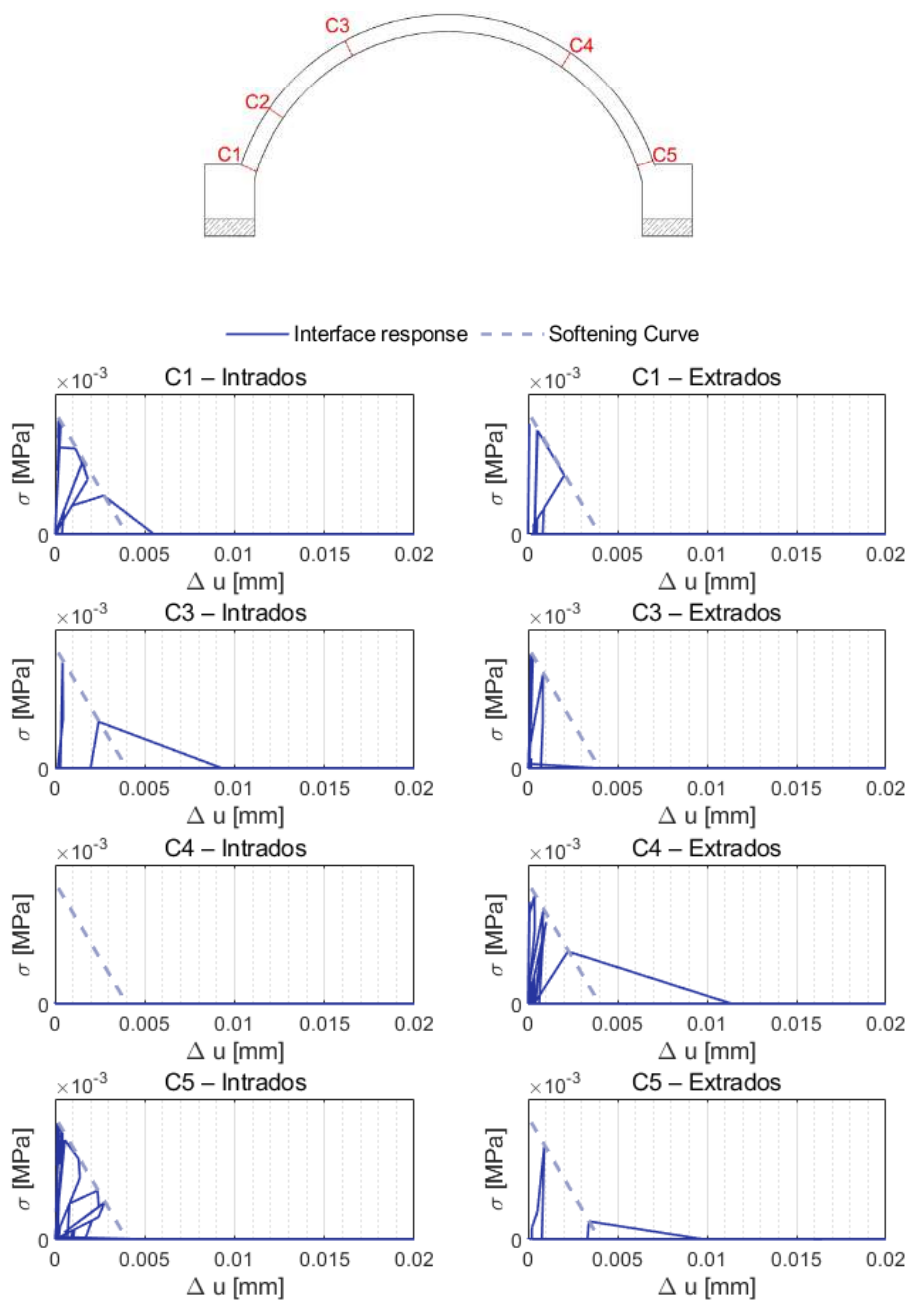


Fig. 6.32: Results considering pre-damaged condition and fracture energy set to $1 \times 10^{-5} \text{ N/mm}$: numerical vs theoretical behaviour at the extreme points of the target interfaces

those located between the two upper cracks (Figures 6.33(a) and 6.34(a)). In addition, the RDTHs for both the keystone point and location M16 are presented in panels (b) of Figures 6.33 and 6.34. Both points exhibit the onset of rocking at the expected time, showing good agreement with the experimental response up to 5 seconds. Beyond this point, the rocking mechanism is qualitatively reproduced, although the displacement amplitudes remain underestimated. In particular, Figure 6.33 shows a relative error of approximately 63% between the maximum numerical and experimental values in the X direction, with the maximum numerical displacement recorded at 7.2 s and equal to 9 mm. For the Z direction, a smaller relative error of 55% was observed.

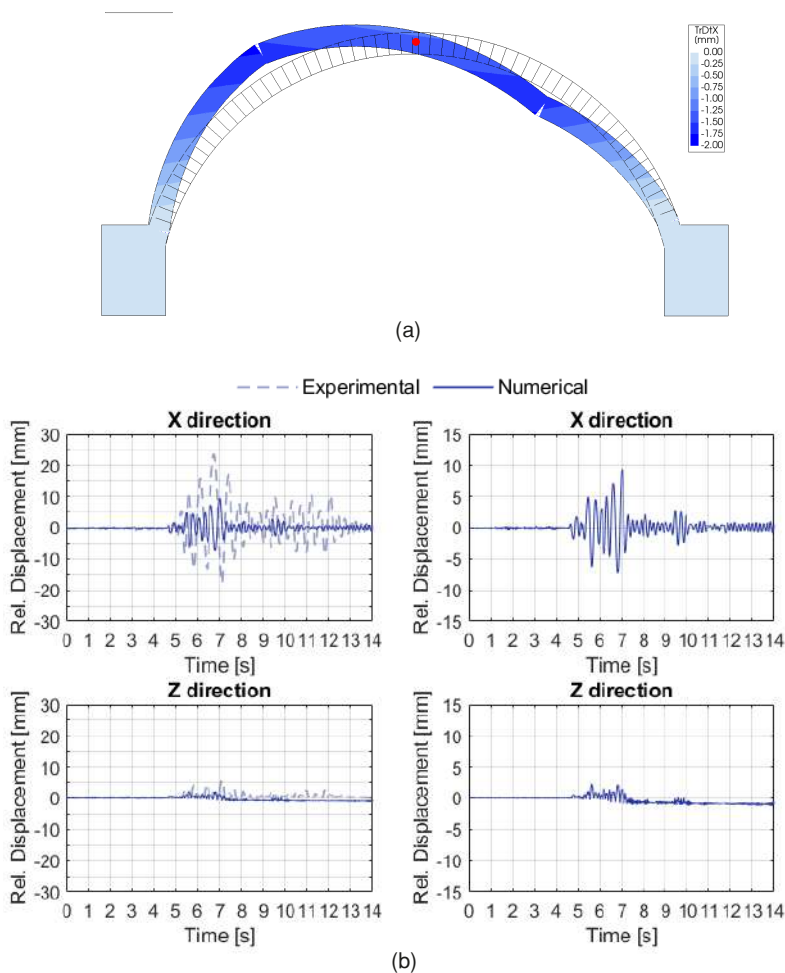


Fig. 6.33: Results considering pre-damaged condition and fracture energy set to 1×10^{-5} N/mm: (a) deformed shape at 7.0 s, and (b) comparison of RDTHs at the keystone (left panels) and corresponding numerical response (right panels)

A similar discrepancy is observed at location M16 (Figure 6.34(b)), although higher relative errors were recorded. The maximum numerical displacements were reached at 7.2 s, equal to 12 mm in both X and Z directions, resulting in errors of approximately 59%. Despite this quantitative mismatch, the rocking trend highlighted in the experimental displacement response is nonetheless captured in the numerical results.

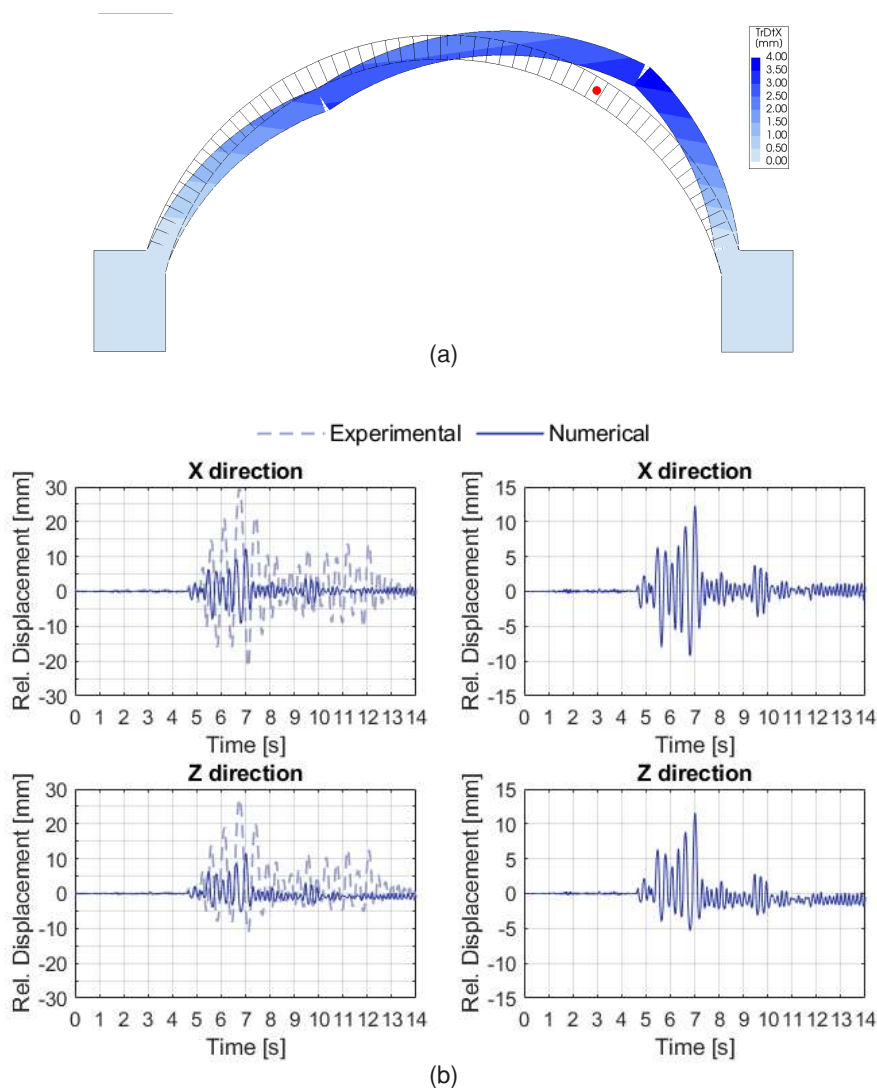


Fig. 6.34: Results considering pre-damaged condition and fracture energy set to 1×10^{-5} N/mm: (a) deformed shape at 7.2 s, and (b) comparison of RDTHs at location M16 (left panels) and corresponding numerical response (right panels)

Finally, the global distribution of relative displacements for all monitored points in both directions at 6.7 seconds is shown in Figure 6.35. While the numerical results do not perfectly replicate the experimental data, they reach a comparable order of magnitude. Specifically, the computed displacements are approximately half the experimental values in both horizontal (X) and vertical (Z) directions. Furthermore, the numerical angular variation indicators begin to deviate from zero, exhibiting small but significant values that qualitatively align with the experimental trends and indicate full angular variation. A similar observation applies to the overall displacement pattern, which reproduced the experimental deformation mode, although still quantitatively underestimated.

This analysis confirmed the assumption and the necessity of adopting a pre-damaged configuration at the target interfaces, which reflects not only a reduced tensile strength but also a reduced fracture energy, in order to capture the rocking trend and activation time.

6.3.6. Results considering pre-damaged condition and brittle behaviour of the target interfaces (fracture energy set to 1×10^{-6} N/mm)

A further analysis was carried out by adopting the same configuration as the previous model, with the assumption of a pre-damaged state characterised by a more brittle response at the target interfaces. The material parameters were kept unchanged, except for the fracture energy, which was reduced to 1×10^{-6} N/mm.

The results highlight the significant influence of fracture energy in rapidly degrading the resistance of the material once the tensile threshold is reached. In particular, Figure 6.36 shows that all monitored extreme points of the target interfaces lose their tensile resistance within the first 1.5 seconds of the analysis, clearly evidencing the loss of both resistance and tensile stiffness for the rest of the simulation. Similarly to the previous analysis, the influence of the saved time steps is more evident in this case. In particular, when considering the normal traction (σ) versus crack width opening (Δ_u) at the interfaces, some points lose resistance almost immediately, within just two to three saved steps, whereas others follow the softening laws more closely. In all cases, however, the attainment of significantly larger displacements is evident, with ultimate crack opening values reaching approximately 4×10^{-4} mm.

Subsequently, the RDTHs at the reference locations were assessed and compared with the experimental recordings. Figure 6.38(b) shows the results at the keystone section, demonstrating good agreement between the numerical and experimental responses. The model accurately reproduces both the onset and development of the rocking mechanism, capturing the correct activation timing and closely matching the relative displacement values. In particular, the simulation yields a maximum relative displacement of 27 mm in the X direction at 7.2 s, compared to 24 mm at 6.7 s in the experiment, corresponding to a relative error of approximately 12.5%. The numerical response follows the ex-

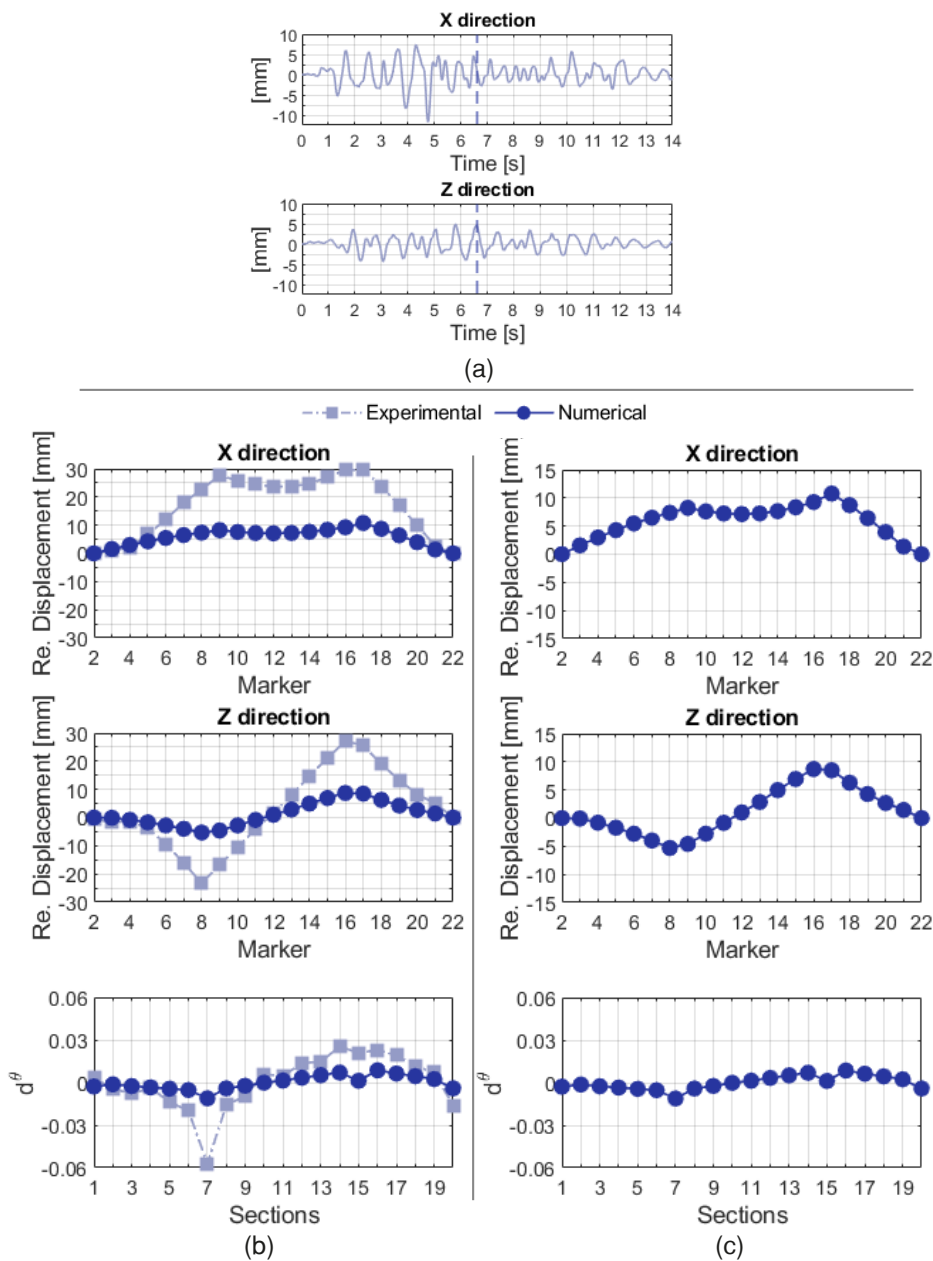


Fig. 6.35: Results considering pre-damaged condition and fracture energy set to 1×10^{-5} N/mm at 6.7 s: (a) input signals, and (b) comparison between numerical and experimental results, (c) numerical results

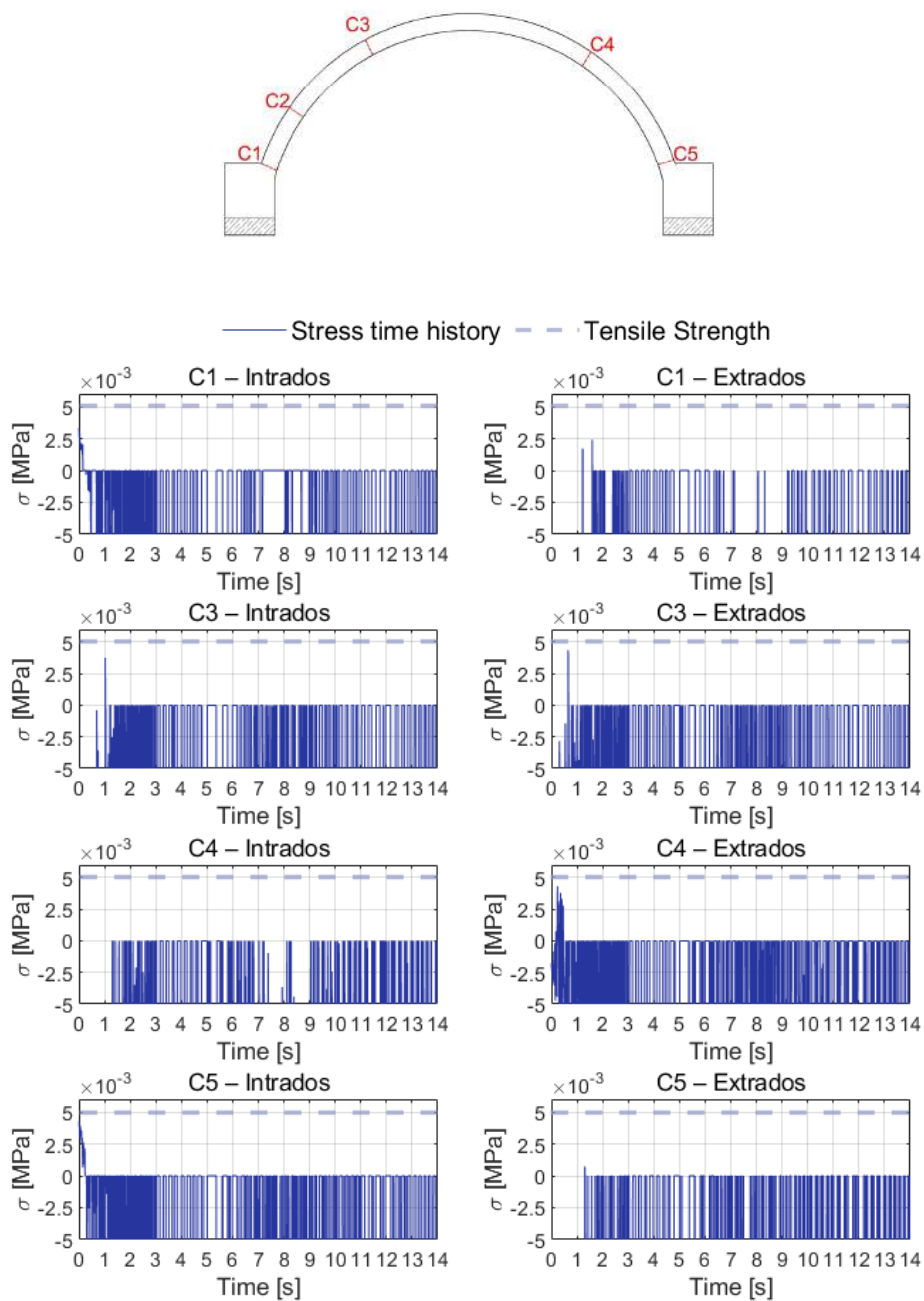


Fig. 6.36: Results considering pre-damaged condition and fracture energy set to 1×10^{-6} N/mm: traction time histories at the extreme points of the target interfaces

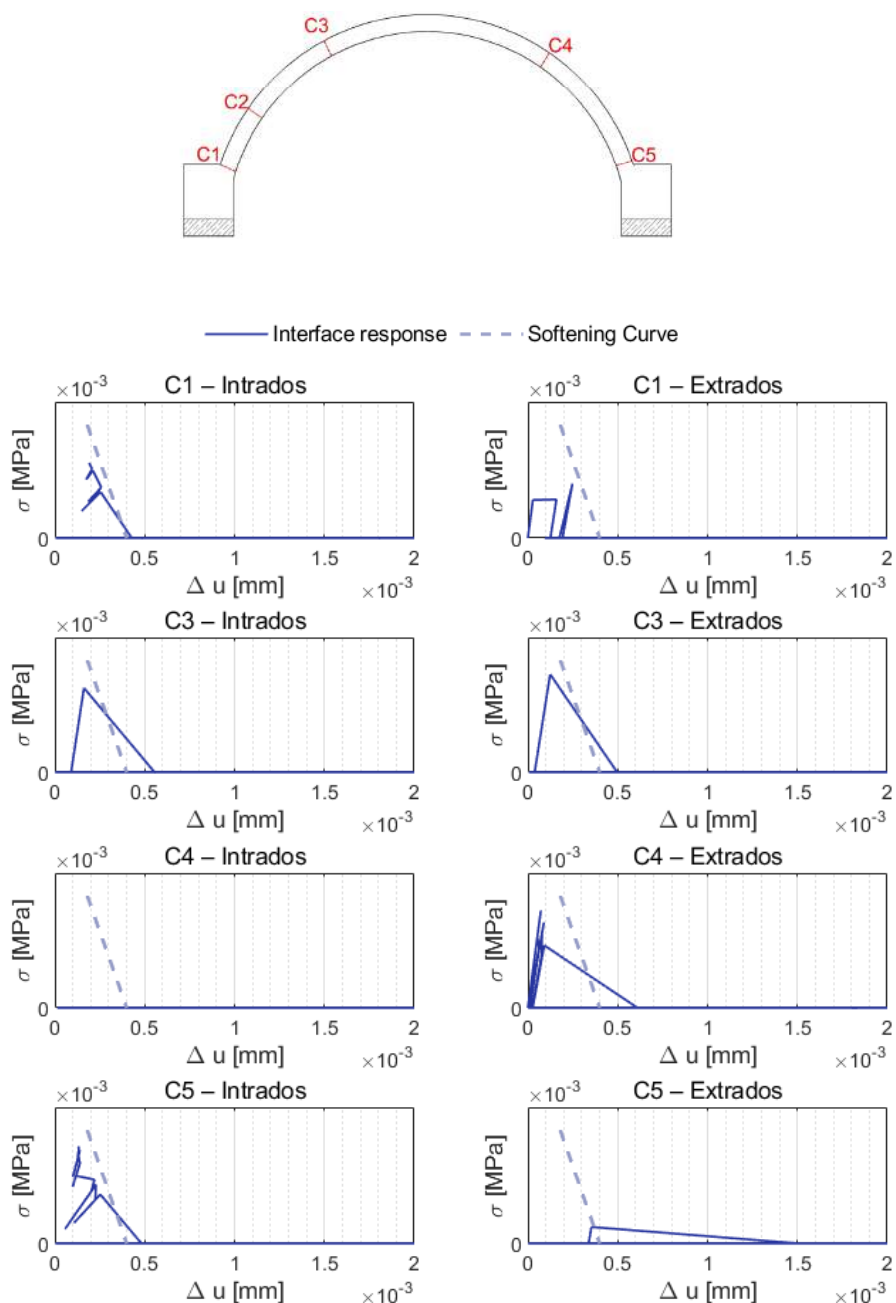


Fig. 6.37: Results considering pre-damaged condition and fracture energy set to 1×10^{-6} N/mm: numerical vs theoretical behaviour at the extreme points of the target interfaces

perimental data closely up to 7 seconds in both directions; beyond this point, discrepancies appear, especially in the Z direction, where the model indicates a permanent sliding of the element that is less evident in the experimental test. Additionally, the comparison of RDTHs presented in Figure 6.39 corresponds to location M16. The numerical results show very good agreement with the experimental data, effectively reproducing both the timing of the rocking mechanism

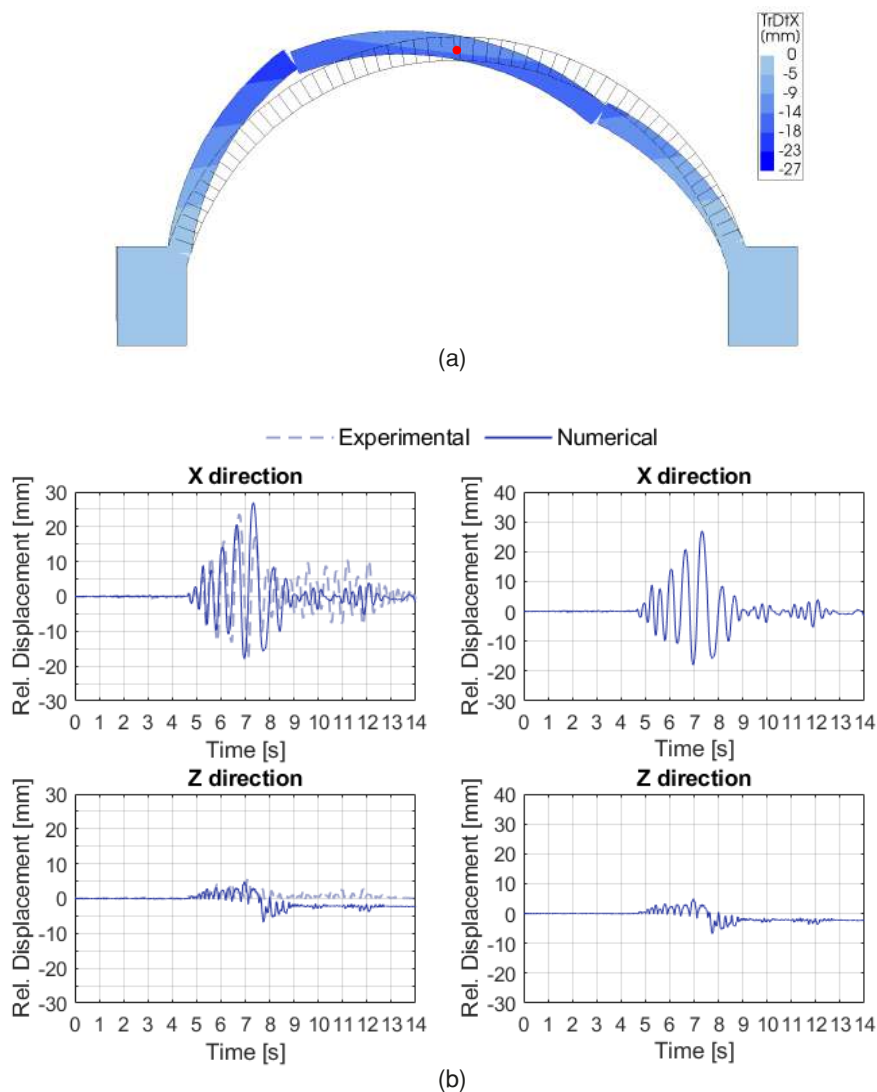


Fig. 6.38: Results considering pre-damaged condition and fracture energy set to 1×10^{-6} N/mm: (a) deformed shape at 7.0 s, and (b) comparison of RDTHs at the keystone (left panels) and corresponding numerical response (right panels)

activation and the associated displacement trends.

The simulation yields a maximum relative displacement of 34 mm in the X direction and 32 mm in the Z direction, against 29 mm and 27 mm recorded experimentally. These differences correspond to relative errors of approximately 17% in the X direction and 18% in the Z direction. As previously noted, the model accurately reproduces the experimental behaviour up to about 7 sec-

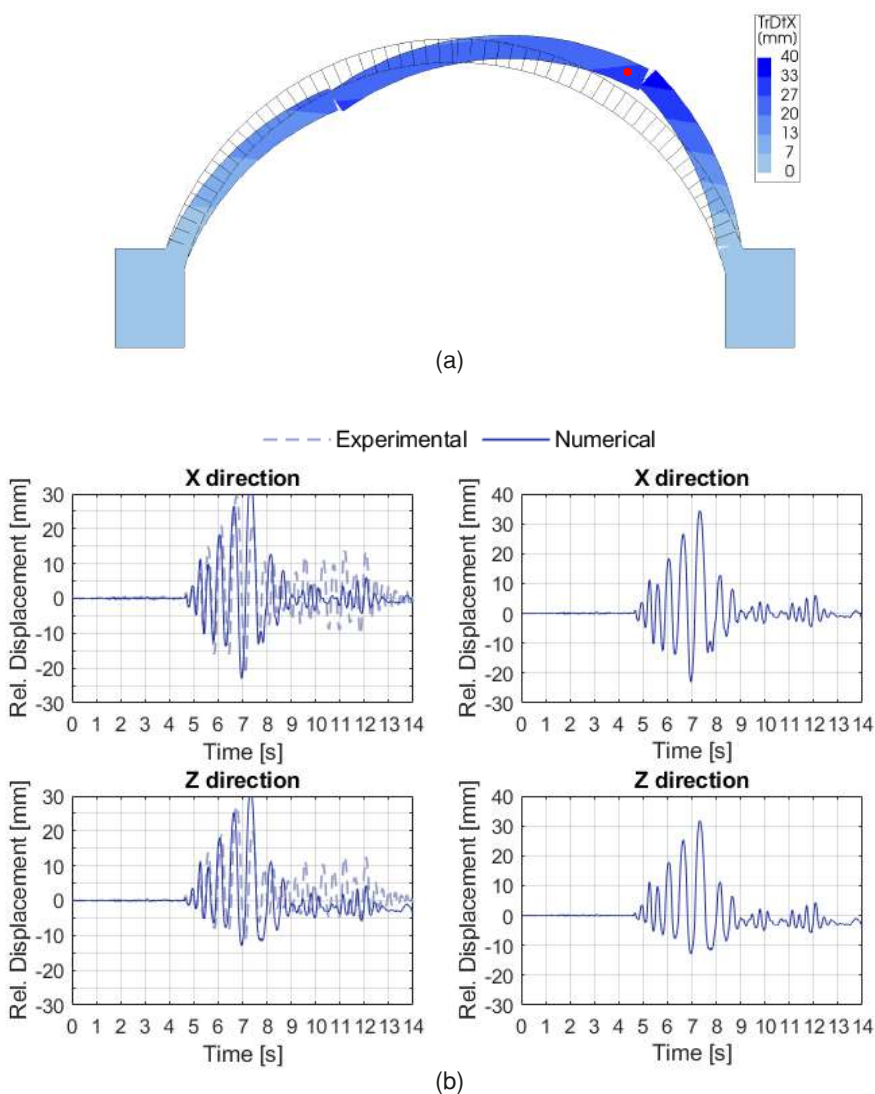


Fig. 6.39: Results considering pre-damaged condition and fracture energy set to 1×10^{-6} N/mm: (a) deformed shape at 7.2 s, and (b) comparison of RDTHs at location M16 (left panels) and corresponding numerical response (right panels)

onds, after which the response starts to differentiate from it, particularly in the later stages of the analysis. Nonetheless, it provides a reliable representation of the early and most critical phases of the dynamic response. Overall, the analysis provides a robust and consistent representation of the experimental behaviour, confirming the model's capability to capture the dynamic response of the structure with good accuracy.

The global performance of the model in terms of relative displacements is shown in Figure 6.40, which reports the displacements of all monitored points along Section A–A in both directions at 6.7 seconds. The numerical results (blue circle markers) exhibit excellent agreement with the experimental data (grey square markers), successfully reproducing both the displacement magnitudes and the overall deformation pattern associated with the rocking mechanism. In addition, the comparison of the angular variation d^θ further confirms the strong agreement between the numerical and experimental responses. The numerical model captures the hinge formation observed in the tests, with only minor discrepancies: namely, a less pronounced hinge at section S7 and the presence of hinges at section S15, which were not reported experimentally. These differences do not compromise the overall validation of the model, which demonstrates high fidelity in reproducing the structural response under the tested conditions.

To further assess the global response of the numerical model, an additional comparison with the experimental results was performed at 5.8 s. Figure 6.41 presents the relative displacements and angular variation at this time instant. The numerical model again shows excellent accuracy in reproducing the experimental behaviour. The relative displacements in both the X and Z directions closely match the experimental values across all monitored points. Likewise, the angular variation indicator (d^θ) proves consistent agreement with the experimental data, further confirming the model's ability to capture the observed deformation mechanisms with high accuracy.

6.4. Conclusive remarks

In this chapter, a formulation to analyse the evolution of rocking mechanism failure in unreinforced masonry structures was presented, based on an implicit time-stepping method and a non-linear representation of both geometric and material properties. The implicit method was selected to accurately capture the development of significant changes in the stiffness matrix, from elastic behaviour to fully degraded non-linear material properties. The HHT method was employed because it allows to introduce numerical dissipation, avoiding the need for additional damping formulations, which are challenging to implement in rocking analyses. A representation of the pure rocking mechanism was considered; i.e., the alternation of opening and closing in cracked interfaces. Therefore, the full non-linearity of the FE model was assigned to the zero-thickness interfaces using a simplified micro-modelling approach. The analysis aimed to evaluate the contribution of each non-linear parameter in determining

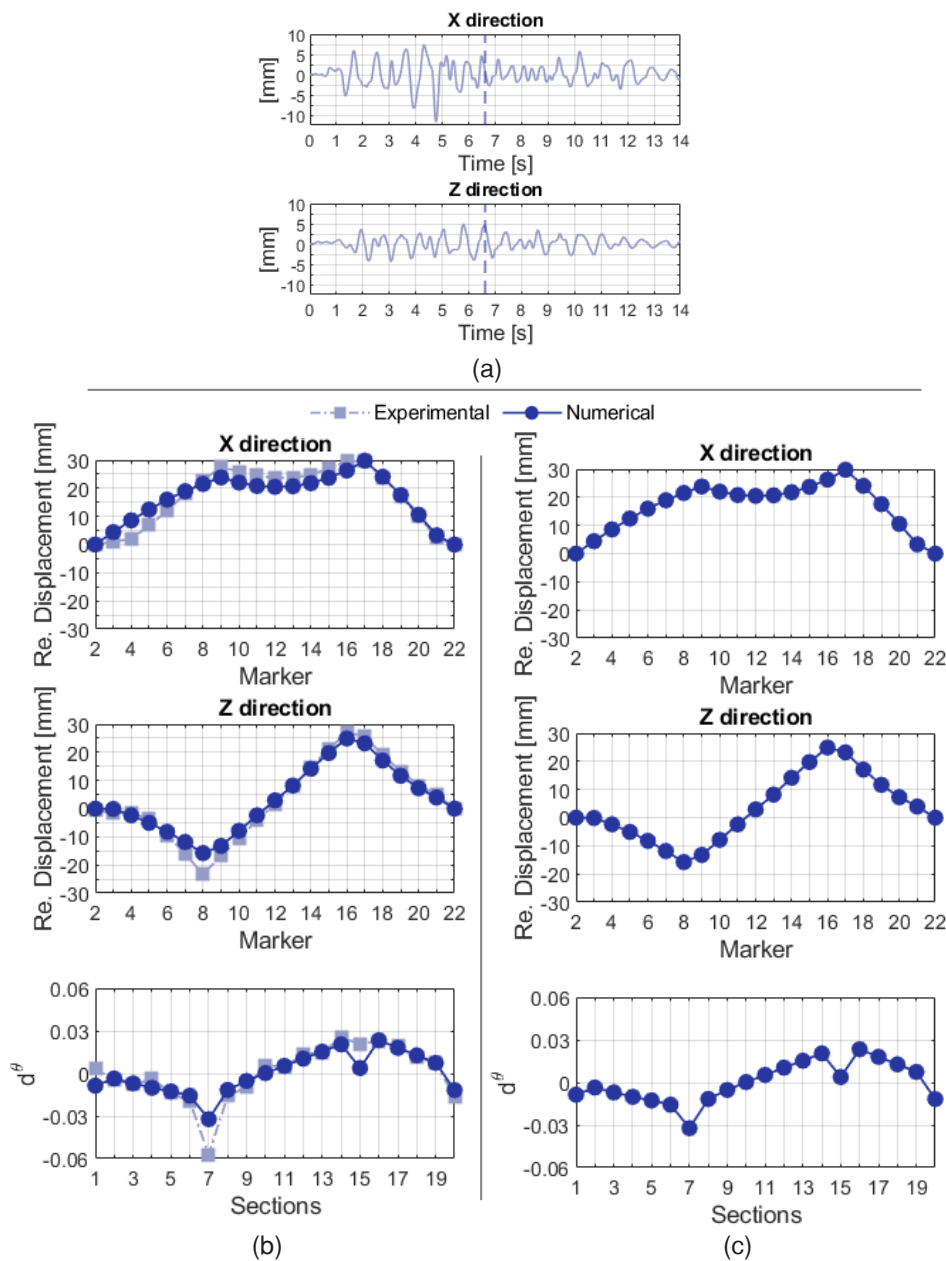


Fig. 6.40: Results considering pre-damaged condition and fracture energy set to 1×10^{-6} N/mm at 6.7 s: (a) input signals, and (b) comparison between numerical and experimental results, (c) numerical results

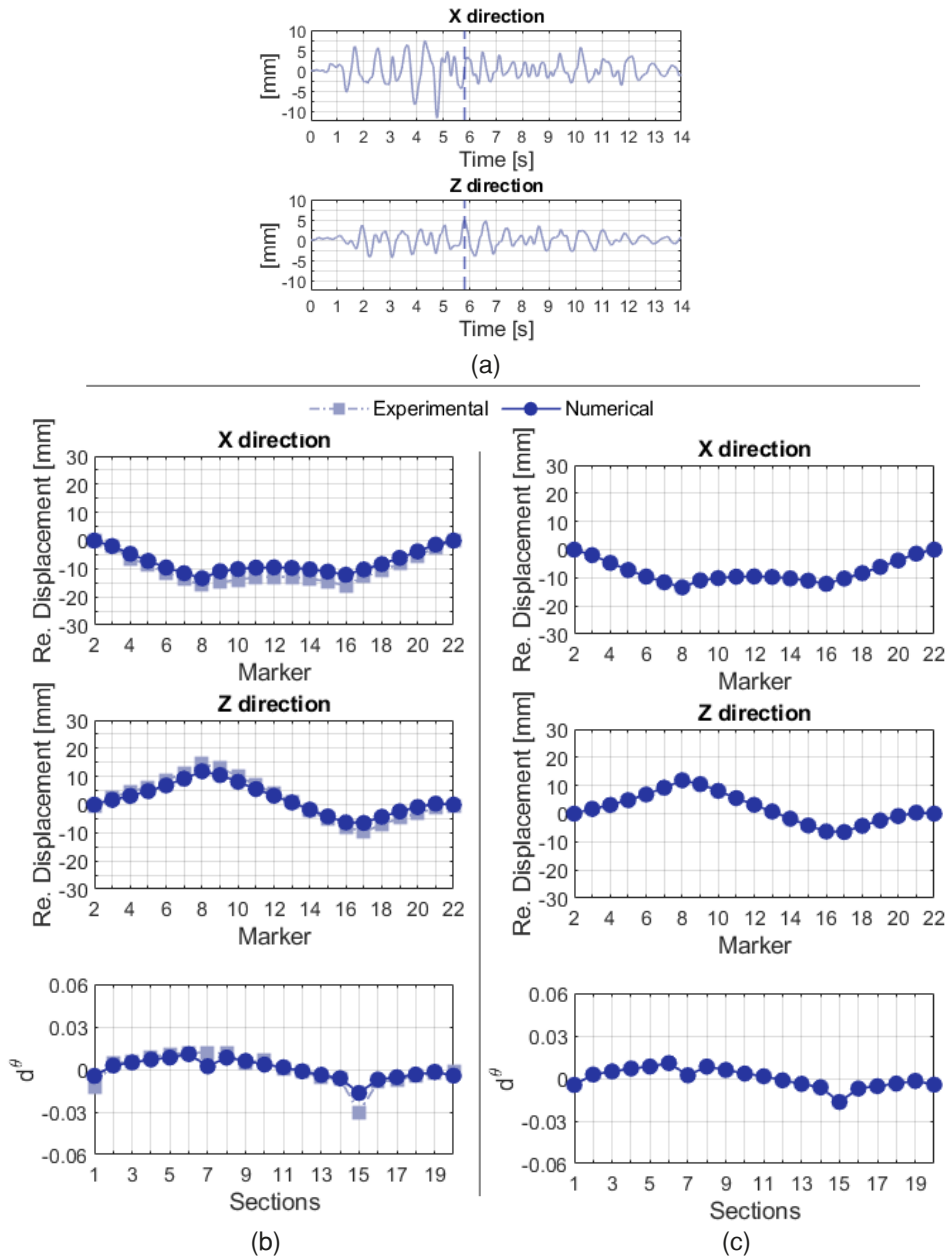


Fig. 6.41: Results considering pre-damaged condition and fracture energy set to 1×10^{-6} N/mm at 5.8 seconds:(a) input signals, and (b) comparison between numerical and experimental results, (c) numerical results

the onset and progression of the rocking mechanism. To this aim, a discrete cracking model, defined by normal stiffness, tensile strength, softening curve, and fracture energy, was chosen as the proposed constitutive law. Its application to assess the rocking mechanism was demonstrated through a specific case study: the unreinforced vault, which failed primarily through rocking. The numerical model was first validated in the elastic range by tuning the interface stiffness values in the normal and tangential directions and comparing the results with the experimental modal properties obtained during the investigation campaign. Subsequently, the influence of the non-linear interface parameters was evaluated, leading to several conclusions:

- Fracture energy is the key parameter influencing the activation of the rocking mechanism, as it governs the rate at which tensile resistance is reduced. This allowed for the observation of the phenomenon for two significantly different tensile strength values, under the same load conditions.
- Tensile strength determines when the rocking mechanism is activated: in a pre-damaged condition, rocking occurs earlier than when a nonzero tensile strength is assumed.
- Using an implicit time-integration scheme allows the rocking mechanism to be represented as a damage-induced process, enabling computation of the updated stiffness matrix.
- The HHT method is a reliable approach to model the rocking mechanism without additional damping formulations.

In conclusion, this investigation demonstrates that the definition of an appropriate fracture energy is critical, as it determines whether the model captures rocking or prevents its activation. When evaluating the seismic dynamic capacity of a structure, setting fracture energy values excessively high can lead to misleading results, since cracks may never fully open and the structural capacity may be overestimated. Conversely, variations in tensile strength have a clear influence on the failure behaviour, underscoring the need for careful parameter selection in modelling.

7. Conclusion

The seismic dynamic behaviour of masonry barrel vaults has been investigated in this thesis through a combined approach involving both experimental and numerical analyses. The current state of the art highlights the still limited number of contributions on this topic. This work builds upon a previous experimental campaign carried out by Gioffré et al. [2023], in which an Unreinforced Vault (UV) and a Reinforced Vault (RV) were seismically tested on shaking tables at the L.E.D.A. (Laboratory of Earthquake engineering and Dynamic Analysis) research centre in Enna, Italy. A novel reinforcement system was introduced, consisting of a hemp-rope net embedded in a cocchiopesto matrix composed of ground clay bricks and an organic resin. Building on this previous work, in which potential results were not exhaustively explored, the present thesis extended the experimental investigation to a Repaired–Damaged Unreinforced Vault (R-DUV), obtained by repairing the damaged UV specimen with the same reinforcement technique adopted in the RV. This new specimen was subjected to an additional shaking table campaign. The aim was to carry out a comparative analysis with the other two tested specimens, assessing the final load capacity, seismic dynamic behaviour, failure mechanisms, modal properties, and damage evolution through analyses based on the variation of modal parameters. Furthermore, displacement-based analyses were developed as a basis for future numerical investigations, with the objective of selecting significant monitoring points for experimental–numerical comparison. The numerical investigations, based on the Finite Element Method (FEM), provided new insights into modelling approaches for the seismic dynamic behaviour of masonry barrel vaults. In conclusion, the systematic analysis and comparative study presented in this thesis addressed several aspects of the seismic response of masonry barrel vaults

that had previously remained underexplored in literature.

7.1. Summary of contributions and findings

This thesis started with the introduction chapter (**Chapter 1**), which provided the main motivations for this work, emphasising how the seismic dynamic behaviour of masonry vaults remains an open research field. The main research goals were then presented. The thesis was based on a multi-directional approach, providing investigations from both experimental and numerical perspectives. For this reason, based on the continuation of a previous seismic shaking table campaign, a new experimental campaign was carried out during the research period, aiming to assess and provide further information on the behaviour of masonry barrel vaults. The thesis aimed to provide a comprehensive analysis of the seismic dynamic behaviour through the comparison of the response of the tested specimens. Additionally, the aim of pursuing numerical investigations was introduced, adding significant information on the modelling of the response of unreinforced structures under seismic excitations. Finally, a summary of the main contributions, as well as the thesis structure, was provided. Subsequently, **Chapter 2** presented a literature review of the current state of the art. The chapter was divided into two main sections. The first provided basic definitions of arch components and discussed the evolution of masonry arches and barrel vaults over time, starting from the pozzolan-based systems initially adopted by the Romans to the more refined brick-and-mortar constructions extensively studied during the Renaissance. The second section of this chapter addressed reinforcement methods and experimental tests available in the literature, with a particular focus on the seismic dynamic response of full-scale specimens tested on shaking tables under natural and artificial seismic excitations. From this section, the main brick arrangements and arch profile configurations were identified, along with other significant findings from the current state of the art. **Chapter 3** reviewed the main analysis methods for masonry barrel vaults, progressing from analytical to numerical approaches. The first part presented the historical development of analytical methods, with particular emphasis on those that led to the formulation of limit analysis for masonry arches. Subsequently, the second part addressed numerical studies, distinguishing between the FEM, the Discrete Element Method, and the Discrete Macro-Element Method. For each numerical approach, selected studies from the literature that were relevant to the scope of this work were reviewed. The chapter concluded with a summary highlighting the current state of the art in numerical modelling. The main research contributions of the thesis were mainly presented in the following three chapters. **Chapter 4** introduced the two experimental campaigns carried out on masonry vaults and addressed in this work. Three vaults were tested under identical input conditions, consisting of seismic tests at increasing input intensity levels based on the ground motion recorded during the October 30, 2016 Norcia earthquake, identified as the Reference Earthquake (RE), and Ambient

Vibration Tests (AVTs) performed without active input. The three specimens, namely the Unreinforced Vault (UV), Reinforced Vault (RV), and Reinforced-Damaged Unreinforced Vault (R-DUV), were tested in two separate shaking table campaigns at the L.E.D.A. laboratory of the University of Enna Kore. The first section of this chapter described the initial experimental campaign carried out by Gioffré et al. [2023], with a focus on the geometry and layout of the specimens, as well as the main construction phases. Particular attention was devoted to the novel hemp-rope-based strengthening system and its mechanical properties. The shaking table facility and the instrumentation layout were presented in detail, together with the selection of the input. The second experimental campaign, carried out during the research period, was performed on the previously tested UV specimen in its damaged state from the first campaign. This Damaged Unreinforced Vault (DUV) was repaired with the same extrados reinforcement used for the RV, creating the R-DUV specimen, which was then tested under the same loading conditions as in the first campaign. The construction phases, instrumentation layout, and test sequence were detailed.

Chapter 5 discussed the results of the shaking table tests for the three specimens, focusing on their seismic performance through a comparative approach and presenting the identification of critical locations as a basis for numerical comparison. The chapter began with a description of the methodology adopted to achieve the assessment of seismic performance in terms of capacity, failure mechanisms, and dynamic behaviour. This methodology combined qualitative and quantitative approaches based on the extensive database recorded during the experimental campaign. Initially, a qualitative description of the damage onset and progression was provided for each specimen, highlighting the failure mechanisms and ultimate load capacity, expressed through the scaling factor of the RE. The UV failed through the development of a rocking mechanism with four main cracks, around which the rocking motion evolved. Collapse occurred at a seismic input equal to 0.4 times the RE. The RV exhibited a completely different failure pattern, with debonding at two significant locations: near the keystone and at the reinforcement–abutment connection on the right side of the specimen. This mechanism was complex, involving a higher number of cracks, and collapse was prevented only by the tensile strength provided by the hemp-rope net embedded in the cocchiopesto layer. This critical condition occurred at an input equal to 2.3 times the RE. The R-DUV developed a collapse mechanism characterized by rocking at the bases and the expulsion of part of the abutment at the strengthened masonry connections, together with debonding close to the keystone. Failure occurred under an input equal to 2.2 times the RE. A comparison of the damage patterns among the specimens provided significant insights into the cracking distribution. Quantitative assessments were also presented based on the variation of modal parameters obtained through the Enhanced Frequency Domain Decomposition (EFDD) method applied to AVTs performed after significant seismic inputs. This analysis tracked the evolution

of damage as a function of vibration frequencies and modes, showing significant variations at higher excitation levels. A comparison of the modal parameters in the undamaged state (i.e., prior to seismic testing) was also presented among the specimens, assessing the compatibility of the intervention in terms of modal properties and showing that the repaired configuration almost fully recovered the initial modal properties of the unreinforced vault. Another part of the analysis focused on the identification of significant locations as a basis for numerical and experimental comparison. A methodology was proposed to detect the most critical locations of the structure during the tests, allowing the global behaviour to be assessed through the deformed shape, relative displacement patterns, and the angular variation indicator, based on the variation of the angle between two monitored locations. This procedure successfully detected the areas of significant structural response, consistent with the observed damage patterns. These locations, identified for significant seismic tests, provide the basis for subsequent numerical and experimental comparisons. Finally, conclusive remarks assessed the effectiveness and compatibility of the bio-composite strengthening solution. The reinforced and repaired vaults reached a final load factor more than 5.5 times higher than that of the UV, delayed damage onset, and prevented widespread cracking until later stages. The most sensitive areas to damage in the reinforced specimens, especially at the extrados, were also summarized. The compatibility between the strengthening solutions in terms of modal properties was confirmed. Overall, this chapter provided a comprehensive description of the behaviour and limitations of masonry vaults under seismic loading. **Chapter 6** presented a new strategy for describing rocking mechanisms in curved unreinforced structures within a FEM formulation, adopting an implicit time-stepping solution scheme and a simplified micro-modelling approach. The chapter first discussed the reasons behind using implicit methods and explained that the Hilber–Hughes–Taylor (HHT) method was selected, as it allows control of numerical damping. All non-linearities of the model were attributed to zero-thickness interfaces, with a discrete cracking model adopted to represent the non-linear law defined by tensile strength, a linear softening curve, and tensile fracture energy, while neglecting crushing–sliding phenomena. After presenting the formulation, the methodology was applied to a case study of the UV whose failure was governed by rocking. A two-dimensional numerical model was developed and calibrated against the vibration modes and frequencies obtained during the experimental campaign, by tuning the interface stiffness to ensure a reliable elastic response. The calibration resulted in frequency errors below 2% for the considered modes, while consistency in mode shapes was verified using a Modal Assurance Criterion (MAC) analysis. The influence of non-linear parameters was then investigated, evaluating configurations with weaker and more brittle interfaces. This second calibration step involved reducing tensile strength and fracture energy at the target interfaces and comparing the relative displacement time histories at two significant lo-

cations: the keystone and the location of maximum displacement. A global assessment was then carried out by evaluating both the relative displacements and angular variation indicators at a representative time step. This analysis provided important insights into the seismic behaviour under brittle interface conditions, where the rocking mechanism could develop, compared to conditions where it was completely prevented when adopting fracture energy values from the literature. The combination of reduced tensile strength and calibrated fracture energy proved effective in reproducing the initiation and progression of the rocking mechanism. Finally, some concluding remarks emphasized the main advantages of the proposed modelling strategy. Finally, **Chapter 7** concluded with a summary the main contributions of this work and highlighted its limitations and possible future research developments.

7.2.Future research directions

This thesis was developed within specific research directions. Although the findings outlined in the previous summary highlighted the main novelty of the work, as with any research, further improvements are possible and future developments can be proposed. The main suggested topics for the continuation of this work are presented below, providing insights for potential applications and future studies.

- **Number of specimens:**

Since the experimental campaigns were limited to testing only one specimen per configuration (UV, RV, R-DUV), as a future direction, it would be worthwhile to extend the investigation to at least three specimens per configuration. This would allow a more reliable evaluation of capacity and failure mechanisms, providing particularly beneficial when dealing with rocking mechanisms, which are inherently chaotic and not easily reproducible.

- **Structural assessment methodology:**

The seismic response in this thesis was evaluated through visual inspection and variation of modal parameters, which provided insights and reliable assessments of the spreading of damage and its accumulation on the specimen. This approach could be implemented by adding information on the structural capacity, developing capacity and hysteresis curves for critical monitored locations, which would provide additional information on the seismic dynamic behaviour, contributing to a more complete assessment of the structural seismic capacity.

- **Numerical modelling approach:**

The numerical model was developed within a FEM framework, which allows the integration of the proposed methodology for modelling other

types of structures that fail through rocking, thus representing a versatile approach. Moreover, FEM enables the adoption of different modelling strategies within the same framework, combining macro- and micro-modelling. This approach could also be applied to the reinforced and repaired vaults, starting from the proposed model and introducing the strengthening layer, simplified as a homogeneous layer in contact with the masonry through interfaces. Such a strategy could serve as a first step in modelling the strengthening technique, but it must be supported by a refinement of the non-linear modelling of the reinforcement–masonry interface to capture complex phenomena such as debonding. Additionally, developing a DEM-based formulation could provide an alternative framework to reproduce discrete mechanisms and allow direct comparison with FEM results.

References

Abbass, A.; Oliveira, D. V.; Lourenço, P. B.; Paiva, M. C. Multi-scale experimental investigation on the structural behaviour of novel nanocomposite/natural textile-reinforced mortars, *Construction and Building Materials*, 2024, Volume 444, pp. 137798. doi: 10.1016/j.conbuildmat.2024.137798.

Acikgoz, S.; Soga, K.; Woodhams, J. Evaluation of the response of a vaulted masonry structure to differential settlements using point cloud data and limit analyses, *Construction and Building Materials*, 2017, Volume 150, pp. 916–931. doi: 10.1016/j.conbuildmat.2017.05.075.

Acito, M.; Bocciarelli, M.; Chesi, C.; Milani, G. Collapse of the clock tower in Finale Emilia after the May 2012 Emilia Romagna earthquake sequence: Numerical insight, *Engineering Structures*, 2014, Volume 72, pp. 70-91. doi: 10.1016/j.engstruct.2014.04.026.

Alecci, V.; Misseri, G.; Rovero, L.; Stipo, G.; De Stefano, M.; Feo, L.; Luciano, R. Experimental investigation on masonry arches strengthened with PBO-FRCM composite, *Composites Part B: Engineering*, 2016, Volume 100, pp. 228-239. doi: 10.1016/j.compositesb.2016.05.063.

Alejo, L.E.; Mendes, N.; Lourenço, P.B.; Martínez, G. Protecting the Historic Buildings of Mexico: The Barrel Vault of San Agustín Church in Morelia, *Journal of Performance of Constructed Facilities*, 2021. doi:10.1061/(ASCE)CF.1943.

Alexakis, H.; Makris, N. Hinging mechanisms of masonry single-nave barrel

vaults subjected to lateral and gravity loads, *Journal of Structural Engineering*, 2017, Volume 143, Issue 6. doi: 10.1061/(asce)st.1943-541x.0001762.

Alforno, M. Investigation of the Role of Constructive Aspects on the Structural Response of Masonry Vaults. *Ph.D. Thesis*, Politecnico di Torino (Italy), 2021.

Alforno, M.; Venuti, F.; Monaco, A.; Calderini, C. Seismic behaviour of cross vaults with different brick pattern, *Bulletin of Earthquake Engineering*, 2022, Volume 20, Issue 8, pp. 3921–3939. doi: 10.1007/s10518-022-01347-6.

AlShawa, O.; Felice, G.; Mauro, A.; Sorrentino, L. Out-of-plane seismic behaviour of rocking masonry walls, *Earthquake Engineering & Structural Dynamics*, 2012, Volume 41, Issue 4, pp. 949-968. doi: 10.1002/eqe.1168.

Arias, A. A measure of earthquake intensity, *Seismic Design for Nuclear Power Plants*, 1970, pp. 438–469.

Aşıkoğlu, A.; D'Anna, J.; Ramirez, R.; Solarino, F.; Romanazzi, A.; Ciocci, M. P.; Bianchini, N. Simulation of blind pre-diction and post-diction shaking table tests on a masonry building aggregate using a continuum modelling approach, *Bulletin of Earthquake Engineering*, 2024, Volume 22, Issue 12, pp. 6163-6193. doi: 10.1007/s10518-023-01666-2.

ASTM International, *ASTM D2258–99. Standard practice for sampling yarn for testing*, In: *Annual Book of ASTM Standards*, West Conshohocken, PA, USA: ASTM International, 1999.

Au, S.-K. Ambient Data Modeling and Analysis. In: *Operational Modal Analysis*, Springer, 1st ed., 2017

Azwa, Z. N.; Yousif, B. F.; Manalo, A. C.; Karunasena, W. A review on the degradability of polymeric composites based on natural fibres, *Materials and Design*, 2013, Volume 47, pp. 424-442. doi: 10.1016/j.matdes.2012.11.025.

Babatunde, S. A. Review of strengthening techniques for masonry using fiber reinforced polymers, *Composite Structures*, 2017, Volume 161, pp. 246-255. doi: 10.1016/j.compstruct.2016.10.132.

Baggio, C.; Trovalusci, P. Stone assemblies under in-plane actions. Comparison between non linear discrete approaches, *Computer Methods in Structural Masonry*, 1995.

Baratta, A.; Corbi, I.; Corbi, O.; Rinaldis, D. Experimental evidence of uncertainty in seismic response of masonry vaults, *Proceedings of the Fourteenth World Conference on Earthquake Engineering*, Beijing (China), 2008.

- Barbhuiya, S.; Das, B. B.; Kapoor, K.; Das, A.; Katare, V. Mechanical performance of bio-based materials in structural applications: A comprehensive review, *Structures*, 2025, Volume 75, Article 108726. doi: 10.1016/j.istruc.2025.108726.
- Bartoli, G. Hagia Sophia in Istanbul: Some Remarks on Displacement Phenomena in Main Piers, *Hagia Sophia: Surveying Project Conference*, 2001. Available online: <https://www.researchgate.net/publication/266244403>.
- Bartoli, G.; Betti, M. Cappella dei Principi in Firenze, Italy: Experimental Analyses and Numerical Modeling for the Investigation of a Local Failure, *Journal of Performance of Constructed Facilities*, 2013, Volume 27, Issue 1, pp. 4-26. doi:10.1061/(ASCE)CF.1943-5509.0000315.
- Bathe, K.-J. *Finite Element Procedures in Engineering Analysis*, Publisher: Prentice Hall, 1982, Chapter 9.
- Bertolesi, E.; Grande, E.; Fagone, M.; Milani, G.; Rotunno, T. Mechanical model based on a BVP for FRPs applied on flat and curved masonry pillars with anchor spikes, *Composite Structures*, 2021, Volume 273. doi: 10.1016/j.compstruct.2021.114251.
- Bianchini, N.; Mendes, N.; Calderini, C.; Lourenço, P. B. Modelling of the dynamic response of a reduced scale dry joints groin vault, *Journal of Building Engineering*, 2023, Volume 66, Article 105826. doi: 10.1016/j.job.2023.105826.
- Bianchini, N. Evaluation of the Seismic Response of Masonry Cross Vaults through Shaking Table Tests and Numerical Analysis, *Ph.D. Thesis*, University of Minho (Portugal), 2023.
- Block, P.; Ochsendorf, J. Thrust Network Analysis: A New Methodology for Three-Dimensional Equilibrium. *J. Int. Assoc. Shell Spat. Struct.*, 2007.
- Block, P. Thrust Network Analysis Exploring Three-dimensional Equilibrium. *Ph.D. Thesis*, Massachusetts Institute of Technology, 2009.
- Boem, I.; Gattesco, N. Cyclic behavior of masonry barrel vaults strengthened through Composite Reinforced Mortar, considering the role of the connection with the abutments, *Engineering Structures*, 2021, Volume 228, Article 111518. doi: 10.1016/j.engstruct.2020.111518.
- Boni, C.; Ferretti, D.; Lenticchia, E. Effects of Brick Pattern on the Static Behavior of Masonry Vaults, *International Journal of Architectural Heritage*, 2022, Volume 16, Issue 8, pp. 1199–1219. doi: 10.1080/15583058.2021.1874565.
- Boore, D.M.; Bommer, J.J. Processing of strong-motion accelerograms: Needs, options and consequences, *Soil Dynamics and Earthquake Engineering*, 2005, 25(2), pp. 93–115. doi: 10.1016/j.soildyn.2004.10.007.

- Borri, A.; Corradi, M.; De Maria, A. The Failure of Masonry Walls by Disaggregation and the Masonry Quality Index, 2020, *Heritage*, Volume 3 , Issue 4, pp. 1162-1198. doi: 10.3390/heritage3040065.
- Bove, M.; Castellano, A.; Fraddosio, A.; Scacco, J.; Milani, G.; Piccioni, M. D. Experimental and numerical analysis of FRCM strengthened parabolic tuff barrel vault, *Key Engineering Materials*, 2019, Volume 817, pp. 213-220. doi: 10.4028/www.scientific.net/KEM.817.213.
- Brincker, R.; Ventura, C.E. Frequency-Domain Identification, *Introduction to Operational Modal Analysis*, John Wiley & Sons, Ltd, 2015, Chapter 10, pp. 261-280. doi: 10.1002/9781118535141.ch10.
- Bui, H.; Boutouil, M.; Levacher, D.; Sebaibi, N. Evaluation of the influence of accelerated carbonation on the microstructure and mechanical characteristics of coconut fibre-reinforced cementitious matrix, *Journal of Building Engineering*, 2021, Volume 39. doi: 10.1016/j.jobbe.2021.102269.
- Buttarazzi, M.A. Hystorical Rules and Theories on Masonry Arches from the Renaissance to Monasterio and Persy, *Ph.D. Thesis*, Dipartimento di Ingegneria Strutturale e Geotecnica, Università degli Studi di Roma "La Sapienza", 2013.
- Buzzetti, M.; Acito, M.; Chesi, C.; Serra, F. Semi-automatic calibration of numerical models for the seismic safety assessment of masonry towers embedded in building aggregates, *Engineering Structures*, 2024, Volume 316. doi: 10.1016/j.engstruct.2024.118553.
- Caddemi, S.; Calì, I.; Cannizzaro, F.; D'Urso, D.; Pantò, B.; Rapisavoli, D.; Occhipinti, G. 3D Discrete Macro-modelling Approach for Masonry Arch Bridges, *Proceedings of the IABSE Symposium, Guimaraes 2019: Towards a Resilient Built Environment Risk and Asset Management*, 2019, pp. 1825-1835. doi:10.2749/guimaraes.2019.1825.
- Calì, I.; Marletta, M.; Pantò, B. A simplified model for the evaluation of the seismic behaviour of masonry buildings, In: *Proceedings of the 10th International Conference on Civil, Structural and Environmental Engineering Computing*, Rome, (Italy) 30 August–2 September 2005; Civil-Comp Press: Stirling, UK, 2005. doi:10.4203/ccp.81.195.
- Calì, I.; Pantò, B. A discrete element approach for the evaluation of the seismic response of masonry buildings, In: *Structural Analysis of Historic Construction. Preserving Safety and Significance, Proceedings of the 6th International Conference on Structural Analysis of Historic Construction (SAHC 2008)*, Bath (UK), 2–4 July 2008; D'Ayala, D.; Fodde, E., Eds.; CRC Press: London, UK, 2008; pp. 91–98.

- Caliò, I.; Cannizzaro, F.; Marletta, M. A discrete element for modeling masonry vaults, *Advanced Materials Research*, 2010, Volumes 133-134, pp. 447–452. doi:10.4028/www.scientific.net/AMR.133-134.447
- Caliò, I.; Marletta, M.; Pantò, B. A new discrete element model for the evaluation of the seismic behaviour of unreinforced masonry buildings, *Engineering Structures*, 2012, Volume 40, pp. 327–338. doi:10.1016/j.engstruct.2012.02.039
- Camata, G.; Di Primio, A.; Sepe, V.; Cantagallo, C. The Effect of the Vertical Component of the Earthquake on a Regular Masonry Wall. *Applied Sciences (Switzerland)*, 2023, Volume 13, Issue 4. MDPI. doi: 10.3390/app13042459.
- Cannizzaro, F. Un nuovo approccio di modellazione della risposta sismica degli edifici storici, *Ph.D. Thesis*, Università degli Studi di Catania, Italy, April 2011. doi:10.13140/RG.2.1.3094.3125
- Cannizzaro, F.; Lourenço, P. Simulation of Shake Table Tests on Out-of-Plane Masonry Buildings. Part (VI): Discrete Element Approach, *International Journal of Architectural Heritage*, 2016, Volume 11. doi:10.1080/15583058.2016.1238973
- Cardinali, V.; Pintucchi, B.; Tanganelli, M.; Trovatielli, F. Settlement of masonry barrel vaults: an experimental and numerical study, *Procedia Structural Integrity*, 2022, Volume 44, pp. 1252-1259. doi: 10.1016/j.prostr.2023.01.161.
- Carozzi, F. G.; Poggi, C.; Bertolesi, E.; Milani, G. Ancient masonry arches and vaults strengthened with TRM, SRG and FRP composites: Experimental evaluation, *Composite Structures*, 2018, Volume 187, pp. 466-480. doi: 10.1016/j.compstruct.2017.12.075.
- Carydis, P.; Castiglioni, C.; Lekkas, E.; Kostaki, I.; Lebesis, N.; Drei, A. The Emilia Romagna, May 2012 Earthquake Sequence. The Influence of the Vertical Earthquake Component and Related Geoscientific and Engineering Aspects, *Ingegneria Sismica*, 2012, Volume 29, pp. 31–58.
- Castellano, A.; Scacco, J.; Fraddosio, A.; Milani, G.; Piccioni, M.D. Experimental and Numerical Analysis of a FRCM Reinforced Parabolic Tuff Barrel Vault, In: *Proceedings of the 12th International Conference on Structural Analysis of Historical Constructions*, 2020; Roca, P.; Pelà, L.; Molins, C., Eds.
- Castellano, A.; Camassa, D.; Fraddosio, A.; Scacco, J.; Piccioni, M.D.; Milani, G. Dynamic damage identification for a full-scale parabolic tuff barrel vault under differential settlements of the supports, *Construction and Building Materials*, 2021, Volume 291. doi: 10.1016/j.conbuildmat.2021.123271.

- Castellano, A.; Fraddosio, A.; Oliveira, D. V.; Piccioni, M. D.; Ricci, E.; Sacco, E. An effective numerical modelling strategy for FRCM strengthened curved masonry structures, *Engineering Structures*, 2023, Volume 274. doi: 10.1016/j.engstruct.2022.115116.
- Castellazzi, G.; Pantò, B.; Occhipinti, G.; Talledo, D. A.; Berto, L.; Camata, G. A comparative study on a complex URM building: part II—issues on modelling and seismic analysis through continuum and discrete-macroelement models, *Bulletin of Earthquake Engineering*, 2022, Volume 20, pp. 2159–2185. doi:10.1007/s10518-021-01147-4
- Cavicchi, A.; Gambarotta, L. Collapse analysis of masonry bridges taking into account arch-fill interaction, *Engineering Structures*, 2005, Volume 27, Issue 4, pp. 605–615. doi: 10.1016/j.engstruct.2004.12.002.
- European Committee for Standardization (CEN). Eurocode 8, design of structures for earthquake resistance. Part 1, General rules, seismic actions and rules for buildings, 1st ed.; 2004.
- European Committee for Standardization (CEN) EN 10027-1:2005 - Designation systems for steels - Part 1: Steel names, 2005.
- European Committee for Standardization (CEN). UNI EN 1015-11:2019, Methods of Test for Mortar for Masonry: Determination of Flexural and Compressive Strength of Hardened Mortar, 2019.
- Chatzis, M. N.; Smyth, A. W. Robust modeling of the rocking problem, *Journal of Engineering Mechanics*, 2012, Volume 138, Issue 3, pp. 247–262. doi: 10.1061/(ASCE)EM.1943-7889.0000329.
- Chen, Z.; Fu, J.; Peng, Y.; Chen, T.; Zhang, L.; Yuan, C. Baseline correction of acceleration data based on a hybrid EMD–DNN method, *Sensors*, 2021, Volume 21, Issue 18. doi: 10.3390/s21186283.
- Chieffo, N.; Mosoarca, M.; Formisano, A.; Lourenço, P. B.; Milani, G. The Effect of Ground Motion Vertical Component on the Seismic Response of Historical Masonry Buildings: The Case Study of the Banloc Castle in Romania, *Engineering Structures*, 2021, Volume 249. Publisher: Elsevier Ltd. doi: 10.1016/j.engstruct.2021.113346.
- Chiozzi, A.; Milani, G.; Grillanda, N.; Tralli, A. Fast Kinematic Limit Analysis of Masonry Walls with Out-of-Plane Loading, *Proceedings of the 6th International Conference on Computational Methods in Structural Dynamics and Earthquake Engineering (COMPDYN 2017)*, Athens, Greece, 2017, pp. 2404–2412. doi:10.7712/120117.5576.17224.

Chopra, A.K. Dynamics of Structures: Theory and Applications to Earthquake Engineering, 4th ed.; Pearson: 2014; Chapter 11.

Ministero delle Infrastrutture e dei Trasporti. Circolare 2019, n. 7, Istruzioni per l'applicazione dell' "Aggiornamento delle "Norme tecniche per le costruzioni"" di cui al decreto ministeriale 17 gennaio 2018, *Gazzetta Ufficiale della Repubblica Italiana*, 2019.

Chung, J.; Lee, J. M. A new family of explicit time integration methods for linear and non-linear structural dynamics, *International Journal for Numerical Methods in Engineering*, 1994, Volume 37, Issue 23, pp. 3961-3976. doi: 10.1002/nme.1620372303.

Civico, R.; Pucci, S.; Villani, F.; Pizzimenti, L.; De Martini, P. M.; Nappi, R.; the Open EMERGEIO Working Group. Surface Ruptures Following the 30 October 2016 Mw 6.5 Norcia Earthquake, Central Italy, *Journal of Maps*, 2018, Volume 14, Issue 2, pp. 151–160. Publisher: Taylor and Francis Ltd. doi: 10.1080/17445647.2018.1441756.

Clemente, P. Introduction to dynamics of stone arches, *Earthquake Engineering & Structural Dynamics*, 1998, Volume 27, Issue 5, pp. 513-522. doi: 10.1002/(SICI)1096-9845(199805)27:5<513::AID-EQE740>3.0.CO;2-O.

Canyonlands Natural History Association (CNHA). Geology: Arch Formation – Arches National Park, *CNHA.org*, 2025. URL: <https://cnha.org/visit/arches-national-park/things-to-know/geology/> (accessed on 5 August 2025).

Coisson, E.; Ferretti, D.; Pagliari, F. A comparison between traditional and modern approaches for the structural modelling of brick masonry barrel vaults, In: *Proceedings of the 12th International Conference on Structural Analysis of Historical Constructions (SAHC 2020)*, 2020. Available online: <http://www.dia.unipr.it> (accessed on 18 August 2025).

Como, M. *Statics of Historic Masonry Constructions*, 3rd Edition, 2016; Publisher: Springer; Chapters 2-4; doi: 10.1007/978-3-319-24569-0.

Como, M.; Coccia, S.; Di Carlo, F. Dynamic response of rocking masonry circular arches, *COMPdyn Proceedings*, 2019, Volume 1, pp. 622-632. doi:10.7712/120119.6944.19177.

Creazza, G.; Saetta, A.V.; Matteazzi, R.; Vitaliani, R.V. Analyses of masonry vaulted structures by using 3-D damage model, *Proceedings of the European Congress on Computational Methods in Applied Sciences and Engineering (EC-COMAS)*, 2000, pp. 11–14.

- Cundall, P. A.; Hart, R. D. A computer model for simulating progressive large scale movements in blocky rock systems, *Proceedings of the Symposium of the International Society for Rock Mechanics*, Nancy, France, 1971, Volume 1, Paper No. II-8.
- Dailamani, S. O.; Croll, J. G. A. Relative Importance of Horizontal and Vertical Components of Earthquake Motion on the Responses of Barrel Vault Cylindrical Roof Shells, In *Proceedings of the 16th World Conference on Earthquake Engineering*, 2017. Publisher: World Conference on Earthquake Engineering.
- D'Altri, A.M.; De Miranda, S.; Castellazzi, G.; Sarhosis, V.; Hudson, J.; Theodossopoulos, D. Historic barrel vaults undergoing differential settlements, *International Journal of Architectural Heritage*, 2020, Volume 14, Issue 8, pp. 1196–1209. doi: 10.1080/15583058.2019.1596332.
- D'Altri, A. M.; Vlachakis, G.; de Miranda, S.; Lourenço, P. B. Rocking block simulation based on numerical dissipation, *Nonlinear Dynamics*, 2024, Volume 112, Issue 20, pp. 17843-17862. doi: 10.1007/s11071-024-09974-1.
- D'Antino, T.; Calabrese, A. S.; Poggi, C. Experimental procedures for the mechanical characterization of composite reinforced mortar (CRM) systems for retrofitting of masonry structures, *Materials and Structures/Materiaux et Constructions*, 2020, Volume 53, Issue 4. doi: 10.1617/s11527-020-01529-1.
- Davino, A.; Meglio, E.; Formisano, A. Hemp FRCM as bio-based technique for seismic consolidation of existing masonry buildings, *International Journal of Masonry Research and Innovation*, 2025, Volume 10, Issue 2, pp. 158–177. doi: 10.1504/IJMRI.2025.144887.
- de Carvalho Bello, C. B.; Boem, I.; Cecchi, A.; Gattesco, N.; Oliveira, D.V. Experimental tests for the characterization of sisal fiber reinforced cementitious matrix for strengthening masonry structures, *Construction and Building Materials*, 2019, Volume 219, pp. 44-55. doi: 10.1016/j.conbuildmat.2019.05.168.
- de Carvalho Bello, C. B.; Baraldi, D.; Cecchi, A.; Oliveira, D.V. Experimental Characterization of Masonry Panels Strengthened with NFRCM, *Key Engineering Materials*, 2021, Volume 898, pp. 43–48. doi: 10.4028/www.scientific.net/KEM.898.43.
- de Felice, G.; De Santis, S.; Lourenço, P.B.; Mendes, N. Methods and Challenges for the Seismic Assessment of Historic Masonry Structures, *International Journal of Architectural Heritage*, Volume 11, Issue 1, January 2017, pp. 143-160. doi: 10.1080/15583058.2016.1238976.
- De Lorenzis, L.; DeJong, M.; Ochsendorf, J. Failure of masonry arches under impulse base motion, *Earthquake Engineering and Structural Dynamics*, 2007, Volume 36, Issue 14, pp. 2119-2136. doi: 10.1002/eqe.719.

- De Matteis, G.; Cacace, D.; Rouhi, J. Masonry vaults: Architectural evolution, structural behaviour and collapse mechanisms, In: *Proceedings of the 7th Structural Engineers World Congress 2019 – Architecture and Structure: From Past to Future*, Istanbul (Turkey), 2019.
- de Nardis, R.; Filippi, L.; Costa, G.; Suhadolc, P.; Nicoletti, M.; Lavecchia, G. Strong Motion Recorded During the Emilia 2012 Thrust Earthquakes (Northern Italy): A Comprehensive Analysis, *Bulletin of Earthquake Engineering*, 2014, Volume 12, Issue 5, pp. 2117–2145. Publisher: Kluwer Academic Publishers. doi: 10.1007/s10518-014-9614-0.
- Dejong, M. J.; Ochsendorf, J. A. Dynamics of in-plane arch rocking: An energy approach, *Proceedings of the Institution of Civil Engineers: Engineering and Computational Mechanics*, 2010, Volume 163, Issue 3, pp. 179–186. doi: 10.1680/eacm.2010.163.3.179.
- Delorme, A.; Grandin, R.; Klinger, Y.; Pierrot-Deseilligny, M.; Feuillet, N.; Jacques, E.; Rupnik, E.; Morishita, Y. Complex Deformation at Shallow Depth During the 30 October 2016 Mw6.5 Norcia Earthquake: Interference Between Tectonic and Gravity Processes?, *Tectonics*, 2020, Volume 39, Issue 2, pp. 1–18. Publisher: Blackwell Publishing Ltd. doi: 10.1029/2019TC005596.
- Di Michele, F.; Cantagallo, C.; Spacone, E. Effects of the vertical seismic component on seismic performance of an unreinforced masonry structure, *Bulletin of Earthquake Engineering*, 2020, Volume 18, Issue 4, pp. 1635–1656. Publisher: Springer. doi: 10.1007/s10518-019-00765-3.
- Di Pasquale, S. L'arte del costruire. Tra conoscenza e scienza, *Marsilio*, 1st ed., 1996.
- Di Primio, A.; Fiorini, N.; Spina, D.; Valente, C.; Vasta, M. Ambient vibrations for the assessment of the strengthening intervention of a masonry barrel vault, *Structural Health Monitoring*, 2021, Volume 20, Issue 3, pp. 1312–1330. doi: 10.1177/1475921720952066.
- DIANA FEA. *DIANA Documentation*, 2025. Available at: <https://manuals.dianafea.com/d107/?lang=en> (accessed on 07 March 2025).
- Dimitri, R.; De Lorenzis, L.; Zavarise, G. Numerical study on the dynamic behavior of masonry columns and arches on buttresses with the discrete element method, *Engineering Structures*, 2011, Volume 33, Issue 12, pp. 3172–3188. doi: 10.1016/j.engstruct.2011.08.018.
- Dokainish, M. A.; Subbaraj, K. A survey of direct time-integration methods in computational structural dynamics—I. Explicit methods, *Computers & Structures*, 1989, Volume 32, Issue 6, pp. 1371–1386. doi: 10.1016/0045-7949(89)90314-3.

- Elfaleh, I.; Abbassi, F.; Habibi, M.; Ahmad, F.; Guedri, M.; Nasri, M.; Garnier, C. A comprehensive review of natural fibers and their composites: An eco-friendly alternative to conventional materials, *Results in Engineering*, 2023, Volume 19. doi: 10.1016/j.rineng.2023.101271.
- Elgawady, M. A.; Ma, Q.; Butterworth, J. W.; Ingham, J. Effects of interface material on the performance of free rocking blocks, *Earthquake Engineering and Structural Dynamics*, 2011, Volume 40, Issue 4, pp. 375-392. doi: 10.1002/eqe.1025.
- Demi L. Fang; Napolitano, R. K.; Michiels, T. L.; Adriaenssens, S. M. Assessing the Stability of Unreinforced Masonry Arches and Vaults: A Comparison of Analytical and Numerical Strategies, *International Journal of Architectural Heritage*, 2019, Volume 13, Issue 5, pp. 648-662. doi:10.1080/15583058.2018.1463413.
- Foraboschi, P. Masonry does not limit itself to only one structural material: Interlocked masonry versus cohesive masonry, *Journal of Building Engineering*, 2019, **26**, 100831. doi: 10.1016/j.jobbe.2019.100831.
- Fossetti, M.; Lo Iacono, F.; Minafò, G.; Navarra, G.; Tesoriere, G. A new large scale laboratory: The LEDA research centre (Laboratory of Earthquake engineering and Dynamic Analysis). In *Proceedings of the International Conference on Advances in Experimental Structural Engineering*, Enna, Italy, September 2017; pp. 699–717. doi: 10.7414/7aese.T6.18.
- Lourenço, P. B. Computational Strategy for Masonry Structures. Ph.D. Thesis, Delft University of Technology, 1996. URL: <https://www.researchgate.net/publication/27344834>
- Lourenço, P. B. Guidelines for the Analysis of Historical Masonry Structures, In: *Finite Elements in Civil Engineering Applications*, 1st ed.; CRC Press, 2002, pp. 241–247. doi: 10.1201/9781003211365-33.
- Lourenço, P. B. Masonry Structures, Overview, In *Encyclopedia of Earthquake Engineering*, Springer Berlin Heidelberg, 2021, pp. 1–9. doi: 10.1007/978-3-642-36197-5_111-1.
- Lourenço, P. B.; Gaetani, A. *Finite Element Analysis for Building Assessment*, Publisher: Routledge, USA, 2022. doi: 10.1201/9780429341564.
- Luger, H.; Ramirez, R.; Pineda, P.; Lourenço, P. B. Field Investigation and Numerical Modeling for the Seismic Assessment of the Castle of Lanjarón, Spain, *Applied Sciences (Switzerland)*, 2025. doi: 10.3390/app15031518.
- Mainstone, R. J. *Hagia Sophia: Architecture, Structure and Liturgy of Justinian's Great Church*, Thames & Hudson: London, UK, 1997. Although the original book was not available, the figure is reported in Bartoli [2001].

Gaetani, A.; Lourenço, P. B.; Monti, G.; Milani, G. A parametric investigation on the seismic capacity of masonry cross vaults, *Engineering Structures*, 2017, Volume 148, pp. 686-703. doi: 10.1016/j.engstruct.2017.07.013.

Gaetani, A. *Seismic Performance of Masonry Cross Vaults. Learning from Historical Developments and Experimental Testing*, Publisher: Sapienza Università Editrice, Italy, 2020.

Galano, L.; Betti, M. *Elements of Statics of Historic Masonry Constructions*, 2nd ed.; Società Editrice Esculapio: Bologna, Italy, 2024. Chapters 9-10.

Galassi, S.; Misseri, G.; Rovero, L.; Tempesta, G. Failure Modes Prediction of Masonry Voussoir Arches on Moving Supports, *Engineering Structures*, 2018, Volume 173, pp. 706–717. doi: 10.1016/j.engstruct.2018.07.015.

Galli, P.; Castenetto, S.; Peronace, E. The Macroseismic Intensity Distribution of the 30 October 2016 Earthquake in Central Italy (Mw 6.6): Seismotectonic Implications, *Tectonics*, 2017, Volume 36, Issue 10, pp. 2179–2191. Publisher: Blackwell Publishing Ltd. doi: 10.1002/2017TC004583.

Galvez, F.; Sorrentino, L.; Dizhur, D.; Ingham, J. M. Damping considerations for rocking block dynamics using the discrete element method, *Earthquake Engineering and Structural Dynamics*, 2022, Volume 51, Issue 4, pp. 935-957. doi: 10.1002/eqe.3598.

Garcia-Ramonda, L.; Pelà, L.; Roca, P.; Camata, G. Cyclic shear-compression testing of brick masonry walls repaired and retrofitted with basalt textile reinforced mortar, *Composite Structures*, 2022, Volume 283. doi: 10.1016/j.compstruct.2021.115068.

Gattesco, N.; Boem, I. Review of experimental tests and numerical study on masonry vaults reinforced through fiber-reinforced mortar coating, *Bulletin of Earthquake Engineering*, 2019, Volume 17, Issue 7, pp. 4027–4048. doi: 10.1007/s10518-019-00619-y.

Gavin, H. P. *Numerical Integration in Structural Dynamics*, CEE 541 Course Notes, Fall 2020, Department of Civil and Environmental Engineering, Duke University. Available online: <https://people.duke.edu/~hpgavin/StructuralDynamics/NumericalIntegration.pdf> (accessed on 14 May 2025).

Giamundo, V. Seismic Assessment and Retrofit of Historical Masonry Barrel Vaults, *Ph.D. Thesis*, University of Naples Federico II, Department of Structures for Engineering and Architecture, 2014.

- Giamundo, V.; Sarhosis, V.; Lignola, G. P.; Sheng, Y. ; Manfredi, G. Evaluation of different computational modelling strategies for the analysis of low strength masonry structures, *Engineering Structures*, Volume 73, 2014, pp. 160-169. doi: 10.1016/j.engstruct.2014.05.007.
- Giamundo, V.; Lignola, G. P.; Maddaloni, G.; Balsamo, A.; Prota, A.; Manfredi, G. Experimental investigation of the seismic performances of IMG reinforcement on curved masonry elements, *Composites Part B: Engineering*, 2015, Volume 70, pp. 53-63. doi: 10.1016/j.compositesb.2014.10.039.
- Giamundo, V.; Lignola, G. P.; Maddaloni, G.; da Porto, F.; Prota, A.; Manfredi, G. Shaking table tests on a full-scale unreinforced and IMG retrofitted clay brick masonry barrel vault, *Bulletin of Earthquake Engineering*, 2016, Volume 14, Issue 6, pp. 1663-1693. doi: 10.1007/s10518-016-9886-7.
- Gilbert, M.; Melbourne, C. Rigid-block analysis of masonry arches, *The Structural Engineer*, 1994, Volume 72, pp. 356-361.
- Gilbert, M.; Casapulla, C.; Ahmed, H. M. Limit analysis of masonry block structures with non-associative frictional joints using linear programming, *Computers and Structures*, 2006, Volume 84, Issue 13-14, pp. 873-887. doi:10.1016/j.compstruc.2006.02.005.
- Girardello, P.; Valluzzi, M. R. Experimental Testing and Numerical modelling of Masonry Vaults, *International Conference on Rehabilitation and Restoration of Structures*, 2013.
- Gobbin, F. Discrete element modelling for the seismic analysis of masonry structures, *Ph.D. Thesis*, Università degli Studi Roma Tre, 2021.
- Gobbin, F.; de Felice, G.; Lemos, J. V. Numerical procedures for the analysis of collapse mechanisms of masonry structures using discrete element modelling, *Engineering Structures*, 2021, Volume 246. doi: 10.1016/j.engstruct.2021.113047.
- Gioffré, M.; Vincenzini, A.; Cavalagli, N.; Gusella, V.; Caponero, M. A.; Terenzi, A.; Pepi, C. A novel hemp-fiber bio-composite material for strengthening of arched structures: Experimental investigation, *Construction and Building Materials*, 2021, Volume 308. doi: 10.1016/j.conbuildmat.2021.124969.
- Gioffré, M.; Navarra, G.; Cavalagli, N.; Lo Iacono, F.; Gusella, V.; Pepi, C. Effect of hemp bio composite strengthening on masonry barrel vaults damage, *Construction and Building Materials*, 2023, Volume 367. doi: 10.1016/j.conbuildmat.2022.130100.

- Gioffrè, M.; Pepi, C. Environmental aging effects on mechanical behavior of an hemp based biocomposite material for structural strengthening, *Journal of Building Engineering*, 2024, Volume 91. doi: 10.1016/j.jobbe.2024.109571.
- Great Runs. Landscape Arch, Arches National Park, *Great Runs*, 2025. URL: <https://greatruns.com/national-parks-arches/> (accessed on 5 August 2025).
- Greco, P. F.; Pepi, C.; Gioffré, M. A novel biocomposite material for sustainable constructions: Metakaolin lime mortar and Spanish broom fibers, *Journal of Building Engineering*, 2024, Volume 83, pp. 108425. doi: 10.1016/j.jobbe.2023.108425.
- Grillanda, N.; Chiozzi, A.; Milani, G.; Tralli, A. Collapse Behavior of Masonry Domes under Seismic Loads: An Adaptive NURBS Kinematic Limit Analysis Approach, *Engineering Structures*, 2019, Volume 200, 109517. doi:10.1016/j.engstruct.2019.109517.
- Hejazi, M.; Sadeghi, R. Effect of brick arrangement on structural behaviour of Persian brick masonry barrel vaults by micro-modelling, *Engineering Failure Analysis*, 2022, Volume 134, Article 106033. doi: 10.1016/j.engfailanal.2022.106033.
- Heyman, J. *The masonry arch*, 1982; Publisher: E. Horwood; Chapter 2, Section Mechanisms of collapse; ISBN: 0853125007.
- Housner, G.W. The behavior of inverted pendulum structures during earthquakes, *Bulletin of the Seismological Society of America*, 1963, Volume 53, Issue 2, pp. 403-417. URL: <http://pubs.geoscienceworld.org/ssa/bssa/article-pdf/53/2/403/5348002/bssa0530020403.pdf> (accessed on 26 August 2025).
- Hudson, J.; Theodossopoulos, D. Gothic barrel vaults under differential settlement: The effects of boundary conditions and FRP on structural behaviour, *Key Engineering Materials*, 2017, Volume 747, pp. 496-503. doi: 10.4028/www.scientific.net/KEM.747.496.
- Huerta, S. Mechanics of Masonry Vaults: The Equilibrium Approach, in *Historical Constructions*, P. B. Lourenço and Roca, P. (Eds.), Guimarães, 2001, pp. 47-70. URL: <https://www.researchgate.net/publication/39424866> (accessed on 14 August 2025).
- Huerta, S. The analysis of masonry architecture: A historical approach, *Architectural Science Review*, 2008, Volume 51, Issue 4, pp. 297-328. doi:10.3763/asre.2008.5136
- ICOMOS. International Charter for the Conservation and Restoration of Monuments and Sites (The Venice Charter 1964), *2nd International Congress of Architects and Technicians of Historic Monuments*, Venice, 1964.

- ICOMOS/ISCARSAH. Guidelines for the Analysis, Conservation and Structural Restoration of Architectural Heritage, 2nd edition, 2024. Available online: <https://iscarsah.org/wp-content/uploads/2025/04/iscarsah-guidelines.-approved-september-2024-1.pdf> (accessed on 30 July 2025).
- Illampas, R.; Oliveira, D. V.; Lourenço, P. B. Design of Strain-Hardening Natural TRM Composites: Current Challenges and Future Research Paths, *Materials*, 2023, Volume 16, Issue 13, 4558. doi: 10.3390/ma16134558.
- Ishiyama, Y. Motions of rigid bodies and criteria for overturning by earthquake excitations, *Earthquake Engineering & Structural Dynamics*, 1982, Volume 10, Issue 5, pp. 635–650. doi: 10.1002/eqe.4290100502.
- Kallioras, S.; Graziotti, F.; Penna, A.; Magenes, G. Effects of Vertical Ground Motions on the Dynamic Response of URM Structures: Comparative Shake-Table Tests, *Earthquake Engineering and Structural Dynamics*, 2022, Volume 51, Issue 2, pp. 347–368. doi: 10.1002/eqe.3569.
- Kim, W.; Lee, J. H. An improved explicit time integration method for linear and nonlinear structural dynamics, *Computers & Structures*, 2018, Volume 206, pp. 42–53. doi: 10.1016/j.compstruc.2018.06.005.
- Kooharian, A. Limit Analysis of Voussoir (Segmental) and Concrete Arches, *ACI Journal Proceedings*, 1952, Volume 49 and Issue 12. doi:10.14359/11822.
- Krajewski, P.; Hojdys, Ł. Experimental Studies on Buried Barrel Vaults, *International Journal of Architectural Heritage*, 2015, Volume 9, Issue 7, pp. 834–843. doi: 10.1080/15583058.2013.860499.
- Krajewski, P.; Hojdys, Ł. Comparative Study of Granite and Expanded Clay Aggregate as Backfill Materials for Masonry Vaults, *Materials*, 2024, Volume 17, Issue 24, pp. 6277. doi: 10.3390/ma17246277.
- Lagomarsino, S. Seismic assessment of rocking masonry structures, *Bulletin of Earthquake Engineering*, 2015, Volume 13, Issue 1, pp. 97–128. doi: 10.1007/s10518-014-9609-x.
- Lancaster, L. C. Innovative Vaulting in the Architecture of the Roman Empire: 1st to 4th Centuries CE. Chapter 3, In *Innovative Vaulting in the Architecture of the Roman Empire*; Lancaster, L. C., Ed.; Cambridge University Press: Cambridge, UK, 2015. doi: 10.1017/CBO97811074444935.
- Lemos, J. Discrete element modelling of the seismic behaviour of stone masonry arches, *Computer Methods in Structural Masonry*, pp. 220–227, 1998.

José V. Lemos Discrete element modeling of masonry structures, *International Journal of Architectural Heritage*, 2007, Volume 1, Issue 2, pp. 190-213. doi:10.1080/15583050601176868. Publisher: Taylor and Francis Inc.

Liberatore, D.; Doglioni, C.; AlShawa, O.; Atzori, S.; Sorrentino, L. Effects of coseismic ground vertical motion on masonry constructions damage during the 2016 Amatrice-Norcia (Central Italy) earthquakes, *Soil Dynamics and Earthquake Engineering*, 2019, Volume 120, pp. 423–435. doi: 10.1016/j.soildyn.2019.02.015.

Lo Iacono, F.; Navarra, G.; Oliva, M.; Cascone, D. Experimental investigation of the dynamic performances of the LEDA shaking tables system. In *AIMETA 2017 - Proceedings of the 23rd Conference of the Italian Association of Theoretical and Applied Mechanics*, Italy, 2017; pp. 897–915.

Majumder, M.; Gangopadhyay, T. K.; Chakraborty, A. K.; Dasgupta, K.; Bhattacharya, D. K. Fibre Bragg gratings in structural health monitoring—Present status and applications, *Sensors and Actuators, A: Physical*, 2008, Volume 147, Issue 1, pp. 150-164. doi: 10.1016/j.sna.2008.04.008.

Majumder, A.; Stochino, F.; Valdes, M.; Concu, G.; Pepe, M.; Martinelli, E. Sustainable Masonry Retrofitting and Upgrading Techniques: A Review, *Fibers*, 2025, Volume 13, Issue 6, pp. 68. doi: 10.3390/fib13060068.

Malomo, D.; Mehrotra, A.; DeJong, M. J. Distinct element modeling of the dynamic response of a rocking podium tested on a shake table, *Earthquake Engineering and Structural Dynamics*, 2021, Volume 50, Issue 5, pp. 1469-1475. doi: 10.1002/eqe.3404.

Marini, A.; Giardina, G.; Giuriani, E. Seismic behavior of barrel vault systems, in *Structural Analysis of Historic Construction: Preserving Safety and Significance. Volume 1. Proceedings of the Sixth International Conference on Structural Analysis of Historic Construction*, Bath, United Kingdom, 2–4 July 2008; CRC Press: Boca Raton, FL, USA, 2008; pp. 413–421. doi:10.1201/9781439828229.ch46.

Marmo, F.; Rosati, L. Reformulation and Extension of the Thrust Network Analysis, *Computers and Structures*, 2017, Volume 182, pp. 104–118. doi:10.1016/j.compstruc.2016.11.016.

Marthys, H.; Noland, L., Eds. *Proceedings of an International Seminar on Evaluation, Strengthening, and Retrofitting Masonry Buildings*, Colorado, 1989; The Masonry Society (TMS).

Note: Although the original source is not directly accessible, it is referenced in several studies, including Babatunde [2017] and Wang et al. [2018] .

- Martínez, G.; Núñez, A.; Jara, J. M.; Olmos, B. A. Expected seismic performance of the San Agustín church at Morelia, Mexico, *Journal of Building Pathology and Rehabilitation*, 2025, Volume 10, Issue 1, Article 4. doi: 10.1007/s41024-024-00488-2.
- Mendes, N. Seismic assessment of ancient masonry buildings: shaking table tests and numerical analysis. *Ph.D. Thesis*, University of Minho (Portugal), 2012.
- Mendes, N.; Lourenço, P. B.; Campos-Costa, A. Shaking table testing of an existing masonry building: assessment and improvement of the seismic performance, *Earthquake Engineering and Structural Dynamics*, 2014, Volume 43, Issue 2, pp. 247-266. doi: 10.1002/eqe.2342.
- Mendes, N.; Lourenço, P. B. Sensitivity analysis of the seismic performance of existing masonry buildings, *Engineering Structures*, 2014, Volume 80, pp. 137–146. doi: 10.1016/j.engstruct.2014.09.005.
- Menna, C.; Asprone, D.; Durante, M.; Zinno, A.; Balsamo, A.; Prota, A. Structural behaviour of masonry panels strengthened with an innovative hemp fibre composite grid, *Construction and Building Materials*, 2015, Volume 100, pp. 111–121. doi: 10.1016/j.conbuildmat.2015.09.051.
- Milani, G. 3D upper bound limit analysis of multi-leaf masonry walls, *International Journal of Mechanical Sciences*, 2008, Volume 50, Issue 4, pp. 817–836. doi:10.1016/j.ijmecsci.2007.10.002.
- Milani, G.; Rossi, M.; Calderini, C.; Lagomarsino, S. Tilting plane tests on a small-scale masonry cross vault: Experimental results and numerical simulations through a heterogeneous approach, *Engineering Structures*, 2016, Volume 123, pp. 300–312. doi: 10.1016/j.engstruct.2016.05.017.
- Modak, S.; Sotelin, E.D. Generalising direct time-stepping algorithms for transient problems in solid mechanics, in *Developments in Scientific Computing Techniques and Applications*, Ed. Topping, B.H.V., Civil-Comp Press, Edinburgh, Scotland, 2000, pp. 161–171.
- O'Dwyer, D. Funicular Analysis of Masonry Vaults, *Computers & Structures*, 1999, Volume 73, Issues 1–5, pp. 187–197. doi:10.1016/S0045-7949(98)00279-X.
- Oliveira, D.V.; Babilio, I.; Lourenço, P.B. Experimental Behavior of FRP Strengthened Masonry Arches, *Journal of Composites for Construction*, 2010, Volume 14, Issue 3, pp. 312-322. doi: 10.1061/(asce)cc.1943-5614.0000086.

- Oliveira, D. V.; Lourenço, P. B.; Lemos, C. Geometric Issues and Ultimate Load Capacity of Masonry Arch Bridges from the Northwest Iberian Peninsula, *Engineering Structures*, 2010, Volume 32, Issue 12, pp. 3955–3965. doi: 10.1016/j.engstruct.2010.09.006.
- Oppenheim, I. J. The masonry arch as a four-link mechanism under base motion, *Earthquake Engineering and Structural Dynamics*, 1992, Volume 21, pp. 1017.
- Orduña, A.; Lourenço, P. B. Cap model for limit analysis and strengthening of masonry structures, *Journal of Structural Engineering*, 2003, Volume 129, Issue 10, pp. 1367–1375. doi: 10.1061/(ASCE)0733-9445(2003)129:10(1367).
- Orduña, A.; Lourenço, P. B. Three-dimensional limit analysis of rigid blocks assemblages. Part I: Torsion failure on frictional interfaces and limit analysis formulation, *International Journal of Solids and Structures*, 2005, Volume 42, Issues 18–19, pp. 5140–5160. doi: 10.1016/j.ijsolstr.2005.02.010.
- Orduña, A.; Lourenço, P. B. Three-dimensional limit analysis of rigid blocks assemblages. Part II: Load-path following solution procedure and validation, *International Journal of Solids and Structures*, 2005, Volume 42, Issues 18–19, pp. 5161–5180. doi: 10.1016/j.ijsolstr.2005.02.011.
- Pagnoni, T. Seismic analysis of masonry and block structures with the discrete element method, *Proc. 10th European Conference on Earthquake Engineering*, 1994, Volume 3, pp. 1674–1694.
- Pantò, B.; Malena, M.; de Felice, G. A Macro-modelling Approach for Arches Strengthened with Externally Bonded Inorganic Matrix Composites, *Structures*, 2021, Volume 33, pp. 4299–4312. doi:10.1016/j.istruc.2021.07.015.
- Pantò, B.; Ortega, J.; Grosman, S.; Oliveira, D. V.; Lourenço, P. B.; Macorini, L.; Izzuddin, B. A. Advanced calibration of a 3D masonry arch bridge model using non-destructive testing and numerical optimisation, *Construction and Building Materials*, 2024, Volume 438. doi: 10.1016/j.conbuildmat.2024.137131.
- Papazoglou, A. J.; Elnashai, A. S. Analytical and field evidence of the damaging effect of vertical earthquake ground motion, *Earthquake Engineering and Structural Dynamics*, 1996, Volume 25, Issue 10, pp. 1109–1137. doi: 10.1002/(SICI)1096-9845(199610)25:10<1109::AID-EQE604>3.0.CO;2-0.
- Parisi, F.; Lignola, G. P.; Augenti, N.; Prota, A.; Manfredi, G. Rocking response assessment of in-plane laterally-loaded masonry walls with openings, *Engineering Structures*, 2013, Volume 56, pp. 1234–1248. doi: 10.1016/j.engstruct.2013.06.041.

- Pelà, L.; Aprile, A.; Benedetti, A. Seismic Assessment of Masonry Arch Bridges, *Engineering Structures*, 2009, Volume 31, Issue 8, pp. 1777–1788. doi:10.1016/j.engstruct.2009.02.012.
- Peña, F.; Prieto, F.; Lourenço, P. B.; Campos Costa, A.; Lemos, J. V. On the dynamics of rocking motion of single rigid-block structures, *Earthquake Engineering and Structural Dynamics*, 2007, Volume 36, Issue 15, pp. 2383–2399. doi: 10.1002/eqe.739.
- Pepi, C. Suitability of Dynamic Identification for Damage Detection in the Light of Uncertainties on a Cable-Stayed Footbridge. *Ph.D. Thesis*, Technische Universität Braunschweig (Germany) and University of Florence (Italy), 2019.
- Pepi, C.; Cavalagli, N.; Gusella, V.; Giofrè, M. Damage detection via modal analysis of masonry structures using shaking table tests, *Earthquake Engineering and Structural Dynamics*, 2021, Volume 50, Issue 8, pp. 2077–2097. doi: 10.1002/eqe.3431.
- Pepi, C.; Pipistrelli, M. E.; Giofrè, M. Random modeling of hemp fibers mechanical properties, *Composites Part A: Applied Science and Manufacturing*, 2024, Volume 183. doi: 10.1016/j.compositesa.2024.108203.
- Petracca, M.; Pelà, L.; Rossi, R.; Zaghi, S.; Camata, G.; Spacone, E. Micro-scale continuous and discrete numerical models for non-linear analysis of masonry shear walls, *Construction and Building Materials*, 2017, Volume 149, pp. 296–314. doi: 10.1016/j.conbuildmat.2017.05.130.
- Pipistrelli, M. E.; Pepi, C.; Greco, P. F.; Tomassoli, L.; Vinti, F.; Quaglia, G.; Latterini, L.; Giofrè, M. Evaluating the impact of coatings and chemical treatments on the tensile behavior of natural fibers, *Construction and Building Materials*, 2025, Volume 485. doi: 10.1016/j.conbuildmat.2025.141724.
- Plavsic, A.; Pantó, B.; Chisari, C.; Macorini, L.; Izzuddin, B. A.; Boem, I.; Gattesco, N. non-linear simulation of masonry vaults under earthquake loading, *COMPdyn Proceedings*, 2021. doi: 10.7712/120121.8511.19581.
- Portioli, F.; Casapulla, C.; Gilbert, M.; Cascini, L. Limit analysis of 3D masonry block structures with non-associative frictional joints using cone programming, *Computers and Structures*, 2014, Volume 143, pp. 108–121. doi: 10.1016/j.compstruc.2014.07.010.
- Prager, W. An introduction to plasticity, *Reading, Mass: Addison Wesley Publication*, 1959.
- Pulatsu, B.; Bretas, E.M.; Lourenço, P.B. Discrete Element Modeling of Masonry Structures: Validation and Application, *Earthquake and Structures*, 2016, Volume 11, Issue 4, pp. 563–582. doi:10.12989/eas.2016.11.4.563.

- Pulatsu, B.; Erdogmus, E.; Bretas, E. M. Parametric study on masonry arches using 2D discrete-element modeling, *Journal of Architectural Engineering*, 2018, Volume 24, Issue 2, 04018012. doi:10.1061/(ASCE)AE.1943-5568.0000305.
- Pulatsu, B.; Erdogmus, E.; Lourenço, P.B.; Lemos, J.V.; Tuncay, K. Simulation of the In-Plane Structural Behavior of Unreinforced Masonry Walls and Buildings Using DEM, *Structures*, 2020, Volume 27, pp. 2274-2287. doi:10.1016/j.istruc.2020.08.026.
- Pulatsu, B.; Erdogmus, E.; Lourenço, P. B.; Lemos, J. V.; Hazzard, J. Discontinuum analysis of the fracture mechanism in masonry prisms and wallettes via discrete element method, *Meccanica*, 2020, Volume 55, Issue 3, pp. 505-523. doi: 10.1007/s11012-020-01133-1.
- Pugi, F.; Mariani, M.; Francioso, A. Vertical Component of the Seismic Action: Amplified Vulnerability of Existing Masonry Buildings, In *Proceedings of the 7th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering*, 2019. URL: <https://www.researchgate.net/publication/345309698> (accessed on 11 February 2025).
- Ramaglia, G.; Lignola, G. P.; Prota, A. Collapse analysis of slender masonry barrel vaults, *Engineering Structures*, 2016, Volume 117, pp. 86-100. doi: 10.1016/j.engstruct.2016.03.016.
- Ramaglia, G.; Lignola, G. P.; Balsamo, A.; Prota, A.; Manfredi, G. Seismic Strengthening of Masonry Vaults with Abutments Using Textile-Reinforced Mortar, *Journal of Composites for Construction*, 2017, Volume 21, Issue 2. doi: 10.1061/(asce)cc.1943-5614.0000733.
- Reboul, N.; Saidi, M.; Mualla, S.; Homoro, O.; Amziane, S. Bond behaviour of Fibre Reinforced Polymers applied on masonry substrate: Analysis based on acoustic emission, digital image correlation and analytical modelling, *Construction and Building Materials*, 2023, Volume 403. doi: 10.1016/j.conbuildmat.2023.132921.
- Reccia, E.; Milani, G.; Cecchi, A.; Tralli, A. Full 3D homogenization approach to investigate the behavior of masonry arch bridges: The Venice trans-lagoon railway bridge, *Construction and Building Materials*, 2014, Volume 66, pp. 567-586. doi:10.1016/j.conbuildmat.2014.05.096.
- Roca, P.; Cervera, M.; Gariup, G.; Pelà, L. Structural Analysis of Masonry Historical Constructions. Classical and Advanced Approaches, *Archives of Computational Methods in Engineering*, 2010, Volume 17, Issue 3, pp. 299-325. doi: 10.1007/s11831-010-9046-1.

- Roca, P.; Lourenço, P. B.; Gaetani, A. *Historic Construction and Conservation*; Routledge: London, UK, 2019; doi: 10.1201/9780429052767.
- Roselli, F.; Alforno, M.; Manuello Bertetto, A.; Venuti, F. Experimental investigation of the effect of brick pattern on the structural response of masonry arches and barrel vaults, *Construction and Building Materials*, 2023, Volume 368, 130434. doi: 10.1016/j.conbuildmat.2023.130434.
- Rossi, M.; Calderini, C.; Lagomarsino, S. Experimental testing of the seismic in-plane displacement capacity of masonry cross vaults through a scale model, *Bulletin of Earthquake Engineering*, 2016, Volume 14, Issue 1, pp. 261–281. doi: 10.1007/s10518-015-9815-1.
- Rupakhety, R.; Sigbjörnsson, R. A note on the L'Aquila earthquake of 6 April 2009: Permanent ground displacements obtained from strong-motion accelerograms, *Soil Dynamics and Earthquake Engineering*, 2010, Volume 30, Issue 4, pp. 215–220. doi: 10.1016/j.soildyn.2009.10.008.
- Rytter, A. Vibrational Based Inspection of Civil Engineering Structures, *Ph.D. Thesis*, Building Technology and Structural Engineering, University of Aalborg, Denmark, April 1993.
- Sadeghi, N. H.; Oliveira, D. V.; Silva, R. A.; Mendes, N.; Correia, M.; Azizi-Bondarabadi, H. Experimental characterization of adobe vaults strengthened with a TRM-based compatible composite, *Construction and Building Materials*, 2021, Volume 271. doi: 10.1016/j.conbuildmat.2020.121568.
- Sarhosis, V.; Forgács, T.; Lemos, J. V. A discrete approach for modelling backfill material in masonry arch bridges, *Computers and Structures*, 2019, Volume 224. doi: 10.1016/j.compstruc.2019.106108.
- Scacco, J.; Milani, G.; Lourenço, P. B. A micro-modeling approach for the prediction of TRM bond performance on curved masonry substrates, *Composite Structures*, 2021, Volume 256. doi: 10.1016/j.compstruct.2020.113065.
- Schiantella, M.; Gilbert, M.; Smith, C. C.; He, L.; Cluni, F. Limit Analysis of 2D Non-Periodic Masonry Walls via Discontinuity Layout Optimization, *International Journal of Architectural Heritage*, 2024. doi: 10.1080/15583058.2024.2437633.
- Schiavoni, M.; Giordano, E.; Roscini, F.; Clementi, F. Numerical modeling of a majestic masonry structure: A comparison of advanced techniques, *Engineering Failure Analysis*, 2023, Volume 149. doi:10.1016/j.engfailanal.2023.107293.
- Sorrentino, L.; Cattari, S.; da Porto, F.; Magenes, G.; Penna, A. Seismic behaviour of ordinary masonry buildings during the 2016 central Italy earthquakes,

Bulletin of Earthquake Engineering, 2019, Volume 17, Issue 10, pp. 5583–5607. doi: 10.1007/s10518-018-0370-4.

Standage, K. Differences between a corbel arch and a true arch, *Kevin Standage Photography*, 2025.

URL: <https://kevinstandagephotography.wordpress.com/2025/01/09/the-forgotten-tomb-of-ghiyas-ud-din-balban/> (accessed on 5 August 2025).

Taranu, N.; Oprisan, G.; Budescu, M.; et al. Improving structural response of masonry vaults strengthened with polymeric textile composite strips, in *Proceedings of the 3rd WSEAS International Conference on Engineering Mechanics, Structures, Engineering Geology*, 2010.

Tarhan, İ. H.; Uysal, H. Topology Optimization of the FRP for strengthening of masonry barrel vaults, *Engineering Failure Analysis*, 2023, Volume 151. doi: 10.1016/j.engfailanal.2023.107390.

Tecchio, G.; Barbisan, S.; Saler, E.; da Porto, F. Effects of Aging and Deterioration on the Seismic Fragility of Masonry Arch Bridges, in *Proceedings of the 8th International Conference on Computational Methods in Structural Dynamics and Earthquake Engineering (COMPDYN 2023)*, National Technical University of Athens, 2023. doi:10.7712/120123.10515.20969.

Tralli, A.; Alessandri, C.; Milani, G. Computational Methods for Masonry Vaults: A Review of Recent Results, *The Open Civil Engineering Journal*, 2014, Volume 8, pp. 272–287.

Travasarou, T.; Bray, J. D.; Abrahamson, N. A. Empirical attenuation relationship for Arias Intensity, *Earthquake Engineering and Structural Dynamics*, 2003, Volume 32, Issue 7, pp. 1133–1155. doi: 10.1002/eqe.270.

Trcala, M.; Němec, I.; Gálová, A. On the non-linear transient analysis of structures, in *Earthquakes – Recent Advances, New Perspectives and Applications*, Ed. Salazar, W., IntechOpen, Rijeka, 2022, Chapter 9. doi: 10.5772/intechopen.108446.

Valluzzi, M. R.; Valdemarca, M.; Modena, C. Behavior of Brick Masonry Vaults Strengthened by FRP Laminates, *Journal of Composites for Construction*, 2001, Issue 163. doi: 10.1061/(ASCE)1090-0268(2001)5:3(163).

Vassiliou, M. F.; Broccardo, M.; Cengiz, C.; Dietz, M.; Dihoru, L.; Gunay, S.; Mosalam, K. M.; Mylonakis, G.; Sextos, A.; Stojadinovic, B. Shake table testing of a rocking podium: Results of a blind prediction contest, *Earthquake Engineering & Structural Dynamics*, 2021, Volume 50, Issue 4, pp. 1043–1062. doi: 10.1002/eqe.3386.

- Vecchio, D.; Vlachakis, G.; Menon, A.; Lourenço, P.B. Seismic vulnerability assessment against rocking and sliding failure using nonlinear dynamic analysis: Application to the temples of Bagan, Myanmar, *Structures*, 2025, Volume 74. doi: 10.1016/j.istruc.2025.108584.
- Wang, C.; Sarhosis, V.; Nikitas, N. Strengthening/Retrofitting Techniques on Unreinforced Masonry Structure/Element Subjected to Seismic Loads: A Literature Review, *The Open Construction and Building Technology Journal*, 2018, Volume 12, pp. 251–268. doi: 10.2174/1874836801812010251.
- Williams, M. S.; Albuerne, A.; Lawson, V.; Yip, F. Model Scale Shaking Table Tests on Masonry Barrel and Cross Vaults, *Proceedings of the 15th World Conference on Earthquake Engineering*, Lisbon, Portugal, 2012. Available online: <https://www.researchgate.net/publication/328837969>.
- Witzany, J.; Pirner, M.; Zigler, R.; Urushadze, S. Experimental research into the response of segmental barrel vaults to repetitive static and dynamic loads, *Engineering Structures*, 2020, Volume 208. doi: 10.1016/j.engstruct.2020.110342.
- Yim, C.-S.; Chopra, A.K.; Penzien, J. Rocking response of rigid blocks to earthquakes, *Earthquake Engineering & Structural Dynamics*, 1980, Volume 8, Issue 6, pp. 565–587. doi: 10.1002/eqe.4290080606.
- Yuan, Y.; Milani, G. Closed-form model for curved brittle substrates reinforced with FRP strips, *Composite Structures*, 2023, Volume 304. doi: 10.1016/j.compstruct.2022.116443.
- Zampieri, P.; Simoncello, N.; Tetougueni, C.D.; Pellegrino, C. A review of methods for strengthening of masonry arches with composite materials, *Engineering Structures*, 2018, Volume 171, pp. 154–169. doi: 10.1016/j.engstruct.2018.05.070.
- Zampieri, P.; Amoroso, M.; Pellegrino, C. The masonry buttressed arch on spreading support, *Structures*, 2019, Volume 20, pp. 226–236. doi: 10.1016/j.istruc.2019.03.008.
- Zhong, C.; Christopoulos, C. Finite element analysis of the seismic shake-table response of a rocking podium structure, *Earthquake Engineering & Structural Dynamics*, 2021, Volume 50, Issue 4, pp. 1223–1230. doi: 10.1002/eqe.3397.
- Zizi, M.; Chisari, C.; De Matteis, G. Effect of the backfill material in the seismic response of multi-span masonry arch bridges under seismic loading, *Procedia Structural Integrity*, 2022, Volume 44, pp. 673–680. doi: 10.1016/j.prostr.2023.01.088.

Bernardo Zuccarello; Carmelo Militello; Francesco Bongiorno. Environmental aging effects on high-performance biocomposites reinforced by sisal fibers, *Polymer Degradation and Stability*, 2023, Volume 211. doi: 10.1016/j.polymdegradstab.2023.110319.