

# Polyakov loop and the color–flavor locked phase of quantum chromodynamics

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## Abstract

We consider the Polyakov–Nambu–Jona-Lasinio model with three massless quarks at high density and moderate temperature in the superconductive color–flavor locking phase. We compute the critical temperature  $T_c$  as a function of the baryonic chemical potential for the phase transition from the superconductive state to the normal phase. We find that  $T_c$  is higher by a factor 1.5–2 in comparison to the model containing no Polyakov loop. We also compute the specific heat  $C_V$  near the second-order phase transition and we show that the inclusion of the Polyakov loop does not change the value of the critical exponent.

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## 1. Introduction

At small hadronic densities and sufficiently high temperature chiral symmetry is restored and the nature of the chiral phase transition can be investigated by various effective approaches. One of the most popular is the Nambu–Jona-Lasinio (NJL) model [1], describing the chiral transition in terms of the  $\langle \bar{q}q \rangle$  order parameter. At high temperatures one also expects a deconfinement transition [2]. Its nature is rather clear in pure gauge theory, because, in absence of quarks, quantum chromodynamics at low temperature possesses a  $Z_3$  global symmetry, which is spontaneously broken at high temperature  $T$ . The order parameter for this phase transition is the Polyakov loop [3] whose expectation value vanishes in the disordered low temperature phase and is different from zero in the high  $T$  phase.

The Polyakov loop is an  $SU(3)_c$  matrix in color space given by ( $\beta = T^{-1}$ )

$$L(\mathbf{x}) = \mathcal{P} e^{-i \int_0^\beta dx_4 A_4(\mathbf{x}, x_4)}. \quad (1)$$

For uniform  $A_4$  one gets an order parameter that can be written as follows in the Polyakov gauge

$$\Phi = \frac{1}{3} \text{Tr} e^{i\beta(\phi_3 \lambda_3 + \phi_8 \lambda_8)}; \quad (2)$$

for  $T \rightarrow \infty$  one has  $\Phi = 1$  and in the confined phase  $\Phi = 0$ .

In presence of dynamical quarks there is no clear order parameter for the deconfinement transition because in this case the  $Z_3$  center of the  $SU(3)_c$  gauge group is not a good symmetry. Though one cannot properly speak of a phase transition in this case, the  $T$ -dependence of the Polyakov loop can nonetheless be studied by numerical simulations and one still observes its rise from low to high temperatures on the lattice.

An interesting and still debated [4] feature of these data is that chiral symmetry breaking and the decrease of the Polyakov loop occur at the same critical temperature [5]. It has been argued that a mixing between the effective models for chiral transition and the Polyakov loop dynamics might account for the

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approximate equality of these temperatures [6]. A step forward has been obtained in Refs. [7] and [8], where the NJL model is studied in presence of a uniform extended gauge field  $A_4$ . Its effect on dynamical quarks is obtained by identifying the parameters appearing in the Polyakov loop (2) with an imaginary quark chemical potential (for related studies see also [9]).

This modified NJL model (called Polyakov–NJL = PNJL) is characterized by a thermodynamical potential  $\Omega$  comprising two terms,  $\Omega_{\text{NJL}}$  and  $\mathcal{U}(T, \phi)$ .  $\Omega_{\text{NJL}}$  contains the NJL thermodynamic potential modified by the inclusion of the imaginary quark chemical potential; in the mean field approximation it therefore depends on the chiral order parameter and on the Polyakov loop  $\Phi$ .  $\mathcal{U}(T, \phi)$  depends only on  $\Phi$  and  $T$  and its parameters can be obtained by fitting pure gauge lattice QCD results. The PNJL model still stands on a conjectural basis. We do not analyze here its theoretical foundations. Nevertheless we do attempt to determine some of its possible physical implications.

In [10] the PNJL model has been extended to high baryonic densities for the case of two flavors ( $u, d$ ) by including a quark chemical potential  $\mu$ . At moderate  $\mu$  and small  $T$  a plausible model describing quark dynamics is the 2SC model [11,12] characterized by condensation in the diquark antisymmetric color channel and decoupling of the strange quark. The two flavor approximation can only be valid at high, but not very high, densities. At these densities  $u$  and  $d$  quark masses can play a role, but their effect is included by considering also condensation in the  $\bar{q}q$  channel.

The aim of the present Letter is to consider the case of higher densities, where all the three light quarks can form color superconductive pairs. The favored phase for sufficiently high density is the color–flavor locking (CFL) state [13], characterized by three massless quarks,  $qq$  condensation in spin 0, color and flavor antisymmetric state (for reviews see [14]). This result was obtained in an NJL model, where the gluon interaction is mimicked by a four fermion interaction and one works in the mean field approximation. The dominance of the CFL phase can also be proved in QCD by way of one-gluon exchange; however this result is valid only at extreme densities ( $\mu \sim 10^8$  GeV) [15]. These densities are much larger than those presumably existing in the core of compact stars, where color superconductivity might be found. For these latter densities perturbative QCD is of little or no help. On the other hand, the standard non-perturbative method, i.e., lattice QCD, is not applicable, as the quark determinant is complex at  $\mu \neq 0$  and Monte Carlo simulations are not directly usable. Therefore the four fermion interaction remains as the only practical way to study the CFL phase.

Though in this approximation gluon interactions are described by an effective four quark interactions, the Polyakov order parameter can nevertheless play a role, similarly to what happens for two flavors [10]. The study of this role is the aim of this Letter, where we present a preliminary study of the second-order phase transition around the critical line  $T_c(\mu)$ . We will consider only the case of massless quarks, even though finite mass effects could be included either as free parameters or by considering condensation in both the diquark and quark–antiquark channels [16]. The reason for this neglect is

that the inclusion of, say, the strange quark mass  $M_s$  considerably complicates the analysis, because for  $M_s \neq 0$  one should include electric and color chemical potentials to enforce electric and color neutrality. We will treat these effects, as well as the extension to the gapless CFL (gCFL) phase [17], in a future publication.

Since we neglect mass effects, the free energy depends only on the order parameter  $\Phi$  and the unique gap parameter  $\Delta$  (the role of the gap parameters due the symmetric color channels will be discussed below). Moreover we are interested only in the transition line between the CFL and the normal phase in the  $T$ – $\mu$  plane. Therefore we can use a Ginzburg–Landau (GL) expansion near the critical line. This approximation is discussed in Section 2. In Section 3 we verify that the transition is continuous and compute the critical temperature as a function of the quark chemical potential  $\mu$ . Our result is that the critical temperature  $T_c$  is higher by a factor 1.5–2 in comparison with the treatment of CFL within the original NJL approximation. We also evaluate the critical exponent  $\beta$  that fixes the relationship between the gap parameter and the temperature near the phase transition and we find that including the Polyakov loop does not change the classical value  $\beta = 1/2$ . The discontinuity in the specific heat at the second-order phase transition is also evaluated and a comparison of results obtained with and without Polyakov loop is performed. Finally, some concluding remarks are contained in Section 4.

## 2. Thermodynamics of the three flavor PNJL model

The model we study is described by the quark Lagrangian

$$\mathcal{L} = \bar{\psi}(iD_\mu\gamma^\mu + \mu\gamma_0)\psi + \mathcal{L}_\Delta. \quad (3)$$

In the above equation we have introduced the coupling of the quarks to a background temporal gauge field  $A_\mu = g\delta_{\mu 0}A_\mu^a T_a$  coupled to the quarks via the covariant derivative  $D_\mu = \partial_\mu - iA_\mu$ ;  $\mu$  is the quark chemical potential. The term  $\mathcal{L}_\Delta$  is responsible for color condensation. It can be obtained in the mean field approximation from a four fermion interaction term. In the CFL model one has

$$\mathcal{L}_\Delta = -\frac{\Delta}{2}(\psi_{\alpha i}^\dagger \gamma_5 \epsilon^{\alpha\beta I} \epsilon_{ij I} C \psi_{\beta j}^* + \text{h.c.}) - \frac{3\Delta^2}{G}. \quad (4)$$

Eq. (4) describes the fact that in the ground state one has a non-vanishing expectation value of the di-quark field operator

$$\langle \psi_{\alpha i} \psi^{\beta j} \rangle \propto \Delta \epsilon_{\alpha\beta I} \epsilon_{ij I} \neq 0. \quad (5)$$

The constant  $G$  in Eq. (4) is the NJL four fermion coupling constant. In Eq. (5) we have neglected the color symmetric channel contribution, as one can prove that it becomes less and less important when one approaches the second-order phase transition.

Once the quark Lagrangian is specified, the mean field free energy of the CFL quark matter is easily obtained by integration over the fermion fields in the generating functional, namely

$$\Omega = \mathcal{U}(T, \phi) + \frac{3\Delta^2}{G} - \frac{T}{2} \text{Tr} \sum_n \int \frac{d\mathbf{p}}{(2\pi)^3} \log \left( \frac{S^{-1}(i\omega_n, \mathbf{p})}{T} \right), \quad (6)$$

where  $\omega_n = \pi T(2n + 1)$  are the fermion Matsubara frequencies, and  $S^{-1}$  is the inverse fermion propagator in the mean field approximation, whose explicit form can be found in Ref. [17].  $S^{-1}$  is in principle a  $72 \times 72$  matrix in color, flavor, spin and Nambu–Gorkov indices. In the high density limit the effect of the antiparticles can be neglected; moreover, one can split the left-handed and the right-handed quark contributions to the free energy, since the quarks are massless and the condensation does not mix quarks with opposite chirality. Thus  $S^{-1}$  is reduced to an  $18 \times 18$  matrix. It can be rearranged to a block diagonal form, with a  $6 \times 6$  matrix describing the propagation of  $u_r, d_g, s_b$  quarks, and three  $4 \times 4$  matrices describing the propagation of  $d_r, u_g,$  and  $s_r, u_b,$  and  $s_g, d_b$  quarks. This allows a straightforward extraction of the quasiparticle dispersion laws, much in the same way as in the analogous evaluation contained in [18], the difference being that there  $\Delta_1 = 0, \Delta_2 = \Delta_3$  and here  $\Delta_1 = \Delta_2 = \Delta_3 = \Delta$ .

In Eq. (6) we have introduced the part of the thermodynamic potential  $\mathcal{U}(T, \phi)$  which describes the dynamics of the Polyakov loops in absence of dynamical quarks. In principle various forms can be used [7,10]; for definiteness we adopt the form proposed in [10]

$$\mathcal{U}(T, \phi) = T^4 \left\{ -\frac{a(T)}{2} \Phi^* \Phi + b(T) \ln[1 - 6\Phi^* \Phi + 4(\Phi^3 + \Phi^{*3}) - 3(\Phi^* \Phi)^2] \right\}, \quad (7)$$

where

$$a(T) = a_0 + a_1 \left( \frac{T_0}{T} \right) + a_2 \left( \frac{T_0}{T} \right)^2, \quad (8)$$

$$b(T) = b_3 \left( \frac{T_0}{T} \right)^3.$$

The Polyakov loop  $\Phi$  can be expressed in terms of one parameter  $\phi \equiv \phi_3$ , as the other parameter  $\phi_8$  can always be absorbed by a redefinition of  $\phi_3$ . It is given by

$$\Phi = \Phi^* = \frac{1 + 2 \cos(\beta\phi)}{3}. \quad (9)$$

Numerical values of the coefficients have been fitted in [10] using lattice data [19]:

$$a_0 = 3.51, \quad a_1 = -2.47, \quad a_2 = 15.2, \quad (10)$$

$$b_3 = -1.75,$$

together with the deconfinement temperature  $T_0 = 270$  MeV. The use of a definite form for  $\mathcal{U}(T, \phi)$  is not a limit of our computation because other functional dependences produce similar results as they are derived from the same lattice data set. The NJL coupling  $G$  should in principle depend on  $\Phi$  because the four fermion coupling is induced by gluon dynamics. However following [6] we will neglect this effect. In final results we will trade  $G$  for the value  $\Delta_0$  of the CFL gap at  $T = 0$  using the weak coupling formula [20]

$$\frac{3}{G} = \frac{6\mu^2}{\pi^2} \ln \frac{2\delta}{2^{1/3}\Delta_0}. \quad (11)$$

In the above relation  $\delta$  is an ultraviolet cutoff, introduced to ensure ultraviolet convergence of the loop integrals. By means of Eq. (11) the coupling strength is represented by the parameter  $\Delta_0$ .

Performing the summation over the Matsubara frequencies in Eq. (6) one gets

$$\Omega = \mathcal{U}(T, \phi) + \frac{3\Delta^2}{G} - 2 \sum_{i=1}^9 \int \frac{d^3p}{(2\pi)^3} (T \ln|1 + e^{-\beta\epsilon_j}| + \Re(\epsilon_j - p + \mu) + T \ln|1 + e^{-\beta\tilde{\epsilon}_j}| + \Re(\tilde{\epsilon}_j - p - \mu)). \quad (12)$$

In Eq. (12)  $\epsilon_j$  are the energies of quasiparticles and  $\tilde{\epsilon}_j$  are obtained from  $\epsilon_j$  by the substitution  $\xi = p - \mu \rightarrow p + \mu$ . The terms with  $\tilde{\epsilon}_j$  correspond to antiparticles. They are put here for completeness but omitted in numerical evaluations. The dispersion laws for the nine quasiparticles can be derived by the standard methods. Since the resulting expressions are cumbersome and our study is limited to the critical line we present here their expression only for small values of the gap parameters, i.e., in the GL approximation. One obtains

$$\epsilon_1 = \epsilon_2^* = \left( \xi + i\phi + \frac{\Delta^2}{2\xi} \frac{8\xi^2 + \phi^2}{4\xi^2 + \phi^2} - i \frac{\Delta^2 \phi}{4\xi^2 + \phi^2} \right),$$

$$\epsilon_3 = \xi \left( 1 + \frac{4\Delta^2}{4\xi^2 + \phi^2} \right),$$

$$\epsilon_4 = \epsilon_5^* = \xi + i\phi + \frac{\Delta^2}{2\xi},$$

$$\epsilon_6 = \epsilon_8^* = \xi + i\phi + \frac{\Delta^2}{4\xi^2 + \phi^2} (2\xi - i\phi),$$

$$\epsilon_7 = \epsilon_9^* = \xi + \frac{\Delta^2}{4\xi^2 + \phi^2} (2\xi - i\phi). \quad (13)$$

We have also computed the coefficients of the  $\mathcal{O}(\Delta^4)$  term but we do not report them here (they are needed to control that the phase transition is continuous at  $T_c$  and to compute the gap, see below).

The gap parameter  $\Delta$  and the background gauge field  $\phi$  at a fixed temperature and chemical potential are obtained solving the equations

$$\frac{\partial \Omega}{\partial \phi} = 0, \quad (14)$$

$$\frac{\partial \Omega}{\partial \Delta} = 0. \quad (15)$$

Near the critical temperature  $T_c$  one can expand  $\Omega$  in Eq. (12) as follows

$$\Omega(\Delta, \phi) - \Omega(0, \phi) \sim \frac{\alpha}{2} \Delta^2 + \frac{\beta}{4} \Delta^4. \quad (16)$$

The critical temperature  $T_c$  is obtained at a fixed  $\mu$  as in the usual BCS theory by solving the equation  $\alpha(T_c) = 0$ , with  $\alpha$  given by

$$\alpha = \frac{6}{G} \left[ 1 + GT \frac{2\mu^2}{3\pi^2} \right]$$

$$\begin{aligned} & \times \sum_n \int_{-\delta}^{\delta} d\xi \frac{[3 \cdot (l_0^2 - \xi^2)^2 + \phi^2 \cdot (l_0^2 + 3\xi^2)]}{(l_0^2 - \xi^2)[(l_0 + \xi)^2 + \phi^2][(l_0 - \xi)^2 + \phi^2]} \Big|_{l_0=i\omega_n} \\ & = \frac{12\mu^2}{\pi^2} \left( \ln \frac{2\delta}{\partial 2^{1/3} \Delta_0} + \frac{1}{3} \int_{-\delta}^{+\delta} d\xi f(\xi, \phi) \right) \end{aligned} \quad (17)$$

and  $f$  defined by

$$\begin{aligned} f(\xi, \phi) = & -\frac{2\xi}{4\xi^2 + \phi^2} \tanh \frac{\beta\xi}{2} \\ & - 2\Re \frac{4\xi - i\phi}{4\xi(2\xi - i\phi)} \tanh \frac{\beta(\xi - i\phi)}{2}. \end{aligned} \quad (18)$$

This expression for  $\alpha$  is identical to the result obtained by (12) using the dispersion laws (13) up to  $\Delta^2$ . On the other hand, the coefficient  $\beta$  is given by

$$\begin{aligned} \beta = & T \frac{\mu^2}{2\pi^2} \\ & \times \sum_n \int_{-\delta}^{\delta} d\xi \frac{8\Delta^4 \cdot \mathcal{F}(l_0, \xi, \Phi)}{\{(l_0^2 - \xi^2)[(l_0 + \xi)^2 + \phi^2][(l_0 - \xi)^2 + \phi^2]\}^2} \Big|_{l_0=i\omega_n}, \end{aligned} \quad (19)$$

where

$$\begin{aligned} \mathcal{F}(l_0, \xi, \Phi) = & 6l_0^8 - l_0^6 \cdot (24\xi^2 + 5\phi^2) \\ & + l_0^4 \cdot (36\xi^4 + 21\xi^2\phi^2 - 4\phi^4) \\ & - l_0^2 \cdot (24\xi^6 + 26\xi^4\phi^2 - 24\xi^2\phi^4 + \phi^6) \\ & + \xi^2 \cdot (6\xi^2 - \phi^2) \cdot (\xi^2 + \phi^2)^2. \end{aligned} \quad (20)$$

The summation over Matsubara frequencies in the expression of  $\beta$  can be performed analytically, but the final expression is involved and we omit it for simplicity.

### 3. Numerical results

To get the critical temperature  $T_c$  we solve the equation  $\alpha(T_c) = 0$  with  $\alpha$  given by Eq. (17) and  $\phi$  obtained by Eq. (14). We have checked that the phase transition is of the second order since  $\beta(T_c) > 0$ . It is well known that in the case  $\phi = 0$  one has in the CFL phase  $T_c/\Delta_0 \simeq 0.71$ , see for example [21]. On the other hand, in the 2SC phase one has  $T_c/\Delta_0 \simeq 0.57$  as in ordinary BCS superconductors. This difference is related to the fact that in the CFL model one has eight gapped modes with gap  $\Delta$  and one mode with gap  $2\Delta$ .

The result of the numerical evaluation of  $T_c$  is shown in Fig. 1, where we plot the ratio  $T_c/\Delta_0$  with (solid line) and without (dashed line) Polyakov loop, at the reference value  $\mu = 500$  MeV. We notice that introducing self-consistently the parameter  $\phi$  implies a significant increase of the critical temperature. This effect has been noticed also in the two flavor model [10].

Next we turn to the behavior of the gap parameter  $\Delta$  for temperature close to  $T_c$ . We find

$$\frac{\Delta(T)}{T_c} = k(\Delta_0) \left( 1 - \frac{T}{T_c} \right)^\beta, \quad T \rightarrow T_c^-, \quad (21)$$

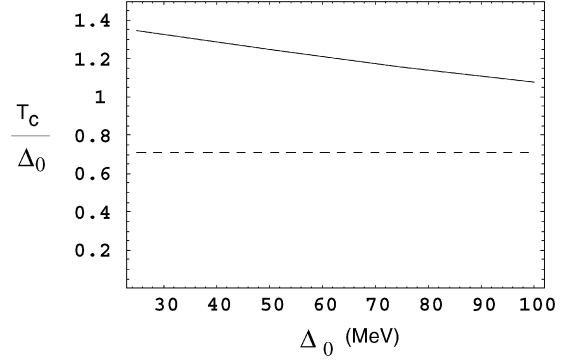


Fig. 1. Ratio  $T_c/\Delta_0$  against  $\Delta_0$  (MeV), with (solid line) and without (dashed line) Polyakov loop, at  $\mu = 500$  MeV.

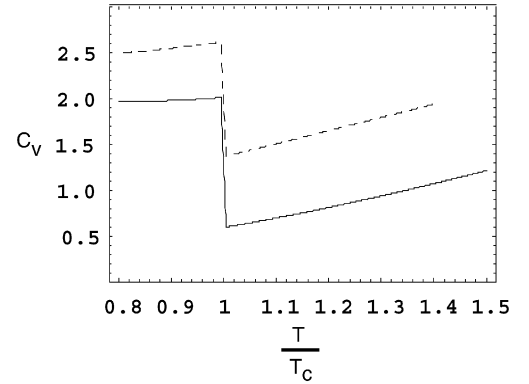


Fig. 2. Specific heat  $C_v$  (units:  $10^7$  MeV<sup>3</sup>) against  $T/T_c$ , with (solid line) and without (dashed line) the Polyakov loop, for  $\mu = 500$  MeV and  $\Delta_0 = 25$  MeV.

with  $\beta = 1/2$ . The value of the critical exponent is the same as in BCS superconductors. However the presence of the Polyakov loop affects the constant  $k$  in two ways. First, it gives a dependence on  $\Delta_0$  that is absent in the BCS and in the two flavor–color superconductor. Second, it changes its numerical values. For example for the 2SC case  $k \simeq 3.1$ ; in the present case  $k = 1.7$  and  $2.2$  for  $\Delta_0 = 40$  and  $100$  MeV, respectively.

The knowledge of  $\Delta(T)$  near  $T_c$  allows to determine some thermal properties of the model. For example we compute the specific heat as a function of the temperature, near  $T_c$ . It is given by

$$C_v = -T \frac{\partial^2 \Omega(\Delta, \phi)}{\partial T^2}. \quad (22)$$

We show the result of this calculation in Fig. 2, with (solid line) and without (dashed line) Polyakov loop, for  $\mu = 500$  MeV and  $\Delta_0 = 25$  MeV (for other values of  $\Delta_0$  we find qualitatively similar results). We notice that including the Polyakov loop slightly decreases the specific heat and increases a bit its discontinuity around  $T_c$ .

### 4. Conclusions

In this Letter we have studied the effect of the inclusion of the Polyakov loop on the NJL description of the CFL model. We have restricted our attention to a temperature range close to the critical temperature of the second-order phase transition.

We have found that introducing the Polyakov loop significantly increases the critical temperature, the effect being more important in the weak coupling regime. This increase may have some phenomenological consequences, both for astrophysical systems and for future experiments at GSI, if the proposed facility SIS100/200 [22] will be able to reach the hadronic densities needed for color superconductivity. Needless to say, one has to stress the heuristic use of the Polyakov loop when quarks are dynamical. Already their presence destroys the center symmetry of pure gauge QCD. More theoretical investigation will be needed on the PNJL model to ascertain its possible regions of validity. Nevertheless we felt that it is useful to investigate the effect of the Polyakov loop in some portions of the QCD phase diagrams where a direct QCD treatment is not available at the present.

We have studied the behavior of the gap parameter  $\Delta(T)$  for  $T \approx T_c$ , showing that the Polyakov loop does not modify the critical exponent  $\beta = 1/2$ , but only the pre-factor. In ordinary superconductor the pre-factor does not depend on  $\Delta_0$ ; on the other hand, the presence of the Polyakov loop results in a pre-factor dependent on the strength of the coupling.

A quantity of interest is the specific heat  $C_v$  since it can be measured experimentally. At the second-order phase transition  $C_v$  is discontinuous, in the superconductive phase being larger than in the normal phase. Although the effect of the Polyakov loop is to decrease the absolute value of  $C_v$ , the discontinuity  $\Delta C_v$  with  $\phi \neq 0$  is larger than the corresponding value at  $\phi = 0$ .

Further developments include the treatment of the strange quark mass, as well as the study of the thermodynamics of the CFL superconductor with Polyakov loop at lower temperatures. While the effect of the Polyakov loop is not expected to modify strongly the thermodynamics at small  $T$ , it is known that the finite strange quark mass affects the phase diagram of QCD and its role in the present model may be of interest as well.

## References

- [1] Y. Nambu, G. Jona-Lasinio, Phys. Rev. 122 (1961) 345; Y. Nambu, G. Jona-Lasinio, Phys. Rev. 124 (1961) 246.
- [2] N. Cabibbo, L. Maiani, G. Parisi, R. Petronzio, Nucl. Phys. B 158 (1979) 295.
- [3] A.M. Polyakov, Phys. Lett. B 72 (1978) 477; L. Susskind, Phys. Rev. D 20 (1979) 2610; B. Svetitsky, L.G. Yaffe, Nucl. Phys. B 210 (1982) 423; B. Svetitsky, Phys. Rep. 132 (1986) 1.
- [4] A. Gocksch, M. Ogilvie, Phys. Rev. D 31 (1985) 877; E.M. Ilgenfritz, J. Kripfganz, Z. Phys. C 29 (1985) 79; S. Digal, E. Laermann, H. Satz, Eur. Phys. J. C 18 (2001) 583, hep-ph/0007175; K. Fukushima, Phys. Lett. B 553 (2003) 38, hep-ph/0209311; K. Fukushima, Phys. Rev. D 68 (2003) 045004, hep-ph/0303225; F. Sannino, Phys. Rev. D 66 (2002) 034013, hep-ph/0204174; Y. Hatta, K. Fukushima, Phys. Rev. D 69 (2004) 097502, hep-ph/0307068; A. Mocsy, F. Sannino, K. Tuominen, Phys. Rev. Lett. 92 (2004) 182302, hep-ph/0308135.
- [5] M. Fukugita, A. Ukawa, Phys. Rev. Lett. 57 (1986) 503; A. Barducci, R. Casalbuoni, S. De Curtis, R. Gatto, G. Pettini, Phys. Lett. B 244 (1990) 311; F. Karsch, E. Laermann, Phys. Rev. D 50 (1994) 6954, hep-lat/9406008; S. Aoki, et al., JLQCD Collaboration, Phys. Rev. D 57 (1998) 3910, hep-lat/9710048; F. Karsch, E. Laermann, A. Peikert, Nucl. Phys. B 605 (2001) 579, hep-lat/0012023; C.R. Allton, et al., Phys. Rev. D 66 (2002) 074507, hep-lat/0204010.
- [6] K. Fukushima, Ann. Phys. 304 (2003) 72, hep-ph/0204302.
- [7] K. Fukushima, Phys. Lett. B 591 (2004) 277, hep-ph/0310121.
- [8] P.N. Meisinger, M.C. Ogilvie, Nucl. Phys. B (Proc. Suppl.) 47 (1996) 519, hep-lat/9509050; P.N. Meisinger, M.C. Ogilvie, Phys. Lett. B 379 (1996) 163, hep-lat/9512011; C. Ratti, M.A. Thaler, W. Weise, Phys. Rev. D 73 (2006) 014019, hep-ph/0506234; C. Ratti, M.A. Thaler, W. Weise, nucl-th/0604025; C. Ratti, S. Roessner, M.A. Thaler, W. Weise, Eur. Phys. J. C 49 (2007) 213, hep-ph/0609218.
- [9] E. Megias, E. Ruiz Arriola, L.L. Salcedo, Phys. Rev. D 74 (2006) 065005, hep-ph/0412308; E. Megias, E. Ruiz Arriola, L.L. Salcedo, Phys. Rev. D 74 (2006) 114014, hep-ph/0607338.
- [10] S. Roessner, C. Ratti, W. Weise, Phys. Rev. D 75 (2007) 034007, hep-ph/0609281.
- [11] R. Rapp, T. Schafer, E.V. Shuryak, M. Velkovsky, Phys. Rev. Lett. 81 (1998) 53, hep-ph/9711396.
- [12] M.G. Alford, K. Rajagopal, F. Wilczek, Phys. Lett. B 422 (1998) 247, hep-ph/9711395.
- [13] M.G. Alford, K. Rajagopal, F. Wilczek, Nucl. Phys. B 537 (1999) 443, hep-ph/9804403.
- [14] K. Rajagopal, F. Wilczek, hep-ph/0011333; M.G. Alford, Annu. Rev. Nucl. Part. Sci. 51 (2001) 131, hep-ph/0102047; G. Nardulli, Riv. Nuovo Cimento 25 (3) (2002) 1, hep-ph/0202037; D.H. Rischke, Prog. Part. Nucl. Phys. 52 (2004) 197, nucl-th/0305030; M. Buballa, Phys. Rep. 407 (2005) 205, hep-ph/0402234; M. Huang, Int. J. Mod. Phys. E 14 (2005) 675, hep-ph/0409167.
- [15] K. Rajagopal, E. Shuster, Phys. Rev. D 62 (2000) 085007, hep-ph/0004074.
- [16] M. Buballa, Phys. Rep. 407 (2005) 205, hep-ph/0402234.
- [17] M. Alford, C. Kouvaris, K. Rajagopal, Phys. Rev. Lett. 92 (2004) 222001, hep-ph/0311286; M. Alford, C. Kouvaris, K. Rajagopal, Phys. Rev. D 71 (2005) 054009, hep-ph/0406137.
- [18] R. Anglani, G. Nardulli, M. Ruggieri, M. Mannarelli, Phys. Rev. D 74 (2006) 074005, hep-ph/0607341.
- [19] G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legeland, M. Lutgemeier, B. Petersson, Nucl. Phys. B 469 (1996) 419, hep-lat/9602007.
- [20] T. Schafer, Nucl. Phys. B 575 (2000) 269, hep-ph/9909574.
- [21] K. Fukushima, C. Kouvaris, K. Rajagopal, Phys. Rev. D 71 (2005) 034002, hep-ph/0408322.
- [22] An International Accelerator Facility for Beams of Ions and Antiprotons, Conceptual Design Report, GSI, 2001.