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# Catch Laffer If You Can: Tax Take in an Evasion-Detection Game

Rosaria Distefano  | Francesco Reito 

Department of Economics and Business, University of Catania, Catania, Italy

**Correspondence:** Rosaria Distefano ([rosaria.distefano@unict.it](mailto:rosaria.distefano@unict.it))**Received:** 30 July 2025 | **Revised:** 10 October 2025 | **Accepted:** 13 November 2025**Keywords:** auditing | contest | policy | tax evasion

## ABSTRACT

In a simple taxation framework, we analyze a taxpayer's decision of whether to report income truthfully or engage in an evasion game with the tax agency. Specifically, taxpayer and tax agency can expend efforts, respectively, to conceal income and detect evasion. These activities are costly, and the final outcome—whether evasion is detected or not—is stochastic, and depends endogenously on the relative abilities of the contestants and on the policy parameters set by the authority. We present two main results: (i) evasion always occurs at relatively low tax rates, and then it may exhibit a U-shaped relationship with the tax rate; (ii) at the revenue-maximizing tax, the government's revenue is invariant to both detection efficiency and penalty rate.

**JEL Classification:** D82, H26, K42

## 1 | Introduction

We present a simple income taxation model in which a taxpayer can either report truthfully or engage in an evasion contest game with the tax agency. We assume that, if taxable income is not declared, the taxpayer and the tax agency exert concealment and auditing efforts, influencing the probability that evasion is detected.<sup>1</sup> Our concealment-detection subgame is built on Bayer (2006), who models tax evasion as a concealment-detection Bayesian contest between the taxpayer and the tax authority, and where both parties can expend effort to influence the probability that a taxable income component is verified. He shows that higher tax rates can lead to more evasion and more wasted effort resources, thus implicitly suggesting that lower rates may be a good policy measure. Our model differs in the following two key respects.

First, in the spirit of Fudenberg and Tirole (1984) and Bulow et al. (1985), we consider a strategic environment with three agents: a government, a taxpayer, and a tax-collecting agency. In stage one, the government sets the tax rate. In stage two, the taxpayer receives a stochastic, non-divisible income component,

which may be either positive or zero. In stage three, if income is positive, the taxpayer decides whether to declare it honestly or attempt to evade the tax. If income is truthfully reported, the game ends; otherwise, the taxpayer exerts unobservable effort trying to cover evasion. If zero income is declared, the tax agency must decide whether to exert detection effort trying to determine the true tax liability or randomize the control, leading to a mixed-strategy equilibrium scenario. In both cases, blatant dishonesty or random compliance, the outcome of the game, that is, whether undeclared income is detected or not, is stochastic and depends on the efforts exerted by the two parties. The second difference is that we use a contest probability function that, unlike Tullock's (1980) formulation, does not allow the agency that expends some positive effort—no matter how small it is—to win with certainty when the taxpayer exerts zero effort. This means that, if, for example, the taxpayer's income is not reported and no effort is made to conceal it, the detection probability will still depend on the resources devoted to auditing.

We derive two main results. First, our model suggests the possibility of a Laffer-type relationship between tax rate and tax

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revenues, from the perspective of tax evasion and enforcement activities. In particular, we show that the taxpayer will *always* attempt to evade for small tax rates, as in this case the net return to audit effort would be particularly low, making the agency a “weaker” contestant and leading to a low probability of successful detection. In contrast, for large tax liabilities, both the returns to concealment and auditing become higher, but at the same time, efforts more costly, so whether the taxpayer decides to evade or not will depend on the relative strengths of the two opposing forces. Finally, for intermediate levels of tax rate, auditing is relatively more cost-effective, making it possible for the taxpayer to mix between truthful and untruthful reporting and thus be randomly compliant. Therefore, it is ambiguous whether a higher tax rate will increase evasion or encourage the taxpayer to be more compliant, as in Yitzhaki (1974). In our model, evasion does not depend on the risk attitude of the taxpayer, assumed to be risk neutral, but on the endogenous detection probability, which may make it preferable to report true income even at higher tax rates.<sup>2</sup>

Our second main result concerns the case where the taxpayer and tax agency end up in the concealment-detection game, and the government’s objective is to maximize expected net revenues. We show that tax revenues are invariant both to the audit efficiency, treated as a parameter influencing the probability of successful detection, and to the fine imposed. The intuition is that higher levels of efficiency and penalty would increase the returns to concealment effort, thereby raising the taxpayer’s propensity to conceal income, and lowering the probability of successful detection.<sup>3</sup> To counteract this effect and not spend too many resources in audit effort, the government is forced to set a lower tax rate, thus maintaining the same level of expected tax returns. In other words, as shown in the analysis, the elasticity of substitution between tax rate and each of the other two “inputs” is constant and equal to unity. This conclusion may provide at least a partial explanation for the observation that in many countries the resources devoted to enforcement and the expected fines are surprisingly very low, as reported, among others, by Slemrod (2004) and Kirchler (2007). Our invariance result is derived under the assumption of risk-neutral taxpayers, so our framework can also be applied, for example, to large firms with multiple businesses or individuals with diversified portfolios, for which the outcome variance of different income components has no or less direct influence on evasion behavior. We show, however, that tax revenues remain invariant to the evasion penalty even when the taxpayer is assumed to be risk-averse, but they may not be invariant to audit frequency, as it directly influences the dispersion of possible income outcomes and thus the evader’s attitude towards risk.

Section 2 introduces the model. Section 3 characterizes the equilibrium. Section 4 derives the results. Section 5 presents some possible extensions. Section 6 draws the conclusions and discusses some empirical evidence in support to our predictions.

## 2 | Model

Consider a one-period risk-neutral economy with three agents: the government, which regulates the taxation scheme; a taxpayer, who earns stochastic taxable income  $\tilde{m}$ , equal to

$\tilde{m} = m > 0$  with probability  $x$  and  $\tilde{m} = 0$  with probability  $1 - x$ ; a tax agency in charge of collecting taxes.<sup>4</sup> The taxpayer can be either honest and pay the due amount or dishonest and try to evade the payment. We assume that the realization of income is unknown both to the government and tax collector, so that the taxpayer can have the possibility to evade without certainty of detection. The taxpayer can thus have two types: type  $m$  with probability  $x$  and type 0 with probability  $1 - x$ , and this distribution is common knowledge.

The taxpayer-evader has access to a “concealment technology” which, through the provision of physical or financial effort,  $e \in [0, 1]$ , can reduce the probability of being caught by the tax agency, at a cost described by the continuous and twice differentiable function  $C : [0, 1] \rightarrow \mathbb{R}_+$ , with  $C'(e) > 0$ ,  $C''(e) \geq 0$  for  $e > 0$ , and  $C(0) = C'(0) = 0$ . The tax agency can make use of a detection technology, with input consisting of the monetary and/or physical resources,  $a \in [0, 1]$ , needed to verify the taxpayer’s true income, at a cost defined by the continuous and differentiable function  $D : [0, 1] \rightarrow \mathbb{R}_+$ , with  $D'(a) > 0$ ,  $D''(a) \geq 0$  for  $a > 0$ , and  $D(0) = D'(0) = 0$ . We also assume that both  $C'(1)$  and  $D'(1)$  are sufficiently large to ensure an interior solution for  $e$  and  $a$ .<sup>5</sup>

We assume that the two contestants are able to infer each other’s ability and costs, and that the final outcome of the game—whether income concealment is detected or not—is stochastic. Specifically, the probability that the taxpayer is caught cheating is described by the function  $p = p(e, a)$ , capturing the interaction between concealment and audit efforts. This relationship is formalized in the following

**Assumption 1.**  $p = p(e, a)$  is a continuous and differentiable function, with  $p_e < 0$ ,  $p_a > 0$ ,  $p_{ea} < 0$ ,  $p_{ee} = 0$ , and  $p_{aa} = 0$ .

Hence,  $p(e, a)$  is decreasing (increasing) in  $e$  ( $a$ ), whereas the sign of the cross derivative implies that each player’s effort weakens the effect of the other’s action.<sup>6</sup> The probability function is separable and linear in each of its arguments, but the convexity of the effort cost functions implies diminishing returns to concealment and detection activities.

**Assumption 2.**  $p(e, 0) = 0$  for all  $e \geq 0$ .

This assumption implies that, if the agency does not invest any auditing resources, the detection probability will be equal to  $p(0, 0) = 0$ , even if the evading taxpayer does not exert any concealment effort after declaring no taxable income. The contest probability function is thus slightly different from the standard Tullock-type function, which would yield a detection probability equal to 1 if, for example, the taxpayer does not exert effort and the agency chooses any positive, though very small, level of detection activity.<sup>7</sup> When evasion is successfully detected, we assume that the taxpayer is forced to pay a penalty  $s > 1$  levied on the evaded tax,  $tm$ .

We will analyze an evasion game with the following sequence of events: 1. The government sets the tax rate  $t$ , which must be lower than an upper bound,  $\bar{t}$  (and below 1); 2. Nature determines the taxpayer’s income, drawn from the common-

knowledge distribution  $\tilde{m} \in \{0, m\}$ ; 3. If the realized income is  $\tilde{m} = 0$ , the taxpayer will report nothing. If the realized income is  $\tilde{m} = m$ , the taxpayer can decide to report either truthfully or report nothing. The game sequence is illustrated in Figure 1.

The equilibrium concept is the perfect Bayesian equilibrium. In particular, the taxpayer's reported income is interpreted as a signal, and the agency's beliefs are updated according to Bayes' rule. In what follows, we discuss the two possible equilibrium configurations. The first is a pooling equilibrium, where the two taxpayer types,  $\tilde{m} = m$  and  $\tilde{m} = 0$ , choose the same action, reporting no income, while the second is a semi-separating (or partially pooling) equilibrium, where the two types use, respectively, mixed and pure strategies, resulting in a partial revelation of private information.

Finally, we consider that audit efficiency—successful detection—is influenced by the resources allocated by the government to fight evasion, through the multiplicative parameter  $q \in (0, 1]$ . In Tullock's terminology, this parameter can be interpreted as the "precision" of the contest, determining how the winning probabilities are influenced by efforts exerted. To keep the analysis simple, we assume that  $q$  can be increased at no cost. This assumption, though extreme, is theoretically useful for demonstrating that the equilibrium audit frequency is kept low even though the government's budget does not pose a constraint.<sup>8</sup>

In our evasion game, the taxpayer maximizes expected net payoff and the agency expected net revenue, whereas we follow a positive approach and do not explicitly specify an objective function for the government. We thus take the view that the policy decisions can be, at least partially, influenced by several environmental factors, such as lobbying, electoral concerns, and institutional variables. In the model, we will not go into detail on how the tax rate is determined, but largely treat it as exogenously given. Nonetheless, we will examine the properties of the equilibrium outcome at the tax rate maximizing net revenues, i.e., when the government and tax agency share the same objective function (placing no social weight on the taxpayer's well-being to simplify the analysis).<sup>9</sup> In this case, the

strategic interaction would be essentially one between the taxpayer and government, and the tax authority treated as a single entity maximizing expected revenues. Hence, this interaction can be viewed as a modified version of the strategic game in Fudenberg and Tirole (1984), where, in our case, the government, in stage one, makes an observable choice, the tax rate, which will influence the actions and payoffs of the taxpayer and the agency. In particular, the tax rate choice can be loosely interpreted as a precommitment strategy, which will either accommodate the taxpayer's "entry" into the concealment-detection subgame with the agency or partially deter it by decreasing the probability of dishonest reporting, though not necessarily to zero in a semi-separating scenario.

As established in the literature on the political economy of taxation, social, economic, or political forces play a large role in shaping the tax legislation, in ways that are not necessarily optimal compared with the allocation that would be chosen by an abstract benevolent planner. However, what we can say is that, if taxpayer and the tax agency end up in the concealment-detection sub-game, the government will never rationally choose to set a tax rate larger than the level maximizing expected revenues, as it could be detrimental for electoral or political reasons, and also lead to lower revenues (a lower tax rate might instead be strategically rational). This is an assumption we will make in the next section, where we show that a trade-off may arise between enforcement investment and tax rate. As in Bayer (2006), the taxpayer and the agency can influence the probability of evasion detection, depending on how much effort they are willing to exert in the contest game. This analysis extends the framework of Cremer and Gahvari (1994), where they allow for tax evaders to affect the probability of being caught cheating, but consider fixed audit strategies. Additionally, we will analyze how setting a tax rate either above or below the revenue-maximizing level can affect the agency's detection effort, as well as impose a constraint on the resources allocated to the collection activity. This may suggest a different perspective on the implementation of antitax avoidance measures, where the interaction between the tax agency and its sovereign political government is viewed as a "conflict", as discussed in Skinner and Slemrod (1985).

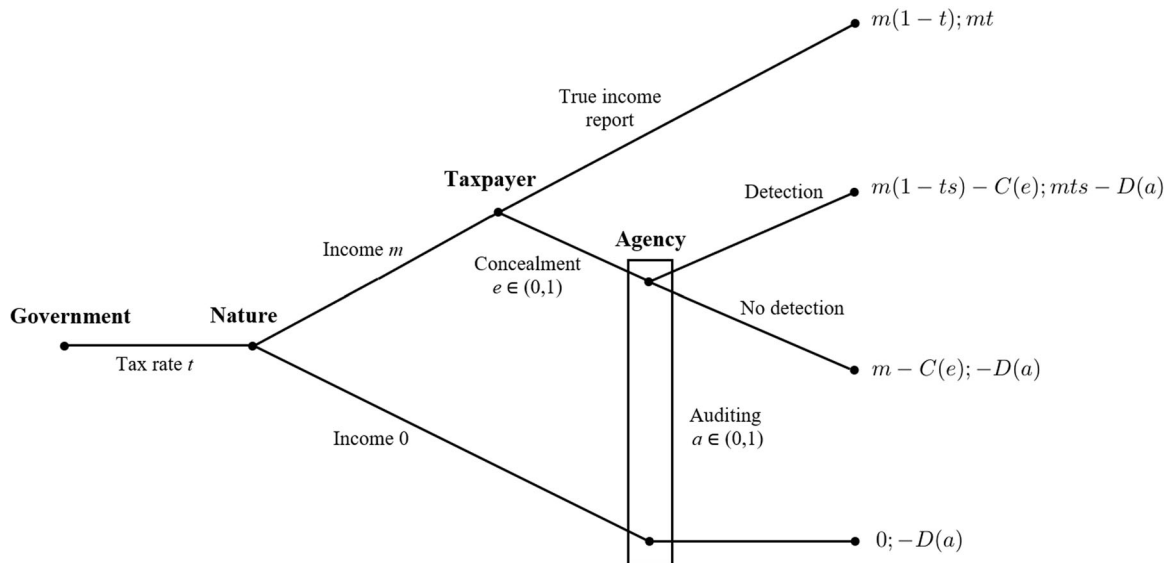


FIGURE 1 | Game timeline.

### 3 | Equilibrium

We will solve the game by backward induction, analyzing first the interaction between the taxpayer and the agency, and then describing the tax-rate choices available to the government. We first treat the concealment and detection efforts as exogenous, as well as the tax rate  $t \leq \bar{t}$  set by the government, and then endogenize the strategic choices in the following subsections.

In stage three of the game, the taxpayer, after obtaining income  $\tilde{m} = m$ , will decide to report it truthfully or not depending on whether the expected payoff from evasion, denoted by  $U_E$ , is higher or lower than the compliance payoff,  $m(1 - t)$ . If  $t$  is such that  $U_E > m(1 - t)$ , reporting zero income is a dominant strategy for the taxpayer, who will engage with the agency in a simultaneous-move concealment-detection contest game. In this case, there can exist a pure-strategy fully pooling equilibrium, where the  $m$ -type taxpayer attempts to evade, and the 0-type will, of course, report nothing. Conversely, if  $t$  is such that  $U_E \leq m(1 - t)$ , the expected payoff from evasion is lower than the compliance payoff, but a pure non-evasion strategy cannot be part of an equilibrium path, as this would be inconsistent with sequential rationality. The reason is that, in the absence of a credible precommitment policy, truthful reporting cannot dominate evasion, because the tax agency would respond by exerting no detection effort, and in turn being honest would no longer be a best response for the taxpayer. In this case, there may be a semi-separating equilibrium, in which the 0-type taxpayer plays a pure non-evasion strategy and the  $m$ -type will randomize.

In the following, the interaction between the  $m$ -type taxpayer and the tax agency is described in more detail.

#### 3.1 | Concealment-Detection Contest

If the tax rate is set at a level such that evasion is a dominant strategy, the  $m$ -type taxpayer will report no income and exert concealment effort  $e$ . Without observing  $e$ , the tax agency will exert detection effort  $a$ , as long as the expected revenue is positive (which we assume throughout). Assuming quasi-linear preferences (linear in money, convex in effort), the expected payoffs in the concealment-detection subgame can be written as

$$U_E = m - qp(e, a)stm - C(e), \tag{1}$$

$$R_E = xqp(e, a)stm - D(a). \tag{2}$$

where: in (1), the taxpayer incurs concealment cost  $C(e)$ , is detected with probability  $qp(e, a)$ , that is when audit is initiated and is successful, and pay the Yitzhaki-type (monetary) sanction  $stm$ ; in (2), the tax agency incurs the cost  $D(a)$  and receives  $stm$  with probability  $xqp(e, a)$ , that is only if the taxpayer has earned income and evasion is successfully detected.

As both players have continuous and quasi-concave payoff functions, their best replies are continuous and single-valued. The strategy sets are compact and convex, so this ensures the existence of a Nash equilibrium. The optimal efforts, denoted by  $e^* = \arg \max_e U_E(e, at)$  and  $a^* = \arg \max_a R_E(e, at)$ , at interior

solutions, must satisfy the following system of first-order conditions:

$$\frac{dU_E}{de} = -qp_e(e^*, a^*)stm - C'(e^*) = 0; \tag{3}$$

$$\frac{dR_E}{da} = xqp_a(e^*, a^*)stm - D'(a^*) = 0. \tag{4}$$

From the assumption  $p_{ea}(e, a) < 0$ , the reaction functions are, respectively, positively and negatively sloped, so the taxpayer views efforts as strategic complements, whereas the agency as strategic substitutes. Hence, the concealment-detection subgame exhibits mixed strategic interactions—complements for one player and substitutes for the other. A classic example of this type of game can be found in the model of crime and law enforcement of Becker (1968), though with opposing strategic effects, as in his case, the offender exhibits strategic substitutes—higher law enforcement leads to lower criminal activity—and the justice system exhibits strategic complements—higher crime leads to higher enforcement. The difference stems from the fact that, while in Becker (1968) the criminal activity can result in some financial gain to the offender and social harm to others, in our case taxpayer and tax agency compete for the same “prize”, the amount of tax evaded.

From (1) and (2), the concealment-detection interaction generates a social loss equal to  $C(e^*) + D(a^*)$ , corresponding to the effort costs incurred by the two parties. Note that, in this equilibrium scenario, there is no belief updating since taxpayers of both types choose the same action.<sup>10</sup>

Given the equilibrium efforts,  $e^*$  and  $a^*$ , the tax rate that maximizes net revenues is obtained from the constrained maximization of (2), with solution

$$\hat{t}_E = \arg \max_{t \in (0, \bar{t})} R_E,$$

derived from

$$\frac{dR_E}{dt} = \frac{\partial R_E}{\partial t} + \frac{\partial R_E}{\partial e} \frac{de}{dt} \tag{5}$$

$$= xq \left[ \underbrace{p(e^*, a^*)}_{\text{direct effect}} + \underbrace{tp_e(e^*, a^*) \frac{de}{dt}}_{\text{strategic effect}} \right] sm \geq 0 \quad \perp t \leq \bar{t}. \tag{6}$$

In (6), the first term is positive, as it represents the marginal revenue from an increase in  $t$ , whereas the second term represents the strategic effect on revenues of the taxpayer's effort, and is negative because  $p_e(e, a) < 0$ . That is, a higher  $t$  also entails a higher concealment effort, which lowers the probability of being caught and therefore the marginal revenue.<sup>11</sup> The following Lemma defines the relationship between tax rate and strategic efforts, taking into account that  $t$  and  $a$  can be either complementary or substitute inputs in the revenue function.

**Lemma 1.** *In the concealment-detection sub-game:*

- a.  $e^*$  is increasing in  $t$ ;
- b.  $a^*$  is bell-shaped in  $t$ , and reaches a maximum at  $t = \hat{t}_E$ .

As expected,  $de^*/dt > 0$ , so a higher tax rate will in general incentivize the taxpayer to expend more effort in the concealment of evasion. For the second part of the Lemma, when the tax rate is below (above) the revenue-maximizing level, it follows that  $da^*/dt > 0$  ( $da^*/dt < 0$ ), meaning that the net returns to detection effort is positive (negative).

### 3.2 | Mixed-Strategy Equilibrium

If the tax rate set by the government is such that evasion is not a dominant strategy, the interaction between the taxpayer and tax agency will result in a semi-separating (partial pooling) equilibrium, in which the 0-type taxpayer plays a pure non-evasion strategy and the  $m$ -type will randomize. In this case, if no income is reported, both the government and tax agency remain uninformed about the true realization, and their beliefs have to be consistent with the updating of prior probabilities,  $x$  and  $1 - x$ , according to Bayes' rule. Denoting by  $\epsilon$  the probability that the  $m$ -type taxpayer declares 0, the posterior probability that  $\tilde{m} = m$  is

$$\pi(\epsilon) = \frac{x\epsilon}{x\epsilon + 1 - x}, \quad (7)$$

so the agency, after observing that no income has been reported, has the expectation that  $\tilde{m} = m$  with probability  $\pi(\epsilon)$  and  $\tilde{m} = 0$  with probability  $1 - \pi(\epsilon)$ . These updated probabilities are determined endogenously by the optimizing behavior of the  $m$ -type taxpayer. Specifically, the  $m$ -type taxpayer forms rational expectations about the agency's behavior, and then chooses concealment effort,  $e = \hat{e} = \arg \max_e U_E(e, \hat{a}|t)$ , taking into account that the agency will set  $a = \hat{a}$  such that

$$m - qp(\hat{e}, \hat{a})stm - C(\hat{e}) = m(1 - t), \quad (8)$$

that is such that the  $m$ -type taxpayer is indifferent between evading and being honest. Whereas, given  $\epsilon$ , the tax agency must be indifferent between auditing and not,

$$\pi(\epsilon)qp(e, a)stm - D(a) = 0. \quad (9)$$

Solving (8) and (9),

$$\hat{e} = \frac{(1 - x)D(a)}{x[qp(e, a)stm - D(a)]}, \text{ and} \quad (10)$$

$$\hat{a} = p^{-1} \left[ \hat{e}, \frac{tm - C(\hat{e})}{qstm} \right],$$

so the  $m$ -type taxpayer evades with probability  $\hat{e}$ , and expects the agency to conduct an audit with probability  $q$  and then exert detection effort  $\hat{a}$ .

As in the hybrid equilibrium of Bayer (2006), the agency's detection effort is deterministic. This is in contrast to the

models with precommitment, where auditing is "perfect" if applied, and thus it can be optimal for the tax-collection authority to mix between auditing and doing nothing. In our case, audits are not perfect, so the tax agency can simply choose to adjust its own detection effort, so as to make the taxpayer indifferent between being honest and dishonest. As argued by Harsanyi (1973), a possible interpretation of a mixed strategy equilibrium is that it can be viewed as the limit of a pure strategy equilibrium in a "disturbed" game when the perturbations vanish. These exogenous random perturbations can make the game appear to be random even if it is deterministic.

In equilibrium, the taxpayer obtains nothing when  $\tilde{m} = 0$ , and is indifferent between honest and dishonest reporting when  $\tilde{m} = m$ , with resulting payoff  $m(1 - t)$ . The tax agency receives the payment  $tm$  with probability  $x(1 - \hat{e})$ , that is, when the taxpayer obtains  $m$  and chooses not to evade. If  $\tilde{m} = 0$ , the agency is indifferent between auditing and not, so with probability  $x\hat{e} + 1 - x$ , the expected revenue is zero. The mixed-strategy equilibrium payoffs are thus

$$U_{ME} = m(1 - t), \quad (11)$$

$$R_{ME} = x(1 - \hat{e})tm = \frac{xqxp(\hat{e}, \hat{a})stm - D(\hat{a})}{qxp(\hat{e}, \hat{a})stm - D(\hat{a})}tm, \quad (12)$$

where the agency's revenue is lower than  $tm$  as the fraction in (12) is strictly lower than 1, and increasing in  $x$ .

Note that the taxpayer obtains an equilibrium payoff equal to the compliance payoff,  $m(1 - t)$ . We will thus refer to  $U_{ME}$  as the mixed-strategy or *random* compliance payoff. There can be none or multiple tax rates such that the taxpayer will find it optimal to play a mixed strategy. The set of such tax rates is defined as  $T_{ME} = \{t|U_E \leq U_{ME}\}$ . If  $T_{ME} \neq \emptyset$ , the highest possible compliance tax rate is  $\hat{t}_{ME} = \max T_{ME}$ , under the constraint that it must not exceed the upper bound  $\bar{t}$ .

## 4 | Results

### 4.1 | Tax-Revenue Relationship

Here, we analyze the nature of the relationship between tax rate and tax revenues, considering the strategic decision of the taxpayer who, depending on the tax rate  $t$ , will either decide to engage in the evasion contest with the agency or play mixed strategies. Regarding the composition of revenues, we will incorporate both the resource costs of enforcing tax collections, as well as the monetary penalty paid by the taxpayer in case of detection.<sup>12</sup>

**Proposition 1.** *The taxpayer is more inclined to evade for relatively low tax rates, and may also be for relatively high tax rates.*

This result relies on the marginal utilities from random compliance and tax evasion for increases in  $t$ , evaluated at the equilibrium effort levels,  $e^*$  and  $a^*$ . Letting  $U_{ME} = U_{ME}(t)$  and  $U_E = U_E(e^*, a^*|t)$ , we have that

$$\frac{dU_{ME}}{dt} = -m, \quad (13)$$

$$\begin{aligned} \frac{dU_E}{dt} &= \frac{\partial U_E}{\partial t} + \frac{\partial U_E}{\partial a} \frac{da^*}{dt} \\ &= -q \left[ p(e^*, a^*) + t p_a(e^*, a^*) \frac{da^*}{dt} \right] sm, \end{aligned} \quad (14)$$

where (14) is derived by applying the envelope theorem, but taking into account how the best response of the tax agency varies to marginal changes in  $t$  (as in Caputo 1998, though in a Stackelberg-type game). In this expression, the first term represents the marginal disutility associated with an increase in  $t$  under the assumption that the penalty amount is proportional to the tax evaded  $tm$ . The second is the strategic effect of the agency's effort on the taxpayer's utility, and this term can be either negative or positive from Lemma 1. The total effect is negative, and the explanation is as follows. Starting from zero taxation, an infinitesimal increase in the tax rate would make evasion strictly more attractive, as

$$\left. \frac{dU_E}{dt} \right|_{t=0} = 0 > -m = \frac{dU_{ME}}{dt}. \quad (15)$$

For very small tax rates, the agency conducts very little audit activity (zero if  $t = 0$ ), resulting in a very low probability of detection, so the taxpayer would *always* choose to evade. Regarding the second part of the proposition, we can show that, at the revenue-maximizing tax, since  $da^*/dt|_{t=\hat{t}_E} = 0$  from Lemma 1,  $dU_E/dt|_{t=\hat{t}_E} = -qp(e^*, a^*)sm$ , which may be lower than  $dU_{ME}/dt = -m$  in absolute terms. This means that the function  $U_E$  may cut the line  $U_{ME}$  from below, suggesting that there can be an interval of tax rates where the marginal utility from evasion is lower in absolute terms than that from random compliance, so that the taxpayer will attempt to evade. Finally, for intermediate levels of  $t$ ,  $dU_E/dt|_{t < \hat{t}_E} < 0$ , as  $p_a(e^*, a^*) > 0$  and  $da^*/dt|_{t < \hat{t}_E} > 0$ , so we may obtain that the taxpayer will find it preferable to randomize. Since random compliance entails a lower social loss than tax evasion, we can therefore state the following

**Corollary 1.** *A higher tax rate may lead to less evasion and social loss.*

In the seminal work of Yitzhaki (1974), a higher tax increases reported income and thus reduces evasion for DARA taxpayers. This occurs because the prevailing income effect makes taxpayers more sensitive to the risk of detection, reducing their propensity to engage in an evasion lottery. We derive the same result, but for risk-neutral taxpayers, and the logic behind is simple. If we suppose an initial evasion equilibrium at a relatively low tax rate, and if the tax rate is increased such that  $U_E \leq U_{ME}$ , this induces a regime shift from an evasion to a random compliance equilibrium. In the model, there are two sources of uncertainty, one associated with the taxpayer's income and the other with the nature of the concealment-detection activities. The following result characterizes the effect on evasion of the risk associated with the earning prospects of the taxpayer.

**Proposition 2.** *Evasion is more likely when the risk of having no income is higher.*

The parameter  $x$  represents the probability that the taxpayer earns a positive income,  $m$ . This probability does not affect directly  $U_{ME}$  and  $U_E$ , as the taxpayer decides whether to report or not after receiving the income, but rather indirectly through a variation in the authority's detection effort. As shown in the Appendix,  $da^*/dx > 0$ , so a more likely income realization creates a stronger incentive for the agency to increase its detection activity. In turn, evaluated at the equilibrium efforts, the effect on the taxpayer's payoff,  $U_E = U_E(e^*, a^*|t, x)$ , from evading is

$$\frac{dU_E}{dx} = \frac{\partial U_E}{\partial a} \frac{da^*}{dx} = -q p_a(e^*, a^*) sm \frac{da^*}{dx} < 0, \quad (16)$$

that is, the profitability of evasion declines as the likelihood of earning a positive income rises. This happens because, when  $x$  increases, the returns to auditing becomes higher, while the returns to concealment lower, resulting in a higher possibility of successful detection. This leads the tax authorities to allocate more resources to detection. On the other hand, when the income realization is less likely –that is,  $x$  is relatively low–enforcement efforts become less effective, and this makes evasion more attractive to the taxpayer.

This result can be related to the topic of the effects of income-taxation policy on occupational choice.<sup>13</sup> Self-employment income, for instance, is usually more uncertain than paid employment, and the tax policy can add one more risky asset to the individual's portfolio, thus altering the occupational decision. Generally, paid employment is subject to third-party tax reporting, precluding noncompliance, whereas in self-employment noncompliance is possible. In our case, where detection outcome is stochastic, it can be argued that the taxation system can encourage entry into those occupations with noncompliance opportunities, and that this choice becomes more profitable the higher the uncertainty in income realizations. For example, physicians, lawyers and other professionals, falling into the category of the “hard-to-tax”, as defined by Alm et al. (2004), can take advantage of the fact that the stream of earnings derived from private practice is considered uncertain, and evade the entire or a considerable fraction of their taxable income, knowing that its aleatory nature tends to discourage the tax enforcement activity.<sup>14</sup>

Finally, our modeling approach may shed some light on how taxation can shape the incentives of firms and individuals to engage in underground economic activities, including the production of goods and services which are *entirely* concealed to avoid the payment of value added and other types of taxes, or to bypass the regulations on working conditions, minimum wages, and legal standards.<sup>15</sup>

#### 4.1.1 | Numerical Example

As in the classic Allingham and Sandmo (1972), the taxpayer evades or not depending on some fundamental parameters, such as the audit efficiency and the magnitude of penalties. In our case, this choice is also influenced by the relative abilities of the two contestants, expressed as parameters in the effort cost functions of the following numerical example.

Consider that the taxpayer's concealment cost function is  $C(e) = \alpha e^2/2$ ,  $\alpha > 0$ , and that the agency's detection cost function is  $D(a) = \beta a^2/2$ ,  $\beta > 0$ . The contest-winning/losing probability function is  $p(e, a) = a(1 - e)$ , where we interpret efforts as probabilities (to simplify, the winning/losing probability and cost functions do not depend directly on  $t$ ).

In Figure 2, we represent the taxpayer's utility from evasion,  $U_E$ , and the tax agency's revenue,  $R_E$ , against the tax rate,  $t$ . The increasing and decreasing straight lines represent the revenue,  $R_{ME} = x(1 - \hat{e})tm$ , and the taxpayer's utility,  $U_{ME} = m(1 - t)$ , under random compliance. The parameters are  $m = 1$ ,  $x = 0.7$ ,  $q = 0.5$ ,  $s = 2$ ,  $\alpha = 1.5$ ,  $\beta = 0.5$ . The exogenous upper bound on the tax rate is set at  $t = \bar{t} = 0.8$ . In plot 2a,  $T_{ME} = \emptyset$ , that is, there exists no  $t \leq \bar{t}$  such that  $U_{ME} \geq U_E$ , so there exist no feasible tax rates such that the taxpayer will choose to be randomly compliant. In this case, if the government chooses the revenue-maximizing tax,  $t = \hat{t}_E = \sqrt{\alpha\beta} / \sqrt{x} qsm = 0.73$ , the equilibrium evasion expected payoffs are  $U_E = 0.43$  and  $R_E = 0.13$ . In plot 2b, a larger penalty rate,  $s = 3$ , shifts the utility from evasion downward, giving rise to a set of tax rates over which the taxpayer plays a mixed strategy. Any  $t$  within the set  $T_{ME} = \{0.2, 0.65\}$  would result in a random compliance equilibrium. If the objective of the government is to maximize revenues, the tax rate is  $\hat{t}_{ME} = \max T_{ME} = 0.65$ , with payoffs  $U_{ME} = 0.35$  and  $R_{ME} = 0.55$ .

Figure 2b shows that starting from sufficiently low tax rates, utility from evasion decreases less rapidly than that from random compliance, making evasion more profitable. As  $t$  increases,  $U_E$  decreases at an increasing rate, eventually reducing utility below  $U_{ME}$ . At higher tax rates, the utility fall from an additional tax increase is smaller, and this can make evasion more profitable than random compliance. Depending on the concavity/convexity of  $U_E$ , evasion may occur at both relatively low and high tax rates, so it is possible to obtain a U-shaped relationship between tax rate and propensity to evade.

To show how the structure of the strategic interaction—contest game or random compliance outcome—between taxpayer and tax agency can vary depending on the level of taxation chosen by the government, consider the following example. Suppose that the tax rate chosen by the government—not necessarily set at the maximizing value—is such that it encourages the taxpayer to evade, as  $t_1 = 0.13$  in Figure 3. If the tax rate is increased, in the example to  $t_2 = 0.56$ , this may result in a shift from an evasion to a random compliance regime, inducing the taxpayer to randomize.

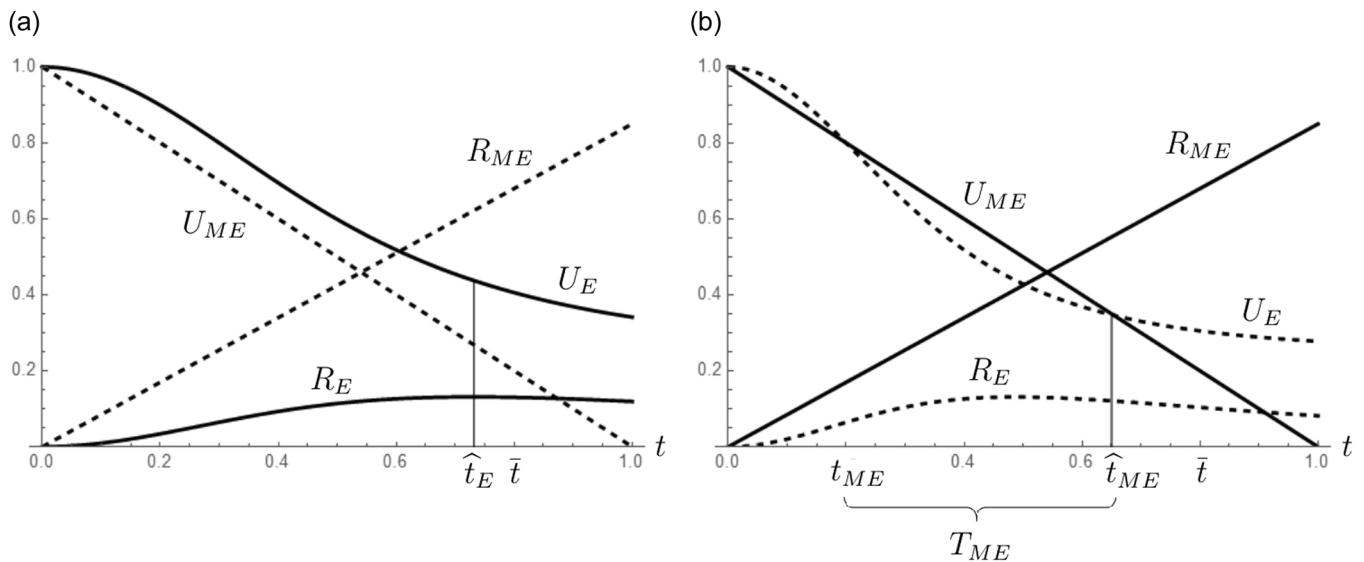
## 4.2 | Inspection Efficiency and Penalty Rate

In this subsection, we examine the effects of  $q$ , the efficiency of each audit inspection, and penalty rate  $s$  on the revenues collected by the government, in the case where the tax rate is set at the revenue-maximizing  $\hat{t}_E$  and taxpayer and tax agency end up in the concealment-detection game. We will treat  $q$  and  $s$  as parameters, for example set by a different public authority, though the results would not change if they were modeled as choice variables for the government. The following Lemma will be useful for the derivation of the main results.

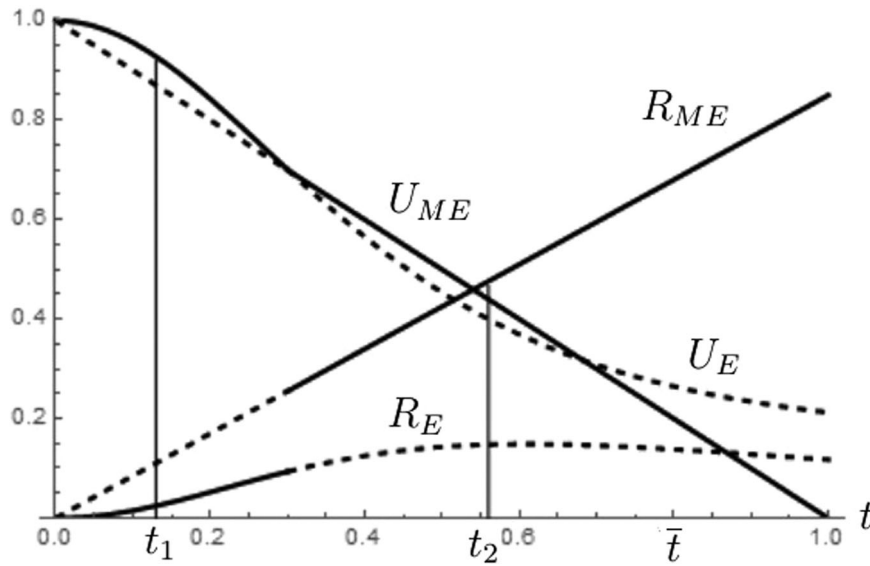
**Lemma 2.** *In the concealment-detection sub-game:*

- a. *the propensity to evade,  $e^*$ , is increasing both in  $q$  and  $s$ ;*
- b.  *$\hat{t}_E$  is decreasing both in  $q$  and  $s$ .*

The intuition is that, if the tax rate is such that the two contestants end up in the evasion subgame, the returns to concealment tend to increase for higher levels of  $q$  and  $s$ , leading to more effort on the part of the taxpayer.<sup>16</sup> If the objective is to maximize revenues, a higher concealment effort will make it optimal for the government to reduce the tax rate, so as to avoid large expenditures in terms of detection effort.



**FIGURE 2** | Tax evasion and random tax compliance. Parameters:  $m = 1$ ,  $x = 0.7$ ,  $q = 0.5$ ,  $\alpha = 1.5$ ,  $\beta = 0.5$ . a) Tax evasion equilibrium:  $s = 2$ ,  $t = \hat{t}_E = 0.73$ ,  $U_E = 0.43$ ,  $R_E = 0.13$ . b) Mixed-strategy equilibrium:  $s = 3$ ,  $x(1 - \hat{e}) = 0.8$ ,  $\hat{t}_{ME} = \max T_{ME} = 0.65$ ,  $U_{ME} = 0.35$ ,  $R_{ME} = 0.55$ .



**FIGURE 3** | Higher tax rates lead to a random compliance equilibrium. Parameters:  $m = 1, x = 0.7, q = 0.5, \alpha = 1.7, \beta = 0.6, s = 2.8, \bar{t} = 0.8$ . Evasion regime:  $t_1 = 0.13, U_E = 0.93 > 0.87 = U_{ME}, R_E = 0.02$ . Mixed-strategy regime:  $x(1 - \hat{e}) = 0.76, t_2 = 0.56, U_{ME} = 0.44 > 0.4 = U_E, R_{ME} = 0.47$ .

Consider first the effect of a change in audit efficiency on tax revenues. At the optimal effort levels,  $e^*$  and  $a^*$ , and at  $\hat{t}_E$ , by the envelope theorem, the effect of  $q$  on revenues  $R_E = R_E(e^*, a^* | \hat{t}_E, q)$  is

$$\frac{dR_E}{dq} = \frac{\partial R_E}{\partial q} + \frac{\partial R_E}{\partial e} \frac{de^*}{dq} = x \left[ p(e^*, a^*) + qp_e(e^*, a^*) \frac{de^*}{dq} \right] stm = 0, \tag{17}$$

where the last equality holds at an interior solution for  $t$  because, as shown in the Appendix,  $qp_e(e^*, a^*) \frac{de^*}{dq} = tp_e(e^*, a^*) \frac{de^*}{dt}$ , that is the marginal effect of an increase in  $q$  is equal to the marginal effect of a variation in  $t$ . Therefore, substituting the first-order condition with respect to  $t$  in (6) into (17), we obtain the result that audit efficiency has no effect on the size of tax revenues. Given the simple but rather general functional form of tax revenues, it is possible to show that the elasticity of substitution between the two “inputs”  $t$  and  $q$  is constant and equal to unity, so the two opposing effects cancel out.

A similar argument applies to the effect of variations in the penalty rate on tax revenues. Since  $sp_e(e^*, a^*) \frac{de^*}{ds} = tp_e(e^*, a^*) \frac{de^*}{dt}$ , the effect of  $s$  on  $R_E = R_E(e^*, a^* | \hat{t}_E, s)$  reduces to

$$\frac{dR_E}{ds} = \frac{\partial R_E}{\partial s} + \frac{\partial R_E}{\partial e} \frac{de^*}{ds} = qx \left[ p(e^*, a^*) + sp_e(e^*, a^*) \frac{de^*}{ds} \right] tm = 0, \tag{18}$$

so we can state the following

**Proposition 3.** *In the concealment-detection subgame, under the assumption of identical risk-neutral taxpayers, the maximum tax take is invariant to both audit efficiency and penalty rate.*

This conclusion, coupled with the above result on the relationship between tax rate and random compliance, can provide

a partial response to the puzzle that in many countries worldwide, tax honesty is rather high despite low enforcement and expected fines. Firstly, based on our model, tax revenues are invariant both to the efficiency with which the government can catch evaders and to the (monetary) penalty paid by them. So, if the government knows that the taxpayer and tax-collecting agency will end up in a concealment-detection game, it may find it preferable to keep audit efficiency and punishment low, for example, for electoral or other political purposes (not modeled here). Secondly, if the outcome is the random compliance regime, the reason may be that the concealment ability of taxpayers is so small that they prefer to pay the due taxes without engaging in a contest, despite the low  $q$  and  $s$ . Note that, as is common in this class of models, the decision to evade or randomize depends on the composite parameter  $qst$ , which can be viewed as the relevant “tax rate” set by the authority. What we show in this paper is that, when the taxpayer is risk neutral, the changes in  $q$  or  $s$  can be offset by adjusting  $t$ , yielding the same revenues.<sup>17</sup>

The logic behind the proposition does not depend on the effect of  $q$  and  $s$  on the cost of auditing, which, by assumption, is zero. So, we can speculate that, if increasing the audit effectiveness and penalty were costly to the government, their marginal effects on tax revenues may become negative. Thus, the choice to allocate a small amount of resources to tax enforcement may result from a mere rational calculation of the net gain in tax revenues, and not—or not just—from budgetary constraints, voting pressure, or ideological considerations. This is in contrast to the implications of Becker (1968) where, if deterrence activities were not costly, the authority would set the inspection rate and fees high enough to deter all criminal behavior.

The conclusion in Proposition 4 applies to the case where the revenue-maximizing  $\hat{t}_E$  is an interior solution. If the optimal tax rate is at the boundary,  $\hat{t}_E = \bar{t}$ , that is equal to the upper level set exogenously, from the first-order condition in (6), the marginal direct effect,  $p(e^*, a^*)$ , would be strictly larger than the strategic

effect,  $p_e(e^*, a^*)t \frac{de^*}{dt}$ . Thus, after substituting the latter term in (17) and (18), the total effect of an increase in  $q$  and  $s$  would be positive.

## 5 | Extensions

### 5.1 | Heterogeneous Taxpayers

Here, we consider a continuum of taxpayers of two types, intrinsically honest ( $H$ ), who never evade taxes, and potentially dishonest ( $D$ ), identical to the one analyzed in the main model. We assume that types are unobservable, so the government and tax agency will face both problems of hidden information and of hidden action on the part of  $D$  taxpayers. The analysis is essentially the same as in Sections 3 and 4, except that the result in favor of low audit enforcement and penalties becomes even more compelling.

In what follows, we will focus on the government's problem and on the effect of the parameters  $q$  and  $s$ . With two types, if  $D$  types engage in the concealment-detection game with the agency, tax revenues can be written as

$$R_E = hxtm + (1 - h)[qxp(e, a)stm - D(a)], \quad (19)$$

where  $h$  and  $1 - h$  are the proportions of honest and (potential) dishonest taxpayers (this distribution is common knowledge), and where we assume that the tax agency does not incur any effort cost inspecting  $H$  types with zero income realization.

The government's first-order condition for  $t$  is

$$\frac{dR_E}{dt} = xm \left\{ h + (1 - h)q \left[ p(e^*, a^*) + tp_e(e^*, a^*) \frac{de^*}{dt} \right] s \right\} \geq 0 \quad \perp t \leq \bar{t}, \quad (20)$$

which, compared to the condition in (6) of the main model, here it is more likely to lead to a corner solution at the upper threshold  $\bar{t}$  (e.g., if  $h$  approaches 1,  $dR_E/dt$  is always positive).

The effect of  $s$  on tax revenues is

$$\frac{dR_E}{ds} = (1 - h)x \left[ p(e^*, a^*) + sp_e(e^*, a^*) \frac{de^*}{ds} \right] stm, \quad (21)$$

where again  $sp_e(e^*, a^*) \frac{de^*}{ds} = tp_e(e^*, a^*) \frac{de^*}{dt}$ . But, differently from the analysis in Section 4, it can be shown that, at an interior solution for  $t$ , since the second term in curly brackets in (20) must be negative, the effect of  $s$  on  $R_E$  is negative (a similar argument can be made in the case of a corner solution, and for the effect of an increase in audit efficiency  $q$ ). An example is shown in Figure 4.

**Proposition 4.** *With heterogeneous taxpayers, the maximum tax take can be decreasing both in audit efficiency and penalty rate.*

### 5.2 | Nonmonetary Evasion Penalties

In the main model, the result that tax revenues in the concealment-detection game do not vary in  $q$  and  $s$  is in part driven by the assumption that the evasion sanction is monetary. In this extension, we assume that detected evaders are subject to a nonmonetary punishment, still denoted by  $s$ . Examples of this type of penalty can include revoking driver licenses, professional licenses, and passports from tax evaders, as discussed by Blank (2014), and Kuchumova (2018).<sup>18</sup>

In this case, tax revenues in the concealment-detection subgame are  $R_E = qxp(e, a)tm - D(a)$ , and the government's first-order condition for  $t$  is

$$\frac{dR_E}{dt} = xq \left[ p(e^*, a^*) + tp_e(e^*, a^*) \frac{de^*}{dt} \right] m \geq 0 \quad \perp t \leq \bar{t}.$$

and the effect of changes in (non-pecuniary) sanctions is

$$\left. \frac{dR_E}{ds} \right|_{t=\hat{t}_E} = \frac{\partial R_E}{\partial e} \frac{de^*}{ds} = xqm p_e(e^*, a^*) \frac{de^*}{ds} \left( \frac{de^*}{dt} \right)^{-1} < 0,$$

where the sign follows from  $de^*/ds > 0$ , as well as  $de^*/dt > 0$  and  $p_e(e^*, a^*) < 0$  from Assumption 1. Therefore, at the revenue-maximizing tax, non-pecuniary sanctions reduce tax revenues from the concealment-detection contest.

**Proposition 5.** *Nonmonetary sanctions can reduce tax revenues.*

### 5.3 | Risk-Averse Taxpayer

Here, we allow for the taxpayer to be risk-averse. In this variant of the main model, the expected revenue of the tax agency in (2) remains unchanged, whereas the taxpayer's evasion payoff in (1) becomes

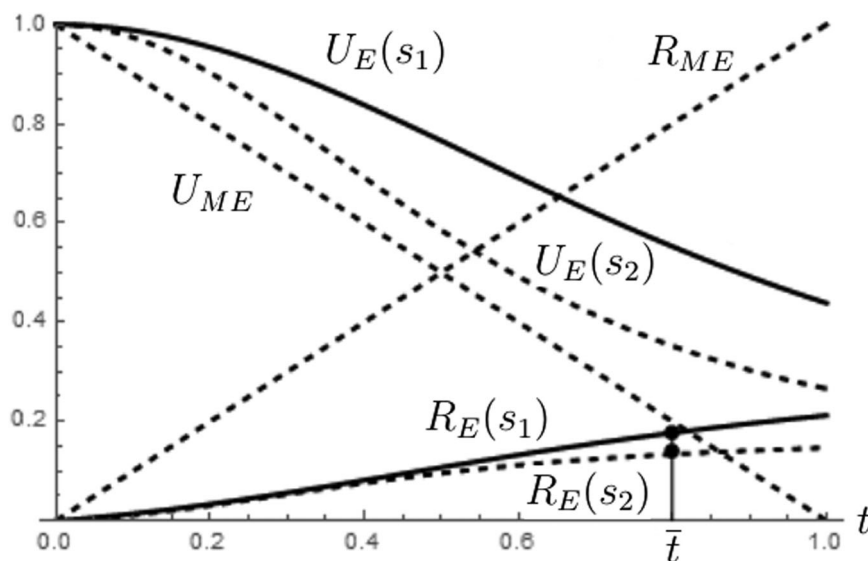
$$U_E = [1 - qp(e, a)]u(m) + qp(e, a)u(m - stm) - C(e),$$

with first-order condition equal to

$$\frac{dU_E}{de} = -qp_e(e, a)[u(m) - u(m - stm)] - C'(e),$$

where  $u(\cdot)$  is a continuous function, increasing and concave in the income realized. As the function  $U_E$  is continuous and quasi-concave, we can again find a unique solution for the best response,  $e^*$ .

To show that expected revenues, evaluated at the optimal effort levels,  $e^*$  and  $a^*$ , and at  $\hat{t}_E$ , are invariant to the penalty rate, first note that the effect of a marginal variation in  $s$  on the value function of  $R_E$  is the same as that described in (18), except for the term  $de^*/ds$ , as  $e^*$  now depends on the utility function,  $u(\cdot)$ . The same holds for the effect of  $t$  on  $R_E$  and, more specifically, for  $de^*/dt$ . Second, as  $e^*$  is increasing in both  $t$  and  $s$ , and these



**FIGURE 4** | Heterogeneous types: tax revenues decrease with the penalty rate. Parameters:  $m = 1$ ,  $x = 0.6$ ,  $q = 0.3$ ,  $\alpha = 1.7$ ,  $\beta = 0.2$ ,  $h = 0.2$ ,  $\bar{t} = 0.8$ . At  $s_1 = 2$ ,  $R_E(\bar{t}|s_1) = 0.17$ . At  $s_2 = 3$ ,  $R_E(\bar{t}|s_2) = 0.13$ .

two policy instruments only affect the taxpayer's payoff in the detection state, we again have the result that, from the government's perspective,  $s$  and  $t$  are perfect substitutes.

In contrast, detection efficiency,  $q$ , increases the variance of the outcomes, and, in particular, the uncertainty associated with successfully engaging in evasion. Although the monetary payoffs of detection and non-detection states remain unchanged, higher levels of  $q$  directly increase the exposure to risk, thus reducing the taxpayer's expected utility from evasion. In this case, the optimal tax rate still decreases with  $q$ . Nevertheless, the effect of a better detection efficiency on expected tax revenues is ambiguous.

**Proposition 6.** *If the taxpayer is risk-averse, tax revenues are invariant to penalty rates but are ambiguous with respect to detection efficiency.*

Although this result differs from that derived in the main analysis—namely, that tax revenues are invariant to  $q$ —it is important to note that, for simplicity, audit frequency is assumed to be costless, so the result may partly be driven by this modeling assumption.

## 6 | Concluding Remarks

In an evasion contest game between a taxpayer and a tax agency, a Laffer-shaped relationship between tax rate and tax revenues is found. Low tax rates reduce audit by so much that it encourages taxpayers to engage in income deception, while at higher tax rates, the profitability of evasion is ambiguous, so what we identify as *random* compliance is likely to occur. Penalties and detection efficiency are crucial policy tools in fighting evasion. However, we show that these interventions may have no effect on total tax revenues. This occurs because the tax rate and either the penalty rate or audit efficiency act as perfect substitutes from the perspective of the government. As a

result, any adjustment in one of these parameters can be offset by a proportional change in the other, leaving overall tax revenues unchanged.

A similar convex relationship is analyzed in the theoretical works on corruption by Sanyal et al. (2000), and Cerqueti and Coppier (2009). In the former paper, the presence of corrupt tax officials can result in a collusive agreement with taxpayers who want to under-report income returns when the tax rate is relatively high. In the latter paper, the decision to engage in corruption depends on a “shame effect”, which can be lower as the tax rate increases. Empirically, the relationship between tax rates and tax evasion remains ambiguous due to the challenges of measuring unobservable behaviors. Our results are consistent with the findings of the calibration analysis of Trabandt and Uhlig (2011), who use data from the US and EU economy and find that an increase in tax pressure leads to more revenues up to a certain level, after which they tend to decrease. While experimental studies typically focus on the effects of sanctions on tax compliance rather than on tax revenues, our result may be supported by the work of Bayer and Sutter (2009). They show that the welfare losses arising from the evasion-detection contest are positively correlated with the tax rate rather than the penalty rate for evasion. Mendoza et al. (2017) find evidence of a U-shaped relationship between tax evasion and audit activity conducted by tax authorities. They argue that excessive auditing may backfire, as it can signal a lack of trust in taxpayers, raising feelings of unfairness and resentment, which in turn reduce compliance. This may provide an explanation for why tax agencies may prefer increasing tax rates over intensifying auditing efforts. In Alm et al. (1995), evasion fines have no effects on compliance when the audit probability is relatively low, but are significant once both instruments are increased at higher levels. In terms of our model, if audit efficiency and/or penalty rate are increased at relatively high levels, the regime can shift from a concealment-detection game to a random compliance equilibrium. In the experiments conducted by Cummings et al. (2009) in South Africa and Botswana, an increase in audit

frequency led to more compliance in South Africa but less in Botswana, though a very strong increase in audit resulted in more honest reporting in both countries.

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## Conflicts of Interest

The authors declare no conflicts of interest.

## Data Availability Statement

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

## Endnotes

<sup>1</sup>As described by Skinner and Slemrod (1985), “*expenditures made by the taxpayer to cover his tax deceptions [...] include costs of offshore banks, laundering schemes, payments to unscrupulous tax lawyers, taxpayer time spent planning evasion, as well as the implicit costs (for risk-averse evaders) of chancing detection and arrest or fines by the tax authorities. [...] [For the tax collection agency] the enforcement process entails real resource costs in standard income records collection, investigative activities, and punishments.*”

<sup>2</sup>In contrast to the literature on optimal incentive contracts and auditing, as for example Reinganum and Wilde (1985), and Mookherjee and Png (1989), we do not allow the tax agency or the government to commit to a given amount of detection resources. As in Khalil (1997), though in a different context, a commitment strategy would not be credible if it is not ex-post incentive compatible.

<sup>3</sup>This result is true as long as taxpayer and agency are stuck in the evasion contest game. As shown in the analysis, if at a given tax rate, the audit frequency and/or penalty rate are increased considerably, then the game scenario may shift from an evasion to a hybrid, random compliance equilibrium.

<sup>4</sup>As in Corchón (1992), Bayer (2006), and Emran and Stiglitz (2005), tax evasion is modeled as a binary choice variable, involving for instance some kind of indivisibility in the occupational status or the filing procedures for the taxpayer “blatant dishonesty”—in the terminology of Cowell (1985). Emran and Stiglitz (2005) focus on the topic of value-added tax evasion in developing countries, and argue that, while formal firms pay their taxes, informal firms evade them completely, thus becoming part of the underground economy. Or, in the example of Corchón (1992), a taxpayer may prefer to conceal the entire income rather than under-report it if, during the tax filing process, the provision of personal information can be used as a signal by the tax authority.

<sup>5</sup>Note that, these functions may be affected by a number of parameters related, for example, to the efficiency of legal institutions or the availability of networks and financial systems to shelter money. In the numerical examples below, we will make the cost functions depend on the ability of the taxpayer to conceal income and on the ability/efficiency of the audit system employed by the agency. To

simplify the notation, we will not indicate the parameters in the general formulation.

<sup>6</sup>To simplify, we assume that this probability does not depend *directly* on the tax rate set by the government and on the income concealed by the taxpayer.

<sup>7</sup>The Tullock contest function is especially suited for analyzing the effect of prize allocations on incentives in rank-order tournaments or other forms of highly competitive situations.

<sup>8</sup>As in Rablen (2014), we thus distinguish between audit effectiveness and audit probability, which can have different effects on compliance.

<sup>9</sup>See Amir et al. (2023) for a case in which the tax authority maximizes social welfare.

<sup>10</sup>In Richter and Boadway (2005), the authors analyze the distortions on the taxpayer’s behavior, in terms of risk taking, induced by the auditing process.

<sup>11</sup>In (6), there is no (first-order) effect of a marginal change in auditing effort on revenue, as the condition for a Nash equilibrium in the third stage in (4) requires this to equal zero. That is, if government and tax agency maximize the same objective function, the envelope theorem applies.

<sup>12</sup>It can be shown that the theoretical conclusions would remain unaffected if we were to consider gross rather than net revenues and if monetary penalties were excluded from the tax collection gains.

<sup>13</sup>This argument, along with others, has been surveyed by Sandmo (2005).

<sup>14</sup>Individuals may underreport their personal wealth in other strategic contexts. For example, in credit markets borrowers may choose the amount of wealth to pledge as collateral to shield assets from creditors with considerable market power, as shown by Capasso and Jappelli (2013) and Distefano and Reito (2025).

<sup>15</sup>On this topic see Straub (2005) and Distefano (2025).

<sup>16</sup>This result is in line with the recent study of Amir et al. (2025), who show that the individual propensity to engage in criminal activities increases with the severity of the punishment.

<sup>17</sup>We show in subsection 5.3 that the invariance result is confirmed with regard to the effects of the penalty,  $s$ , even when the taxpayer is risk averse. In contrast, tax revenues may be not invariant to the audit frequency  $q$ , as it affects the probability of being caught and, in particular, the evader’s risk profile, for which evasion may be less attractive.

<sup>18</sup>Blank (2014) argues that non-pecuniary sanctions may be more effective in discouraging tax evasion as, among others reasons, they may create a social stigma from noncompliance. Kuchumova (2018) corroborates these findings, showing that by targeting indirect measures of income, such as consumption or the ability to engage in certain activities, non-pecuniary evasion penalties lead to a better redistribution of income.

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### Appendix A

*Proof of Lemma 1.* Let  $\Phi$  be the Hessian matrix of the taxpayer's utility function, that is

$$\Phi = \begin{bmatrix} \frac{\partial^2 U_E}{\partial e^2} & \frac{\partial^2 U_E}{\partial e \partial a} \\ \frac{\partial^2 R_E}{\partial e \partial a} & \frac{\partial^2 R_E}{\partial a^2} \end{bmatrix} = \begin{bmatrix} -C''(e) & -qp_{ea}(e, a)stm \\ xqp_{ea}(e, a)stm & -D''(a) \end{bmatrix},$$

which is negative-definite as, from the convexity of the effort cost functions, the minors are strictly negative, whereas the determinant,  $\det(\Phi) = C''(e)D''(a) + x[qp_{ea}(e, a)stm]^2$ , is strictly positive.

By applying the implicit function theorem to the system of first-order conditions in (3) and (4), the effect of a change in  $t$  on the taxpayer's concealment effort derives from

$$\frac{de^*}{dt} = - \frac{\det \begin{bmatrix} \frac{\partial^2 U_E}{\partial e \partial t} & \frac{\partial^2 U_E}{\partial e \partial a} \\ \frac{\partial^2 R_E}{\partial a \partial t} & \frac{\partial^2 R_E}{\partial a^2} \end{bmatrix}}{\det(\Phi)} = - \frac{\det \begin{bmatrix} -qp_e(e, a)sm & -qp_{ea}(e, a)stm \\ xqp_a(e, a)sm & -D''(a) \end{bmatrix}}{\det(\Phi)},$$

which reduces to

$$\frac{de^*}{dt} = -\varphi q [p_e(e, a)D''(a) + xqp_a(e, a)p_{ea}(e, a)stm]sm > 0, \tag{A1}$$

where

$$\varphi = \frac{1}{C''(e)D''(a) + x[qp_{ea}(e, a)stm]^2} > 0. \tag{A2}$$

In (A1), the term in the square brackets is negative as  $p_e(e, a)$  and  $p_{ea}(e, a)$  are negative, while  $p_a(e, a)$  is positive, as stated in Assumption 1. The denominator in (A2) is  $\det(\Phi) > 0$ . Therefore, the final effect of a variation of  $t$  on  $e^*$  is positive.

From the agency's first-order condition in (4), the effect of a variation in  $t$  on the agency's detection effort is

$$\frac{da^*}{dt} = -\frac{\det \begin{bmatrix} \frac{\partial^2 R_E}{\partial a \partial t} & \frac{\partial^2 R_E}{\partial e \partial a} \\ \frac{\partial^2 U_E}{\partial e \partial t} & \frac{\partial^2 U_E}{\partial e^2} \end{bmatrix}}{\det(\Phi)} = -\frac{\det \begin{bmatrix} -xqp_a(e, a)sm & xqp_{ea}(e, a)stm \\ -qp_e(e, a)sm & -C''(e) \end{bmatrix}}{\det(\Phi)},$$

which reduces to

$$\frac{da^*}{dt} = \varphi xq [p_a(e^*, a^*)C''(e) - qp_e(e^*, a^*)p_{ea}(e^*, a^*)stm]sm. \tag{A3}$$

The sign of (A3) depends on the magnitude of the tax rate. We will now show that the best-response function of the agency,  $a^*$ , is first increasing and then decreasing in  $t$  and reaches a maximum at  $t = \hat{t}_E$ . Specifically, at  $t = 0$ ,

$$\left. \frac{da^*}{dt} \right|_{t=0} = \frac{xqp_a(e^*, a^*)sm}{D''(a)} > 0, \tag{A4}$$

meaning that the equilibrium auditing effort is initially increasing in  $t$ . The sign of (A4) indicates that, at low levels of the tax rate, auditing activities and the tax rate set are strategic complements. Since from condition (A1) the taxpayer's concealment effort is monotonically increasing in the tax rate, the tax agency will need to exert a greater detection effort to catch evaders.

To show that  $a^*$  reaches a maximum at  $\hat{t}_E$ , we rewrite the first-order condition of the revenue function with respect to  $t$  in (4) as

$$t = -\frac{p(e^*, a^*)}{p_e(e^*, a^*) \frac{de^*}{dt}}.$$

From the separability of the probability function and substituting  $\frac{de^*}{dt}$ , it yields

$$t = \frac{p_a(e^*, a^*)C''(e)}{qp_e(e^*, a^*)p_{ea}(e^*, a^*)sm} \perp \frac{da^*}{dt} = 0. \tag{A5}$$

Thus, the best-response function of the agency is maximized at  $\hat{t}_E$ .

Finally, at  $t_E = 1$ , the revenue-maximizing tax rate,

$$\left. \frac{da^*}{dt} \right|_{t=1} = \varphi xq [p_a(e^*, a^*)C''(e) - qp_e(e^*, a^*)p_{ea}(e^*, a^*)stm]sm < 0, \tag{A6}$$

because the second—negative—term in square brackets prevails for  $t > \hat{t}_E$ . In this case, the detection effort and the tax rate are strategic substitutes. This is because, at high levels of the tax rate, it may become excessively costly to the agency to detect a taxpayer who is exerting a high concealment effort. Therefore, the agency may prefer to lower the intensity of monitoring, thereby making the taxpayer a weaker duelist, and impose a significantly high tax rate.  $\square$

*Proof of Proposition 1.* To prove that evasion may occur for low levels of  $t$ , we begin by analyzing the slope of  $U_{ME}$  and  $U_E$  at  $t = 0$ .

From (13),  $dU_{ME}/dt = -m$ , that is the marginal utility from random compliance is constant and negative for any  $t \in (0, \bar{t}]$ , as the marginal disutility increases linearly as the tax rate raises. The marginal utility from evasion, instead, depends on both the probability of detection and how the best response of the tax agency varies to marginal changes in  $t$ . For convenience, we rewrite here the marginal utility from evasion in (14)

$$\frac{dU_E}{dt} = -q \left[ p(e^*, a^*) + tp_a(e^*, a^*) \frac{da^*}{dt} \right] sm.$$

Evaluated (14) at  $t = 0$ , the second term inside the square brackets goes to zero. The first term instead depends on  $a^*$ . At  $t = 0$ , the first-order condition in (4) is strictly negative and the corner solution gives  $a^* = 0$ . This is because when the tax rate is infinitesimally small, it would be extremely costly to invest resources in detection, so no audit is conducted. From Assumption 2, condition in (15) applies, so that,  $U_{ME}$  decreases faster than  $U_E$ . Looking at the value of the functions, it is straightforward to see that evaluated at  $t = 0$ ,  $U_E = U_{ME}$ .

Given that the two utility functions assume the same value but the marginal utility from evasion is strictly lower than that from random compliance in absolute terms, it follows that, at  $t = 0$ ,  $U_E > U_{ME}$ .  $\square$

*Proof of Proposition 2.* From the Cramer's rule, applying the implicit function theorem to the system of first-order conditions in (3) and (4), the marginal variation in  $a^*$  to changes in  $x$  is given by

$$\frac{da^*}{dx} = -\frac{\det \begin{bmatrix} \frac{\partial^2 R_E}{\partial a \partial x} & \frac{\partial^2 R_E}{\partial e \partial a} \\ \frac{\partial^2 U_E}{\partial e \partial x} & \frac{\partial^2 U_E}{\partial e^2} \end{bmatrix}}{\det(\Phi)} = -\frac{\det \begin{bmatrix} qp_a(e, a)stm & xqp_{ea}(e, a)stm \\ 0 & -C''(e) \end{bmatrix}}{\det(\Phi)},$$

which reduces to

$$\frac{da^*}{dx} = \varphi qp_a(e^*, a^*)C''(e^*)stm > 0,$$

whose sign derives from the condition in Assumption 1 and the determinant of the Hessian matrix which is positive for quasi-concave functions. Thus, the sign in equation (16) follows. The proof of the Proposition can also be derived by analyzing the taxpayer evasion rule for which evasion occurs if  $U_E > U_{ME}$ . In terms of concealment and compliance costs, such a rule can be rewritten as

$$qp(e^*, a^*)stm + C(e^*) < tm,$$

that is, evasion occurs if the costs from income concealment are higher than the total tax burden. Total differentiating with respect to  $x$  yields

$$\frac{d[qp(e^*, a^*)stm + C(e^*) - tm]}{dx} = qp_a(e^*, a^*)stm \frac{da^*}{dx} > 0,$$

that is, a higher probability of positive income realization discourages evasion as it raises its cost.  $\square$

*Proof of Lemma 2.*

a. From the Cramer's rule, by applying the implicit function theorem to the system of first-order conditions in (3) and (4), the effect of a change in  $q$  on the taxpayer concealment effort is

$$\begin{aligned} \frac{de^*}{dq} &= -\frac{\det \begin{bmatrix} \frac{\partial^2 U_E}{\partial e \partial q} & \frac{\partial^2 U_E}{\partial e \partial a} \\ \frac{\partial^2 R_E}{\partial a \partial q} & \frac{\partial^2 R_E}{\partial a^2} \end{bmatrix}}{\det(\Phi)} \\ &= -\frac{\det \begin{bmatrix} -p_e(e, a)stm & -qp_{ea}(e, a)stm \\ xp_a(e, a)stm & -D''(a) \end{bmatrix}}{\det(\Phi)}, \end{aligned}$$

which reduces to

$$\frac{de^*}{dq} = -\varphi [p_e(e, a)D''(a) + xqp_a(e, a)p_{ea}(e, a)stm]stm > 0, \tag{A7}$$

where  $p_e(e, a) < 0$ ,  $p_a(e, a) > 0$  and  $p_{ea}(e, a) < 0$  from Assumption 1 and  $D''(a) > 0$  from the assumptions on the effort cost function. Similarly, the effect of a marginal variation in  $s$  on  $e^*$  derives from

$$\begin{aligned} \frac{de^*}{ds} &= -\frac{\det \begin{bmatrix} \frac{\partial^2 U_E}{\partial e \partial \theta} & \frac{\partial^2 U_E}{\partial e \partial a} \\ \frac{\partial^2 R_E}{\partial a \partial s} & \frac{\partial^2 R_E}{\partial a^2} \end{bmatrix}}{\det(\Phi)} \\ &= -\frac{\det \begin{bmatrix} -qp_e(e, a)tm & -qp_{ea}(e, a)stm \\ xqp_a(e, a)tm & -D''(a) \end{bmatrix}}{\det(\Phi)}, \end{aligned}$$

which is equal to

$$\frac{de^*}{ds} = -\varphi q [p_e(e, a)D''(a) + xqp_a(e, a)p_{ea}(e, a)stm]tm > 0, \tag{A8}$$

where the sign follows again from Assumption 1. So, the effect of both  $q$  and  $s$  on  $e^*$  is positive.

- b. Applying the implicit-function theorem to the first-order condition in (2), the effect of a change in  $q$  on  $\hat{t}_E$  is

$$\frac{d\hat{t}_E}{dq} = -\frac{xqp_e(e, a)\frac{de^*}{dq}sm}{xqp_e(e, a)\left(2\frac{de^*}{dt} + t\frac{d^2e^*}{dt^2}\right)sm} < 0,$$

where the numerator is negative as  $p_e(e, a) < 0$  from Assumption 1 and  $\frac{de^*}{dq} > 0$  as in (A7), whereas the denominator is negative as it is the second-order condition for a maximum which is always satisfied for an interior solution. Similarly,

$$\frac{d\hat{t}_E}{ds} = -\frac{xqp_e(e, a)\frac{de^*}{ds}sm}{xqp_e(e, a)\left(2\frac{de^*}{dt} + t\frac{d^2e^*}{dt^2}\right)sm} < 0,$$

where the numerator is negative as  $p_e(e, a) < 0$  from Assumption 1 and  $\frac{de^*}{ds} > 0$  from (A8). □

*Proof of Proposition 3.* Comparing expression in (A7) with expression in (A1), it is immediate to note that

$$qp_e(e, a)\frac{de^*}{dq} = tp_e(e, a)\frac{de^*}{dt}. \tag{A9}$$

Changing  $t$  and  $q$  while holding  $R_E$  fixed yields

$$\frac{dt}{dq} = -\frac{q \left[ p(e, a) + qp_e(e, a)\frac{de^*}{dq} \right]}{t \left[ p(e, a) + tp_e(e, a)\frac{de^*}{dt} \right]},$$

From (6), the terms in the square brackets of both the numerator and denominator are equal and the elasticity of substitution between  $t$  and  $q$  is

$$\frac{dt}{dq} \frac{t}{q} = -1.$$

Similarly, comparing (A8) with (A1), we get

$$sp_e(e, a)\frac{de^*}{ds} = tp_e(e, a)\frac{de^*}{dt}. \tag{A10}$$

Therefore, the value function of the tax take is invariant to both  $q$  and  $s$ . □

*Proof of Proposition 4.* The government's first-order condition for  $t$  in (20) can be rewritten as

$$p(e^*, a^*) = -\frac{h}{(1-h)qs} - tp_e(e^*, a^*)\frac{de^*}{dt}.$$

At the revenue-maximizing tax rate satisfying condition (A), the effect of  $s$  on total tax take reduces to

$$\left. \frac{dR_E}{ds} \right|_{t=\hat{t}_E} = -\frac{hxm\hat{t}_E}{q} < 0,$$

where the result follows from condition (A10) which also holds for the heterogeneous-type case. □

*Proof of Proposition 6.* Under risk aversion, the procedure to show that tax revenues are invariant to the penalty rates is similar to that used in the proof of Proposition 3, except for the terms  $de^*/dt$  in (A1) and  $de^*/ds$  in (A8). In particular, if the taxpayer is risk-averse, the effect of a marginal variation in  $t$  on the optimal concealment effort is

$$\frac{de^*}{dt} = -\frac{\det \begin{bmatrix} -qp_e(e, a)u'(m-stm)sm & -qp_{ea}(e, a)[u(m) - u(m-stm)] \\ xqp_a(e, a)sm & -D''(a) \end{bmatrix}}{\varphi},$$

which reduces to

$$\frac{de^*}{dt} = -\varphi q \{ p_e(e, a)u'(m-stm)D''(a) + xp_a(e, a)p_{ea}(e, a)[u(m) - u(m-stm)] \} sm > 0, \tag{A11}$$

where

$$\varphi = \frac{1}{C''(e)D''(a) + xq^2p_{ea}(e, a)^2[u(m) - u(m-stm)]stm} > 0.$$

Whereas the effect of a change in  $s$  on  $e^*$  is

$$\frac{de^*}{ds} = -\frac{\det \begin{bmatrix} -qp_e(e, a)u'(m-stm)tm & -qp_{ea}(e, a)[u(m) - u(m-stm)] \\ xqp_a(e, a)tm & -D''(a) \end{bmatrix}}{\varphi},$$

which is equal to

$$\frac{de^*}{ds} = -\varphi q \{ p_e(e, a) u'(m - stm) D''(a) + x p_a(e, a) p_{ea}(e, a) [u(m) - u(m - stm)] \} tm > 0. \tag{A12}$$

Comparing (A11) with (A12), it can easily be noted that

$$s p_e(e, a) \frac{de^*}{ds} = t p_e(e, a) \frac{de^*}{dt}.$$

Therefore, the value function of the tax take is invariant to  $s$ .

As for the audit efficiency, the effect of a change in  $q$  on  $e^*$

$$\frac{de^*}{dq} = - \frac{\det \begin{bmatrix} -p_e(e, a) u'(m - stm) & -q p_{ea}(e, a) [u(m) - u(m - stm)] \\ x p_a(e, a) stm & -D''(a) \end{bmatrix}}{\varphi},$$

which is equal to

$$\frac{de^*}{dq} = -\varphi [u(m) - u(m - stm)] [p_e(e, a) D''(a) + x q p_a(e, a) p_{ea}(e, a) stm] > 0.$$

The effect of  $q$  on revenues  $R_E = R_E(e^*, a^*, \hat{t}_E|q)$  is

$$\frac{dR_E}{dq} = \frac{\partial R_E}{\partial q} + \frac{\partial R_E}{\partial e} \frac{de^*}{dq} = x \left[ p(e^*, a^*) + q p_e(e^*, a^*) \frac{de^*}{dq} \right] \tag{A13}$$

where at  $\hat{t}_E$ ,  $p(e^*, a^*) = -t p_e(e^*, a^*) \frac{de^*}{dt}$ . Replacing  $p(e^*, a^*)$  into (A13), it yields

$$\frac{dR_E}{dq} = -\varphi x q [u(m) - u(m - stm) - u'(m - stm) stm] p_e(e, a)^2 D''(a) stm,$$

which is ambiguous. □