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Ginzburg–Landau approach to the three flavor LOFF phase of QCD

R. Casalbuoni ^a, R. Gatto ^b, N. Ippolito ^c, G. Nardulli ^c, M. Ruggieri ^c^a *Dipartimento di Fisica, Università di Firenze, and INFN, Sezione di Firenze, I-50019 Firenze, Italy*^b *Département de Physique Théorique, Université de Genève, CH-1211 Genève 4, Switzerland*^c *Università di Bari, and INFN, Sezione di Bari, I-70126 Bari, Italy*

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Abstract

We explore, using a Ginzburg–Landau expansion of the free energy, the Larkin–Ovchinnikov–Fulde–Ferrell (LOFF) phase of QCD with three flavors, using the NJL four-fermion coupling to mimic gluon interactions. We find that, below the point where the QCD homogeneous superconductive phases should give way to the normal phase, Cooper condensation of the pairs u – s and d – u is possible, but in the form of the inhomogeneous LOFF pairing.

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1. Introduction

At high quark density and small temperatures Quantum Chromo-Dynamics (QCD) predicts Cooper pairing of quarks due to the existence of an attractive quark interaction in the color antisymmetric channel, see [1–3] and for reviews [4,5]. At extreme densities the energetically favored phase is the Color–Flavor–Locking (CFL) phase, characterized by a spin 0 diquark condensate antisymmetric in both color and flavor [6]; at intermediate densities the situation is much more involved, because one cannot neglect the strange quark mass and the differences $\delta\mu$ in the quark chemical potentials induced by β equilibrium. Several ground states have been considered in the literature, from the 2SC phase [2], to the gapless phases g2SC [7] and gCFL [8,9]. The gapless phases are unstable as shown by imaginary gluon Meissner masses (for g2SC see [10], for gCFL see [11,12]). This seems to be connected

E-mail address: giuseppe.nardulli@ba.infn.it (G. Nardulli).

to the existence of gapless modes in these phases [13]. An instability is present also in the 2SC phase [10]. Though this phase has no gapless mode, imaginary gluon masses are present when the gap Δ and $\delta\mu$ satisfy the condition $\Delta/\sqrt{2} \leq \delta\mu \leq \Delta$.

Another superconductive state discussed in the literature is the Larkin–Ovchinnikov–Fulde–Ferrell (LOFF) [14] phase. The relevance of this phase is based on the possibility that, for appropriate values of $\delta\mu$, it can be advantageous for quarks to form pairs with non-vanishing total momentum: $\mathbf{p}_1 + \mathbf{p}_2 = 2\mathbf{q} \neq 0$, see [15,16] and for a review [17]. As far as instability is concerned, the authors in [18] have shown that, with two flavors, the instability of 2SC implies that the LOFF phase is energetically favored. Moreover, in the LOFF phase with two flavors the gluon Meissner masses are real [19].

Thus far only the case of two species for the LOFF phase has been studied. This is not justified in QCD. At intermediate densities all the three quarks: u , d and s should be considered. The three flavor problem is, however, much more involved and difficult to work out. We present here a first attempt to study the three flavor LOFF phase of QCD. Our approach is based on a Ginzburg–Landau (GL) expansion of the free energy. Differently from the CFL phase, where quark matter is in β equilibrium while being also electrically and color neutral, here we should impose these conditions. We shall consider in the sequel only β -equilibrated and electrically neutral quark matter, while assuming that the color chemical potentials vanish. This is an approximation we discuss below.

2. Gap equation

To get the gap equation in the Ginzburg–Landau approximation, we start with the Lagrangean density for three flavor ungapped quarks:

$$\mathcal{L} = \bar{\psi}_{i\alpha} (i\not{D}_{ij}^{\alpha\beta} - M_{ij}^{\alpha\beta} + \mu_{ij}^{\alpha\beta} \gamma_0) \psi_{\beta j}. \quad (1)$$

$M_{ij}^{\alpha\beta} = \delta^{\alpha\beta} \text{diag}(0, 0, M_s)$ is the mass matrix and $D_{ij}^{\alpha\beta} = \partial\delta^{\alpha\beta}\delta_{ij} + igA_a T_a^{\alpha\beta}\delta_{ij}$, $\mu_{\alpha\beta}^{ij}$ is a diagonal color–flavor matrix depending, in general, on μ (the average quark chemical potential), μ_e (the electron chemical potential), and μ_3, μ_8 , related to color [8]. We do not require color neutrality and we work in the approximation $\mu_3 = \mu_8 = 0$, which might be justified by the results of Ref. [8] for the gCFL phase showing that μ_3 and μ_8 assume, in general, small values (at least in the region of interest, see later). Therefore, in this Letter,

$$\mu_{ij}^{\alpha\beta} = (\mu\delta_{ij} - \mu_e Q_{ij})\delta^{\alpha\beta} = \mu_i\delta_{ij}\delta^{\alpha\beta}, \quad (2)$$

where Q is the quark electric charge matrix.

We treat the strange quark mass at the leading order in the $1/\mu$ expansion; this corresponds to a shift in the chemical potential of the s quark: $\mu_s \rightarrow \mu_s - \frac{M_s^2}{2\mu}$. This is the same approximation used in Refs. [8,11] for the study of the gCFL phase. Therefore,

$$\mu_u = \mu - \frac{2}{3}\mu_e, \quad \mu_d = \mu + \frac{1}{3}\mu_e, \quad \mu_s = \mu + \frac{1}{3}\mu_e - \frac{M_s^2}{2\mu}. \quad (3)$$

Another approximation we employ is the High Density Effective Theory (HDET), see [20–22] and, for a review, [5]. Here one decomposes the quark momentum into a large component, proportional to μ , and a residual small component: $\mathbf{p} = \mu\mathbf{n} + \ell$; \mathbf{n} is a unit vector and ℓ is the small residual momentum. Moreover, one introduces \mathbf{n} -dependent fields $\psi_{\mathbf{n}}$ and $\Psi_{\mathbf{n}}$ by the Fourier decomposition

$$\psi(x) = \int \frac{d\mathbf{n}}{4\pi} e^{i\mu\mathbf{n}\cdot\mathbf{x}} (\psi_{\mathbf{n}}(x) + \Psi_{\mathbf{n}}(x)), \quad (4)$$

$\psi_{\mathbf{n}}$ and $\Psi_{\mathbf{n}}$ correspond to positive and negative energy solutions of the Dirac equation.

Substituting the expression (4) in the Eq. (1) one gets at the leading order in $1/\mu$

$$\mathcal{L} = \int \frac{d\mathbf{n}}{4\pi} \psi_{\mathbf{n},i\alpha}^\dagger (iV \cdot D_{ij}^{\alpha\beta} + \bar{\mu}_i \delta^{\alpha\beta} \delta_{ij}) \psi_{\mathbf{n},\beta j}, \quad (5)$$

where $V^\mu = (1, \mathbf{n})$, $\tilde{V}^\mu = (1, -\mathbf{n})$ and $\bar{\mu}_i = \mu_i - \mu$.

It is convenient to change the basis for the spinor fields by defining $\psi_A = (\psi_{ur}, \psi_{dg}, \psi_{bs}, \psi_{dr}, \psi_{ug}, \psi_{sr}, \psi_{ub}, \psi_{sg}, \psi_{db})$. This change of basis is performed by unitary matrices F_A , whose explicit expression can be found in Ref. [11]. To the Lagrangean in Eq. (5) we add a Nambu–Jona-Lasinio four fermion coupling treated in the mean field approximation. This corresponds to the same coupling and the same approximation used in Ref. [9]. The gap term in the resulting Lagrangean is conveniently treated by introducing the Nambu–Gorkov field

$$\chi_A = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_{\mathbf{n}} \\ C\psi_{-\mathbf{n}}^* \end{pmatrix}_A, \quad (6)$$

so that the Lagrangean reads

$$L = \frac{1}{2} \sum_{A,B} \int \frac{d\mathbf{n}}{4\pi} \int \frac{dE d\xi}{(2\pi)^2} \chi_A^\dagger \begin{pmatrix} (E - \xi + \bar{\mu}_A) \delta_{AB} & -\Delta_{AB}(\mathbf{r}) \\ -\Delta_{AB}^*(\mathbf{r}) & (E + \xi - \bar{\mu}_A) \delta_{AB} \end{pmatrix} \chi_B, \quad (7)$$

where E is the energy, $\xi \equiv \ell \cdot \mathbf{n}$ is the component of the residual momentum along \mathbf{n} and satisfies: $|\xi| < \delta$, with δ an ultraviolet cutoff. Moreover, $(\bar{\mu})_A = (\bar{\mu}_u, \bar{\mu}_d, \bar{\mu}_s, \bar{\mu}_d, \bar{\mu}_u, \bar{\mu}_s, \bar{\mu}_u, \bar{\mu}_s, \bar{\mu}_d)$.

We assume the pairing ansatz

$$\langle \psi_{i\alpha} C \gamma_5 \psi_{\beta j} \rangle = \sum_{I=1}^3 \Delta_I(\mathbf{r}) \epsilon^{\alpha\beta I} \epsilon_{ijI} \quad (8)$$

with

$$\Delta_I(\mathbf{r}) = \Delta_I \exp(2i\mathbf{q}_I \cdot \mathbf{r}). \quad (9)$$

In other words, for each inhomogeneous pairing we assume a Fulde–Ferrell ansatz; $2\mathbf{q}_I$ represents the momentum of the Cooper pair. The gap matrix Δ_{AB} in (7) can be expressed in terms of the three independent functions $\Delta_1(\mathbf{r})$, $\Delta_2(\mathbf{r})$, $\Delta_3(\mathbf{r})$ describing, respectively, d - s , u - s and u - d pairing. The explicit expression of Δ_{AB} can be found in [8,11].

To write down the gap equation it is useful to introduce the following components of the free quark propagator:

$$[S_0^{11}]_{AB} \equiv \frac{\delta^{AB}}{E - \xi + \bar{\mu}_A}, \quad [S_0^{22}]_{AB} \equiv \frac{\delta^{AB}}{E + \xi - \bar{\mu}_A}. \quad (10)$$

The quark propagator is the matrix

$$S_{AB} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}_{AB}, \quad (11)$$

whose components satisfy the Gorkov equations

$$S_{11} = S_0^{11} + S_0^{11} \Delta(\mathbf{r}) S_{21}, \quad S_{21} = S_0^{22} \Delta^*(\mathbf{r}) S_{11}. \quad (12)$$

S_{21} is the anomalous propagator involved in the gap equation.

The wave vectors \mathbf{q}_I should be derived by minimizing the free energy. We will fix the norms $|\mathbf{q}_I|$ by a minimization procedure. As to their directions, we will limit the analysis to four structures, choosing among them the one with the smallest value of the energy. The first structure has all \mathbf{q}_I along the positive z -axis. The structures 2, 3, 4 have, respectively, $\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3$ along the positive z -axis (the remaining two momenta along the negative z -axis).

This is obviously a limitation. It is justified by our final results that show the existence of a range of values of the strange quark mass where the LOFF phase, even with these limitations, is favored in comparison with other QCD phases.

The gap equation in the HDET formalism can be written as follows [5]:

$$\Delta_{AB}^*(\mathbf{r}) = i3GV^\mu \tilde{V}^\nu \sum_{C,D=1}^9 h_{AaC}^* h_{DbB} \int \frac{d\mathbf{n}}{4\pi} \int \frac{d^3\ell}{(2\pi)^3} \int \frac{dE}{2\pi} S_{21}(E, \ell)_{CD} g_{\mu\nu} \delta_{ab}, \quad (13)$$

where S_{21} is given in Eq. (12); in the above equation h_{DbB} is a Clebsch–Gordan coefficient. It is expressed by the formula $h_{DbB} = \text{Tr}[F_D^\dagger T_b F_B]$ in terms of the unitary matrices F_A used to write the quark fields as in (6), i.e., in the basis $A = 1, \dots, 9$. G is the Nambu–Jona-Lasinio coupling constant, of dimension mass^{-2} . In what follows, we shall get rid of G introducing the value of the CFL gap parameter Δ_0 as a measure of the strength of the interaction (see below, Eq. (18)).

3. Ginzburg–Landau expansion

Performing the Ginzburg–Landau expansion of the propagator

$$S_{21} = S_0^{22} \Delta^* S_0^{11} + S_0^{22} \Delta^* S_0^{11} \Delta S_0^{22} \Delta^* S_0^{11} + O(\Delta^5) \quad (14)$$

we get

$$\Delta_I = \Pi_I \Delta_I + \sum_J J_{IJ} \Delta_I \Delta_J^2 + O(\Delta^5), \quad I = 1, 2, 3. \quad (15)$$

Let us comment on the functions Π_I and J_{IJ} appearing in this expansion. Π_I are defined as follows: $\Pi_1 = \Pi(q_1, \delta\mu_{ds})$, $\Pi_2 = \Pi(q_2, \delta\mu_{us})$, $\Pi_3 = \Pi(q_3, \delta\mu_{ud})$, with

$$\delta\mu_{ud} \equiv \frac{\bar{\mu}_d - \bar{\mu}_u}{2} = \frac{\mu_e}{2}, \quad \delta\mu_{us} \equiv \frac{\bar{\mu}_s - \bar{\mu}_u}{2} = \frac{\mu_e}{2} - \frac{M_s^2}{4\mu}, \quad \delta\mu_{ds} \equiv \frac{\bar{\mu}_s - \bar{\mu}_d}{2} = -\frac{M_s^2}{4\mu} \quad (16)$$

and

$$\Pi(q, \delta\mu) = 1 + \frac{2G\mu^2}{\pi^2} \left(1 - \frac{\delta\mu}{2q} \log \left| \frac{q + \delta\mu}{q - \delta\mu} \right| - \frac{1}{2} \log \left| \frac{4(q^2 - \delta\mu^2)}{\Delta_0^2} \right| \right). \quad (17)$$

We note that Π is analogous to the function determining the behavior of the free energy in the GL approximation of the LOFF phase with two flavors. We have introduced the parameter Δ_0 to get rid of the ultraviolet cutoff δ . It is defined by

$$\Delta_0 \equiv 2\delta \exp \left\{ -\frac{\pi^2}{2G\mu^2} \right\}. \quad (18)$$

Δ_0 is equal to the CFL gap for $M_s = 0$ and $\mu_e = 0$ in the weak coupling limit, with no sextet condensation. As for J_{IJ} , we have, for the diagonal components: $J_{11} \equiv J_1 \equiv J(q_1, \delta\mu_{ds})$, $J_{22} \equiv J_2 \equiv J(q_2, \delta\mu_{us})$, $J_{33} \equiv J_3 \equiv J(q_3, \delta\mu_{ud})$, with

$$J(q, \delta\mu) = -\frac{G\mu^2}{2\pi^2} \frac{1}{q^2 - \delta\mu^2}. \quad (19)$$

The off-diagonal term J_{12} is

$$J_{12} = \frac{3}{2} \frac{G\mu^2}{\pi^2} \int \frac{d\mathbf{n}}{4\pi} \frac{1}{(2\mathbf{q}_1 \cdot \mathbf{n} + \mu_s - \mu_d)(2\mathbf{q}_2 \cdot \mathbf{n} + \mu_s - \mu_u)}, \quad (20)$$

J_{13} is obtained from J_{12} in (20) by the exchange $\mathbf{q}_2 \rightarrow \mathbf{q}_3$ and $\mu_s \leftrightarrow \mu_d$; J_{23} from J_{12} by $\mathbf{q}_1 \rightarrow \mathbf{q}_3$ and $\mu_s \leftrightarrow \mu_u$.

4. Free energy

Let us now consider the free energy Ω . It is obtained by integrating the gap equation. The result is

$$\Omega = \Omega_n + \sum_{I=1}^3 \left(\frac{\alpha_I}{2} \Delta_I^2 + \frac{\beta_I}{4} \Delta_I^4 + \sum_{J \neq I} \frac{\beta_{IJ}}{4} \Delta_I^2 \Delta_J^2 \right) + O(\Delta^6) \quad (21)$$

with

$$\Omega_n = -\frac{3}{12\pi^2} (\mu_u^4 + \mu_d^4 + \mu_s^4) - \frac{\mu_e^4}{12\pi^2}, \quad (22)$$

where the chemical potentials for quarks are defined in Eq. (3) and the coefficients are given by

$$\alpha_I = \frac{2(1 - \Pi_I)}{G}, \quad \beta_I = -\frac{2J_I}{G}, \quad \beta_{IJ} = -\frac{2J_{IJ}}{G}. \quad (23)$$

Electric neutrality is obtained by imposing the condition

$$-\frac{\partial \Omega}{\partial \mu_e} = 0, \quad (24)$$

which, together with the gap equations, gives, for each value of the strange quark mass, the electron chemical potential μ_e and the gap parameters Δ_I . Moreover, one should determine \mathbf{q}_I by searching for the energetically favored solution. This is a complex task as it would require the simultaneous solution of the previous equations (24) and (15) together with:

$$0 = \frac{\partial \Omega}{\partial q_I} = \Delta_I \frac{\partial \alpha_I}{\partial q_I} + \Delta_I \sum_{J=1}^3 \Delta_J^2 \frac{\partial \beta_{IJ}}{\partial q_I}, \quad I = 1, 2, 3. \quad (25)$$

Moreover, one should look for the most energetically favored orientations of the three vectors \mathbf{q}_I in space. A complete analysis is postponed to a future paper; as discussed above we have limited the analysis to the four structures characterized by all vectors \mathbf{q}_I parallel or antiparallel to the same axis. Even with this limitation we are able to prove that there exists a window of values of M_s where the LOFF phase is favored in comparison with other phases of QCD, as it will be seen below. As to the norms $|\mathbf{q}_I|$, since we work in the GL approximation, we can neglect the $\mathcal{O}(\Delta^2)$ terms in (25). As a consequence we simply get $\partial \alpha_I / \partial q_I = 0$, which, being identical to the condition for two flavors, gives the result $q_I = 1.1997 |\delta \mu_I|$ [14,15].

5. Results and discussion

Our results are summarized in Figs. 1–4. In Fig. 1 we give $\Omega_{\text{LOFF}} - \Omega_{\text{norm}}$ (in units 10^6 MeV^4) as a function of M_s^2/μ (in MeV) for the four structures considered above. The curves correspond to different minima of the free energy. The solid line corresponds to the structures 3 and 4 with $\Delta_1 = 0$, $\Delta_2 = \Delta_3$. Since it has the lowest free energy it is the energetically favored solution. Note that the structure 4 has \mathbf{q}_3 along the positive z -axis and $\mathbf{q}_1, \mathbf{q}_2$ in the opposite direction, but if $\Delta_1 = 0$ it cannot be distinguished from the structure 3. The dot-dashed, long-dashed, double-dot-dashed and dotted curves refer to the structures 1, 2, 3, and 4, respectively, and have, in general, $\Delta_2 \neq \Delta_3$. We shall see below that the kinematical range where the LOFF state is favored corresponds to $M_s^2/\mu > 115 \text{ MeV}$. In this range there is no appreciable difference between the solid line, the double-dot-dashed and the dotted line. The results in this figure and in the subsequent ones are obtained for $\mu = 500 \text{ MeV}$ (for $\mu = 400 \text{ MeV}$ the results are qualitatively similar). The value of the CFL gap for zero strange quark mass is fixed at $\Delta_0 = 25 \text{ MeV}$. This is the same value used in Ref. [9]. This choice, as well as the same form of the NJL coupling,

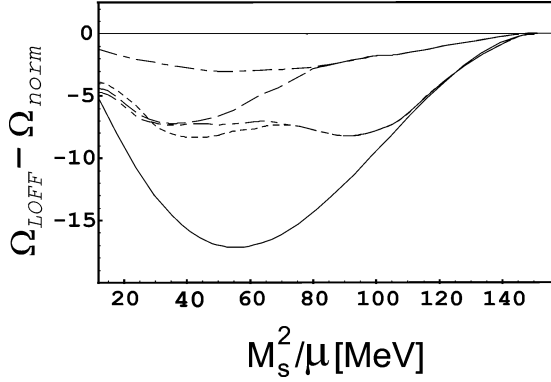


Fig. 1. Free energy differences $\Omega_{\text{LOFF}} - \Omega_{\text{norm}}$ in units of 10^6 MeV^4 plotted versus M_s^2/μ (in MeV). The curves correspond to different minima of the free energy. The results are obtained for $\mu = 500 \text{ MeV}$ and $\Delta_0 = 25 \text{ MeV}$. The solid line corresponds to the structures 3 and 4 with $\Delta_1 = 0$, $\Delta_2 = \Delta_3$. The dot-dashed, long-dashed, double-dot-dashed and dotted refer to the structures 1, 2, 3, and 4, respectively, and have, in general, $\Delta_2 \neq \Delta_3$.

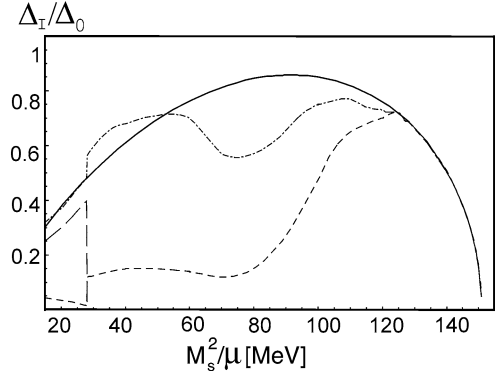


Fig. 2. Gaps Δ_I/Δ_0 as functions of M_s^2/μ (in MeV). The solid line represents the solution $\Delta_2 = \Delta_3$ for the structures 3 and 4 (in this case $\Delta_1 = 0$). It corresponds to the solid line in Fig. 1. The other curves are as follows: dashed line: Δ_1 ; dot-dashed line: Δ_2 ; dotted line, Δ_3 . These solutions of the gap equations correspond to the double dot-dashed and dotted curves of the free energy in Fig. 1. The discontinuity at $M_s^2/\mu \sim 30 \text{ MeV}$ corresponds to the change from the structure 3 to the structure 4 (see text).

with the same approximation, allows a comparison between our results and those of Ref. [9], see the discussion below.

In Fig. 2 we give the gaps Δ_I/Δ_0 as functions of M_s^2/μ (in MeV). The solid line represents the solution $\Delta_2 = \Delta_3$ for the structures 3 and 4 (in this case $\Delta_1 = 0$). It corresponds to the solid line in Fig. 1. The other lines represent other solutions of the gap equations. The dashed line is Δ_1 , dot-dashed line is Δ_2 and the dotted line Δ_3 , corresponding to the double dot-dashed and dotted curves of the free energy in Fig. 1. The discontinuity at $M_s^2/\mu \sim 30 \text{ MeV}$ corresponds to the change from the structure 3 to the structure 4. However, as discussed below, for these values of M_s^2/μ the favored phase is the CFL state, not the LOFF state. We also notice that, for M_s^2/μ larger than 30 MeV, Δ_1 vanishes. For this reason above this threshold we have either u - s or u - d pairing; \mathbf{q}_1 plays no role, which is why the structures 3 and 4, that would differ only for the role of \mathbf{q}_1 , have the same free energy. Again we note that in the region where the LOFF state is favored the solution is unique and has $\Delta_1 = 0$, $\Delta_2 \approx \Delta_3$.

In Fig. 3 we present results for the electron chemical potential μ_e . The solid line corresponds to the energetically favored solution, the dotted line corresponds to the other solution for the structure 3 \equiv 4. In the region where the LOFF state prevails ($M_s^2/\mu > 115 \text{ MeV}$, see below) they are almost identical, within the approximations (we have neglected terms suppressed in the $1/\mu$ expansion, consistently with the HDDET scheme). A glance at Eq. (16) shows that μ_e is given by $\mu_e \approx M_s^2/(4\mu)$, which corresponds to a symmetric splitting of the s and d Fermi surfaces around the u Fermi surface. Therefore, in this kinematical region we have us and du pairings, with

$$\mathbf{p}_u + \mathbf{p}_s = 2\mathbf{q}_2, \quad \mathbf{p}_u + \mathbf{p}_d = 2\mathbf{q}_3 \approx -2\mathbf{q}_2.$$

The gaps Δ_2 (us pairing) and Δ_3 (ud pairing) have to be almost equal since they depend only on the absolute values of the splittings. This is confirmed by Fig. 2. Since the separation between the d and s Fermi surfaces is higher, one does not expect ds pairing, which is confirmed again by Fig. 2 ($\Delta_1 = 0$ in this region).

In Fig. 4 we present comparison of different phases of QCD. In order to comment this figure, let us start assuming that all the other phases are stable, meaning that in some way it is possible to cure the instability due to the imaginary gluon masses. In this case, following the graph for decreasing values of M_s^2/μ , we see that at about $M_s^2/\mu = 150 \text{ MeV}$ the LOFF phase has a free energy lower than the normal one. This is a second-order transition as it can be seen from Fig. 2. Then the LOFF state is energetically favored till the point where it meets the gCFL line at about $M_s^2/\mu = 115 \text{ MeV}$. This is a first-order transition since all the gaps are different in the two phases

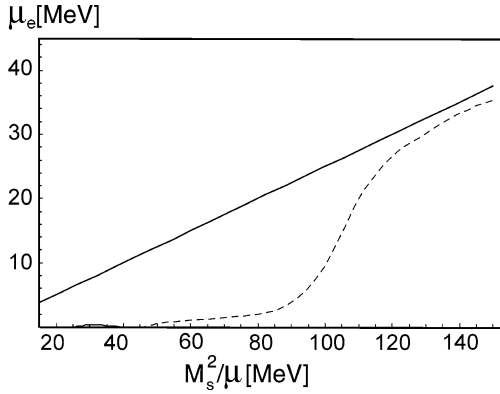


Fig. 3. The electron chemical potential μ_e as a function of M_s^2/μ . Units are MeV. Solid line corresponds to the favored solution with $\Delta_2 = \Delta_3$; dotted line to the solution with $\Delta_2 \neq \Delta_3$.

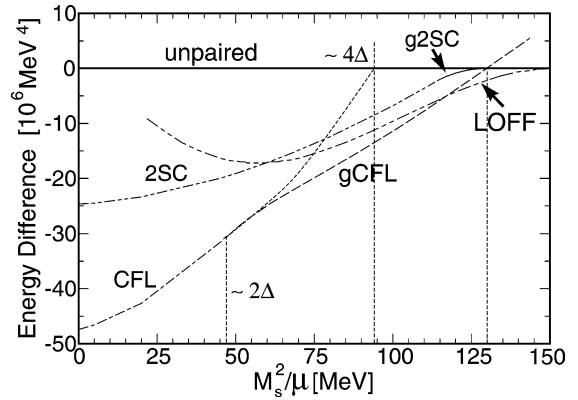


Fig. 4. Free energy differences $\Omega_{\text{LOFF}} - \Omega_{\text{norm}}$ in units of 10^6 MeV^4 plotted versus M_s^2/μ (in MeV) for various QCD phases.

(for the gCFL case, see [8]). Then the system stays in the gCFL phase up to $M_s^2/\mu \approx 48 \text{ MeV}$ where it turns into the CFL phase via a second-order transition (see [8]).

However, if the gapless phases are unstable, then they should not be considered, and the LOFF phase is the stable phase from $M_s^2/\mu = 150 \text{ MeV}$ down to about $M_s^2/\mu = 75 \text{ MeV}$ where the LOFF line meets the CFL line, with a first-order transition (this can be seen by comparing our gaps with the $\Delta_{\text{CFL}} \approx 23 \text{ MeV}$ at this value of M_s^2/μ).

We should also add that at the moment it is still unknown if the LOFF phase with three flavors suffers of chromo-magnetic instabilities. This problem is left to future investigations.

6. Conclusion and outlook

We have explored in the framework of the Ginzburg–Landau expansion the LOFF phase of QCD with three flavors, using the NJL four-fermion coupling to mimic the gluon interactions. We have worked on the ansatz of a single plane wave behaviour for each quark pairing, which is the simplest generalization of the gCFL phase that takes into account the possibility of anisotropic condensation. We found that near the point where the CFL phase should give way to the normal phase, Cooper condensation takes place in the form of the LOFF pairing. Our analysis has some limitations. First, we have assumed vanishing color chemical potentials μ_3, μ_8 . The results of Ref. [9] show that in the region where the LOFF state dominates the color chemical potential have rather small values, in particular smaller than μ_e . However, non-vanishing values of μ_3 and μ_8 are expected to increase the LOFF free energy and therefore a more complete calculation is necessary. Second, we have considered the three possible momenta \mathbf{q}_I all along the same direction. Third, more than one plane wave might be present in each condensate. Finally, we have treated the strange quark mass at its leading effect, i.e., by a shift in its chemical potential, which is also an approximation [22]. We plan to address all these issues by a more refined study in the future.

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Erratum

Erratum to: “Ginzburg–Landau approach to the three flavor LOFF phase of QCD” [Phys. Lett. B 627 (2005) 89]

R. Casalbuoni ^{a,b}, R. Gatto ^c, N. Ippolito ^{d,e}, G. Nardulli ^{d,e,*}, M. Ruggieri ^{d,e}

^a *Dipartimento di Fisica, Università di Firenze, I-50019 Firenze, Italy*

^b *INFN, Sezione di Firenze, I-50019 Firenze, Italy*

^c *Département de Physique Théorique, Université de Genève, CH-1211 Genève 4, Switzerland*

^d *Università di Bari, I-70126 Bari, Italy*

^e *INFN, Sezione di Bari, I-70126 Bari, Italy*

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Eq. (20) of the Letter should be substituted by the following one:

$$J_{12} = \frac{G\mu^2}{\pi^2} \int \frac{d\mathbf{n}}{4\pi} \frac{1}{(2\mathbf{q}_1 \cdot \mathbf{n} + \mu_s - \mu_d - i\epsilon)(2\mathbf{q}_2 \cdot \mathbf{n} + \mu_s - \mu_u - i\epsilon)}.$$

Although this correction does not affect the main conclusion of the Letter, the following changes take place:

The favored configurations are now 1 and 2, with $\mathbf{p}_d + \mathbf{p}_s = 0$, $\mathbf{p}_u + \mathbf{p}_s = 2\mathbf{q}_2$, $\mathbf{p}_u + \mathbf{p}_d = 2\mathbf{q}_3 = 2\mathbf{q}_2$. The results for the other configurations are practically unchanged. On the other hand the configurations 3 and 4, with \mathbf{q}_2 and \mathbf{q}_3 antiparallel, are disfavored. This can be understood in simple terms as follows. In the Ginzburg–Landau limit, if \mathbf{q}_2 , \mathbf{q}_3 are parallel, the pairing region on the u -quark surface is formed by two distinct rings (in the northern and southern hemisphere, respectively), while for antiparallel \mathbf{q}_2 , \mathbf{q}_3 the two rings overlap, which reduces the phase space available for pairing.

In Fig. 1 only the two upper lines (configurations 1 and 2) remain, whereas the other three should be discarded. Since in the region of interest the free energy of configurations 1 and 2 coincide (for $M_s^2/\mu > 80$ MeV) and $\Delta_1 = 0$, $\Delta_2 = \Delta_3$, for the discussion we consider only this situation.

Fig. 2 should be substituted by one with a single line corresponding to the gap $\Delta_2 = \Delta_3$. This line can be obtained by the continuous line of Fig. 2 by scaling the maximum by a factor 0.42.

In Fig. 3 only the continuous line survives.

Fig. 4 is now as shown on the next page of this erratum.

The transition between gCFL and LOFF occurs at about $M_s^2/\mu = 128$ MeV (previously 115 MeV). On the other hand the critical point where the LOFF phase goes to the normal one remains at 150 MeV. Therefore the conclusions of the Letter remain unchanged.

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* Corresponding author.

E-mail address: giuseppe.nardulli@ba.infn.it (G. Nardulli).

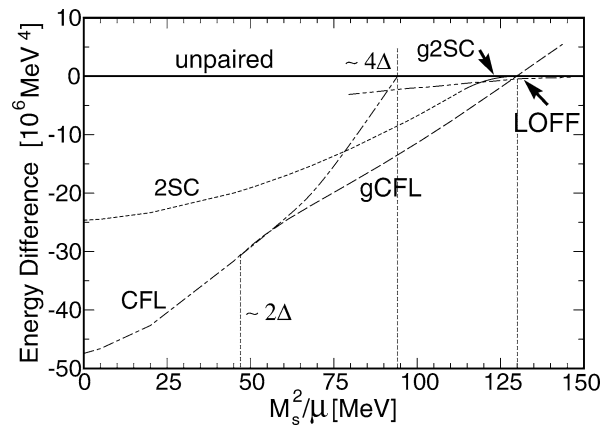


Fig. 4. Free energy differences $\Omega_{\text{LOFF}} - \Omega_{\text{norm}}$ in units of 10^6 MeV^4 plotted versus M_s^2/μ (in MeV) for various QCD phases.