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Simplified evaluation of the shear strength of slender rectangular r.c. members with shear reinforcement

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Abstract

This paper describes a simple analytical tool for the calculation of the shear strength of r.c. rectangular members with shear reinforcement and subjected to only truss action. The proposed method considers simplified stress fields and evaluates the shear strength of members subjected to axial force, bending moment and shear force by means of the application of the static theorem of limit analysis. Unlike most of the formulations proposed in codes and in other research studies, the proposed method considers a single physical model to explain the resistance to axial force, bending moment and shear force, and simultaneously satisfies the equilibrium under all the above internal forces. The paper identifies basic points of the N-M-V ultimate domain and reports relations and procedures to calculate the values of the internal forces of these points as well as the internal forces of the points in between. The method is applied to a set of beam and column members and a comparison is drawn between the shear strength resulting from the simplified method and that from a more complex non-linear mathematical program proposed in the past by the same author. Finally, to prove the value of the method and define the field of reliable application, the proposed method is applied to members tested in laboratory by other researchers and characterised by different geometric and mechanical properties.

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Keywords: Reinforced concrete; rectangular cross-section; shear force; axial force; bending moment

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1. Physical model

The considered method simulates the sole truss action and is applied to members characterised by a rectangular cross-section endowed with longitudinal and transverse reinforcements (Fig. 1). The member is subjected to constant axial force and to bending moment and shear force. The longitudinal steel bars are distinguished into flange and web bars. The flange bars are concentrated at the centroid of their cross-section. The longitudinal bars of the web are distributed over the cross-sectional area between the longitudinal bars of the opposite flanges and are characterised by the reinforcement ratio $\rho_{\mathrm{lw}} = A_{\mathrm{slw}} / b\left(h - c_1 - c_2\right)$, where A_{slw} is the cross-sectional area of the longitudinal bars of the web, b is the width of the cross-section, h is the depth of the cross-section, c_1 is the mechanical cover to the longitudinal reinforcement in tension, i.e. the distance between the external surface of the cross-section and the centroid of the entire longitudinal reinforcement of the flange in tension, and c_2 is the mechanical cover to the longitudinal reinforcement in compression.

The transverse reinforcement consists of hoops and ties. The cross-sectional area of this reinforcement is considered through the transverse reinforcement ratio $\rho_{\mathrm{sw}} = A_{\mathrm{sw}} / b\,s$, where s is the spacing of the hoops and A_{sw} is the projection of the cross-sectional area of the transverse reinforcement per layer in the direction of the applied shear force.

The cross-section of the member is divided into three parts, named F_1, F_2 and F_3, as shown in Figure 1a. In each of these parts the response of concrete and steel is defined by means of simplified stress fields. The stress-strain constitutive behaviour of concrete and steel is considered to be perfectly plastic. However, while longitudinal and transverse steel bars are assumed to resist both compression and tension, concrete is assumed to resist compression only. In the following sections, steel stresses are considered positive when tensile whereas stresses of concrete are considered positive when compressive. The geometry of zones F_1, F_2 and F_3 of the cross-section is identified by the separation lines of the central part F_3. The position of these two lines is defined by coordinates y_1 and y_2 (see Fig. 1a).

1.1. Stress fields

In the outermost parts of the cross-section (called F_1 and F_2) longitudinal reinforcement and concrete are subjected to stresses that are parallel to the longitudinal member axis. The stresses of the longitudinal (flange and web) reinforcement in F_1 and F_2 are called $\sigma_{\ell 1}$ and $\sigma_{\ell 2}$, respectively. The stresses of concrete in zones F_1 and F_2 are called σ_{c1} and σ_{c2}. Stresses $\sigma_{\ell 1}$, $\sigma_{\ell 2}$, σ_{c1} and σ_{c2} are constant within the single part of the cross-section (i.e. F_1 and F_2).

In the central part of the cross-section (called F_3) longitudinal and transverse reinforcements experience stresses that are constant and parallel to the axis of the steel bars. In particular, the stress field relative to the transverse reinforcement is inclined at an angle equal to 90° with respect to the longitudinal axis of the member as hoops and ties are assumed here orthogonal to the longitudinal axis of the member. Still in F_3, concrete is assumed to experience compressive stresses σ_{c3} inclined at an unknown angle θ with respect to the longitudinal member axis. To derive the

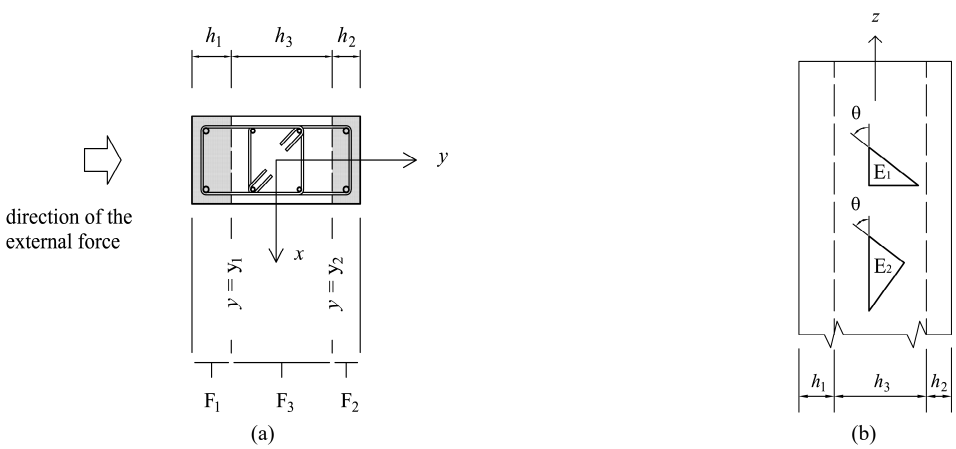

Figure 1. (a) Parts F1, F2 and F3 of the cross-section and (b) elements E1 and E2 of the member

equilibrium equations that involve stresses of concrete and steel of the central part of the cross-section (F_3) two elements are considered (E_1 and E_2), which are obtained by cutting the reinforced concrete member by means of three (see Fig. 1b). Element E_1 is obtained by cutting the member with one plane parallel to the compressive stress of concrete and with two other planes that are parallel and orthogonal to the longitudinal member axis. This element is subjected to the stress $\sigma_{\ell 3}$ of the longitudinal reinforcement, to the stress σ_{s3} of the hoops, to the tangential stress τ and to the equivalent normal stress σ_3 of concrete and steel bars.

The equilibrium equations along y and z-axes give

$$\sigma_{s3}\,\rho_{sw}\cos\theta - \tau\sin\theta = 0 \qquad (1)$$

$$\sigma_{\ell 3}\,\rho_{lw}\sin\theta - \tau\cos\theta - \sigma_3\sin\theta = 0 \qquad (2)$$

To obtain equilibrium equations involving the normal stress of concrete σ_{c3} the element E_2 is considered, which is obtained by cutting the member with two planes parallel and orthogonal to the compressive stress of concrete and with a plane parallel to the longitudinal member axis. This element is subjected to the stress σ_{s3} of the hoops and to the compressive stress σ_{c3} of concrete. The condition $\sigma_y = 0$ applied to this element states that

$$\sigma_{s3}\,\rho_{sw} - \sigma_{c3}\sin^2\theta = 0 \qquad (3)$$

The normal stress of concrete in F_3 in the direction of the longitudinal member axis ($\sigma_{c3,l}$) is linked to the compressive stress σ_{c3} by means of the relation $\sigma_{c3,l} = \sigma_{c3}\cos^2\theta$.

1.2. Internal forces

The axial force of the generic cross-section is calculated as the sum of the contributions of the parts F_1, F_2 and F_3. Specifically, the first contribution N_1 is given by the expression

$$N_1 = \sigma_{\ell 1}\left[A_{slf1} + \rho_{lw}\, b\left(y_{\ell 1,lim} + y_1\right)\right] - \sigma_{c1} b\left(y_{c,lim} + y_1\right) \qquad (4)$$

where A_{slf1} is the cross-sectional area of the longitudinal bars of the flange in tension, $y_{c,lim}$ is the distance from the geometric centre of the cross-section to the external surface of the cross-section and $y_{\ell 1,lim}$ is the distance from the geometric centre of the cross-section to the centroid of the longitudinal bars of the flange in tension.

The second contribution N_2 is defined by the relation

$$N_2 = \sigma_{\ell 2}\left[A_{slf2} + \rho_{lw}\, b\left(y_{\ell 2,lim} - y_2\right)\right] - \sigma_{c2} b\left(y_{c,lim} - y_2\right) \qquad (5)$$

where A_{slf2} is the cross-sectional area of the longitudinal bars of the flange in compression and $y_{\ell 2,lim}$ is the distance from the geometric centre of the cross-section to the centroid of the longitudinal bars of the flange in compression.

The third contribution N_3 is defined by the relation

$$N_3 = b\left(y_2 - y_1\right)\left(\sigma_{\ell 3}\,\rho_{lw} - \sigma_{s3}\,\rho_{sw}\frac{\cos^2\theta}{\sin^2\theta}\right) \qquad (6)$$

Similar to the axial force, the bending moment of the generic cross-section is calculated as the sum of the contributions of the parts F_1, F_2 and F_3. These contributions may be evaluated by means of the following relations

$$M_1 = \sigma_{\ell 1}\left[A_{\mathrm{slf1}} y_{\ell 1,\mathrm{lim}} + \frac{\rho_{\mathrm{lw}} b}{2}\left(y_{\ell 1,\mathrm{lim}}^2 - y_1^2\right)\right] - \sigma_{\mathrm{c1}}\frac{b}{2}\left(y_{\mathrm{c,lim}}^2 - y_1^2\right) \tag{7}$$

$$M_2 = -\sigma_{\ell 2}\left[A_{\mathrm{slf2}} y_{\ell 2,\mathrm{lim}} + \frac{\rho_{\mathrm{lw}} b}{2}\left(y_{\ell 2,\mathrm{lim}}^2 - y_2^2\right)\right] + \sigma_{\mathrm{c2}}\frac{b}{2}\left(y_{\mathrm{c,lim}}^2 - y_2^2\right) \tag{8}$$

$$M_3 = -\frac{b}{2}\left(y_2^2 - y_1^2\right)\left(\sigma_{\ell 3}\rho_{\mathrm{lw}} - \sigma_{\mathrm{s3}}\rho_{\mathrm{sw}}\frac{\cos^2\theta}{\sin^2\theta}\right) \tag{9}$$

Finally, the shear force V is given by the integral of the shear stresses in F_3, i.e.

$$V = \tau\, b\left(y_2 - y_1\right) \tag{10}$$

2. N-M-V ULTIMATE INTERACTION DOMAIN

The N-M-V ultimate domain is identified by the envelope of M-V interaction domains characterized by different values of the axial force. The construction of the single quadrant of each M-V domain is based on five points, later called PV, PM, P1, P2 and P3. As shown in Figure 1 (where the interactions curves are derived by means of the proposed simplified method with the design values of the mechanical properties of the materials and cotgθ in the range from 1 to 2.5), point PV and PM are at the ends of the quadrant as corresponding to a null bending moment or shear force, respectively. Point P2 identifies the point with the maximum shear strength. If more than one point is characterised by the same maximum shear strength, point P2 identifies the point of this group with the maximum bending moment. Point P3 identifies the point of the interaction domain with the same shear strength as point P2 but with the lowest bending moment. Point P1 does not correspond to the maximum value of either the internal forces. However, it identifies a change in the cross-section behaviour that has been noted when examining the results of the reference nonlinear mathematical programming problem (Rossi and Recupero 2013, Rossi 2013). The central zone F_3, which is generally quite large in P2 and has a null area in PM, does not shrink linearly from P2 to PM. In particular, in the first part of this route from P2 to PM one of the ending lines of the central zone F3 tends to remain close to one of the limit positions of F3 (i.e. the y-coordinate of this line is either $-y_{\ell 1,\mathrm{lim}}$ or $y_{\ell 2,\mathrm{lim}}$) while the other ending line slowly tends to the first one and reduces the area of the central zone. Then, the two ending lines move closer to each other and reach the common position corresponding to point PM. The point corresponding to the end of the first behaviour and to the beginning of the second is reported here as P1.

Owing to this, the generic M-V ultimate interaction curve may be characterised by a number of distinct basic points ranging from two to five.

3. Ultimate state of stress in zone F3

With the sole exception of PM, the shear force of the single basic point is calculated as the maximum of the shear forces corresponding to angles θ variable in an assigned range of values. Once a value of the angle θ has been selected, the normal stresses of concrete (σ_{c3}) and hoops (σ_s) in zone F_3 are calculated by Equation (3). To this end, the normal stress σ_{c3} is first fixed equal to the concrete strength under biaxial stress state and the normal stress σ_s is obtained by Equation (3). If this latter value is lower than the yield strength of the transverse reinforcement, the above values of σ_{c3} and σ_s are assumed as the normal stresses of concrete and hoops in zone F_3. If this is not the case, σ_s is assumed equal to the yield strength of the transverse reinforcement and σ_{c3} is obtained by Equation (3). The normal stress of concrete $\sigma_{c3,1}$ is calculated as $\sigma_{c3}\cos^2\theta$.

This paper shows how the internal forces corresponding to PV are calculated. For the internal forces corresponding to the other points and to the intermediate points, readers are referred to (Rossi, 2021).

4. Internal forces corresponding to point PV

Point Pv is characterised by a null bending moment and by an assigned axial force. The shear force corresponding to a value of the angle θ is evaluated taking into account six combinations of values of the variables $y_1, y_2, \sigma_{\ell 1}, \sigma_{\ell 2}, \sigma_{\ell 3}, \sigma_{c1}, \sigma_{c2}$ (see Table 1). In each combination, the values of one or two variables are not fixed as they are to be calculated case by case to ensure a null bending moment on the cross-section. These free variables are highlighted in Table 1 by grey hatches. The proposed combinations intend to maximize the size of zone F3 for assumed limit stresses of concrete or steel. The corresponding axial resistances are identified by $N_R^{(V,\text{i})}$ (i=1 to 6) and increase with the number in the superscript of the parameter. In particular, the first combination of the variables identifies the maximum axial compressive resistance whereas the latter identifies the maximum axial tensile resistance of the cross-section.

If the maximum stress resultant in the longitudinal reinforcement is symmetric with respect to the x-axis, the limits of zone F3 and the distributions of the normal stresses are symmetric with respect to the x-axis. If this is not the case, the proposed combinations allow limits of zone F3 and distributions of normal stresses that are non-symmetric. In particular, if the maximum contribution of A_{slf1} to the rotational equilibrium is not lower than that of A_{slf2}, i.e. $A_{\text{slf1}}\, f_y\, y_{\ell 1,\text{lim}} \geq A_{\text{slf2}}\, f_y\, y_{\ell 2,\text{lim}}$, the proposed combinations of $y_1, y_2, \sigma_{\ell 1}, \sigma_{\ell 2}, \sigma_{\ell 3}, \sigma_{c1}, \sigma_{c2}$ are reported in Table 1. The axial resistance $N_R^{(V,1)}$ is calculated assuming that $\sigma_{\ell 2}$ and $\sigma_{\ell 3}$ are equal to the compressive yield stress of steel while σ_{c2} is equal to the compressive strength of concrete f_c. The variables $y_1^{(V,1)}$ and $y_2^{(V,1)}$ are null if $A_{\text{slf1}}\, f_y\, y_{\ell 1,\text{lim}} = A_{\text{slf2}}\, f_y\, y_{\ell 2,\text{lim}}$ while they are equal to $-y_{\ell 1,\text{lim}}$ if $A_{\text{slf1}}\, f_y\, y_{\ell 1,\text{lim}} > A_{\text{slf2}}\, f_y\, y_{\ell 2,\text{lim}}$. The free variables σ_{c1} and $\sigma_{\ell 1}$ must be specified to provide a null bending moment. To this end, the stress $\sigma_{\ell 1}$ is first set equal to the compressive yield stress of steel and σ_{c1} is calculated by the following equation to ensure a null bending moment

$$\sigma_{c1} = \frac{M_2 + M_3 + \sigma_{\ell 1}\left[A_{\text{slf1}} y_{\ell 1,\text{lim}} + 0.5\,\rho_{\text{lw}}\, b\left(y_{\ell 1,\text{lim}}^2 - y_1^2\right)\right]}{b\left(y_{c,\text{lim}}^2 - y_1^2\right)} \tag{14}$$

where the bending moment contributions M2 and M3 are reported in Equations (8) and (9). If the value of σ_{c1} resulting from Equation (14) is positive and lower than f_c, it is accepted as the value of σ_{c1} and $\sigma_{\ell 1}$ is assumed equal to the compressive yield stress of steel. If this is not the case, σ_{c1} is set equal to the limit of the concrete strength (0 or f_c) nearer to the value obtained by the above calculation and $\sigma_{\ell 1}$ is calculated by means of the rotational equilibrium equation

$$\sigma_{\ell 1} = \frac{-M_2 - M_3 + \sigma_{c1}\, b\left(y_{c,\text{lim}}^2 - y_1^2\right)}{A_{\text{slf1}} y_{\ell 1,\text{lim}} + 0.5\,\rho_{\text{lw}}\, b\left(y_{\ell 1,\text{lim}}^2 - y_1^2\right)} \tag{15}$$

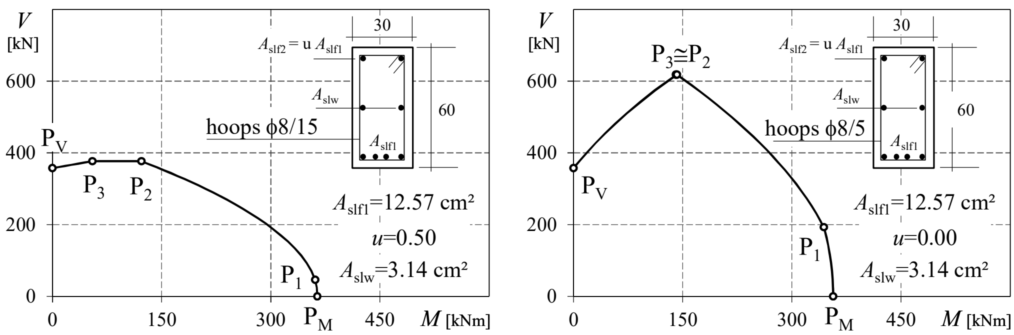

Figure 2. Simplified ultimate M-V interaction curves (f_c=30 MPa; f_{c2}=15.8 MPa; f_y=f_{yw}=450 MPa; N=0)

Table 1. Reference axial resistances for point P_V:
$A_{slf1} f_y y_{l1,lim} \geq A_{slf2} f_y y_{l2,lim}$ (Notes: grey hatches identify the free variables)

Symbol	y_1	y_2	σ_{l1}	σ_{l2}	σ_{l3}	σ_{c1}	σ_{c2}
$N_R^{(V,1)}$	$y_1^{(V,1)}$	$y_2^{(V,1)}$		$-f_y$	$-f_y$		f_c
$N_R^{(V,2)}$	$-y_{l1,lim}$			$-f_y$	$-f_y$	0	0
$N_R^{(V,3)}$	$-y_{l1,lim}$			0	0	0	0
$N_R^{(V,4)}$	$-y_{l1,lim}$			f_y	f_y	0	0
$N_R^{(V,5)}$	$y_1^{(V,5)}$	0		f_y	f_y	0	0
$N_R^{(V,6)}$	$y_1^{(V,6)}$	$y_2^{(V,6)}$		f_y	f_y	0	0

The axial resistance $N_R^{(V,2)}$ is determined assuming that the variable y_1 is equal to $-y_{\ell 1,lim}$, the stresses $\sigma_{\ell 2}$, $\sigma_{\ell 3}$ are equal to the compressive yield stress of steel and the stress of concrete in zones F_1 and F_2 is null. The free stress $\sigma_{\ell 1}$ is first assumed equal to the compressive yield stress of steel and the value of the variable y_2 is calculated by rotational equilibrium, i.e.

$$y_2 = \left\{ \frac{\sigma_{\ell 2}\left[A_{slf2}\, y_{\ell 2,lim} + 0.5\rho_{lw}\, b\, y_{\ell 2,lim}^2\right] - 0.5\sigma_{c2}\, b\, y_{c,lim}^2 - 0.5b\left(\sigma_{\ell 3}\,\rho_{lw} - \sigma_{c3,1}\right) y_1^2 - M_1}{-0.5\sigma_{c2}\, b + 0.5\sigma_{\ell 2}\,\rho_{lw}\, b - 0.5b\left(\sigma_{\ell 3}\,\rho_{lw} - \sigma_{c3,1}\right)} \right\}^{\frac{1}{2}} \tag{16}$$

where the bending moment contribution M_1 is reported in Equation (7). If the value of y_2 resulting from Equation (16) is not higher than $y_{\ell 2,lim}$, it is assumed as the value of the variable y_2. If this is not the case, y_2 is fixed equal to $y_{\ell 2,lim}$ and $\sigma_{\ell 1}$ is calculated by Equation (15).

The axial resistance $N_R^{(V,3)}$ corresponds to null values of variables $\sigma_{\ell 2}$, $\sigma_{\ell 3}$, σ_{c1}, σ_{c2} and to a value of y_1 equal to $-y_{\ell 1,lim}$. The free stress $\sigma_{\ell 1}$ is first assumed equal to the tensile yield stress of steel and the value of the variable y_2 is calculated by Equation (16). If this value of y_2 is higher than $y_{\ell 2,lim}$, y_2 is fixed equal to $y_{\ell 2,lim}$ and $\sigma_{\ell 1}$ is calculated by Equation (15).

The axial resistance $N_R^{(V,4)}$ is calculated assuming that the variable y_1 is equal to $-y_{\ell 1,lim}$, $\sigma_{\ell 2}$ and $\sigma_{\ell 3}$ are equal to the tensile yield stress of steel and the stresses of concrete in zones F_1 and F_2 are null. The values of the free variables y_2 and $\sigma_{\ell 1}$ are calculated as proposed for the axial resistance $N_R^{(V,2)}$, except that in this case the tentative value of $\sigma_{\ell 1}$ is equal to the tensile yield stress of steel. The axial resistance $N_R^{(V,5)}$ is calculated assuming that the variable y_2 is null, $\sigma_{\ell 2}$ and $\sigma_{\ell 3}$ are equal to the tensile yield stress of steel and the stresses of concrete in zones F_1 and F_2 are null. The stress $\sigma_{\ell 1}$ is first assumed equal to the tensile yield stress of steel and the value of the variable y_1 is calculated by the rotational equilibrium, i.e.

$$y_1 = \left\{ \frac{-\sigma_{\ell 1}\left[A_{slf1}\, y_{\ell 1,lim} + 0.5\rho_{lw}\, b\, y_{\ell 1,lim}^2\right] + 0.5\sigma_{c1}\, b\, y_{c,lim}^2 + 0.5b\left(\sigma_{\ell 3}\,\rho_{lw} - \sigma_{c3,1}\right) y_2^2 - M_2}{0.5\sigma_{c1}\, b - 0.5\sigma_{\ell 1}\,\rho_{lw}\, b + 0.5b\left(\sigma_{\ell 3}\,\rho_{lw} - \sigma_{c3,1}\right)} \right\}^{\frac{1}{2}} \tag{17}$$

If the solution of Equation (17) is not lower than $-y_{\ell 1,lim}$, it is assumed as the value of the variable y_1. If this is not the case, y_1 is fixed equal to $-y_{\ell 1,lim}$ and $\sigma_{\ell 1}$ is calculated by Equation (15).

Finally, the axial force $N_R^{(V,6)}$ is the tensile axial resistance of the cross-section and is calculated assuming that the variables y_1 and y_2 are null if $A_{slf1} f_y y_{\ell 1,lim} = A_{slf2} f_y y_{\ell 2,lim}$ while they are equal to $-y_{\ell 1,lim}$ if $A_{slf1} f_y y_{\ell 1,lim} > A_{slf2} f_y y_{\ell 2,lim}$. Stresses $\sigma_{\ell 2}$ and $\sigma_{\ell 3}$ are equal to the tensile yield stress of steel and the stress of concrete in zones F_1 and F_2 is null. The variable $\sigma_{\ell 1}$ is free and is calculated by Equation (15). If the maximum contribution of A_{slf1} to the rotational equilibrium is lower than that of A_{slf2}, i.e. $A_{slf1} f_y y_{\ell 1,lim} < A_{slf2} f_y y_{\ell 2,lim}$, other combinations reported in (Rossi, 2021) must be used and considerations similar to those described above may be applied to calculate the free variables or set the value of the variables y_1 and y_2.

To obtain the values of the variables corresponding to the assigned axial force, the axial resistances N_R^V are compared to the axial force N. Independently of the value of the shear force, the cross-section is able to sustain the assigned axial force N only if this value is in the range from $N_R^{(V,1)}$ to $N_R^{(V,6)}$. If this is the case, the values of the axial resistances that are immediately lower and higher than the assigned axial force are first identified. Then, to obtain the value of the variables corresponding to the assigned axial force and to a null bending moment, a simple method reported is applied (Rossi, 2021).

5. Comparison with results of the reference nonlinear mathematical programming problem

The method is applied to some reinforced concrete members and the results are compared with those deriving from the reference nonlinear mathematical programming problem. The cross-section of the members considered in the first set of tests is rectangular (30x60 cm²) and representative of beams subjected to shear force and bending moment acting in the plane where the lateral stiffness of the member is maximum. The transverse reinforcement consists of rectangular hoops. The longitudinal reinforcement of the tension side consists of bars with cross-sectional area equal to either 6.28, 12.57 or 28.27 cm² and mechanical cover equal to 5 cm. The longitudinal reinforcement on the compression side is defined by means of the geometric ratio $u=A_{slf2}/A_{slf1}$, which is equal to either 1, 0.50 or 0. The longitudinal reinforcement in the web consists of bars with cross-sectional area equal to 6.28 cm². The hoops consist of 8 mm diameter bars with spacing equal to either 5, 10, 15 or 25 cm. The yield stress f_y of longitudinal and transverse bars is equal to 45 MPa. The cylindrical compression strength of concrete f_c is equal to 30 MPa. The reduced compressive strength of concrete under biaxial state of stress f_{c2} is derived from the compressive strength f_c as $f_{c2} = 0.6\left(1 - f_c/250\right) f_c$ and is equal to 15.8 MPa.

In accordance with other researchers (Walther and Miehlbradt, 1990), the minimum and maximum values of the cotangent of the angle θ are suggested to be calculated by means of the following relations

$$\text{cotg}\theta_{\min} = \text{cotg}\left(\theta_I + \Delta\theta\right) \qquad \text{cotg}\theta_{\max} = \text{cotg}(\theta_I - \Delta\theta) \tag{18}$$

where θ_I is the angle of inclination of the first crack with respect to the longitudinal axis of the member and Δθ is the maximum excursion allowed for the angle θ. The angle Δθ is assumed equal to 23.2°. Owing to this, in the absence of any axial load, $\theta_I = 45°$ and the values obtained by Equation (18) are 0.4 and 2.5, as sometimes considered in codes.

The ultimate M-V interaction domains of the members are reported in Figure 2 (exemplary derived, as in Fig. 1, with reference to the design values of the mechanical properties of the materials and assuming cotgθ in the range from 1 to 2.5). The comparison with the results of the reference nonlinear mathematical programming problem proves the accuracy of the simplified method in reproducing the results of the non-linear programming problem. In particular, the domains reflect the expected variations in the shear strength because of longitudinal and transverse reinforcements.

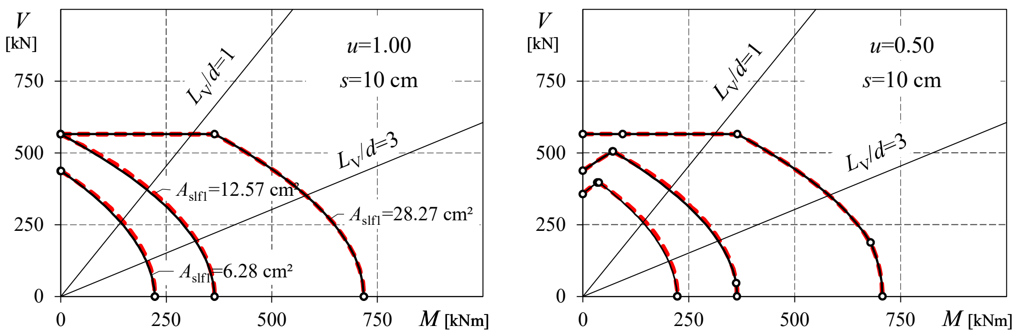

Figure 9. Simplified ultimate M-V interaction domains for beams according to the refined (dashed red line) and simplified methods (continuous black line)

To validate further this method, the ultimate interaction domains have also been calculated for members subjected to axial force. The cross-section considered in this second set of tests is rectangular (30x50 cm²) and representative of columns subjected to axial force, shear force and bending moment acting in the plane where the lateral stiffness is maximum. The cross section is endowed with equal longitudinal reinforcement on its opposite sides. The longitudinal reinforcement on the tension (and compression) side consists of either 4.62, 7.84 or 12.50 cm² with mechanical cover equal to 5 cm, whereas the longitudinal reinforcement in the web consists of bars with cross-sectional area equal to 6.28 cm². The hoops consist of 8 mm diameter bars with spacing equal to either 5, 10, 15 or 25 cm. The normalised axial force $N/(A_c f_c)$ is equal to either 0, 0.1, 0.3 and 0.5. Again, the yield stress f_y of longitudinal and transverse bars is equal to 45 MPa and f_c=30 MPa. Further, the reduced compressive strength of concrete under biaxial state of stress f_{c2} is equal to 15.8 MPa. The ultimate interaction domains of these members are reported in Figure 3 where they are compared to those resulting from the reference nonlinear mathematical programming problem. Again, the comparison proves the ability of the simplified method to reproduce the results of the more complicated method and reflects the variations because of the different axial force.

6. Comparison with results of laboratory tests

The method has been validated against results of 73 laboratory tests on members with different properties (Rossi, 2021). The parameter adopted for comparison of theoretical and experimental results is $R_V = V_{exp} / V_{num}$, where V_{num} is the ultimate shear force obtained by means of the proposed method and V_{exp} is the maximum shear force recorded during the laboratory test. The values of R_V vary from 0.83 to 1.41, with a mean value of 1.11, a standard deviation of 0.139 and a coefficient of variation equal to 0.125. In view of these results, the application of the proposed method is suggested for normalised axial force not higher than 0.45 and for shear span ratios not lower than 2.5.

7. Conclusion

The paper proposes a simple procedure for the calculation of the shear strength resulting from the only truss action in reinforced concrete rectangular members with shear reinforcement and subjected to axial force, bending moment and shear force.

The main conclusions of the study are:

- the proposed procedure provides the N-M-V ultimate interaction domain of the cross-section by means of simple equations or procedures and is easy to implement within structural programs to perform safety checks of members.
- the proposed procedure identifies points of the N-M-V ultimate interaction domain characteristic of limits of behaviour of steel and concrete.
- the comparison between the results of the proposed method and those of a more refined non-linear programming problem highlights that the differences between the results of the two methods are negligible.
- the comparison between the results of the proposed method and those of laboratory tests highlights that the proposed method can be reliably applied to predict the shear strength of members characterised by normalised axial force not higher than 0.45 and by shear span ratios not lower than 2.5.

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