Triple α resonances in the ^{6}Li + ^{6}Li → 3α reaction at low energy

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ABSTRACT

The ^{6}Li + ^{6}Li → 3α reaction has been measured in a kinematically complete experiment at 3.1 MeV of beam energy. The reaction mainly proceeds via intermediate ^{8}Be states. The interaction between any two of the three α particles provides events with one, two or three ^{8}Be interfering levels, with strong enhancement in the α–α coincidence yield. Evidence of three ^{8}Be levels within the same 3α event suggests that one α particle is exchanged between the other two. This is a condition for Efimov states to occur in nuclei, for which no observation exists yet. The hyperspherical formalism for the low-energy three-body problem has been applied to point out the 3α particle correlation.

1. Introduction

The basic condition for the Efimov effect is the existence of resonant two-body forces [1,2]. The so-called Efimov trimers appear for a system of three particles with resonant two-body interactions in a way that three (or more) interacting particles may form bound states even when any two of the particles are unable to bind. When the two-body scattering length a is much larger than the range of the interaction potential r_0, the three-body physics becomes independent of the details of the short-range interaction and takes kind of universal character. This universality has been experimentally exploited in ultracold atomic systems. Three-body and even four-body composites naturally form in an ultra cold gas of alkali atoms ([3–6] and references therein) with resonant interactions and are detected after they decay into hot atoms and dimers, which rapidly leave the ultra cold sample. Quantum mechanics explains the existence of these few-body systems in a quite simple way [7]. Let us assume to have a weakly bound pair of two identical bosons, much greater in size (a) than the range r_0 of the forces between the bosons; if a third identical boson comes across the pair at a distance of the order of a, one boson of the

pair can jump closer to the third boson and build a similar bound pair with it. Sharing one boson between two others can happen time after time, giving rise to an effective attractive interaction of range a between all three bosons. This might be the case for a system of three α-particles. Efimov [1] has predicted the possibility of existence of trimers in such a system, with a maximum radius of attraction of the order of the Bohr radius, $a_C = 1/e^2$, due to the repulsive long-range Coulomb potential that leads to important modifications and to a scaling violation for large distances (small energies) among the bosons. His prescription refers mainly to ^{12}C levels in the vicinity of the threshold of breakup into three α-particles or $\alpha + {}^8$Be, taking into account the Coulomb force among α particles which destroys the $1/r_0^2$ scaling at large distance where Coulomb force is dominant [1]. The question is now if (multi) resonant states can only arise through the Efimov mechanism, or there could be other conditions for which we can have similar phenomena. In the present paper, we present results from a coincidence measurement of the ^{6}Li + ^{6}Li → 3α reaction at 3.1 MeV of beam energy on the existence of α-trimers at an excitation energy of ^{12}C of about 29.6 MeV (^{6}Li + ^{6}Li threshold: 28.17 MeV; center-of-mass beam energy at half target: 1.4 MeV). We observe in some kinematical regions three α particles with mutual two α resonances through excited states of ^{8}Be at the same time: this is consistent with a mechanism where one α boson is

exchanged between the two other α's, i.e. very similar to the celebrated Efimov mechanism in spite of the high excitation energy of ^{12}C. This work can be linked to earlier studies of three-body reactions such as ^{10}B + $d \rightarrow 3\alpha$ in kinematical plane and space-star conditions [8] and ^{11}B + $p \rightarrow 3\alpha$ to look for the effect of the single rescattering amplitude, the so-called triangle diagram [9]. No definite conclusion was drawn from those experimental studies, partially due to a somewhat model dependent procedure applied in the data analysis. Nonetheless, they stimulate further experimental and theoretical work in the three-cluster problems.

2. The experiment

The experiment was performed at the Ruđer Bošković Institute in Zagreb (Croatia). A 3.1 MeV ^{6}Li^{++} Tandem beam with intensity of about 3 nA, was delivered onto an isotopically enriched ^{6}LiF target, 87 μg/cm^2 thick, evaporated on a 27 μg/cm^2 carbon backing. The beam energy at half target was 2.74 MeV and this was the energy used for further calculations. The α–α coincidences were measured by four 1000 μm position sensitive detectors (PSD – intrinsic α resolution quoted as 0.3 mm for the position and about 0.5% for the energy). Two of them were placed symmetrically at either side of the beam direction, each covering laboratory angles 52° to 68°. Laboratory angles for the other two PSD's were 74° to 98° and 92° to 120° from opposite sides, each one devoted to the α–α coincidence detection with one of the symmetric PSD's not placed on the same side.

3. Data analysis and results

Channel selection was accomplished without particle identification, since the high Q-value (theoretical value 20.899 MeV) of the ^{6}Li(^{6}Li, $\alpha\alpha$)^{4}He reaction allows the α–α coincident events to be easily identified, being the largest among the other possible three-body reactions occurring on lithium, carbon, oxygen and/or impurities in the target. The kinematics were reconstructed under the assumption of a third α as undetected particle. We refer to [10] for a thorough description of the experimental setup and channel selection.

As reported in [10], another run was performed at 2.5 MeV of beam energy but with much less statistics (less than 20% of the experiment). Results from the two runs are very much consistent with each other. Here, we will refer only to the run at 3.1 MeV. In the following, the detected α particles from any of the selected coincidences will be indicated with numbers 1 and 2, while 3 will indicate the undetected α particle.

If one considers a two-dimensional spectrum with a kinematical variable, such as the energy or the angle of any one of the involved particles as a function of the Q-value, coincidence events of interest should lie on a vertical line that cuts the Q-value axis at the expected value. A typical spectrum for the present experiment is reported in Fig. 1 where the laboratory angle of one of the two detected particles is shown as a function of the Q-value. A dominant sharp vertical line shows up, crossing the Q-value axis at about 20.9 MeV, in very good agreement with the expected value for the 3α channel. It makes us confident of the quality of the calibration and of the possibility to identify the 3α channel. Events within this vertical region were selected for further analysis.

To study the nature of the selected events, they were projected onto two-dimensional plots θ_1 vs. E_2 for a fixed value of θ_2. This is a way similar to the Dalitz plot distribution, but more incisive for the present case. A typical spectrum obtained for $\theta_2 = 60° \pm 1°$ is shown in Fig. 2.

Colored solid lines superimposed onto experimental data represent calculated kinematical loci associated to ^{8}Be levels that can
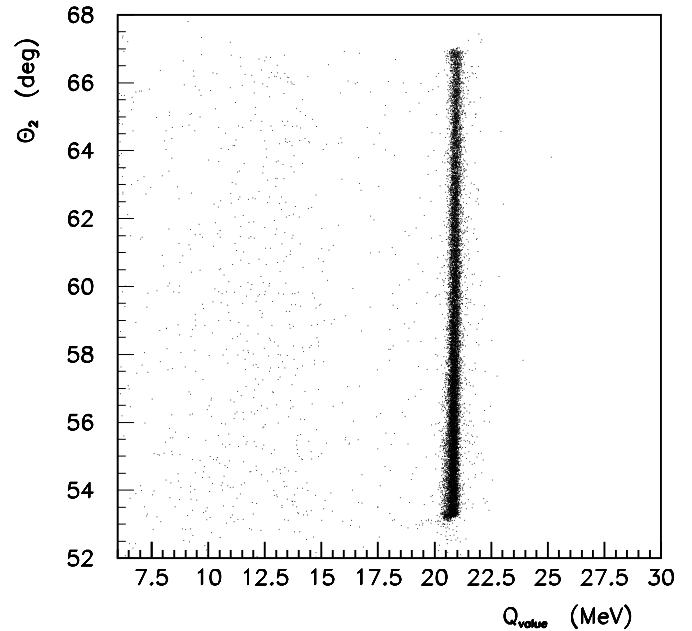

Fig. 1. Two-dimensional spectrum showing the angle of one of the two detected α particles *vs* the Q-value. The sharp vertical region at about 20.9 MeV of Q-value, corresponds to the ^{6}Li + ^{6}Li $\rightarrow 3\alpha$ reaction.
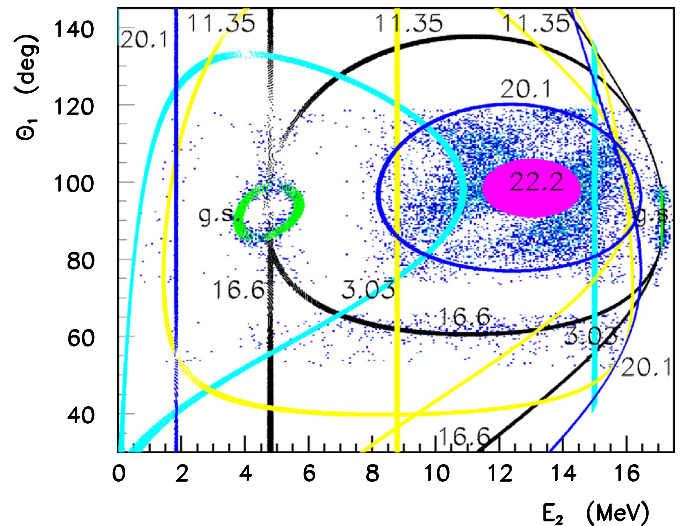

Fig. 2. Two-dimensional plot θ_1 vs. E_2 at $\theta_2 = 60° \pm 1°$. See text for the description of colored solid lines superimposed onto experimental data. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

contribute in the phase space region populated in the present experiment. There is more than one line for each level, three at most, corresponding to relative energies E_{12}, E_{13}, E_{23} of any two of the three α particles involved. We have evidence for the g.s. (green, $\Gamma_{cm} = 5.57 \pm 0.25$ keV), 3.03 (aqua, $\Gamma_{cm} = 1513 \pm 15$ keV), 11.35 (yellow, $\Gamma_{cm} \approx 3500$ keV), 16.62 (nearly the same for 16.93) (black, $\Gamma_{cm} = 108.1 \pm 0.5$ keV), 20.1 (blue, $\Gamma_{cm} = 880 \pm 20$ keV) and 22.2 MeV (purple, $\Gamma_{cm} \approx 800$ keV) excited states in ^{8}Be. Lines do not account for the natural widths of the levels but they are useful to guide the eye. From the spectrum, one can notice the intersection between lines, pointing out that events with correlated α's fed by two and even three ^{8}Be at the same time can exist. However, due to the limited phase space region populated in the present experiment, clear signatures can be accessible only in few

cases. One of those cases corresponds to the formation of three ^{8}Be, one in its g.s. and two at 16.62 MeV of excitation energy.

From this figure, though very illustrative, it is not straightforward to gather a global information from all experimental data. To better investigate the correlation between the three α particles from the available data in a whole, the hyperspherical formalism for the low-energy three-body problem [2] was applied in the momentum space and used to calculate the so-called "Delves hyperangle". A complete description of the hyperspherical formalism to understand the universal aspect of the three-body problem is reported in [2].

A set of Jacobi momenta consists of the relative momentum $\vec{p}_{ij}$ between the two detected α particles and the momentum vector $\vec{p}_{k,ij}$ of the undetected third α particle with respect to the center-of-mass of the other two. For particles of equal mass, the Jacobi momenta are [2]:

$$\vec{p}_{ij} = \vec{p}_i - \vec{p}_j,$$
$$\vec{p}_{k,ij} = \vec{p}_k - \frac{1}{2}(\vec{p}_i + \vec{p}_j). \qquad (1)$$

The Delves hyper-angle α_k is defined as [2]:

$$\alpha_k = \arctan\left(\frac{\sqrt{3}\,p_{ij}}{2\,p_{k,ij}}\right), \qquad (2)$$

where (i, j, k) is a permutation of $(1, 2, 3)$ and p_{ij} and $p_{k,ij}$ are the magnitudes of the momentum vectors. The range of α_k is from 0 to $\frac{\pi}{2}$. It is near 0 when particle k is far from particles i and j, and near $\frac{\pi}{2}$ when k is close to the center-of-mass of particles i and j. This angle was calculated assuming that k is the undetected third α particle. Thus, we refer to this angle as α_3. Following the prescriptions of [2] to determine α_3, the reference system defined by the Jacobi momenta was rotated to bring $\vec{p}_{3,12}$ to the x-axis. Coincidence events were projected onto the α_3 variable and the result is shown in Fig. 3(a) as black dots. Error bars represent only statistical errors. Phase-space effects were divided out by performing a Monte Carlo simulation of the experimental setup, which provides a rather smooth α_3 spectrum (solid line in the figure) as the result of experimental and kinematics constraints.

The region spanned by α_3 goes from about 40° to 90°, with two sharp peaks centered at $\alpha_3 = 45°$ and 60° and a huge bump at about 90°. The corresponding three-body geometrical configurations are shown on top of them, with α particles 1 and 2 at the endpoints of a diameter of a circle with radius P given by:

$$P^2 = \frac{1}{2}p_{ij}^2 + \frac{2}{3}p_{k,ij}^2, \qquad (3)$$

whose value is about 406 MeV/c for all 3α particle events.

To supply the meaning of correlation function to the α_3 spectrum, a similar spectrum was reconstructed with uncorrelated events, built up with the three α tracks belonging to different coincidence events, disregarding any restriction from energy and momentum conservation for the undetected third α particle. The resulting behavior is shown in Fig. 3(a) as green dots, normalized to the correlated one around 88°. The ratio between correlated (black dots) and uncorrelated (green dots) data is reported in Fig. 3(b). It turns out that the bulk of uncorrelated events is barely contributing to the α_3 peaks at 45° and 60°, thus suggesting that they can be a clear signature of triple α correlation.

To support this interpretation, events within those peaks were projected onto the E_{ij} ($i, j = 1, 2, 3$) relative energy axis, as shown in Figs. 4(a) ($\alpha_3 = 60°$) and (b) ($\alpha_3 = 45°$) with black dots. Two sharp peaks around 90 keV and 16.7 MeV appear in Fig. 4(a), corresponding to the ground state and to the 16.62 MeV state of ^{8}Be. The 16.7 MeV peak is actually fed twice with respect to the 90 keV
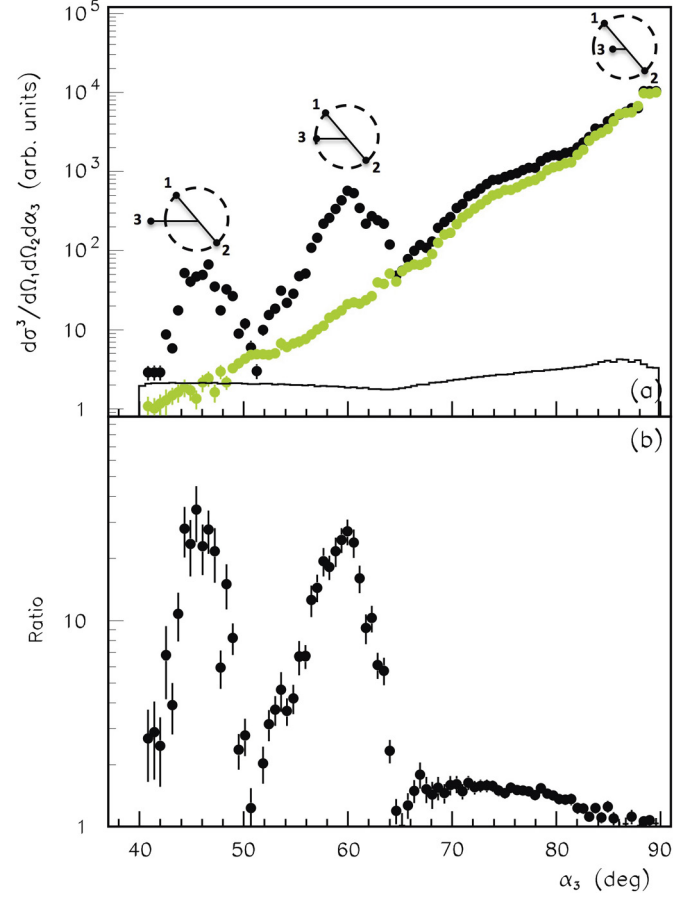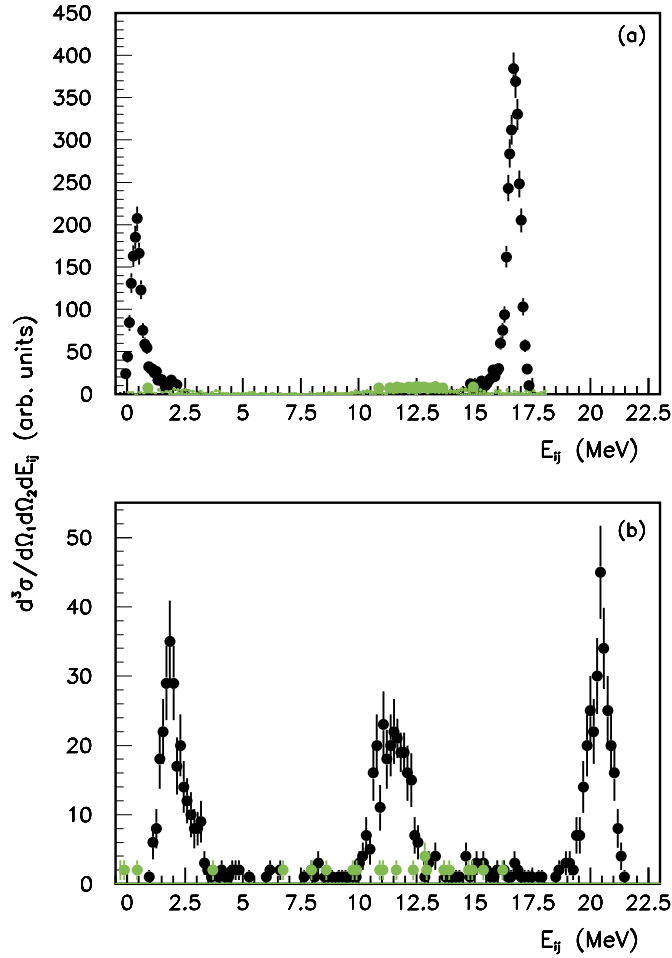

Fig. 3. (a) Coincidence events projected onto the Delves hyperangle α_3 axis (black dots). The solid line is the result of a Monte Carlo calculation accounting for experimental and kinematics constraints. Green dots refer to uncorrelated events as explained in the text. (b) Ratio between coincidence events (black dots) and uncorrelated ones (green dots). (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

one, in agreement with the interpretation provided by the crossing lines in Fig. 2. From Fig. 4(b), one more possibility is foreseen to correlate the three α particles to form three ^{8}Be states at 20.1, 11.35 and 3.03 MeV, taking advantage of their huge widths. The corresponding distributions of the random coincidences for both spectra are shown as green dots. Their negligible contribution to the peaks is consistent with the interpretation of triple α correlation.

All other events fall within the huge shoulder in Fig. 3 extending up to $\alpha_3 = 90°$ and do not correspond to triple α resonances. The reason why the highest coincidence yield pertains to the 90° region is twofold. First, the 3α channel is mainly fed by the 22.2 MeV level of ^{8}Be because the ^{6}Li + d threshold is on top of this state. Thus, the bulk of recombination into 3α particles goes through it, with two α's sharing the full amount of energy and the third α particle with essentially zero energy. Second, α_3 approaching 90° corresponds to 3α's in a line and can be linked to two interacting ^{6}Li in a stretched α–d–d–α configuration that corresponds to the minimum value of the Coulomb barrier. This will be the subject of a forthcoming paper.

Values which are taken from E_{ij} obey a simple formula obtained from energy and momentum conservation principles and given by:

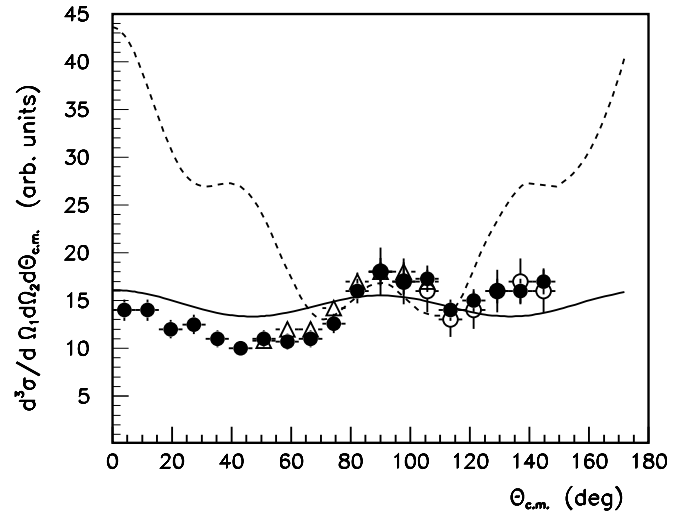$$E_{12} + E_{13} + E_{23} = \frac{3}{2}(E_{beam} + Q), \qquad (4)$$

Fig. 5. $\theta_{\text{c.m.}}$ angular distributions spectra for $\alpha_3 = 45°$ (open circles) 60° (solid dots) and 90° (open triangles). The solid and dashed lines correspond to calculations assuming a decay from an excited level of ^{12}C with $J^\pi = 2^+$ and $J^\pi = 4^+$ respectively (see text for details).

Fig. 4. E_{ij} spectra with $i, j = 1, 2, 3$ (black dots) obtained projecting events within the 60° (a) and 45° (b) peaks in α_3 (Fig. 3). Several contributing resonances from different states in ^{8}Be are observed, which explain the enhancement for those angles. The corresponding distributions from uncorrelated events are shown as green dots in both spectra. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

with E_{beam} the beam energy in the center-of-mass system and Q the Q-value of the ^{6}Li(^{6}Li, $\alpha\alpha$)^{4}He reaction. This means that the sum of the three relative energies is fixed once the reaction and the beam energy are chosen and in the present case totals about 33.5 MeV.

Thus, the possibility for the three α particles to correlate is first subjected to this formula that can be used as a kind of prescription to determine the appropriate beam energy. Then, one needs a feasible experimental setup covering the correct phase space to catch the three ^{8}Be events. These considerations apply to any other reaction feeding the three α particle channel. This means that the three ^{8}Be events are likely to be observed at other beam energies and with different reactions. Consequence of that is the peculiar unbound nature of ^{8}Be with a full-bodied level scheme feeding the two decaying α channel.

Eq. (3) lends to a consideration regarding the Hoyle state in ^{12}C, at 7.65 MeV of excitation energy, whose Q-value for the decay into 3α particles is about 380 keV. Applying Eq. (3) to the Hoyle state, it turns out that it cannot decay into three correlated α particles. This is confirmed by several results available in the literature, proving that the Hoyle state decays almost entirely through a ^{8}Be$_{g.s.}$ + α [11,12].

The present results are also consistent with [13]. In that work, a coincidence detection of three α particles was performed to in-

vestigate the breakup of ^{12}C in the 3α continuum, following ^{12}C population by β-decay of ^{12}N and ^{12}B. In a ^{12}C excitation energy region up to 15 MeV, and with a density of levels reduced by the selectivity of the β decay, 3α continuum states were found to be dominated by the ^{8}Be$_{g.s.}$ + α and ^{8}Be$^*_{1ecc}$ + α decays. One can therefore imagine that moving higher in ^{12}C excitation energy, as in the present experiment, into the region of overlapping resonances, the 3α decay may sample even more excited configurations of the intermediate ^{8}Be states. Indeed, the level structure at the excitation energy of the present experiment is characterized by a superposition of several states having widths larger than 1 MeV. Depending on spin and parity and on the nature of these interfering resonances, the triple α channel would go through different states of ^{8}Be although being originated by the same excited system. Before drawing any conclusion, it is thus necessary to check if the structure of overlapping resonances may influence the angular distribution of the triple α decay.

Based on the appearance of essentially two peaks (at 45° and 60°) and a huge bump (around 90°) in the α_3 spectrum, and since in all of them there is at least one excited ^{8}Be with $J^\pi = 2^+$ fed by two α particles, we have reconstructed the invariant scattering angle, $\theta_{\text{c.m.}}$, as the one between the relative momenta of the final $(\alpha + {}^8\text{Be}^*)$ and initial $(^6\text{Li} + {}^6\text{Li})$ particles. In the center-of-mass system of the reaction, such an angle is the one between the momentum of any of the two fragments (α or ^{8}Be*) and the beam direction.

Angular distributions were extracted for each peak in Fig. 3(a) (cutoff of ±2.5°) and compared with each other and to the results obtained from the general expression of the angular distribution of the fragments for a resonance reaction, as given in [14]. Typical angular distributions are reported in Fig. 5 for ^{8}Be excitation energy of 20.1 MeV at $\alpha_3 = 45°$ (open circles), 16.62 MeV at $\alpha_3 = 60°$ (solid dots) and 22.2 MeV at $\alpha_3 = 90°$ (open triangles). Such distributions are divided by the phase-space contribution and normalized with each other at $\theta_{\text{c.m.}} = 90°$. Error bars represent statistical errors (14.6% for open circles, 7.6% for solid dots and 4.4% for open triangles). The solid lines in the same figure correspond to calculations assuming a decay from an excited level of ^{12}C with $J^\pi = 2^+$ (solid line) and $J^\pi = 4^+$ (dashed line), which account for all combinations of total initial and final channel spins and angular momenta.

The experimental behaviors are all consistent with each other within experimental errors, suggesting a similar effect due to a possible influence of the structure of overlapping resonances in the excitation region of ^{12}C around 29.6 MeV. The peak at $\theta_{c.m.} = 90°$ indicates an even J assignment and the periodicity of experimental data is closer to the one corresponding to $J = 2$ (solid line), initial and final channel spins $S_i = 0, 2$, $S_f = 0$, and angular momenta $L_i = 0, 2$, $L_f = 2$ respectively. However, the main result from the analysis of the angular distributions is that the effect of overlapping resonances in the excited ^{12}C is the same in and off the regions corresponding to the triple α correlated events and, thus, it cannot be responsible for the peaks in the α_3 spectrum.

This allows for further interpretation. Indeed, in the present case the ^{6}Li + ^{6}Li interaction with a center-of-mass beam energy at half target of 1.4 MeV has itself something special. This energy is barely enough to break one ^{6}Li and the entrance channel can be seen as a ^{6}Li + α + d trimer at zero energy with two (of the three) large scattering lengths, e.g. those for ^{6}Li + d and α + d larger than 25 fm [15]. This is considered as a necessary condition to observe a "canonical" Efimov. Thus, systems with correlated α particles can be seen as recombinations or escape channels from Efimov, which carry away the energy surplus. This evidence might represent the evolution of Efimov when virtual states are involved, corresponding to positive energy variable K and negative scattering length a, for which no description exists yet in the Efimov diagram [2]. Additional theoretical and experimental work is required to accomplish with this interpretation.

4. Conclusions

The ^{6}Li + ^{6}Li → 3α reaction was measured in a coincidence experiment at 3.1 MeV of beam energy. The 3α decay channel is mainly populated via intermediate ^{8}Be states with events involving even three of them. In spite of the high excitation energy in ^{12}C, such an evidence might inspire the idea of a possible connection with the existence of Efimov trimers in nuclei, so far never observed. What we observed is the presence in the same event of three excited levels in ^{8}Be when making correlations of the emitted three α particles. Because of favorable phase space conditions and basic quantum mechanics, one could envisage that one α particle is exchanged between the other two with all possible permutations since we have three identical particles. To study the correlation between the three α particles, we have applied the hyperspherical formalism in momentum space. This gives us an idea of the geometry of the reaction in the exit channel. By selecting peaks at certain values of the α_3 hyperangle, we have built the corresponding E_{12}, E_{13} and E_{23} relative energies and shown that they are fed in the same event by states of ^{8}Be. In particular, we have defined a correlation function that shows very clearly the existence of triple-α resonances otherwise buried by the phase space. From the analysis of the angular distributions in and off the regions corresponding to the triple-α correlated events, it turns out that the enhanced triple-α correlations do not seem to be influenced by the structure of overlapping resonances at the ^{12}C excitation energy populated in the present experiment. To make any statement that goes beyond these, can only be addressed based upon a model approach. Further measurements at different beam energies and with different initial channels such as ^{10}B + d, ^{11}B + p and ^{9}Be + ^{3}He will add information on possible 3α resonant channels.

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