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NETWORK MODELS FOR THE HUMAN MIGRATION PHENOMENON

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Nothing in life is to be feared, it is only to be understood. Now is the time to understand more, so that we may fear less.
-Marie Curie-

Contents

Acknowledgement

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CHAPTER 1

Introduction

OR is a relatively new mathematical field, that has its roots in quantitative analysis of real-world phenomena with the aim of supporting military operations, business tactics and strategy, social policy interventions, as well as many other applications. In 1978, the Operational Research Society of the UK, sought to provide a widespread definition of Operations Research (OR) as *the application of the methods of science to the complex problems arising in the direction and management of large systems of men, machines, materials and money in industry, business, government and defence. The distinctive approach is to develop a scientific model of the system, incorporating measurements of factors such as chance and risk, with which to predict and compare outcomes of alternative decisions, strategies or controls. The purpose is to help management determine its policy and actions scientifically* ([20]).
ORs official appearance in the scientific world can be traced back to the Second World War, when it was used to solve strategic and tactical problems in military operations. For the first time there was a convergence of scientists from different disciplines all with the aim of determining the most efficient use of the limited resources using quantitative techniques. Following the Second World War OR grew as scientists realised the potential of further applications outside of military oper-

ations, more specifically to solve issues within the civilian sector. Many scientists efficiently implemented the approach to solve short and long term problems such as scheduling, inventory control and strategic planning, resource allocation. In recent times the OR applications are those that intertwine with social sciences. In [39], the authors highlight how OR can help social scientists to make their findings suitable for general purpose into practice, and how social science can assist operational researchers to achieve improvements in their applications and from this upgrade their models.
The application of OR is a relatively new concept within humanitarian problems such as the development of complex supply chains networks in the case of critical needs ([76]) such as food ([78]) or the novel application to competitive pharmaceutical supply chains and quality issues (see [71]); to blood supply chains (see [70]) or in the social phenomenon, as defined in [57], of cybersecurity (see [14]); to disaster relief (see [34]); and to migration of which some examples are proposed in this thesis.
The main focus of this dissertation is on the timely topic of the migration phenomenon. In particular, here it is advanced the modelling, analysis, and solution of human migration problems by developing network models that differ substantially in the perspectives proposed for the inspection of the phenomenon of migration. Furthermore the results presented add to the literature on operations research for societal impact, inspired by the real world.

The International Organization for Migration (IOM) defines a *migrant* as any person who is moving or has moved across an international border or within a state regardless of legal status, whether the movement is voluntary or involuntary, what the causes for the movement are, and what the length of the stay is ([61]).
The reasons for recent migrations include: persecution, conflict, violence, human rights violations or events seriously disturbing public order, poverty and economic inequality but also climate change, natural disasters, and with the latter driving humans to seek better lives for their families and themselves.
The highest level of worldwide forcibly displacement has been registered at 79.5 million people in 2020 according to the United Nations High Commissioner for Refugees (UNHCR) ([82]), as shown in Figure 1.1.

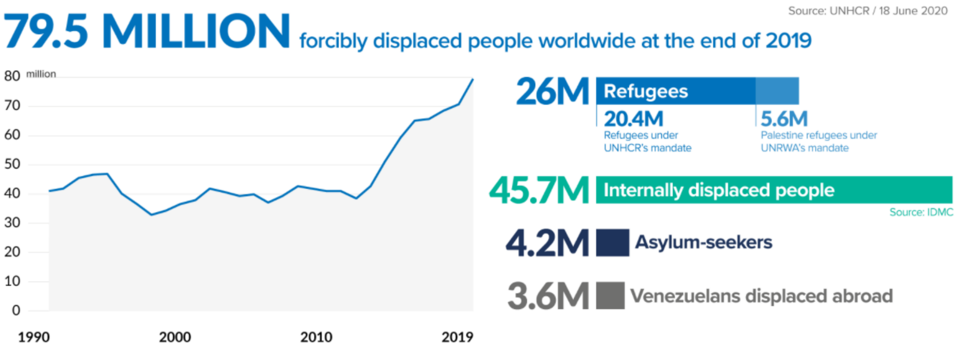

Figure 1.1: Global displacement data. Source: UNHCR, 18 June 2020.

The International Migrant Stock 2019, a dataset released by the Population Division of the UN Department of Economic and Social Affairs (DESA), is reporting that the number of international migrants was an estimated 272 million in 2019, an increase of 51 million since 2010 ([28]). The number of refugees and asylum seekers has increased by almost 50% since the new millennium.

The increasing role of migration in the social, economic and demographic development of countries, regions and the whole of the world, is becoming more and more evident. Especially in the Mediterranean Basin, which has become the theatre of a humanitarian crisis that has challenged the collective leadership around the sea. The Mediterranean sea received 66,890 arrivals in 2020 and the data includes sea arrivals in Italy, Cyprus and Malta and both sea and land arrivals in Greece and Spain, with most migrants from the north of Africa ([81]), as shown in Figure 1.2.
According to Jones ([40]), Italy has now become the main route into Europe for economic migrants and asylum seekers, with hundreds of thousands risking their lives in their journeys from North Africa each year and thousands dying at sea. Kitsantonis in [46], states that the number of asylum seekers making the short but often treacherous journey from Turkey across the Aegean Sea to Greece has seen a rise again, with Greek officials looking toward replacing overcrowded migrant camps with centres and hoping to restrict the migrants' movements. Recent data are showing that Cyprus is now hosting the most refugees per capita in the

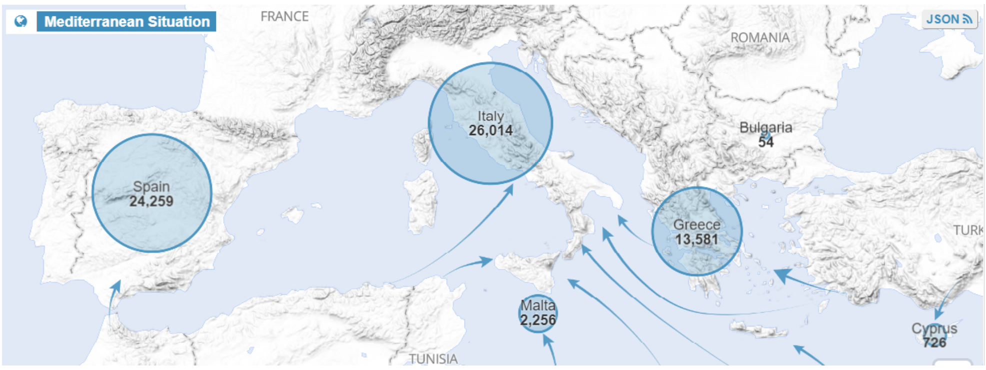

Figure 1.2: Mediterranean situation. Source: UNHCR, 19 October 2020.

European Union ([93]). As emphasized in Nagurney and Daniele in [68], closer attention must be paid to human migration problems in economic, political, sociological, and even environmental dimensions.

The vulnerability of millions of international migrants may be exacerbated in crisis situations, such as the current COVID-19 (COrona VIrus Disease 2019) pandemic (see [85], [96]). The infectious respiratory disease emerged in Wuhan, China, and rapidly spread around the world, posing enormous health, economic, environmental, and social challenges to the entire human population (see [12], [31]). More specifically, the COVID-19 pandemic has also affected global mobility in the form of blockages, restrictions, and travel disruptions, as risk mitigation measures are being implemented by numerous countries (see [9]).
The United Nations, Department of Economic and Social Affairs, emphasizes that economic and social factors are the main reasons why people migrate. On the other hand, if supported by appropriate policies, migration can contribute to inclusive and sustainable economic growth and development in both origin and destination locations ([95]). The Organization for Economic Co-operation and Development (OECD) countries in response to the COVID-19 pandemic worked on the development of short-term policy responses and longer-term challenges to migration management (see [29]).

How to effectively manage human migration flows has become one of the major challenges of the new millennium. The governments of many nations, hence, are now faced with identifying suitable policies and regulations to address a variety of human migration flows. In managing international migration flows, governments usually focus on distinct classes of migrants such as highly skilled workers, dependents of migrant workers, irregular migrants, and refugees and asylum seekers ([42]). In 2017, the United Nations Department of Economic and Social Affairs, Population Division ([95]) has compiled a list of high-level policies of various countries regarding international migration. For example, policies associated with "irregular migration" include fines, detention, or deportation of migrants in an irregular situation, as well as penalizing employers of such migrants. The International Organization of Migration ([59]) defines irregular migration as the movement of persons that takes place outside the laws, regulations, or international agreements governing the entry into or exit from the origin, transit or destination location; see also Karagiannis ([42]). The United Nations in [94], emphasize that migration policies in both origin and destination countries play an important role in determining the migratory flows. In this respect, migration interactions in all aspects of economic and social development will be the key to achieving the 2030 Sustainable Development Goals (SDGs) adopted by the member states of the United Nations. In the 2030 SDGs, proving the importance of improving migration with over 50% of the targets focused on migration or mobility (see [60]). Moreover, the Organization for Economic Co-operation and Development (OECD) countries in response to the COVID-19 pandemic, worked on the development of short-term policy responses and longer-term challenges to migration management (see [29]).

This thesis explores the theme of human migrations, contributing to research on human migration network systems. In this work we propose different model-based human migration problems. For each of them we formulate the associated nonlinear constrained problems which allows us to solve the decision problems with different optimization tools.

1.1 Overview on the migration network

This section offers an overview of the relevant literature on human migration models, with a focus on networks. It is important to mention that the network models proposed, to-date, have primarily been from the perspective of individual migrants making their decisions, which can be characterized as being that of "user-optimization" and leading to an equilibrium that includes the utilities associated with migration.
In [37], Rahmati and Tularam provide a critical review of a variety of theoretical frameworks for human migration models, which also reference foundational network equilibrium models. These authors highlight various mathematical theories of migration, focusing on macro-level, micro-level, and meso-level approaches.
The classical network equilibrium model of human migration is that of Nagurney ([64]), in which a new theoretical model for the analysis of human migration within an equilibrium framework is introduced. It is a multiclass model, and is isomorphic to a traffic network equilibrium with special structure (see also [65]). In that model, migrants of each class distribute themselves among the locations according to maximal utilities associated with the locations. The population of each class is assumed to be fixed and the utility functions are assumed to be concave and a function of the populations at the locations. Subsequently, Nagurney ([63]) generalized that model to include migration costs that depend on the flows along with the governing equilibrium conditions.

Nagurney, Pan, and Zhao (see [73]), continuing in the user-optimization vein of human migration modelling, constructed a multiclass human migration model, which they later extended, in [74], to include class transformations. Pan and Nagurney (see [79]) developed a multi-stage, Markov chain model and identified the connection between a sequence of variational inequalities and a non-homogeneous Markov chain. In all of the above papers, the specific governing equilibrium conditions were formulated as finite dimensional variational inequality problems. In [72], Pan and Nagurney were the first to apply evolutionary variational inequalities to model the underlying dynamics of human migration and also to discuss associated algorithms (see also [21]).

A conjectural variations equilibrium (CVE) for human migration was promulgated by Kalashnikov et al. (see [41]). The authors reported results of numerical experiments based on population data in locations in Mexico. In [13], Cojocaru formulates the human migration problem in terms of a transportation network and applies the double-layer dynamics theory. It is also worth mentioning that some network equilibrium models of human migration have even been applied to the migration of animals in ecology (see [62] and [55]). Davis et al. (see [26]), in turn, utilize a complex network approach for human migration and utilize an international dataset for their quantitative analysis. Cui and Bai (see [15]) present a mathematical model in which the population density varies when the spatial movement of individuals is a function of the departure and arrival locations. In [97], the authors make a comparison between human migration and wealth distribution. They present a model with equations for the population density and for the wealth distribution. It is based on perturbation methods and on the spectral properties of the linearised operators. The authors prove that, in the absence of cross diffusion terms, the dynamics of solutions can be described by travelling wave solutions of the corresponding reaction diffusion systems of equations. They also show the persistence of such solutions for sufficiently small cross diffusion coefficients.

As noted in Nagurney and Daniele ([69]), Causa, Jadamba, and Raciti ([11]) extended the Nagurney ([63]) model to include uncertainty in the utility functions, the migration cost functions, and the populations. However, none of the above-cited models of human migration included regulations and/or policies. Furthermore, they are all essentially user-optimized models.

As mentioned earlier, there has been very limited modelling work done on including policies and/or regulations in human migration networks. To date, the only papers that we are aware of are those by Nagurney and Daniele ([68]) and Nagurney et al. ([75]). These models incorporated explicit constraints that could be imposed by governmental authorities to put capacities on the flows of different classes of migrants from certain origins to specific destinations. In these models it was found that the utility of those that were restricted decreased, whereas those that were allowed to migrate increased. The latter paper also allowed for routes of migration consisting of links that capture congestion and associated costs and can handle even migration through different nations. These models were inherently user-optimized ones.

CHAPTER 2

Background

Mathematical modelling is a useful tool both to formalize, objectively analyse and solve real life problems and to understand how to manage them optimally. Mathematical modelling can be regarded as an iterative process commonly divided up into five major phases, as shown in Figure 2.1 below.

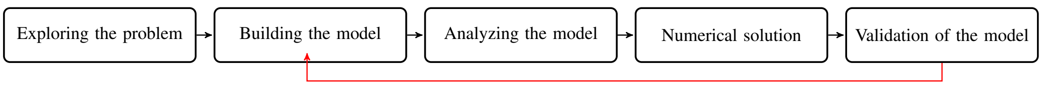

Figure 2.1: Modellistic approach steps

The first phase is about examining the structure of the problem in order to identify objectives, logical-functional links and to collect data.
In the second phase, also called formulation, the model is defined formally. Once the main characteristics of the problem are captured, they can be selected and combined into abstract representations and mathematically described.
The third step provides to analytically deduce:

- existence and uniqueness of the optimal solution;
- optimality conditions, that are an analytical characterization of the optimal solution;

- stability of the solutions, when the data or any parameters change.

The subsequent numerical solution phase takes place by means of suitable algorithms implemented to calculate the optimal solution. This is the area on which a huge amount of research and development in O.R. has been focused, and there is a plethora of methods for analysing a wide range of models.
Once the solution has been obtained there is one final step, the validation of the model, which needs to be passed. The expected accuracy of the model and the domain of validity are the points to be considered in the validation phase. If the used model used is still not accurate or does not capture some major issue, it will be reformulated until the results are sensible and come from a valid system representation.

2.1 Optimization Problems

Decisions or predictions in a variety of different context are devised by maximizing or minimizing a determined function, taking into account possible constraints of the problem. *Optimization*, or *Mathematical Programming*, entails the theory, use and computational solution of mathematical models to assist in making decisions, typically about the optimal use of resources.
Given a real-valued function of n real variables, $f : \mathbb{R}^n \to \mathbb{R}$, whose variables are bound to belong to a predetermined subset of $\mathbb{R}^n$, $K \subseteq \mathbb{R}^n$, *Optimization problem* or *mathematical programming problem* consists in determining an n-dimensional vector $x = (x_1, x_2, \ldots, x_n)$, if it exists, among the points of the set $\mathbb{K}$, that minimizes the function f.
Formally, the optimization problem can be written as follows:

$$\begin{aligned} &\min f(x) \\ &\text{subject to } x \in K. \end{aligned} \tag{2.1}$$

The function f, the set K and the points $x \in K$ are called *objective function, feasible set* and *feasible solutions*, respectively.
Indifferently we consider a minimum or maximum problem as the equality $\min f(x) =$

$-\max(-f(x))$ holds.
Let's briefly introduce some definitions.

Problem (2.1) is said *infeasible* if the feasible set is empty, $K = \emptyset$.

A point $x^* \in K$ is said *optimal solution* or *global minimum* for the (2.1) problem if $f(x^*) \leq f(x)$,$\forall x \in K$. The corresponding value $f(x^*)$ is called *optimal value*.

The feasible set K is usually expressed by a finite number of equality and/or inequality relations called *constraints*, formally

$$K = \{x \in \mathbb{R}^n | \, g_i(x) \geq 0 \, \forall i = 1, \dots, p \text{ and/or } h_j = 0 \, \forall j = 1, \dots, q\} \qquad (2.2)$$

where $g_i : \mathbb{R} \rightarrow \mathbb{R}$, $\forall i = 1 \dots, p$ and $h_j : \mathbb{R} \rightarrow \mathbb{R}$, $\forall j = 1, \dots, q$, are real-valued functions of n real variables.
Optimization problems are classically classified according to:

- the properties of the feasible set K. In particular (2.1) problem is called
  - Continuous Optimization Problem, if the variables x can take values in $\mathbb{R}^n$ (continuous values). We can further distinguish in unconstrained problems if $K = \mathbb{R}$ or constrained problem if $K \subset \mathbb{R}$.
  - Discrete Optimization Problem, if the variables x can take values only on a discrete set. We can further distinguish in integer programming problem if $K \subseteq \mathbb{Z}^n$ or boolean optimization problem if $K = \{0, 1\}^n$.
  - Mixed Optimization problem, if some of the variables are continuous and some are discrete.
- the structure of the objective and constraint functions. In particular (2.1) problem is called
  - Linear Programming (LP) problem, if the objective and constraint functions are linear.

- Nonlinear Programming (NLP) problem, if at least one among the objective and constraint functions, is not linear.

- the randomness of the variables as data. In this case, (2.1) problem is called Stochastic Programming (SP) problem.

Within these categories of programming problem there are special cases such as the assignment, transportation, network flow, travelling salesman problems, among an outsize number of real problems.
In this thesis we present different multiclass human migration models under user-optimizing and system-optimizing behaviour, in which, therefore, the migration phenomenon is analysed from different perspective. For each of them we formulate the associated non linear constrained problem which allows us to solve the decision-making problems related to the specific applications.
The models here introduced are formulated, studied with different suitable approaches such as network problem, game theory problem and equilibrium problem approach. For all of them an equivalent formulation and analysis are also conducted using variational inequality theory.

2.1.1 Convex Optimization Problem

An important subclass of optimization problems are the convex optimization problems, that represent the largest class of tractable optimization problems ([87]). Consider the generic optimization problem (2.1), in which the objective function f is convex and continuously differentiable on K, that is assumed to be closed and convex. Such a problem is termed *convex optimization problem*.
We recall that

- $K \subset \mathbb{R}^n$ is said a *convex set* if $\lambda x + (1-\lambda)y \in K,\ \forall x,y \in K,\ \lambda \in [0,1]$. That means that for any two points of K, the segment joining them belongs to K.

- $f : K \to \mathbb{R}$, where $K \subset \mathbb{R}^n$ is a convex set, is said a *convex function* if $\lambda f(x + (1-\lambda)y) \le \lambda f(x) + (1-\lambda)f(y)\ \forall x,y \in K,\ \lambda \in [0,1]$. When the inequality is strict, f is said *strictly convex function.*

The importance of the convex optimization can be attributed not only to their multiple applications present in literature, but also to the powerful analytical and algorithmic tools available for their study (see [4],[7]).

2.2 Optimality conditions

Consider the (2.1) optimization problem in which the feasible set $K = \mathbb{R}^n$, and assume that the function f is at least continuously differentiable. It is useful to outline which are the conditions that may give insight to understand whether a feasible point is an optimal solution and, in general, about the problem. This kind of conditions, called *optimality conditions*, constitute the foundations for the theoretical study of the optimization problem and its numerical solution.

The basic necessary condition for unconstrained optimality of x^* is formalized by the following theorem.

Theorem 2.2.1. *Given* (2.1) *problem, if $x^* \in \mathbb{R}^n$ is a local minimum problem and f is continuously differentiable in a neighbourhood of x^*, then $\nabla f(x^*) = 0$*

Such a point x^*, satisfying the $\nabla f(x^*) = 0$ condition, is called *stationary point*. Consider the programming problem (2.1) with $K \subset \mathbb{R}^n$ as in (2.2).
The following theorem provides the necessary optimality conditions in the case in which the feasible set is defined by inequalities and equalities. Such conditions are more commonly known as *Karush-Kuhn-Tucker* (KKT) conditions (see [43] and [51]).

Theorem 2.2.2. *Given an optimization problem as in* (2.1)*, where the feasible set is defined as in* (2.2) *and $\bar{x} \in K$ is a local minimum point. Suppose that the functions f, g_i, $h_j, \forall i = 1,\dots,p$ and $\forall j = 1,\dots,q$ are continuously differentiable in $\bar{x}$ and that $\nabla g_1(x^*), \nabla g_2(x^*),\dots,\nabla g_p(x^*), \nabla h_1(x), \nabla h_2(x^*),\dots,\nabla h_q(x^*)$ are linearly independent vectors.*
Than, there exist $\lambda^* = (\lambda_1^*, \lambda_2^*,\dots,\lambda_p^*)$ and $\mu^* = (\mu_1^*, \mu_2^*,\dots,\mu_q^*)$ vectors, such

that

$$f(x^*) + \sum_{i=1}^{p} \lambda_i^* g_i(x^*) + \sum_{j=1}^{q} \mu_j^* h_j(x^*) = 0 \tag{2.3}$$

$$g_i(x^*) \leq 0 \quad \forall i = 1, 2, \ldots, p \tag{2.4}$$

$$h_j(x^*) = 0 \quad \forall j = 1, 2, \ldots, q \tag{2.5}$$

$$\lambda_i^* g_i(x^*) = 0 \quad \forall i = 1, 2, \ldots, p \tag{2.6}$$

$$\lambda_i^* \geq 0 \quad \forall i = 1, 2, \ldots, p \tag{2.7}$$

where $\lambda^* = (\lambda_1^*, \lambda_2^*, \ldots, \lambda_p^*)$ *and* $\mu^* = (\mu_1^*, \mu_2^*, \ldots, \mu_q^*)$ *are called Lagrange multipliers vectors.*

The (2.3) condition is that of the cancellation of the gradient of the Lagrangian function[1] associated with the problem. It ensures that there is no feasible direction that could potentially improve the objective function. The (2.4) and (2.5) conditions are a statement that the constraints must not be violated by x^*, while the (2.6) condition, called *complementarity condition*, enforces a zero Lagrange multiplier if associated with an inactive constraint. Finally, the (2.7) is the non-negativity condition of the multiplier associated with the inequality constraints.

2.2.1 Optimality conditions for a convex optimization problem

The fundamental optimality conditions for convex optimization problems are called the *minimum principle*, formally given in (2.8).
Consider the convex optimization problem. A feasible point $x^* \in K$ is an optimal solution if and only if

$$\nabla f(x^*)^T (x - x^*) \geq 0 \quad \forall x \in K. \tag{2.8}$$

[1]Given the optimization problem (2.1) with feasible set K as defined in (2.2), the *Lagrange function* is given by:

$$\mathscr{L}(x, \lambda, \mu) = f(x) + \sum_{i=1}^{p} \lambda_i g_i(x) + \sum_{j=1}^{q} \mu_j h_j(x) \qquad \forall x \in K$$

where $\lambda = (\lambda_1, \ldots, \lambda_p)$ and $\lambda = (\mu_1, \ldots, \mu_q)$ are the *Lagrange multipliers*

It turns out that convexity makes the necessary conditions introduced, both for constrained and unconstrained problems, also sufficient for optimality.
Indeed, in the case in which $K = \mathbb{R}^n$, the minimum principle (2.8) reduces to the necessary condition given in theorem 2.2.1 (and sufficient for convex f) for unconstrained optimality of x^*: $\nabla f(x^*) = 0$.
The case in which the feasible set $K \subset \mathbb{R}^n$ is defined by inequalities and equalities it can be shown that, under some additional conditions, the minimum principle is equivalent to the necessary KKT optimality conditions given in theorem 2.2.2, where the convexity of the objective function f makes them also sufficient (see [4], [7]).

2.3 Brief recall of Variational Inequality theory

Variational inequality theory was introduced by Hartman and Stampacchia in 1966 ([36]) as a tool for the study of partial differential equations with applications principally drawn from mechanics. Such variational inequalities were infinite-dimensional.
An important piece in the finite-dimensional theory literature found his place in 1980 when Dafermos recognized that the traffic network equilibrium conditions, as stated by Smith one year before (see [90]), had a structure of a variational inequality. As a result of this breakthrough problems management science/operations research, and also in economics and engineering, with a focus on transportation, have been studied and solved by means of variational inequality theory.
At the end of the nineties, researchers started to investigate optimization problems, through a variational approach, by considering also time dependence. In [24] and [25] (see also [32]), for the first time, Daniele, Maugeri and Oettli presented a traffic network equilibrium problem with path flows capacity constraints dependent from time and traffic demands.
The concept of *equilibrium* is central in numerous disciplines including economics, operations research, engineering. Variational inequality theory is also a powerful tool for:

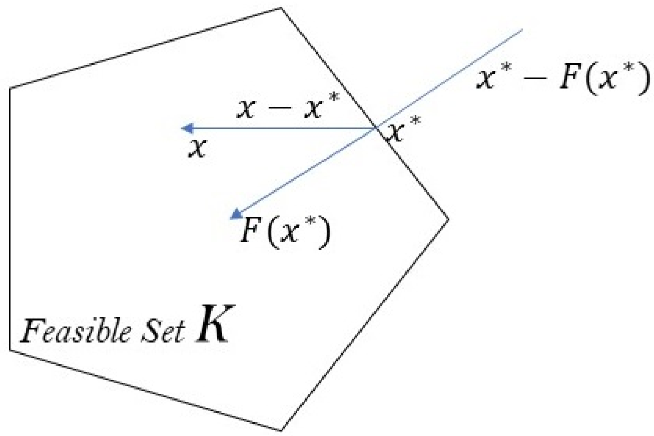

Figure 2.2: Geometrical interpretation of variational inequality definition

- formulating a variety of equilibrium problems;
- qualitative analysis of equilibria in terms of existence and uniqueness of solutions, stability and sensitivity analysis;
- providing us with algorithms accompanying convergence analysis for computational purposes.

To-date problems which have been formulated and studied as variational inequality problems include traffic network equilibrium problems (see e.g. [33], [23]), spatial price equilibrium problems (see e.g. [22], [86]), migration equilibrium problems, as well as (example of which are treated in this thesis).
We now introduce the definition of variational inequality (VI) problem.

Definition 2.3.1. The finite-dimensional variational inequality problem, $VI(F,K)$, is to determine a vector $x^* \in K \subseteq \mathbb{R}^n$, such that

$$< F(x^*), x - x^* > \geq 0, \quad \forall x \in K, \tag{2.9}$$

where $F : K \to \mathbb{R}$ is a given function, K is a given closed convex set, and $< \bullet, \bullet >$ denotes the inner product in n-dimensional Euclidean space.

From a geometric point of view the variational inequality (2.9) states that $F(x^*)^T$ is "orthogonal" to the feasible set K at the point x^*.
Furthermore, according to the definition of variational inequality (2.9), if a feasible point x^* is a solution of the $VI(K,F)$ than $F(x^*)$ and $x-x^*$, $\forall x \in K$, form an nonobtuse angle[2].
It is interesting to outline the relationships between variational inequalities and optimization problems. Indeed, a variational inequality is related to an optimization problem when F involved in the first one can be expressed as the gradient of the objective function of the second one. In this case, as stated in the following proposition, the optimization problems can be formulated as VIPs.

Proposition 2.3.1. *Let x^* be a solution to the optimization problem:*

$$\begin{aligned} &min f(x) \\ &s.t. x \in K, \end{aligned} \tag{2.10}$$

where f is continuously differentiable and K is closed and convex. Then x^* is a solution to the variational inequality problem:

$$< \nabla f(x^*)^T, x - x^* > \geq 0, \ \forall x \in K. \tag{2.11}$$

It is clear that in the case in which the $F = \nabla f$, for some suitable function f that is convex in K, the variational inequality problem (2.9) coincides with the problem of finding a point satisfying the minimum principle (2.8) and therefore with the problem of finding an optimal solution of the convex optimization problem. This result, formalized in the following proposition, implies that the viceversa of proposition (2.3.1) holds.

Proposition 2.3.2. *If $f(x)$ is a convex function and x^* is a solution to $VI(\nabla f, K)$, then x^* is a solution to the optimization problem (1.3).*

In the following results, conditions for existence and uniqueness of solutions

[2]We recall that the inner product between two vectors $v, w \in R^n$ can be geometrically characterized by the $< v^T, w >= ||v|| \, ||w|| cos\theta$ equation, where θ measures the angle between the two vectors. That means $< v^T, w >$ scalar is grater or equal to 0 when the angle θ falls in the range $0 - 90$.

to $VI(F,K)$ are provided.
Continuity of the function F entering the variational inequality and compactness of the feasible set K ensure the existence of a solution to a variational inequality problem. Indeed, we have the following:

Theorem 2.3.1. *If $\mathbb{K}$ is a compact set and $F(x)$ is continuous on $\mathbb{K}$, then the variational inequality problem* (2.9) *admits at least one solution x^*.*

Qualitative properties of existence and uniqueness follows from certain monotonicity conditions on the function F. We briefly recall some definitions. A function $F : \mathbb{K} \subseteq \mathbb{R}^n \to \mathbb{R}$ is

- *monotone* on $\mathbb{K}$ if $[F(x^1) - F(x^2)]^T (x^1 - x^2) \geq 0 \quad \forall x^1, x^2 \in \mathbb{K}$;

- *strictly monotone* on $\mathbb{K}$ if $[F(x^1) - F(x^2)]^T (x^1 - x^2) > 0 \quad \forall x^1, x^2 \in \mathbb{K}, x^1 \neq x^2$;

Theorem 2.3.2. *Assume that $F(X)$ is continuous and strictly monotone on K, that is a nonempty, convex and compact subset of $\mathbb{R}^n$. Then there exists precisely one solution x^* to VI(F,K).*

Moreover, using the strong monotonicity and coerciveness properties, we introduce the following theorem on the existence and uniqueness of the solutions of the $VI(F,K)$, without requiring the compactness of the feasible set K.

Theorem 2.3.3. *Given $\mathbb{K} \subseteq \mathbb{R}^n$, closed convex and nonempty. Assume that $F(X) : \mathbb{K} \to \mathbb{R}$ is*

- *strongly monotone on $\mathbb{K}$, that means that for some $\alpha \geq 0$ $[F(x^1) - F(x^2)]^T (x^1 - x^2) \geq \alpha ||x^1 - x^2||^2 \quad \forall x^1, x^2 \in \mathbb{K}$*

- *and coercive on K, which means if $\exists\, M > \alpha$ such that $||F(x^1) - F(x^2)|| \leq M||x^1 - x^2|| \quad \forall x^1, x^2 \in \mathbb{K}$*

then there exists only one solution x^* to VI(F,K).

2.3.1 Variational inequality and game theory problems

Game theory deals with the analysis and resolution of conflicts among interacting decision makers (called players) which are assumed to be rational and which have different interests, quantified in general through an objective function. The decision (called strategy) of each player can produce different results depending on the strategies chosen by the other players.
Noncooperative game theory is a branch of game theory in which each player behaves selfishly to optimize their own well being.
Assume there are M players each controlling the variables x_i, that we group into the vector $x = (x_1, x_2, \dots, x_M)$. Given by M_i the set of ith player strategies, the aim of player i, provided the other players' strategies $x_{-i} = (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_M)$, is to choose an $x_i \in M_i$ that minimizes his payoff function that we mean by $f(x_i, x_{i-1})$[3]: $M_1 \times M_2 \times \cdots \times M_M \to \mathbb{R}$. In other words the aim of each player i consists in solving the optimization problem

$$\begin{aligned} &\min f(x_i, x_{i-1}) \\ &\quad s.t. x_i \in M_i. \end{aligned} \tag{2.12}$$

We introduce now the definition of *Nash Equilibrium problem* (NEP). A vector $x^* \in \mathbb{M}$, where $M = \prod_{i=1}^{M} M$, is called a *Nash equilibrium* (or simply a solution) of the NEP if

$$f(x_i^*, x_{-i}^*) \leq f(x_i, x_{-i}^*), \quad \forall x_i \in M_i. \tag{2.13}$$

or equivalently, x^* is NEP if and only if it solves the minimization problem (2.12), $\forall\, i = 1, 2, \dots, M$.
In other words, x^* is a Nash equilibrium if no single player can obtain higher payoff by deviating unilaterally to any other feasible point.

The next result establishes, under appropriate conditions, the equivalence between an NEP and a suitably defined VI.
Let $G =< M, f >$ the game defined by the (2.12) problems, where $f = f_i(x)_{i=1}^{M}$.

[3] $(x_i, x_{i-1}) = (x_1, \dots, x_{i_1}, x_i, x_{i+1}, \dots, x_M)$ for the notation used.

Proposition 2.3.3. *Suppose that for each player i the strategy set M is a closed and convex subset of* $M \subseteq \mathbb{R}^{n_i}$ *and the payoff function* $f_i(x_i, x_{-i})$ *is continuously differentiable. Than the game* $G =< M, f >$ *is equivalent to the* $VI(M,F)$*, where* $F(x) = (\nabla_{x_i} f_i(x))_{i=1}^M$.

2.3.2 Network equilibrium problems and Variational inequality

The purpose of (static) network equilibrium models is to predict traffic flows in a congested network, with minimal cost of users travel paths from origin to destination node of the network. Their first appearance occurs in 1920 when Pigou (see [80]) studied two-node, two-link (or path) transportation network, further developed in the 1924 by Knight (see [47]).
In the 1952 Wardrop (see [98]) stated two principles that formalize the traffic equilibrium conditions:

1. *First principle*: "the journey times of all routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route";

2. *Second principle*: "the average journey time is minimal".

These conditions were first rigorously mathematically formulated in [3] by Beckmann, McGuire, and Winsten, in which the authors established the equivalence between the network equilibrium problem and a convex optimization problem with a single objective function.

In the context of transportation, user-optimization (U-O) and system optimization (S-O) are seminal concepts and constructs. These are so named by Dafermos and Sparrow in the 1969 (see [19]) and correspond, respectively, to the aforementioned Wardrop's two principles. In the case of system-optimization a central controller routes the traffic between origin/destination pairs of nodes, so that the total cost to society is minimized. In the case of U-S, travellers act in a unilateral way, seeking their individual cost-minimal routes of travel between an origin/destination pair of nodes.
In the human migration network context, in contrast, as we will see in details in

Section 5 and 6, we are concerned with total utility maximization in the case of S-O and individual utility maximization in the case of U-O behaviour and the selection of locations.
We now formally formulate the Wardrop's traffic model.
Let $\mathscr{N} = \{P_1, P_2, \dots, P_p\}$ the nodes set and $\mathscr{A} = \{a_i,\ i = 1, 2, \dots, n\} \subset \mathscr{N} \times \mathscr{N}$ the unidirectional links set. Assume that there are l Origin/Destination (O/D) pairs, with a typical O/D pair denoted by w_j. $\mathscr{W} \subseteq \mathscr{N} \times \mathscr{N}$ denotes O/D pairs set.
A Traffic Network is identified by the triple $(\mathscr{N}, \mathscr{A}, \mathscr{W})$.
Given an O/D pair $w_j = (P_h, P_k)$ we introduce both the set $\mathscr{R}_j$ of the paths that join P_h with P_k and the set of all the paths of the network with $\mathscr{R} = \bigcup_{j=1}^{l} \mathscr{R}_j = \{R_r,\ r = 1, 2, \dots, m\}$.
It is reasonable to assume that:

- $R_j \neq \emptyset \quad \forall j = 1, 2, \dots, l$;
- the paths outnumber the O/D pairs ($m > l$), so that the user can choose the most convenient path.

For each link a_i and for each path R_r of the network, we respectively associate a nonnegative number $f_i \geq 0$, called link flow, and a nonnegative number $F_r \geq 0$ called path flow. We group the link flows into a vector $f = (f_1, \dots, f_n) \in \mathbb{R}^n_+$ and the path flows into a vector $F = (F_1, \dots, F_m) \in \mathbb{R}^m_+$.
We must have the flow on a link is equal to the sum of the flows on all paths that contain that link, namely

$$f_i = \sum_{r=1}^{m} \delta_{ir} F_r \quad \forall i = 1, 2, \dots, n. \tag{2.14}$$

where δ_{ir} are the link-path incidence matrix components, Δ, defined as

$$\delta_{ir} = \begin{cases} 1 & \text{if } a_i \in R_r, \\ 0 & \text{otherwise.} \end{cases} \tag{2.15}$$

Thus (2.14) in matrix form is $f = \Delta F$.

For each link of the network is introduced the link cost $c_i(f) \geq 0$, that depends upon the flows on every link in the network. We group the link costs into a vector $c(f) = (c_1(f), \ldots, c_n(f)) \in \mathbb{R}^n_+$. Similarly, with each path we associate the path cost $C_r(F) \geq 0$ that we group into the path cost vector $C(F) = (C_1(F), \ldots, C_m(F)) \in \mathbb{R}^m_+$.
We must have that the cost on a path is equal to the sum of the link costs of links comprising that path and using that mode. It is formally expressed as follows

$$C_r(F) = \sum_{i=1}^{n} \delta_{ir} c_i(f), \quad \forall r = 1, 2, \ldots, m. \tag{2.16}$$

or equivalently $C(F) = \Delta^T c(f)$, in matrix form.

We introduce the ρ_j demand of potential users travelling between O/D pair w_j. For each $j = 1, 2, \ldots, l$ we group ρ_j into the demand vector $\rho = (\rho_1, \rho_2, \ldots, \rho_l) \in \mathbb{R}^l_+$. The *flow conservation constraint* states that each demand must be equal to the sum of the flows on the paths joining the O/D pair, that is,

$$\sum_{R_r \in R_j} F_r = \rho_j, \quad \forall j = 1, 2, \ldots, l. \tag{2.17}$$

Introducing the O/D pairs-path incidence matrix, Φ whose elements are defined as follows

$$\phi_{jr} = \begin{cases} 1 & \text{if } R_r \in \mathscr{R}_j, \\ 0 & \text{otherwise;} \end{cases} \tag{2.18}$$

we write (2.17) in matrix form as $\Phi F = \rho$.
Therefore, the path flows feasible set is given by $\mathbb{K} = \{F \in \mathbb{R}^m_+ : \;\; \Phi F = \rho\}$.

Definition 2.3.2. (Equilibrium, Wardrop (1952))
$H \in \mathbb{K}$ is an *equilibrium configuration* if $\forall w_j \in \mathscr{W}$ and $\forall R_q, R_s \in \mathscr{R}_j$ we have that $C_q(H) < C_s(H)$, than $H_s = 0$.

The following result (see [90] and [16]) allows to characterize an equilibrium configuration according to Wardrop's definition as a solution to a suitable variational inequality.

Theorem 2.3.4. *$H \in \mathbb{K}$ is an equilibrium distribution according to the 2.3.2 definition if and only if H is a solution to the variational inequality "Finding $H \in \mathbb{K}$ such that $< C(H), F - H > \geq 0 \ \forall F \in \mathbb{K}$"*

CHAPTER 3

A variational formulation for a human migration problem

In this chapter a human migration network based model is provided. In the model here considered, the aim of each migration class is to maximize the attractiveness of the origin country, which is given by the sum of its utility and its expected increment of utility value, with respect to the destination one.
This work has focused on the modelling of migration flows assuming user-optimizing (U-O) behaviour, inspired by the one originating with the work of Nagurney in 1989 (see [64]). In other words, here it has been assumed that the migrants act selfishly and independently; see also [63], [73], [74], [79], [72], [38], [41], [11], [68], [69], for a spectrum of U-O migration models from a period ranging from 1990 to 2019.
The human migration optimization problem, unlike other works in the literature, is analysed and studied in terms of Nash equilibrium problem. The solution of the model proposed is defined as a Nash Equilibrium and it is established the equivalence between the Nesh Equilibrium problem and an appropriate variational inequality.
Finally, some illustrative numerical results applied to the human migration from

Africa to Europe are presented and analysed.

3.1 The mathematical model

We present a network consisting of n nodes, that are countries or, more generally, locations, and H classes of the population. As depicted in Figure 3.1, the n locations are both origin nodes (where migrants are initially located) and destination nodes (where the migrants may be interested in migrating to).
We assume that at each location i; $i = 1, \dots, n$, there is an initial fixed population of the general class k, denoted by $\bar{p}_i^k$.
We denote the nonnegative population of migrant class k at node i by p_i^k and by f_{ij}^k we denote the nonnegative migration flow out of the node i, and into the node j of the network, with $i \neq j$. It means that if a volume of population of a typical class decides to migrate, then the destination node differs from the origin one; otherwise, it remains in the same origin node.
The population of class k at location i is determined by the initial population of class k at location i plus the migration flow, f_{ji}^k, into i of that population, minus the migration flow, f_{ij}^k, out of i. Indeed, the flow conservation constraints, given for each class k and each location i, are given as follows:

$$p_i^k = \bar{p}_i^k - \sum_{\substack{j=1 \\ j \neq i}}^{n} f_{ij}^k + \sum_{\substack{j=1 \\ j \neq i}}^{n} f_{ji}^k \quad \forall i = 1, ..., n, \quad \forall k = 1, \dots, J. \tag{3.1}$$

We group the populations k in each location i into the vector p^k and the migration flows of population k from each origin node i to each destination node j into the vector f^k. Each location i is characterized by

- a destination utility function, v_i^k, that is indicative of the attractiveness of that location intended as an idealization of the opportunities that this node can offer, as perceived by the migration class k;
- an origin utility function, u_i^k, that is indicative of the attractiveness of that location intended as the awareness of the opportunities that this node can offer, as perceived by the migration class k.

Both groups of functions, u_i^k and v_i^k, depend on the population p^k.
Let us introduce $w_{ij}^{k+} \geq 0$, which denotes the influence coefficient taken into account by an individual of the migration class k moving from node i to j. It is the expected variation of the population at node j after a migratory flow, as perceived by the migration class k. Similarly, we introduce $w_{ij}^{k-} \geq 0$, that is the expected rate of change of the total population at node i, as perceived by the migration class k.
After the potential movement of migrants from location i to location j, each person of class k expects a variation of the utility function value at j:

$$f_{ij}^k w_{ij}^{k+}(p^k, f^k) \frac{\partial v_j^k(p^k)}{\partial p_j^k},$$

and a variation of the utility function value at i:

$$-f_{ij}^k w_{ij}^{k-}(p^k, f^k) \frac{\partial u_i^k(p^k)}{\partial p_i^k},$$

where the negative sign denotes the loss for the migration class k, when choosing to abandon the origin node.
Both groups of such variations are assumed concave.
In addition, we denote by c_{ij}^k the movement cost function from i to j for the population class k that depends on the entire migration flows vector of population k, namely

$$c_{ij}^k = c_{ij}^k(f^k), \quad \forall i, j = 1, \ldots, n, \ j \neq i, \ \forall k = 1, \ldots, J.$$

Such costs are assumed to be convex and continuously differentiable.

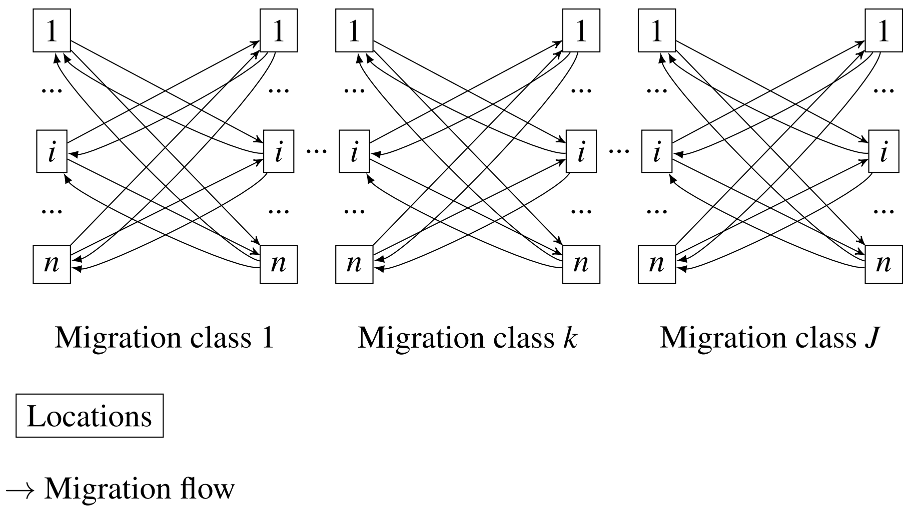

Figure 3.1: **Multiclass migration network**

Symbol	**Definition**
$\bar{p}_i^k$	Initial population of class k in location i
p_i^k	Population at location i of class k
f_{ij}^k	Migration flow from i to j of class k, with $i \neq j$
$v_i^k(p^k)$	Destination utility function of any location i as perceived by the class k
$u_i^k(p^k)$	Origin utility function of any location i as perceived by the class k
$c_{ij}^k(f^k)$	Movement cost from i to j for the class k
$w_{ij}^{k+}(p^k, f^k)$, $w_{ij}^{k-}(p^k, f^k)$	Influence coefficient

Table 3.1: Functions, parameters and variables of the model

In order to reduce the migration phenomenon and to encourage people to remain in their own country the attractiveness of location i (which is given by the sum of the utility of location i and its expected increment of utility value consequently to the class k migratory movement in the network) must exceed the sum between the attractiveness of location j (which is given by the sum of the utility of location j and its expected increment of utility value consequently to the class k migratory movement in the network) and the transportation costs from location i to location j. Hence, in our model, the aim of each migration class k, $k = 1, ..., J$, in each departing node i, $i = 1, ..., n$ is to maximize its net utility, namely the following difference:

$$\max_{(p^k,f^k)\in\mathbb{K}^k} U^k(p^k,f^k) = \max_{(p^k,f^k)\in\mathbb{K}^k} \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} \Bigg(u_i^k(p^k) - f_{ij}^k w_{ij}^{k-}(p^k,f^k)\frac{\partial u_i^k(p^k)}{\partial p_i^k} - c_{ij}^k(f^k) - v_j^k(p^k) - f_{ij}^k w_{ij}^{k+}(p^k,f^k)\frac{\partial v_j^k(p^k)}{\partial p_j^k}\Bigg) \quad (3.2)$$

where

$$\mathbb{K}^k = \Bigg\{(p^k,f^k)\in\mathbb{R}^{n+n(n-1)} : p_i^k = \bar{p}_i^k - \sum_{\substack{j=1\\ j\neq i}}^{n} f_{ij}^k - \sum_{\substack{j=1\\ j\neq i}}^{n} f_{ji}^k,\ \forall i=1,\ldots,n; \; p_i^k \geq 0,\ f_{ij}^k \geq 0, \forall i,\ j=1,\ldots,n,\ j\neq i; \; \sum_{\substack{j=1\\ j\neq i}}^{n} f_{ij}^k \leq \bar{p}_i^k, \forall i=1,...,n\Bigg\}. \quad (3.3)$$

We also assume that the migration classes compete in a noncooperative manner, so that each maximizes its utility, given the actions of the other classes.
Under the imposed assumptions, the objective function in (3.2) is concave and continuously differentiable. We also define the feasible set

$$\mathbb{K} = \Bigg\{(p,f)\in\mathbb{R}^{Jn+Jn(n-1)} : p_i^k = \bar{p}_i^k - \sum_{\substack{j=1\\ j\neq i}}^{n} f_{ij}^k - \sum_{\substack{j=1\\ j\neq i}}^{n} f_{ji}^k,\ \forall i=1,\ldots,n,\ \forall k=1,\ldots,J \; p_i^k \geq 0,\ f_{ij}^k \geq 0, \forall i,\ j=1,\ldots,n,\ j\neq i,\ \forall k=1,\ldots,J; \; \sum_{\substack{j=1\\ j\neq i}}^{n} f_{ij}^k \leq \bar{p}_i^k,\ \forall i=1,...,n,\ \forall k=1,\ldots,J\Bigg\} = \prod_{k=1}^{J}\mathbb{K}^k \quad (3.4)$$

and the total utility as:

$$U(p,f) = \sum_{k=1}^{J} U^k(p^k,f^k) \quad \forall (p,f)\in\mathbb{K}.$$

Hence, the above game theory model, in which the migration classes compete noncooperatively, is a Nash equilibrium problem. Therefore, we can state the following definition.

Definition 3.1.1. A population and migration flow pattern $(p^*, f^*) \in \mathbb{K}$ is said to be a Nash equilibrium if for each migration class k

$$U(p^{k*}, f^{k*}, \hat{p}^{k*}, \hat{f}^{k*}) \geq U(p^k, f^k, \hat{p}^{k*}, \hat{f}^{k*}) \quad \forall (p^k, f^k) \in \mathbb{K}^k,$$

where

$$\hat{p}^{k*} = (p^{1*}, \ldots, p^{k-1*}, p^{k+1*}, \ldots, p^{J*}) \quad \textit{and} \quad \hat{f}^{k*} = (f^{1*}, \ldots, f^{k-1*}, f^{k+1*}, \ldots, f^{J*}).$$

Hence, according to the above definition, a Nash equilibrium in established if no migration class can unilaterally improve its expected utility by choosing an alternative vector of population and migration flow.
The optimality conditions (3.2) for all migration classes $k, k = 1, \ldots, J$ simultaneously can be expressed by means of a variational inequality as follows.

Theorem 3.1.1 (Variational formulation)**.** *Under the above assumptions,* $(p^*, f^*) \in \mathbb{K}$ *is an equilibrium according to Definition 3.1.1 if and only if it satisfies the following variational inequality:*

Find $(p^*, f^*) \in \mathbb{K}$ *such that:*

$$\sum_{l=1}^{n}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{k=1}^{J}\left(\frac{\partial v_j^k(p^{k*})}{\partial p_l^k} + f_{ij}^k \frac{\partial w_{ij}^{k+}(p^{k*}, f^{k*})}{\partial p_l^k}\frac{\partial v_j^k(p^{k*})}{\partial p_j^k} + f_{ij}^k w_{ij}^{k+}(p^{k*}, f^{k*})\frac{\partial^2 v_j^k(p^{k*})}{\partial p_j^k \partial p_l^k}\right.$$
$$\left. - \frac{\partial u_i^k(p^{k*})}{\partial p_l^k} + f_{ij}^k \frac{\partial w_{ij}^{k-}(p^{k*}, f^{k*})}{\partial p_l^k}\frac{\partial u_i^k(p^{k*})}{\partial p_i^k} + f_{ij}^k w_{ij}^{k-}(p^{k*}, f^{k*})\frac{\partial^2 u_i^k}{\partial p_i^k \partial p_l^k}\right)(p_l^k - p_l^{k*})$$
$$+\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{k=1}^{J}\left(w_{ij}^{k+}(p^{k*}, f^{k*})\frac{\partial v_j^k(p^*)}{\partial p_j^k} + f_{ij}^{k*}\frac{\partial w_{ij}^{k+}(p^{k*}, f^{k*})}{\partial f_{ij}^k}\frac{\partial v_j^k(p^{k*})}{\partial p_j^k} + \frac{\partial c_{ij}^k(f^{k*})}{\partial f_{ij}^k}\right.$$
$$\left. + w_{ij}^{k-}(p^{k*}, f^{k*})\frac{\partial u_i^k(p^*)}{\partial p_i^k} + f_{ij}^k \frac{\partial w_{ij}^{k-}(p^{k*}, f^{k*})}{\partial f_{ij}^k}\frac{\partial u_i^k(p^{k*})}{\partial p_i^k}\right)(f_{ij}^k - f_{ij}^{k*}) \geq 0,$$
$$\forall (p, f) \in \mathbb{K}. \tag{3.5}$$

3.2 Numerical Examples

In order to perform numerical experiments, we consider the flow of migrants from Africa through the Mediterranean sea to Italy during 2018. The network, as we can see in Figure 3.2, is composed by six nodes: the first three ones are the origin nodes (Tunisia, Eritrea and Sudan, with a percentage of $23,8\%$, 15.0%, and 7.3% of departures, respectively), the fourth node is Italy, which is the destination chosen by most of the migrants (see [82]), the last two nodes are Germany and France, with a percentage of 31% and 15%, respectively, of arrivals, and represent the countries where most migrants have been relocated from Italy ([58]).
We collected the necessary data, referred to 2018, about population, average movement, measures of the quality of life, transportation costs for each node of the network. We assume that the migration class is only one ($k = 1$), and it represents the African population that moves from its continent to Europe.
We assume also that the migration class in question is interested in evaluating the displacement between the nodes $1-4;\quad 2-4;\quad 3-4;\quad 4-5;\quad 4-6$; and this is because, starting from real data, the other displacements never occur due to political or economic issues, as we can see in Fig. 2.

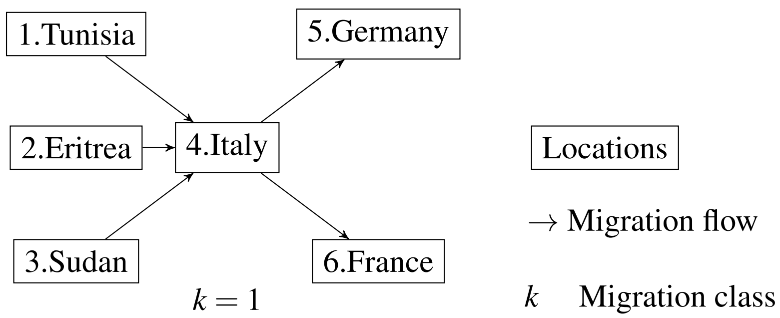

Figure 3.2: Multiclass migration network for the first numerical example

Let us consider the data in Table 3.2.

Origin utility function	$u_1 = -0.8(p_1^1)^2 + 2p_1^1$
	$u_2 = -0.7(p_2^1)^2 + 3p_2^1$
	$u_3 = -0.85(p_3^1)^2 + 2.5p_3^1$
	$u_4 = -0.5p_4^1$
Destination utility function	$v_4 = -0.5p_4^1$
	$v_5 = 0.3(p_5^1)^2 - 6p_5^1$
	$v_6 = 0.35(p_6^1)^2 - 7p_6^1$
Movement cost	$c_{14} = 3f_{14}^1$
	$c_{24} = 4f_{24}^1$
	$c_{34} = 5f_{34}^1$
	$c_{45} = 0.25f_{45}^1$
	$c_{46} = 0.25f_{46}^1.$
Inizial population	$\bar{p}_1 = 55$
	$\bar{p}_2^1 = 45$
	$\bar{p}_3^1 = 40$
	$\bar{p}_4^1 = 35$
	$\bar{p}_5^1 = 50$
	$\bar{p}_6^1 = 52$

Table 3.2: Data for the numerical example.

Then, optimality problem (3.2) becomes:

$$
\begin{aligned}
&\max_{(p^1,f^1)\in\mathbb{K}^1} U^1(p^1,f^1) = \\
&= \max_{(p^1,f^1)\in\mathbb{K}^1} \Big\{ u_1(p^1) - f^1_{14} w^-_{14}(p^1,f^1)\frac{\partial u_1(p^1)}{\partial p^1_1} - c_{14}(f^1) - v_4(p^1) - f^1_{14} w^+_{14}(p^1,f^1)\frac{\partial v_4(p^1)}{\partial p^1_4} + \\
&+ u_2(p^1) - f^1_{24} w^-_{24}(p^1,f^1)\frac{\partial u_2(p^1)}{\partial p^1_2} - c_{24}(f^1) - v_4(p^1) - f^1_{24} w^+_{24}(p^1,f^1)\frac{\partial v_4(p^1)}{\partial p^1_4} + \\
&+ u_3(p^1) - f^1_{34} w^-_{34}(p^1,f^1)\frac{\partial u_3(p^1)}{\partial p^1_3} - c_{34}(f) - v_4(p^1) - f^1_{34} w^+_{34}(p^1,f^1)\frac{\partial v_4(p^1)}{\partial p^1_4} + \\
&+ u_4(p^1) - f^1_{45} w^-_{45}(p^1,f^1)\frac{\partial u_4(p^1)}{\partial p^1_4} - c_{45}(f^1) - v_5(p^1) - f^1_{45} w^+_{45}(p^1,f^1)\frac{\partial v_5(p^1)}{\partial p^1_5} + \\
&+ u_4(p^1) - f^1_{46} w^-_{46}(p^1,f^1)\frac{\partial v_4(p^1)}{\partial p^1_4} - c_{46}(f^1) - u_6(p^1) - f^1_{46} w^+_{46}(p^1,f^1)\frac{\partial u_6(p^1)}{\partial p^1_6} \Big\}
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbb{K}^1 = \mathbb{K} \;=\; & \Big\{ (p^1_1,p^1_2,p^1_3,p^1_4,p^1_5,p^1_6,f^1_{14},f^1_{24},f^1_{34},f^1_{45},f^1_{46}) \in \mathbb{R}^{11}; \\
& p^1_1,p^1_2,p^1_3,p^1_4,p^1_5,p^1_6,f^1_{14},f^1_{24},f^1_{34},f^1_{45},f^1_{46} \geq 0 \\
& p^1_1 = 55 - f^1_{14};\; p^1_2 = 45 - f^1_{24};\; p^1_3 = 40 - f^1_{34}; \\
& p^1_4 = 35 + f^1_{14} + f^1_{24} + f^1_{34} - f^1_{45} - f^1_{46}; p^1_5 = 50 + f^1_{45}; \\
& p^1_6 = 52 + f^1_{46}\; f^1_{14} \leq 55;\; f^1_{34} \leq 40;\; f^1_{45} + f^1_{46} \leq 35 \Big\} \qquad (3.6)
\end{aligned}
$$

and the associated variational inequality (3.5) becomes:

$$
\begin{aligned}
&\textit{Find } (p^*, f^*) \in \mathbb{K} \quad \textit{such that:}\\
&(1.6p_1^1 - 2 - 1.6f_{14}^1 w_{14}^-) \times (p_1^1 - p_1^{*1}) + (1.4p_2^1 - 3 - 1.4f_{24}^1 w_{24}^-) \times (p_2^1 - p_2^{*1})\\
&+(1.7p_3^1 - 2.5 - 1.7f_{34}^1 w_{34}^-) \times (p_3^1 - p_3^{*1}) - 0.5 \times (p_4 - p_4^*)\\
&+(0.6p_5^1 - 6 + 0.6f_{45}^1 w_{45}^+) \times (p_5^1 - p_5^{*1}) + (0.7p_6^1 - 7 + 0.7f_{46}^1 w_{46}^+) \times (p_6^1 - p_6^{*1})\\
&+(w_{14}^-(-1.6p_1^1 + 2) + 3 - 0.5w_{14}^+) \times (f_{14}^1 - f_{14}^{*1})\\
&+(w_{24}^-(-1.4p_2^1 + 3) + 4 - 0.5w_{24}^+) \times (f_{24}^1 - f_{24}^{*1})\\
&+(w_{34}^-(-1.7p_3^1 + 2.5) + 5 - 0.5w_{34}^+) \times (f_{34}^1 - f_{34}^{*1})\\
&+(-0.5w_{45}^- + 0.25 + w_{45}^+(0.6p_5^1 - 6)) \times (f_{45}^1 - f_{45}^{*1})\\
&+(-0.5w_{46}^+ + 0.25 + w_{46}^+(0.7p_6^1 - 7)) \times (f_{46}^1 - f_{46}^{*1}) \geq 0\\
&\forall (p_1, p_2, p_3, p_4, p_5, p_6, f_{14}, f_{24}, f_{34}, f_{45}, f_{46}) \in \mathbb{K}.
\end{aligned}
\tag{3.7}
$$

It was solved using the modified projection method (see [92]), implemented in Matlab. The computational time to obtain the optimal flows according to the terms $w_{ij}^{\pm}$ was 9.01 seconds. The machine used for the simulation is a 4 GB RAM Asus Intel (R) Core (TM) i5-3317U CPU@1.10 GHz.

The optimal flows as solution of the VI (3.7) obtained considering different values of $w_{ij}^{\pm}$ $i = 1,2,3,4;$ $j = 4,5,6$ are shown in Table 3.2. We observed that the optimal flows, from the poorest countries to the richest are high comparing with the initial population, even thought the indices $w_{ij}^{\pm}$ let the migration class to conjecture an improvement of utility in the countries of departure and a worsening of the utility of the richer countries due to the migration.

w_{ij}^+	w_{ij}^-	Optimal flows
$w_{14}^+ = 0$	$w_{14}^- = 0$	$f_{14}^1 = 52,18706322$.
$w_{14}^+ = 0$	$w_{14}^- = 0$	$f_{24}^1 = 14,64240764$
$w_{34}^+ = 0$	$w_{34}^- = 0$	$f_{34}^1 = 35,8821929$
$w_{45}^+ = 0$	$w_{45}^- = 0$	$f_{45}^1 = 0$
$w_{46}^+ = 0$	$w_{46}^- = 0$	$f_{46}^1 = 0$
$w_{14}^+ = 1$	$w_{14}^- = 1$	$f_{14}^1 = 35,41646178$.
$w_{14}^+ = 1$	$w_{14}^- = 1$	$f_{24}^1 = 19,2855584$
$w_{34}^+ = 1$	$w_{34}^- = 1$	$f_{34}^1 = 24,90181913$
$w_{45}^+ = 1$	$w_{45}^- = 1$	$f_{45}^1 = 0$
$w_{46}^+ = 1$	$w_{46}^- = 1$	$f_{46}^1 = 0$
$w_{14}^+ = 0.5$	$w_{14}^- = 1$	$f_{14}^1 = 35,36437875.$
$w_{14}^+ = 0.5$	$w_{14}^- = 1$	$f_{24}^1 = 19,22603507$
$w_{34}^+ = 0.5$	$w_{34}^- = 1$	$f_{34}^1 = 24,8527998$
$w_{45}^+ = 0.2$	$w_{45}^- = 0.2$	$f_{45}^1 = 0$
$w_{46}^+ = 0.2$	$w_{46}^- = 0.2$	$f_{46}^1 = 0$
$w_{14}^+ = 0.1$	$w_{14}^- = 0.8$	$f_{14}^1 = 53,04662606.$
$w_{14}^+ = 0.1$	$w_{14}^- = 0.8$	$f_{24}^1 = 18,12476595$
$w_{34}^+ = 0.1$	$w_{34}^- = 0.8$	$f_{34}^1 = 27,64697497$
$w_{45}^+ = 0.2$	$w_{45}^- = 0$	$f_{45}^1 = 0$
$w_{46}^+ = 0.2$	$w_{46}^- = 0$	$f_{46}^1 = 0$
$w_{14}^+ = 0.2$	$w_{14}^- = 1$	$f_{14}^1 = 35,33312893.$
$w_{14}^+ = 0.2$	$w_{14}^- = 1$	$f_{24}^1 = 19,19032108$
$w_{34}^+ = 0.2$	$w_{34}^- = 1$	$f_{34}^1 = 24,8233882$
$w_{45}^+ = 0.1$	$w_{45}^- = 0.3$	$f_{45}^1 = 0$
$w_{46}^+ = 0.1$	$w_{46}^- = 0.3$	$f_{46}^1 = 0$
$w_{14}^+ = 0$	$w_{14}^- = 1$	$f_{14}^1 = 35,31234671.$
$w_{14}^+ = 0$	$w_{14}^- = 1$	$f_{24}^1 = 20,53554511$
$w_{34}^+ = 0$	$w_{34}^- = 1$	$f_{34}^1 = 24,80381573$
$w_{45}^+ = 0$	$w_{45}^- = 1$	$f_{45}^1 = 0$
$w_{46}^+ = 0$	$w_{46}^- = 1$	$f_{46}^1 = 0$
$w_{14}^+ = 0.25$	$w_{14}^- = 1$	$f_{14}^1 = 35,33833724.$
$w_{14}^+ = 0.25$	$w_{14}^- = 1$	$f_{24}^1 = 19,19627341$
$w_{34}^+ = 0.25$	$w_{34}^- = 1$	$f_{34}^1 = 24,82829013$
$w_{45}^+ = 0.5$	$w_{45}^- = 1$	$f_{45}^1 = 0$
$w_{46}^+ = 0.5$	$w_{46}^- = 1$	$f_{46}^1 = 0$
$w_{14}^+ = 0.5$	$w_{14}^- = 1$	$f_{14}^1 = 35,36437875.$
$w_{14}^+ = 0.5$	$w_{14}^- = 1$	$f_{24}^1 = 19,22603507$
$w_{34}^+ = 0.5$	$w_{34}^- = 1$	$f_{34}^1 = 24,8527998$
$w_{45}^+ = 0.25$	$w_{45}^- = 1$	$f_{45}^1 = 0$
$w_{46}^+ = 0.25$	$w_{46}^- = 1$	$f_{46}^1 = 0$
$w_{14}^+ = 0$	$w_{14}^- = 0.5$	$f_{14}^1 = 39,53106448.$
$w_{14}^+ = 0$	$w_{14}^- = 0.5$	$f_{24}^1 = 18,03548139$
$w_{34}^+ = 0$	$w_{34}^- = 0.5$	$f_{34}^1 = 27,57344578$
$w_{45}^+ = 0$	$w_{45}^- = 0.2$	$f_{45}^1 = 0$
$w_{46}^+ = 0$	$w_{46}^- = 0.2$	$f_{46}^1 = 0$

Table 3.3: Optimal flows obtained from VI (3.7) for different values of $w_{ij}^{\pm}$

3.3 Summary and Conclusion

In this chapter, we introduce a new multiclass network model of human migration that assumes user-optimizing behaviour. Here differing from the classic user-optimized models, the human migration optimization problem is analysed and studied in terms of Nash equilibrium problem.

We provide, for completeness, the variational inequality formulations of the Nash equilibrium problem. The optimal distribution patterns across multiple locations in the network, as shown in Numerical Examples section, are calculated using the modified projection method, implemented in Matlab.

CHAPTER 4

Human Migration Networks and Policy Interventions: Bringing Population Distributions in Line with System-Optimization

In this chapter, we first propose a system-optimized multiclass network model of human migration. The model assumes no births and no deaths and there is a fixed population of each class of migrant. There are no costs associated with migration, since we are interested in the long term. There is a utility function associated with each class of migrant and each location in the network economy. Each such utility is a function of the populations of the classes at that location and at other locations, in general. The objective in the network economy is to optimize the system and the societal welfare by maximizing the total utility, subject to the flow conservation constraints. The total utility is equal to the sum for all classes and all locations of the product of the individual utility at the location and class times are the population of that class at the location.We assume that the total utility function is concave and continuously differentiable and show that the system-optimized solution satisfies a VI problem. We then recall the user-optimized analogue of this human migration network, which was introduced in 1989 by Nagurney (see [64]). Therein, the governing equilibrium conditions state that migrants of a class will keep on moving from location to location until the individual utility of each class at each location that is populated by that class is maximal and equalized. Hence, at the equilibrium, none has any incentive to change his location. We also, for completeness, provide the VI formulation of the user-optimized (equilibrium) solution. We propose a ratio for assessing the societal welfare loss if migrants select their locations based on U-O rather than S-O behavior. The price of anarchy, introduced by Koutsoupias and Papadimitriou ([50]), is an inspiration for the societal welfare loss ratio. The price of anarchy was originally constructed to measure the total cost evaluated at the U-O solution and divided by the total cost evaluated at the S-O solution (see also [84]). Here, in contrast, we focus on total utility maximization in the network economy. In this chapter, we provide a procedure for computing subsidies that a government (or governments) can impose to guarantee that the system-optimized multiclass population distribution in the network economy is also user-optimized. These policies, when imposed, guarantee that individuals will choose their locations in a manner so that the societal welfare is maximized.

The chapter is organized as follows. In Section 4.1, the multiclass system-optimized model of human migration and its user-optimized analogue is presented. The optimal solutions to both satisfy an appropriate VI problem. The societal welfare loss ratio is proposed and an illustrative example is presented in order to reinforce the basic concepts. In Section 4.2, we outline the procedure for determining the subsidies for the different migrant classes and locations, so that, when applied, the S-O multiclass population distribution in the network economy is, at the same time, U-O. We also discuss how the framework can be used post a disaster. An algorithm is proposed in Section 4.3 and convergence results are provided. The algorithm is then applied in Section 4.4 to compute the solutions to a series of numerical examples, with changes in the demands, and in the utility functions, in order to also address the possible impacts of a pandemic, followed by a natural disaster. We summarize our results and present our conclusions in Section 4.5.

4.1 The System-Optimization Migration Network Model

In the network economy there are n locations at which the different classes in the population can locate. We assume freedom of movement between locations at zero cost, since here we are focusing on the long-term equilibrium population distributions at the various locations. We consider H classes of migrants with a typical class denoted by k. The network representation is given in Figure 4.1. Note that we associate locations with links (rather than nodes). Each link i; $i = 1, \ldots, n$, has an associated utility for each class denoted by U_i^k. The utility functions capture how attractive location i, $\forall i$, is for an individual of class k, $\forall k$. The relevant notation is given in Table 4.1. All vectors here are assumed to be column vectors.

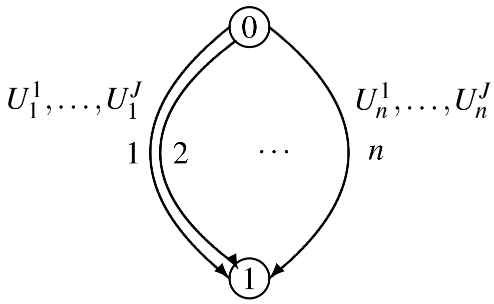

Figure 4.1: Network Structure of the Human Migration Model

Table 4.1: Notation for the Human Migration Models

Notation	**Definition**
p_i^k	the population of class k at location i. The $\{p_i^k\}$ elements are grouped into the vector $p^k \in R_+^n$. We then further group the p^k vectors; $k = 1, \ldots, J$, into the vector $p \in R_+^{Jn}$.
P^k	the population of class k in the network economy; $k = 1, \ldots, J$.
$U_i^k(p)$	the utility perceived by individuals of class k at location i; $i = 1, \ldots, n$. We group the utility functions for each k into the vector $U^k \in R^n$ and then group all such vectors for all k into the vector $U \in R^{Jn}$.
$\hat{U}_i^k(p)$	the total utility perceived by class k at location i; $i = 1, \ldots, n$. The total utility $\hat{U}_i^k(p) = U_i^k(p) \times p_i^k$; $k = 1, \ldots, J; i = 1, \ldots, n$.

The flow conservation constraint for each class k; $k = 1, \ldots, J$, is:

$$\sum_{i=1}^{n} p_i^k = P^k. \tag{4.1}$$

Hence, the population distribution of each class among the various locations must sum up to the population of that class in the network economy.

Moreover, the populations must be nonnegative, that is,

$$p_i^k \geq 0, \quad \forall i; \forall k. \tag{4.2}$$

Note that, according to Table 4.1, the total utility associated with a location and class, may, in general, depend upon the distribution of not only that particular class but also on that of the other classes. This is very reasonable: migrants may wish to locate where there is a certain concentration of those similar to themselves at the same location and, in proximity. At the same time, because of cultural and even economic factors they may wish to distance themselves from other classes.

We define the feasible set $K^1 \equiv \{p | (4.1) \text{ and } (4.2) \text{ hold.}\}$.

4.1.1 The System-Optimization (S-O) Problem

In the system-optimization (S-O) problem, the goal is to maximize the total utility in the network economy. This is achieved when the following optimization problem is solved:

$$\text{Maximize} \quad \sum_{k=1}^{H} \sum_{i=1}^{n} \hat{U}_i^k(p) = \sum_{k=1}^{H} \sum_{i=1}^{n} U_i^k(p) \times p_i^k \tag{4.3}$$

subject to the flow conservation constraints (4.1) for all k and the nonnegativity ones (4.2).

Observe that the objective function (4.3) captures the total utility of the society/economy under study. Locations i can correspond to different countries; to different regions in different countries, or to regions within a country, if the focus is on within country movements/migrations.

Under the assumption that the total utility functions for all the classes at all the locations are concave, and are continuously differentiable, we know that the optimal solution, denoted by p', satisfies the following variational inequality (VI): determine $p' \in K^1$, such that

$$-\sum_{k=1}^{H}\sum_{i=1}^{n}\left[\sum_{l=1}^{H}\sum_{j=1}^{n}\frac{\partial \hat{U}_j^l(p')}{\partial p_i^k}\right]\times(p_i^k - p_i^{k'}) \geq 0, \quad \forall p \in K^1. \tag{4.4}$$

Furthermore, if all the utility functions are strictly concave, it follows that the optimal population distribution p' is unique.

It is interesting to remark that, since $\hat{U}_i^k(p) = U_i^k(p) \times p_i^k$, then VI (4.4) is equivalent to

$$-\sum_{k=1}^{H}\sum_{i=1}^{n}\sum_{l=1}^{H}\sum_{j=1}^{n}\frac{\partial U_j^l(p')}{\partial p_i^k}\times(p_i^k - p_i^{k'}) - \sum_{k=1}^{H}\sum_{i=1}^{n}U_i^k(p')\times(p_i^k - p_i^{k'}) \geq 0, \quad \forall p \in K^1$$

and the second term coincides with (4.6).

4.1.2 The User-Optimization (U-O) Problem

As mentioned in the Introduction, it may be challenging, if nearly impossible, to reallocate the various class populations so that the system-optimal solution is achieved (unless one is living in a totalitarian state). This is especially the case, since migrants may not individually care about such a solution but may act selfishly in order to achieve an optimal solution for themselves “individually." In the case of user-optimization (U-O), it is assumed that the migrants are rational and that migration will continue until no individual of any class has any incentive to move since a unilateral decision will no longer yield an increase in the utility. The governing solution will satisfy the following migration equilibrium conditions proposed by Nagurney in [64] (see also [65]). Mathematically, a multiclass population vector $p^* \in K^1$ is said to be in equilibrium if for each class k; $k = 1,\ldots,H$; $i = 1,\ldots,n$:

$$U_i^k(p^*)\begin{cases} = \lambda^k & \text{if} \quad p_i^{k*} > 0 \\ \leq \lambda^k & \text{if} \quad p_i^{k*} = 0 \end{cases} \tag{4.5}$$

Equilibrium conditions (4.5) state that, for a given class k, only those locations

i with maximal utility equal to an indicator λ^k will have a positive population of the class. Furthermore, the utilities for a given class at populated locations by that class are equalized, that is, equilibrated across the locations. We note that λ^k is, in actuality, the Lagrange multiplier associated with constraint (4.1) for k. Indeed, from the duality theory (see [2] and [14]), we have that the multipliers $\overline{\lambda}$ and $\overline{\mu}$ associated with constraints (4.1) and (4.2), respectively, satisfy the following conditions:

$$U_i^k(p^*) = -\overline{\lambda}^k + \overline{\mu}_i^k,\ i = 1,\ldots,n,\ k = 1,\ldots,J; \quad \overline{\mu}_i^k p_i^{k*} = 0, \quad \overline{\mu}_i^k \geq 0$$

and, hence, we obtain 4.5.

As shown in the above noted references, the equilibrium p^* satisfies the variational inequality problem: determine $p^* \in K^1$ such that

$$-\sum_{k=1}^{H}\sum_{i=1}^{n} U_i^k(p^*) \times (p_i^k - p_i^{k*}\rangle \geq 0, \quad \forall p \in K^1. \tag{4.6}$$

Clearly, the solution p^* to VI (4.6) can be expected to be distinct from the solution p' to VI (4.4).

4.1.3 A Simple Example

In order to reinforce the above concepts we present an example for which both the system-optimized and the user-optimized solutions are provided. We consider a network economy consisting of two locations and a single class; hence, we suppress the superscript notation. The total population is: $P = 100$ and the utility functions at the two locations are:

$$U_1(p) = -p_1 + 200, \quad U_2(p) = -p_2 + 220.$$

The user-optimized solution is:

$$p_1^* = 40, \quad p_2^* = 60,$$

yielding $\lambda = 160$, since

$$U_1(p^*) = U_2(p^*) = 160.$$

Observe that VI (6) is satisfied by this p^*. Note that, at this population distribution, the total utility: $\hat{U}_1 + \hat{U}_2 = 16,000$.

On the other hand, the system-optimized solution is:

$$p'_1 = 45, \quad p'_2 = 55,$$

and VI (4) is satisfied since

$$-\frac{\partial \hat{U}_1(p')}{\partial p_1} = -\frac{\partial \hat{U}_2(p')}{\partial p_2} = -110.$$

The corresponding total utility is: $16,050$. Clearly, $16,050 > 16,000$. Furthermore, at the system-optimized solution, we have that:

$$U_1(p') = 155, \quad U_2(p') = 165$$

and, clearly, the S-O solution is not U-O. Hence, without appropriate policy interventions, and, if humans are “free" to move/migrate, the S-O solution would be difficult to sustain. Indeed, one can expect that some of those at location 1 will migrate to location 2 until the U-O solution is achieved since the utility in location 2 is higher than in location 1 and people will move; so the initial S-O solution $(45, 55)$ would become $(40, 60)$.

4.1.4 The Societal Welfare Loss Ratio

We now construct the societal welfare loss ratio. Note that the classical price of anarchy focused on total cost minimization in the congested transportation/telecommunications network context. Here, in contrast, we are concerned with total utility maximization in a network economy and the associated societal welfare loss under U-O as opposed to S-O behaviour.

Our societal welfare loss ratio π is defined as follows:

$$\pi = \frac{TU(p^*)}{TU(p')}, \tag{4.7}$$

where TU denotes the total utility such that

$$TU = \sum_{k=1}^{H} \sum_{i=1}^{n} \hat{U}_i^k; \tag{4.8}$$

and recall that p^* is the population distribution pattern under the U-O solution for a given migration network problem and p' is that for the S-O solution for the problem. The ratio quantifies the societal welfare loss if migrants select their U-O destinations rather than being allocated to locations under S-O. Note that here we consider multiple classes of migrants in our ratio.

For the above numerical example, we have that:

$$\pi = 16{,}000/16{,}050 = .997.$$

The value of π in(4.7) can never exceed 1, since the highest societal total utility TU is achieved at p', the S-O solution. The smaller the value of the societal welfare loss ratio π, the greater the welfare loss to society under the U-O solution.

4.2 Policy Intervention in the Form of Subsidies

We now proceed to ask the following question. Is there a migration policy that, when applied, can make the system-optimized solution also a user-optimized one? If so, an application of such a policy would result in no users having any incentive to switch their locations, and the population distribution would be one that is also optimal for the society (system-optimized).

The answer is: Yes! The derivation and construction of such a migration policy is as follows. We first solve for the system-optimized solution p'. For each class k, we denote those locations with a positive population by $k_1, \ldots, k_{n_k}$, where n_k is the number of locations in the network economy with a positive population of class k. We also introduce notation for subsidies associated with the different locations for each class denoted by class k by: $(subsidy)_{k_1}$, $(subsidy)_{k_2}$, ..., $(subsidy)_{k_{n_k}}$. We can then list those location as:

$$\begin{aligned} U^k_{k_1}(p') + subsidy^k_{k_1} &= \mu^k, \\ U^k_{k_2}(p') + subsidy^k_{k_2} &= \mu^k, \\ \text{and so on until} \\ U^k_{k_{n_k}}(p') + subsidy^k_{k_{n_k}} &= \mu^k, \end{aligned} \tag{4.9}$$

where μ^k is the incurred utility for class k after the subsidies are distributed for the class at the locations with positive populations of that class. Also, we can number those locations for that class with zero populations of that class (if there are any) as follows:

$$\begin{aligned} U^k_{k_{n_{k+1}}}(p') + subsidy^k_{k_{n_k+1}} &\le \mu^k, \\ \text{and so on until} \\ U^k_{k_n}(p') + subsidy^k_{k_n} &\le \mu^k. \end{aligned} \tag{4.10}$$

According to (4.9) and (4.10), the cognizant government authority selects the μ^k for each class k, and then the subsidy for each location for that class is easily determined via subtraction.

The question now arises as to what value is reasonable for μ^k? We propose that μ^k be set as: $\max_{k_l; l=1,\ldots,n_k} U^k_{k_l}(p')$. This procedure guarantees that all the subsidies will be nonnegative and that all enjoy the maximal utility for each class at all the populated locations. Also, for the subsidies associated with locations with no populations of a class k (see (4.10)), those subsidies are set to zero.

Returning to the above simple example and, again suppressing superscripts since there is a single class, the above subsidy formulae simplify to:

$$U_1(p') + subsidy_1 = \mu,$$

$$U_2(p') + subsidy_2 = \mu,$$

or

$$155 + subsidy_1 = 165,$$

$$165 + subsidy_2 = 165.$$

Location 2 has a subsidy of zero, whereas those at location 1 receive a subsidy of 10. It is always the case that, with the above procedure, the location(s) with the highest utility at the system-optimized solution are not subsidized, since it is wasteful to do so. Of course, according to (4.9) the government is “free" to set μ^k as high as it is willing to and the budget allows.

In terms of practical implementation, we note that congestion pricing in the form of tolls has achieved success around the world with notable examples including the cities of Gothenburg and Stockholm in Sweden, London in the United Kingdom, as well as Singapore, with tolls even on the horizon in New York City (see [89], [91]).

We view the provision of the above subsidies as investments by government(s) that might help to alleviate various migrant and refugee crises around the globe. As for the budgets, if an individual government falls short, provision should be provided by a supra authority such as the World Bank, the United Nations, or if in Europe, the European Union.

4.2.1 Applying the Framework Post a Disaster

Disasters, as mentioned in the introduction of this chapter, may lead to migrations of humans. Of course, it is important to emphasize that there are both slow-onset disasters (certain wars, droughts, famine, pestilence, etc.), as well as sudden-onset disasters (earthquakes, hurricanes, tsunamis, floods, etc.). For an edited volume on dynamics of disasters, see [49].

In the case of a disaster, one would expect to encounter changes in the utility functions associated with locations that the disaster has impacted and such changes may affect certain classes more or less. In particular, one would expect that, in general, the location(s) would become less attractive because of compromised infrastructure, loss of resources and amenities, and even perhaps dangerous conditions. The above S-O model can then be adapted to incorporate the modified utility functions and appropriate subsidies provided.

Similarly, once a disaster strikes, there may be major loss of life and that would then affect the population in the network economy of one or more classes of migrants. The S-O model can then be rerun to identify the new optimal multiclass population distribution and assign the subsidies accordingly.

The above S-O model and policy intervention are relevant to a network economy even prior to a disaster since individuals may wish to seek better lives for themselves by changing their locations within a country or across countries and governments may wish to intervene to enhance the societal welfare in terms of the population distribution and associated total utility.

4.3 Computation of the S-O Multiclass Population Distribution Pattern

In order to determine the subsidies to make the S-O behavior, after imposition, also U-O, we first must compute the system-optimizing flow pattern.

For purposes of standardizing the mechanism, we put VI (4.4) into standard form (see [65]): determine $X^{**} \in \mathscr{K} \subset R^N$ such that:

$$\langle F(X^{**}), X - X^{**} \rangle \geq 0, \quad \forall X \in \mathscr{K}, \tag{4.11}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space. $F(X)$ is a given continuous function such that $F(X) : X \to \mathscr{K} \subset R^N$. $\mathscr{K}$ is a closed, bounded, and convex set.

We define the vector $X \equiv p$ and the vector $F(X)$ with elements: $F_{k,i}(p) \equiv \sum_{l=1}^{H} \sum_{j=1}^{n} -\frac{\partial \hat{U}_j^l(p)}{\partial p_i^k}$; $k = 1, \ldots, H$; $i = 1, \ldots, n$. The feasible set $\mathscr{K} \equiv K^1$ and $N = Jn$. Then, clearly, VI (4.4) can be put into the standard form (4.11) with $X^{**} = p'$. Similarly, VI (4.6) can also be put into standard form with X and $\mathscr{K}$ as above and with the components of its $F(X)$ given by $-U_i^k(p)$; $\forall k$, $\forall i$, and with $X^{**} = p^*$.

We emphasize that there exists a solution to both VI (4.4) and VI (4.6) since the underlying feasible set is compact and the corresponding function that enters the variational inequality, $F(X)$ is continuous under our imposed assumptions (cf. [45]).

We apply the Euler method to compute the solutions to the numerical examples in the next section. The Euler method is induced by the general iterative scheme presented in [27] by Dupuis and Nagurney. Specifically, iteration τ of the Euler method ([77]) is given by:

$$X^{\tau+1} = P_{\mathscr{K}}(X^{\tau} - a_{\tau}F(X^{\tau})), \tag{4.12}$$

where $P_{\mathscr{K}}$ is the projection on the feasible set $\mathscr{K}$ and F is the function that enters the VI problem (4.11).

As discussed in [77], the Euler method is, in fact, a discrete-time approximation to the continuous-time trajectories associated with a projected dynamical system, whose set of stationary points coincides with the set of solutions to the corresponding VI problem. In the multiclass human migration network context here, this means that there is an associated projected dynamical system to both VI (4.4) and to VI (4.6). Projected dynamical systems, as noted by Dupuis and Nagurney in [27], are nonclassical in that the right-hand side is discontinuous, but capture the underlying feasible set corresponding to the constraints of the given problem.

In addiction, in [27], Dupuis and Nagurney proved that, for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$. Specific conditions for convergence of the Euler method within many network-based models can be found in [77] and in [67] and the references therein.

We now provide the convergence result.

Theorem 4.3.1. ***Convergence.*** *In the S-O model of human migration constructed above let $F(X)$ be strictly monotone at any equilibrium pattern. Also, assume that F is uniformly Lipschitz continuous. Then there exists a unique S-O population distribution pattern $p' \in \mathscr{K}$ and any sequence generated by the Euler method as given by* (4.12)*, with $\{a_{\tau}\}$ satisfies $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$ converges to p', satisfying VI* (4.4).

Proof. The proof follows from Theorem 5.8 in [77]. □

The analogue of the above Theorem for VI (4.6) easily follows.

In view of the special network structure underlying our multiclass model (cf. Figure 4.1), it makes sense to use an algorithm for the solution of the encountered separable quadratic programming problems at each iteration of the above Euler method. Note that (4.12), because of the network structure of the feasible set $\mathcal{K}$, consists of H separable quadratic programming problems, one for each class k, and subject to the flow conservation constraints (4.1) and the nonnegativity constraints (4.2) for each class k. Specifically, for these network subproblems of special structure, we propose the use of the exact equilibration algorithm (cf. [19] and [65]). This algorithm yields the exact solution at each iteration and guarantees that the conservation of equations (4.1) and (4.2) are satisfied.

4.4 Numerical Examples

We now present several numerical examples, which are solved using the Euler method outlined in the preceding section. The algorithm was implemented in FORTRAN and the system used was a Unix system at the University of Massachusetts Amherst. The series $\{a_\tau\}$ in the algorithm was set to: $.1(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots$. the convergence tolerance ε was 10^{-5}, that is, the algorithm was deemed to have converged when the absolute value of each of the computed population values at two successive iterations was less than or equal to .00001.

There are two classes of migrants and five locations in all the numerical examples. We report the computed U-O and the S-O solutions, as well as the subsidies, for each of them.

Numerical Example 1

This example serves as the baseline for all the other examples in this section.
The fixed populations in the network economy of the two classes are, respectively:

$$P^1 = 1000.00 \quad P^2 = 2000.00.$$

The utility functions and the total utility functions for class 1 are:

$$U_1^1(p) = -2p_1^1 - .2p_1^2 + 2000, \quad \hat{U}_1^1(p) = -2(p_1^1)^2 - .2p_1^2p_1^1 + 2000p_1^1$$
$$U_1^1(p) = -2p_1^1 - .2p_1^2 + 2000, \quad \hat{U}_1^1(p) = -2(p_1^1)^2 - .2p_1^2p_1^1 + 2000p_1^1,$$
$$U_2^1(p) = -3p_2^1 - .1p_2^2 + 1500, \quad \hat{U}_2^1(p) = -3(p_2^1)^2 - .1p_2^2p_2^1 + 1,500p_2^1,$$
$$U_3^1(p) = -p_3^1 - .3p_3^2 + 3000, \quad \hat{U}_3^1(p) = -(p_3^1)^2 - .3p_3^2p_3^1 + 3000p_3^1,$$
$$U_4^1(p) = -p_4^1 - .2p_4^2 + 2500, \quad \hat{U}_4^1(p) = -(p_4^1)^2 - .2p_4^2p_4^1 + 2500p_4^1,$$
$$U_5^1(p) = -2p_5^1 - .3p_5^2 + 4000, \quad \hat{U}_5^1(p) = -2(p_5^1)^2 - .3p_5^2p_5^1 + 4000p_5^1.$$

The utility functions and the total utility functions for class 2 are:

$$U_1^2(p) = -p_1^2 - .4p_1^1 + 4000, \quad \hat{U}_1^2(p) = -(p_1^2)^2 - .4p_1^1p_1^2 + 4000p_1^2,$$
$$U_2^2(p) = -2p_2^2 - .6p_2^1 + 3000, \quad \hat{U}_2^2(p) = -2(p_2^2)^2 - .6p_2^1p_2^2 + 3000p_2^2,$$
$$U_3^2(p) = -p_3^2 - .2p_3^1 + 5000, \quad \hat{U}_3^2(p) = -(p_3^2)^2 - .2p_3^1p_3^2 + 5000p_3^2,$$
$$U_4^2(p) = -2p_4^2 - .3p_4^1 + 4000, \quad \hat{U}_4^2(p) = -2(p_4^2)^2 - .3p_4^1p_4^2 + 4000p_4^2,$$
$$U_5^2(p) = -p_5^2 - .4p_5^1 + 3000, \quad \hat{U}_5^2(p) = -(p_5^2)^2 - .4p_5^1p_5^2 + 3000p_5^2.$$

We first computed the U-O solution since it is interesting to compare it with the S-O solution. If the migrants are free to move between locations (and no subsidies are provided), the U-O solution satisfying VI (4.6) is:

Class 1 **U-O population distribution**

$$p_1^{1*} = 0.00, \quad p_2^{1*} = 0.00, \quad p_3^{1*} = 167.31, \quad p_4^{1*} = 41.68, \quad p_5^{1*} = 791.01.$$

Class 2 **U-O population distribution**

$$p_1^{2*} = 415.89, \quad p_2^{2*} = 0.00, \quad p_3^{2*} = 1382.41, \quad p_4^{2*} = 201.69, \quad p_5^{2*} = 0.00.$$

For class 1, the incurred utility at equilibrium at the populated locations 3, 4, and 5 is 2417.98 and it is lower at locations 1 and 2 - 1916.82 and 1500.00, respectively. For class 2, the incurred utility at equilibrium at the populated locations 1, 3, and 4 is 3584.11. At location 2, the utility of class 2 is 3000.00 and at location 5 it is 2683.60.

Neither class, under U-O behaviour, elects to migrate to and locate at location 2. Class 1 migrants only locate at locations 3 through 5. Only those of class 2 locate at location 1, in equilibrium, whereas only those of class 1 locate at location 5.

Now we present the S-O solution for this problem, which satisfies VI (4.4).

Class 1 **S-O population distribution**

$$p_1^{1'} = 0.00, \quad p_2^{1'} = 0.00, \quad p_3^{1'} = 120.43, \quad p_4^{1'} = 314.39, \quad p_5^{1'} = 565.19.$$

Class 2 **S-O population distribution**

$$p_1^{2'} = 606.48, \quad p_2^{2'} = 53.23, \quad p_3^{2'} = 1,076.35, \quad p_4^{2'} = 263.94, \quad p_5^{2'} = 0.00.$$

We see that in the S-O solution, location 2 is now populated by class 2. Locations 1 and 2 remain unpopulated under S-O for class 1 as does location 5 for class 2.

Using the procedure outlined in Section 4.2 (cf. (4.9) and (4.10)), we set $\mu^1 = 2869.63$ since the highest utility under the S-O flow pattern for class 1 is that at location 5 and it is equal to the above value. Also, we set $\mu^2 = 3899.56$ since the highest utility for class 2 under the S-O flow pattern is achieved at location 3 and it is equal to 3899.56.

We then, via subtraction of the particular location and class utility evaluated at the S-O population pattern, obtain the following subsidies:

Class 1 subsidies

$$subsidy_1^1 = 0.00, \; subsidy_2^1 = 0.00, \; subsidy_3^1 = 312.96, \; subsidy_4^1 = 736.80, \; subsidy_5^1 = 0.00,$$

Class 2 subsidies

$$subsidy_1^2 = 506.04, \; subsidy_2^2 = 1006.03, \; subsidy_3^2 = 0.00, \; subsidy_4^2 = 521.75, \; subsidy_5^2 = 0.00.$$

In order to verify the above theory we modified the original utility functions by adding the above subsidies and solved for the U-O pattern, and the answer, as expected, was identical to the above reported S-O pattern. Hence, the policy of subsidies accomplishes what it was designed for.

Numerical Example 2

Numerical Example 2 has the same data as Numerical Example 1, except that the demands are now switched so that:

$$P^1 = 2,000.00 \quad P^2 = 1,000.00.$$

The U-O solution is now:

Class 1 **U-O population distribution**

$$p_1^{1*} = 0.00, \quad p_2^{1*} = 0.00, \quad p_3^{1*} = 631.39, \quad p_4^{1*} = 412.41, \quad p_5^{1*} = 956.21.$$

Class 2 **U-O population distribution**

$$p_1^{2*} = 63.16, \quad p_2^{2*} = 0.00, \quad p_3^{2*} = 936.84, \quad p_4^{2*} = 0.00, \quad p_5^{2*} = 0.00.$$

For class 1, the incurred utility at equilibrium at the populated locations 3, 4, and 5 is 2087.59 and it is lower at locations 1 and 2 - 1987.37 and 1500.00, respectively. For class 2, the incurred utility at equilibrium at the populated locations 1 and 3 is 3936.89. At location 2, the utility of class 2 is 3000.00, at location 4 it is 3876.28, and at location 5 it is 2617.52.

The computed S-O solution, which is also utilized to determine the subsidies is:

Class 1 **S-O population distribution**

$$p_1^{1'} = 205.08, \quad p_2^{1'} = 75.92, \quad p_3^{1'} = 265.89, \quad p_4^{1'} = 714.23, \quad p_5^{1'} = 738.89.$$

Class 2 **S-O population distribution**

$$p_1^{2'} = 225.39, \quad p_2^{2'} = 0.00, \quad p_3^{2'} = 720.43, \quad p_4^{2'} = 54.18, \quad p_5^{2'} = 0.00.$$

Again, we see in this example that the S-O population pattern is quite different from the U-O one. In fact, at the U-O solution, class 1 only located at three locations: 3 – 5, whereas, at the S-O solution, there was a positive population of this class at the S-O solution at all locations. Hence, both quantitatively and qualitatively, we can expect the U-O and the S-O solutions to differ, demonstrating the need for appropriate migration policies.

We now report the subsidies, noting that in this example, we set $\mu^1 = 2522.23$ since the highest utility under the S-O flow pattern for class 1 is that at location 5 and it is equal to the above value. Also, we set $\mu^2 = 4226.39$ since the highest utility for class 2 under the S-O flow pattern is achieved, again, at location 3.

The subsidies are now:

Class 1 subsidies

$$subsidy_1^1 = 977.46,\ subsidy_2^1 = 1250.00,\ subsidy_3^1 = 4.25,\ subsidy_4^1 = 747.29,\ subsidy_5^1 = 0.00,$$

Class 2 subsidies

$$subsidy_1^2 = 533.81,\ subsidy_2^2 = 0.00,\ subsidy_3^2 = 0.00,\ subsidy_4^2 = 549.01,\ subsidy_5^2 = 1521.94.$$

Numerical Example 3

In this example, we consider a healthcare disaster hitting the network economy in the form of a pandemic. This example is inspired, in part, by the coronavirus outbreak emanating from Wuhan, China ([88]). The data in this example was as in Numerical Example 1, except that now we assumed that 50% of the population of each class has perished, so that:

$$P^1 = 500.00 \quad P^2 = 1,000.00.$$

Note that in this example the utility functions remain unchanged since the disaster does not affect infrastructure, *per se*, and involves "only" loss of life.

Next, we report the complete results for this example, as we have done for the others in this section. The U-O population distribution is now

Class 1 U-O population distribution

$$p_1^{1*} = 0.00, \quad p_2^{1*} = 0.00, \quad p_3^{1*} = 0.00, \quad p_4^{1*} = 0.00, \quad p_5^{1*} = 500.00.$$

Class 2 **U-O population distribution**

$$p_1^{2*} = 0.00, \quad p_2^{2*} = 0.00, \quad p_3^{2*} = 1,000.00, \quad p_4^{2*} = 0.00, \quad p_5^{2*} = 0.00.$$

This solution is quite interesting. Note that all migrants of class 1 choose to locate exclusively at location 5 whereas those of class 2 all migrate, post the disaster, to location 3. There is a complete separation of these two classes under U-O behaviour in the network economy.

Those of class 1 have a utility of 3000.00 at location 5, whereas those of class 1 have a utility of 4000.00 at location 3.

The computed S-O solution is

Class 1 **S-O population distribution**

$$p_1^{1'} = 0.00, \quad p_2^{1'} = 0.00, \quad p_3^{1'} = 47.98, \quad p_4^{1'} = 43.17, \quad p_5^{1'} = 408.85.$$

Class 2 **S-O population distribution**

$$p_1^{2'} = 206.96, \quad p_2^{2'} = 0.00, \quad p_3^{2'} = 694.96, \quad p_4^{2'} = 98.08, \quad p_5^{2'} = 0.00.$$

In the S-O solution, there is more “spreading out" of the classes among the locations than in the U-O solution.

For this example, we set $\mu^1 = 3182.31$ since the highest utility under the S-O flow pattern for class 1 is that at location 5 and it is equal to the above value. Also, we set $\mu^2 = 4295.44$ since the highest utility for class 2 under the S-O flow pattern is achieved, again, at location 3.

The subsidies (see, again, (4.9) and (4.10)) are:

Class 1 subsidies

$$subsidy_1^1 = 0.00,\ subsidy_2^1 = 0.00,\ subsidy_3^1 = 438.78,\ subsidy_4^1 = 745.10,\ subsidy_5^1 = 0.00,$$

Class 2 subsidies

$$subsidy_1^2 = 502.40,\ subsidy_2^2 = 0.00,\ subsidy_3^2 = 0.00,\ subsidy_4^2 = 504.56,\ subsidy_5^2 = 0.00.$$

Numerical Example 4

Numerical Example 4 had the same data as that in Numerical Example 3 except that we now consider the impact of a natural disaster following the healthcare disaster. We assume that locations 3 and 5 are impacted so that the utility functions of both classes of migrants associated with these locations are modified as follows. The fixed term in each of the noted utility functions in Examples 1 through 3 is reduced by 50% yielding new associated utility functions for the classes at those locations of

$$U_3^1(p) = -p_3^1 - .3p_3^2 + 1500, \quad U_5^1(p) = -2p_5^1 - .3p_5^2 + 2000,$$

$$U_3^2(p) = -p_3^2 - .2p_3^1 + 2500, \quad U_5^2(p) = -p_5^2 - .4p_5^1 + 1500.$$

The computed U-O solution is

Class 1 **U-O population distribution**

$$p_1^{1*} = 0.00, \quad p_2^{1*} = 0.00, \quad p_3^{1*} = 0.00, \quad p_4^{1*} = 480.97, \quad p_5^{1*} = 19.03.$$

Class 2 **U-O population distribution**

$$p_1^{2*} = 714.75, \quad p_2^{2*} = 0.00, \quad p_3^{2*} = 0.00, \quad p_4^{2*} = 285.25, \quad p_5^{2*} = 0.00.$$

With the negatively impacted by the disaster locations 3 and 5, the majority of class 1 migrates from location 5 to location 4; whereas those of class 2 completely leave location 3 (as do those of class 1). Those of class 2 leave location 3 for locations 1 and 4.

The computed S-O solution is:

Class 1 **S-O population distribution**

$$p_1^{1'} = 6.85, \quad p_2^{1'} = 0.00, \quad p_3^{1'} = 0.00, \quad p_4^{1'} = 389.05, \quad p_5^{1'} = 104.10.$$

Class 2 **S-O population distribution**

$$p_1^{2'} = 648.28, \quad p_2^{2'} = 75.17, \quad p_3^{2'} = 0.00, \quad p_4^{2'} = 276.54, \quad p_5^{2'} = 0.00.$$

The S-O population distribution is also affected by the disaster with neither class locating at location 3. No-one of class 2 remains at location 5, whereas those of class 1 have a higher population at location 5 under the S-O solution, than under the U-O one.

In Numerical Example 4, $\mu^1 = 2055.64$ and $\mu^2 = 3348.98$.

The subsidies are now:

Class 1 subsidies

$subsidy_1^1 = 199.00,\ subsidy_2^1 = 0.00,\ subsidy_3^1 = 0.00,\ subsidy_4^1 = 0.00,\ subsidy_5^1 = 263.83,$

Class 2 subsidies

$subsidy_1^2 = 0.00,\ subsidy_2^2 = 499.33,\ subsidy_3^2 = 848.98,\ subsidy_4^2 = 18.78,\ subsidy_5^2 = 1,890.62.$

4.5 Summary and Conclusions

Mathematical models of human migration networks have advanced over the past three decades to include, among others, multiple classes of migrants, costs associated with migration between locations, the incorporation of dynamics, etc. However, essentially all of the rigorous operations research based migration modelling work has focused on selfish, that is, user-optimizing, behaviour. Such a perspective is rich in theory and scope but such behaviour may lead to migratory flows and the resulting population distributions among regions and countries that are far from optimal from a societal perspective.

In this chapter, we introduce a new multiclass network model of human migration that assumes system-optimizing behaviour. The model fills a research gap in the literature. We then, using its classical user-optimizing analogue, demonstrate how governments can provide subsidies in order to make the system-optimizing population distribution pattern across multiple locations, also user-optimizing. Hence, once the subsidies are provided, migrants will independently locate themselves where it is also best from a societal perspective.

We provide, for completeness, the variational inequality formulations of both models and draw analogues to traffic network models and policies of tolls that alter travellers' behaviour to make drivers selects routes of transport that are system-optimizing. We also propose a societal welfare loss ratio, inspired by the price of anarchy.

An algorithm is proposed that is a time-discretization of the underlying dynamics until the optimal population distribution is achieved/computed. The algorithm is then applied to compute solutions to a series of multiclass numerical examples and the population distributions reported under user-optimization, under system-optimization, along with the subsidies for the different classes at the different locations.

CHAPTER 5

Capacitated Human Migration Networks and Subsidization

In this chapter, we develop user-optimized (U-O) and system-optimized (S-O) multiclass models of human migration under capacities associated with the migrant classes and locations. The work here presented takes up that of the previous chapter, but with the generalization of the inclusion of capacities. Such a generalization is especially timely, as noted above. Moreover, to-date, the majority of research on human migration networks, from an operations research and mathematical modelling perspective, has focused on the modelling of migration flows assuming user-optimizing behaviour, originating with the work of Nagurney (see [64]).

System-optimization in multiclass human migration networks is also important since governments may wish to maximize societal welfare and hope that migrants locate accordingly. However, the latter may be extremely challenging unless proper policies/incentives are put into place. Indeed, in [1], the authors have argued for an effective cost-efficient mechanism for the distribution of refugees in the European Union, for example. Clearly, that would require some form of central control and cooperation/coordination.

In this chapter, we provide a quantitative mechanism, in the form of subsidies, that, when applied, guarantees that the system-optimized solution of our multiclass capacitated human migration network problem is also user-optimized. This is very important, since it enables governments, and policy-making bodies, to achieve optimal societal welfare in terms of the location of the migrants in the network economy, while the migrants locate independently in a U-O manner! It is also provided an alternative variational inequality formulations of both the new U-O and S-O models, which include Lagrange multipliers associated with the location capacity constraints as explicit variables. Their values at the equilibrium/optimal solutions provide valuable economic information for decision-makers.

This chapter is organized as follows. In Section 5.1, the capacitated multiclass human migration network models, under S-O and under U-O behaviours is presented. Associated with each location as perceived by a class, is an individual utility function, that, when multiplied by the population of that class at that location, yields the total utility function for that location and class. As in the previous chapter, the utility associated with a location and class can, in general, depend upon the vector of populations of all the classes at all the locations in the network economy. We assume a fixed population of each class in the network economy and are interested in determining the distributions of the populations among the locations under S-O and U-O behaviours. For each model, we provide alternative variational inequality formulations. It is also highlighted the role that is played by the Lagrange multipliers associated with the class capacities on the locations in the network economy.

In Section 5.2, it is outlined the procedure for the calculation of the multiclass subsidies in order to guarantee, even in the capacitated case, that the system-optimized solution is, simultaneously, also user-optimized. Hence, once the subsidies are applied, the migrants will locate themselves individually in the network economy in a manner that is optimal from a societal perspective. As argued in the preceding chapter there are analogues of our subsidies to tolls in transportation science. In the case of congested transportation networks, the imposition of tolls (see [19], [17], [18], [54]), results in system-optimized flows also being user-optimized. In other words, once the tolls are imposed, travellers, acting independently, select routes of travel which result in a system optimum, that minimizes the total cost to the society. In this chapter, we construct policies for human migration networks that maximize societal welfare but in the case of capacities.

In Section 5.3, it is outlined the computational algorithm, which then is applied to compute solutions to numerical examples in Section 5.3.2, that illustrate the theoretical results in this chapter in a practical format. We summarize the results and present the conclusions in Section 5.4.

5.1 The Capacitated Multiclass Human Migration Network Models

In this section, we construct the capacitated multiclass network models of human migration. We first present the system-optimized model and then the user-optimized one. The notation follows that in the previous chapter, where, no capacities on the populations at the locations were imposed.

We assume that the human migrants have no movement costs associated with migrating from location to location since we are concerned with the long-term population distribution behaviours under both principles of system-optimization and user-optimization. The network representation of the models is given in Figure 5.1.

There are H classes of migrants, with a typical class denoted by k, and n locations corresponding to locations that the multiclass populations can migrate to, with a typical location denoted by i.

In the network representation, locations are associated with links. A link can correspond to a country or a region within a country and the network economy can capture multiple countries. If a government is interested in within country migration, exclusively, then the network economy (network) would correspond to that country.

Table 5.1 contains the notation for the models. All vectors here are assumed to be column vectors.

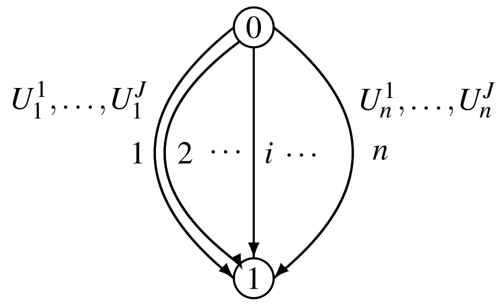

Figure 5.1: The Network Structure of the Multiclass Human Migration Models

Notation	Definition
p_i^k	the population of class k at location i. We group the $\{p_i^k\}$ elements into the vector $p^k \in R^n_+$. We then further group the p^k vectors; $k=1,\dots,H$, into the vector $p \in R^{Hn}_+$.
cap_i^k	the nonnegative capacity at location i for class k; $k=1,\dots,H; i=1,\dots,n$.
β_i^k	the Lagrange multiplier associated with capacity constraint for k at i; $k=1,\dots,J; i=1,\dots,n$. We group all these Lagrange multipliers into the vector $\beta \in R^{Hn}_+$.
P^k	the fixed population of class k in the network economy; $k=1,\dots,J$.
$U_i^k(p)$	the utility of individuals of class k at location i; $i=1,\dots,n$. We group the utility functions for each k into the vector $U^k \in R^n$ and then group all such vectors for all k into the vector $U \in R^{Hn}$.
$\hat{U}_i^k(p)$	the total utility of class k at location i; $i=1,\dots,n$. The total utility $\hat{U}_i^k(p) = U_i^k(p) \times p_i^k$; $k=1,\dots,J; i=1,\dots,n$.

Table 5.1: Notation for the Multiclass Human Migration Models

According to Table 5.1, there is a utility function U_i^k associated with each class k; $k=1,\dots,H$, and location i; $i=1,\dots,n$, which captures how attractive location i is for that class k. Observe that (see Table 5.1), the utility, and, hence, the total utility, $\hat{U}_i^k$, associated with location i and class k, may, in general, depend upon the population distribution of all the classes at all the locations. The Organization for Economic Co-operation and Development (OECD) ([30]), for example, recognizes that different locations may be more or less attractive to distinct classes of migrants.

We now present the constraints. In this model, in order to ensure that no changes in the population occur, it is assumed a conservation of population in the network economy. The population distribution of each class among the various locations must sum up to the population of that class in the network economy, that is, for each class k; $k = 1,\ldots,H$:

$$\sum_{i=1}^{n} p_i^k = P^k. \tag{5.1}$$

Furthermore, the population of each class at each location must be nonnegative, that is,

$$p_i^k \geq 0, \quad \forall i; \forall k, \tag{5.2}$$

and not exceed the capacity:

$$p_i^k \leq cap_i^k, \quad \forall i; \forall k. \tag{5.3}$$

The feasible set $K^1 \equiv \{p|\,(5.1), (5.2), (5.3)\,\text{hold.}\}$.

We assume here that

$$\sum_{i=1}^{n} cap_i^k \geq P^k, \tag{5.4}$$

for all classes k. In other words, we assume that the network economy has sufficient capacity to accommodate the population of each class. Hence, the feasible set K^1 is nonempty. Moreover, it is compact.

5.1.1 The Capacitated System-Optimization (S-O) Problem

The government (or governments), in the case of system optimization, wishes to maximize the total utility in the network economy, which reflects the societal welfare, subject to the constraints. The capacitated system-optimization (S-O) problem is:

$$\text{Maximize} \quad \sum_{k=1}^{H}\sum_{i=1}^{n} \hat{U}_i^k(p) = \sum_{k=1}^{H}\sum_{i=1}^{n} U_i^k(p) \times p_i^k, \tag{5.5}$$

subject to constraints (5.1) through (5.3).

We assume that the total utility functions for all the classes at all the locations are concave and continuously differentiable. Then, from classical results (cf. [45] and [65]), we know that the optimal solution, denoted by p', satisfies the variational inequality (VI) problem: determine $p' \in K^1$, such that

$$-\sum_{k=1}^{H}\sum_{i=1}^{n}\left[\sum_{l=1}^{H}\sum_{j=1}^{n}\frac{\partial \hat{U}_j^l(p')}{\partial p_i^k}\right]\times(p_i^k - p_i^{k'}) \geq 0, \quad \forall p \in K^1. \tag{5.6}$$

A solution p' to VI (5.6) is guaranteed to exist under our imposed assumptions since the feasible set K^1 is compact and the total utility functions are continuously differentiable. Uniqueness of the solution p' then follows under the assumption that all the utility functions are strictly concave.

We now present an alternative variational inequality to the one in (5.6), which we utilize to compute the S-O solution in numerical examples. Furthermore, the solution of the alternative VI allows us to determine the optimal Lagrange multipliers associated with the location class capacities in the S-O context. The Lagrange multipliers at the optimal solution provide valuable economic information. We define the feasible set $K^2 \equiv \{(p,\beta)|(5.1),(5.2)\,\text{hold and}\,\beta \in R_+^{Hn}\}$.

Alternative Variational Inequality Formulation of the Capacitated S-O Problem

A solution to the S-O problem also satisfies the VI: determine $(p',\beta') \in K^2$ such that

$$-\sum_{k=1}^{H}\sum_{i=1}^{n}\left[\sum_{l=1}^{H}\sum_{j=1}^{n}\frac{\partial \hat{U}_j^l(p')}{\partial p_i^k} - \beta_i^{k'}\right]\times(p_i^k - p_i^{k'}) + \sum_{k=1}^{H}\sum_{i=1}^{n}\left[cap_i^k - p_i^{k'}\right]\times(\beta_i^k - \beta_i^{k'}) \geq 0,$$
$$\forall (p,\beta) \in K^2 \tag{5.7}$$

The above result follows from the work of Bertsekas and Tsitsiklis, [5] (page 287). Capacities have also been applied to links in various supply chain system-optimization problems and variational inequality formulations constructed ([66] and [76]).

5.1.2 The Capacitated User-Optimization (U-O) Problem

We now introduce the capacitated user-optimized version of the above S-O model. The new model extends the classical one introduced in [64] to include capacities.

The Capacitated Equilibrium Conditions

Mathematically, a multiclass population vector $p^* \in K^1$ is said to be U-O or, equivalently, a capacitated equilibrium, if for each class k; $k = 1,\ldots,J$; and all locations $i; i = 1,\ldots,n$:

$$U_i^k(p^*) \begin{cases} \geq \lambda^k, \text{ if } & p_i^{k^*} = cap_i^k, \\ = \lambda^k, \text{ if } & 0 < p_i^{k^*} < cap_i^k, \\ \leq \lambda^k, \text{ if } & p_i^{k^*} = 0. \end{cases} \tag{5.8}$$

From (5.8) one can see that locations with no population of a class are those with the lowest utilities; those locations with a positive population of a class, with the population not at the capacity for the location and class will have equalized utility for that class and higher than the unpopulated locations of that class. Moreover, the equalized utility will be equal to an indicator λ^k. The indicator λ^k is, actually, the Lagrange multiplier associated with constraint (5.1) for k with the value at the equilibrium. Those locations with a class k at its capacity have a utility greater than or equal to λ^k.

A capacitated U-O solution p^* satisfies the VI: determine $p^* \in K^1$ such that

$$\sum_{k=1}^{H}\sum_{i=1}^{n} -U_i^k(p^*) \times (p_i^k - p_i^{k*}) \geq 0, \quad \forall p \in K^1. \tag{5.9}$$

We now prove the equivalence of the solution to the Capacitated Equilibrium Conditions (5.8) and the VI (5.9).

Indeed, it is easy to see that, according to (5.8), for a fixed k and i, the equilibrium conditions imply that

$$\left[\lambda^k - U_i^k(p^*)\right] \times \left[p_i^k - p_i^{k*}\right] \geq 0, \quad \forall p_i^k : 0 \leq p_i^k \leq cap_i^k. \tag{5.10}$$

Observe that, if $p_i^{k*} = 0$, (5.10) holds true; if $p_i^{k*} = cap_i^k$, then (5.10) also holds, and (5.10) also holds if $0 < p_i^{k*} < cap_i^k$.

Summing now (5.10) over all k and all i, yields:

$$\sum_{k=1}^{J}\sum_{i=1}^{n}\left[\lambda^k - U_i^k(p^*)\right] \times \left[p_i^k - p_i^{k*}\right] \geq 0, \quad \forall p \in K^1. \tag{5.11}$$

But, because of (5.1), (5.11) simplifies to precisely (5.9).

Furthermore, we now show that if p^* satisfies VI (5.9), then the p^* also satisfies the Capacitated Equilibrium Conditions (5.8).

In (5.9), we set $p_i^l = p_i^{l*}$ for all $l \neq k$, which yields:

$$\sum_{i=1}^{n} -U_i^k(p^*) \times (p_i^k - p_i^{k*}) \geq 0, \quad \forall p_i^k : 0 \leq p_i^k \leq cap_i^k; \quad \sum_{i=1}^{n} p_i^k = P^k. \tag{5.12}$$

If there are two locations, say, r and s with positive populations not at their capacities, set for a sufficiently small $\varepsilon > 0$:

$$p_r^k = p_s^{k*} - \varepsilon; \quad p_s^k = p_r^{k*} + \varepsilon$$

and all other p_i^ks equal to p_i^{k*}. Clearly, such a population distribution is also feasible. Substitution into (5.12) yields, after algebraic simplification:

$$(-U_r^k(p^*) + U_s^k(p^*)) \times (p_s^{k*} - p_r^{k*} - \varepsilon) \geq 0. \tag{5.13}$$

Similarly, by constructing another feasible population pattern:

$$p_r^k = p_s^{k*} + \varepsilon, \quad p_s^k = p_r^{k*} - \varepsilon,$$

with all other $p_i^k = p_i^{k*}$, and substitution into (5.12) yields

$$(U_r^k(p^*) - U_s^k(p^*)) \times (p_s^{k*} - p_r^{k*} - \varepsilon) \geq 0. \tag{5.14}$$

(5.13) and (5.14) can only hold true if

$$U_r^k(p^*) = U_s^k(p^*)$$

which we call λ^k. Hence, the second condition in (5.8) has been established.

On the other hand, suppose that $p_i^{k*} \geq 0$ for all i, but $p_r^{k*} > 0$ and $p_s^{k*} = 0$. For a sufficiently small $\varepsilon > 0$, construct $p_r^k = p_r^{k*} - \varepsilon$ and $p_s^k = p_s^{k*} + \varepsilon$, with all other p_i^ks equal to p_i^{k*} and substitute these values into (5.12). After, algebraic simplification, we obtain:

$$(U_r^k(p^*) - U_s^k(p^*))\varepsilon \geq 0,$$

hence,

$$U_r^k(p^*) \geq U_s^k(p^*)$$

and the third condition in (5.8) is verified.

Now, in order to verify that a solution to VI (5.9) also satisfies the top condition in (5.8), if for some location r: $p_r^{k*} = cap_r^k$, then we construct a feasible distribution pattern such that:

$$p_r^k = p_r^{k*} - \varepsilon, \quad p_s^k = p_s^{k*} + \varepsilon,$$

with $\varepsilon > 0$ sufficiently small and all other $p_i^k = p_i^{k*}$. Substitution into (5.12), after algebraic simplification yields:

$$U_r^k(p^*) \geq U_s^k(p^*)$$

and the conclusion follows. With the above arguments, we have shown that a capacitated equilibrium p^* is equivalent to the solution of the VI (5.9).

We now provide an alternative VI formulation of the capacitated equilibrium conditions. This result is immediate by making note of [64], demonstrating that the U-O human migration model (without capacities) is isomorphic to a traffic network equilibrium problem (cf. [19] and [16]) and, hence, in the case of capacities, also isomorphic to a traffic network equilibrium problem with side constraints (see [53]) and with special structure.

Alternative Variational Inequality Formulation of the U-O Problem
The U-O solution satisfies the variational inequality problem: determine $(p^*, \beta^*) \in K^2$ such that

$$\sum_{k=1}^{H}\sum_{i=1}^{n}\left[-U_i^k(p^*)+\beta_i^{k*}\right]\times(p_i^k - p_i^{k*}) + \sum_{k=1}^{H}\sum_{i=1}^{n}\left[cap_i^k - p_i^{k*}\right]\times(\beta_i^k - \beta_i^{k*}) \geq 0,$$
$$(p,\beta)\in K^2. \quad (5.15)$$

5.1.3 Illustrative Examples

We first present an uncapacitated example for which we provide U-O and S-O solutions. We then add capacities to the locations and report the new U-O and S-O solutions. There is a single class in the network economy and three locations. The total population is: $P^1 = 120$ and the utility functions at the three locations are:

$$U_1^1(p) = -p_1^1 + 190, \quad U_2^1(p) = -p_2^1 + 200, \quad U_3^1(p) = -p_3^1 + 210.$$

The user-optimized solution is:

$$p_1^{1*} = 30.00, \quad p_2^{1*} = 40.00, \quad p_3^{1*} = 50.00,$$

yielding $\lambda^1 = 160$, since

$$U_1^1(p^*) = U_2^1(p^*) = U_3^1(p^*) = 160.00.$$

The S-O solution, on the other hand, is:

$$p_1^{1'} = 35.00, \quad p_2^{1'} = 40.00, \quad p_3^{1'} = 45.00.$$

We now impose capacities as follows:

$$cap_1^1 = 60.00, \quad cap_2^1 = 60.00, \quad cap_3^1 = 30.00,$$

and solve for the U-O and S-O solutions.

The new U-O solution, satisfying VI (5.15), is:

$$p_1^{1*} = 40.00, \quad p_2^{1*} = 50.00, \quad p_3^{1*} = 30.00,$$

with Lagrange multipliers associated with the capacities of:

$$\beta_1^{1*} = 0.00, \quad \beta_2^{1*} = 0.00, \quad \beta_3^{1*} = 30.00.$$

The new S-O solution, satisfying VI (5.7), is:

$$p_1^{1'} = 42.50, \quad p_2^{1'} = 47.50, \quad p_3^{1'} = 30.00,$$

with Lagrange multipliers associated with the capacities of:

$$\beta_1^{1'} = 0.00, \quad \beta_2^{1'} = 0.00, \quad \beta_3^{1'} = 45.00.$$

Observe that the S-O solution is distinct from the U-O solution in both the uncapacitated and the capacitated versions.

Remark

We now show how the optimal Lagrange multipliers can be utilized. For example, if one modifies the utility functions by reducing each of them by the value of the optimal Lagrange multiplier associated with the location and the class then the same user-optimizing solution is obtained as the one for the problem with the corresponding capacities. Indeed, proceeding as above, we modify the utility functions as:

$$\tilde{U}_1^1(p) = -p_1^1 + 190 - 0 = -p_1^1 + 190,$$

$$\tilde{U}_2^1(p) = -p_2^1 + 200 - 0 = -p_2^1 + 200,$$

$$\tilde{U}_3^1(p) = -p_3^1 + 210 - 30 = -p_3^1 + 180,$$

and observe that the capacitated U-O solution: $p_1^{1*} = 40.00$, $p_2^{1*} = 50.00$, $p_3^{1*} = 30.00$ remains optimal.

Similarly, one can modify the utility functions in the same manner, but by using the optimal Lagrange multipliers for the S-O problem, to obtain the same

S-O solution as for the problem with the capacities.

Hence, government decision-makers, in order to limit the population of certain (or all) classes at certain (or all) locations can accomplish this through regulations corresponding to the capacities or by modifying the utility functions accordingly to yield the same result.

Now, we describe how subsidies (which may be viewed as a positive intervention) can, once imposed, make the capacitated S-O solution also a capacitated U-O one.

5.2 Subsidies to Guarantee the Capacitated S-O Solution is Also a Capacitated U-O Solution

In the previous chapter a procedure is introduced for the calculation of subsidies that, once applied to the locations with a positive population of a class under S-O, guaranteed that migrants operating under the U-O behavioural principle would select locations that were also optimal from a societal standpoint; that is, they were system-optimized.

Here we show that the same general construct is also applicable to capacitated problems of human migration.

The procedure is as follows. We first solve for the capacitated system-optimized solution p' satisfying VI (5.7), or, equivalently, VI (5.6). For each class k, we denote those locations with a positive population by $k_1, \ldots, k_{n_k}$, where n_k is the number of locations in the network economy with a positive population of class k. We also introduce notation for subsidies associated with the different locations for each class denoted by class k by: $(subsidy)_{k_1}$, $(subsidy)_{k_2}$, ..., $(subsidy)_{k_{n_k}}$. We then enumerate those locations in a list as follows:

$$U^k_{k_1}(p') + subsidy^k_{k_1} = \mu^k,$$
$$U^k_{k_2}(p') + subsidy^k_{k_2} = \mu^k,$$

and so on until

$$U^k_{k_{n_k}}(p') + subsidy^k_{k_{n_k}} = \mu^k. \tag{5.16}$$

Note that μ^k is the incurred utility for class k after the subsidies are distributed for the class at the locations with positive populations of that class. Also, we can number those locations for that class with zero populations of that class (if there are any) as follows:

$$U^k_{k_{n_{k+1}}}(p') + subsidy^k_{k_{n_k+1}} \leq \mu^k,$$

and so on until

$$U^k_{k_n}(p') + subsidy^k_{k_n} \leq \mu^k. \tag{5.17}$$

Expressions (5.16) and (5.17) reveal that the appropriate governmental authority chooses the μ^k for each class k, and then the subsidy for each location for that class is determined by straightforward subtraction.

In order to select an appropriate μ^k, as already noted in the previous chapter for the uncapacitated case, the μ^ks can be set as: $\max_{k_l;l=1,\ldots,n_k} U^k_{k_l}(p')$. All thus calculated are nonnegative and, furthermore, all migrants enjoy the maximal utility for each class at all the populated locations. Also, for the subsidies associated with locations with no populations of a class k (see (5.17)), we set those subsidies zero.

Returning to the above simple example, we note that $\mu^1 = 180.00$, and the above subsidy formulae simplify to:

$$U^1_1(p') + subsidy^1_1 = \mu^1,$$

$$U^1_2(p') + subsidy^1_2 = \mu^1,$$

$$U^1_3(p') + subsidy^1_3 = \mu^1,$$

or

$$147.50 + subsidy^1_1 = 180.00,$$

$$152.50 + subsidy^1_2 = 180.00,$$

$$180.00 + subsidy^1_3 = 180.00,$$

which yields:

$$subsidy_1^1 = 32.50, \quad subsidy_2^1 = 27.50, \quad subsidy_3^1 = 0.00.$$

Observe that an application of the above subsidies modifies the utility functions as follows:

$$\tilde{U}_1^1(p) = -p_1^1 + 190 + 32.50, \quad \tilde{U}_2^1(p) = -p_2^1 + 200 + 27.50, \quad \tilde{U}_3^1(p) = -p_3^1 + 210 + 0.$$

Clearly, the S-O solution

$$p_1^{1'} = 42.50, \quad p_2^{1'} = 47.50, \quad p_3^{1'} = 30.00,$$

is at the same time U-O, since the utilities are equalized (and maximal) under this S-O pattern and, hence, migrants will select locations, although acting selfishly and individually, accordingly, because of the subsidies.

The above subsidies are investments by government(s) that might help to alleviate various migrant and refugee crises around the globe. As for the budgets, if an individual government experiences a budgetary shortfall, additional financing may be provided by a supra authority such as the World Bank, the United Nations, or if in Europe, the European Union. In [1], the authors have argued for closer cooperation among countries regarding migration crises and also advocated for an economic approach as to distribution of the migrants. Here, we provide a quantitative approach with explicit formulae for implementation.

As noted earlier, climate change as well as disasters may act as drivers of human migrations. In [83], Robinson, Dilkina, and Moreno-Cruz, for example, provide a machine learning approach to migration in the United States under sea level rise but emphasize that their approach is not yet ready for policy making. They, as Bier, Zhou, and Du in [6], consider sea level rise due to climate change, and migration within a country - the United States. The latter authors observe that offering a subsidy (e.g., a partial buyout) can be effective if the government has a significantly lower discount rate than residents. However, they assume homogeneous residents, whereas we consider multiclass ones and we also allow for multiple countries and not just regions within a country. For edited volumes on dynamics of disasters, see [49] and [48]. Once a disaster or disasters strike, one would modify the fixed populations of the various classes in the economy, as need be, along with the utility functions and rerun the model(s), along with the subsidies. In the case of disasters, we can expect that populations will decrease and so would utility functions associated with locations that have been negatively impacted.

5.3 The Algorithm and Numerical Examples

We apply the Euler method of Dupuis and Nagurney ([27]) for the solution of the capacitated network models of human migration. As discussed therein (see also [77]), the Euler method is induced by a general iterative scheme, and was inspired by the theory of projected dynamical systems, whose set of stationary points coincides with the set of solutions to an appropriate variational inequality problem. The Euler method, in fact, can be viewed as a time-discretization of the underlying continuous time trajectories of the projected dynamical system until a solution is achieved. It has been applied to numerous network problems, including supply chain ones (see [67]).

5.3.1 The Algorithm

For the purposes of standardizing the mechanism, we utilize similar notation to that in previous chapter and put variational inequality (5.7) into standard form (see [65]): determine $X^{**} \in \mathscr{K} \subset R^N$ such that:

$$\langle F(X^{**}), X - X^{**} \rangle \geq 0, \quad \forall X \in \mathscr{K}, \tag{5.18}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space. $F(X)$ is a given continuous function such that $F(X) : X \to \mathscr{K} \subset R^N$. $\mathscr{K}$ is a closed and convex set.

We define the vector $X \equiv (p, \beta)$ and the vector $F(X)$ with elements: $F^1_{k,i}(p, \beta) \equiv \sum_{l=1}^{H} \sum_{j=1}^{n} -\frac{\partial \hat{U}^l_j(p)}{\partial p^k_i}$ and $F^2_{k,i}(p, \beta) \equiv cap^k_i - p^k_i$; $k = 1, \ldots, H$; $i = 1, \ldots, n$. We define the feasible set $\mathscr{K} \equiv K^2$ and $N = 2Hn$. Thus, VI (5.7) can be put into the standard form (5.18) with $X^{**} = (p', \beta')$. Similarly, VI (5.15) can also be put into standard form with X and $\mathscr{K}$ as above and with the components of its $F(X)$ given by $-U_i^k(p, \beta)$, $cap_i^k - \beta_i^k$; $\forall k$, $\forall i$, and with $X^{**} = (p^*, \beta^*)$.

At iteration τ, the statement of the Euler method is:

$$X^{\tau+1} = P_{\mathscr{K}}(X^\tau - a_\tau F(X^\tau)), \tag{5.19}$$

where $P_{\mathscr{K}}$ is the projection on the feasible set $\mathscr{K}$ and F is the function that enters the variational inequality problem (5.18).

Dupuis and Nagurney in [27], proved that, for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$. Specific conditions for convergence of the Euler method within many network-based models can be found in [77] and in [67] and the references therein.

The Euler method nicely exploits the special network structure of the models as depicted in Figure 5.1 and allows for closed form expressions at each iteration for the computation of the Lagrange multipliers associated with the capacity constraints. We solve the network subproblems of special structure, which are separable quadratic programming problems, using the exact equilibration algorithm (cf. [19] and [65]). This algorithm yields the exact solution at each iteration for the populations.

5.3.2 Numerical Examples

The algorithm was implemented in FORTRAN and a Unix system at the University of Massachusetts Amherst used for the computations. The series $\{a_\tau\}$ in the algorithm was set to: $1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \dots$ with the convergence tolerance ε equal to 10^{-5}. In other words, the algorithm was considered to have converged when the absolute value of each of the computed population values for each class at two successive iterations was less than or equal to .00001.

For continuity, and cross comparison purposes, we recall the data for the uncapacitated examples taken from the previous chapter and to these we added capacities. For completeness, we report both the uncapacitated and the capacitated versions, reported for the first time here.

In the following numerical examples, the network economy consists of two classes of migrants and five locations.

Utility Function and Fixed Population Data

We recall the data introduced in the Numerical Examples section 4.4, in the previous chapter.

The fixed populations in the network economy of the two classes are, respectively:

$$P^1 = 1{,}000.00 \quad P^2 = 2{,}000.00.$$

The utility functions and the total utility functions for class 1 are:

$$U_1^1(p) = -2p_1^1 - .2p_1^2 + 2{,}000, \quad \hat{U}_1^1(p) = -2(p_1^1)^2 - .2p_1^2 p_1^1 + 2{,}000p_1^1,$$

$$U_2^1(p) = -3p_2^1 - .1p_2^2 + 1{,}500, \quad \hat{U}_2^1(p) = -3(p_2^1)^2 - .1p_2^2 p_2^1 + 1{,}500p_2^1,$$

$$U_3^1(p) = -p_3^1 - .3p_3^2 + 3{,}000, \quad \hat{U}_3^1(p) = -(p_3^1)^2 - .3p_3^2 p_3^1 + 3{,}000p_3^1,$$

$$U_4^1(p) = -p_4^1 - .2p_4^2 + 2{,}500, \quad \hat{U}_4^1(p) = -(p_4^1)^2 - .2p_4^2 p_4^1 + 2{,}500p_4^1,$$

$$U_5^1(p) = -2p_5^1 - .3p_5^2 + 4{,}000, \quad \hat{U}_5^1(p) = -2(p_5^1)^2 - .3p_5^2 p_5^1 + 4{,}000p_5^1.$$

The utility functions and the total utility functions for class 2 are:

$$U_1^2(p) = -p_1^2 - .4p_1^1 + 4{,}000, \quad \hat{U}_1^2(p) = -(p_1^2)^2 - .4p_1^1 p_1^2 + 4{,}000p_1^2,$$

$$U_2^2(p) = -2p_2^2 - .6p_2^1 + 3{,}000, \quad \hat{U}_2^2(p) = -2(p_2^2)^2 - .6p_2^1 p_2^2 + 3{,}000p_2^2,$$

$$U_3^2(p) = -p_3^2 - .2p_3^1 + 5{,}000, \quad \hat{U}_3^2(p) = -(p_3^2)^2 - .2p_3^1 p_3^2 + 5{,}000p_3^2,$$

$$U_4^2(p) = -2p_4^2 - .3p_4^1 + 4{,}000, \quad \hat{U}_4^2(p) = -2(p_4^2)^2 - .3p_4^1 p_4^2 + 4{,}000p_4^2,$$

$$U_5^2(p) = -p_5^2 - .4p_5^1 + 3{,}000, \quad \hat{U}_5^2(p) = -(p_5^2)^2 - .4p_5^1 p_5^2 + 3{,}000p_5^2.$$

We first recall the uncapacitated U-O and S-O solutions obtained in the previous chapter, as said before, and then report the capacitated solutions based on the new models constructed here. We also report the calculated subsidies in the more general capacitated case introduced in this chapter. We provide two numerical examples.

Numerical Example 1

The uncapacitated U-O solution for the numerical example with the above data is:
Class 1 **Uncapacitated U-O Population Distribution**

$$p_1^{1*} = 0.00, \quad p_2^{1*} = 0.00, \quad p_3^{1*} = 167.31, \quad p_4^{1*} = 41.68, \quad p_5^{1*} = 791.01.$$

Class 2 **Uncapacitated U-O Population Distribution**

$$p_1^{2*} = 415.89, \quad p_2^{2*} = 0.00, \quad p_3^{2*} = 1,382.41, \quad p_4^{2*} = 201.69, \quad p_5^{2*} = 0.00.$$

The uncapacitated S-O solution is:
Class 1 **Uncapacitated S-O Population Distribution**

$$p_1^{1'} = 0.00, \quad p_2^{1'} = 0.00, \quad p_3^{1'} = 120.43, \quad p_4^{1'} = 314.39, \quad p_5^{1'} = 565.19.$$

Class 2 **Uncapacitated S-O Population Distribution**

$$p_1^{2'} = 606.48, \quad p_2^{2'} = 53.23, \quad p_3^{2'} = 1,076.35, \quad p_4^{2'} = 263.94, \quad p_5^{2'} = 0.00.$$

We now impose the following capacities on the locations for the classes in the above problem.

$$cap_1^1 = 500.00, \quad cap_2^1 = 500.00, \quad cap_3^1 = 500.00, \quad cap_4^1 = 500.00, \quad cap_5^1 = 200.00,$$

$$cap_1^1 = 500.00, \quad cap_2^1 = 500.00, \quad cap_3^1 = 400.00, \quad cap_4^1 = 500.00, \quad cap_5^1 = 500.00.$$

The capacitated U-O solution is:
Class 1 **Capacitated U-O Population Distribution**

$$p_1^{1*} = 0.00, \quad p_2^{1*} = 0.00, \quad p_3^{1*} = 500.00, \quad p_4^{1*} = 300.00, \quad p_5^{1*} = 200.00.$$

Class 2 **Capacitated U-O Population Distribution**

$$p_1^{2*} = 500.00, \quad p_2^{2*} = 226.67, \quad p_3^{2*} = 400.00, \quad p_4^{2*} = 500.00, \quad p_5^{2*} = 373.33.$$

The optimal Lagrange multipliers are:

Class 1 **Capacitated U-O Lagrange Multipliers**

$$\beta_1^{1*} = 0.00, \quad \beta_2^{1*} = 0.00, \quad \beta_3^{1*} = 280.00, \quad \beta_4^{1*} = 0.00, \quad \beta_5^{1*} = 1,388.01.$$

Class 2 **Capacitated U-O Lagrange Multipliers**

$$\beta_1^{2*} = 953.33, \quad \beta_2^{2*} = 0.00, \quad \beta_3^{2*} = 1,953.33, \quad \beta_4^{2*} = 363.33, \quad \beta_5^{2*} = 0.00.$$

One can see, from this example, that at all the locations with populations of a class at the capacity, there is an associated positive Lagrange multiplier. Also, it is clear that the capacitated U-O solution is quite distinct from the uncapacitated one. For example, all the locations have a positive population of class 2 under the capacitated solution. Moreover, in the uncapacitated case, location 5 is most attractive for class 1, whereas location 3 is most attractive for class 2. In contrast, in the capacitated case, location 3 is now most popular for class 1, whereas locations 1 and 4 are most popular (and at the capacities) for class 2.

The capacitated S-O solution is:

Class 1 **Capacitated S-O Population Distribution**

$$p_1^{1'} = 88.82, \quad p_2^{1'} = 0.00, \quad p_3^{1'} = 242.55, \quad p_4^{1'} = 468.63, \quad p_5^{1'} = 200.00.$$

Class 2 **Capacitated S-O Population Distribution**

$$p_1^{2'} = 500.00, \quad p_2^{2'} = 244.65, \quad p_3^{2'} = 400.00, \quad p_4^{2'} = 436.07, \quad p_5^{2'} = 419.29.$$

The optimal Lagrange multipliers are:

Class 1 **Capacitated S-O Lagrange Multipliers**

$$\beta_1^{1'} = 0.00, \quad \beta_2^{1'} = 0.00, \quad \beta_3^{1'} = 0.00, \quad \beta_4^{1'} = 0.00, \quad \beta_5^{1'} = 1,561.77.$$

Class 2 Capacitated S-O Lagrange Multipliers

$$\beta_1^{2'} = 925.30, \quad \beta_2^{2'} = 0.00, \quad \beta_3^{2'} = 2,057.33, \quad \beta_4^{2'} = 0.00, \quad \beta_5^{2'} = 0.00.$$

Under the uncapacitated S-O, location 5 is most attractive for class 1 and location 3 is for class 2. However, in the capacitated case, location 4 is best for class 1 and location 1 for class 2, with locations 3 through 5 also quite competitive.

We now report the calculated subsidies, which are obtained using the described procedure in Section 5.2. We note that $\mu^1 = 3,474.21$ and $\mu^2 = 4,551.50$ - these values represent the highest utility of each class at a location evaluated at the S-O solution, which are obtained for class 1 at location 5 and for class 2 at location 3. The calculated subsidies are:

Class 1 Subsidies

$subsidy_1^1 = 1751.85$, $subsidy_2^1 = 1998.67$, $subsidy_3^1 = 836.76$, $subsidy_4^1 = 1530.05$,

$subsidy_5^1 = 0.00$.

Class 2 Subsidies

$subsidy_1^2 = 1087.03$, $subsidy_2^2 = 2040.79$, $subsidy_3^2 = 0.00$, $subsidy_4^2 = 1564.22$,

$subsidy_5^2 = 2,050.79$.

Numerical Example 2

Numerical example 2 takes up the scenario proposed in numerical example 3 of Section 4.4 in which a disaster does not affect infrastructure, but involves loss of life, and now studied in the case of capacitated network. As argued in the previous section, this scenario could occur in the form of a “pandemic", that is, a healthcare disaster hitting the network economy. We note that the novel coronavirus outbreak that originated in Wuhan, China ([88]), was officially declared a pandemic by the World Health Organization on March 11, 2020 (cf. [8]). This coronavirus causes the disease known as Covid-19. In this numerical example data were as in the first one except now we consider a sizeable decrease in the populations of each of the two classes due to a disaster. More specifically, the utility functions remain unchanged and we assume that 50% of the population of each class has perished, that is,

$$P^1 = 500.00 \quad P^2 = 1,000.00.$$

The uncapacitated U-O solution for the numerical example with the above data is:

Class 1 **Uncapacitated U-O Population Distribution**

$$p_1^{1*} = 0.00, \quad p_2^{1*} = 0.00, \quad p_3^{1*} = 0.00, \quad p_4^{1*} = 0.00, \quad p_5^{1*} = 500.00.$$

Class 2 **Uncapacitated U-O Population Distribution**

$$p_1^{2*} = 0.00, \quad p_2^{2*} = 0.00, \quad p_3^{2*} = 1,000.00, \quad p_4^{2*} = 0.00, \quad p_5^{2*} = 0.00.$$

The uncapacitated computed S-O solution is:

Class 1 **S-O Uncapacitated Population Distribution**

$$p_1^{1'} = 0.00, \quad p_2^{1'} = 0.00, \quad p_3^{1'} = 47.98, \quad p_4^{1'} = 43.17, \quad p_5^{1'} = 408.85.$$

Class 2 **S-O Uncapacitated Population Distribution**

$$p_1^{2'} = 206.96, \quad p_2^{2'} = 0.00, \quad p_3^{2'} = 694.96, \quad p_4^{2'} = 98.08, \quad p_5^{2'} = 0.00.$$

As noted in the previous chapter, in the S-O solution one sees a greater “spreading out" of the classes among the locations than in the U-O solution.

We kept the same capacities as in the first numerical example. The Euler Method now yielded the following solution:

The capacitated U-O solution for the numerical example with the above data is:

Class 1 **Capacitated U-O Population Distribution**

$$p_1^{1*} = 0.00, \quad p_2^{1*} = 0.00, \quad p_3^{1*} = 300.00, \quad p_4^{1*} = 0.00, \quad p_5^{1*} = 200.00.$$

Class 2 **Capacitated U-O Population Distribution**

$$p_1^{2*} = 0.00, \quad p_2^{2*} = 400.00, \quad p_3^{2*} = 0.00, \quad p_4^{2*} = 400.00, \quad p_5^{2*} = 200.00.$$

The optimal Lagrange multipliers are:

Class 1 **Capacitated U-O Lagrange Multipliers**

$$\beta_1^{1*} = 0.00, \quad \beta_2^{1*} = 0.00, \quad \beta_3^{1*} = 0.00, \quad \beta_4^{1*} = 0.00, \quad \beta_5^{1*} = 1,020.00.$$

Class 2 **Capacitated U-O Lagrange Multipliers**

$$\beta_1^{2*} = 0.00, \quad \beta_2^{2*} = 0.00, \quad \beta_3^{2*} = 940.00, \quad \beta_4^{2*} = 0.00, \quad \beta_5^{2*} = 0.00.$$

The capacitated computed S-O solution is:

Class 1 **S-O Capacitated Population Distribution**

$$p_1^{1'} = 0.00, \quad p_2^{1'} = 0.00, \quad p_3^{1'} = 124.08, \quad p_4^{1'} = 175.91, \quad p_5^{1'} = 200.00.$$

Class 2 **S-O Capacitated Population Distribution**

$$p_1^{2'} = 414.66, \quad p_2^{2'} = 0.00, \quad p_3^{2'} = 400.00, \quad p_4^{2'} = 185.34, \quad p_5^{2'} = 0.00.$$

The optimal Lagrange multipliers are:

Class 1 **Capacitated S-O Lagrange Multipliers**

$$\beta_1^{1'} = 0.00, \quad \beta_2^{1'} = 0.00, \quad \beta_3^{1'} = 0.00, \quad \beta_4^{1'} = 0.00, \quad \beta_5^{1'} = 1,144.49.$$

Class 2 **Capacitated S-O Lagrange Multipliers**

$$\beta_1^{2'} = 0.00, \quad \beta_2^{2'} = 0.00, \quad \beta_3^{2'} = 967.27, \quad \beta_4^{2'} = 0.00, \quad \beta_5^{2'} = 0.00.$$

We now report the subsidies that, when imposed, guarantee that the capacitated S-O solution obtained above for the second numerical example is also U-O. Here we had that $\mu^1 = 3,599.99$ and $\mu^2 = 4,575.18$.

Class 1 **Subsidies**

$subsidy_1^1 = 1682.92$, $subsidy_2^1 = 2099.99$, $subsidy_3^1 = 844.07$, $subsidy_4^1 = 1312.97$,

$subsidy_5^1 = 0.00$.

Class 2 **Subsidies**

$subsidy_1^2 = 989.84$, $subsidy_2^2 = 1575.18$, $subsidy_3^2 = 0.00$, $subsidy_4^2 = 998.63$,

$subsidy_5^2 = 1655.18$.

5.4 Summary and Conclusions

Problems of human migration are issues of global concern and are presenting immense challenges to governments around the world. Many are dealing with different classes of migratory flows and the ensuing difficulties when faced with capacities at locations under their jurisdictions. Rigorous, appropriate policies may help to better reallocate migrants across suitable locations.

Historically, many of the mathematical models of human migration have utilized a network formalism and have assumed user-optimizing behavior, that is, that migrants select locations, which are best for themselves, as revealed through utility functions that depend on the population distributions among the locations of the different classes of migrants. However, such behavior may lead to costs to society and even reduced societal welfare.

Hence, in this chapter, we build upon the work in the previous chapter, in which we proposed both system-optimized and user-optimized multiclass migration network models, and in which we demonstrated how incentives, in the form of subsidies, when applied, guarantee that the system-optimized solution, which maximizes the total utility in the network economy, becomes, at the same time, user-optimizing. Migrants, thus, under such subsidies, and acting selfishly and independently, would select locations to migrate to and locate at that are optimal from the system perspective.

In this chapter, we propose a novel extension of that work, in the form of capacities at different locations associated with the classes of migrants. This brings a greater realism in capturing challenges faced by various governments who are dealing with refugees, asylum seekers, etc. For each U-O and S-O model we provide alternative variational inequality formulations of the governing equilibrium/optimality conditions. We then utilize the variational inequality formulations with Lagrange multipliers associated with the multiclass capacity constraints to gain deeper insights into appropriate policies. We show that the Lagrange multipliers can be utilized to modify the utility functions so that the capacities are made implicit. Moreover, we show how, through the use of appropriately constructed formulae for subsidies, once applied, the system-optimized solution becomes, at the same time, user-optimized. This provides a more positive approach to the redistribution of human migrants and enhances societal welfare.

In addition, in this chapter, we provide an effective computational procedure, which exploits the underlying special network structure of our models. The algorithm is implemented, and the solutions to a series of numerical examples computed. We report the user-optimized and the system-optimized solutions, both uncapacitated and capacitated, along with the subsidies for the latter. Our theoretical framework can be applied in practice under different scenarios, along with sensitivity analysis, as, for example, in the case of disasters, when there are population changes and/or modifications to utility functions because of impacted infrastructure.

CHAPTER 6

A System-Optimization Model for Multiclass Human Migration with Migration Costs and Regulations inspired by the Covid-19 Pandemic

In this chapter, the system-optimized models of human migration noted in the overview chapter are extended to include novel utility functions, migration costs, and more general regulations. Specifically, in the objective function it is taken into account the changes in the utility functions of the multiple classes caused by the migratory flows and policies adopted by governments. Further, in determining the optimal flows, the government policies are considered a priori, thanks to a suitable coefficient influence vector w. Finally, the capacities and the regulations of the flows are included in a single formulation. The aim of this work is to find a system-optimized solution, which is a social optimum, in that an organization, such as the United Nations, maximizes the attractiveness of the origin countries, which for an individual origin is given by the sum of its utility and its expected increment of utility value, with respect to the destination one, for each migration class and each pair of countries (or locations). An equivalent formulation of the variational inequality by means of Lagrange theory are provided. Several numerical examples are presented and analysed.

6.1 Presentation of the Model

We consider, as introduced in the previous chapters models, a network consisting of n nodes, that are countries or, more generally, locations, and H classes of the population, depicted in Figure 6.1.

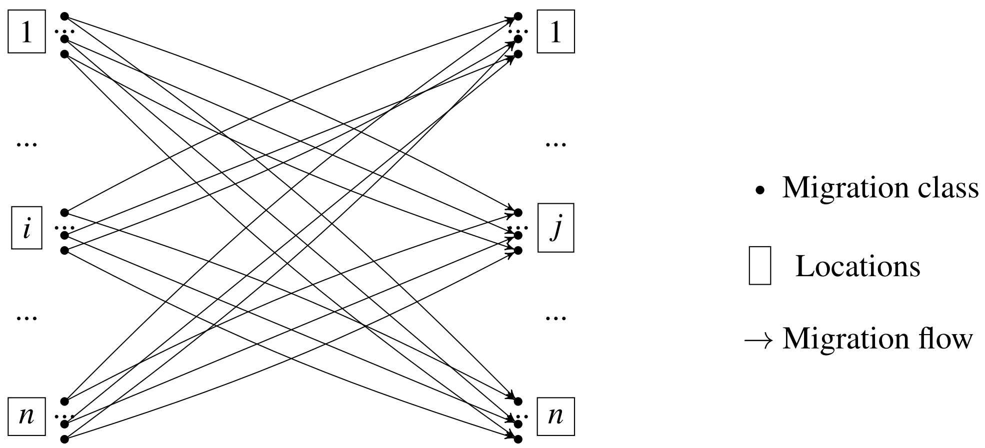

Figure 6.1: Network structure of the multiclass human migration model

In Table 6.1 we summarize the notation adopted for the model, briefly recalling some of the symbols already introduced in Chapter 3.

Notation	Definition
p_i^k	the population of class k at location i. We group the populations for all the migration classes k, in each location i, into the vector population $p = (p_i^k)_{\substack{i=1,\dots,n \\ k=1,\dots,H}}$.
$\bar{p}_i^k$	the fixed population of class k in the network economy; $k = 1,\dots,J$.
f_{ij}^k	migration flow out of the node i, and into the node j of the network, with $i \neq j$. We group the migration flows of each migration class k, from each origin node i to each destination node j into the vector flow $f = (f_{ij}^k)_{\substack{i,j=1,\dots,n, i\neq j \\ k=1,\dots,H}}$.
$v_j^k(p)$	Destination utility function of location j as perceived by an individual of class k
$u_i^k(p)$	Origin utility function of location i as perceived by an individual of class k
U_{ij}^k	Total net utility function for class k with respect to the route from i to j
$c_{ij}^k(f)$	Unit migration cost from i to j for an individual of class k
$C_{ij}^k(f)$	Total migration cost from i to j for class k
$w_{ij}^{k\pm} \in [-1,1]$	Policy influence coefficients

Table 6.1: Functions, parameters, and decision variables of the model.

It is clear that if a volume of population of a typical class decides to migrate, then the destination node differs from the origin one; otherwise, it remains in the same origin node. Hence, the volume of population of each class k at each node i, after the migration takes place, is given by the following flow conservation constraints:

$$p_i^k = \bar{p}_i^k - \sum_{\substack{j=1 \\ j\neq i}}^{n} f_{ij}^k + \sum_{\substack{j=1 \\ j\neq i}}^{n} f_{ji}^k, \quad i = 1,...,n; \quad k = 1,\dots,H. \tag{6.1}$$

We consider that the sum of the flows out of each node in the network, must not exceed the initial population in that node; in other words:

$$\sum_{\substack{j=1 \\ j\neq i}}^{n} f_{ij}^k \leq \bar{p}_i^k, \qquad i = 1,\dots,n; \; k = 1,\dots,H. \tag{6.2}$$

We assume that both the vector population and vector flow components are positive, namely

$$\begin{aligned} p &= (p_i^k) \; i = 1,\dots,n; \; k = 1,\dots,H \in \mathbb{R}_+^{nH}, \\ f &= (f_{ij}^k) \; i,j = 1,\dots,n, i \neq j; \; k = 1,\dots,H \in \mathbb{R}_+^{n(n-1)H} \end{aligned} \tag{6.3}$$

Furthermore, as in the Chapter 5, we introduce the population capacity constraint for each node in the network. Here, unlike in the aforementioned chapter, we consider that the total population at each node i must not exceed the capacity, as follows:

$$\sum_{k=1}^{H} p_i^k \leq cap_i, \quad i = 1,\dots,n, \tag{6.4}$$

where cap_i, for all nodes i, is the population capacity. The constraints (6.4) ensure that all the nodes are not overpopulated.

We assume that the sum of the capacities is greater than the total population of the network of all the classes of migrants.

We now introduce the origin and destination utility functions; specifically, u_i^k and v_j^k, which capture the attractiveness from an economic and/or political and/or social point of view of the origin node i and of the destination node j, respectively, as perceived by a single individual of the migration class k. In other words, such functions reflect the liveability of each node of the network as perceived by an individual of the migration class k; hence, these functions are expressed in a different way depending on whether the node is analysed as an origin or as a destination one. We assume that the u_i^k and v_j^k are functions of the entire population vector: $u_i^k = u_i^k(p)$ and $v_j^k = v_j^k(p)$, $i,\ j = 1,\dots,n$; $k = 1,\dots,H$. Such functions are assumed to be continuously differentiable and concave.

To interpret the concavity condition on the utility functions in terms of applications, we assume that, without loss of generality, there is a population threshold at which utility functions stop growing. The excess of population leads to a decrease in the economic growth of the node (see [44]) due to an increase in pollution, competition for jobs, housing, etc. (see [52]). For each individual of class k, we introduce the net utility function, which is given by the difference between the individual origin and destination utility functions. Hence, the total net utility functions for the class k, for all $k = 1,\dots,H$, with respect to the route from i to j are defined as:

$$U_{ij}^k(p,f) = (u_i^k(p) - v_j^k(p)) \times f_{ij}^k, \quad i,j = 1,\dots,n; k = 1,\dots,H, \tag{6.5}$$

and are assumed to be continuously differentiable and concave.

Let $c_{ij}^k(f)$ and $C_{ij}^k(f)$ denote the unit migration cost function and the total migration cost function between locations i and j, respectively. We have:

$$C_{ij}^k(f) = c_{ij}^k(f) \times f_{ij}^k, \quad i,j = 1,\dots,n;\ \ k = 1,\dots,H. \tag{6.6}$$

Such costs are assumed to be convex and continuously differentiable. This assumption is justified by the concept of *diminishing marginal utility* without requiring utility functions, according to which, roughly speaking, averages are better than the extremes.

In our model we assume that there is a government or an organization of governments, such as the United Nations, whose interest is to guarantee the respect of the right to choose migration and at the same time a high level of welfare for each individual living in each location node, in order to improve the quality of life.

Each node in the network differs in terms of migration policies and ideologies, how it is experienced by a class, and how ready it is to integrate immigrants (see [35]).

It is reasonable to suppose that the migration policies that are adopted in the various nodes of the network by governments influence the migration routes. Such policies can be, for example, inclusive or not, and depend more generally on choices based on the economic, social, and/or political features of the network nodes.

Hence, we introduce for each possible migration route from node i to node j, the influence coefficients w_{ij}^{k-} and w_{ij}^{k+}, which allow us to take into account the migration policies implemented in nodes i and j as a consequence of the utility changes when individuals of the class k choose to migrate from node i towards node j, respectively. We assume that such influence coefficients range in the interval $[-1,1]$. When the influence coefficient value is close to the upper bound 1, it is indicative of a more inclusive policy. In other words, these coefficients in the objective function will be related to the changes of the utility caused by the migration flows.

Therefore, considering any origin node i and any other node j in the network, the possible variations of the utility functions u_i^k and v_j^k and the subsequent policies undertaken by the governments in the aforementioned nodes will be considered in determining the optimal flows, respectively, through the following terms:

$$\delta_i^-(p,f) = \sum_{k=1}^{H} \left(\sum_{\substack{j=1 \\ j\neq i}}^{n} w_{ij}^{k-} f_{ij}^k \right) \times \frac{\partial u_i^k(p)}{\partial p_i^k}, \quad i=1,\dots,n, \tag{6.7}$$

and

$$\delta_j^+(p,f) = \sum_{k=1}^{H} \left(\sum_{\substack{i=1 \\ i\neq j}}^{n} w_{ij}^{k+} f_{ij}^k \right) \times \frac{\partial v_j^k(p)}{\partial p_j^k}, \quad j=1,\dots,n. \tag{6.8}$$

Remark 6.1.1. *Governments hope to minimize or maximize the terms* (6.7) *and* (6.8) *since they represent, respectively, a deficit or a surplus to the starting utility functions depending on both the sign of the derivatives of the utility functions (which we assume to be concave) and the sign of the influence coefficients, that are the variation in attractiveness in terms of welfare, quality of life, and so on of* *a node with respect to the population and the adopted migration policies.*

Let $\mathbb{K}$ denote the feasible set such that:

$$\mathbb{K} = \left\{(p,f) \in \mathbb{R}^{n^2H} | (6.1), (6.2), (6.3), (6.4) \text{ hold}\right\}. \tag{6.9}$$

6.1.1 Regulations

As mentioned in the introduction, a global emergency situation, such as the COVID-19 pandemic, highlights the importance of assessing and analyzing the management of human migration, in the event that flow regulations are applied. For this reason, in our model we introduce, as in [68], the flow regulations in terms of constraints.

Suppose that the typical destination node j applies a restriction R_j on the flows of some classes k coming from some nodes i of the network, which we will group together in the set C^j of pairs (i,k) to which restrictions are imposed by j, as follows:

$$\sum_{(i,k)\in C^j} f_{ij}^k \leq R_j. \tag{6.10}$$

As noted in [68], the (6.10) restrictions, for each node j represent the most general case of flow regulations which, depending on the adopted policies, may be more specific, such as:

- restrictions for a single class $\bar{k}$ and coming from a single node in the network $\bar{i}$:

$$f_{\bar{i}j}^{\bar{k}} \leq R_j, \tag{6.11}$$

- restrictions for a single class $\bar{k}$

$$\sum_{(i,\bar{k})\in C^j} f_{ij}^{\bar{k}} \leq R_j, \quad \forall j, \tag{6.12}$$

- restrictions for every class coming from origin node $\bar{i}$

$$\sum_{(\bar{i},k)\in C^j} f_{\bar{i}j}^{k} \leq R_j, \quad \forall j. \tag{6.13}$$

We denote by $\mathbb{K}^1$ the feasible with the above regulations as follows:

$$\mathbb{K}^1 \quad = \quad \left\{(p,f) \in \mathbb{R}^{n^2H} |\ (6.10) \text{ holds}\right\}. \tag{6.14}$$

6.1.2 The Multiclass Human Migration Network System-Optimization Problem and its Variational Formulation

The multiclass human migration network system-optimization problem can be expressed as follows. The cognizant organization seeks to determine the optimal flows, as well as the optimal populations at each node in the network, subject to the convenience to remain, given by the difference between the total net utility function and the migration costs and also trying to take into account the choices of policies by the governments/organization and the potential variations of the utility function as closely as possible. As a consequence, we are dealing with a system-optimized model. Therefore, the optimization problem is constructed as follows:

$$\text{Maximize} \sum_{k=1}^{H} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \left[U_{ij}^k(p,f) - C_{ij}^k(f) + \delta_i^-(p,f) + \delta_j^+(p,f) \right] \tag{6.15}$$

subject to: constraints (6.1), (6.2), (6.3), (6.4), and (6.10). We introduce the feasible set for the optimization problem under regulations

$$\mathbb{K}^2 \quad = \quad \left\{(p,f) \in \mathbb{R}^{n^2H} | (6.1), (6.2), (6.3), (6.4) \text{and } (6.10) \text{ hold}\right\}. \tag{6.16}$$

Under the above assumptions, the objective function in (6.15) is concave and continuously differentiable and so, using the classical variational theory (see [45] and [65]), it is easy to prove that an optimal solution for the optimization problem, denoted by $(p^*, f^*) \in \mathbb{K}^2$, satisfies the following variational inequality: find

$(p^*, f^*) \in \mathbb{K}^2$, such that

$$-\sum_{q=1}^{H}\sum_{l=1}^{n}\left(\sum_{k=1}^{H}\sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n}\frac{\partial U_{ij}^{k}(p^*,f^*)}{\partial p_l^q}+\frac{\partial \delta_i^{k+}(p^*,f^*)}{\partial p_l^q}+\frac{\partial \delta_i^{k-}(p^*,f^*)}{\partial p_l^q}\right)\times(p_l^q-p_l^{q*})$$
$$-\sum_{q=1}^{H}\sum_{l=1}^{n}\sum_{\substack{s=1\\s\neq l}}^{n}\left(\sum_{k=1}^{H}\sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n}\frac{\partial U_{ij}^{k}(p^*,f^*)}{\partial f_{ls}^q}-\frac{\partial C_{ij}^{k}(f^*)}{\partial f_{ls}^q}+\frac{\partial \delta_i^{k+}(p^*,f^*)}{\partial f_{ls}^q}\right.$$
$$\left.+\frac{\partial \delta_i^{k-}(p^*,f^*)}{\partial f_{ls}^q}\right)\times(f_{ls}^q-f_{ls}^{q*})\geq 0, \qquad \forall (p,f)\in\mathbb{K}^2. \tag{6.17}$$

Applying the well-known results about variational inequalities in finite dimension (see [45] and [10], [14] and [68]), we can find an equivalent formulation of the variational inequality using the Lagrange multipliers associated with the constraints defining the feasible set $\mathbb{K}^2$ and proving the strong duality.

Indeed, variational inequality (6.17) can be rewritten as a minimization problem, since, if we denote by $V(p,f)$ the left-hand side of (6.17), then we have:

$$V(p,f)\geq 0 \text{ in } \mathbb{K}^2 \text{ and } \min_{\mathbb{K}^2} V(p,f)=V(p^*,f^*)=0.$$

Now, denoting by $\lambda^1 \in \mathbb{R}_+^{nH}$, $\lambda^2 \in \mathbb{R}_+^{n(n-1)H}$, $\varepsilon \in \mathbb{R}^{nH}$, $\nu \in \mathbb{R}_+^{nH}$, and $\mu,\gamma \in \mathbb{R}_+^n$, the Lagrange multiplier vectors associated with the nonnegativity constraints (6.3), and constraints (6.1), (6.2), (6.4), and (6.10), respectively, we can consider the following Lagrange function:

$$\mathscr{L}(p,f,\lambda^1,\lambda^2,\varepsilon,\nu,\mu,\gamma)=V(p,f)+\sum_{i=1}^{n}\sum_{k=1}^{H}\lambda_i^{k1}(-p_i^k)$$
$$+\sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{k=1}^{H}\lambda_{ij}^{k2}(-f_{ij}^k)+\sum_{i=1}^{n}\sum_{k=1}^{H}\varepsilon_{ik}\left(p_i^k-\bar{p}_i^k+\sum_{\substack{j=1\\j\neq i}}^{n}f_{ij}^k-\sum_{\substack{j=1\\j\neq i}}^{n}f_{ji}^k\right) \tag{6.18}$$
$$+\sum_{i=1}^{n}\sum_{k=1}^{H}\nu_{ik}\left(\sum_{\substack{j=1\\j\neq i}}^{n}f_{ij}^k-\bar{p}_i^k\right)+\sum_{i=1}^{n}\mu_i\left(\sum_{k=1}^{H}p_i^k-cap_i\right)+\sum_{j=1}^{n}\gamma_j\left(\sum_{(i,j)\in C^j}f_{ij}^k-R_j\right).$$

Making use of the Lagrange theory, if (p^*,f^*) is a solution to variational inequality (6.17), we are able to prove that the following KKT conditions (6.19)-(6.20) hold and vice versa. Moreover, we show that strong duality (6.23) holds.

Theorem 6.1.1. *The Lagrange multipliers in (6.18) do exist and, for all i, $j = 1,\dots,n$, and $k = 1,\dots,H$, the following conditions hold true:*

$$\overline{\lambda}_i^{k1}(-p_i^{k*}) = 0, \quad \overline{\lambda}_{ij}^{k2}(-f_{ij}^{k*}) = 0, \quad \overline{\nu}_{ik}\left(\sum_{\substack{j=1 \\ j\neq i}}^{n} f_{ij}^{k*} - \bar{p}_i^k\right) = 0, \tag{6.19}$$

$$\overline{\mu}_i\left(\sum_{k=1}^{H} p_i^{k*} - cap_i\right) = 0, \quad \overline{\gamma}_j\left(\sum_{(i,j)\in C^j} f_{ij}^{k^*} - R_j\right) = 0, \tag{6.20}$$

$$\frac{\partial U_{ij}^k(p^*,f^*)}{\partial p_i^k} + \frac{\partial \delta_i^{k+}(p^*,f^*)}{\partial p_i^k} + \frac{\partial \delta_i^{k-}(p^*,f^*)}{\partial p_i^k} - \overline{\lambda}_i^{k1} + \overline{\varepsilon}_{ik} + \overline{\mu}_i = 0, \tag{6.21}$$

$$\frac{\partial U_{ij}^k(p^*,f^*)}{\partial f_{ij}^k} - \frac{\partial C_{ij}^k(f^*)}{\partial f_{ij}^k} + \frac{\partial \delta_i^{k+}(p^*,f^*)}{\partial f_{ij}^k} + \frac{\partial \delta_i^{k-}(p^*,f^*)}{\partial f_{ij}^k} - \overline{\lambda}_{ij}^{k2} + \overline{\varepsilon}_{ik} + \overline{\gamma}_j = 0, \tag{6.22}$$

where $\overline{\lambda}^1$, $\overline{\lambda}^2$, $\overline{\varepsilon}$, $\overline{\nu}$, $\overline{\mu}$, $\overline{\gamma}$ are the optimal Lagrange multiplier vectors. Moreover, the strong duality also holds true; namely:

$$\begin{aligned} V(p^*,f^*) &= \min_{\mathbb{K}^2} V(p,f) \\ &= \max_{\substack{\lambda^1\in\mathbb{R}_+^{nH},\lambda^2\in\mathbb{R}_+^{2nH} \\ \varepsilon\in\mathbb{R}^{nH},\nu\in\mathbb{R}^{nH},\mu,\gamma\in\mathbb{R}_+^n}} \ \min_{(p,f)\in\mathbb{R}^{nH+2nH}} \mathscr{L}(p,f,\lambda^1,\lambda^2,\nu,\mu,\gamma). \end{aligned} \tag{6.23}$$

Proof. See Theorem 3.1 in [10].

The existence of at least one solution to variational inequality (3.5) is guaranteed from the classical theory of variational analysis (see Th.3.1. in [45]), since the feasible set is compact and the function that enters the variational inequality is continuous (see [56] for additional existence results).

6.2 Numerical Examples

In this section we present two illustrative examples for the optimization problem (6.15), without and with regulations, respectively. Specifically, for the regulation model three different situations are analysed.

For each example we consider two migration classes and two locations in the network, as depicted in Figure 6.2.

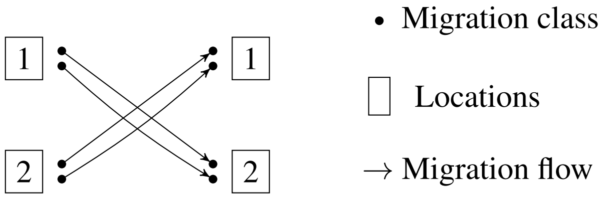

Figure 6.2: Network structure of 2-class human migration model.

Let us consider the data as in Table 6.2, that are the common data for the two examples.

We also introduce the population node capacities $\text{cap}_1 = 150$, $\text{cap}_2 = 180$ and, consequently, the capacity constraints as follows:

$$p_1^1 + p_1^2 \leq 150 \tag{6.24}$$

$$p_2^1 + p_2^2 \leq 180. \tag{6.25}$$

In the case without regulations, the feasible set $\mathbb{K}$ (6.16), is given by:

$$\begin{aligned} \mathbb{K} \;=\; & \big\{(p,f) \in \mathbb{R}^8_+ : p_1^1 = 40 - f_{12}^1 + f_{21}^1;\, p_2^1 = 30 - f_{21}^1 + f_{12}^1, \\ & p_1^2 = 25 - f_{21}^2 + f_{12}^2, p_2^2 = 15 - f_{21}^2 + f_{12}^2\; f_{12}^1 \leq 40,\; f_{21}^1 \leq 30, \\ & f_{12}^2 \leq 25,\; f_{21}^2 \leq 15;\; p_1^1 + p_1^2 \leq 150,\; p_2^1 + p_2^2 \leq 180\big\}. \end{aligned} \tag{6.26}$$

The solution to variational inequality (6.17) is:

$$(p_1^{1*}, p_2^{1*}, p_1^{2*}, p_2^{2*}, f_{12}^{1\;*}, f_{21}^{1\;*}, f_{12}^{2\;*}, f_{21}^{2\;*}) = (45.92, 24.07, 33.27, 6.72, 0, 5.92, 0, 8.27).$$

As we can see from the solution, for both migrant classes, there is no flow from node 2 to node 1.

Origin utility functions	$u_1^1 = -0.54p_1^1 - 0.11p_2^1 - 0.17p_1^2$
	$u_2^1 = 0.49p_2^1 - 0.39p_2^2$
	$u_1^2 = 0.26p_2^2 - 1.77p_1^2 - 0.47p_1^1$
	$u_2^2 = p_2^1 + 0.08p_1^2 + 0.64p_2^2$
Destination utility functions	$v_1^1 = 0.02p_1^1$
	$v_2^1 = 0.02p_2^2 - 0.51p_2^1$
	$v_1^2 = p_2^1$
	$v_2^2 = 0.15p_2^1 + p_2^2$
Migration costs	$c_{12}^1 = 5.54f_{12}^1$
	$c_{21}^1 = 5.08f_{21}^1$
	$c_{12}^2 = 1.72f_{12}^2$
	$c_{21}^2 = 5.00f_{21}^2$
Initial populations	$\bar{p}_1^1 = 40$
	$\bar{p}_2^1 = 30$
	$\bar{p}_1^2 = 25$
	$\bar{p}_2^2 = 15$
Influence coefficients	$w_{12}^{1-} = 0.3$
	$w_{21}^{1-} = -0.002$
	$w_{12}^{1+} = 0.26$
	$w_{21}^{1+} = 0.9$
	$w_{12}^{2-} = -0.9$
	$w_{21}^{2-} = 0.15$
	$w_{12}^{2+} = 0.4$
	$w_{21}^{2+} = 0.2$

Table 6.2: Data for the two numerical examples

We now introduce the regulation constraints (6.10), for every class coming from the origin node 2, as follows:

$$f_{21}^1 + f_{21}^2 \leq R_1, \tag{6.27}$$

where R_1 is the typical restriction applied by the destination node 1. We give three different values to R1 and, then, make a comparison with the solution obtained in the case without regulations. In the case with regulations, the feasible set $\mathbb{K}^1$ (6.14) is:

$$\begin{aligned}\mathbb{K} \;=\; &\big\{(p,f) \in \mathbb{R}_+^8 : p_1^1 = 40 - f_{12}^1 + f_{21}^1;\, p_2^1 = 30 - f_{21}^1 + f_{12}^1,\\ &p_1^2 = 25 - f_{21}^2 + f_{12}^2, p_2^2 = 15 - f_{21}^2 + f_{12}^2\; f_{12}^1 \leq 40,\; f_{21}^1 \leq 30,\, f_{12}^2 \leq 25,\\ &f_{21}^2 \leq 15;\; p_1^1 + p_1^2 \leq 150,\, p_2^1 + p_2^2 \leq 180;\; f_{21}^1 + f_{21}^2 \leq R_1\big\}.\end{aligned} \tag{6.28}$$

We consider three cases:

- **case 1**, $R_1 = 5$: The solution to variational inequality (6.17) is:

$$({p_1^1}^*, {p_2^1}^*, {p_1^2}^*, {p_2^2}^*, {f_{12}^1}^*, {f_{21}^1}^*, {f_{12}^2}^*, {f_{21}^2}^*)$$

$$= (41.21, 28.78, 28.78, 11.21, 0, 1.21, 0, 3.78).$$

- **Case 2**, $R_1 = 10$: The solution to variational inequality (6.17) is:

$$({p_1^1}^*, {p_2^1}^*, {p_1^2}^*, {p_2^2}^*, {f_{12}^1}^*, {f_{21}^1}^*, {f_{12}^2}^*, {f_{21}^2}^*)$$

$$= (43.77, 26.22, 31.22, 8.77, 0, 3.77, 0, 6.22).$$

- **Case 3**, $R_1 = 37$: The solution to variational inequality (6.17) is:

$$({p_1^1}^*, {p_2^1}^*, {p_1^2}^*, {p_2^2}^*, {f_{12}^1}^*, {f_{21}^1}^*, {f_{12}^2}^*, {f_{21}^2}^*)$$

$$= (45.91, 24.08, 33.27, 6.72, 0, 5.91, 0, 8.27).$$

As we can see from the solution, also in these three cases, for both classes of population, there is no flow from node 2 to node 1. We obtain a reduction of the flows from node 2 to 1 for every migration class in each cases in which restrictions are introduced. Note that, when R_1 increases, which means the movement possibility in the network increases, then also the optimal flows for both classes from node 2 to node 1 increase till the values without regulations.

The variational inequalities of the optimization problems both without and with regulations were solved using the Projection-Contraction method (see [92]). The algorithm was coded using Matlab and was run on a PC with 8 GB RAM, Asus Intel (R) Core (TM) i5-10210U CPU@1.60 GHz.

6.3 Conclusions and Further Research

The Covid-19 pandemic, a global healthcare disaster, has dramatically influenced the movement of humans over space and time in 2020. It has, also, impacted international migration as governments institute regulations banning travel. The world has seen immense migratory flows over the past decades with migrants seeking more amenable locations for themselves and their families. The topic of human migration has assumed further attention during the pandemic.

Migration can have positive as well as negative effects on the lives of the migrants. Positive aspects include: potentially the reduction of unemployment, a better quality of life, learning about a new culture, customs, and languages, and/or economic growth of the region. On the other hand, negative effects can include: increasing competition for jobs; possibly, growth in poverty, criminality, and exploitation, as well as pollution.

In this chapter, we introduced a network-based model for multiclass human migration with the objective of improving the system, that is, the society. Unlike previous system-optimization models for human migration, the new model includes migration costs as well as novel utility functions. We, nevertheless, retain regulations introduced earlier by the authors on the migratory flows. The model is studied qualitatively and numerical examples also provided.

Conclusion

Humans throughout history have sought to migrate to locations where they could enjoy a greater quality of life with enhanced safety, security, prosperity, and access to resources. With climate change, the increasing number of natural disasters, and their major impact, as well as wars, violence, and persecution in many countries around the globe, the world is witnessing some of the largest human migrations. The aim of this thesis is to analyse human migration and applications to real situation by using models based on networks.
Different perspectives were considered to investigate the phenomenon of migration as well as different optimization tools used to solve the proposed problems.

In Chapter 3, the migration is analysed as a noncooperative game. Each migration classes compete in a noncooperative manner so that each maximizes its utility function, given the actions of the other classes. We introduce the definition of population and migration flow pattern as Nash equilibrium and we provide the equivalent formulation of the Nash equilibrium as a solution to a variational inequality problem.

Migration is a continuous process that has been the subject of political debate worldwide: its increasing role in the social, economic and demographic development is becoming more and more evident. Governments, are feeling increasing pressure and stress to respond to the challenges of migratory flows through appropriate policies and regulations. In Chapter 4, indeed, we introduce a new multiclass network model of human migration in which we demonstrate how governments can provide subsidies in order to make the system-optimizing population distribution pattern across multiple locations, also user optimizing. In Chapter 5, we extend the model presented in Chapter 4, introducing capacities at different locations associated with the classes of migrants. Also in this case we provide alternative variational inequality formulations of the governing equilibrium/optimality conditions for each U-O and S-O model. Moreover, we show how, through the use of appropriately constructed formulae for subsidies, once applied, the system-optimized solution becomes, at the same time, user optimized. This provides a more positive approach to the redistribution of human migrants and enhances societal welfare.

A current phenomenon that is conditioning the migration movement in the world is the Covid-19 pandemic. It has, also, impacted international migration as governments institute regulations banning travel. In Chapter 6, indeed, we introduced a network-based model for multiclass human migration with the objective of improving the system, that is, the society. The proposed model includes migration costs as well as novel utility functions and regulations on the migratory flows.

The formulation is conducted using variational inequality theory. Due to the high complexity of this problem, in order to efficiently solve realistic instances a heuristic method is proposed. The presented algorithms are tested and compared over a number of randomly generated instances.

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