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A variational approach for supply chain networks with environmental interests

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ABSTRACT

Nowadays, the supply chain networks, consisting of different tiers of decision-makers, provide an effective framework for the production, the distribution, and the consumption of goods. In this paper we propose a supply chain network optimization model where manufacturers, retailers and consumers in the demand markets have a degree of interest in environmental sustainability. The manufacturers can improve their energy level (assumed as variables), aim to minimize their environmental emissions (for production and transport) and can also establish the amount of quantity of the production waste to dispose in a eco-sustainable way. The retailers, who are also profit-maximizers, aim to minimize their environmental emissions (which depend on the chosen shipping methods). The consumers at demand markets make their own choices according to the prices and to their degree of aversion to the environmental emissions. We describe the behavior of each decision-maker and we present the mathematical model for each of them, deriving the variational inequality problems. Furthermore, we derive a unique variational inequality formulation for the entire network for whose solution an existence and uniqueness result is obtained. Finally, we illustrate some numerical simulations that highlight how the use of UAVs and

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the presence of waste sorting centers in the supply chain reduce environmental emissions and related costs.

1. Introduction

A Supply Chain represents the process that allows a product or service to be brought to market, transferring it from the supplier to the customer. It is therefore a complex process that involves several professional figures, activating numerous processes of the business ecosystem: from the flow of raw materials linked to the production processes, up to the distribution logistics which ensures that the purchased goods reach the customer. The distinctive features of the Supply Chain concept can be summarized as follows:

- it is a cooperative system which is developed and managed within a unitary strategic design;
- it is aimed at the satisfaction of the final customers;
- it is to be pursued through the integration of one's corporate processes and the development of adequate relationships of interdependence;
- it is governed by suitable coordination mechanisms.

Until the end of the 1980s, the concept of "environmental quality" was not widespr-ead in the Mediterranean area of Europe. Sensitivity towards the quality of the environment and of life was very low and consumption was essentially conditioned by three selection criteria: appearance, price and quality of function. Starting from the 90s, also in the Mediterranean area the evolution of the concept of quality begins towards aspects related to naturalness, the impact on health and the environment, the reduction of resources, the social consequences of the production, distribution and consumption. This evolution has been accelerated by two phenomena: the issuing of European and national regulations that protect the safety of people and the environment (see [20]) and the diffusion of international brands and the start of globalization that accelerate the development of consumerism even in countries where until then the citizen defense movements met with modest support.

With the advent of globalization, the theme of corporate social responsibility (CSR) explodes, which consists in the voluntary integration of social and environmental issues in commercial operations and in relations with all interested parties (the so-called stakeholders). The company must respect an economic responsibility, which corresponds to the duty to create value and profit but also has legal, environmental and ethical responsibilities. The management processes, therefore, integrate new environmental and social issues, to contribute positively to sustainable development and social responsibility shared by all the actors involved.

The supply chain concept evolves towards that of Green Supply Chain: the purchasing manager (buyer) has to manage the needs of reduction, recovery, reuse but also energy saving, low environmental impact materials, low consumption systems and machines. Then, from the idea of Green Supply Chain derives that of Green Supply Chain Management, understood as the management of the supply chain useful for improving the company-environment relationship, with particular attention to the characteristics of both the product and the process (from emissions to consumption, from logistic impact to the visual one).

In general, the introduction of sustainability policies pushes companies to raise the prices of their products or services compared to the competition. Among the causes of this phenomenon we can find, for instance, the higher cost of raw materials and industrial manufacturing processes, or the lengthening of the production chain times. Carrying out an environmental transition process requires the company to invest in its supplying, production, storage and transport processes. However, several governments are decisive in support of the ecological transition of industries, guaranteeing incentives to finance research and development projects aimed at protecting the environment, reducing waste and consumption (see [9]).

However, it is known that consumers in demand markets are willing to pay a higher price if they are aware of the eco-sustainability of production processes or product transport operations. Indeed, consumers develop a degree of aversion to producers or retailers with a bad "environmental" reputation (see [23]).

In existing literature, several optimization models have been developed to design Supply Chain Networks with environmental interests (see [25] and [10] for an extensive review on the role of sustainability in supply chain network with identification of strategies and various methodologies used by the academicians). Nagurney et al. in [16] developed a rigorous modeling and analytical framework for the design of sustainable supply chain networks, proposing an optimization model that allows for the simultaneous determination of supply chain network link capacities, through capital investments, and the product flows on various links, i.e. the manufacturing, storage, distribution/shipment links, etc. coupled with the emissions generated. In [17], the authors proposed a multi-objective fuzzy mathematical programming model for designing an environmental supply chain under inherent uncertainty of input data in such a problem. In [21], authors studied a supply chain network design problem with environmental concerns, placing interest in the environmental investments decisions in the design phase. They proposed a multi-objective optimization model that captures the trade-off between the total cost and the environment influence. Yu et al., in [24], established a three-level supply chain composed of plants, distribution centers, and retailers, and studied the location of distribution centers in the supply chain network and the carbon emissions during processing and transportation, proposing a multi-objective optimization model of green supply chain under random and fuzzy environment.

In this paper, we propose a supply chain network optimization model where manufacturers, retailers and consumers in the demand markets have a degree of interest in environmental sustainability. Particularly, the manufacturers aim to maximize their total profit, given by the difference between the revenues associated with the product sale to the retailers and the consumers at the demand markets and the costs due to the transport of the product to retailers or demand markets and due to the production process management as well as other contributes described below. In this context, we suppose that manufacturers can improve their energy level (to be interpreted as a weighted average of various factors, such as environmental emissions related to production and transport, the use of forms of renewable energy and so on) here assumed as variables for the model, investing in this increase by paying a cost with the possibility of receiving an economic incentive from a government organization in such a eventuality. Moreover, manufacturers seek to minimize, at the same time, the penalty associated with the total environmental emissions related to the production processes and the transportation of the finished product to retailers (with different shipment methods) and demand markets and the penalty associated with a not eco-sustainable disposal of the production waste as well as the costs due to an eco-sustainable disposal of it.

Retailers of the network are involved in the product transition with the manufacturers and the consumers at the demand markets and they aim to maximize their own profit, given by the difference between the revenues obtained by the product sale to the demand markets and the total costs given by the sum between management and transport costs. Moreover, they seek to minimize, at the same time, the penalty associated with the total environmental emissions related to the choice of the shipment method to transport the sold product.

Consumers at the demand markets can buy the finished product from manufacturers, directly, or from the retailers of the network. Their consumption decisions depend on the price charged on the products by the manufacturers or the retailers, on their transaction costs associated with obtaining the product and, decisively, on the emissions associated with their purchase. This aspect has a crucial importance in this model. Indeed, depending on their environmental interests, consumers in demand markets may choose not to buy the product from a certain highly polluting producer or retailer, even though the purchase price of the product is lower than other less polluting producers or retailers in the supply chain. Consumers, therefore, have a degree of environmental interest associated with the total emissions caused by their purchase. Such total emissions are related to the emission on the transactions and to the perception the consumers have of the manufacturer or retailer.

For each of the three levels of decision makers, we provide a variational inequality formulation (see [2], [3], [4], [5], [6], [12], [13], [14] for other optimization models for which a variational formulation is provided) associated with profit maximization problems, for producers and retailers, and with equilibrium conditions, for the consumers at the demand markets. Moreover, we provide a unique variational inequality formulation for the entire network that allows us to determine, simultaneously, the optimal values of

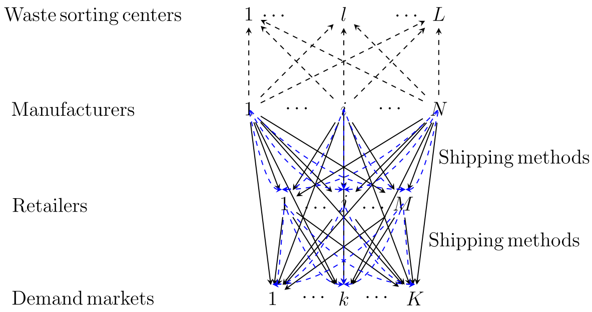

Fig. 1. Supply chain network.

the variables of the model for manufacturers, retailers and consumers at the demand markets.

The rest of the paper is as follows. In Section 2, we describe the supply chain network on which our model is based. In Section 3, we present the mathematical model and we describe, in a detailed way, the behavior of the manufactures, of the retailers and of the consumers at the demand markets. For each of them, we derive the variational inequality problems associated with the optimization decision processes and in Section 4 we derive a unique variational inequality formulation for the entire network for whose solution an existence and uniqueness result is obtained. In Section 5, we present the numerical results of different simulations in order to illustrate key aspects of the optimization model and to validate its effectiveness. Finally, Section 6 is devoted to our conclusions.

2. Supply chain network description

A general supply chain network consists of some manufacturers which produce goods and send them to the retailers. Each retailer, after buying the products from manufacturers, sends the products to consumers at demand markets.

Therefore, we consider a supply chain network with three levels of decision-makers, consisting of:

- N manufacturers;
- M retailers;
- K demand markets.

The topology of the network is depicted in Fig. 1, where the typical manufacturer is denoted by i, the typical retailer by j and the typical demand market by k. Moreover, we suppose that the decision makers (i.e. the manufacturers, the retailers and the demand markets) are spatially distributed in different parts of the world and can be of different types. Further, we assume that it is also allowed to consumers at demand markets to buy the products directly from manufacturers.

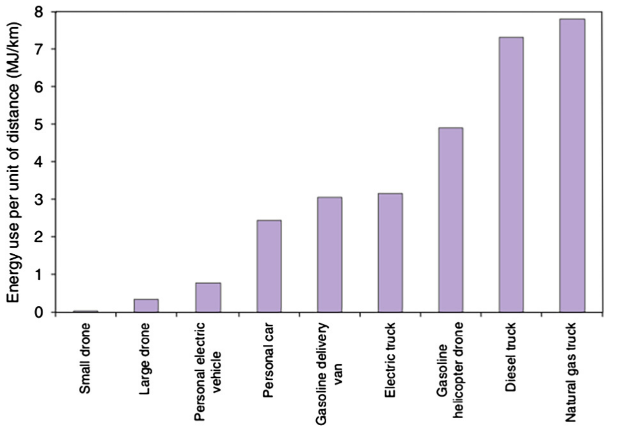

Fig. 2. Energy usage through different shipping methods. Source: Stolaroff et al. ([19]).

In particular, as previously mentioned, we pay attention on two very important aspects: the emissions (for production and transmissions) and the eco-sustainable disposal of the production waste.

Therefore, we assume there are also different shipping methods between each manufacturer and each retailer and from each retailer to each demand market. These shipping methods are represented by parallel links (see the blue dashed links in Fig. 1), since they are not decision-makers in our network (they do not decide on the transactions). Moreover, one of these arcs should represent the possibility for retailers to buy directly the products, using no carriers. Analogously, the possibility for demand markets to buy from retailers themselves. We highlight that each shipping method, not only, has a different cost of use, but also has a different environmental emission, which the decision maker aims to minimize. In this paper, we introduce the possibility to also use, among the different shipment methods, the Unmanned Aerial Vehicles (UAVs), such as drones. The efficiency of using UAVs as means of transport in terms of costs and, especially, environmental emissions is demonstrated by a lot of studies. Stolaroff et al., in their work (see [19]), showed that, comparing the energy use required per km of distance traveled, carrying a package, for example from a retail store to a customer, electric drones are far more efficient than trucks, vans, larger gasoline drones, and passenger cars. The data are shown in Fig. 2, from which we can observe that the electric drone is clearly a lower-energy-impact solution. Furthermore, Stolaroff et al. showed that UAVs (across all U.S. regions) have lower GreenHouse Gas (GHG) emissions than conventional delivery trucks powered by diesel and natural gas, electric vehicle (EV) trucks, and gasolinepowered vans. Moreover, drones are shown to have lower emissions than use of a personal vehicle to pick-up a single package.

Rogrigues et al. in [18] have compared the small quadcopter drone with different transportation modes in terms of energy consumption and CO_2 emissions. They showed that energy per package delivered by drones can be up to 94% lower than conventional trans-

Table 1
Base-case energy consumption and GHG emissions for different vehicles. Source: Rodrigues et al. ([18]).

Vehicle class	Energy consumption (MJ/km)	Fuel GHG emissions (g/km)	Upstream GHG emissions (g/km)	Battery GHG emission s (g/km)	Energy consumption (MJ/package)	GHG emission (g/package)
Medium-duty diesel truck	11.00	762.8	168.7	–	5.24	443.6
Small diesel van	4.90	339.8	75.2	–	1.41	119.2
Medium-duty electric truck	3.80	674.3	83.7	24.5	1.81	372.6
Small electric van	1.65	293.0	36.4	14.1	0.47	98.7
Electric cargo bicycle	0.10	17.9	2.2	3.3	0.10	23.4
Small quadcopter drone	0.08	14.4	1.8	1.3	0.33	70.1

portation modes as well as the environmental emissions of drones are the lowest (see Table 1).
Chiang et al. (see [1]) analytically and numerically showed that UAV delivery cut down energy use and CO_2 emissions. Particularly, they affirm that an emission reduction of over twenty percent, on average, can be realized with the use of UAVs. The latter also have a considerable impact on the fixed costs of routing as well as on the variable costs that are reduced by an average of over thirty percent through the use of UAVs.
However, drones also have negative characteristics (such as their limited battery life, load capacity, short distance that they can reach, and so on), therefore, it is necessary to establish which shipping method is more appropriate to use.

Furthermore, in this paper we also assume that the manufacturers could decide if to dispose in a sustainable way or not the production waste. For this reason at the highest level of the network we represented the L waste sorting centers (the typical one is denoted by l). Observe that the direction of the arrows is from the manufacturers to the waste sorting centers (see the black dashed link in the network Fig. 1).

3. The model

In this Section we present the mathematical model which allows us to determine the optimal flows of product among the tiers of the network previously described.

The aim of manufacturers and retailers is to maximize their own profit, while consumers at demand markets aim at minimizing their expenses. Therefore, in this paper we shall focus on the behavior of manufacturers, the retailers and the demand markets. A detailed presentation of the optimality or equilibrium conditions and the characterization by means of variational inequality problems follow.

3.1. The behavior of the manufacturers

In this section, we describe manufacturers' behavior. As previously discussed, each manufacturer seeks to maximize its profit and its environmental sustainability, minimizing the total emissions related to the production and to the transport to retailers and demand markets and, simultaneously, minimize the total penalty associated with the amount of production not transacted and not disposed of in a sustainable way (recycled or recovered).

Let us introduce the variables for the manufacturers. We denote by:

- $q_i \geq 0$ the amount of goods produced by manufacturer i and we group these quantities, for all $i = 1, \dots, N$, into the vector $q^0 = (q_i)_{i=1,\dots,N} \in \mathbb{R}^N_+$;
- $q_{ij}^{v_i} \geq 0$ the flow of product that manufacturer i, $i = 1, \dots, N$, sells to the retailer j, $j = 1, \dots, M$, through the shipment method v_i, $v_i = 1, \dots, V_i$. We group these quantities, for all $v_i = 1, \dots, V_i$ and for all $j = 1, \dots, M$, into the vector $q_i^1 = (q_{ij}^{v_i})_{\substack{j=1,\dots,M \\ v_i=1,\dots,V_i}} \in \mathbb{R}^{V_i M}_+$ and we group these vectors, for all manufacturers, into the vector $q^1 = (q_i^1)_{i=1,\dots,N} \in \mathbb{R}^{V_i MN}_+$;
- $\hat{q}_{ik} \geq 0$ the flow of product sold by manufacturer i, $i = 1, \dots, N$, to demand market k, $k = 1, \dots, K$. We group these quantities, for all demand markets, into the vector $\hat{q}_i^2 = (\hat{q}_{ik})_{k=1,\dots,K} \in \mathbb{R}^K_+$ and, in turn, we group these vectors, for all manufacturers, into the vector $\hat{q}^2 = (\hat{q}_i^2)_{i=1,\dots,N} \in \mathbb{R}^{KN}_+$;
- $q_{il} \geq 0$ the amount of product that manufacturer i, $i = 1, \dots, N$, disposes of in a sustainable way in a waste sorting center l, $l = 1, \dots, L$, and we group these quantities, for all l, into the vector $q_i^3 = (q_{il})_{l=1,\dots,L} \in \mathbb{R}^L_+$. In turn, we group these vectors into the vector $q^3 = (q_i^3)_{i=1,\dots,N} \in \mathbb{R}^{LN}_+$;
- $\mathcal{E}_i \in [1; 10]$ the energy level of the producer i, $i = 1, \dots, N$. We group these quantities into the vector $\mathcal{E} \in [1; 10]^N$.

Let c_i be the production cost associated with manufacturer i, $i = 1, \dots, N$, and we assume c_i as a function of the total production of manufacturer i, q_i, i.e.:

$$c_i := c_i(q_i), \quad \forall i = 1, \dots, N. \tag{1}$$

Associated with the sale of product to retailer j, $j = 1, \dots, M$, through the shipment method v_i, $v_i = 1, \dots, V_i$, there is a transportation cost that the manufacturer i, $i = 1, \dots, N$, has to bear. Let $c_{ij}^{v_i}$ be such a cost and we suppose $c_{ij}^{v_i}$ as a function of the flow of product sold by manufacturer i to retailer j and transported with the shipment method v_i, $q_{ij}^{v_i}$, i.e.:

$$c_{ij}^{v_i} := c_{ij}^{v_i}(q_{ij}^{v_i}), \quad \forall i = 1, \dots, n, j = 1, \dots, M, \ v_i = 1, \dots, V_i. \tag{2}$$

We underline that, as previously mentioned, among shipment methods v_i, $v_i = 1, \dots, V_i$; $i = 1, \dots, N$, we also consider UAVs, as a novelty in this context of application. Moreover, the choice to differentiate the set of possible shipment methods for manufacturers, through the different number V_i, $i = 1, \dots, N$, allows us to better fit the reality, since each manufacturer could have a different number of means of transport available.

Similarly, we denote by c_{ik} the transportation cost that manufacturer i, $i = 1, \dots, N$, has to bear in the sale to demand market k, $k = 1, \dots, K$ and we consider c_{ik} as a function of the flow of product sold by the manufacturer i to the demand market k, $\hat{q}_{ik}$, i.e.:

$$c_{ik} := c_{ik}(\hat{q}_{ik}), \quad \forall i = 1, \dots, N, \ k = 1, \dots, K. \tag{3}$$

Moreover, we denote by $\hat{c}_{ik}$ the cost function on the packing and packaging that manufacturer i, $i = 1, \dots, N$, has to spend when it sells product directly to demand market k, $k = 1, \dots, K$ and we consider $\hat{c}_{ik}$ as a function of the flow of product sold by the manufacturer i to the demand market k, $\hat{q}_{ik}$, i.e.:

$$\hat{c}_{ik} := \hat{c}_{ik}(\hat{q}_{ik}), \quad \forall i = 1, \dots, N, \ k = 1, \dots, K. \tag{4}$$

Finally, let $c^1 > 0$ and $c^2 > 0$ be the unit cost to dispose the wastage amount of product in a sustainable or not-sustainable way, respectively. Let $\delta_i \in [0, 1]$ be the rate of produced good that must be wasted (the percentage of production waste). Therefore, we have that $\delta_i q_i$ is the total amount of product wasted. Moreover, if q_{il} is the amount of discharged product in a suitable way, then, $\delta_i q_i - \sum_{l=1}^{L} q_{il}$ is the amount discharged in a not-suitable way, $\forall i = 1, \dots, N$.

The total costs incurred by manufacturer i ($\forall i = 1, \dots, N$), related to the production of goods, the transport to retailers and demand markets and the sustainable disposal of unsold product read as follows:

$$c_i(q_i) + \sum_{v_i=1}^{V_i} \sum_{j=1}^{M} c_{ij}^{v_i}(q_{ij}^{v_i}) + \sum_{k=1}^{K} c_{ik}(\hat{q}_{ik}) + \sum_{k=1}^{K} \hat{c}_{ik}(\hat{q}_{ik}) + c^1 \sum_{l=1}^{L} q_{il} + c^2(\delta_i q_i - \sum_{l=1}^{L} q_{il}). \tag{5}$$

The revenues of the manufacturer i, $i = 1, \dots, N$, derive from the sale of products to retailers and demand markets. If we denote by $\rho_{1ij}^{v_i,*}$ and ρ_{1ik}^{*} the unit revenue obtained by the sale of a unit of product to a retailer j, $j = 1, \dots, M$, with the shipment method v_i, $v_i = 1, \dots, V_i$, and the unit revenue obtained by the sale of a unit of product to a demand market k, $k = 1, \dots, K$, respectively, the total revenue for the retailer i corresponds to the quantity:

$$\sum_{v_i=1}^{V_i} \sum_{j=1}^{M} \rho_{1ij}^{v_i*} q_{ij}^{v_i} + \sum_{k=1}^{K} \rho_{1ik}^{*} q_{ik}. \tag{6}$$

Hence, the profit for a manufacturer i related to the production and the sale of product to all retailers and all demand markets reads as follows:

$$\sum_{v_i=1}^{V_i}\sum_{j=1}^{M}\rho_{1ij}^{v_i,*}q_{ij}^{v_i}+\sum_{k=1}^{K}\rho_{1ik}^{*}\hat{q}_{ik}-c_i(q_i)-\sum_{v_i=1}^{V_i}\sum_{j=1}^{M}c_{ij}^{v_i}(q_{ij}^{v_i})-\sum_{k=1}^{K}c_{ik}(\hat{q}_{ik})+$$
$$-\sum_{k=1}^{K}\hat{c}_{ik}(\hat{q}_{ik})-c^1\sum_{l=1}^{L}q_{il}-c^2(\delta_i q_i-\sum_{l=1}^{L}q_{il}). \tag{7}$$

As previously mentioned, manufacturers are interested, in addition to the sale to retailers and demand markets, in the environmental sustainability of the production and transport chain. Therefore it is plausible to consider a penalty associated with the not eco-sustainable disposal of the production waste. Let $\beta_i > 0$ the unit penalty incurred by the manufacturer i, $i = 1, \ldots, N$, in case of not eco-sustainable disposal of the production waste. Hence, the total penalty, which manufacturer i seeks to minimize, corresponds to the quantity:

$$\beta_i\left[\delta_i q_i-\sum_{l=1}^{L}q_{il}\right],\quad \forall i=1,\ldots,N. \tag{8}$$

We note that the parameter β_i enables distinct manufacturers to have different losses related to the unsustainable disposal of the production waste. This difference could be dictated, for instance, by the geographical location of the producer and the taxes applied in that area.

In addition to the penalty in the event of unsustainable disposal of the discarded product, the manufacturer i, $i = 1, \ldots, N$, seeks to minimize also the environmental emissions, in terms of costs, related to the production and shipment of goods. Let e_i be the environmental emissions related to the production of manufacturer i, $i = 1, \ldots, N$, and we assume e_i as a function of the total amount of goods produced by i, i.e.:

$$e_i := e_i(q_i),\quad \forall i=1,\ldots,N. \tag{9}$$

Similarly, we denote by $e_{ij}^{v_i}$ and e_{ik} the environmental emissions of manufacturer i, $i = 1, \ldots, N$, related to the transport to a retailer j, $j = 1, \ldots, M$, with the shipment method v_i, $v_i = 1, \ldots, V_i$, and the environmental emissions of manufacturer i related to the transport to a demand market k, $k = 1, \ldots, K$, respectively and we suppose $e_{ij}^{v_i}$ as a function of the flow of product transacted between i and j in the v_i shipment mode, namely $q_{ij}^{v_i}$, and e_{ik} as a function of the flow of product sent by manufacturer i to the demand market k, namely q_{ik}. Hence, we have:

$$e_{ij}^{v_i} := e_{ij}^{v_i}(q_{ij}^{v_i}),\quad \forall i=1,\ldots,N,\ j=1,\ldots,M,\ v_i=1,\ldots,V_i; \tag{10}$$
$$e_{ik} := e_{ik}(\hat{q}_{ik}),\quad \forall i=1,\ldots,N,\ k=1,\ldots,K. \tag{11}$$

Therefore, the total environmental emissions, in terms of costs, related to the production and shipment of goods for manufacturer i, $i = 1, \dots, N$, are:

$$\xi_i(q_i, q_i^1, q_i^2) := \alpha_i \left[e_i(q_i) + \sum_{v_i=1}^{V_i} \sum_{j=1}^{N} e_{ij}^{v_i}(q_{ij}^{v_i}) + \sum_{k=1}^{K} e_{ik}(\hat{q}_{ik}) \right], \quad \forall i = 1, \dots, N, \tag{12}$$

where $\alpha_i > 0$ is a parameter that allows us to express environmental emissions in terms of costs. Moreover, the parameter α_i enables distinct manufacturers to have different emissions in terms of costs (different production plants, use environmentally friendly machinery).

Considering that in recent decades sustainable production has become one of the main characteristics of supply chains, in this model we assume that, through a series of improvements in the production process, each producer can increase their initial energy level $\overline{\mathcal{E}}_i$, $i = 1, \dots, N$. The Government Institutions, considering the emission limits of their own Country and the constraints imposed by the International Agreements in force, are therefore committed to raising awareness of an eco-sustainable production process, encouraging producers through financing them. To this end, we suppose that for each deviation of energy level, producers receive funding from their Government. We denote by f the unit funding received by the producer in the event that she/he increases her/his energy level. Therefore, denoting by $\mathcal{E}_i$ the non-negative variable on the energy level of producer i, $i = 1, \dots, N$ (and by $\mathcal{E} = (\mathcal{E}_i)_{i=1,\dots,N} \in \mathbb{R}_+^N$), to the producer's profit (7), we need to add the following quantity:

$$f \cdot \left(\mathcal{E}_i - \overline{\mathcal{E}}_i \right). \tag{13}$$

Observe that we are assuming that the energy level could only be increased (not reduced). Therefore we have that $\mathcal{E}_i \geq \overline{\mathcal{E}}_i$ that is $\mathcal{E}_i - \overline{\mathcal{E}}_i \geq 0$. Therefore, $f \cdot \left(\mathcal{E}_i - \overline{\mathcal{E}}_i \right) \geq 0$ is a revenue.

However, increasing its energy level involves costs for producers linked, for example, to the installation of solar panels in its production plants, the purchase of cutting-edge production machines, the adaptation of the means of transport to the legislation in terms of circulation etc. Therefore, we introduce the cost function γ_i related to the increase of the energy level of producer i, $i = 1, \dots, N$, and we suppose it as a function of $\mathcal{E}_i$, that is:

$$\gamma_i \equiv \gamma_i(\mathcal{E}_i). \tag{14}$$

Particularly, we assume that such a function depends on the difference $\mathcal{E}_i - \overline{\mathcal{E}}_i$, between the current energy level and the initial one. Note that if the energy level is not varied from the beginning, the manufacturer has neither funding nor costs.

Each manufacturer i seeks to maximize its own profit, given by the difference between the revenues obtained by the sale of product and the costs associated with the production and the shipment of goods and, simultaneously, to minimize its total environmental emissions in terms of costs. Therefore, each manufacturer is faced with the following maximization problem:

$$\text{Max}\left\{\sum_{v_i=1}^{V_i}\sum_{j=1}^{M}\rho_{1ij}^{v_i,*}q_{ij}^{v_i}+\sum_{k=1}^{K}\rho_{1ik}^{*}\hat{q}_{ik}+f\cdot\left(\mathcal{E}_i-\overline{\mathcal{E}}_i\right)-c_i(q_i)-\sum_{v_i=1}^{V_i}\sum_{j=1}^{M}c_{ij}^{v_i}(q_{ij}^{v_i})\right.$$
$$-\gamma_i(\mathcal{E}_i)-\sum_{k=1}^{K}c_{ik}(\hat{q}_{ik})-\sum_{k=1}^{K}\hat{c}_{ik}(\hat{q}_{ik})-c^1\sum_{l=1}^{L}q_{il}-c^2(\delta_i q_i-\sum_{l=1}^{L}q_{il}) \tag{15}$$
$$\left.-\beta_i\left[\delta_i q_i-\sum_{l=1}^{L}q_{il}\right]-\alpha_i\left[e_i(q_i)+\sum_{v_i=1}^{V_i}\sum_{j=1}^{M}e_{ij}^{v_i}(q_{ij}^{v_i})+\sum_{k=1}^{K}e_{ik}(\hat{q}_{ik})\right]\right\},$$

subject to constraints:

$$q_i=\sum_{v_i=1}^{V_i}\sum_{j=1}^{M}q_{ij}^{v_i}+\sum_{k=1}^{K}\hat{q}_{ik}, \tag{16}$$

$$\sum_{l=1}^{L}q_{il}\le\delta_i q_i, \tag{17}$$

$$\frac{\xi_i(q_i,q_i^1,\hat{q}_i^2)}{\alpha_i}\le\Xi_i, \tag{18}$$

$$\overline{\mathcal{E}}_i\le\mathcal{E}_i\le\mathcal{E}^M, \tag{19}$$

$$\sum_{i=1}^{I}\sum_{v_i=1}^{V_i}q_{ij}^{v_i}+\sum_{k=1}^{K}\hat{q}_{ik}\le\overline{Q}_i, \tag{20}$$

$$q_i,\ q_{ij}^{v_i},\ \hat{q}_{ik},\ q_{il}\ge 0,\quad \forall j=1,\dots,M,\ v_i=1,\dots,V_i,\ k=1,\dots,K,\ l=1,\dots,L. \tag{21}$$

Constraint (16) states that the manufacturer i, $i=1,\dots,N$, produces exactly the quantity sold to all retailers and all demand markets. Therefore, such a conservation law affirms that neither overproduction nor underproduction are allowed. Indeed, the overproduction is not convenient in economics terms, and underproduction is forbidden for clear reasons (it is not possible to sell goods not produced).

We can observe that the conservation law (16) allows us to express q_i in terms of the vector $(q_i^1,\hat{q}_i^2)$. Therefore, problem (15)-(21) could be rewritten as follows:

$$\text{Max}\left\{\sum_{v_i=1}^{V_i}\sum_{j=1}^{M}\rho_{1ij}^{v_i,*}q_{ij}^{v_i}+\sum_{k=1}^{K}\rho_{1ik}^{*}\hat{q}_{ik}+f\cdot\left(\mathcal{E}_i-\overline{\mathcal{E}}_i\right)-c_i(q_i^1,\hat{q}_i^2)\right.$$
$$-\sum_{v_i=1}^{V_i}\sum_{j=1}^{M}c_{ij}^{v_i}(q_{ij}^{v_i})-\gamma_i(\mathcal{E}_i)-\sum_{k=1}^{K}c_{ik}(\hat{q}_{ik})-\sum_{k=1}^{K}\hat{c}_{ik}(\hat{q}_{ik})-c^1\sum_{l=1}^{L}q_{il}$$

$$
-(c^2+\beta_i)\left[\delta_i\left(\sum_{v_i=1}^{V_i}\sum_{j=1}^{M} q_{ij}^{v_i}+\sum_{k=1}^{K}\hat{q}_{ik}\right)-\sum_{l=1}^{L} q_{il}\right] \tag{22}
$$

$$
\left.-\alpha_i\left[e_i(q_i^1,\hat{q}_i^2)+\sum_{v_i=1}^{V_i}\sum_{j=1}^{M} e_{ij}^{v_i}(q_{ij}^{v_i})+\sum_{k=1}^{K} e_{ik}(\hat{q}_{ik})\right]\right\},
$$

subject to constraints:

$$
\sum_{l=1}^{L} q_{il} \le \delta_i\left(\sum_{v_i=1}^{V_i}\sum_{j=1}^{M} q_{ij}^{v_i}+\sum_{k=1}^{K}\hat{q}_{ik}\right), \tag{23}
$$

$$
\frac{\xi_i(q_i^1,\hat{q}_i^2)}{\alpha_i} \le \Xi_i, \tag{24}
$$

$$
\overline{\mathcal{E}}_i \le \mathcal{E}_i \le \mathcal{E}^M, \tag{25}
$$

$$
\sum_{i=1}^{I}\sum_{v_i=1}^{V_i} q_{ij}^{v_i}+\sum_{k=1}^{K}\hat{q}_{ik} \le \overline{Q}_i, \tag{26}
$$

$$
q_{ij}^{v_i},\ \hat{q}_{ik},\ q_{il} \ge 0, \quad \forall j=1,\dots,M,\ v_i=1,\dots,V_i,\ k=1,\dots,K,\ l=1,\dots,L. \tag{27}
$$

Constraint (23) asserts that the manufacturer i, $i = 1, \dots, N$, can decide to dispose in a eco-sustainable way at most the amount of waste produced.

Constraint (24) is an environmental emissions constraint and it affirms that manufacturer i, $i = 1, \dots, N$ cannot exceed the maximum limit of environmental emissions, denoted by Ξ_i (kg/km^2). We note that different values of Ξ_i may depend, for instance, on the different geographical locations of the manufacturers and, therefore, on the different environmental limitations of these areas. We observe that the parameters α_i allow us to express environmental emissions in terms of costs. Therefore, their unit of measure is (km^2·€)/kg.

Constraint (25) assures that the energy level is increased or not varied. It is clear that the energy level cannot exceed the maximum allowed $\mathcal{E}^M$.

Constraint (26) affirms that the quantity of product sent to all retailers (through all the shipping methods) and all demand markets, cannot exceed the maximum amount of product that the manufacturer i is able to produce $\overline{Q}_i$ (for example, since the limited amount of raw materials).

Finally, the latest family of constraints (27) defines the domain of the variables of the model.

3.1.1. Variational formulation for manufacturers

To obtain a variational formulation of the previous constrained optimization model, we make the following fundamental assumption.

Assumption 3.1. We assume that all the involved functions are convex and continuously differentiable.

Assuming that, we obtain that the objective function is continuously differentiable and concave. Therefore, since the objective function is concave and the feasible set is closed and convex, it is easy to verify that the optimality conditions for all manufacturers simultaneously are characterized by a variational inequality, as expressed by the following Theorem (for the proof, see [11]).

Theorem 3.1. *A vector $(q^{1*}, \hat{q}^{2*}, q^{3*}, \mathcal{E}^*) \in \mathbb{K}$ is an optimal solution to the problem (22)-(27) if and only if such a vector is a solution to the variational inequality:*
Find $(q^{1*}, \hat{q}^{2*}, q^{3*}, \mathcal{E}^*) \in \mathbb{K}$ such that:

$$\begin{aligned}
&\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{v_i=1}^{V_i}\left[\frac{\partial c_i(q_i^{1*},\hat{q}_i^{2*})}{\partial q_{ij}^{v_i}} + \frac{\partial c_{ij}^{v_i}(q_{ij}^{v_i *})}{\partial q_{ij}^{v_i}} + \delta_i(c^2+\beta_i)\right.\\
&\qquad\left. +\alpha_i\frac{\partial e_i(q_i^{1*},\hat{q}_i^{2*})}{\partial q_{ij}^{v_i}} + \alpha_i\frac{\partial e_{ij}^{v_i}(q_{ij}^{v_i *})}{\partial q_{ij}^{v_i}} - \rho_{1ij}^{v_i,*}\right]\times(q_{ij}^{v_i} - q_{ij}^{v_i *})\\
&+\sum_{i=1}^{N}\sum_{k=1}^{K}\left[\frac{\partial c_i(q_i^{1*},\hat{q}_i^{2*})}{\partial \hat{q}_{ik}} + \frac{\partial c_{ik}(\hat{q}_{ik}^{*})}{\partial \hat{q}_{ik}} + \frac{\partial \hat{c}_{ik}(\hat{q}_{ik}^{*})}{\partial \hat{q}_{ik}} + \delta_i(c^2+\beta_i)\right.\\
&\qquad\left. +\alpha_i\frac{\partial e_i(q_i^{1*},\hat{q}_i^{2*})}{\partial \hat{q}_{ik}} + \alpha_i\frac{\partial e_{ik}(\hat{q}_{ik}^{*})}{\partial \hat{q}_{ik}} - \rho_{1ik}^{*}\right]\times(\hat{q}_{ik} - \hat{q}_{ik}^{*})\\
&+\sum_{i=1}^{N}\sum_{l=1}^{L}\left[c^1-(c^2+\beta_i)\right]\times(q_{il}-q_{il}^*)\\
&+\sum_{i=1}^{N}\left[\frac{\partial \gamma_i(\mathcal{E}_i^*)}{\partial \mathcal{E}_i} - f\right]\times(\mathcal{E}_i - \mathcal{E}_i^*) \geq 0\\
&\qquad\qquad\qquad\forall (q^1,\hat{q}^2,q^3,\mathcal{E}) \in \mathbb{K},
\end{aligned} \tag{28}$$

where

$$\mathbb{K} := \left\{(q^1,\hat{q}^2,q^3,\mathcal{E}) \in \mathbb{R}_+^{N[MV+K+L+1]} : \text{(23)-(26) hold}\right\} \text{ and } V = \sum_{i=1}^{N} V_i. \tag{29}$$

The variational inequality (28) represents the optimality conditions for all manufacturers simultaneously. The solution of (28) gives the optimal amount of product each manufacturer i, $\forall i = 1, \dots, N$, has to sell to all retailers (q^{1*}) and to all demand markets ($\hat{q}^{2*}$); the optimal amount of waste production disposed in eco-sustainable way (q^{3*}), and the energy level to reach ($\mathcal{E}^*$), in equilibrium. We now provide an existence result for a solution to variational inequality (28).

Theorem 3.2. *Under the Assumption 3.1, variational inequality (28) admits at least one solution.*

Proof. The result follows from the classical theory of variational inequalities (see [8]), since the feasible set is compact and the operator of the variational inequality is continuous. □

3.2. The behavior of the retailers

At the middle level of the network, the retailers are involved in the product transition both with the manufacturers and with the consumers at the demand markets. Particularly, the retailers buy the finished product from the manufacturers and sell it to the demand markets, representing the last level of the network.

A retailer j has to incur costs to manage the storage of the products purchased from the manufacturers of the network. We denote by c_j such a cost and we suppose it as a function of the total amount of product purchased from all manufacturers via all shipment methods, i.e. $\sum_{i=1}^{N}\sum_{v_i=1}^{V_i} q_{ij}^{v_i}$, for all $j = 1, \dots, M$. However, for sake of generality, we suppose that this cost depends on the whole vector q^1. Thereby, the management cost for a retailer j depends also on the amounts of product held by all others retailers. This assumption allows us to better describe the competition among the retailers of the network. Hence, we have:

$$c_j = c_j(q^1), \quad \forall j = 1, \dots, M.$$

We denote by $\tilde{c}_{ij}^{v_i}$ the costs incurred by retailer j in the transaction of products with manufacturer i via the shipment method v_i and we assume that the function can depend upon the manufacturer/retailer pair product transaction, that is,

$$\tilde{c}_{ij}^{v_i} \equiv \tilde{c}_{ij}^{v_i}(q_{ij}^{v_i}), \quad \forall i = 1, \dots, N,\ j = 1, \dots, M,\ v_i = 1, \dots, V_i. \tag{30}$$

As previously discussed, the retailers transact the product to demand markets. Similarly to manufacturers, the retailers can decide to send the product to demand markets via different shipment methods. Let:

- $\tilde{q}_{jk}^{u_j} \geq 0$ be the amount of goods transacted with the shipment method u_j between the retailers j and the demand market k and we group these quantities, for all $u_j = 1, \dots, U_j$, into the vector $\tilde{q}_{jk} = (\tilde{q}_{jk}^{u_j})_{u_j=1,\dots,U_j} \in \mathbb{R}_+^{U_j}$. In turn, we group these quantities, for all $k = 1, \dots, K$, into the vector $\tilde{q}_j^4 = (\tilde{q}_{jk})_{k=1,\dots,K} \in \mathbb{R}_+^{KU_j}$ and, finally, for all $j = 1, \dots, M$, into the vector $\tilde{q}^4 = (\tilde{q}_j)_{j=1,\dots,M} \in \mathbb{R}_+^{MKU}$, where $U = \sum_{j=1}^{M} U_j$.

As previously discussed, the different numbers of shipment methods U_j, $j = 1, \ldots, M$, allow us to differentiate the retailers in terms of shipment methods.

Associated with the transaction between retailers and demand markets, the retailers have to incur transportation costs $c_{jk}^{u_j}$, which differ in relation to the chosen shipment method. Clearly, these costs depend on the quantity of transacted product, that is

$$c_{jk}^{u_j} \equiv c_{jk}^{u_j}(\tilde{q}_{jk}^{u_j}), \quad \forall j = 1, \ldots, M, \ \forall k = 1, \ldots, K, \ \forall u_j = 1, \ldots, U_j. \tag{31}$$

Unlike other models, even at the level of retailers we will consider the emissions related to the various transport methods to transact products with demand markets. Let $e_{jk}^{u_j}$ be the environmental emissions of retailer j, $j = 1, \ldots, M$ related to the transport to a demand market k, $k = 1, \ldots, K$, and we suppose $e_{ik}^{u_j}$ as a function of the flow of product transacted between j and k with u_j shipment method, namely $\tilde{q}_{jk}^{u_j}$. Therefore, we have:

$$e_{jk}^{u_j} := e_{jk}^{u_j}(\tilde{q}_{jk}^{u_j}), \quad \forall j = 1, \ldots, M, \ k = 1, \ldots, K, \ u_j = 1, \ldots, U_j. \tag{32}$$

Therefore, the total environmental emissions, in terms of costs, related to the shipment of goods for retailers j, $j = 1, \ldots, M$, is:

$$\xi_j(\tilde{q}_j^4) = \tilde{\alpha}_j \sum_{k=1}^{K} \sum_{u_j=1}^{U_j} e_{jk}(\tilde{q}_{jk}^{u_j}), \quad \forall j = 1, \ldots, M,$$

where $\tilde{\alpha}_j > 0$ is a parameter that allows us to express environmental emissions in terms of costs. Moreover, the parameter $\tilde{\alpha}_j$ enables distinct retailers to have different emissions in terms of costs.

The retailers associate a price with the product at their retail outlet, which is denoted by ρ_{2j}^*, for retailer j and each of them seeks to maximize its profit, minimizing, simultaneously, the environmental emissions related to the sale with demand markets, that is:

$$\begin{aligned} \max \Bigg\{ & \rho_{2j}^* \sum_{k=1}^{K} \sum_{u_j=1}^{U_j} \tilde{q}_{jk}^{u_j} - c_j(q^1) - \sum_{i=1}^{N} \sum_{v_i=1}^{V_i} \tilde{c}_{ij}^{v_i}(q_{ij}^{v_i}) - \sum_{k=1}^{K} \sum_{u_j=1}^{U_j} c_{jk}^{u_j}(\tilde{q}_{jk}^{u_j}) \\ & - \sum_{i=1}^{N} \sum_{v_i=1}^{V_i} \rho_{1ij}^{v_i,*} q_{ij}^{v_i} - \tilde{\alpha}_j \sum_{k=1}^{K} \sum_{u_j=1}^{U_j} e_{jk}^{u_j}(\tilde{q}_{jk}^{u_j}) \Bigg\} \end{aligned} \tag{33}$$

subject to constraints:

$$\sum_{k=1}^{K} \sum_{u_j=1}^{U_j} \tilde{q}_{jk}^{u_j} \le \sum_{i=1}^{N} \sum_{v_i=1}^{V_i} q_{ij}^{v_i} \tag{34}$$

$$\sum_{v_i=1}^{V_i} q_{ij}^{v_i} \le \overline{Q}_{ij}, \forall i = 1, \ldots, N \tag{35}$$

$$\frac{\xi_j(\tilde{q}_j^4)}{\tilde{\alpha}_j} \leq \tilde{\Xi}_j \tag{36}$$

$$q_{ij}^{v_i},\ \tilde{q}_{jk}^{u_j} \geq 0, \quad \forall i = 1,\dots,N,\ \forall v_i = 1,\dots,V_i,\ \forall k = 1,\dots,K,\ u_j = 1,\dots,U_j. \tag{37}$$

Constraint (34) simply expresses that retailer j, $j = 1,\dots,M$, cannot sell to demand markets more than bought from all the manufacturers of the network.

Constraint (35) guarantees that retailer j cannot buy from each manufacturer more product than that allowed, that is the maximum quantity that each manufacturer is willing to sell at j, $\overline{Q}_{ij}$.

Constraint (36) is an environmental emissions constraint and it affirms that retailer j, $j = 1,\dots,M$, cannot exceed the maximum limit of environmental emissions $\tilde{\Xi}_j$ (kg/km^2). We note that different values of $\tilde{\Xi}_j$ may depend, for instance, on the different geographical locations of the retailers and, therefore, on the different environmental limitations of their areas. Similarly to the previous Section, parameters α_j are expressed in (km^2·€)/kg.

Finally, the latest family of constraints (37) defines the domain of the variables of the model.

3.2.1. Variational formulation for retailers

Since the objective function is concave (by virtue of Assumption 3.1) and the feasible set is closed and convex, the optimality conditions for all retailers simultaneously are characterized by the following Theorem.

Theorem 3.3. *A vector* $(q^{1*}, \tilde{q}^{4*}) \in \tilde{\mathbb{K}}$ *is an optimal solution to the problem* (33)-(37) *if and only if such a vector is a solution to the variational inequality:*
Find $(q^{1*}, \tilde{q}^{4*}) \in \tilde{\mathbb{K}}$ *such that:*

$$\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{v_i=1}^{V_i}\left[\frac{\partial c_j(q^{1*})}{\partial q_{ij}^{v_i}} + \frac{\partial \tilde{c}_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}} + \rho_{1ij}^{v_i,*}\right] \times (q_{ij}^{v_i} - q_{ij}^{v_i*})$$
$$+\sum_{j=1}^{M}\sum_{k=1}^{K}\sum_{u_j=1}^{U_j}\left[\frac{\partial c_{jk}^{u_j}(\tilde{q}_{jk}^{u_j*})}{\partial \tilde{q}_{jk}^{u_j}} + \tilde{\alpha}_j\frac{\partial e_{jk}^{u_j}(\tilde{q}_{jk}^{u_j*})}{\partial \tilde{q}_{jk}^{u_j}} - \rho_{2j}^*\right] \times (\tilde{q}_{jk}^{u_j} - \tilde{q}_{jk}^{u_j*}) \geq 0$$
$$\forall (q^1, \tilde{q}^4) \in \tilde{\mathbb{K}}, \tag{38}$$

where

$$\tilde{\mathbb{K}} := \left\{(q^1, \tilde{q}^4) \in \mathbb{R}_+^{M[NV+KU]} :\ (34)\text{-}(36)\ \textit{hold}\right\}, V = \sum_{i=1}^{N} V_i\ \textit{and}\ U = \sum_{j=1}^{M} U_j. \tag{39}$$

The variational inequality (38) represents the optimality conditions for all retailers simultaneously. The solution of (38) gives the optimal amount of product each retailer j, $\forall j = 1,\dots,M$, has to buy from all manufacturers (q^{1*}) and to sell to all demand markets ($\tilde{q}^{4*}$), in equilibrium. Analogously to the variational formulation for manufacturers and Theorem 3.2, we can provide an existence result for a solution to variational inequality (38), as follows:

Theorem 3.4. *Under the Assumption 3.1, variational inequality (38) admits at least one solution.*

3.3. The behavior of the consumers at the demand markets

At the lower level of the network, the consumers at the demand markets, as previously mentioned, can decide to buy finished product both directly from the manufacturers and/or from the retailers. Their consumption decisions depend on the price charged on the products by the manufacturers or the retailers, on their transaction costs associated with obtaining the product and, decisively, on the emissions associated with their purchase. This aspect has a crucial importance in this model. Indeed, depending on their environmental interests, consumers in demand markets may choose not to buy the product from a certain highly polluting producer or retailer, even though the purchase price of the product is lower than other less polluting producers or retailers in the supply chain. Consumers, therefore, have a degree of environmental interest associated with the total emissions caused by their purchase. Such total emissions are related to the emission on the transactions and to the perception the consumers have of the manufacturer or retailer.

We denote by $\mu_k, \tilde{\mu}_k \geq 0$ (to be interpreted as weights) the degrees of environmental interest of consumers at demand market k, for all $k = 1, \dots, K$, about the emissions associated with their transactions with the manufacturers and the retailers, respectively. Moreover, we denote by ψ_{ki} and $\tilde{\psi}_{kj}$ the perception of consumers at k for the total emissions of manufacturer i and retailer j, respectively. Observe that the last ones are parameters and do not depend on the variables, since, to better fit the realty, we are assuming that the consumers do not know the exact quantity of goods produced by each manufacturer (and, hence, the total emission for his production), or the manufacturer chosen by each retailer, or the energy level of each manufacturer, and so on.

Therefore, the aversion to buy from a non-sustainable manufacturer i (in terms of production and transport) depends on the emission on the transaction between i and k, $e_{ik}(\hat{q}_{ik})$, and the perception such a market has on the total emission of manufacturer i, ψ_{ki} (due to the production, the energy level of i and to its disposal rate in an eco-sustainable way). Hence, the aversion for consumers at demand market $k = 1, \dots, K$ in buying from a manufacturer $i = 1, \dots, N$ is as follows:

$$a_{ik}(\hat{q}_{ik}) := \mu_k \cdot [e_{ik}(\hat{q}_{ik}) + \psi_{ki}] \,. \tag{40}$$

We group the quantities $\hat{q}_{ik}$, for all manufacturers, into the vector $\hat{q}^2_k = (\hat{q}_{ik})_{i=1,\dots,N} \in \mathbb{R}^N_+$.

Similarly, the aversion in buying from a retailer j is expressed by the emission on the transaction between j and k, $e_{jk}^{u_j}(\tilde{q}_{jk}^{u_j})$, and the perception such a market has on the total emission of retailer j, $\tilde{\psi}_{kj}$ (which not only depends on the retailer himself from whom the consumers at demand market k buy the product, but actually also on the manufacturers the chosen retailer bought the product):

$$a_{jk}^{u_j}(\tilde{q}_{jk}^{u_j}) := \tilde{\mu}_k \cdot \left[e_{jk}^{u_j}(\tilde{q}_{jk}^{u_j}) + \tilde{\psi}_{kj} \right], \quad \forall j = 1, \dots M, \forall k = 1, \dots, K. \tag{41}$$

We group the quantities $\tilde{q}_{jk}^{u_j}$, for all retailers, into the vector $\tilde{q}_k^4 = (q_{jk}^{u_j})_{\substack{j=1,\dots,M \\ u_j=1,\dots,U_j}} \in \mathbb{R}_+^{MU}$, where $U = \sum_{j=1}^{M} U_j$.

In the previous expression, the first term refers to the total emission of retailer j on the transmission to k (through the different shipping methods), while the last term, $\tilde{\psi}_{kj}$, expresses the perception of the environmental emission the consumers at k associate to the retailer j.

Let $\hat{c}_{ik}$ and $\hat{c}_{jk}^{v_j}$ be the unit transaction cost associated with consumers at demand market k that buy finished product from manufacturer i and retailer j in the v_j-th mode, respectively, for all $k = 1, \dots, K$, $i = 1, \dots, N$, $j = 1, \dots, M$, $u_j = 1, \dots, U_j$. To express the fact that these costs depend both on the amount of product obtained from all manufacturers and all retailers of the network, we suppose that:

$$\hat{c}_{ik} := \hat{c}_{ik}(\hat{q}^2, \tilde{q}^4), \quad \forall i = 1, \dots, N, \ k = 1, \dots, K$$

and

$$\hat{c}_{jk}^{u_j} := \hat{c}_{jk}^{u_j}(\hat{q}^2, \tilde{q}^4), \quad \forall j = 1, \dots, M, \ k = 1, \dots, K, \ u_j = 1, \dots, U_j.$$

Let ρ_{3k} be the price of the product the consumers at the demand market k are willing to pay, and we group these quantities into the vector $\rho_3 = (\rho_{3k})_{k=1,\dots,K} \in \mathbb{R}_+^K$. To better describe the competition among demand markets, denoting by d_k the request for the product at the demand market k, we will suppose that such a demand depends on the whole vector ρ_3. Thereby, the requests at the demand markets depend not only on the price made in each demand market but, also, on the prices of the other demand markets. Hence,

$$d_k := d_k(\rho_3), \quad \forall k = 1, \dots, K.$$

Therefore, in purchasing the product from a manufacturer, the consumers at demand markets bear the manufacturer's selling cost and the transport cost due to them. Furthermore, the aversion on the environmental emissions of the manufacturer from whom they buy is decisive (including the perception of such a manufacturer that cusumers have). In a similar way, when consumers at demand markets buy products from a retailer, they have to bear the price fixed by the retailer and the transaction costs due to them. Also in this case, the aversion to buy from a non-sustainable retailer j is of fundamental importance.

Hence, we have the following Definition for the *equilibrium conditions*.

Definition 3.1. A vector $(\hat{q}_k^{2*}, \tilde{q}_k^{4*}, \rho_{3k}^*) \in \mathbb{R}_+^{N+MU+1}$ (where $U = \sum_{j=1}^{M} U_j$) is of equilibrium for consumers at the demand market k if the following conditions hold:

$$\rho_{1ik}^* + \hat{c}_{ik}(\hat{q}_k^{2*}, \tilde{q}^{4*}) + a_{ik}(\hat{q}_{ik}^*) \begin{cases} = \rho_{3k}^* & \text{if} \quad \hat{q}_{ik}^* > 0 \\ \geq \rho_{3k}^* & \text{if} \quad \hat{q}_{ik}^* = 0, \end{cases} \quad \forall i = 1, \ldots, N \tag{42}$$

$$\rho_{2j}^* + \hat{c}_{jk}^{u_j}(q^{2*}, \tilde{q}^{4*}) + a_{jk}^{u_j}(\tilde{q}_{jk}^{u_j*}) \begin{cases} = \rho_{3k}^* & \text{if} \quad q_{jk}^{u_j*} > 0 \\ \geq \rho_{3k}^* & \text{if} \quad q_{jk}^{u_j*} = 0, \end{cases}$$
$$\forall j = 1, \ldots, M, \, \forall u_j = 1, \ldots, U_j \tag{43}$$

and

$$d_k(\rho_3^*) \begin{cases} = \displaystyle\sum_{i=1}^{N} q_{ik}^* + \sum_{j=1}^{M} \sum_{u_j=1}^{U_j} q_{jk}^{u_j*} & \text{if} \quad \rho_{3k}^* > 0 \\ \leq \displaystyle\sum_{i=1}^{N} q_{ik}^* + \sum_{j=1}^{M} \sum_{u_j=1}^{U_j} q_{jk}^{u_j*} & \text{if} \quad \rho_{3k}^* = 0. \end{cases} \tag{44}$$

Condition (42) states that consumers at the demand market k buy from a manufacturer if the sum of the manufacturer's selling price, the transaction costs (incurred by the consumers) and the marginal costs of emissions associated with the selling is equal to the price that the demand markets are willing to pay. Otherwise, the consumers at the demand market k decide not to buy from the specific manufacturer.

Condition (43) is analogous to the previous condition for each retailer (and each shipping method) and the demand market k.

Finally, condition (44) expresses that, if the price imposed by the consumers at demand market k is positive (and, hence, a purchase has been made), the quantity purchased by the consumers at the demand market k is equal to the demand.

3.3.1. Variational formulation for demand markets

The equilibrium conditions (42)-(44) for consumers at all the demand markets simultaneously are characterized by the following Theorem (see [11] and [22]).

Theorem 3.5. *A vector* $(\hat{q}^{2*}, \tilde{q}^{4*}, \rho_3^*) \in \overline{\mathbb{K}}$ *satisfies the equilibrium conditions* (42)-(44) *if and only if such a vector is a solution to the variational inequality:*
Find $(\hat{q}^{2*}, \tilde{q}^{4*}, \rho_3^*) \in \overline{\mathbb{K}}$ *such that:*

$$\sum_{i=1}^{N}\sum_{k=1}^{K}\left[\rho_{1ik}^{*}+\hat{c}_{ik}(\hat{q}^{2*},\tilde{q}^{4*})+a_{ik}(\hat{q}_{ik}^{*})-\rho_{3k}^{*}\right]\times(\hat{q}_{ik}-\hat{q}_{ik}^{*})$$

$$+\sum_{j=1}^{M}\sum_{k=1}^{K}\sum_{u_j=1}^{U_j}\left[\rho_{2j}^{*}+\hat{c}_{jk}^{u_j}(\hat{q}^{2*},\tilde{q}^{4*})+a_{jk}^{u_j}(\tilde{q}_{jk}^{u_j*})-\rho_{3k}^{*}\right]\times(\tilde{q}_{jk}^{u_j}-\tilde{q}_{jk}^{u_j*})$$

$$+\sum_{k=1}^{K}\left[\sum_{i=1}^{N}\hat{q}_{ik}^{*}+\sum_{j=1}^{M}\sum_{u_j=1}^{U_j}q_{jk}^{u_j*}-d_k(\rho_3^{*})\right]\times[\rho_{3k}-\rho_{3k}^{*}]\geq 0$$

$$\forall(\hat{q}^{2},\tilde{q}^{4},\rho_3)\in\overline{\mathbb{K}}, \tag{45}$$

where

$$\overline{\mathbb{K}}:=\mathbb{R}_{+}^{K[N+MU+1]} \text{ and } U=\sum_{j=1}^{M}U_j. \tag{46}$$

The variational inequality (45) represents the equilibrium for all demand markets simultaneously. The solution gives the optimal amount of product that each demand market $k=1,\dots,K$ has to buy from manufacturers ($\hat{q}^{2*}$) and retailers through each shipping method ($\tilde{q}^{4*}$). Moreover, the solution gives the optimal price that each demand market is willing to pay (ρ_3^{*}), in equilibrium.

4. Unique variational formulation

In this Section we provide a unique variational formulation for the entire network.

Since the quantity of product sold by a manufacturer i (through the v_i shipping method) to a retailer j must be equal to the quantity that j buys from i, the optimal solution $q_{ij}^{v_i*}$ to variational inequality (28) must be the same of variational inequality (38). Analogously, it happens for the optimal quantities of product sold from manufacturers to demand markets (and, hence, bought by demand markets from manufacturers), that is the optimal solutions $\hat{q}_{ik}^{*}$ in (28) and (45), and the optimal solutions $q_{jk}^{u_j*}$ in (38) and (45).

Therefore, we can obtain the optimal solutions, for the entire network, by solving the variational inequality given by the sum of the three variational inequalities (28), (38) and (45), in order to formalize the agreements between the tiers. We state formally this concept in the following definition.

Definition 4.1. An equilibrium state of the presented network is reached if the flows between the tiers of the network coincide and the product shipments and prices satisfy the sum of the optimality conditions (28), (38) and (45).

The following result represents a variational inequality formulation of the governing equilibrium conditions according to Definition 4.1 (see [15] for the proof).

Theorem 4.1. *A vector $(q^{1*}, \hat{q}^{2*}, q^{3*}, \tilde{q}^{4*}, \mathcal{E}^*, \rho_3^*) \in \mathcal{K}$ is an optimal solution to the complete problem of the network if and only if such a vector is a solution to the variational inequality:*
Find $(q^{1*}, \hat{q}^{2*}, q^{3*}, \tilde{q}^{4*}, \mathcal{E}^*, \rho_3^*) \in \mathcal{K}$ such that:

$$
\begin{aligned}
&\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{v_i=1}^{V_i}\left[\frac{\partial c_i(q_i^{1*}, \hat{q}_i^{2*})}{\partial q_{ij}^{v_i}} + \frac{\partial c_{ij}^{v_i}(q_{ij}^{v_i *})}{\partial q_{ij}^{v_i}} + \delta_i(c^2 + \beta_i)\right. \\
&\qquad \left. +\alpha_i \frac{\partial e_i(q_i^{1*}, \hat{q}_i^{2*})}{\partial q_{ij}^{v_i}} + \alpha_i \frac{\partial e_{ij}^{v_i}(q_{ij}^{v_i *})}{\partial q_{ij}^{v_i}} + \frac{\partial c_j(q^{1*})}{\partial q_{ij}^{v_i}} + \frac{\partial \tilde{c}_{ij}^{v_i}(q_{ij}^{v_i *})}{\partial q_{ij}^{v_i}}\right] \times (q_{ij}^{v_i} - q_{ij}^{v_i *}) \\
&+ \sum_{i=1}^{N}\sum_{k=1}^{K}\left[\frac{\partial c_i(q_i^{1*}, \hat{q}_i^{2*})}{\partial \hat{q}_{ik}} + \frac{\partial c_{ik}(\hat{q}_{ik}^*)}{\partial \hat{q}_{ik}} + \frac{\partial \hat{c}_{ik}(\hat{q}_{ik}^*)}{\partial \hat{q}_{ik}} + \delta_i(c^2 + \beta_i) + \alpha_i \frac{\partial e_i(q_i^{1*}, \hat{q}_i^{2*})}{\partial \hat{q}_{ik}}\right. \\
&\qquad \left. +\alpha_i \frac{\partial e_{ik}(\hat{q}_{ik}^*)}{\partial \hat{q}_{ik}} + \hat{c}_{ik}(\hat{q}^{2*}, \tilde{q}^{4*}) + a_{ik}(\hat{q}_{ik}^*) - \rho_{3k}^*\right] \times (\hat{q}_{ik} - \hat{q}_{ik}^*) \\
&+ \sum_{i=1}^{N}\sum_{l=1}^{L}\left[c^1 - (c^2 + \beta_i)\right] \times (q_{il} - q_{il}^*) \\
&+ \sum_{j=1}^{M}\sum_{k=1}^{K}\sum_{u_j=1}^{U_j}\left[\frac{\partial c_{jk}^{u_j}(\tilde{q}_{jk}^{u_j *})}{\partial \tilde{q}_{jk}^{u_j}} + \tilde{\alpha}_j \frac{\partial e_{jk}^{u_j}(\tilde{q}_{jk}^{u_j *})}{\partial \tilde{q}_{jk}^{u_j}}\right. \\
&\qquad \left. + \hat{c}_{jk}^{u_j}(\hat{q}^{2*}, \tilde{q}^{4*}) + a_{jk}^{u_j}(\tilde{q}_{jk}^{u_j *}) - \rho_{3k}^*\right] \times (\tilde{q}_{jk}^{u_j} - \tilde{q}_{jk}^{u_j *}) \\
&+ \sum_{i=1}^{N}\left[\frac{\partial \gamma_i(\mathcal{E}_i^*)}{\partial \mathcal{E}_i} - f\right] \times (\mathcal{E}_i - \mathcal{E}_i^*) \\
&+ \sum_{k=1}^{K}\left[\sum_{i=1}^{N} q_{ik}^* + \sum_{j=1}^{M}\sum_{u_j=1}^{U_j} q_{jk}^{u_j *} - d_k(\rho_3^*)\right] \times (\rho_{3k} - \rho_{3k}^*) \geq 0, \\
&\qquad\qquad \forall (q^1, \hat{q}^2, q^3, \tilde{q}^4, \mathcal{E}, \rho_3) \in \mathcal{K},
\end{aligned} \tag{47}
$$

where the feasible set

$$
\mathcal{K} := \mathbb{K} \cap \tilde{\mathbb{K}} \cap \overline{\mathbb{K}} \subseteq \mathbb{R}_+^{NMV+NK+NL+MKU+N+K}. \tag{48}
$$

We now prove that, under some assumptions, variational inequality (47) represents the optimality conditions for all decision-makers of the whole supply chain network, that is, conditions (28), (38) and (45) are all satisfied.

Theorem 4.2. *If a vector $(q^{1*}, \hat{q}^{2*}, q^{3*}, \tilde{q}^{4*}, \mathcal{E}^*, \rho_3^*) \in \mathcal{K}$ is an optimal solution to the system of variational inequalities (28), (38) and (45) then it is a solution to variational* *inequality* (47)*. Moreover, a solution to variational inequality* (47) *is also a solution to the system of variational inequalities* (28)*,* (38) *and* (45) *if the following conditions hold*

$$-\frac{\partial c_j(q^{1*})}{\partial q_{ij}^{v_i}}-\frac{\partial \tilde{c}_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}} \le \rho_{1ij}^{v_i*} \le \frac{\partial c_i(q_i^{1*},\hat{q}_i^{2*})}{\partial q_{ij}^{v_i}}+\frac{\partial c_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}}+\delta_i(c^2+\beta_i) + \alpha_i\frac{\partial e_i(q_i^{1*},\hat{q}_i^{2*})}{\partial q_{ij}^{v_i}}+\alpha_i\frac{\partial e_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}}, \tag{49}$$

$$\rho_{3k}^*-\hat{c}_{ik}(\hat{q}^{2*},\tilde{q}^{4*})-a_{ik}(\hat{q}_{ik}^*) \le \rho_{1ik}^* \le \frac{\partial c_i(q_i^{1*},\hat{q}_i^{2*})}{\partial \hat{q}_{ik}}+\frac{\partial c_{ik}(\hat{q}_{ik}^*)}{\partial \hat{q}_{ik}}+\frac{\partial \hat{c}_{ik}(\hat{q}_{ik}^*)}{\partial \hat{q}_{ik}}+\delta_i(c^2+\beta_i) +\alpha_i\frac{\partial e_i(q_i^{1*},\hat{q}_i^{2*})}{\partial \hat{q}_{ik}}+\alpha_i\frac{\partial e_{ik}(\hat{q}_{ik}^*)}{\partial \hat{q}_{ik}} \tag{50}$$

$$\rho_{3k}^*-\hat{c}_{jk}^{u_j}(\hat{q}^{2*},\tilde{q}^{4*})-a_{jk}^{u_j}(\tilde{q}_{jk}^{u_j*}) \le \rho_{2j}^* \le \frac{\partial c_{jk}^{u_j}(\tilde{q}_{jk}^{u_j*})}{\partial \tilde{q}_{jk}^{u_j}}+\tilde{\alpha}_j\frac{\partial e_{jk}^{u_j}(\tilde{q}_{jk}^{u_j*})}{\partial \tilde{q}_{jk}^{u_j}}. \tag{51}$$

Proof. Let the vector $(q^{1*},\hat{q}^{2*},q^{3*},\tilde{q}^{4*},\mathcal{E}^*,\rho_3^*)\in\mathcal{K}$ be a solution to the system of variational inequalities (28), (38) and (45), then such a vector also satisfies variational inequality (47) (since the latter is the sum of the previous ones). Hence the necessary condition is proved.

Let $(q^{1*},\hat{q}^{2*},q^{3*},\tilde{q}^{4*},\mathcal{E}^*,\rho_3^*)\in\mathcal{K}$ be a solution to variational inequality (47). In order to prove the sufficient condition, observe that (47) holds $\forall(q_{ij}^{v_i},\hat{q}^2,q^3,\tilde{q}^4,\mathcal{E},\rho_3)\in\mathcal{K}$. If we let $(\hat{q}^2,q^3,\tilde{q}^4,\mathcal{E},\rho_3)\equiv(\hat{q}^{2*},q^{3*},\tilde{q}^{4*},\mathcal{E}^*,\rho_3^*)$ and substitute into (47) the resulting inequality is equivalent to the statement that: For all i, j, v_i, in equilibrium, we must have that:

$$\frac{\partial c_i(q_i^{1*},\hat{q}_i^{2*})}{\partial q_{ij}^{v_i}}+\frac{\partial c_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}}+\delta_i(c^2+\beta_i)+\alpha_i\frac{\partial e_i(q_i^{1*},\hat{q}_i^{2*})}{\partial q_{ij}^{v_i}} +\alpha_i\frac{\partial e_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}}\begin{cases}\ge -\dfrac{\partial c_j(q^{1*})}{\partial q_{ij}^{v_i}}-\dfrac{\partial \tilde{c}_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}} & \text{if } q_{ij}^{v_i*}=0\\ = -\dfrac{\partial c_j(q^{1*})}{\partial q_{ij}^{v_i}}-\dfrac{\partial \tilde{c}_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}} & \text{if } q_{ij}^{v_i*}>0.\end{cases} \tag{52}$$

Under hypothesis (49) and condition (52), we obtain that:
if $q_{ij}^{v_i*}>0$

$$\frac{\partial c_i(q_i^{1*},\hat{q}_i^{2*})}{\partial q_{ij}^{v_i}}+\frac{\partial c_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}}+\delta_i(c^2+\beta_i)+\alpha_i\frac{\partial e_i(q_i^{1*},\hat{q}_i^{2*})}{\partial q_{ij}^{v_i}}+\alpha_i\frac{\partial e_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}}=\rho_{1ij}^{v_i*} \text{ and} \tag{53}$$

$$\frac{\partial c_j(q^{1*})}{\partial q_{ij}^{v_i}}+\frac{\partial \tilde{c}_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}}=-\rho_{1ij}^{v_i*}, \tag{54}$$

if $q_{ij}^{v_i*} = 0$

$$\frac{\partial c_i(q_i^{1*}, \hat{q}_i^{2*})}{\partial q_{ij}^{v_i}} + \frac{\partial c_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}} + \delta_i(c^2 + \beta_i) + \alpha_i \frac{\partial e_i(q_i^{1*}, \hat{q}_i^{2*})}{\partial q_{ij}^{v_i}} + \alpha_i \frac{\partial e_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}} \geq \rho_{1ij}^{v_i*} \text{ and} \tag{55}$$

$$\frac{\partial c_j(q^{1*})}{\partial q_{ij}^{v_i}} + \frac{\partial \tilde{c}_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}} \geq -\rho_{1ij}^{v_i*}. \tag{56}$$

Conditions (53) and (55) mean that:

$$\begin{aligned} &\frac{\partial c_i(q_i^{1*}, \hat{q}_i^{2*})}{\partial q_{ij}^{v_i}} + \frac{\partial c_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}} + \delta_i(c^2 + \beta_i) + \alpha_i \frac{\partial e_i(q_i^{1*}, \hat{q}_i^{2*})}{\partial q_{ij}^{v_i}} \\ &\quad + \alpha_i \frac{\partial e_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}} \begin{cases} \geq \rho_{ij}^{v_i*} & \text{if } q_{ij}^{v_i*} = 0 \\ = \rho_{ij}^{v_i*} & \text{if } q_{ij}^{v_i*} > 0, \end{cases} \end{aligned} \tag{57}$$

that is:

$$\begin{aligned} &\left[\frac{\partial c_i(q_i^{1*}, \hat{q}_i^{2*})}{\partial q_{ij}^{v_i}} + \frac{\partial c_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}} + \delta_i(c^2 + \beta_i) + \alpha_i \frac{\partial e_i(q_i^{1*}, \hat{q}_i^{2*})}{\partial q_{ij}^{v_i}} \right. \\ &\quad \left. + \alpha_i \frac{\partial e_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}} - \rho_{ij}^{v_i*} \right] (q_{ij}^{v_i} - q_{ij}^{v_i*}) \geq 0 \quad \forall i, j, v_i; \end{aligned} \tag{58}$$

while conditions (54) and (56) are equivalent to the following:

$$\frac{\partial c_j(q^{1*})}{\partial q_{ij}^{v_i}} + \frac{\partial \tilde{c}_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}} \begin{cases} \geq -\rho_{ij}^{v_i*} & \text{if } q_{ij}^{v_i*} = 0 \\ = -\rho_{ij}^{v_i*} & \text{if } q_{ij}^{v_i*} > 0, \end{cases} \tag{59}$$

that is:

$$\left[\frac{\partial c_j(q^{1*})}{\partial q_{ij}^{v_i}} + \frac{\partial \tilde{c}_{ij}^{v_i}(q_{ij}^{v_i*})}{\partial q_{ij}^{v_i}} + \rho_{ij}^{v_i*} \right] (q_{ij}^{v_i} - q_{ij}^{v_i*}) \geq 0 \quad \forall i, j, v_i. \tag{60}$$

Therefore, the first row of (28) and (38) are non-negative.
If we now let $(q_{ij}^{v_i}, q^3, \tilde{q}^4, \mathcal{E}, \rho_3) \equiv (q_{ij}^{v_i*}, q^{3*}, \tilde{q}^{4*}, \mathcal{E}^*, \rho_3^*)$ and substitute into (47) the resulting inequality is equivalent to the statement that: For all i, k, in equilibrium, we must have that:

$$\begin{aligned} &\frac{\partial c_i(q_i^{1*}, \hat{q}_i^{2*})}{\partial \hat{q}_{ik}} + \frac{\partial c_{ik}(\hat{q}_{ik}^*)}{\partial \hat{q}_{ik}} + \frac{\partial \hat{c}_{ik}(\hat{q}_{ik}^*)}{\partial \hat{q}_{ik}} + \delta_i(c^2 + \beta_i) + \alpha_i \frac{\partial e_i(q_i^{1*}, \hat{q}_i^{2*})}{\partial \hat{q}_{ik}} \\ &\quad + \alpha_i \frac{\partial e_{ik}(\hat{q}_{ik}^*)}{\partial \hat{q}_{ik}} \begin{cases} \geq -\hat{c}_{ik}(\hat{q}^{2*}, \tilde{q}^{4*}) - a_{ik}(\hat{q}_{ik}^*) + \rho_{3k}^* & \text{if } \hat{q}_{ik}^* = 0 \\ = -\hat{c}_{ik}(\hat{q}^{2*}, \tilde{q}^{4*}) - a_{ik}(\hat{q}_{ik}^*) + \rho_{3k}^* & \text{if } \hat{q}_{ik}^* > 0. \end{cases} \end{aligned} \tag{61}$$

Under hypothesis (50) and condition (61), we obtain the following:

$$\frac{\partial c_i(q_i^{1*}, \hat{q}_i^{2*})}{\partial \hat{q}_{ik}} + \frac{\partial c_{ik}(\hat{q}_{ik}^*)}{\partial \hat{q}_{ik}} + \frac{\partial \hat{c}_{ik}(\hat{q}_{ik}^*)}{\partial \hat{q}_{ik}} + \delta_i(c^2 + \beta_i) + \alpha_i \frac{\partial e_i(q_i^{1*}, \hat{q}_i^{2*})}{\partial \hat{q}_{ik}} + \alpha_i \frac{\partial e_{ik}(\hat{q}_{ik}^*)}{\partial \hat{q}_{ik}} \begin{cases} \geq \rho_{1ik}^* & \text{if } \hat{q}_{ik}^* = 0 \\ = \rho_{1ik}^* & \text{if } \hat{q}_{ik}^* > 0. \end{cases} \tag{62}$$

$$\hat{c}_{ik}(\hat{q}^{2*}, \tilde{q}^{4*}) + a_{ik}(\hat{q}_{ik}^*) - \rho_{3k}^* \begin{cases} \geq -\rho_{1ik}^* & \text{if } \hat{q}_{ik}^* = 0 \\ = -\rho_{1ik}^* & \text{if } \hat{q}_{ik}^* > 0, \end{cases} \tag{63}$$

that is:

$$\left[\frac{\partial c_i(q_i^{1*}, \hat{q}_i^{2*})}{\partial \hat{q}_{ik}} + \frac{\partial c_{ik}(\hat{q}_{ik}^*)}{\partial \hat{q}_{ik}} + \frac{\partial \hat{c}_{ik}(\hat{q}_{ik}^*)}{\partial \hat{q}_{ik}} + \delta_i(c^2 + \beta_i) + \alpha_i \frac{\partial e_i(q_i^{1*}, \hat{q}_i^{2*})}{\partial \hat{q}_{ik}} + \alpha_i \frac{\partial e_{ik}(\hat{q}_{ik}^*)}{\partial \hat{q}_{ik}} - \rho_{1ik}^*\right] (\hat{q}_{ik} - \hat{q}_{ik}^*) \geq 0 \quad \forall i, k, \tag{64}$$

$$\left[\hat{c}_{ik}(\hat{q}^{2*}, \tilde{q}^{4*}) + a_{ik}(\hat{q}_{ik}^*) - \rho_{3k}^* + \rho_{1ik}^*\right] (\hat{q}_{ik} - \hat{q}_{ik}^*) \geq 0 \quad \forall i, k. \tag{65}$$

Therefore, the second row of (28) and the first row of (45) are non-negative.
If we now let $(q_{ij}^{v_i}, \hat{q}^2, \tilde{q}^4, \mathcal{E}, \rho_3) \equiv (q_{ij}^{v_i *}, \hat{q}^{2*}, \tilde{q}^{4*}, \mathcal{E}^*, \rho_3^*)$ and substitute into (47) the resulting inequality is equivalent to the statement that: For all i, l, in equilibrium, we must have that:

$$c^1 - (c^2 + \beta_i) \begin{cases} \geq 0 & \text{if } q_{il}^* = 0 \\ = 0 & \text{if } q_{il}^* > 0, \end{cases} \tag{66}$$

that is:

$$\left[c^1 - (c^2 + \beta_i)\right] (q_{il} - q_{il}^*) \geq 0 \quad \forall i, l, \tag{67}$$

hence, the third row of (28) is non-negative.
Now, we let $(q_{ij}^{v_i}, \hat{q}^2, q^3, \mathcal{E}, \rho_3) \equiv (q_{ij}^{v_i *}, \hat{q}^{2*}, q^{3*}, \mathcal{E}^*, \rho_3^*)$ and substitute into (47) the resulting inequality is equivalent to the statement that: For all j, k, u_j, in equilibrium, we must have that:

$$\frac{\partial c_{jk}^{u_j}(\tilde{q}_{jk}^{u_j *})}{\partial \tilde{q}_{jk}^{u_j}} + \tilde{\alpha}_j \frac{\partial e_{jk}^{u_j}(\tilde{q}_{jk}^{u_j *})}{\partial \tilde{q}_{jk}^{u_j}} \begin{cases} \geq -\hat{c}_{jk}^{u_j}(\hat{q}^{2*}, \tilde{q}^{4*}) - a_{jk}^{u_j}(\tilde{q}_{jk}^{u_j *}) + \rho_{3k}^* & \text{if } \tilde{q}_{jk}^{u_j *} = 0 \\ = -\hat{c}_{jk}^{u_j}(\hat{q}^{2*}, \tilde{q}^{4*}) - a_{jk}^{u_j}(\tilde{q}_{jk}^{u_j *}) + \rho_{3k}^* & \text{if } \tilde{q}_{jk}^{u_j *} > 0. \end{cases} \tag{68}$$

Under hypothesis (51) and condition (68), we obtain that:

$$\frac{\partial c_{jk}^{u_j}(\tilde{q}_{jk}^{u_j *})}{\partial \tilde{q}_{jk}^{u_j}} + \tilde{\alpha}_j \frac{\partial e_{jk}^{u_j}(\tilde{q}_{jk}^{u_j *})}{\partial \tilde{q}_{jk}^{u_j}} \begin{cases} \geq \rho_{2j}^* & \text{if } \tilde{q}_{jk}^{u_j *} = 0 \\ = \rho_{2j}^* & \text{if } \tilde{q}_{jk}^{u_j *} > 0, \end{cases} \tag{69}$$

$$\hat{c}_{jk}^{u_j}(\hat{q}^{2*}, \tilde{q}^{4*}) + a_{jk}^{u_j}(\tilde{q}_{jk}^{u_j*}) - \rho_{3k}^* \begin{cases} \geq -\rho_{2j}^* & \text{if } \tilde{q}_{jk}^{u_j*} = 0 \\ = -\rho_{2j}^* & \text{if } \tilde{q}_{jk}^{u_j*} > 0, \end{cases} \tag{70}$$

that is:

$$\left[\frac{\partial c_{jk}^{u_j}(\tilde{q}_{jk}^{u_j*})}{\partial \tilde{q}_{jk}^{u_j}} + \tilde{\alpha}_j \frac{\partial e_{jk}^{u_j}(\tilde{q}_{jk}^{u_j*})}{\partial \tilde{q}_{jk}^{u_j}} - \rho_{2j}^* \right] (\tilde{q}_{jk}^{u_j} - \tilde{q}_{jk}^{u_j*}) \geq 0 \quad \forall j, k, u_j \text{ and} \tag{71}$$

$$\left[\hat{c}_{jk}^{u_j}(\hat{q}^{2*}, \tilde{q}^{4*}) + a_{jk}^{u_j}(\tilde{q}_{jk}^{u_j*}) - \rho_{3k}^* + \rho_{2j}^* \right] (\tilde{q}_{jk}^{u_j} - \tilde{q}_{jk}^{u_j*}) \geq 0 \quad \forall j, k, u_j. \tag{72}$$

Therefore, the second row of (38) and (45) are non-negative.
If we let $(q_{ij}^{v_i}, \hat{q}^2, q^3, \tilde{q}^4, \rho_3) \equiv (q_{ij}^{v_i*}, \hat{q}^{2*}, q^{3*}, \tilde{q}^{4*}, \rho_3^*)$ and substitute into (47) the resulting inequality is equivalent to the statement that: For all i, in equilibrium, we must have that:

$$\frac{\partial \gamma_i(\mathcal{E}_i^*)}{\partial \mathcal{E}_i} - f \begin{cases} \geq 0 & \text{if } \mathcal{E}_i^* = 0 \\ = 0 & \text{if } \mathcal{E}_i^* > 0, \end{cases} \tag{73}$$

that is:

$$\left[\frac{\partial \gamma_i(\mathcal{E}_i^*)}{\partial \mathcal{E}_i} - f \right] (\mathcal{E}_i - \mathcal{E}_i^*) \geq 0 \quad \forall i. \tag{74}$$

Hence, the fourth row of (28) is non-negative.
Finally, if we let $(q_{ij}^{v_i}, \hat{q}^2, q^3, \tilde{q}^4, \mathcal{E}) \equiv (q_{ij}^{v_i*}, \hat{q}^{2*}, q^{3*}, \tilde{q}^{4*}, \mathcal{E}^*)$ and substitute into (47) the resulting inequality is equivalent to the statement that: For all k, in equilibrium, we must have that:

$$\sum_{i=1}^{N} \hat{q}_{ik}^* + \sum_{j=1}^{M} \sum_{u_j=1}^{U_j} q_{jk}^{u_j*} - d_k(\rho_3^*) \begin{cases} \geq 0 & \text{if } \rho_{3k}^* = 0 \\ = 0 & \text{if } \rho_{3k}^* > 0, \end{cases} \tag{75}$$

that is:

$$\left[\sum_{i=1}^{N} \hat{q}_{ik}^* + \sum_{j=1}^{M} \sum_{u_j=1}^{U_j} q_{jk}^{u_j*} - d_k(\rho_3^*) \right] (\rho_{3k} - \rho_{3k}^*) \geq 0 \quad \forall k. \tag{76}$$

Therefore, the third row of (45) is non-negative.
Summing up (58), (64), (67) and (74), we obtain variational inequality (28). Summing up (60) and (71), we obtain variational inequality (38). Summing up (65), (72) and (76), we obtain variational inequality (45).
Hence, also the sufficient condition is proved. □

Note that Theorem 4.2 also clarifies how the prices $\rho_{1ij}^{v_i*}$, ρ_{1ik}^* and ρ_{2j}^* (which are endogenous parameters) are determined, especially when the flows between layers are (strictly) positive. Indeed, we have that, according to (69), if the product shipment transacted between a retailer and a demand market (through a shipment method) is positive, then the marginal transaction costs plus the marginal environmental emission (multiplied by the parameter that allows us to express environmental emissions in terms of costs) is equal to the price of the product associated to the retailer. If the sum of all those marginal cost and emission exceeds the price, then there will be a zero amount of the product transacted between that retailer and demand market pair and via that shipment method. Similarly, according to (70), we have that consumers at demand market k will purchase the product from retailer j, transacted via shipment method u_j, if the price that the consumers are willing to pay for the product minus the transaction cost (from the perspective of the consumers) and the aversion in buying from retailer j is equal to the price charged by the retailer (or, equivalently, the price that the consumers are willing equals the price charged by the retailer plus the transaction cost and the aversion); otherwise there is no amount of product transacted between that retailer and demand market pair. Therefore, in accordance with assumption (51) (and condition (68)), if the product shipment transacted between a retailer and a demand market (through a shipment method) is positive, the price charged by that retailer is given by:

$$\rho_{2j}^* = \frac{\partial c_{jk}^{u_j}(\tilde{q}_{jk}^{u_j*})}{\partial \tilde{q}_{jk}^{u_j}} + \tilde{\alpha}_j \frac{\partial e_{jk}^{u_j}(\tilde{q}_{jk}^{u_j*})}{\partial \tilde{q}_{jk}^{u_j}} = \rho_{3k}^* - \hat{c}_{jk}^{u_j}(\hat{q}^{2*}, \tilde{q}^{4*}) - a_{jk}^{u_j}(\tilde{q}_{jk}^{u_j*}).$$

Analogous interpretations could be given, in accordance with assumptions (49) and (50), to the prices $\rho_{1ij}^{v_i*}$ and ρ_{1ik}^*.

Therefore, under assumptions (49), (50) and (51), variational inequality (47) represents the optimality conditions for all decision-makers of the whole supply chain network, simultaneously. The solutions of (47) give:

- the optimal amount of product all manufacturers have to sell to all retailers (q^{1*}), that clearly is also the optimal amount all retailers have to buy from all manufacturers;
- the optimal amount of product all manufacturers have to sell to all demand markets, and that all consumers (at demand markets) have to buy from all manufacturers, through each shipping method (q^{2*});
- the optimal amount of waste production disposed in eco-sustainable way, for all manufacturers (q^{3*});
- the optimal amount of product all the retailers have to sell to all demand markets, that clearly is also the optimal amount all consumers have to buy from all retailers ($\tilde{q}^{4*}$);
- the energy level that each manufacturer has to reach ($\mathcal{E}^*$);
- the optimal price that each demand market is willing to pay (ρ_3^*), in equilibrium.

Variational inequality (47) can be put in standard form, as follows:
find $X \in \mathcal{K}$ such that

$$\langle F(X), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{77}$$

where:

- $F(X) = (F^z(X))_{z=1,\dots,6}$ is a vector function with

$$F^{1,v_i}_{ij}(X) := \frac{\partial c_i(q^1_i, q^2_i)}{\partial q^{v_i}_{ij}} + \frac{\partial c^{v_i}_{ij}(q^{v_i}_{ij})}{\partial q^{v_i}_{ij}} + \delta_i(c^2 + \beta_i) + \alpha_i \frac{\partial e_i(q^1_i, q^2_i)}{\partial q^{v_i}_{ij}}$$
$$+ \alpha_i \frac{\partial e^{v_i}_{ij}(q^{v_i}_{ij})}{\partial q^{v_i}_{ij}} + \frac{\partial c_j(q^1)}{\partial q^{v_i}_{ij}} + \frac{\partial \tilde{c}^{v_i}_{ij}(q^{v_i}_{ij})}{\partial q^{v_i}_{ij}}, \quad \forall i,\ j,\ v_i,$$

representing the (i, j, v_i)-th component of $F^1(X)$,

$$F^2_{ik}(X) := \frac{\partial c_i(q^1_i, q^2_i)}{\partial q_{ik}} + \frac{\partial c_{ik}(q_{ik})}{\partial q_{ik}} + \frac{\partial \hat{c}_{ik}(q_{ik})}{\partial q_{ik}} + \delta_i(c^2 + \beta_i) + \alpha_i \frac{\partial e_i(q^1_i, q^2_i)}{\partial q_{ik}}$$
$$+ \alpha_i \frac{\partial e_{ik}(q_{ik})}{\partial q_{ik}} + \hat{c}_{ik}(q^2, \tilde{q}^4) + a_{ik}(q_{ik}) - \rho_{3k}, \quad \forall i,\ k,$$

representing the (i, k)-th component of $F^2(X)$,

$$F^3_{il}(X) := c^1 - (c^2 + \beta_i), \quad \forall i,\ l,$$

representing the i-th component of $F^3(X)$,

$$F^{4,u_j}_{jk}(X) := \frac{\partial c^{u_j}_{jk}(\tilde{q}^{u_j}_{jk})}{\partial \tilde{q}^{u_j}_{jk}} + \tilde{\alpha}_j \frac{\partial e^{u_j}_{jk}(\tilde{q}^{u_j}_{jk})}{\partial \tilde{q}^{u_j}_{jk}}$$
$$+ \hat{c}^{u_j}_{jk}(q^2, \tilde{q}^4) + a^{u_j}_{jk}(\tilde{q}^{u_j}_{jk}) - \rho_{3k}, \quad \forall j,\ k,\ u_j$$

representing the (j, k, u_j)-th component of $F^4(X)$,

$$F^5_i(X) := \frac{\partial \gamma_i(\mathcal{E}_i)}{\partial \mathcal{E}_i} - f, \quad \forall i,$$

representing the i-component of $F^5(X)$ and

$$F^6_k(X); = \sum_{i=1}^{N} q_{ik} + \sum_{j=1}^{M} \sum_{u_j=1}^{U_j} q^{u_j}_{jk} - d_k(\rho_3), \quad \forall k,$$

representing the k-th component of $F^6(X)$;

- X the $NMV + NK + NL + MKU + N + K$-dimensional vector given by $X \equiv (q^1, q^2, q^3, \tilde{q}^4, \mathcal{E}, \rho_3)$;
- $\mathcal{K} \equiv K \subseteq \mathbb{R}_+^{NMV+NK+NL+MKU+N+K}$ the feasible set.

We now provide the following theorem, representing an existence and uniqueness result for the solution to variational inequality (47), or variational inequality (77), for whose proof we refer to [8] or [11].

Theorem 4.3 *(Existence and uniqueness). Under the Assumption 3.1, variational inequality (47), or equivalently (77), admits at least one solution. Moreover, if the operator of the variational inequality (77) is strictly monotone, that is:*

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, \quad X^1 \neq X^2,$$

then the solution $(q^{1*}, q^{2*}, q^{3*}, \tilde{q}^{4*}, \mathcal{E}^*, \rho_3^*) \in \mathcal{K}$ to variational inequality (77) is unique.

We conclude the analysis of qualitative properties of variational inequality (47), or equivalently variational inequality (77), by providing the following result that constitutes a sufficient condition to the strict monotonicity of function F.

Theorem 4.4. *Suppose that, for each manufacturer i the cost functions c_i and the emission function e_i are additive, that is*

$$\begin{aligned} c_i(Q) &= c_i^1(Q_i) + c_i^2(\underline{Q}_i), \\ e_i(Q) &= e_i^1(Q_i) + e_i^2(\underline{Q_i}), \end{aligned} \tag{78}$$

where $\underline{Q}_i = (Q_1, \ldots, Q_{i-1}, Q_{i+1}, \ldots, Q_m)$ and that c_i^1, e_i^1 are families of strictly convex functions. Moreover, we suppose that, for each manufacturer i, for each retailer j and for each demand market k, $c_{ij}^{v_i}$, $e_{ij}^{v_i}$, c_{ik}, e_{ik}, γ_i, c_j, $\tilde{c}_{ij}^{v_i}$, $c_{jk}^{u_j}$, $e_{jk}^{u_j}$ are families of strictly convex functions and, finally, that $\hat{c}_{ik}$, a_{ik}, $\hat{c}_{jk}^{u_j}$ and $a_{jk}^{u_j}$ are families of strictly monotone increasing functions and d_k is a family of strictly monotone decreasing functions. Then, the operator F of the variational inequality (77) is strictly monotone.

Proof. Let X^a, $X^b \in \mathcal{K}$ be two feasible vectors such that $X^a \neq X^b$. We evaluate the quantity

$$\begin{aligned} &\langle F(X^a) - F(X^b), X^a - X^b \rangle = \\ &\sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{v_i=1}^{V_i} \left[\frac{\partial c_i(q_i^{1,a}, \hat{q}_i^{2,a})}{\partial q_{ij}^{v_i}} - \frac{\partial c_i(q_i^{1,b}, \hat{q}_i^{2,b})}{\partial q_{ij}^{v_i}} \right] \times (q_{ij}^{v_i,a} - q_{ij}^{v_i,b}) \\ &+ \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{v_i=1}^{V_i} \left[\frac{\partial c_{ij}^{v_i}(q_{ij}^{v_i,a})}{\partial q_{ij}^{v_i}} - \frac{\partial c_{ij}^{v_i}(q_{ij}^{v_i,b})}{\partial q_{ij}^{v_i}} \right] \times (q_{ij}^{v_i,a} - q_{ij}^{v_i,b}) \end{aligned}$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{v_i=1}^{V_i}\left[\alpha_i\left(\frac{\partial e_i(q_i^{1,a},\hat{q}_i^{2,a})}{\partial q_{ij}^{v_i}}-\frac{\partial e_i(q_i^{1,b},\hat{q}_i^{2,b})}{\partial q_{ij}^{v_i}}\right)\right]\times(q_{ij}^{v_i,a}-q_{ij}^{v_i,b})$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{v_i=1}^{V_i}\left[\alpha_i\left(\frac{\partial e_{ij}^{v_i}(q_{ij}^{v_i,a})}{\partial q_{ij}^{v_i}}-\frac{\partial e_{ij}^{v_i}(q_{ij}^{v_i,b})}{\partial q_{ij}^{v_i}}\right)\right]\times(q_{ij}^{v_i,a}-q_{ij}^{v_i,b})$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{v_i=1}^{V_i}\left[\frac{\partial c_j(q^{1,a})}{\partial q_{ij}^{v_i}}-\frac{\partial c_j(q^{1,b})}{\partial q_{ij}^{v_i}}\right]\times(q_{ij}^{v_i,a}-q_{ij}^{v_i,b})$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{v_i=1}^{V_i}\left[\frac{\partial \tilde{c}_{ij}^{v_i}(q_{ij}^{v_i,a})}{\partial q_{ij}^{v_i}}-\frac{\partial \tilde{c}_{ij}^{v_i}(q_{ij}^{v_i,b})}{\partial q_{ij}^{v_i}}\right]\times(q_{ij}^{v_i,a}-q_{ij}^{v_i,b})$$

$$+\sum_{i=1}^{N}\sum_{k=1}^{K}\left[\frac{\partial c_i(q_i^{1,a},\hat{q}_i^{2,a})}{\partial \hat{q}_{ik}}-\frac{\partial c_i(q_i^{1,b},\hat{q}_i^{2,b})}{\partial \hat{q}_{ik}}\right]\times(q_{ik}^{a}-q_{ik}^{b})$$

$$+\sum_{i=1}^{N}\sum_{k=1}^{K}\left[\frac{\partial c_{ik}(\hat{q}_{ik}^{a})}{\partial \hat{q}_{ik}}-\frac{\partial c_{ik}(\hat{q}_{ik}^{b})}{\partial \hat{q}_{ik}}\right]\times(q_{ik}^{a}-q_{ik}^{b})$$

$$+\sum_{i=1}^{N}\sum_{k=1}^{K}\left[\frac{\partial \hat{c}_{ik}(\hat{q}_{ik}^{a})}{\partial \hat{q}_{ik}}-\frac{\partial \hat{c}_{ik}(\hat{q}_{ik}^{b})}{\partial \hat{q}_{ik}}\right]\times(q_{ik}^{a}-q_{ik}^{b})$$

$$+\sum_{i=1}^{N}\sum_{k=1}^{K}\left[\alpha_i\left(\frac{\partial e_i(q_i^{1,a},\hat{q}_i^{2,a})}{\partial \hat{q}_{ik}}-\alpha_i\frac{\partial e_i(q_i^{1,b},\hat{q}_i^{2,b})}{\partial \hat{q}_{ik}}\right)\right]\times(q_{ik}^{a}-q_{ik}^{b})$$

$$+\sum_{i=1}^{N}\sum_{k=1}^{K}\left[\alpha_i\left(\frac{\partial e_{ik}(q_{ik}^{a})}{\partial \hat{q}_{ik}}-\frac{\partial e_{ik}(q_{ik}^{b})}{\partial \hat{q}_{ik}}\right)\right]\times(q_{ik}^{a}-q_{ik}^{b})$$

$$+\sum_{i=1}^{N}\sum_{k=1}^{K}\left[\hat{c}_{ik}(\hat{q}^{2,a},\tilde{q}^{4,a})-\hat{c}_{ik}(\hat{q}^{2,a},\tilde{q}^{4,b})\right]\times(q_{ik}^{a}-q_{ik}^{b})$$

$$+\sum_{i=1}^{N}\sum_{k=1}^{K}\left[a_{ik}(q_{ik}^{a})-a_{ik}(q_{ik}^{b})\right]\times(q_{ik}^{a}-q_{ik}^{b})$$

$$+\sum_{j=1}^{M}\sum_{k=1}^{K}\sum_{u_j=1}^{U_j}\left[\frac{\partial c_{jk}^{u_j}(\tilde{q}_{jk}^{u_j,a})}{\partial \tilde{q}_{jk}^{u_j}}-\frac{\partial c_{jk}^{u_j}(\tilde{q}_{jk}^{u_j,b})}{\partial \tilde{q}_{jk}^{u_j}}\right]\times(\tilde{q}_{jk}^{u_j,a}-\tilde{q}_{jk}^{u_j,b})$$

$$+\sum_{j=1}^{M}\sum_{k=1}^{K}\sum_{u_j=1}^{U_j}\left[\tilde{\alpha}_j\left(\frac{\partial e_{jk}^{u_j}(\tilde{q}_{jk}^{u_j,a})}{\partial \tilde{q}_{jk}^{u_j}}-\tilde{\alpha}_j\frac{\partial e_{jk}^{u_j}(\tilde{q}_{jk}^{u_j,b})}{\partial \tilde{q}_{jk}^{u_j}}\right)\right]\times(\tilde{q}_{jk}^{u_j,a}-\tilde{q}_{jk}^{u_j,b})$$

$$+\sum_{j=1}^{M}\sum_{k=1}^{K}\sum_{u_j=1}^{U_j}\left[\hat{c}_{jk}^{u_j}(\hat{q}^{2,a},\tilde{q}^{4,a})-\hat{c}_{jk}^{u_j}(\hat{q}^{2,b},\tilde{q}^{4,b})\right]\times(\tilde{q}_{jk}^{u_j,a}-\tilde{q}_{jk}^{u_j,b})$$

$$+\sum_{j=1}^{M}\sum_{k=1}^{K}\sum_{u_j=1}^{U_j}\left[a_{jk}^{u_j}(\tilde{q}_{jk}^{u_j,a})-a_{jk}^{u_j}(\tilde{q}_{jk}^{u_j,b})\right]\times(\tilde{q}_{jk}^{u_j,a}-\tilde{q}_{jk}^{u_j,b})$$

$$+\sum_{i=1}^{N}\left[\frac{\partial\gamma_i(\mathcal{E}_i^a)}{\partial\mathcal{E}_i}-\frac{\partial\gamma_i(\mathcal{E}_i^b)}{\partial\mathcal{E}_i}\right]\times(\mathcal{E}_i^a-\mathcal{E}_i^b)$$

$$+\sum_{k=1}^{K}\left[-d_k(\rho_3^a)+d_k(\rho_3^b)\right]\times(\rho_3^a-\rho_3^b). \tag{79}$$

In the previous expression, the nineteen addends are greater than zero, indeed:

- the cost functions c_i and the emission function e_i are additive and, then, the first, the third, the seventh and the tenth addends can be rewritten as:

$$\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{v_i=1}^{V_i}\left[\frac{\partial c_i^1(q_i^{1,a},\hat{q}_i^{2,a})}{\partial q_{ij}^{v_i}}-\frac{\partial c_i^1(q_i^{1,b},\hat{q}_i^{2,b})}{\partial q_{ij}^{v_i}}\right]\times(q_{ij}^{v_i,a}-q_{ij}^{v_i,b}); \tag{80}$$

$$\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{v_i=1}^{V_i}\left[\alpha_i\left(\frac{\partial e_i^1(q_i^{1,a},\hat{q}_i^{2,a})}{\partial q_{ij}^{v_i}}-\frac{\partial e_i^1(q_i^{1,b},\hat{q}_i^{2,b})}{\partial q_{ij}^{v_i}}\right)\right]\times(q_{ij}^{v_i,a}-q_{ij}^{v_i,b}); \tag{81}$$

$$\sum_{i=1}^{N}\sum_{k=1}^{K}\left[\frac{\partial c_i^1(q_i^{1,a},\hat{q}_i^{2,a})}{\partial\hat{q}_{ik}}-\frac{\partial c_i^1(q_i^{1,b},\hat{q}_i^{2,b})}{\partial\hat{q}_{ik}}\right]\times(q_{ik}^a-q_{ik}^b); \tag{82}$$

$$\sum_{i=1}^{N}\sum_{k=1}^{K}\left[\alpha_i\left(\frac{\partial e_i^1(q_i^{1,a},\hat{q}_i^{2,a})}{\partial\hat{q}_{ik}}-\alpha_i\frac{\partial e_i^1(q_i^{1,b},\hat{q}_i^{2,b})}{\partial\hat{q}_{ik}}\right)\right]\times(q_{ik}^a-q_{ik}^b). \tag{83}$$

  Since c_i^1 and e_i^1 are assumed to be strictly convex functions, the expressions (80)-(83) are greater than zero;
- since $c_{ij}^{v_i}$, $e_{ij}^{v_i}$, c_{ik}, e_{ik}, γ_i, c_j, $\tilde{c}_{ij}^{v_i}$, $c_{jk}^{u_j}$, $e_{jk}^{u_j}$ are families of strictly convex functions, the second, fourth, fifth, sixth, eighth, ninth, eleventh, fourteenth, fifteenth and the nineteenth addends are greater than zero;
- the cost functions $\hat{c}_{ik}$, a_{ik}, $\hat{c}_{jk}^{u_j}$, $a_{jk}^{u_j}$ are supposed strictly monotone increasing functions and, therefore, the twelfth, thirteenth, sixteenth, seventeenth addends are greater than zero;
- the request functions d_k are supposed strictly monotone decreasing functions and, therefore, the last addend is greater than zero.

Hence, we can conclude that the expression (79) is greater than zero. □

5. Numerical simulations

In this Section we propose some illustrative simulations in order to highlight key aspects of the optimization model and to validate its effectiveness. We first illustrate the supply chain network topology and the numerical setting. Then, we detail the optimal results and provide their analysis and comparison among the different simulations.

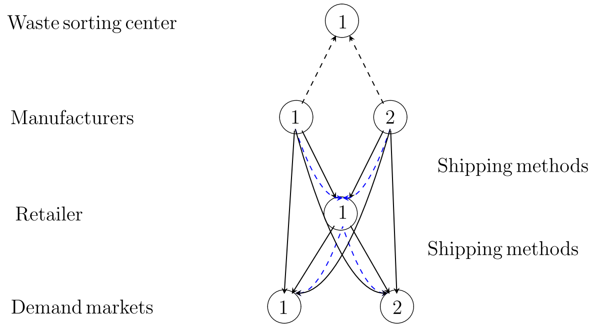

Fig. 3. Supply chain network for the numerical simulations.

5.1. Numerical setting

We report some numerical results for a supply chain network consisting of a waste sorting center, two manufacturers, a retailer and two demand markets, as depicted in Fig. 3. We also take into account two different shipping methods to transport goods between the manufacturers and the retailer and between the retailer and the demand markets. Particularly, we assume that one of the shipping methods is high-emission (that is, the conventional transportation method), while the other one consists in UAVs and, hence, it is environmentally friendly.

We analyze four different simulations, combining the presence or absence of UAVs (as a shipping method) and of waste sorting centers, as follows:

- without UAVs and without waste sorting centers (SIM1);
- with UAVs and without waste sorting centers (SIM2);
- without UAVs and with the waste sorting center (SIM3);
- with UAVs and with the waste sorting center (SIM4).

We assume that the typical cost function c, depending on the variable x, has a general quadratic expression, as follows:

$$c(x) = p_1 x^2 + p_2 x, \tag{84}$$

where $p_1 > 0$ and $p_2 \geq 0$. Observe that in the previous section we assumed that all the cost functions are continuously differentiable and convex and the choice of such an expression and parameters satisfies such assumptions.

The parameters, p_1 and p_2, of the functions used in the numerical simulations are reported in Table 2 and we assumed linear the emission functions. The used values for all the other parameters are shown in Table 3.

Table 2
Function parameters.

Functions		p_1	p_2
$c_i(q_i)$	$i = 1$	0.2	0.2
	$i = 2$	0.2	0.2
$c_{ij}^{v_i}(q_{ij}^{v_i})$	$i = 1, j = 1, v_i = 1$	0.2	0.3
	$i = 1, j = 1, v_i = 2$	0.1	0
	$i = 2, j = 1, v_i = 1$	0.3	0.4
	$i = 2, j = 1, v_i = 2$	0.1	0.05
$c_j(q^1)$	$j = 1$	0.05	0.1
$\tilde{c}_{ij}^{v_i}(q_{ij}^{v_i})$	$i = 1, j = 1, v_i = 1$	0.2	0.1
	$i = 1, j = 1, v_i = 2$	0.1	0
	$i = 2, j = 1, v_i = 1$	0.3	0.1
	$i = 2, j = 1, v_i = 2$	0.1	0
$c_{ik}(\hat{q}_{ik})$	$i = 1, k = 1$	0.1	0.15
	$i = 1, k = 2$	0.1	0.2
	$i = 2, k = 1$	0.15	0.15
	$i = 2, k = 2$	0.15	0.2
$\hat{c}_{ik}(\hat{q}_{ik})$	$i = 1, k = 1$	0.05	0.1
	$i = 1, k = 2$	0.05	0.1
	$i = 2, k = 1$	0.05	0.1
	$i = 2, k = 2$	0.05	0.1
$\hat{c}_{ik}(\hat{q}^2, \tilde{q}^4) = \hat{c}_{ik}(\hat{q}_{ik})$	$i = 1, k = 1$	0.04	0.04
	$i = 1, k = 2$	0.05	0.05
	$i = 2, k = 1$	0.06	0.06
	$i = 2, k = 2$	0.07	0.07
$c_{jk}^{u_j}(q_{jk}^{u_j})$	$j = 1, k = 1, u_j = 1$	0.5	0.5
	$j = 1, k = 1, u_j = 2$	0	0
	$j = 2, k = 1, u_j = 1$	0.6	0.6
	$j = 2, k = 1, u_j = 2$	0.1	0.1
$c_{jk}^{u_j}(\hat{q}^2, \tilde{q}^4)$	$j = 1, k = 1, 2, u_j = 1, 2$	0	0
$\gamma_i(\mathcal{E}_i)$	$i = 1$	0.05	0.05
	$i = 2$	0.1	0.1

5.2. Result analysis

We execute the simulations having the same supply chain network topology, functions and parameters as previously described, and which differ only in the presence or absence of UAVs and waste sorting centers.

The optimal results for all the simulations are computed by solving the variational inequality given in the previous section via the Euler Method (see [7]). We implemented the algorithm in Matlab on an LG laptop with a 12th Gen Intel(R) Core(TM) i7-1260P, 16 GB RAM. The optimal solutions are obtained in less than one second. Simulation SIM1 consists of 12 variables, SIM2 consists of 16 variables, SIM3 consists of 14 variables, while SIM4 consists of 18 variables. The optimal solutions are reported in Table 4.

We first focus our attention on the environmental emissions and transportation costs, and then on the amount of product that each manufacturer disposes in a sustainable way (in the waste sorting centers). Finally, we also analyze the final energy levels.

Table 3
Parameters for numerical simulations.

Parameter		Value
N		2
M		1
K		2
L		1
V_i	$i = 1$	2
	$i = 2$	2
U_j	$j = 1$	2
c^1		0.5
c^2		3
δ_i	$i = 1$	0.2
	$i = 2$	0.1
β_i	$i = 1$	2
	$i = 2$	3
α_i	$i = 1$	0.5
	$i = 2$	0.5
$\mathcal{E}_i$	$i = 1$	2
	$i = 2$	3
f		1
Ξ_i	$i = 1, 2$	100
$\tilde{\Xi}_j$	$j = 1$	100
$\mathcal{E}^M$		10
$\overline{Q}_i$	$i = 1, 2$	100
$\overline{Q}_{ij}$	$i = 1, 2, j = 1$	100
$\tilde{\alpha}_j$	$j = 1$	0.5
μ_k	$k = 1$	2
	$k = 2$	0.2
$\tilde{\mu}_k$	$k = 1$	1
	$k = 2$	0.1
ψ_{ki}	$k = 1, i = 1$	1
	$k = 1, i = 2$	0.5
	$k = 2, i = 1$	1
	$k = 2, i = 2$	0.5
$\tilde{\psi}_{kj}$	$k = 1, j = 1$	1
	$k = 2, j = 1$	0.5

The environmental emissions related to the production of manufacturers is the same for all the simulations, since it depends on the amount of goods produced (or, equivalently, on the amount of goods sent to retailers and demand markets), that does not change. On the contrary, since the used shipping methods could be different in simulations, the environmental emissions of manufacturers and retailers related to the transport of products to retailers and demand markets, respectively, undergo variations on the basis of the related simulation. Particularly, the total environmental emissions for the transmission of goods have higher values in simulations in which UAVs cannot be used (SIM1 and SIM3) and lower values in simulations where UAVs could be used (SIM2

Table 4
Numerical solutions for simulations.

Variables		SIM1	SIM2	SIM3	SIM4
$q_{ij}^{v_i *}$	q_{11}^{1*}	35.07	15.11	35.07	15.11
	q_{11}^{2*}	-	31.47	-	31.47
	q_{21}^{1*}	31.27	11.70	31.27	11.70
	q_{21}^{2*}	-	36.47	-	36.47
$\hat{q}_{ik}^*$	$\hat{q}_{11}^*$	17.73	14.40	17.73	14.40
	$\hat{q}_{12}^*$	47.20	39.02	47.20	39.02
	$\hat{q}_{21}^*$	21.53	16.13	21.53	16.13
	$\hat{q}_{22}^*$	47.20	35.70	47.20	35.70
q_{il}^*	q_{11}^*	-	-	20	20
	q_{21}^*	-	-	25	25
$\tilde{q}_{jk}^{u_j *}$	$\tilde{q}_{11}^{1*}$	66.35	0	66.35	0
	$\tilde{q}_{11}^{2*}$	-	94.75	-	94.75
	$\tilde{q}_{12}^{1*}$	0	0	0	0
	$\tilde{q}_{12}^{2*}$	-	0	-	0
$\mathcal{E}_i^*$	$\mathcal{E}_1^*$	9.50	9.50	9.50	9.50
	$\mathcal{E}_2^*$	4.50	4.50	4.50	4.50
ρ_{3k}^*	ρ_{31}^*	41.13	44.87	41.13	44.87
	ρ_{32}^*	38.84	34.47	38.84	34.47

and SIM4). More specifically, the total environmental emission for the transmission of goods, $e^{(T)} = \sum_{i=1}^{M}\sum_{v_i=1}^{V_i}\sum_{j=1}^{N} e_{ij}^{v_i *}(q_{ij}^{v_i}) + \sum_{i=1}^{M}\sum_{k=1}^{K} e_{ik}(\hat{q}_{ik}^*) + \sum_{j=1}^{N}\sum_{k=1}^{K}\sum_{u_j=1}^{U_j} e_{jk}^{u_j}(\tilde{q}_{jk}^{u_j *})$ is equal to 86.63 kg/km^2 for SIM1 and SIM3 (where only the first conventional shipping method exists), while, it is equal to 47.46 kg/km^2 for SIM2 and SIM4 (where both the shipping methods are used). Therefore, we obtain a percentage variation of -45.22%, that is a reduction of almost half of total emissions due to transport activities.

Furthermore, from the optimal solutions (Table 4), we can also observe that if we can use UAVs (SIM2 and SIM4), they are widely used; especially in the case of transportation between the retailer and the demand markets ($\tilde{q}_{jk}^{u_j}$), when only UAVs are used, while the other conventional method is not used.

The transportation costs of products that the decision makers have to bear, as previously discussed, are functions of the flows of products. Hence, the total transportation cost of products (for the entire supply chain), $c^{(Tr)}$, is given by the sum of the following terms: $c^{(MR)} = \sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{v_i=1}^{V_i} c_{ij}^{v_i}(q_{ij}^{v_i *})$, $\tilde{c}^{(MR)} = \sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{v_i=1}^{V_i} \tilde{c}_{ij}^{v_i}(q_{ij}^{v_i *})$, $c^{(MD)} = \sum_{i=1}^{N}\sum_{k=1}^{K} c_{ik}(\hat{q}_{ik}^*)$, $\hat{c}^{(MD)} = \sum_{i=1}^{N}\sum_{k=1}^{K} \hat{c}_{ik}(\hat{q}^{2*}, \tilde{q}^{4*})$, $c^{(RD)} = \sum_{j=1}^{M}\sum_{k=1}^{K} \hat{c}_{jk}^{u_j}(\hat{q}^{2*}, \tilde{q}^{4*})$). The values assumed by such costs in each simulation are reported in Table 5.

Note that the second and fourth simulations have lower transportation costs than SIM1 and SIM3. Particularly, comparing the total transportation costs of products ($c^{(Tr)}$, the

Table 5
Total transportation costs of products.

	$c^{(MR)}$	$\tilde{c}^{(MR)}$	$c^{(MD)}$	$\hat{c}^{(MD)}$	$c^{(RD)}$	$c^{(Tr)}$
SIM1	2894.95	546.08	682.63	315.36	2234.18	6673.20
SIM2	486.65	321.47	422.70	195.23	0.00	1426.05
SIM3	2894.96	546.08	682.63	315.36	2234.18	6673.21
SIM4	486.65	321.47	422.69	195.23	0.00	1426.05

last column of Table 5), we can observe a reduction of about 78.63% of such costs in SIM2 and SIM4, respect to the other simulations. It is motivated by the use of UAVs for product transportation in SIM2 and SIM4; especially in the transaction between the retailer and the demand markets ($c^{(RD)}$), but also between the manufacturers and the retailers ($c^{(MR)}$).

By comparing the first two simulations (in which there is not the waste sorting center) with the SIM3 and SIM4 simulations (in which a waste sorting center can be used), we observe that the amount of product that each manufacturer ($\forall\ i = 1, 2$) disposes of in a sustainable way in the waste sorting center assumes maximum value (that is, 20% and 25% of the quantity produced by the first and second manufacturer, respectively) when the waste sorting center can be used (on the contrary, clearly, $q^*_{il} = 0$ in SIM1 and SIM2). In this way, in SIM3 and SIM4 the wastage amount of product is totally disposed in a sustainable way and the total cost for disposing the wastage to pay is $c^{1(T)} = \sum_{i=1}^{N} c^1 \sum_{l=1}^{L} q^*_{il} = 22.5$ €. On the contrary, in SIM1 and SIM2, the wastage amount of product is totally disposed in a not-sustainable way and the total cost is $c^{2(T)} = \sum_{i=1}^{N} c^2 \left(\delta_i q^*_i - \sum_{l=1}^{L} q^*_{il} \right) = 135$ €. In such a way, we obtain a reduction of more than 83%. Furthermore, using the waste sorting center, in SIM3 and SIM4, manufacturers do not pay the penalty they would have had to pay in case of not eco-sustainable disposal of the production waste (incurred in SIM1 and SIM2).

Fig. 4 shows, for each simulation, the total environmental emission costs (due to the transactions of goods) and the waste disposal costs (in sustainable and/or not-sustainable ways). It clearly emerges that the best case (with the lowest costs) is represented by the SIM4 simulation, in which UAVs can be used as means of transport and in which there is a waste sorting center (hence, it is possible to dispose of waste in a sustainable way), with a cost-reduction of 74% compared to the worst case, that is SIM1 (where no UAVs can be used and there are not waste sorting centers). Observe that the costs due to the environmental emissions are obtained by multiplying the total emission by the parameter $\alpha_i = 0.5$ (km^2·€)/kg, $\forall i = 1, 2$ that allows us to express environmental emissions in terms of costs, as previously described.

From the optimal solutions, it can be seen that, in all the simulations, despite the costs related to the increase of the energy level of manufacturers, $\gamma_i(\mathcal{E}_i)$, the energy levels are always increased, from $\overline{\mathcal{E}}_1 = 2$ to $\mathcal{E}^*_1 = 9.5$ for the first manufacturer, and from

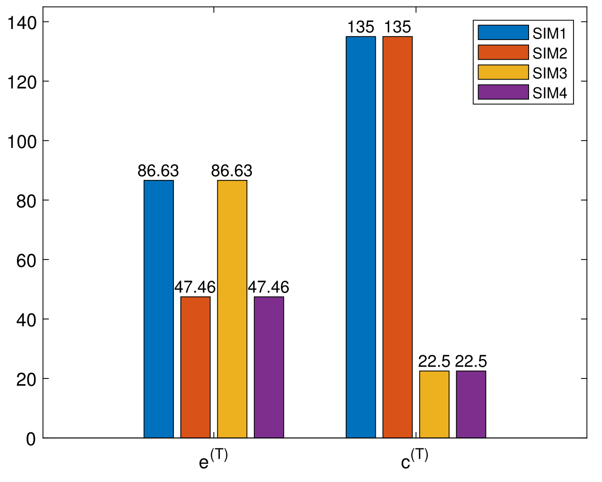

Fig. 4. Environmental emissions (kg/km^2) and waste disposal costs (€).

$\overline{\mathcal{E}}_2 = 3$ to $\mathcal{E}_2^* = 4.5$ for the second manufacturer. Therefore, the total funding received by the manufacturers (from the Government Institutions) for increasing their energy levels $f^{(T)} = \sum_{i=1}^{N} f \cdot (\mathcal{E}_i^* - \overline{\mathcal{E}}_i) = 9$. Clearly, the energy levels are not increased at their allowed maximum ($\mathcal{E}^M = 10$), because of the costs to increase them. Note that the first manufacturer has increased the energy level more than the second one, because she/he has lower investment costs (see parameters of functions $\gamma_i(\mathcal{E}_i)$ in Table 2). Moreover, the sum of such costs is $\gamma_1(\mathcal{E}_1^*) + \gamma_2(\mathcal{E}_2^*) = 3.56$€. Hence, with the optimal values, the total cost to increase the energy level of each manufacturer is lower than the funding they received from the Government Institutions.

6. Conclusions

In this paper, we have described an optimization model based on a supply network in which the three levels of decision makers, producers, retailers and consumers at the demand markets, express a certain degree of interest in environmental policies. From a sustainability point of view, we assumed that the manufacturers, who are profit-maximizers, have the possibility of implementing an ecological transition by investing in their energy level, with the possibility of receiving an economic incentive from a government organization. Furthermore, manufacturers pay attention to the total emissions associated with the production and transport processes of the goods sold to the retailers and the demand markets, for which they pay a penalty. Finally, the environmental interest of the manufacturers also concerns the production process: we have assumed that a monetary penalty is associated in the event that the production waste is not disposed of in an eco-sustainable manner.

The environmental interest of retailers lies in wanting to minimize the total emissions due to the transport of the goods sold to the demand markets. They are too profit maximizers and they determine the optimal values of their variables according to this objective.

Finally, consumers at demand markets represent the lower level of decision makers in the network and make their own consumption choices according to the price they are willing to pay, the selling price of the product established by the manufacturers or retailers and on the degree of aversion to them. This degree of aversion is defined for both manufacturers and retailers. Particularly, consumers have a perception regarding the degree of eco-sustainability of the manufacturers' production processes and they also apply this perception on the retailers who get their supplies from the manufacturers. In this way, consumers are not inclined to purchase products from retailers who have in turn purchased from manufacturers that have a low degree of environmental concern.

For manufacturers' and retailers' constrained optimization problems and for consumers' equilibrium conditions we have derived a variational formulation which, as shown in Section 4, can be solved through a unique variational inequality obtained by summing the previous three. For the last formulation, we have also provided a result of existence and uniqueness of its solution.

Furthermore, the proposed numerical simulations explored the utilization of UAVs as a shipping method of products within the supply chain with the aim of reducing environmental emissions. Additionally, the study investigated the integration of waste sorting centers into the system to enhance sustainability and waste management practices. Through a comprehensive analysis, the numerical results demonstrated that employing UAVs for product transportation offers significant potential for minimizing GHG emissions compared to conventional shipping methods. Moreover, the integration of waste sorting centers into the supply chain proved beneficial in several ways. Indeed, it reduced the environmental impact (using appropriate recycling or disposal facilities, in a sustainable way), the costs to dispose the waste materials and the penalty associated with the not eco-sustainable disposal. Therefore, the findings of this research highlight the potential of UAVs in revolutionizing supply chain operations and environmental sustainability. The optimized UAV transportation system not only reduces emissions but also offers improved efficiency and cost-effectiveness. Additionally, the integration of waste sorting centers enhances the overall environmental impact by promoting responsible waste management practices.

However, it is important to acknowledge certain limitations of the study. The research primarily focused on the optimization aspects of UAV transportation and waste management integration, and further research is required to address practical implementation challenges. Factors such as airspace regulations, safety considerations, and infrastructure requirements need to be carefully evaluated and accounted for in future studies.

In conclusion, the optimization and utilization of UAVs within a supply chain, coupled with the integration of waste sorting centers, hold great promise in reducing environmental emissions and enhancing sustainability in product transportation. This research provides valuable insights and sets the foundation for future investigations in this rapidly evolving field.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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