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# Decisions Dynamics in ICT systems: the influence of a context-aware and social approach on the multiple criteria decision making processes

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Ph. D. thesis

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XXIX CYCLE

*To my little great warrior Anna and Gianni*

*The ideal engineer is a composite...*

*He is not a scientist, he is not a mathematician, he is not a sociologist or a writer; but he may use the knowledge and techniques of any or all of these disciplines in solving engineering problems.*

**Nathan W. Dougherty**

# Abstract

In an Information and Communication Technology (ICT) system, information and knowledge have a key role in the development as well as in the evolution of processes. Due to the continuous improvement of the ICT, there are no limits on when, where and how each process has to take place. In fact, there is no need that the individuals, involved in the process itself, have to be physically and directly connected, via a face-to-face contact for example. This condition, if on one hand permits that all the processes can be easily performed, on the other hand it increases the complexity level of each process itself. The peculiarities of each process and the differences that characterise each process can be appreciated only considering a greater detail level and analysing for each individual the process in which it is involved. Furthermore the development of each process is much more complicated considering the concepts of social networking. In fact, taking into consideration the mechanism of social influence and social contagion as well as the capability and knowledge of each individual, the network node is affected, in a positive or in a negative way, not only by the other nodes of the network connected to it but also by its position and importance within the network.

Considering an ICT system, there are a lot of processes that can take place within a network. The main focus of my Ph. D. research activity has been to analyse the decision making process taking place in a social network and, in particular, the main features that influence the development of the process itself. In fact, due to events and objectives that an individual, the decision maker (DM), had to face and deal, it becomes necessary to take decisions. A decision making process consists on the evaluation of a set of alternatives/actions with respect to a family of criteria. In an ICT system, each decision making process is characterised by four main features: dynamism, context-dependence, multiple criteria and social influ-

ence. Dynamism expresses the continuous change of the characteristics of both environment and of the decision maker who has to perform the process at each time step. Context-dependence, instead, means the importance of the context, defined as the information that is necessary to describe the situation where a decision maker performs its processes. In fact, for example, if the same decision making process is performed in two different context, the decisions taken by the decision maker could not be the same in the two different context. As expressed before and as a confirmation of the importance of a multiple criteria decision analysis, the paradigm of the decision making process is the evaluation of each alternative on the basis of a set of criteria. In this way the advantages and the drawbacks of each alternative are highlighted. Social influence has to be taken into consideration in the development of the decision making process, because the decision maker performs its process not alone but it is surrounded by other individuals that have a minor or a greater, a positive or a negative, influence on it, leading its decisions near or far from its initial inclination, as a results of social interactions among individuals. These four aspects have to be considered together with the personal features of each decision maker, like, for example, its psychological and psycophysical state. Thus, considering the aspects previously introduced, this Ph. D. dissertation proposes a multiple criteria and context-aware decision making model being able to represent the decision making process of an individual in a social network. This model is able to represent the dynamics of decision taken by an individual within a social network, considering the variation of the context and the influence that the individual perceives from its neighborhood. The behaviour of each individual is represented by a set of parameters, whose variation influences the dynamics of decision within the social network. Successively, applying the same perspective to the process of knowledge transfer and learning, it is possible to consider these processes as individual decision making process where each individual has to decide if accept or not knowledge from its neighboring nodes.

In the Ph. D. dissertation the concepts and the analytical instruments provided by the multiple criteria decision analysis (MCDA) are applied to social networks in order to represent as much as possible realistic decision making processes involving individuals that are parts of social networks in different contexts.

**Keywords:** Multiple Criteria Decision Making, Social Networks, Context-Awareness, Dynamism, Knowledge Transfer, Knowledge Learning.

# Contents


**1 Introduction** **4**

1.1 Decision making process in ICT Systems . . . . . . . . . . . . . . . . . 4

1.2 Research Guidelines . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 6

1.3 Dissertation Outline . . . . . . . . . . . . . . . . . . . . . . . . . . . . 7

1.4 List of publications . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 10

**2 Literature Review** **12**

2.1 Decision Theory: Main Concepts . . . . . . . . . . . . . . . . . . . . . 12

2.2 From the beginning of decision theory to the cognitive problems . . . 19

2.2.1 Beyond Expected Utility Theory: Prospect theory and heuristic 22

2.3 Paradigms of the decision making process . . . . . . . . . . . . . . . . . 29

2.3.1 Dynamic decision making . . . . . . . . . . . . . . . . . . . . . 29

2.3.2 Context and Context-dependant applications . . . . . . . . . . . 31

2.3.3 Multiple Criteria Decision Making . . . . . . . . . . . . . . . . . 34

2.3.4 Social Networking for Decision Making . . . . . . . . . . . . . . 37

2.4 Network Models . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 40

2.4.1 Erdös-Rényi model . . . . . . . . . . . . . . . . . . . . . . . . . 41

2.4.2 Watts and Strogatz model . . . . . . . . . . . . . . . . . . . . . 42

2.4.3 Barabási-Albert model . . . . . . . . . . . . . . . . . . . . . . . 42

**3 A Dynamic Context-Aware Multiple Criteria Decision Making Model in Social Networks** **46**

3.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 46

3.2 Multiple Criteria Decision Making with a social perspective . . . . . . 49

3.3 Dynamic context-aware decision model in a social network . . . . . . 51

3.3.1 Context-Awareness in a social network . . . . . . . . . . . . . 51

3.3.2 Dynamic criteria interaction . . . . . . . . . . . . . . . . . . . 54

3.3.3 Decision making model . . . . . . . . . . . . . . . . . . . . . 56

3.4 Summary remarks . . . . . . . . . . . . . . . . . . . . . . . . . . . 61

**4 A context-aware and social model of dynamic multiple criteria preferences** **63**

4.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 63

4.2 Social Networking and model of influence in Decision-Making Process 66

4.2.1 A unified framework of Social Influence and Context-Awareness 70

4.3 A Dynamic Multiple Criteria Decision Making model with a social perspective . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 71

4.3.1 Main components of the model . . . . . . . . . . . . . . . . . 78

4.4 Model Performance and discussion . . . . . . . . . . . . . . . . . . 79

4.5 Summary remarks . . . . . . . . . . . . . . . . . . . . . . . . . . . 94

**5 Multiple Criteria Decision Making and Supernetwork** **96**

5.1 The science of Supernetwork . . . . . . . . . . . . . . . . . . . . . 96

5.2 Supplier selection and supply chain model . . . . . . . . . . . . . . 100

5.2.1 Supernetwork and Multiple Criteria Decision Making: a unified framework . . . . . . . . . . . . . . . . . . . . . . . . . 103

5.3 Model Description . . . . . . . . . . . . . . . . . . . . . . . . . . 107

5.4 Model Perfomance . . . . . . . . . . . . . . . . . . . . . . . . . . . . 124
5.5 Summary remarks . . . . . . . . . . . . . . . . . . . . . . . . . . . . 136

**6 A dynamic and context-aware model of knowledge transfer and learning using a decision making perspective 142**

6.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . 142
6.2 Knowledge and its Processes within a Network . . . . . . . . . . . . . 144
6.3 Model Description . . . . . . . . . . . . . . . . . . . . . . . . . . . . 148
6.4 Results and Discussion . . . . . . . . . . . . . . . . . . . . . . . . . . 154
6.5 Summary remarks . . . . . . . . . . . . . . . . . . . . . . . . . . . . 161

**7 Final remarks 165**

# Chapter 1

# Introduction

## 1.1 Decision making process in ICT Systems

Everyday individuals have the need of making decisions, different in order of importance and in their consequence. The decision making process varies in importance and complexity and depends on the number of decision makers, on the decision to take, on the parameters and conditions that characterise the process itself. Most of times, individuals do not know consequences and they have to deal with uncertainty. This is particularly true in complex and dynamic environment, where the conditions continuously change.

Hence, this statement of Thomas Saaty (Saaty, 2005) well summarises what is decision making and which is the aim of this research field:

*"The purpose of decision-making is to help people make decisions according to their own understanding. They would then feel that they really made the decision themselves justified completely according to their individual or group values, beliefs, and convictions even as one tries to make them understand these better. Because*

*decision-making is the most frequent activity of all people all the time, the techniques used today to help people make better decisions should probably remain closer to the biology and psychology of people than to the techniques conceived and circulated at a certain time and that are likely to become obsolete, as all knowledge does, even though decisions go on and on forever. This suggests that methods offered to help make better decisions should be closer to being descriptive and considerably transparent. They should also be able to capture standards and describe decisions made normatively. Natural science, like decision-making, is mostly descriptive and predictive to help us cope intelligently with a complex world."* .

One of the causes of the "complex world" mentioned above is the rapidly growth in popularity, size and complexity of social networks, where often there are no limits for each single node on the possibility to create new connections and relationships. For this reason, there is the necessity to analyse in depth every process that takes place in this environment, because each single entity involved in the process contributes and has an impact on the whole process dynamics. Each individual, which is the decision maker has to perform each process by taking into account not only its personal capabilities and its indole (to be more or less rational) but also the influence exercised, directly or indirectly, from the other individuals, composing its neighborhood.
In this scenario, information plays a central role. A right or a wrong information, can lead the decision making process to a direction rather than another, because it has a great impact on the creation of the decision maker's knowledge. Its intrinsic characteristic is represented by the value that it can generate in a network and for a process, due to its constant and continuous rate of growth. In fact, from this perspective, knowledge permits individuals to acquire more and more awareness of

the context, which represents the knowledge background that each individual has to take into consideration in doing its decisions. The same decision making problem considered in two different contexts can induce the decision makers to take different decisions. Hence, the introduction of the concept of context-awareness leads to rethink and redesigned the concept of space, that becomes “Smart”. In fact, “Smart space is able to acquire and apply knowledge about its environment and to adapt to its inhabitants in order to improve their experience in that environment” (Cook and Das, 2007).
In this perspective, the process of decision making if on one hand can sometimes be immediate, fast and automatic, in most cases the decision is the result of a complex process taking place in a complex environment.

## 1.2 Research Guidelines

Observing the world around and the phenomena that take place was the starting point of the research activities. In fact, understanding what happens around us is a crucial aspect to analyse in order to predict and then optimise processes that characterise real world phenomena.At this purpose, the main aim of the Ph. D. course activities has been the research of a conjunction point between the pure mathematical theories and instruments, provided by multiple criteria decision analysis, and the constraints and straining to which realistic processes are subjected to. Hence, the starting point has been the consideration of an individual decision making process taking place in a social environment. The fundamental research questions have been:

- **Which are the main features that characterise this process?**
- **Is it possible to create a model that takes into consideration all the**

**features and the constraints that characterise a realistic environment?**

- **Is it possible to apply a decision making model with a social and context-aware perspective to a supply chain problem, represented by a supernetwork?**

- **Knowledge guides and directs every process in a society, a process of knowledge transfer and learning can be look and represented as a decision making one?**

Hence, below in Figure 1.1 it is reported a conceptual map in order to give a better representation of the main research area covered by this Ph. D. dissertation.

## 1.3 Dissertation Outline

The structure of this Ph. D. dissertation is as follows:

- **Chapter 2** briefly introduces the main and basic concepts of decision theory: the classification of different processes (depending on the number of decision makers, on the decision conditions, etc.) and the main features which characterise the process evolution. To give a more deepen insight on the origins of decision theory, a little overview of the different theories that have followed over the years is given. Furthermore, the main four features characterising a decision making process (multiple criteria, dynamism, context-dependance and social influence) are in-depth analysed while explaining their influence and importance on the development of a decision making process.

- In **Chapter 3** the main features, whose importance has been highlighted in the previous chapter, are joined together in order to establish a decision making

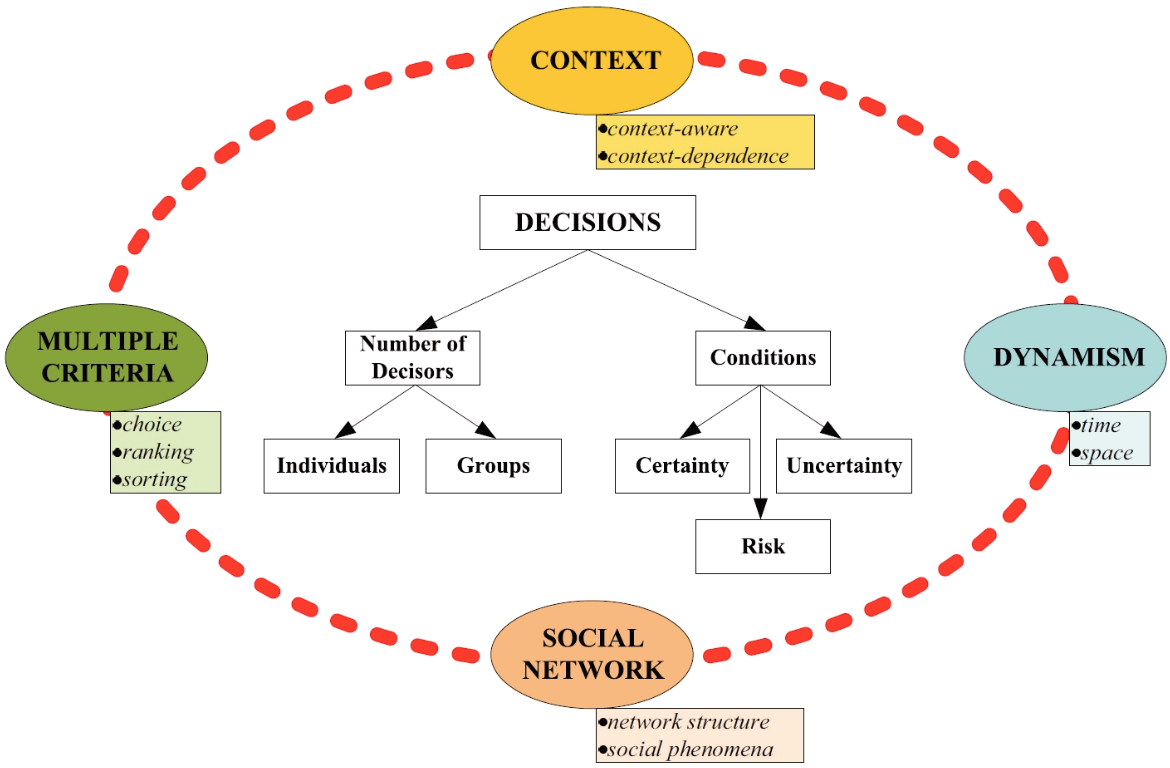


Figure 1.1: Conceptual map of the keywords used in the research activities

model. The research of a conjunction point among the mathematical instruments of multiple criteria decision analysis and the social network analysis has been performed. It can be considered the first step to build a tool exploitable for the analysis of network dynamics as well as for the prediction of individual or community behaviour and decisions. A fully versatility can be obtained by varying the conditions and the parameters characterising the individual and the network.

- **Chapter 4** contains the analytical formulation of the model introduced in the previous chapter. Context and the mechanism of social influence play a central role in the dynamics of multiple criteria preferences. Furthermore, each single node of the network takes into account the variability of the context at each single time step. This variability of the context, together with the influence perceived by each single node from the rest of its social network, affects the weights of each decision criterion while modifying its importance as well as the final decision of each node of the network.

- **Chapter 5** describes an application of the previous model to the supernetworks, in which a supply chain problem is considered. The decision makers represent the nodes of a network composed of various levels, where relationships intra- and inter-levels exist. In particular, nodes that perform the same task belong to the same network level. Each relationship between a couple of nodes is represented by arches. To each arc it is associated a flow that has to be optimised maximizing the utility function, allowing each node to decide to whom and the quantity to sell to the other nodes.

- In **Chapter 6** knowledge and the processes in which it can be involved are presented. In particular, a context-aware multiple criteria decision making perspective is applied to the processes of knowledge transfer and learning. The mechanisms and the patterns of knowledge diffusion within a social network are analysed, in order to evaluate the impact of a decision making approach to the knowledge diffusions patterns.

- **Chapter 7** includes conclusions about the research activities carried out during the Ph. D. studies. It also contains the future research directions and the open questions of the Ph. D. studies.

## 1.4 List of publications

- La Corte A., Scatá M., Giacchi, E.;"A Bio-Inspired Approach for Risk Analysis of ICT Systems", Book Chapter in Computational Science and Its Applications - ICCSA 2011, Lecture Notes in Computer Science,Springer Berlin (Heidelberg), Santander (Spain), Isbn: 978-3-642-21927-6, vol.6782, pp. 652-666, doi: 10.1007/978-3-642-21928-3-48

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- Giacchi E., Corrente S., Di Stefano A., Greco S., La Corte A., Scatá M., "A context-aware approach of multiple criteria decision making for social network analysis", 23rd International Conference on Multiple Criteria Decision Making (MCDM 2015), Hamburg (DE), 2 - 7 Agosto 2015, Book of Abstract p.132

- Giacchi E., Corrente S., Di Stefano A., Greco S., La Corte A., Scatá M., "A context-aware and social model of dynamic multiple criteria preferences", Decision Analytics, 3(1), pp. 1-24, Springer, DOI: 10.1186/s40165-016-0020-3

- Giacchi E., La Corte A., Di Pietro E., "A Dynamic and Context-aware Model of Knowledge Transfer and Learning using a Decision Making Perspective", Proc. 1st International Conference on Complex Information Systems (COMPLEXIS 2016), Science and Technology Publications, Lda (SCITEPRESS) 2016, Rome (Italy), 22-24 Aprile 2016, ISBN: 978-989-758-181-6

- Di Stefano A., Scatá M., La Corte A., Giacchi E., "A Dynamic and Context-aware Social Network Approach for Multiple Criteria Decision Making through a Graph-based Knowledge Learning ", Book chapter in Graph Theoretic Approaches for Analyzing Large-Scale Social Networks, IGI Publisher (to appear, 2017)

# Chapter 2

# Literature Review

## 2.1 Decision Theory: Main Concepts

Understanding the reasons that are at the basis of an individual's actions can help to comprehend individual and collective behaviours within a society. For this purpose, *Decision theory provides a rational framework for choosing between alternative courses of action when the consequences resulting from this choice are imperfectly known* (North, 1968). From the ancient Greek philosophers Aristotle and Plato, decision theory has taken contributions from a lot of disciplines, i.e. mathematics, statistics, economy, sociology, psychology and management.

A decision making process involves an individual or a group of individuals, that are named decision maker(s), and it produces a final choice, that is the result of the evaluation process of the set of alternatives with respect to a finite and coherent family of criteria. Within decision theory and depending on the approach that it is used, it is possible to distinguish between (Oliverio, 2007):

- Normative decision theory, which studies the ideal decision making process and suggests the best decision so that the decision maker(s), supposed to be fully rational, can have the maximum utility;

- Descriptive decision theory, which studies the real decision making process performed by individual(s) acting under some rules and constraints that are intrinsic of the process itself .

Hence, to give a first mathematical definition, assuming that $\{A_i \mid i \in I\}$is a collection of disjoint set representing the alternatives, $A_i \subseteq U$, where $U$ is the universe, and $A_i \neq \emptyset$, a function $c : \{A_i\} \rightarrow A_i, i \in I$ is named choice function if $c(A_i) = a_i, a_i \in A_i$ ($I$ represents a set of natural numbers). The decision $d$ represents the selected alternative $a_i \in A$, non-empty set of alternatives (with $A \subseteq U$), on the basis of a given criteria set $C$, hence (Wang and Ruhe, 2007):

$$d = f(A, C) = f : A \times C \rightarrow A, with A \subseteq U, A \neq \emptyset \tag{2.1}$$

The set of alternatives must include at least two different alternatives, otherwise, if the set $A$ is composed of only one element, the decision making process cannot be performed because the decision is obliged and it cannot be considered a decision. On the other hand, if there are different alternatives, the decision making process should be point out the best choice for the individual, after the evaluation alternatives process. The tool through which the decision maker can evaluate all the alternatives is the decision's criterion. A criterion *is a tool constructed for evaluating and comparing potential actions according to a point of view which must be (as far as it is possible) well-defined* (Figueira et al., 2005). This evaluation must take into account, for each action, all the pertinent effects or attributes linked to the point of view considered. For a long time horizon, many scientists, for the sake of simplicity, thought that a monocriterion approach (that is a single criterion that joins the multiple aspects that characterise the decision situations in order to use a single scale of measures) would be the best way to perform a decision-making process. But, with this approach, the decision making process looses its paradigm and its intrinsic nature, that is the comparison of different points of view. In fact if a monocriterion approach is taken into consideration there are lots of limitations, such as ignoring certain features characterising the real process leading to a wrong evaluation of alternatives (setting up of equivalencies of alternatives).

On the contrary, using a set of criteria, that is a multicriteria approach to perform the decision making process, permits to avoid such limitations because the family of criteria taken into consideration represents different points of view, as the paradigm of the decision process is. The evaluation criteria represent different points of view taken into account by the Decision Maker to highlight the advantages and the drawbacks of each single alternative (Figueira et al., 2005). Hence, indicating with $g$ the criterion, $g(a)$ is the evaluation of alternative $a$ with respect to the criterion $g$. The family of criteria $F$ must satisfy important requirements such as completeness, cohesiveness and non-redundancy (Roy et al., 2005). Furthermore, within the set $F$ of criteria can exist some dependency relationship or there could be some mutual as well as antagonistic interactions (Figueira et al., 2009).

The decision making process is characterised by a finite or infinite set of actions (the alternatives), two or more decision's criteria and one or more decision makers. This is the research field of Multiple Criteria Decision Making (MCDM).

In the decision making process the decision maker has the role to take the final decision, depending on the evaluation of each single alternative. It is also important to highlight that the number of decision makers has a strong influence on the dynamics of the decision making process itself. Hence, it is possible to distinguish between individual and group decision making process. To the first category belongs the decision making process that involves only one decision maker that can be considered isolated or in a social environment (this issue will be further examined later). To the second category, instead, belongs the process that involves a group of individuals that, as a result of dynamic interactions among them, at the end of the process achieves only one decision for the whole group. In this case, it is necessary much more time to reach the final decision, due to the multitude of opinions within the group of individuals. Among the different aspects to be taken into account in the mechanisms of interaction for group decisions there are the different relationships that exist among individuals, especially if there is a hierarchical relationship. The possible configurations are shown in Figure 2.1(Cioffi-Revilla, 2013).

In the case of a chain network, each individual can only communicate with its neighbors and not with everyone. In this way the comparison and the exchange of ideas among all members of the group cannot occur, thus slowing down the decision-making process. In the case of a star network, the central node is the leader of the discussion, that can communicate with everyone directly, but if the other members of the group have to communicate, they must make it through the leader node. The case of a Y-shaped structure results from the combination of the two previous structures, trying to contain the centralization of a node with respect to the other. In the case of a circle network everyone can express its own ideas and it is the most open structure and also the most efficient, and it is the most widely used for solving the most complex problems. In a complete network everyone can speak with all the other members of the group, there is not a leader and the complete information exchange is reached within a few rounds of the minimum possible (Knödel, 1975), (Farrag and Dawson, 1987), (Lakshman and Agrawala, 1986), (Sunderam and Winkler, 1993). In case of opinion dynamics and in many real networks, not all the members have the same point of view, e. g. a two-party political system. In this case two opposite parties are present in a structural balanced network what often can happen is

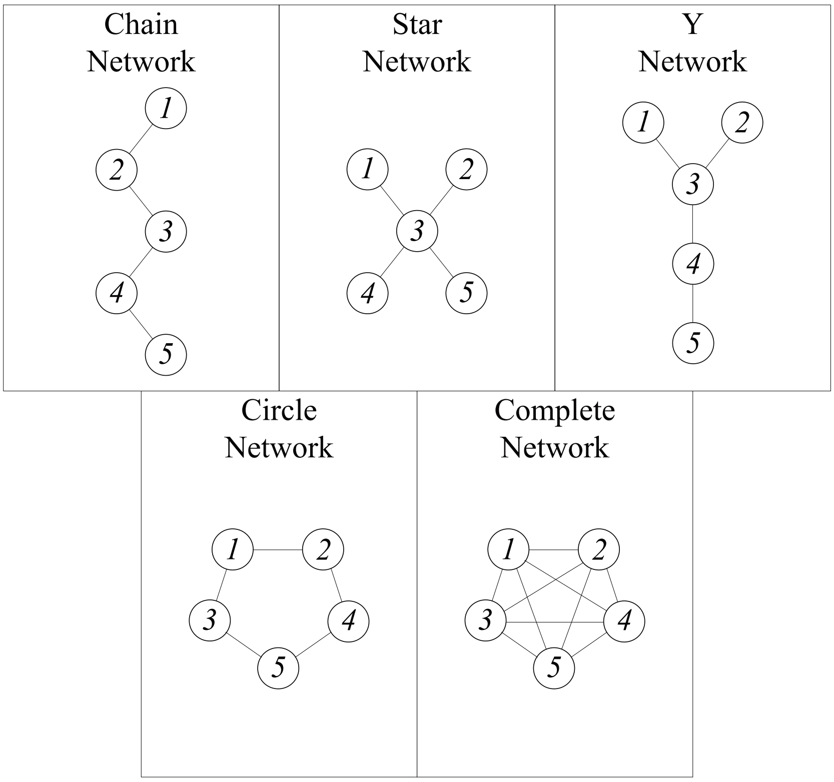



Figure 2.1: Network configurations for Group Decision Making processes

a gridlock (e.g. the US congress in 2011 on raising the national debt ceiling: the antagonism and ideological divide between the two main political factions leads to a legislative gridlock)(Altafini, 2012). Also psychologically, the possibility to take a unanimous decision involves a sense of spread of responsibilities. In fact, while in the case of an individual decision, the responsibility lies with the individual, in the case of a collective decision, the responsibility is divided or “widespread” among all the members who participated in the process, leading some members, for example, to contribute less, not having a direct responsibility.

For the human mind the decision-making process is considered as one of the 37 fundamental cognitive processes in a layered model (Wang and Ruhe, 2007). Hence, the decision-making process is a process of selection among the alternatives satisfying certain criteria, aimed at achieving a specific objective (decision goal). In particular, the number of possible decisions $n$ may be determined by the size of $A$ and from that of $C$:

$$n = \#A \cdot \#C \tag{2.2}$$

denoting by # the cardinality of the sets, and $A \cap C = \emptyset$.
Therefore, the result of the decision-making process is determined by the strategies chosen by the decision maker when the alternatives of choice are identified. It can also be defined a taxonomy of strategies and decision criteria used in the process, divided into 4 categories: intuitive, empirical, heuristic and rational. The first two are identifiable within the intuitive cognitive human psychology. The rational is divisible into two sub-categories: static and dynamic. The heuristic was primarily used by human decision makers. The strategies and criteria are shown in Figure 2.2

To reach the final decision, the conditions in which the decision making process takes place have to be taken into consideration. At this purpose it is possible to distinguish among three decision conditions:

- Decision under certainty;
- Decision under uncertainty;
- Decision under risk.

| No. | Category | Strategy | Criterion (C) |
|---|---|---|---|
| **1** | **Intuitive** | | |
| 1.1 | | Arbitrary | Based on the most easy or familiar choice |
| 1.2 | | Preference | Based on propensity, hobby, tendency, expectaction |
| 1.3 | | Common senses | Based on axioms and judgment |
| **2** | **Empirical** | | |
| 2.1 | | Trial and error | Based on exhaustive trial |
| 2.2 | | Experiment | Based on experiment results |
| 2.3 | | Experience | Based on existing knowledge |
| 2.4 | | Consultant | Based on professional consultation |
| 2.5 | | Estimation | Based on rough evaluation |
| **3** | **Heuristic** | | |
| 3.1 | | Principles | Based on scientific theories |
| 3.2 | | Ethics | Based on philosophical judgment and belief |
| 3.3 | | Representative | Based on common rules of thumb |
| 3.4 | | Availability | Based on limited information or local maximum |
| 3.5 | | Anchoring | Based on presumption or bias and their justification |
| **4** | **Rational** | | |
| 4.1 | Static | | |
| 4.1.1 | | Maximum cost | Based on minimizing energy, time, money |
| 4.1.2 | | Maximum benefit | Based on maximizing gain of usability, functionality, reliability, quality, dependability |
| 4.1.3 | | Maximum utility | Based on cost-benefit ratio |
| 4.1.3.1 | | - Certainty | Based on maximum probability, statistic data |
| 4.1.3.2 | | - Risks | Based on minimum loss or regret |
| 4.1.3.2 | | - Uncertainty | |
| 4.1.3.3 | | - Pessimist | Based on maximin |
| 4.1.3.4 | | - Optimist | Based on maximax |
| 4.1.3.5 | | - Regretist | Based on minimax of regrets |
| 4.2 | Dynamic | | |
| 4.2.1 | | Interactive events | Based on automata |
| 4.2.2 | | Games | Based on conflict |
| 4.2.2.1 | | - Zero sum | Based on $\sum$ (gain + loss) = 0 |
| 4.2.2.2 | | - Non zero sum | Based on $\sum$ (gain + loss) $\neq$ 0 |
| 4.2.3 | | Decision grids | Based on a series of choices in a decision grid |

Figure 2.2: Taxonomy of strategies and criteria for decision-making

Depending on the conditions in which the process takes place, the decision is not always the right and rational one through which the decision maker can reach the maximum utility, because the decision's conditions modify and sometimes alter the problem's perception of the decision maker.

## 2.2 From the beginning of decision theory to the cognitive problems

To better understand the modern theories and the great interest on this research field from a lot of disciplines, a quick overview on the origins of decision theory could be very useful (Oliverio, 2007). One of the first pioneers was Blaise Pascal, who developed the theory of the expected value (or mathematical expectation) for which, considering two events $a$,$b$ having probability $\alpha$ and $(1-\alpha)$, the expected value of a game $x$ is:

$$VA(x) = (a) \cdot (\alpha) + (b) \cdot (1-\alpha) \tag{2.3}$$

Daniel Bernoulli later perfected the previously theory exposed by Pascal, providing the solution of the so-called St. Petersburg paradox, invented by his cousin Nicolas. He considered a type of game whereby a dime (no fixed) is launched so many times until "head" is out and the winner will receive as prize $2n$, where $n$ is the number of throws made until it comes out "head". Applying the theory of Pascal and calculating the expected value which is equal to infinity, a player should be willing to pay any amount of money to be part of the game, even though "head" should come out on the first roll, thus bringing the payout equal to 2 . This paradox, therefore, emphasised the limits of the expected value theory. For this reason, Bernoulli proposed to distinguish between the expected value of a result from its

expected utility, which represents the importance that may have the given results for the individual multiplied by its probability of recurrence. This is because the utility depends on the individual himself that expresses its own preferences. For example, the expectation of wealth utility grows with the increase of the same wealth, but in a way inversely proportional to the quantity of wealth possessed (for a wealthy individual a capital increase equal to $2,000$ is less significant compared to the same amount given to an individual less wealthy). Hence, a problem, until then purely mathematical, was enriched considering the psychological and moral dimensions, acquiring a complexity and a greater variability, introducing new criteria and aspects strictly related to individuality. This leads to the introduction of the notion of risk in the decision making process and furthermore the consideration the individual attitude towards the risk. For these reasons, the utility function is no longer considered as a linear function but a logarithmic function.
Later the theory was perfected by von Neumann and Morgenstern and their expected utility theory (Von Neumann and Morgenstern, 2007). Such theory is based on two axioms:

- Completeness: if there are two possible results $x_1$ and $x_2$, $x_1$ could be preferred to $x_2$ ($x_1 > x_2$) or $x_2$ could be preferred to $x_1$ ($x_2 > x_1$) or $x_1$ and $x_2$ are indifferent ($x_1 \sim x_2$);

- Transitivity: if there are three possible results, if $x_1 > x_2$ and $x_2 > x_3$ then $x_1 > x_3$ (the same can be reported in case of relations of equality).

According to this theory, the decision maker is considered as a rational individual which assigns a certain probability to each event. Von Neumann and Morgenstern have shown the existence of an expected utility function that is equal to the sum of the utilities associated to each alternative multiplied by the probability of occurrence of that particular situation. According to this theory, there are three possible functions of the expected utility, classified on the basis of the behaviour undertaken by the decision maker: aversion, indifference or propension. An individual is risk averse if he prefers to get a payment rather than try the game in a lottery. On the contrary, he is indifferent to the risk if he has no preference in receiving a payment or playing the lottery. Moreover, he has propensity to risk if he prefers to play the lottery rather than accept a certain gain. An important aspect that needs more explanation is that the individual is not considered to have a preliminary particular attitude towards risk. The expected utility theory developed by von Neumann and Morgenstern shows some limitations, mainly related to the knowledge degree and to the rational being of the individual. In many real context such conditions are not feasible.

As a demonstration of these limitations, Simon (Simon, 1955) stated that the real human behaviour is characterised by a bounded rationality, which takes into account the limits of the individual selection, acquisition, processing and information storage processes. These limitations have resulted either from lack of information, for example if it is known only a limited number of alternatives so that the evaluation of them fails since not taking into account all of them. Sometimes these limitations come from an individual inability to perform the decision-making process when, although all the alternatives are available, the decision maker cannot perform all the calculations necessary for a complete evaluation of them and of the corresponding utility. In this way the decision maker can only reach a satisfactory solution, according to which he analyses the possible alternatives and identifies one good enough considering the acceptability thresholds: such alternative becomes the real choice. Later, Simon also introduced the concept of 'procedural rationality', for which the good decision is not the best in terms of results, but the one coming from the resolution process tailored according to the representation that an individual makes of the decision making process.

### 2.2.1 Beyond Expected Utility Theory: Prospect theory and heuristic

After the studies conducted by Simon, many experiments were conducted, especially from Tversky, to demonstrate the limitations of the expected utility theory of von Neumann and Morgenstern.

One of these, which pointed out the violation of the principle of transitivity, compared pairs of bets belonging to set $\{a, b, c, d, e\}$, whose chances of recurrence were sorted in a descending scale from $a$ to $e$. Hence, when comparing pairs of adjacent bettings, the individuals looked to the possible win and not to the probability, while for non-adjacent couples the preference was expressed on the basis of the likelihood of recurrence.

A further example of violation of the expected utility theory is represented by the Allais paradox (Allais, 1953):

There are three possible prizes and the decision makers had to make two independent choices between the situation $A$ and $B$, and later between the situation $C$ and $D$, as listed below:

<u>Choice 1</u>

A = certainty obtain 100

B → 10 % chance of winning 500, 89 % chance of winning 100, 1 % chance of winning nothing

<u>Choice 2</u>

C → 11 % chance of winning 100, 89 % chance of winning nothing

D $\rightarrow$ 10 % chance of winning 500, 90 % chance of winning nothing

This experiment showed that the majority of individuals chose in the first case the alternative $A$ to $B$, while in the second case they preferred the alternative $D$ to $C$, thus violating the independence axiom as if $A > B \rightarrow C > D$. Whereas, considering also the expected utilities, the violation of the affirmation made by von Neumann and Morgenstern becomes even clearer, as computing the expected utilities in the first case it is obtained:

$$U(100) > 0.10 \cdot U(500) + 0.89 \cdot U(100) + 0.01 \cdot U(0) \rightarrow 0.11 \cdot U(100) > 0.10 \cdot U(500) \tag{2.4}$$

while in the second case:

$$0.10 \cdot U(500) + 0.90 \cdot U(0) > 0.11 \cdot U(100) + 0.89 \cdot U(0) \rightarrow 0.10 \cdot U(500) > 0.11 \cdot U(100) \tag{2.5}$$

Observing the inequalities it can be noted that they contradict each other and violate the independence axiom.

With another experiment, Tversky and Shafir (Tversky and Shafir, 1992) demonstrated the violation of the axiom of independence of the alternatives by observing that introducing another alternative to the already existing set, the volition of the individual was to delay the decision and, in some cases, the decision was different than the previous case. This is due to the fact that the individual has not a single order of preferences.

Also the inversion of preferences is a clear violation of the notion of rationality underlying the expected utility theory. In this case, there was an experiment conducted by Lichtenstein and Slovic (Lichtenstein and Slovic, 1971) on a sample of individuals, who were asked to choose between two bets. In the first there was a high probability of winning a small sum of money and a low chance to lose an even smaller sum, while in the second there was a low probability of winning a high sum and a high probability of losing a low sum money. The results showed that individuals, when asked to choose between the two bets, preferred the one which offers the highest probability of winning. On the contrary, in the case in which they were asked to establish the price at which they were willing to give up the bets in question, they attributed the higher price to bet characterised by the lower probability of winning rather than to the one whose probability was higher. What is revealed is a reversal direction of preferences depending on the order of presentation of alternatives, therefore, going to clash with the axiom of the expected utility theory according to which the decision maker has its own order of preferences.

For this reason, Kahneman and Tversky formulated prospect theory (Kahneman and Tversky, 1979) that, starting from the expected value theory, took into account the aspects that characterise the real decision-making process. The prospect theory does not refers to utility but to the value, defined as the loss or gain relative to a certain position, considered as the neutral reference point. The representation of what has just been said is shown in Figure 2.3.

The curve has a "S" shape, concave for the evaluation of gains and convex for the losses. This function thus expresses both the risk aversion in the area of gains and the trend of risk propension in the area of losses. In the area of gains the value function grows more slowly than the decrease of the same function in the area of losses. The effect of a marginal increase decreases as the distance from the reference point that is represented by the origin of Cartesian axes. Near to the reference point the steepness of the curve is greater in the case of loss since the latter will have a greater impact on the individual. According to this theory the preferences of

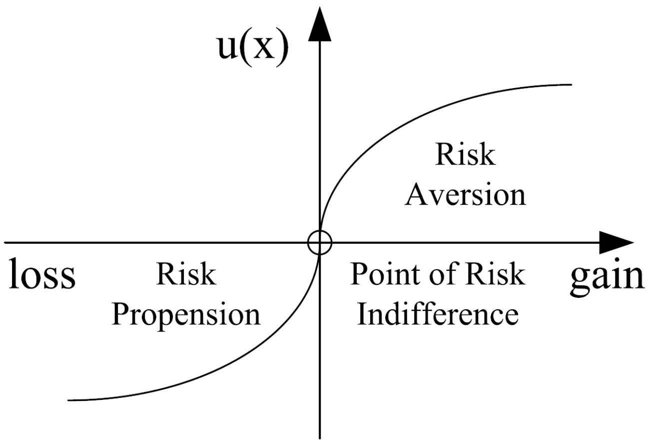



Figure 2.3: Decision maker's behaviour towards risk

individuals are treated as "decision weights" that do not always correspond to the probabilities and that reflect the relative importance of them, overestimating the low probabilities and underestimating the high chances.

In their studies Kahneman and Tversky gave a significant importance to the cognitive aspect and to the reasoning related to the decision-making process. As very often the data available for the decision making process are numerous and also the time to process them is very restricted. In some cases, the decision maker uses its reasoning capabilities to reduce both time and complexity of the information to perform the process in the necessary time. In fact, the intuitive judgments are located between the operations of perception and reasoning. This type of reasoning is also applied to the majority of the daily life decisions, such as for example the estimation of a measure (size, length, etc ..).

Kahneman has outlined the distinction between intuition and reasoning processes admitting the existence of two different systems that are present in the cognitive process: System 1 and System 2 (Kahneman, 2003b). System 1 handles all the processes in which it is involved in a fast and automatic way, using few resources;

it is associative, implicit and influenced by emotions. It is affected by the routine and difficult to control or modify. System 1 generates "impressions", voluntary and non-verbally explicable, linked to the attributes of the thought objects and perceptions.

System 2 handles all the transactions in which it is involved in a slow and serial way, with the use of many resources, it is easily controllable, flexible and governed by strict rules. System 2 is involved in the formulation of "judgments", always intentional and explicit, which originate from the combination of impressions and thoughts. The judgments are referred to as "intuitive", when they directly reflect the impressions.

As in all the duals processes, one of the functions of System 2 is to monitor and control the quality of all the operations that use both mental systems. As is indicated by Kahneman and Frederick (Kahneman and Frederick, 2002), the control performed by System 2 on System 1 is realised in a rather lax, allowing the individual the expression of its intuitive judgments, some of which may sometimes be erroneous.

Another key aspect that characterises information is its "*availability*" (Tversky and Kahneman, 1973). In fact there are some information that come to mind much more easily than other. Availability depends on the cognitive process which has produced that particular information and on the stimulus/event that it has invoked. What determines and has a strong influence on the availability of information is the "physical relevance" of an object, the "emotional significance" of a stimulus, familiarity, emotional salience and also the temporal distance of the event taken as a reference.

In addition to the availability, "*representativeness*" of information may change its perception for the individual (Tversky and Kahneman, 1981). In particular, it is the mechanism by which to judge if an event $x$ belongs to a class of events $y$, it is evaluated how much $x$ is representative of $y$ (Oliverio, 2007). In other words, it is evaluated how much $x$ and $y$ are similar. In particular, the representative may induce decision makers to make two systematic errors “conjunction fallacy” and “basic probability fallacy” (Tversky and Kahneman, 1983). The first consists in the representation of information so that is judged as most likely the conjunction of two events rather than a single event, thus violating the assumption of the theory of probability. An example of the conjuction fallacy is the following:

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?

- Linda is a bank teller.
- Linda is a bank teller and is active in the feminist movement.

Most of individuals judges more probable the second affirmation although its probability is less than the first one.

The second fallacy consists of expressing the evaluation of an event only on the basis of probabilities related to the individual case and not making reference to the entire population.

An additional systematic error which the human mind can commit is the cognitive bias called “anchoring” (Kahneman, 2003a), according to which the evaluations of a certain event is carried out from a specific point, taken as a reference and which is called “anchor”. The other judgments are made depending on their distance from that anchor, and the bias consists in the interpretation of the other information around the anchor. A further bias that can lead to an error is represented by the so-called “framing effect” (Tversky and Kahneman, 1985). In fact, the way in which information is presented, or as it is “framed”, may affect significantly preferences and opinions. In fact, depending on whether the information is presented in terms of losses or gains, the decision maker changes its preferences. An example of the framing effect is the “problem of the Asian disease”, formulated by Kahneman and Tversky in 1981. To a group of people was asked to provide an answer to the following decision problem, according to which the health system of the USA is preparing to face an outbreak of a rare Asian disease that will bring 600 victims. It is possible to choose between two alternatives:

1. With the program $A$, 200 individuals will be saved;
2. With the program $B$, with a probability of 1 by 3 will be saved 600 individuals, while with a probability of 2 by 3 no one will be saved.

The 72% of the interview people chose the program $A$, as it was expressed in positive terms of saved live (highlighting an attitude of risk aversion).
On the contrary, by changing the arrangement of the questions, the answers were very different than the previous case. If the problem was place in the following way:

1. With the program $C$, 400 will die
2. With the program $D$, with a probability of 1 by 3 no one will die, while with a probability of 2 by 3 will die 600 individuals.

In this case the 78% of people preferred the program $D$ to $C$ (highlighting a risk tolerance attitude). The difference in the two formulation was related to the perspective used, in the first case in terms of saved lives, in the second in terms of lost lives. Hence, the expected utility theory is violated since a way to present a problem should have no effect on the ordering of the preferences of individuals.

## 2.3 Paradigms of the decision making process

A decision making process that tries to represent in the best possible way a realistic scenario is characterised by some constraints and features, typical of this environment. In particular, the four main features that define a realistic decision making process are: dynamism, context-dependance, multiple criteria analysis and the mechanism of social influence. Each of this characteristic is in-depth analysed and explained in the following sections.

### 2.3.1 Dynamic decision making

The first aspect that is here analysed is dynamism. As reported in the (Busemeyer, 1999) (Edwards, 1962) a dynamic decision-making process is characterised by three basic features:

- A set of actions (alternatives) among which make the decision in order to achieve objective(s);
- The actions at time $t$ depend on the previous time instant;
- The environment in which decisions are made changes spontaneously or as a consequence of the actions that have been undertaken in previous time instants.

Hence, in a Dynamic Decision Making (DDM) process the decisions made are dependent one another and the environment changes depending on the decisions sequence (Gonzalez et al., 2005). Being then the environment a dynamic one, also the decision has to be taken in real-time. In dynamic systems, the state of the system at time $t$ depends on its state at time $(t-1)$ and, more specifically, it is influenced by both endogenous causes (which depend, therefore, on the decisions made) and exogenous (factors that are beyond the decision makers control). The continuous variation within a dynamic system can give rise to loops, in the sense that some variables can have effect and influence on themselves. The dynamic decision making processes involve a number of cognitive processes, such as monitoring, recognition, causal inference, search, planning, judgment and choice. The ability to coordinate these processes with each other interrelated is one of the main components of dynamic decision making. The cognitive model used in (Gonzalez et al., 2005) is the IBLT (Instance-Based Learning Theory), which describes the decision-making process based on 4 main stages: Recognition, Judgment, Choice and Feedback. In particular, each individual stores in its mind phrases about the particular decision-making process, phrases that will be used to make its decision in a dynamic environment. The decision maker updates the utility associated with each of this statement following the response of the system and uses these phrases to make better decisions and take them as a reference point for the future in case of a similar scenario. The schematic representation reported in (Busemeyer, 1999), brings a medical dynamic decision-making process represented by a feedback diagram, where the block $S$ represents the dynamic environment that takes into consideration both the disturbance $w$ and $T$, that is, the action of decision-makers and provides as output the health status $H$ of the patient. The $D$ block indicates the intention of the patient in the decision-making process, as it takes as input the current status of health $H$ and the desired one $H^*$ and produces as output the action $T$.

The results of this experiment showed that most of the interviewed doctors lost control of their patients, no longer able to assign the correct therapy to patients, always moving away from the more optimal solution that would be provided by the so schematised system. This result is due to both the behavioural component and

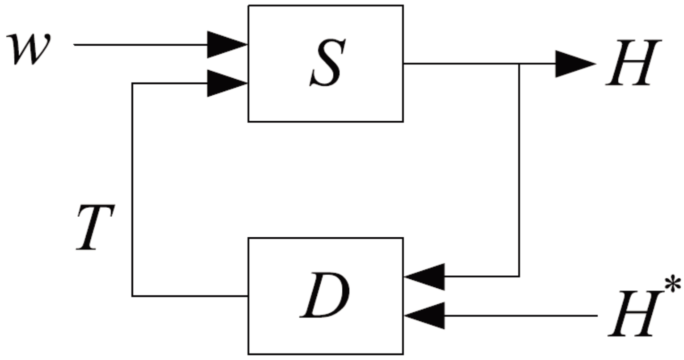



Figure 2.4: Dynamic decision making process

the influences that the decision suffers from other individuals who may represent his network of contacts.

### 2.3.2 Context and Context-dependant applications

The importance of the influence of context in all processes has created a growing interest from a lot of disciplines during the last decades and several attempts to formalise its definition have been made in literature. Schilit et al. (Schilit et al., 1994) stated that context is linked to the location, nearby person, host or objects, and their evolution over time. Moreover, the three main features of the context definition are "where you are", "who you are with" and "what resources are nearby". Brown et al. (Brown et al., 1997) defined *context* as the information regarding location, time of the day, season of the year, temperature, etc. Liu et al. (Liu et al., 2011) modified the previous definitions of context, adding information regarding emotional state, attention focus, objects and people in the user's environment. Snowdon and Grasso

(Snowdon and Grasso, 2000) considered context as a multi-layered set, where each element consists of people and of their expertise, information sources, informational documents, the evaluation of their relevance and relevant pragmatic documents. Ahn and Kim (Ahn and Kim, 2006) defined context as a set of interrelated events with logical and timing relations among them, distinguishing among discrete and continuous events.

Despite all previous examples, there is not a standard definition of context. Several researchers accept that the definition given in (Abowd et al., 1999), where context is defined as *any information that can be used to characterise the situation of an entity. An entity is a person, place or object that is considered relevant to the interaction between a user and an application, including the user and applications themselves.*

Furthermore, some researchers have also divided the concept of context into categories, according to its features. In particular, Henricksen (Henricksen, 2003) used four categories: sensed (data collected directly from sensors), static (information that does not change over time), profiled (information that change over time with a low frequency) and derived (information obtained using primary context). Another categorisation of context can be done as in (Yürür et al., 2016):

- Device context, which includes net connectivity, communication cost and resources,
- User context, which include profile, geographical position, etc.,
- Physical context, which includes temperature, noise level, etc.,
- Temporal context, which includes day, week, month, season, etc.

Given a specific process, the aspects that characterise each of its tasks will have a different impact on the perception of the context that, in turn, will affect the way the process is performed and the obtained output. Therefore, a system must be able to distinguish and elaborate information coming from the environment, in order to react and adapt its behaviour to the new conditions. For example, raw sensor data incoming directly from the source of information are processed in order to obtain context information (Perera et al., 2014). Hence, as pointed out in (Abowd et al., 1999), a system is context-aware *if it uses context to provide relevant information and/or services to the user, where relevancy depends on the user's task*. Consequently, "context-awareness", firstly introduced in (Schilit and Theimer, 1994), indicates the ability of mobile user's applications to discover and react to changes in the environment they are in. According to the different ways the system reacts to the changing conditions of the environment, context-awareness has been classified in (Chen et al., 2000) as follows:

- Active context-awareness: the system adapts itself to the changing environment modifying its behaviour,
- Passive context-awareness: the system presents the new or updated context to an interested user without modifying its behaviour.

As in (Liu et al., 2011), five different classes of context-awareness can be highlighted: Context Acquisition and Sensing, Context Modeling and Representation, Context Filtering and Fusion, Context Storage and Retrieval, Context Application.

Due to the increasing computational capabilities of smart devices, context-aware applications, being able to recognise user's social and cognitive activities anytime and anywhere (Yürür et al., 2016), have gained a central importance nowadays. By means of these applications, users share their information to create a common knowledge and a large community within a smart environment.

Context-aware applications have several fields of application, like healthcare and

well-being, transportation and location, social networking and environmental monitoring. This has been made possible through the integration of context ubiquitous sensing, micro-sensors and the geographic information systems (GIS).
For example, regarding the healthcare field, several E-health applications have been proposed to monitor patients with different diseases, in order to guarantee an adequate level of assistance. Particularly in these cases, it is important to choose necessary context information in order to extract useful information and obtain knowledge that can permit to adapt dynamically the behaviour of the system according to the environment characteristics (Guermah et al., 2013). Considering, instead, the social networking, the SAMOA (Socially Aware and Mobile Architecture) framework (Bottazzi et al., 2007), allows mobile users to create social networks, following its movements. It is based on two kinds of context visibility: place visibility (place awareness) and profile visibility (profile awareness).

### 2.3.3 Multiple Criteria Decision Making

In a Multiple Criteria Decision Making (MCDM) problem (see (Figueira et al., 2005) for a collection of surveys on MCDM), a set of alternatives/actions $A = \{a, b, \ldots\}$ is evaluated with respect to a finite and coherent family of criteria $G = \{g^1, g^2, \ldots, g^p\}$ (Roy, 1996), that is exhaustive (all relevant criteria are taken into account), cohesive (if $a$ is at least as good as $b$ for all but one criteria and $a$ is better than $b$ on the remaining criterion, then $a$ should be preferred to $b$) and non-redundant (removing one criterion from the family renders it not exhaustive or cohesive). It is possible to suppose that each criterion is a real valued function $g^i : A \to \mathbb{R}$ having an increasing direction of preference (the higher the evaluation of $a$ on criterion $g^i$, the better $a$ is) or a decreasing direction of preference (the higher the evaluation of $a$ on criterion $g^i$, the worse $a$ is). For example, in a project evaluation problem, different projects (the alternatives using MCDM terminology) can be evaluated with respect to different aspects such as Opportunity, Potential Risks, Technology, Finance and Employment (Tavana et al., 2015). Investment cost and Return on Investment can be considered subcriteria of the financial aspect, while Impact and Technology Importance can be highlighted as subcriteria in the Technology aspect. Investment Cost has a decreasing direction of preference, while Return on Investment, Impact, and Technology Importance have an increasing direction of preference.

Three main problems are considered in MCDM: choice, ranking and sorting. Choice problems consist into choosing one or more alternatives (actions) considered the best; ranking problems consist into rank ordering all alternatives from the best to the worst, while sorting problems consist into assigning each alternative to one or more contiguous classes preferentially ordered from the decision maker. In the considered example, the decision maker can be interested in choosing the best project, in ranking all of them, or in assigning to classes, such as "bad", "medium", "good", ordered with respect to their reliability.

Looking at the evaluations of the alternatives on the considered criteria, the only information that can be gathered is the dominance relation[1] but, especially in case of a great number of criteria, this relation is really poor since comparing alternatives $a$ and $b$, $a$ is preferred to $b$ on some criteria while $b$ is preferred to $a$ on the remaining ones. For this reason, one needs to aggregate the evaluations of the alternatives to get some recommendations with respect to the problem at hand. Three different ways of aggregating the evaluations are the most known in MCDM:

- assigning a real number to each alternative being representative of its degree of desirability as in the Multiple Attribute Value Theory (MAVT) (Keeney

[1] An alternative $a$ dominates an alternative $b$ if $a$ is at least as good as $b$ on all criteria and better on at least one criterion.

and Raiffa, 1993);

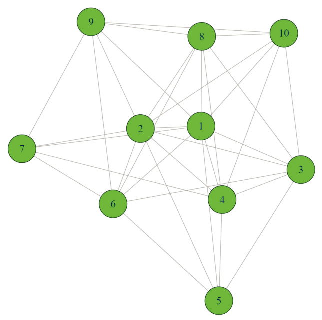
- comparing alternatives pairwise by means of binary relations, to check if one is at least as good as the other or viceversa as in the outranking methods (Brans and Vincke, 1985; Figueira et al., 2013);
- using a set of "*if...then*" decision rules as in the Dominance Based Rough Set (DRSA) approach that, starting from preferences provided by the DM, induces some rules expressed in a natural language (Greco et al., 2001).

The first two families of aggregation methods are based on some parameters such as weights of criteria, marginal value functions, indifference, preference and veto thresholds, etc. that can be obtained in a direct or in an indirect way. In the first case, the decision maker is able to provide directly values to all of these parameters, while in the second one the decision maker provides some preference on reference alternatives, from which parameters compatible with these preferences can be elicited. Since the direct preference information involves a great cognitive effort from the part of the decision maker, the indirect technique is the most used in practice (Jacquet-Lagrèze and Siskos, 2001). In the indirect way of providing preference information and calling *compatible model* a set of value parameters restoring the preferences provided by the decision maker, more than one model could be compatible with them. Each of these models provides the same recommendations on the reference alternatives but different recommendations on the other alternatives on which the decision maker did not provide any preference information. Since, using only one compatible model can be considered arbitrary to some extent, Robust Ordinal Regression (ROR) (Corrente et al., 2014; Greco et al., 2008) takes into account simultaneously all models compatible with the preferences provided by the DM building a necessary and a possible preference relation. The necessary and possible preference relations hold between two alternatives $a$ and $b$ if $a$ is at least as good as $b$ for all or for at least one compatible model, respectively.

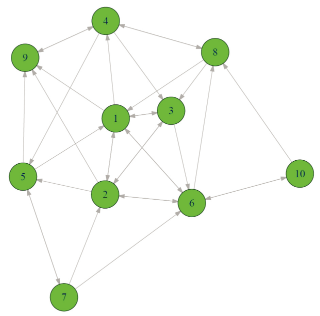
### 2.3.4 Social Networking for Decision Making

As previously introduced, a decision making process could regard an individual or a group of individuals. Even if it is an individual decision making process, to make the more realistic representation of the process, the decision maker is not usually isolated, but it is necessary to consider him as a single part of a network of social relationships. A network is then composed of different parts: entities (actors, ideas, attributes), relations (link, ties) and aggregations (dyads, triads). In order to give a more formally definition, *a network $\mathscr{N}$ consists of a finite set N of entities, called nodes or vertices, denoted by $\{n_1, n_2, ..., n_g\}$ and a set of relations L, called links or edges, $\{l_1, l_2, ..., l_l\}$ defined on the set of nodes.* To have a more immediate visual representation, a network can be represented as a graph, where the entities become nodes and the relations become edges (Cioffi-Revilla, 2013). Graph theory was firstly introduced by Leonhard Euler to solve the Königsberg bridge problem (Biggs et al., 1976) and from then it is used to solve many real practical problems regarding physics, biology, computer science and so on. The study of network started to develop in the 1920s and Social Network Analysis is born to study relationships among network entities also with the instruments provided by graph theory. Due to the complexity, dynamism, irregularity and the evolution along the time axis of the structures, they are very often named *complex networks* (Boccaletti et al., 2006).
Taking into consideration the definition of network given before, a graph can be indicated as $G(N, K)$. A graph can be classified as: undirected, directed and weighted. Considering two general nodes of the set $N$ denoted by $i$ and $j$, in an undirected graph the link connecting the two nodes is indicated as $l_{ij}$ and the nodes are re-

ferred as adjacent or neighboring, as it is indicated in Figure 2.5(a). In a directed graph the order of the two nodes is important and $l_{ij}$ indicates a link from node $i$ to node $j$ (graphically each link is represented from an arrow) and $l_{ij} \neq l_{ji}$.




(a) Undirected Graph




(b) Directed Graph

Figure 2.5: Graphs Typologies

In a weighted graph, that could be undirected or directed, to each link it is associated a weight $w_{i,j}$, that can represent importance, cost, distance (depending on the considered application case). To give a mathematical representation of a graph, it is used a matrix, called *adjacency matrix* $A$. Hence, a graph $G(N, K)$ can be represented by a square matrix $A$ having dimension $N \times N$ whose elements $a_{ij}$ (i,j=1,...N) is equal to 1 if the link $l_{ij}$ exists, 0 otherwise (Boccaletti et al., 2006).
More in detail, there are two kind of measures characterising the network: nodal measures (micro-level measures) and network measures (macro-level measures) (Cioffi-Revilla, 2013). To the first category belong the centrality measures: degree, closeness, betweenness, eigenvector.
Degree centrality for a generic node $i$ of the network $N$ is defined as (Freeman, 1978):

$$C_i = \frac{\sum_{j \in G} a_{ij}}{N - 1} \tag{2.6}$$

This measure is based on the concept that the most important node of the network has to be the most active, having the greater number of connections.

Closeness centrality is defined as (Freeman, 1978; Wasserman and Faust, 1994):

$$C_i = \frac{N-1}{\sum_{j \in G d_{ij}}} \tag{2.7}$$

where $d_{ij}$ is the geodesic distance (minimal number of links that connects node $i$ to node $j$) between nodes $i$ and $j$, i.e. the minimum number of edges from $i$ to $j$. According to the previous definition a node is central if it can communicate with all the others, not only with its neighbor.

To define betweenness centrality for the node $i$ of the network $N$, it is necessary to consider other two nodes $j$ and $k$. It is defined as (Freeman, 1977; Freeman, 1978):

$$C_i = \frac{\frac{\sum_{j<k \in G} n_{jk}(i)}{n_{jk}}}{(N-1)(N-2)} \tag{2.8}$$

where $n_{jk}$ is the number of geodesic between the two nodes $j$ and $k$ and $n_{jk}(i)$ is the number of geodesics in which node $i$ is contained. This measure considers a node central if it lies between many of the other nodes of the network or, more specifically, if it is a bridge among the shortest path between $j$ and $k$.

Eigenvector centrality for a node $i$ is defined as (Bonacich, 1987)

$$\lambda e_i = \sum_j A_{ij} e_j \tag{2.9}$$

or, using a matrix notation:

$$\lambda e = Ae \tag{2.10}$$

where $e$ is an eigenvector of $A$ and $\lambda$ is the associated eigenvalue. The largest eigenvalue is the preferred one. This measure calculates the node's centrality as its summed connections to others, weighted by their centralities.

Instead, among the principal network measures it is possible to include: size, length,

density, average degree and compactness.
The size $S$ is defined as:

$$S = card(N) = |N| \tag{2.11}$$

that is the total number of nodes that constitutes the network and then the set $N$.
The length $L$ is defined as the number of links that constitutes the network and the set $L$. It is defined as:

$$L = card(L) = |L|\,. \tag{2.12}$$

Then it is possible to define the density $Q$ as:

$$Q = \frac{L}{S(S-1)} = \frac{L}{(S^2 - S)} \tag{2.13}$$

which represents the ratio between the number of links actually present and the number of possible links in the network $N$.
The average degree is a measure of the general connectedness of the nodes of the network $N$ and it is defined as:

$$\overline{\delta} = \frac{2L}{S} = Q(S-1) \tag{2.14}$$

Compactness is then defined as:

$$C = \frac{\sum_{i \neq j} \frac{1}{d_{ij}}}{S(S-1)} \tag{2.15}$$

$d_{ij}$ represents the dyadic distance in the network $N$ (Cioffi-Revilla, 2013).

## 2.4 Network Models

In the scientific literature there are three main network models which regroup most of the real systems.

### 2.4.1 Erdös-Rényi model

The first one is the Erdös-Rényi random graph model, firstly introduced by Paul Erdös and Alfréd Rényi in 1959 (Erdös and Rényi, 1959). There are two possible representation of random graphs. The first one, indicated as $G(n, m)$, is characterised by $n$ nodes and $m$ edges, e.g. $G(5, 3)$ is a graph composed by 5 nodes and 3 edges. The second one, instead, is indicated as $G(n, p)$ where $n$ is the number of nodes and $p$ represents the probability of having an edge between two nodes of the network (the lower the value of $p$, the smaller the number of connections in the network). It is important to highlight that each edge is generated in a uniformly random way. Furthermore, the probability that a node $i$ of a network composed of $n$ has a degree $k$ follows a binomial distribution:

$$\binom{n-1}{k} p^k (1-p)^{n-1-k} \tag{2.16}$$

Furthermore, for large value of $n$ this distribution becomes a Poisson distribution:

$$\frac{(\mu)^k \exp^{-\mu}}{k!} \tag{2.17}$$

where $\mu = np = cost$.
The Erdös-Rényi random graph model is not able to represent most of real systems. In fact in most of real systems the degree is not a Poisson distribution but follows a power-law degree distribution. Furthermore, it shows a very low clustering coefficient, in contrast to what happen in social networks.
Most of real systems are, in fact, composed of a lot of entities (nodes) and they are characterised by properties and constraints that makes difficult and complex its representation without losing important information.

Due to the interest on network theory of a lot of disciplines and the representation of biological, social and communication systems as a network, allows to discover that, despite their differences, all the systems show common properties (path lengths, degree distribution, clustering effects, etc). Two main models are able to represent real systems and, in particular, they are the Watts-Strogatz model and the Barabási-Albert model.

### 2.4.2 Watts and Strogatz model

Watts and Strogatz in 1998 (Watts and Strogatz, 1998) noticed that most networks, despite their large dimensions, had some links that connect more quickly different area of the network, allowing an acceleration of communications. This characteristic is named *small-world* property and the network is characterised by a high clustering coefficient and a short path length.
To implement a Watts-Strogatz model the starting point is a ring composed of $N$ nodes. Each node is connected to its $k$ neighbors. After that, each link that connects to a clockwise neighbor is rewired to a randomly chosen node with a probability $p$. If $p = 0$ the network created is a regular lattice, instead if $p = 1$ the graph obtained is totally random. In this case, the degree distribution for $p = 0$ is a delta function centered at $K$, while for $p = 1$ it is a Poisson distribution, as for the Erdös-Rényi model. For $0 < p < 1$ the degree distribution is as follows:

$$P(k) = \sum_{n=0}^{f(k,K)} C^{n}_{\frac{K}{2}} (1-p)^n p^{\frac{K}{2-n}} \frac{p\frac{K}{2}^{\left(k-\frac{K}{2}-n\right)}}{\left(\frac{k-K}{2-n}\right)!} \exp^{-p\frac{K}{2}} \tag{2.18}$$

The shape is similar to the random graph but it has a peak for $k = K$ and after this point it decays exponentially.

### 2.4.3 Barabási-Albert model

The third network model is represented by the Barabási-Albert model. The two scientists in 1999 (Barabási and Albert, 1999) proposed a network model whose generation and growth mechanism are similar to the World Wide Web. In fact this model is based on two key aspects: preferential attachment and growth mechanisms. The concept of growth indicates the increase in the number of nodes constituting the network over time.

The second aspect is the preferential attachment mechanism that can be explained by this affirmation "*rich-get-richer*". That is to say, nodes with high degree acquire connections at a higher rate compared to nodes with lower degree. Hence, considering a network composed by $m_0$ nodes, at each time step a new node joins the network with $m \leq m_0$ links. The probability that a link from the new node to $i$ will exist depends on the degree of $i$:

$$\Pi(k_i) = \frac{k_i}{\sum_l k_l} \tag{2.19}$$

where $l$ is the number of pre-existing nodes. After $t$ time steps the network will be composed by $N = m_0 + t$ nodes and $mt$ links. The degree distribution, which indicates that a node will interact with other $k$ other nodes decays with a power-law as follows:

$$P(k) \sim k^{-\gamma} \tag{2.20}$$

and after a long time period leads to a value of $\gamma$ equal to 3. The mechanisms of growth and preferential attachment explain and are adapt to describe many complex systems as social networks, business networks and transportation networks. Furthermore, these features can help to explain also some phenomena like economic disparities in society, due to the biased information that are available to the more visible nodes (richer) that leads to individual and local decisions that create inhomogeneities and disparities in the network.

Figures 2.6, 2.7, 2.8 represent three examples of the network models discussed

above. All the three models are characterised by a set of nodes composed by $n = 50$ and differ for the other parameters characterising the network.
In the Erdös-Rényi network model the probability of having a connection between two nodes $p$ is set equal to 0.3. In the Watts-Strogatz network model the rewiring probability is set equal to 0.05. At last, in the Barabási-Albert network model the preferential attachment is linear.

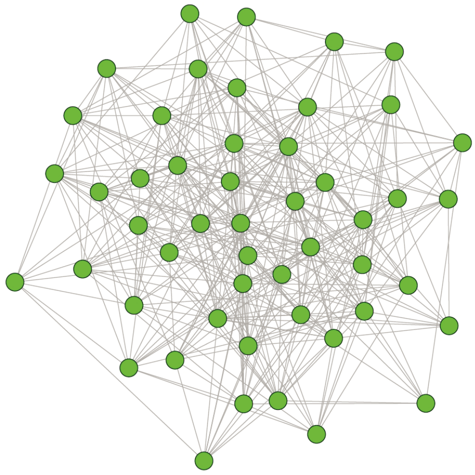


Figure 2.6: Erdös-Rényi network model with $n = 50$ and $p = 0.3$

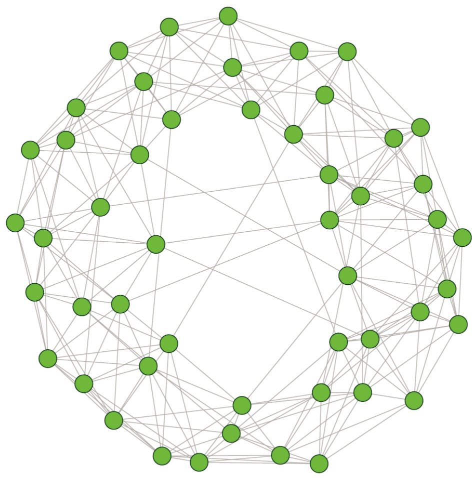


Figure 2.7: Watts and Strogatz network model with $n = 50$

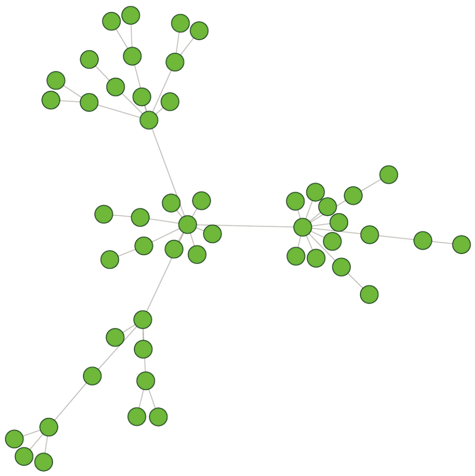


Figure 2.8: Barabási-Albert network model with $n = 50$ and linear preferential attachment

# Chapter 3

# A Dynamic Context-Aware Multiple Criteria Decision Making Model in Social Networks

## 3.1 Introduction

The future interest of ICT resides in study and analysis of innovative tools and methodologies to extract knowledge from many heterogeneous sources and processes mainly linked to social networks, smart environments and to the necessity to have big data available for complex bio-inspired analysis. The innovation is in the usage of the bio-inspired approach for the social networking, making nodes more human through the introduction of modules which operate with cognitive and smart capabilities. The ubiquitous and dynamic nature of the network requires smart entities (node, data, internet of things), which can decide using strategies linked to the specific context. These entities actively participate to dynamic of social influence and contagion (Christakis and Fowler, 2007), to interaction among communities and to decision process, which involves single entity or groups of them.

These characteristics of the network produce a large amount of data that have to be handled and organised with more complex and appropriate analysis. In fact Big data have changed the way to do business and also industry functions (Marz and Warren, 2015), also in order to improve decision making processes and the whole performance. Big data *"'refers to datasets whose size is beyond the ability of typical database software tools to capture, store, manage, and analyze"'* (Manyika et al., 2011). This huge amount of data is created by emails, videos, audios, online transactions, mobile sensors, social networks and so on. Big data have four principal characteristics: volume (and then complexity), variety, velocity (Douglas, 2001) and veracity (Data, 2015). The first one represents the data dimensions, but this is a necessary but not sufficient condition. In fact a large amount of data in terms of volume has not always the degree of complexity to be considered Big data with respect to a small dataset with a higher degree of complexity that can be considered as a big data. The second one refers to the different types of data that can be collected, ranging from videos, texts, images, report of online transactions and so on. On the basis of the variety, data can be divided in three categories: structured (coming from labelled data warehouse and easily classified), semi structured (where elements are separated by tags) and unstructured (not classified and organised). Velocity indicates the quickness to make available data for analysis (the greater velocity the greater the value that can be generated). Veracity indicates the integrity and trustworthiness of data in order to use them for their purpose (University, 2017) (Sagiroglu and Sinanc, 2013) (Zikopoulos et al., 2011).

The importance of big data is also represented by the the value that they can generate, giving transparency and usability of information with a very high rate. This permits the performance improvement (and then more profits) and more customised products, making processes (especially the decision making ones) more accurate and updated depending on the data collected. In this sense, with the large amount of data available also a prediction on the next generation products and services can be made (Manyika et al., 2011).

But, if on one hand big data generate benefits and value, on the other hand it is necessary to understand and comprehend the intrisic meaning of data. In fact, deploying new algorithms requires high skills in order to understand and extract the full meaning and insight of each single datum, always guaranteeing the respect of security and privacy.

Social networks are one of the source of big data. In fact in a social network the information regarding each single user is not only the personal one but concerns his interests and preferences and also the ones of his friends. In fact, as defined by (Cioffi-Revilla, 2013), a social network is formed by different parts, which include entities (actors, values, sentiment, ideas, etc..), relations (links, ties, etc..) and aggregations (dyads, triads, etc..). Sociality then can be considered as a paradigm which characterises the individuals life as every individual plays a role within a social community (Bottazzi et al., 2007). Each entity, the network node, with its personal knowledge together with its cognitive and reasoning capabilities, thinks, decides and acts inside a social network, characterised by the heterogeneity of nodes and relationships between them, so that each node is unique within the network. All these parameters contribute to determine the network complexity but also its structure and function.

Due to the dynamism and evolution of the network, all the processes become themselves complex and dynamic. In this context, decision-making process plays a central role, because the network node, as a result of each change of the network, ranging from its structure to its security, has to take decisions that will affect present and future network processes. But in a social context, where a node, the decision maker (DM), is not alone, a decision will be the output of the system that will consider not only the personal capabilities of the node, but also the influences from its neighboring nodes. These influences, in the proposed model, will alter the stimulus perception in a way that can be positive or negative, leading to a knowledge and a context-awareness that can be near or far from reality. The decision criteria will be affected by these dynamics, modifying its relations, both in terms of typology and importance according to time, decision's context and awareness. In this way the decision cannot be always the best one and, as a consequence, it could be dangerous for the node and for the network itself, e.g. in terms of security.

## 3.2 Multiple Criteria Decision Making with a social perspective

The decision-making process has been subject of study in a lot of disciplines, from psychology to mathematics, with the target to have a model that would serve to represent the individual or group decision-making process. In this way a significant support in a lot of fields can be given: from electronic and telecommunications to electrical engineer, from biology to psychology, etc... The decision-maker, both a human or a network node, makes choices among a set of alternatives, considered in different contexts and conditions. As defined in (Wang and Ruhe, 2007), a decision $d$ is the result of a choice of an alternative $a_j$ belonging to a set of $n$ alternatives $A = \{a_1, a_2, ..., a_n\}$, based on a set of criteria $C = \{c_1, c_2, ..., c_n\}$ to achieve one or more objectives.

So, in the decision-making process the set C of criteria plays a central and important role in addition to environment in which the decision has to be taken.

From the view of SNA, the social environment can be expressed as patterns or regularities in relationships among interacting units. The resulting structures are complex graphs connecting social contacts and, exploiting the graph theory, it is possible to describe these relationships using metrics structurally, using metrics, such as centrality measures, clustering coefficient, etc. Social network analysis produces a different perspective, where the relational ties among actors within the network are more important than the attributes of actors (Scatà et al., 2014). The behavioural dimension means that the individuals actions have to be evaluated not in isolation, but considering the connections with the other nodes. Moreover all these structural and behavioural aspects have to be considered dynamically, so that connections and behaviours between nodes change over the time and space (Easley and Kleinberg, 2010). The study of social networks, primarily designed on the basis of the interactions between the different actors, is also a social influence analysis, as different interacting actors often influence one another in terms of their behaviour. On one hand, SNA means a study of network in terms of structure, links, relationships, and for this aim of understanding networks, graph theory helps to develop a language for talking about the typical structural features of networks. On the other hand, SNA goes beyond the structure, who is linked whom, looking for behaviours, the fact that the nodes actions have implicit consequences for the outcomes of everyone in the system. For this reason, in addition to a language for discussing the structure of networks, we also need a framework for reasoning about behaviour and interaction in network contexts (Easley and Kleinberg, 2010). SNA is functional to the decision-making process, as it allows to discover social patterns, behaviours, and structural properties which influence the strategies and the decisions of the single node, as a connected entity, and of the clusters it belongs to.

## 3.3 Dynamic context-aware decision model in a social network

### 3.3.1 Context-Awareness in a social network

Considering a node $n_1$ in a social network $\mathscr{N}$, composed of a finite set of $N$ entities, a set of relations for $n_1$ can be defined, characterised by an intensity, an importance, an influence level and an uniqueness within the network.

When $n_1$ is subjected to a stimulus, its perception can vary and may be influenced not only by its personal capabilities but also by what it receives from the rest of the network. These causes influence the actions of the node and they can be represented in the process as a noise, which intensity is different depending on the level of influence. This noise alters, in a positive or negative way, the stimulus perception as described by the Weber-Fechner Law (Dehaene, 2003). So, it is possible to distinguish two types of noise:

- "Constructive" noise: it contains and conveys information, and it exercises a positive influence on the node behaviour and actions. This can be defined as a Positive Awareness;
- "Destructive" noise: it influences in a negative way because it conveys only disorder. The stimulus perception is altered and not enriched of information. This can be defined as a Negative Awareness.

So for a network $\mathscr{N}$, in addition to the adjacency matrix $A_{ij}$, it is possible to define a new network matrix, the Awareness Matrix $AW^K_{i\leftrightarrows j}$, that represents the awareness that each single node perceives from all the rest of the network to which it is connected. As the expression suggests, the awareness matrix is defined for elements

of the space of decisions context $K = \{K_1, K_2, .....K_n\}$. Each element $aw^K_{i\to j}$ of the matrix $AW^K_{i\leftrightarrows j}$ is characterised by a magnitude, a sign($\pm$), and, in a graph, it is represented by an arrow with a versus, as it will be reported in the following. Each element is not symmetric, so the awareness that the *i-th* node has on the *j-th* node is different from the awareness that the *j-th* has on the *i-th* node ($aw^K_{i\to j} \neq aw^K_{i\leftarrow j}$).
In order to evaluate the awareness on the node $i$ from the other nodes $j$, in a decision-making process of the node $i$, given a particular decisional context $K_k$, we have to take into account both the similarity measure (homophily), which considers metrics to evaluate bio-inspired features of each node and in terms of genotype-phenotype, and also the centrality measures of the connected nodes to the node $i$, with regards to the context $K_k$, so which nodes are best connected to others or have most influence. It is important to note that a node $j$ could be central in a particular decisional context $K_k$, but not in another one ($K_q$), so the influence on the node $i$ from the node $j$ could be negligible if different decisional contexts are considered.
The awareness from node $i$ to $j$, given the decisional context $K_k$, depends on various parameters: the similarity between $i$ and $j$, the centrality of the node $i$ in the context $K_k$, and the centrality of the node $i$ in its community. Severe centrality measures have been proposed over the time to quantify the importance and so the influence produced by a node in a social network. These measures are based on two different conceptual ideas and therefore we can distinguish two classes of centrality measures (Latora and Marchiori, 2004). In the first class the centrality of a node in a network is related to how is it near to the other nodes (degree and closeness centralities). The second class of centrality measures is based on the idea that central nodes stand between others on the path of communication, these centrality measures include betweenness, eigenvector and Katz centralities. The information centrality (Latora and Marchiori, 2004), which is a combination of the two ideas of centrality

discussed above, takes into account the efficient propagation of information over the network, so it is the ability of the network to respond to the deactivation of the node. Centralization is the process by which the activities of an organization, in particular those regarding planning decision-making, become concentrated within a particular location and/or a group. Another important aspect is the centrality in the clusters. In fact, one may notice that in each community there are usually some members (or leaders) which play a key role in that community while having the greatest structural importance in a network. Therefore, these leader nodes are better able to influence the nodes in the cluster even if their centrality could change according to the decisional context.

To weight awareness, the idea is to consider the different centrality measures and apply a new metric, starting from MCA (Multiple Centrality Assessment) (Porta et al., 2008), moving from spatial networks and from the metric computation of distances in the urban planning to influence networks, using the different centrality measures, creating an *influence map for decision-making.*

A central problem for social influence is to understand the interplay between similarity and social ties (Crandall et al., 2008). *Homophily* (Lazarsfeld et al., 1954) is one of the most fundamental characteristics of social networks. This suggests that an actor in the social network tends to be similar to their connected neighbors or "friends". The phenomenon of *homophily* can originate from many different mechanisms: (a) social influence: this indicates that people tend to follow the behaviours of their friends. The social influence effect leads people to adopt behaviours exhibited by their neighbors; (b) selection: this indicates that people tend to create relationships with other people who are already similar to them; (c) confounding variables: other unknown variables exist, which may cause friends to behave similarly with one another. These three factors are often interweaved in real social networks, and the overall effect is to provide a strong support for the *homophily* phenomenon.

*Social influence* refers to the behavioural change of individuals affected by the others in a network. Social influence is an intuitive and well-accepted phenomenon in social networks (Easley and Kleinberg, 2010). The strength of social influence depends on many factors such as the strength of relationships between people in the network, the network distance between users, temporal effects, characteristics of network and individuals in the network. So, in a social context, the data that an individual perceives about a situation or an event are enriched, but sometimes also distorted, by the level of awareness that it receives from its neighborhood. It is also important to stress the importance of the variable time $t$ when analyzing and modeling the stimulus perception and the influence of the network (Norwich, 1993). Time cannot be supposed to be a constant variable, because the stimuli perception is deeply linked with the time instant in which it is considered.

### 3.3.2 Dynamic criteria interaction

In a multiple criteria decision-making process, the criteria have not always the same priority and they are not independent each other (Yu et al., 2013). Hence, in this case, if a criterion $c_1$ has a priority that is higher than the criterion $c_2$, an alternative will not be chosen until the decision maker will not have a minimal level of satisfaction to $c_1$, and in particular it is not sufficient to have only a gain in criteria $c_2$ (Yager, 2004). Especially in a social network, where a node receives influences by its neighborhood, the criteria relations and dominance are subjected to change dynamically following the network evolution. As for the representation of human belief systems (Cioffi-Revilla, 2013), the criteria can be represented as a network, in which each node is a decision's criterion of the cognitive system and each edge represents a cognitive association among the criteria. Dynamically, they are all connected, weakly or strongly, depending on these three important dimensions: Time (T), Decision's Context (K) and Awareness (AW).

In Fig. 3.1 it is represented an example considering a set $C = \{c_1, c_2, c_3, c_4, c_5, c_6\}$ of six decision's criteria, whose relations vary along the three axis. To give an example, considering the criterion $c_1$, in the first block, it is connected only with $c_2$ and $c_3$, instead in the second one it is connected with $c_4$, $c_5$, and $c_6$ losing all the connections with $c_2$ and $c_3$.

The evaluation of alternatives in a decision-making process at a given time instant $t_1$, will depend on the level of awareness and on the decision's context that will determine which criterion prevails among the others. The explanation is that the social network evolves along the temporal dimensions, modifying its structure, adding new nodes and cutting off others, and functionality. So, the way to perform each process and, in this case, a decision-making process, depends on when it takes place, because the priority and the dominance of a criterion over all the others and their ties can vary substantially.

The personal social relationships determine the awareness of a node in the network and, in conjunction with the decisions context considered, modify the criteria relations and dominance. For example, considering a context $K_1$ at a time $t_1$, where most of the network nodes consider more important a criterion $c_1$ rather than all the others, this affects the way for a single node to perceive the world, to decide and to act, leading it, probably, to conform to the others, through processes of adaptation (Cioffi-Revilla, 2013) and social contagion (Christakis and Fowler, 2007). On the contrary, in a context $K_2$ and at a time $t_2$, the criterion $c_4$ may have most importance and acquires a greater level of importance in the network. All these three dimensions depend on each other and affect the personal perception of the world, creating images from which each individual extracts data regarding the real world

(Cioffi-Revilla, 2013) and the problem taken into account.

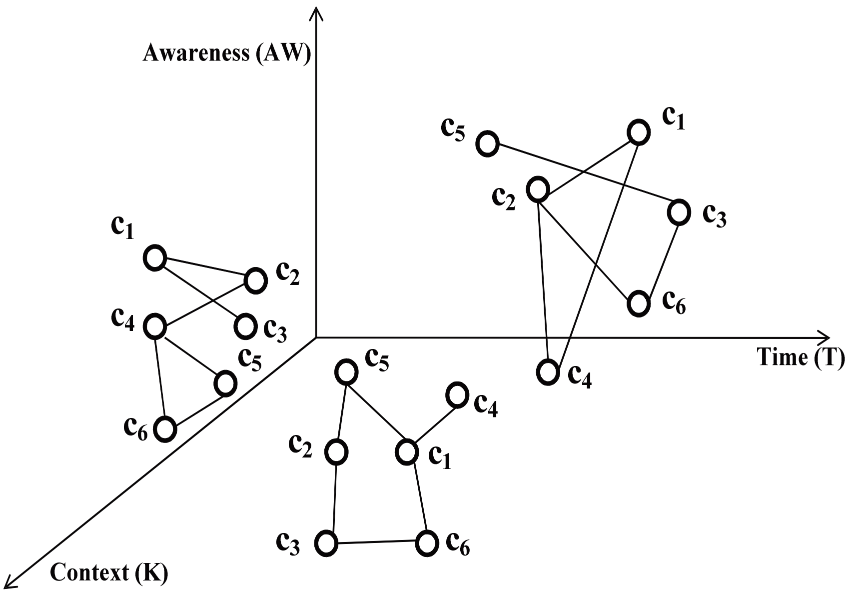



Figure 3.1: Modification of criteria relationships depending on time, decision's context and awareness

### 3.3.3 Decision making model

As said previously, each decisional process is strictly connected to the elements of the space $K$ of the decisions context in which it is considered. An element $K_1 \in K$ may have contexts closer or more distant, through which influences can spatially propagate. For example, it is possible to consider a network portion $\mathscr{N}_1$, as reported in Fig. 3.2, composed by a set $N = \{A, B, C, D, E, F\}$ of six nodes. For each node of the network all the metrics that characterise itself are defined, such as centrality, betweenness, degree, etc.. All of these also contribute to define the role of the node within the network. At a network level, the adjacency matrix for $\mathscr{N}_1$ is the following:

$$A_{ij} = \begin{bmatrix} - & 1 & 0 & 1 & 1 & 1 \\ 1 & - & 1 & 1 & 1 & 0 \\ 0 & 1 & - & 1 & 0 & 1 \\ 1 & 1 & 1 & - & 1 & 0 \\ 1 & 1 & 0 & 1 & - & 1 \\ 1 & 0 & 1 & 0 & 1 & - \end{bmatrix}$$

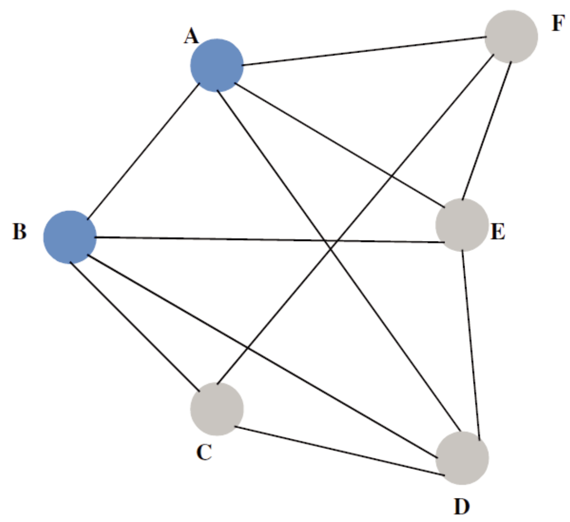



Figure 3.2: Example of network with $N = 6$ nodes. The two different colors represent the levels of similarity among the network nodes. In this case, $A$ is similar only to $B$, instead $C$, $D$, $E$ and $F$ are similar each other.

But, considering the space $K$, it is possible to introduce a greater level of accuracy, diversifying each relationship among all the nodes, according to the element $K_k \in K$, in order to understand how and why certain decisions have been taken. In Fig. 3.3, the same 6 nodes are considered in four different situations: $K_1$, $K_2$, $K_3$, that are contexts belonging to space $K$ and the fourth in which they do not belong to any context of the space $K$.

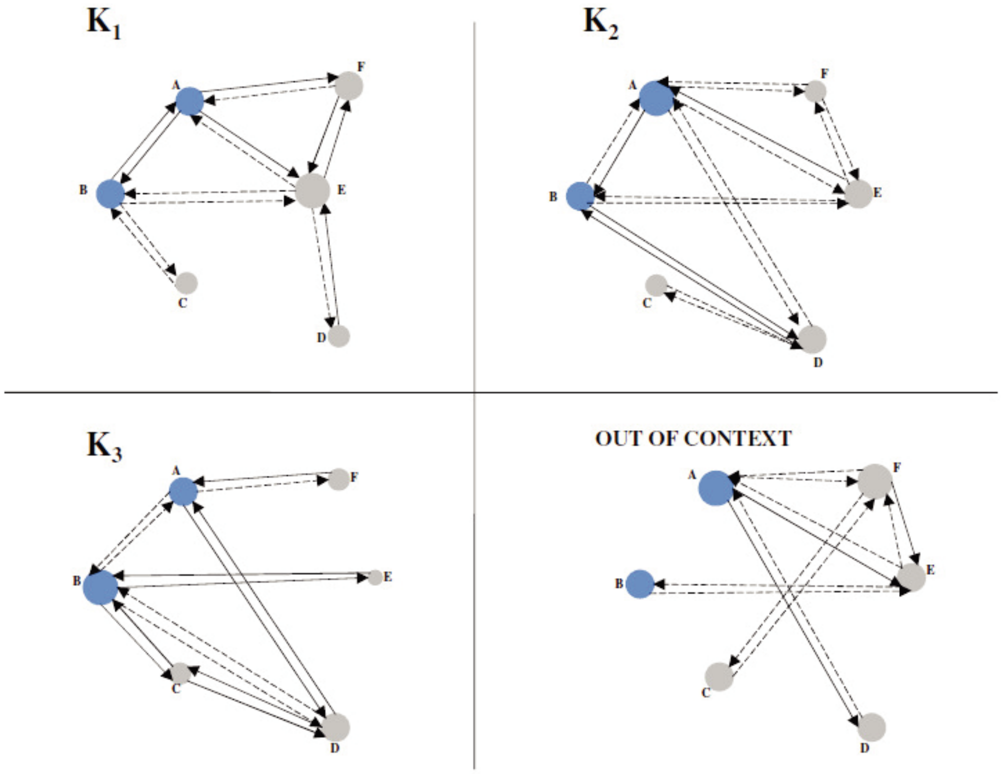



Figure 3.3: Network representation in terms of awareness. The dashed line represents a negative awareness, instead the continuous line represents a positive awareness. The different dimensions of the node represent its centrality $C_i^K$ depending on the context.

Hence, each context modifies the network structure and its relations, giving a significant contribution to the modification of the entire network, the node parameters and measures, changing how the decision-making process performs and then the decision itself. For more than one context, e.g. $K_1$, $K_2$ and $K_3$, the awareness matrix can be defined, but here, for the sake of simplicity, it is defined only the awareness matrix for the context $K_1$:

$$AW_{ij}^{K_1} = \begin{bmatrix} - & +aw_{A\to B}^{K_1} & 0 & 0 & +aw_{A\to E}^{K_1} & +aw_{A\to F}^{K_1} \\ +aw_{B\to A}^{K_1} & - & -aw_{B\to C}^{K_1} & 0 & -aw_{B\to E}^{K_1} & 0 \\ 0 & -aw_{C\to B}^{K_1} & - & 0 & 0 & 0 \\ 0 & 0 & 0 & - & +aw_{D\to E}^{K_1} & 0 \\ -aw_{E\to A}^{K_1} & -aw_{E\to B}^{K_1} & 0 & -aw_{E\to D}^{K_1} & - & +aw_{E\to F}^{K_1} \\ -aw_{F\to A}^{K_1} & 0 & 0 & 0 & +aw_{F\to E}^{K_1} & - \end{bmatrix}$$

For each element $K_k \in K$, the block diagram in Fig. 3.4 represents how the decision-making process takes place, according to what expressed in the previous paragraphs.

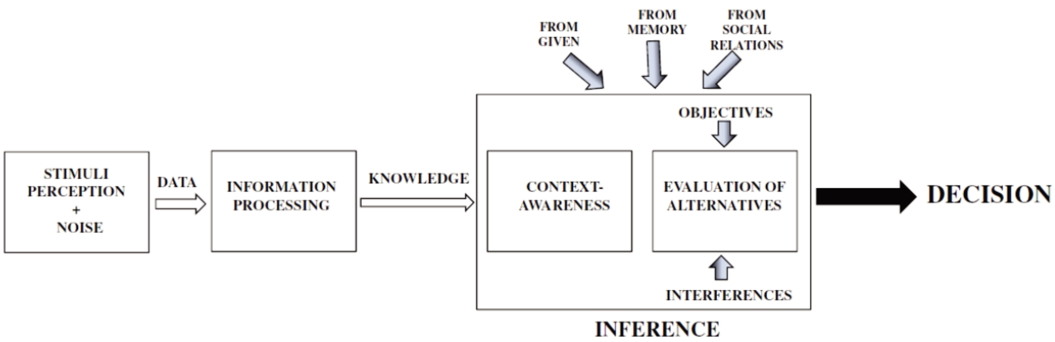



Figure 3.4: Dynamic Context-Aware Multiple Criteria Decision-Making Process

For each block, are listed in the following the functionalities:

- Stimuli perception + Noise: It is the block that gives as output the data regarding the problem. These data can represent a true or a distort image of reality and of the problem itself, depending on the influences received from the social network;

- Information Processing: considering a layered architecture, from the incoming data, information, having a central role to understand the process is extracted in order to create knowledge and a greater awareness about the problem considered;

- Inference: this block is composed by context-awareness and the evaluation of alternatives. Having the knowledge about the problem and the real world, the node becomes aware of the context. The node acquires the ability to adapt according to the location, the collection of neighboring nodes, hosts and accessible devices, as well as to changes to such things over the time. So the node will be able to define where it is, with whom it is and what resources it has (Schilit et al., 1994). So, at this time, defined a space of context $K = \{K_1, K_2, .....K_k\}$ for a network $\mathscr{N}$ composed by a set $N = \{n_1, n_2, .....n_n\}$ of nodes and a set $L = \{l_1, l_2, .....l_l\}$ of relationships, it is possible to define a function $\phi_i^K(t)$, which represents the decisions state of the node at a time $t$:

$$\phi_i^K(t) = \phi_i^K(0) + A_{ij} \cdot \sum_{j=1}^{N-1} |AW_{ij}^K|$$

where $\phi_i^K(0)$ represents the initial condition, $A_ij$ is the adjacency matrix of $\mathscr{N}$, $AW_{ij}^K$ is the awareness matrix. The state of the node at a time $t$ will vary the criteria relations and positions in a scale of dominance, influencing the decision. It is important to highlight that the state of the node cannot be the real representation of the problem. After that, defined the objectives and become aware of its state, the node can evaluate the alternatives, taking into account the possible interferences during the inference process.

There are two types of inference, as defined in (Di Stefano et al., 2013), (Gigerenzer and Goldstein, 1996):

- Inference from given: decisions are taken considering data and information extracted from a calculation or data extracted from an experiment;

- Inference from memory: decisions are taken considering declared knowledge, studies, memory and history.

At this time, we add another kind of inference, as it follows:

- Inference from social relationships: decisions are not only a consequence of our history and/or information acquired, but they are related to cue values deriving also from the social influence produced by the social relationships and the dynamical and structural properties of the network, such as similarity and centrality measures, which characterise the awareness of a node in the network.

## 3.4 Summary remarks

In this chapter it has been described a new paradigm of decision-making process. It can be considered a first research of the conjunction point among the mathematical model of multiple criteria decision analysis and the social network analysis in order to build a tool useful for the analysis of network dynamics, exploitable to predict individual or community behaviour and decisions, while varying the initial conditions and the structure of the network. Social networks, in fact, have had and continue to have a rapid growth in popularity, size and complexity, allowing network nodes to create new connections and relationships. Hence, a huge amount of information and then knowledge is generated, allowing individuals to acquire more and more context-awareness, positive or negative. The model presented in this chapter has its core idea in taking into account the dynamics and context-awareness. In fact, the model dynamic is represented by the variation of decision's criteria relationships due to the changes of the awareness matrix which, in turn, varies based on the bio-inspired features and structure of the node and of the network.

# Chapter 4

# A context-aware and social model of dynamic multiple criteria preferences

## 4.1 Introduction

In order to extract knowledge from many heterogeneous sources and to study the phenomena within a social network, it becomes fundamental to analyse in detail each process that takes place within it. Indeed, every process that involves a certain number of entities, cannot be analysed only in a macroscopic way, because each single entity contributes to establish the path to follow for the whole system. Looking much more in depth, due to the interactions and the relationships within the network, each entity is influenced in its opinion and, consequently, in its actions (Asavathiratham et al., 2001; Grabisch and Rusinowska, 2010a; Grabisch and Rusinowska, 2010b; Barjis et al., 2011; Pachidi et al., 2014).

As reported in (López-Pintado, 2008), *individual decisions are often influenced by* *the decisions of other individuals*. Without considering any interaction, nodes, involved in a decision making process, easily rank criteria in terms of importance following an individual cognitive model (Korhonen and Wallenius, 1997). However, every individual cannot be considered as an isolated entity deliberating carefully considered decision. Instead the behaviour of each entity is the result of the interaction between its preferences and the dynamic social effects that affect every individual decision (Pentland, 2014). These processes of influence are present in different social phenomena such as diffusion of innovations, cultural fads, local variability in crime activities and other conventions that share the contagion logic. As a consequence of the influence exercised by the nodes in the network, the preferences of each node can change during the decision making process bringing therefore to different decisions at different time instants. Focusing on the psychological, social and behavioural aspects of a decision making problem, as done in the Behavioural Operational Research (BOR), would help in making a better use of operational research models (Hämäläinen et al., 2013). A first input to this research field has been done proposing an interactive multiple criteria decision making method, paying much more attention to the behavioural realities of decision making (Korhonen et al., 1990). This is why many subjects such as economics, finance and game theory have addressed their attention to the behavioural research topics (Ackert and Deaves, 2009; Camerer et al., 2003).

As highlighted in the previous chapter, a fundamental aspect in each decision making process is represented by the context in which the decision has to be taken. Although different definitions of context have been given in the literature (Liu et al., 2011), for the social decision-making model described below the most suitable is "*any information that can be used to characterise the situation of an entity*" (Abowd et al., 1999). As a consequence, the decisions taken from each node are not dependent on

its preferences only but also on the context in which the decisions have to be taken and, more specifically, on the context-awareness. Indeed, a different awareness of the context in which the decision making process takes place can bring to different decisions. For this reason, it is necessary for the single node taking into account the context and, in particular, its variability.
In particular, it has been considered a decision making problem in which a node has to decide among different alternatives evaluated on the basis of several evaluation criteria. The evaluation criteria represent different points of view taken into account by the Decision Maker (DM) to highlight the advantages and the drawbacks of each single alternative (Figueira et al., 2005). The model is based on a weighted sum in which a weight is assigned to each evaluation criterion. The weight represents the importance assigned to the criterion by the node.
This model presents two novelties:

- The variability of the weights of criteria depending on the context in which the decision has to be taken;

- The variability of the context during the time.

On one hand, it is assumed that the preferences of each node in the network and, consequently, the weights assigned to the different criteria are not fixed during the decision making process. The preferences of the single nodes will evolve depending on their inclination to be more or less influenced by the other components of the network. In particular, the preferences will change according to the similarity between nodes. The more the preferences, expressed in terms of weights and past choices, are similar between nodes $n_h$ and $n_k$, the more the nodes will influence each other.
On the other hand, we assume that the decisions taken by the nodes in the network

at previous instants can influence the decision of each node at the current time. Indeed, the consideration of the decisions taken previously by the components of the network brings the node to be more or less aware of the context in which the decision has to be taken.
We shall point out that the inclination of each node to be influenced by the other components of the network, as well as the consideration of the context, cause different dynamics of the decisions taken by the nodes showing that the two different aspects are therefore really relevant in a decision making process.
The proposed model could be applied to different network structures. For this reason, the model has been applied to the celebrated El Farol bar problem (Arthur, 1994), supposing that the network follows, on one hand, the Erdös-Rényi model (Erdös and Rényi, 1959) and, on the other hand, the Barabási-Albert model (Barabási and Albert, 1999). In the following, for the sake of simplicity, the abbreviation ER model and BA model will be used instead of Erdös-Rényi model and Barabási-Albert model.

## 4.2 Social Networking and model of influence in Decision-Making Process

Several research works have analysed the importance of social networking in the decision making process. Some of these underline the problem of influence inside networks. In particular, recently some scientists have considered the influence maximisation in viral marketing applications, in which competing entities try to expand their market and maximise their share (Kempe et al., 2015). To give an example a model for the diffusion of competing alternatives in a social network, in which nodes decide between some different alternatives has been presented (Anagnostopoulos et al., 2015). Nodes usually interact and influence each other, furthermore this influence is not only restricted to the connected neighbours, but also includes those nodes affected by their behaviours, due to social connectedness and contagion (Christakis and Fowler, 2013). Social network analysis allows to describe and analyse the interconnections among individuals and how these relationships drive the processes and phenomena inside the network. Therefore, it represents a central analytical tool for understanding the dynamics and diffusion of social behaviours. It allows to unveil how highly connected systems and entities, which form a complex social structure, operate (Aggarwal, 2011). In terms of network theory, nodes represent the individual actors, while ties, referred also as edges, links, or connections, are the relationships among individuals. The resulting structures could be different complex graphs. For this reason, graph theory could be applied to describe structurally the relationships between nodes using metrics, such as betweenness, centrality, degree, closeness, clustering coefficient, community detection, etc (Fortunato, 2010; Wasserman and Faust, 1994). The power of social network analysis is that it produces a different view, where the attributes of individuals are less important than their relationships and ties with other actors within the network. Furthermore, the behavioural dimension means that the individual's actions have to be evaluated not in isolation, but considering the connections with the other players, who can use different strategies (Easley and Kleinberg, 2010). All these structural and behavioural aspects have to cope with the network dynamics, so that connections and behaviours between nodes change over the time.

Large amount of data is available for the case of online social networks. Thus these networks have become much more robust in terms of statistical significance and useful for the verification of some structural properties, such as the small world phenomenon (Watts and Strogatz, 1998), preferential attachment (Barabási and Albert, 1999), and other structural dynamics.

From the decision-making perspective, these relationships, together with the structural properties of the network, could affect the node's decisions also more than the features of the single individual, when considered isolated. Some research studies shed light on the difference between the social mechanisms represented by social selection or homophily (similarity breeds connection (McPherson et al., 2001) (Di Stefano et al., 2015)), and influence (the tendency for characteristics and behaviours to spread through social ties such that friends increasingly resemble one another over time, and this influence may affect the choices (Lewis et al., 2012)). Recent empirical analysis with social network data has suggested that social influence plays an important role in the spread of some behaviours and psychological states (Cacioppo et al., 2009),(Christakis and Fowler, 2007),(Christakis and Fowler, 2008). In fact, Christakis and Fowler have suggested how social influence is significant in some phenomena, such as the spread of obesity, smoking, or happiness. These behaviours spread through the network, producing a social contagion effect. More than social influence, this social contagion process is able to amplify the spread of information in a social network, and this is the reason why understanding the mechanics of social contagion is crucial to predict how far it will spread and with what intensity.

In terms of decision making, social influence mechanisms have been analysed in several works. In (Hoede and Bakker, 1982), an acceptance-rejection decision-making problem, in which each node in a social network has to choose between being in favor or against a certain decision, is taken into account. The basic assumption is that each node has its own inclination towards a certain decision and its final choice can be different from this inclination, due to the influence of the other nodes in the network. Not taking into account any external cause of influence, the final choice of the node can be different from its inclination only due to the influence exercised from the other nodes in the network. The social influence mechanism can also be described, taking into account the example reported in the following (Rusinowska and de Swart, 2006). The authors use a vector to represent the inclination of each node; different values imply different degrees of influence among nodes. In another work(Grabisch and Rusinowska, 2010a), a direct and an opposite influence are defined. On one hand, the direct influence is ruled from a coalition when it succeeds in leading the decision of a single node to the coalition's inclination, even if the node's inclination was different. On the other hand, a coalition exercises an opposite influence on a single node when, although they have the same inclination, the node decides in a different way. In contrast with the work previous cited, in another model the authors considered that the node has an ordered set of possible actions to choose from, and the concepts of direct and opposite influences are generalised using the concepts of positive and negative influences. The positive influence measures the attraction carried on by a coalition on a node having at the beginning an inclination different from that one of the coalition while, eventually, the final choice is closer to the coalition's inclination. The negative influence is instead exercised in the opposite direction, but also in this case the final decision is a consequence of the coalition's influence (Grabisch and Rusinowska, 2010b).

The mechanism of social influence is also considered in the model suitable for large and complex engineered networks, like power grids, communication networks, etc. In the model, the authors try to understand the basic features of the network's global behaviour and the reason of certain spatial and temporal organisation. Based on a network of interacting Markov chains, where each chain corresponds to a node of the network and it is admitted that each chain can differ from one site to another, one of the most relevant aspects is the influence of the other chains on the dynamics of each one related to its neighboring sites on the network (Asavathiratham et al., 2001). In the same direction, a dynamic model is presented in which the choice of a node to adopt or not a particular behaviour is a function of the actions made by its neighbors, which are a random sample of the total population in the previous step (López-Pintado, 2008).

### 4.2.1 A unified framework of Social Influence and Context-Awareness

The model presented in chapter 3 is examined in depth and the analytical formulation is given. To make a brief summary of the key aspects of the model, it is important to highlight that each process taking place in a social network is characterised by two main features: complexity and dynamism. To perform the different processes, each node has to take into account not only its personal knowledge but also the influences perceived from its surroundings. Considering this scenario, every decision will be the result of a complex and dynamical process, affecting the present and the future status of the node.
Consequently, given a set of alternatives $A$ and a set of criteria $G$ within social network, the definition of decision (Wang and Ruhe, 2007)

$$d = f(A, G) = f : A \times G \to A \tag{4.1}$$

has to be extended including the influence that a node can have on the preferences of the other nodes in the network. In this context, it is important to distinguish between positive and negative influences. On one hand, a node $x$ positively influences a node $y$, if $x$ supports $y$ in its decisions while, on the other hand, $x$ negatively influences $y$, if $x$ acts leading $y$ to wrong decisions. For this reason, before making its decisions, the node has to become aware of what it has nearby and what resources it has. A crucial part of the decision process is therefore the context-awareness, that allows a node to make an aware cognitive decision on the basis of the available information.

Furthermore, the previous model took into account a particular scheme of the decision's criteria that, as a result of the interaction among nodes in the network, can assume different priorities depending on three dimensions: Time, Context and Awareness. For example, in a context $K_1$ and at time $t_1$, as a consequence of the interactions among the nodes in the network and through processes of adaptation (Cioffi-Revilla, 2013) and social contagion (Christakis and Fowler, 2007), criterion $g^{i_1}$ may be perceived as the most important one, while in another context $K_2$ and at a different time $t_2$, criterion $g^{i_2}$ may be considered as the most important one.

So, applying this assumptions, the inference process not only has a dependence from given and from memory as indicated in (Gigerenzer and Goldstein, 1996), but also from social relationships, determined by the network properties and structure and by the perceived influences.

## 4.3 A Dynamic Multiple Criteria Decision Making model with a social perspective

As previously described, a social decision making process is characterised by two fundamental aspects, that is, the dynamism and the context-awareness.

We shall suppose that $m$ nodes are individually involved in a decision making choice problem in which a finite set of alternatives is evaluated with respect to $p$ criteria.

In the description of our model we shall use the following notation:

- $N = \{n_1, \ldots, n_h, \ldots, n_m\}$, a finite set of nodes;
- $A = \{a, b, \ldots\}$, a finite set of alternatives;

- $G = \{g^1, \ldots\ldots, g^p\}$, a finite set of criteria.

Since each decision depends on the context in which it has to be taken and, as defined in (Abowd et al., 1999), context is dependent on the information at hand that varies over time, the variability of the context should be taken into account in each decision problem. For this reason, in our model we consider a further criterion $g^{p+1}$, such that $g^{p+1}(a;t)$ is the evaluation of $a$ relative to the considered context at time $t$. This evaluation is not fixed over time but it varies according to the variability of the context, following a rule that we shall describe later. Note that the introduction of criterion $g^{p+1}$ implies that the new set of criteria that has to be considered in the decision problem is $G = \{g^1, \ldots\ldots, g^p, g^{p+1}\}$.

The preferences of node $n_h$ are represented by the vector of weights $w_h = \left(w_h^1, \ldots, w_h^p, w_h^{p+1}\right)$, where $w_h^i$ is the importance given to criterion $g^i$ by node $n_h$. As can be noticed, we introduced also the weight $w_h^{p+1}$ of criterion $g^{p+1}$, which represents the importance given by node $n_h$ to the variability of the context and being dependent on its context-awareness.

**Definition 4.3.1.** *Given a node $n_h$, its vector of weights $w_h = \left(w_h^1, \ldots, w_h^p, w_h^{p+1}\right)$ and the vector $g(a;t) = (g^1(a), \ldots, g^i(a), \ldots, g^p(a), g^{p+1}(a;t))$ composed of the evaluations of alternative $a \in A$ at time $t$, the comprehensive value of $a$ is obtained as follows:*

$$U_h(a;t) = \sum_{i=1}^{p} \left[w_h^i \cdot g^i(a)\right] + w_h^{p+1} \cdot e^{-g^{p+1}(a;\, t)}. \tag{4.2}$$

On the basis of equation (4.2), we shall consider the preference relation $\succsim_h^t$ of node $n_h$ at time $t$ defined as follows:

$$a \succsim_h^t b \qquad \text{iff} \qquad U_h(a;t) \geq U_h(b;t), \qquad a, b \in A.$$

Consequently, node $n_h$ will choose the alternative $a \in A$ such that $a \succsim_h^t b$ for all $b \in A$, that is $U_h(a;t) = \max\limits_{b \in A} U_h(b;t)$. Without loss of generality, in equation (4.2) we shall suppose that criteria $g^1, \ldots, g^p$ have an increasing direction of preference while criterion $g^{p+1}$ has a decreasing direction of preference. In the following, we shall provide two examples to explain the meaning of the variability of the context and to justify the decreasing direction of preference of criterion $g^{p+1}$.

**Example 4.3.1.** *Suppose that a consumer has to buy a good, choosing it among a set of alternative goods. These goods are evaluated with respect to different criteria, such as quality, aesthetics and price. While the evaluation of the goods with respect to quality and aesthetics can be supposed constant over time, the price of the goods evolves in consequence of the choices made by the other DMs in the considered market. Such a variability of the price can be included in our model as the variability of the context in which the choice has to be made. Obviously, in this problem price will have a decreasing direction of preference.*

**Example 4.3.2.** *Being inspired by the El Farol bar problem (Arthur, 1994), suppose that a consumer has to choose a bar to spend the evening. The bars are evaluated on the basis of criteria such as location, quality of service and people attendance. Moreover, suppose that DMs prefer less crowded bar. While the location of the bars and the quality of the provided service can be supposed not variable, the frequency of people going in the bars changes over time in consequence of the choices made by the other DMs. In this case, the variability of frequency of people in each bar can be interpreted as the variability of the context. Furthermore, the preference of the DM for less crowded bars justifies the decreasing direction of preference of criterion* $g^{p+1}$.

**Note 4.3.1.** *it is worth noting that in the two examples above mentioned, a great number of nodes choosing an alternative a will affect negatively the comprehensive evaluation of a. In the first example, the increase of the price is obviously not appreciated by the buyer and, analogously, in the second example, the increase of the* *number of people going in the bar will reduce the appreciation of the customer for the same bar.*

*In some other cases, the increase of the number of nodes choosing an alternative a will affect positively the comprehensive evaluation of a. For example, in the fashion market, the increase of the number of people choosing a good will increase the appeal of the same good generating, therefore, an increase of the good demand and an imitation effect in the other buyers. In these cases, equation (4.2) should be modified replacing* $w_h^{p+1}e^{-g^{p+1}(a;t)}$ *with* $w_h^{p+1}\cdot\left(1-e^{-g^{p+1}(a;t)}\right)$ *so that the comprehensive evaluation of alternative a will increase due to the fact that several nodes have chosen this alternative. These cases have been studied in the models of herd behaviour (Avery and Zemsky, 1998; Banerjee, 1992; Bikhchandani et al., 1992; Brunnermeier, 2001). Of course we can have cases in which an attractive and a repulsive effect of increasing the number of customers can be simultaneously present.*

Being the node $n_h$ part of a network, its preferences can change during the decision making process as a consequence of the influence that the nodes in the network can exercise on it and on the node's inclination to be affected by these influences. On one hand, a node that is not influenced at all by any other node in the network will not change its preferences. On the other hand, a node more or less influenced by the other nodes will modify its preferences taking more into account the preferences of the nodes closer to it and the preferences of the nodes that have made similar decisions in the past. As a consequence of the previous remarks, the weight $w_h^i(t)$ assigned to criterion $g^i$ by node $n_h$ at time $t$ will change according to the following law:

$$w_h^i(t)=\delta_h w_h^i(t-1)+(1-\delta_h)\frac{\sum\limits_{k\neq h} w_k^i(t-1)\cdot f(d_{hk}(t-1))\cdot a_{kh}}{\sum\limits_{k\neq h} f(d_{hk}(t-1))\cdot a_{kh}} \tag{4.3}$$

where:

- $a_{hk} \in \{0, 1\}$ is an element of the adjacency matrix $A_{hk}$ representing the considered network; if $a_{hk} = 0$, then nodes $n_h$ and $n_k$ are not linked and do not influence each other while, if $a_{hk} = 1$, then nodes $n_h$ and $n_k$ are linked and they can influence each other. For the sake of simplicity, we shall suppose that $A_{hk}$ is symmetric and therefore the influence exercised by $n_h$ over $n_k$ is the same as the influence exercised by $n_k$ over $n_h$. Consequently, for each node $n_h$ we can define the set $N_h = \{n_k \in N : a_{hk} = 1\}$, that is the set of nodes that are linked to $n_h$ and that could influence its decisions.

- $\delta_h \in [0, 1]$ represents the node's inclination to be influenced by the nodes belonging to $N_h$; the less $\delta_h$, the more nodes in $N_h$ will influence the preferences of $n_h$; in particular, if $\delta_h = 0$, then the preferences of $n_h$ will be completely dependent on the preferences of the nodes in $N_h$, while, in the opposite case, if $\delta_h = 1$, then the preferences of $n_h$ are not affected by the nodes in $N_h$. For the sake of simplicity we shall suppose that $n_h$ can be influenced in the same way by nodes in $N_h$; however, it could be reasonable to consider an inclination $\delta_{hk}$, representing the inclination of $n_h$ to be influenced by $n_k$;

- $f(d_{hk}(t-1))$ is the importance given by $n_h$ to the preference of node $n_k$ on criterion $g^i$ at time $t-1$ ($w_k^i(t-1)$); moreover $f$ is a non-decreasing function of the distance $d_{hk}(t-1)$ that will be described later. For the moment, we assume that:
$$f(d_{hk}(t-1)) = \frac{1}{d_{hk}^2(t-1)}. \tag{4.4}$$

The idea under equation (4.3) is that the preferences'dynamics of node $n_h$ is dependent on its inclination to be influenced by nodes belonging to $N_h$. In particular, on one hand, preferences of $n_h$ at time $t$ will be dependent on its preferences at time $t-1$ and, on the other hand, $n_h$ will give a weight to the preferences of node $n_k \in N_h$ depending on the distance $d_{hk}(t-1)$ between the two nodes. This distance is computed in terms of similarity between the preferences of the two nodes and in terms of similarity between the choices made by the two nodes at previous times. The idea is that the more the preferences (the weights and the past choices) are similar between nodes $n_h$ and $n_k$, the more $n_k$ influences node $n_h$.

Obviously, the more similar are preferences and choices of nodes $n_h$ and $n_k$, the more importance will be assigned to the preferences of $n_k$ from $n_h$.
From an analytical point of view, $d_{hk}(t)$, that is the distance between nodes $n_h$ and $n_k$ at time $t$, is computed in the following way:

$$d_{hk}(t) = \sqrt{\sum_{i=1}^{p} \left[w_h^i(t-1) - w_k^i(t-1)\right]^2 + x_{hk}(t)}. \tag{4.5}$$

The first part is the Euclidean distance between the weights vectors of $n_h$ and $n_k$ representing the distance between the preferences of the two nodes. The second part, instead, is a measure of the number of times nodes $n_h$ and $n_k$ have taken different decisions in the previous considered time instants. With respect to the second part, the importance given to the decisions will be dependent on the instants in which they have been taken. In particular, the more recent they are, the more importance they have. Formally, $x_{hk}(t)$ can be expressed as:

$$x_{hk}(t) = \sum_{r=1}^{\#PT} \beta\gamma^{r-1}, \tag{4.6}$$

where:

- $\#PT$ is the number of considered previous time instants and it can be interpreted as the memory of the system. If $\#PT$=0 then the system will be memory less. Therefore the output of the system will be based only on the current system state and it will not take into account its history. Instead, if the system has a memory, as proposed in our model, the output of the system is not dependent on the current state only but also on some previous instants. This can be considered an important property of the system, because having memory of what previously happened, influences the behaviour of each single node, contributing to increase or decrease the distance between nodes, as indicated in equation (4.5).

- $\beta = \left\{ \begin{array}{ll} 1 & \text{if at the considered time instant, nodes } n_h \text{ e } n_k \\ & \text{have not taken the same decision} \\ & \\ 0 & \text{otherwise} \end{array} \right\}.$
- $\gamma \in [0, 1]$ is a damping coefficient used to weigh the decisions taken in different time instants.

It is worth noting that the two parts of equation (4.5) are not expressed in the same scale. Indeed, the distance between the weight vectors can assume values in the interval $[0, \sqrt{2}]$, while $x_{hk}(t)$ can take a value in the set $\{0, \gamma, \ldots, \gamma^{\#PT-1}\}$. To make commensurable these two values, $x_{hk}(t)$ has been normalised using the following equation:

$$x_{hk}(t) \cdot \frac{\sqrt{2}(1-\gamma)}{1-\gamma^{\#PT}}. \tag{4.7}$$

In this way $x_{hk}(t)$ will assume values in the interval $[0, \sqrt{2}]$ as the first part of equation (4.5).

As explained above, the variability of the context plays a central role in the dynamics of node's preferences. Since the modification of the context is due to the decisions taken by all nodes in the network, it is assumed that the evaluation of an alternative at time $t$ and, therefore, in a certain context, depends on its evaluation at time $t-1$ and on the number of times the alternative has been chosen by the nodes in the network at times $t-1$ and $t-2$. Formally, the evaluation of alternative $a$ at time $t$ in a considered context will be obtained as follows:

$$g^{p+1}(a;t) = g^{p+1}(a;t-1) + \alpha \frac{[M(a;t-1) - M(a;t-2)]}{|N|} \tag{4.8}$$

where $M(a;t-1)$ and $M(a;t-2)$ indicate the number of times that the alternative $a$ has been chosen at the time instants $t-1$ and $t-2$, while $\alpha$ is a coefficient that represents the relevance of the increase or decrease of the number of times that alternative $a$ has been chosen in the variation of the context.

Referring to examples 4.3.1 and 4.3.2, in the first one the variation of the context will provide a variation on the price of the considered goods while, in the second one, the variation of the context will affect the number of people going into the bar. On one hand, the increase of the demand of a particular good will cause an increase of its price while, on the other hand, the increase of the number of people going into a bar will convince more other people to go there.

### 4.3.1 Main components of the model

For a given network, i.e. for given values $a_{hk}$, the two components that affect the dynamics of the model are $\delta_h$ and $\alpha$.

As introduced in the section above, $\delta_h$, $h = 1, \ldots, m$, represents the node's inclination to be influenced by the other nodes in the network connected to it. So, its introduction, as a parameter that influences the dynamics of the entire system, is representative of the behavioural sphere, and in particular of the node volition to be part of a community and to take into account the others'opinion. $\delta_h$ can assume values in the interval [0, 1]. If $\delta_h = 0$, then node $n_h$ is totally influenced by other nodes in the network to which it is connected. Notice that, in equation (4.3) the

first term will be equal to 0, and the weights updating process will continue till the new weight will be equal to the barycenter of the group. Instead if $\delta_h = 1$, then $n_h$ will not take into account the others'opinion and its preferences will not change. Unlike the previous case ($\delta_h = 0$), and according to (Schelling, 1969), this can be considered a point of stable equilibrium, where a mechanism of complete segregation is active and no node will move towards another choice.
In addition to $\delta_h$, another important parameter of the model is $\alpha$. It is a coefficient that represents the relevance of the increase or decrease of the number of times that an alternative has been chosen in the variation of the context. The introduction of $\alpha$ changes the behaviour of the nodes, inducing the whole system to oscillate much more before stabilizing itself. As pointed out in the literature review section, context plays a central role because different issues can have different perception and representation, due to different contexts, leading to different level of awareness about the task. The parameter $\alpha$ can assume values greater than zero. If $\alpha$ is equal to zero, the context has no importance on the system's dynamics, because it assumes always the same value, and it can be compared to a static model.

## 4.4 Model Performance and discussion

In this section, the proposed model is applied to Example 4.3.2 considering two network structures. In the first part of the section, it is assumed that the network follows the ER model (Erdös and Rényi, 1959). It will be highlighted how the variation of the main components of our model, that are the nodes'inclination and the variability of the context, will affect the dynamics of the preferences.
In the second part, instead, the network will follow the BA model (Barabási and

Albert, 1999) and it will be underlined how the dynamics of preferences is subject to the network structure modification.

As shown in Figure 4.1, in the ER model, all nodes have approximately the same number of connections, while in the BA model only some nodes have high degree. For example, considering node $n_{13}$ in Figure 4.1, one can see that in the ER model, shown in the left side, its degree is 16, while in the BA model, shown in the right side, its degree is 1.

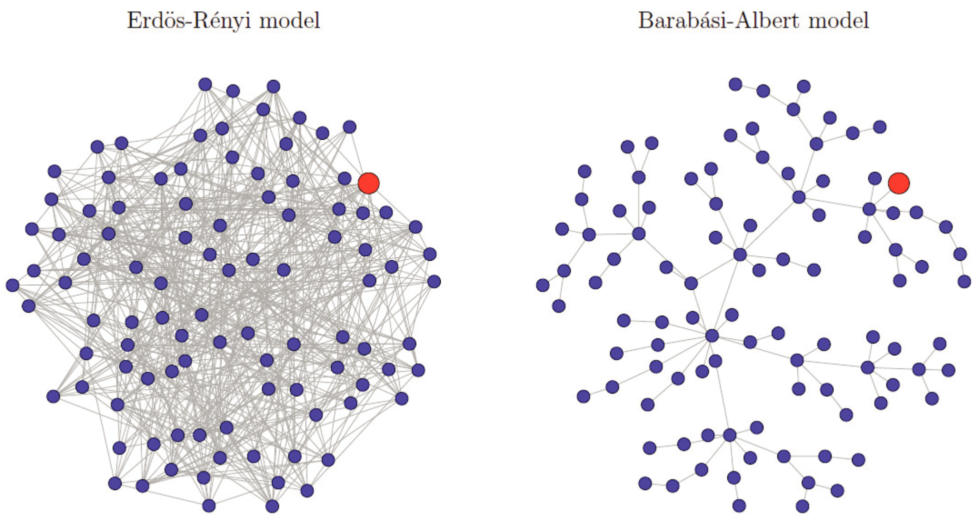



Figure 4.1: Degree of node $n_{13}$, the red one having the greatest size, in the ER model and in the BA model

The network is composed of $m = 100$ nodes representing customers that have to choose a bar to spend the night. Each bar is evaluated on three criteria Location (L), Quality of Service (QoS) and People Attendance (PA) as shown in Table 4.1. It has been supposed that the evaluations of the four bars with respect to L and QoS are expressed on a $[0, 1]$ scale and both of them have an increasing direction of preference. Since PA expresses the variability of the context in which nodes have to make their choices, the evaluations of the bars on this criterion are variable over time, while evaluations on L and QoS are supposed fixed. In particular, it is assumed that the evaluations of the four bars with respect to PA presented in Table 4.1 are based on an estimate of the frequency of the customers in the considered bars. To each node a vector of weights is associated and each of them represents the importance given by the node to the corresponding criterion.

Table 4.1: Evaluations of the bars on the three considered criteria

| Bar/Criteria | Location (L) | Quality of Service (QoS) | People Attendance (PA) |
|---|---|---|---|
| $bar_1$ | 0.684 | 0.086 | 0.058 |
| $bar_2$ | 0.452 | 0.682 | 0.192 |
| $bar_3$ | 0.259 | 0.851 | 0.177 |
| $bar_4$ | 0.203 | 0.891 | 0.400 |

The considered network configuration has a probability $p = 0.1$, where $p = 0.1$ is the probability of having a connection between two nodes. The network will be represented by a graph whose vertices are the nodes of the network. Each node will be colored according to the choice made. In all Figures in this section, the colors associated to the four alternatives are those shown in Table 4.2.

To study the behaviour of the model in a simulation environment, it is assumed that:

- the inclination of each node $n_h$ to be influenced by the other nodes in the network is represented by a value $\delta_h$ taken uniformly in the interval $[0, 1]$;
- $\alpha = 1$;

Table 4.2: Colors associated to the nodes depending on the choice made

| Bar | Color choice |
|---|---|
| $bar_1$ | Red |
| $bar_2$ | Yellow |
| $bar_3$ | Blue |
| $bar_4$ | Green |

- there is not any external cause of influence and, therefore, the preference of the nodes will change only as the effect of the influence exercised by the other network's nodes;
- the network configuration does not change over time in the sense that the nodes in the considered network, as well as their mutual connections, do not change over time;
- starting from time $t_0$, in order to update the evaluations of the four bars with respect to the considered context (the evaluations of the alternatives on criterion PA), the choices made by the network's nodes at time $t_0 - 1$ are represented by the vector $(26 - 22 - 24 - 28)$ meaning that, 26 people choose $bar_1$, 22 choose $bar_2$ and so on. Moreover, the choices made by the nodes at time $t_0$ are the consequence of the application of equation (4.2) to the evaluations in Table 4.1. For example, considering the starting vector of weights $(w_1^1, w_1^2, w_1^3) = (0.491, 0.173, 0.336)$ and the bars'global evaluations $(0.668, 0.617, 0.556, 0.479)$, node $n_1$ will therefore choose $bar_1$.

Note that in all performed simulations, it is considered $\#PT$ equal to 17, since it was observed that considering a value greater than 17 does not affect the dynamics of the choices.

The first characteristic it is important to highlight is the dynamic behaviour of the nodes'choices, as shown in Figure 4.2, where the configurations of the network with respect to the choices made by its nodes, for the first iterations, are reported. As one can see, at time $t_0$, the choices made by the nodes in the network are $(41 - 15 - 37 - 7)$ while, as a consequence of the influence mechanism and due to influences on nodes'inclinations, at time $t_0+5$ the choices made by the nodes became $(3 - 79 - 0 - 18)$.

The variation in the choices made by the nodes at different iterations is due to three main reasons: the variation of the importance assigned to the different criteria, the inclination of each node to be influenced by the other nodes to which it is connected in the network, and the variation of the context. Regarding the first point it is reported, as example, the dynamics of weights of node $n_{30}$ during the first considered iterations, shown in Figure 4.3 on the left. As one can see, at the beginning, the most important criterion for this node is L followed by PA and QoS; as a consequence of the influence mechanism, at iteration $t_0 + 9$, criteria L and QoS assume, more or less, the same importance. Then, at iteration $t_0 + 13$ criterion QoS becomes more important than PA. In the end, QoS is the most important criterion for $n_{30}$ while, at the beginning it was the lowest important one.
As already mentioned before, applying equation (4.3), iteration after iteration, the preferences of a node will be always closer to the preferences of the nodes that have taken similar decisions in the previous iterations and more different from the preferences of the nodes that have taken different decisions. Therefore, just after a certain number of iterations, a cluster effect will appear in the network so that

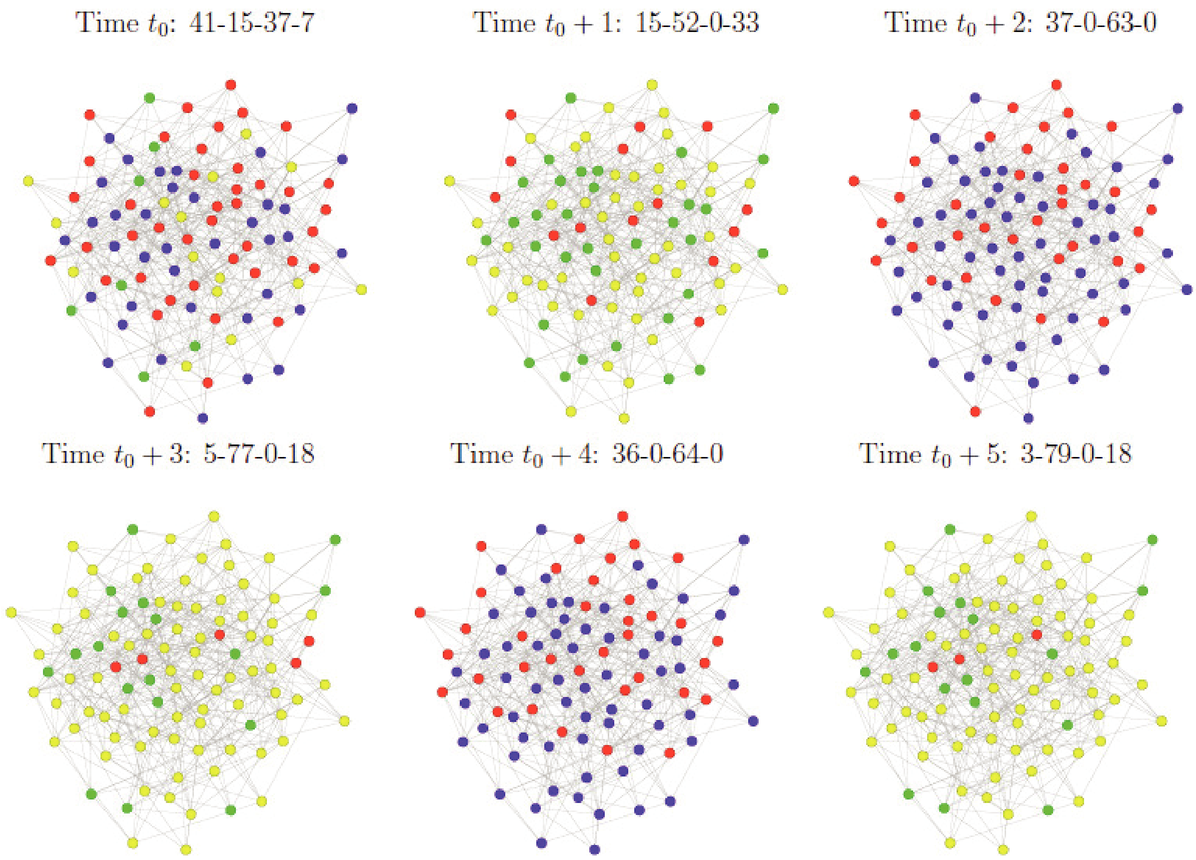



Figure 4.2: Dynamics of the decisions in a ER network with $p = 0.1$; different $\delta_h$ for all $h$ and $\alpha = 1$

nodes that have taken similar decisions in the past will have approximately the same preferences and the weights assigned to the different criteria will be subject only to very slight modifications as shown in Figure 4.3 on the right.

The second reason for the dynamics of choices is the inclination of each node to be influenced. To highlight this aspect, two networks are compared. They differ for the inclination of 10 nodes only, that is, 90 nodes have the same value of $\delta_h$ in both cases while $\delta_h$ of the 10 remaining nodes are pairwise exchanged. In this way the

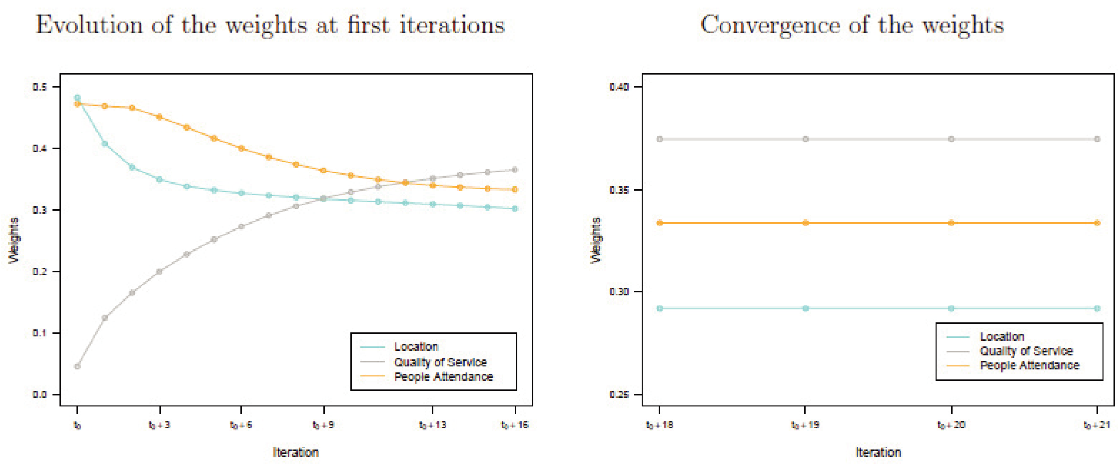



Figure 4.3: Dynamics of the weights for node $n_{30}$

average inclination $\delta_h$ of the nodes in the two networks is the same.

**Note 4.4.1.** *When a modification to a parameter is done, all the other parameters remain fixed as listed in the above assumption. For example, in this comparison, only $\delta_h$ is modified , while $\alpha = 1$ and the vector representing the choices made by the nodes at time $t_0 - 1$ is* $(26 - 22 - 24 - 28)$.

As shown in Figure 4.4, only one node ($n_{40}$), the node having the greatest size in the Figure, makes a different choice in the two networks, because it chooses the first alternative in the first case, while it chooses the third alternative in the second case. Even if this change could appear not significant, it is possible to observe that in the two networks the number of connections and the nodes to which they are connected do not change. Consequently in modifying its preferences, each node will be influenced by the same nodes. The only difference is that 10 out of the 100 nodes change their inclination $\delta_h$ to be influenced and, in particular, $n_{40}$ is exactly one of them.

The third aspect, causing the dynamics of the choices and that represents one of

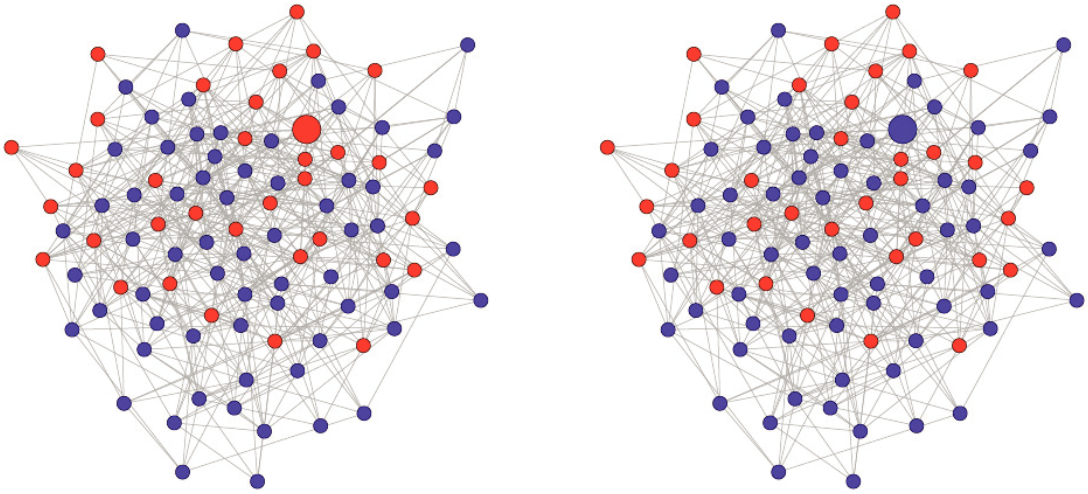


Figure 4.4: Comparison between two networks differing for the values of $\delta_h$ of ten nodes only

the novelty introduced in the paper, is the variability of the context. In particular, it is shown how different values of $\alpha$ in equation (4.8) affect the dynamics of the preferences. For this reason the dynamics of the same network varying only the value of $\alpha$ are compared. In the first network $\alpha = 1$ is considered, while in the second network, $\alpha$ is equal to 2. Therefore the variability of the context is greater in the second case.

As one can see in Figure 4.5, different values of $\alpha$ have a different impact on the dynamics of the choices made by the nodes in the network since 28 of them make different choices in the two cases.

It is worth noting that the importance of taking into account the variability of the context in this model. As already observed above, according to the dynamics of weights described by equation (4.3), after a certain number of iterations the weights of the criteria will not change anymore (see Figure 4.3 on the right) and this implies

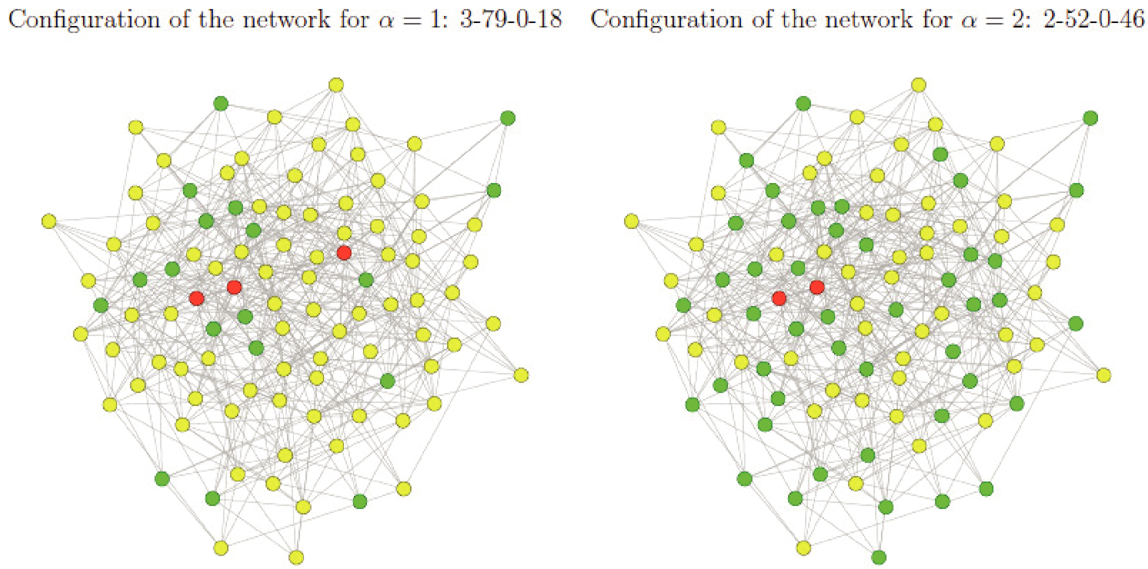



Figure 4.5: Comparison between the same network considering different values of $\alpha$ in equation (4.8)

that the preferences of the node for the considered criteria will not be subject to any modification. Nevertheless, the introduction of the variability of the context may bring to an oscillation of the decisions taken from the different nodes as observed in Table 3(a).

In the first iterations, the configurations of the network will vary more deeply since two phenomena occur simultaneously, that are, the modification of the preferences due to dynamics of the weights described by equation (4.3) and the variability of the context described by equation (4.8). In Table 3(b), one can instead observe a different dynamics of the network due to the value of $\alpha = 0$, implying that the variability of the context is not taken into account in the dynamic of the network. At the beginning, the network passes through different configurations in consequence of the variation of the preferences of the nodes, preferences that become more similar

Table 4.3: Dynamics of the decisions taken by the nodes in the network considering $\alpha = 1$ and $\alpha = 0$, respectively

(a) $\alpha = 1$

| Time | Configuration |
|---|---|
| $t_0$ | 41-15-37-7 |
| $t_0 + 1$ | 15-52-0-33 |
| $t_0 + 2$ | 37-0-63-0 |
| $t_0 + 3$ | 5-77-0-18 |
| $t_0 + 4$ | 36-0-64-0 |
| $t_0 + 5$ | 3-79-0-18 |
| $t_0 + 6$ | 36-0-64-0 |
| $t_0 + 7$ | 3-79-0-18 |

(b) $\alpha = 0$

| Time | Configuration |
|---|---|
| $t_0$ | 41-15-37-7 |
| $t_0 + 1$ | 37-17-44-2 |
| $t_0 + 2$ | 37-17-44-2 |
| $t_0 + 3$ | 36-18-45-1 |
| $t_0 + 4$ | 36-18-46-0 |
| $t_0 + 5$ | 36-17-47-0 |
| $t_0 + 6$ | 36-17-47-0 |
| $t_0 + 7$ | 36-17-47-0 |

to the preferences of the nodes that have taken similar decisions in the past. At iteration $t_0 + 5$ the configuration of the network becomes $36 - 17 - 47 - 0$ and from then on, it will be always the same. Indeed, once that the weights are not subject to great modifications, the different nodes will take always the same decision since the context does not play anymore a role in the decisions taken by the nodes. It is important to highlight that when it has been stated that the configuration of the network reached at iteration $t_0 + 5$, that is $36 - 17 - 47 - 0$, does not change in the following iterations, it means that not only that the number of nodes taking a

certain decision does not change (that is, there will be always 36 nodes choosing the first bar, 17 nodes choosing the second bar and so on) but also that the same nodes will take always the same decision. This means that, beginning from iteration $t_0+5$, each node $n_h$ will always take the same decision.
Before concluding this part, it is necessary to interpret the oscillations in the configurations of the network observed in Table 4.3(a). Indeed, as previously explained, starting from a certain iteration, the weights of the different nodes are not subject to any modification. Nevertheless, the network oscillates always between two configurations (see the configurations at the time instants $t_0 + 4$ and $t_0 + 6$ as well as the configurations at time instants $t_0 + 5$ and $t_0 + 7$). It is possible to start the analysis from the time instant $t_0 + 4$ considering the network configuration at this time instant, that is, $36 - 0 - 64 - 0$. At this time instant, $bar_1$ and $bar_3$ are the most crowded, while $bar_2$ and $bar_4$ are empty. Due to an increase of their customers, according to eq. (4.8), the evaluations got by $bar_2$ and $bar_4$ will decrease at the time instant $t_0 + 5$ rendering the two bars more appealing for the different customers that, consequently, will decide to leave $bar_1$ and $bar_3$ in favor of $bar_2$ and $bar_4$. For this reason, the configuration of the network at the time instant $t_0+5$ will be $3-79-0-18$. Analogous reasons explain why at the next time instant $(t_0+6)$, the customers decide to leave $bar_2$ and $bar_4$ in favor, again of $bar_1$ and $bar_3$.

In the second part of this section how the structure of the network affects the dynamics of the preferences is shown. For this purpose, it is assumed that the network follows the BA model (Barabási and Albert, 1999), with a linear preferential attachment. In a first moment it is assumed that the inclinations of the nodes, as well as the starting weights and the parameter $\alpha$, are those considered in the ER network. By applying the model to this network structure one can observe the dynamics of preferences shown in Figure 4.6.

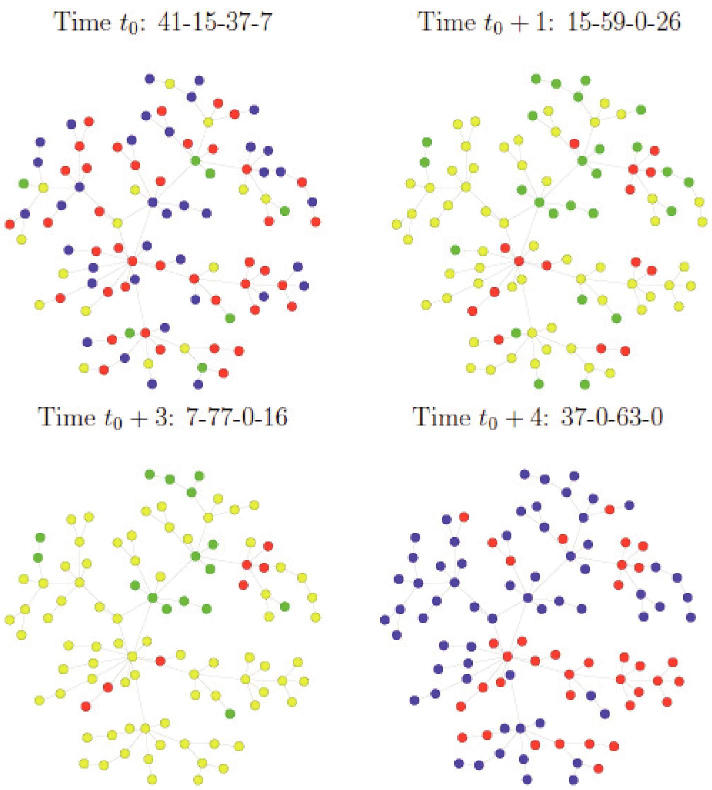



Figure 4.6: Dynamics of the decisions in a BA network with a linear preferential attachment; different $\delta_h$ for all $h$ and $\alpha = 1$

In Table 4.4 the dynamics of the preferences for the first iterations in the two different network models are reported. As one can see, at the time instant $t_0$, the decisions taken by the nodes in the two networks are the same decisions taken at the time instant $t_0$ in the ER network, since it is supposed that the nodes have initially the same starting weights. Already at iteration $t_0+1$, it is notable observing that the choices done by the nodes in the two networks are different. Indeed, in the ER model, 52 customers decided to go to $bar_2$ while, in the BA model, 59 customers decided to go to the same bar. Moreover, while the number of customers going in bars 1 and 3 is the same in the two network models, the number of customers deciding to go to $bar_4$ is different (33 in the ER model and 26 in the BA model). Another aspect really relevant is that, while the vector of preferences begins to oscillate between two configurations already at iterations $t_0 + 4$ and $t_0 + 5$ in the ER model, the same behaviour can not be observed for the BA model. Indeed, in this model, the vector of preferences oscillates between different configurations (36-0-64-0 and 3-79-0-18 in the ER model and 5-79-0-16 and 33-0-67-0 in the BA model) and, moreover, these oscillations begin later than in the ER model (at iterations $t_0+42$ and $t_0+43$ instead of iterations $t_0 + 4$ and $t_0 + 5$ in the ER model). Because, as previously underlined, the main components of the network (nodes'inclinations and $\alpha$) are the same, the different dynamics of the preferences in the two networks is due to their structures and, in particular, to the number of connections of single node. This implies that while, on average, ten nodes can influence the variation of the weights of each node in the ER model[1], in the BA model the preferential attachment law implies that not all the nodes are connected to the same number of nodes.

As already done for the ER model, it is shown here that the variation of the main components of the model affects the dynamics of the preferences also with a different network model. To highlight the influence of nodes'inclinations on the dynamics of the preferences, three different simulations, in which the inclinations of ten nodes are swapped, are performed. In particular, in the first case the inclination of the same nodes already considered in the ER model have been swapped. What one can observed is that the dynamics of preferences in this case is very similar to that one obtained with the starting inclinations. Presumably this behaviour is due to the fact that the ten nodes considered in the swapping had a low degree and, consequently, they were influenced and influenced, only a very limited number of nodes. For this reason, in the second and in the third simulations the inclinations of the nodes have

[1]It has been considered an ER model with p=0.1

Table 4.4: Comparison between the dynamics of the decisions taken by the nodes in the two network models

(a) ER model

| Time | Configuration |
|---|---|
| $t_0$ | 41-15-37-7 |
| $t_0 + 1$ | 15-52-0-33 |
| $t_0 + 2$ | 37-0-63-0 |
| $t_0 + 3$ | 5-77-0-18 |
| $t_0 + 4$ | 36-0-64-0 |
| $t_0 + 5$ | 3-79-0-18 |
| $t_0 + 6$ | 36-0-64-0 |
| $t_0 + 7$ | 3-79-0-18 |

(b) BA model

| Time | Configuration |
|---|---|
| $t_0$ | 41-15-37-7 |
| $t_0 + 1$ | 15-59-0-26 |
| $t_0 + 2$ | 34-0-66-0 |
| $t_0 + 3$ | 7-77-0-16 |
| $t_0 + 4$ | 37-0-63-0 |
| $t_0 + 5$ | 4-80-0-16 |
| $t_0 + 6$ | 36-0-64-0 |
| $t_0 + 7$ | 5-79-0-16 |

been swapped not in a random way but following a certain scheme. More precisely, in the second simulation, the inclinations of ten nodes, seven presenting an high degree and three presenting a low degree, have been swapped. In particular, the inclinations of five nodes having degrees 10, 8, 8, 7 and 5 have been swapped with the inclinations of five nodes having degrees 1, 7, 2, 6 and 2, in this order. This means that the inclination of the node with degree 10 has been swapped with the inclination of the node having degree 1; the inclination of the first node having degree 8 has been swapped with the inclination of the node with degree 7, and so on. In the third simulation, instead, the inclinations of five nodes having high degree with the inclination of five nodes having low degree, have been swapped. In particular, the inclinations of five nodes having degrees 10, 8, 8, 7 and 7 have been swapped with the inclinations of five nodes having degrees 1, 3, 1, 1 and 2.

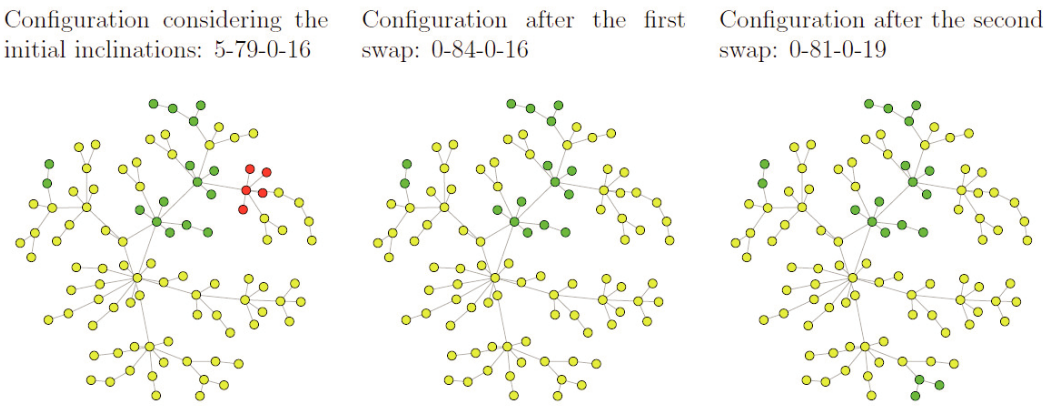



Figure 4.7: Comparison of the different network configurations obtained swapping the inclination of ten nodes

As one can see in the middle picture and on the right picture of Figure 4.7 the last two swapping of the inclinations causes a different dynamics of preferences. In particular, in the first swap, the network decisions oscillate between the configurations 0-84-0-16 and 29-0-71-0 while, in the second swap, the oscillations are between 0-81-0-19 and 37-0-63-0. In these two cases, the different dynamics of the decisions is due to the fact that at least one of the nodes involved in the swapping of the inclination has a high degree and, consequently, more nodes influence its decisions than in the first swapping.

In the end, the impact of the parameter $\alpha$ on the dynamics of preferences also for the BA model has been analysed. To this aim it has been considered two different values of $\alpha$ that are, $\alpha = 1$ and $\alpha = 2$. In Figure 4.8, one can observe that the value

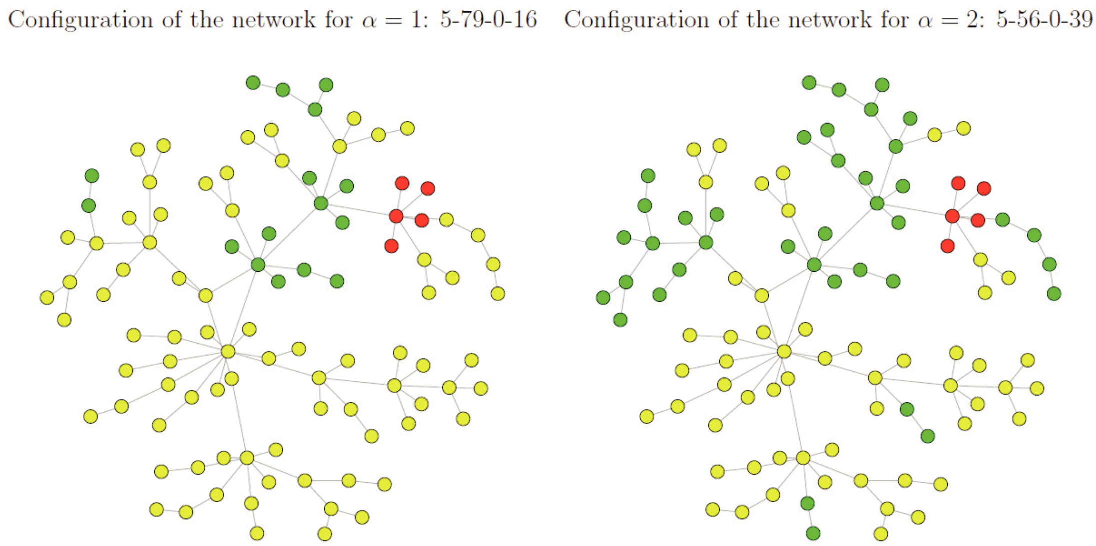



Figure 4.8: Decisions taken by the nodes for two different values of $\alpha$ for the BA model

of $\alpha$ affects the dynamics of preferences and, consequently, the decisions taken by the nodes in the network. In particular, at the same time instant, 23 nodes take different decisions in the two network configurations. Moreover, while in the first case the decisions oscillate between the configurations 5-79-0-16 and 33-0-67-0, in the second one, the oscillations of decisions are between the configurations 5-56-0-39 and 42-0-58-0.

## 4.5 Summary remarks

In this chapter a new social network model in the Multiple Criteria Decision Making framework has been proposed. Assuming that *individual decisions are often influenced by the decisions of other individuals* (López-Pintado, 2008), the new model is characterised by two main novelties that are the variability of the preferences of the nodes in the considered network, and the variability of the context in which the same preferences have to be taken. On one hand, the preferences of each node are subject to the influence exercised by the nodes in the network to which it is connected. The node will be more or less subject to this influence depending on its own inclination that is represented by the parameter $\delta_h$. On the other hand, the variability of the context and, in particular, the context-awareness of each node, are dependent on the decisions taken by the nodes in the network at the previous instants that will also influence the decisions at the current time. To show the applicability of our model to different network structures, it has been applied to the El Farol bar problem (Arthur, 1994) supposing that the network follows two different models that are, the ER model (Erdös and Rényi, 1959) and the BA model (Barabási and Albert, 1999). Simulation results show that the variation of the inclination $\delta_h$ of each node, as well as the variability of the context represented by the parameter $\alpha$, and the number of connections between nodes in the network, affect the dynamics of the decisions in both network models. Moreover, different dynamics of the decisions have been observed in the two models as a consequence of the network's structure.

# Chapter 5

# Multiple Criteria Decision Making and Supernetwork

## 5.1 The science of Supernetwork

Networks contribute to the correct functioning of a lot of processes in our societies. It is for example possible to count transportation networks, that permit to travel all over the world, communication networks, through which the exchange of messages among individuals is possible, and logistical networks, that allow the circulation and exchange of goods. As already introduced in the previous chapters, networks are characterised by the main features of complexity, dynamism, large-scale nature with a trend to have an increasing congestion and whose individuals have behaviours that affected not only the single but also can have effect, positive or negative, on the other individuals of the network. To give examples of real networks and its large-scale, it is possible to consider that Chicago's Regional Transportation Network counts 12.982 nodes, 39.018 links, and 2.297.945 origin/destination (O/D) pairs (Bar-Gera, 2002), instead the Internet users around the world are about 3.675.824.813 (datum provided on June $30, 2016$) (Group, 2016). In order to avoid problems and malfunctioning of the network, processes of resources optimization are needed. In this sense, the decision making process, whose main features are complexity and dynamism, has a central role in the science of network. Hence, in this chapter the decision making process is analysed and performed with the analytical tools provided by the Supernetwork theory. It exploits the analytical instruments of optimization theory, network theory, game theory, multiple criteria decision-making, the theory of variational inequalities (Nagurney, 2013), as well as projected dynamical systems theory (Zhang and Nagurney, 1995; Nagurney and Zhang, 2012).

Supernetworks are defined as "networks of networks" that are "above and beyond" the existing classical networks and they are composed of nodes, that represent the locations in space, links, indicating connections of roads or cables, and flows, representing vehicles or data. In particular, through the supernetwork framework the alternatives available are shown to decision makers and their individual behaviour is reproduced with the typical volition to optimise a particular criterion. Furthermore the flows are computed on the supernetwork, which may consist of product shipments, travelers between origins and destinations, financial flows, information flows, resource and energy flows, as well as the associated costs and "prices". Supernetworks are able to represent issues not only referred to physical networks but also to abstract networks (Nagurney, 2011).

Supernetworks allow to solve problems such as:

- Financial Problems, allowing, after the initial investment, the optimisation of distribution of assets and liabilities of goods, minimising, at the same time, the risk aversion (Nagurney and Siokos, 2012);
- Electrical Networks, modeling the process of energy distribution, from the producers to the final users (Liu and Nagurney, 2009);

- Transportation and Telecommunication Network, providing the best path from source to destionation, taking into account costs, length and traffic conditions (Nagurney and Qiang, 2007);
- Management and Business, studying the pros and cons of companies mergers, where each company is seen as a network of economic activities ranging from production, distribution and storage;
- Humanitarian Logistic Activities, describing the best path to send essential goods to people affected by catastrophic events (Nagurney and Qiang, 2012).

In the Supernetwork framework it is possible to distinguish two phases: problem definition and problem analysis.
After that it has been identified the problem to solve, in the first phase the problem is analysed and it is represented by a multilevel graph $G = [N, L]$, where $N$ and $L$ denote the set of nodes and arches respectively. An example of Supernetwork is shown in Figure 5.1.

Each node of the set $N$ represents a decision maker and, furthermore, nodes that perform the same task independently are placed on the same network level, as it is possible to see in Figure 5.1, where three levels are present: Manufacturers, Retailers and Demand Market. The relationships among decision makers intra- and inter-level are represented by the arches, to which it is associated a flow, representing an inter-level service transition from one decision maker to another. This flow indicates the objective of the optimization process.
The second phase of the Supernetwork framework is represented by the problem analysis. In this step, resources involved in the process and the decision criteria are identified in order to start the optimization process. To each arc, representing a temporal relation between two events $A$ and $B$, it is associated a set of variables,

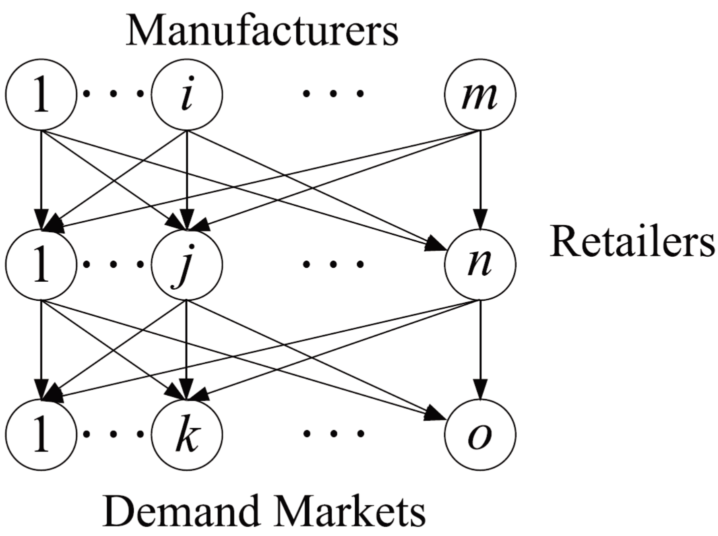



Figure 5.1: Model of Supernetwork

earlier identified, that permits the transition from $A$ to $B$. Furthermore, it is associated a cost to go from $A$ to $B$ and it is dependent on the resources involved; the cost function can be one for each decision criteria to be optimised (e.g. time, cost, risk, etc.) and to which can be associated a weight. Each arc has a global cost function given by the sum of each single cost function and the set of all the cost function of each arc represents the objective function of the problem. The variational formulation of the objective function and finding the solution to the optimisation problem permits to find the "optimal" resources values to involve in the process in order to offer the best services to each decision maker to improve the system efficiency. It is important to highlight that the flows, obtained after the first phase, are not only admittable; in fact, often, it is necessary to define a set of constraints, called feasible set, that the flows have to respect.

## 5.2 Supplier selection and supply chain model

Supplier selection is an issue that regards and involves resources from the client side. Choosing the right supplier implicates an evaluation based not only on the goods price but several other criteria could have an impact and an influence on the process. It can, in fact, be considered as a multiple criteria decision making problem and it can be modeled through the mathematical and analytical tools provided by MCDM. The other factors or criteria that may impact the supplier selection process can be, in addition to price, quality, reliability and delivery performance for example. Especially in Management Science, supplier selection has received a lot of interest, due to its classification as strategic in the Operation Management decision area (Verma and Pullman, 1998), having a major impact on companies costs.

In one of the first works on this research area, Dickson identified over twenty selection criteria, such as quality, delivery, performance history, warranties, price, technical capability and financial position, for supplier selection (Dickson, 1996). Despite the large number of selection criteria identified, several works ((Ansari and Modarress, 1988),(Benton and Krajewski, 1990),(Bernard, 1989),(Weber et al., 1991)) have shown that very often the decision is taken searching a tradeoff among three criteria: quality, cost and delivery performance.

It is possible to distinguish two categories of supplier selection problems. To the first one belong the processes in which the decision maker chooses the best supplier, that satisfies its needs. It is called "single sourcing". To the second category, named "multiple sourcing", belong the processes in which the decision maker cannot find only one supplier that satisfies all its needs and in this case it decides to separate its order among different suppliers, also increasing the level of competitiveness.

This sentence well summarises the importance that has the efficiency of the supply chain and its influence on the whole system: "*When firms make mistakes anywhere* *within a supply chain, the effects can ripple through the chain in both directions* (Flint, 2004). *These effects include disruption to production, forecasting errors, inventory imbalances, stock-outs or damaged goods, all of which usually result in increased costs that may have to be passed on to end users, thus reducing their satisfaction and loyalty* (Ellis, 2010)".

Supply chain is a network composed by suppliers, manufacturers, transportation service providers, retailers, consumers, whose aim is to move products and goods from the supplier to the customers. Supply chains are the backbone of modern economy and they permit the production, distribution and consumption of goods as well as services (Nagurney, 2006). There are two categories of supply chains, depending on the number of decision makers that manages the network. In a centralised supply chain, there is only one central entity or decision maker, a firm for example, that controls all the activities inherent the supply chain. In a decentralised supply chain, instead, there are several decision makers that, more or less, cooperate in a competition regime.

A supply chain can be graphically represented by a network, composed of nodes and links and whose topology and structure reflects the real systems. Through this representation, the supply chain properties and behaviours can be better analysed. For example, the economic impact of the addition or removal of decision makers, represented as nodes of the network or, equivalently, the addition or removal of different modes of transactions or transportation, indicated by the links among nodes. Furthermore, different supply chain, after their representation through a network, can highlight common properties, thus facilitating their analysis based on cases already studied (Nagurney, 2006). An example of supply chain network is depicted in Figure 5.3.

A node origin, indicated as Organization (indicating government, corporations,

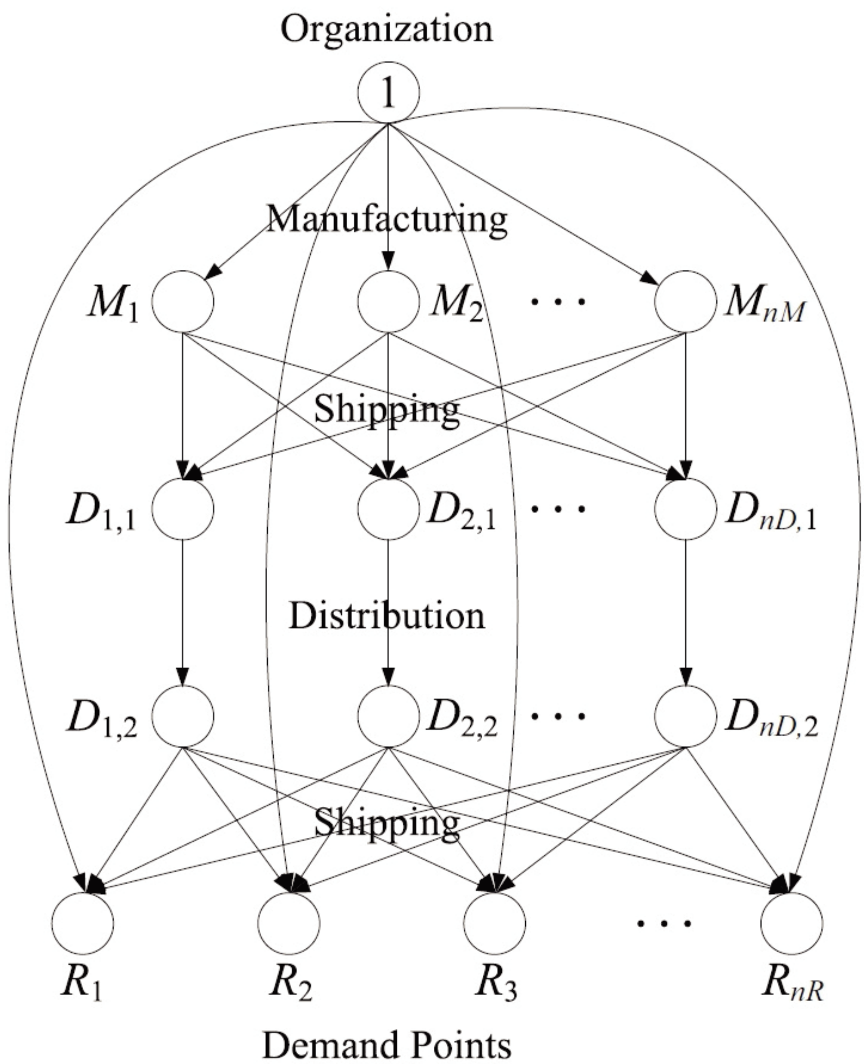



Figure 5.2: Supply Chain Network

humanitarian organizations, etc.) is considering $n_M$ manufacturing facilities/plants; $n_D$ distribution centers and it has to serve the $n_R$ demand points with respective demands given by $d_{R_1}$ , $d_{R_2}$ ,..., $d_{R_{n_R}}$. The links from the top-tiered node 1 are connected to the possible manufacturing nodes of the organization, which are denoted, respectively, by $M_1$,...,$M_{n_M}$, and these links represent the manufacturing links. The links from the manufacturing nodes, in turn, are connected to the possible distribution center nodes of the organization, and are denoted by $D_{1,1}$,...,$D_{n_D,1}$. These links correspond to the possible shipment links between the manufacturing plants and the distribution centers where the product will be stored. The links joining nodes $D_{1,1}$,...,$D_{n_D,1}$ with nodes $D_{1,2}$,...,$D_{n_D,2}$ correspond to the possible storage links. Finally, there are possible shipment links joining the nodes $D_{1,2}$,...,$D_{n_D,2}$ with the demand nodes: $R_1$,...,$R_{n_R}$.

The supply chain network consisting of the graph is indicated as $G = [N, L]$, where $N$ denotes the set of nodes and $L$ the set of links. The objective of the supply chain is to optimise resources and costs. In particular, the organization wishes to determine which manufacturing plants it should operate and at what level; the same for the distribution centers and how much of the product should be outsourced. In addition, the organization seeks to determine the capacity levels of the mode of transportation/shipment it is necessary to use to have the best efficiency. Due to continuous change of market conditions, supply chains change along time, modifying dynamically the network structure.

### 5.2.1 Supernetwork and Multiple Criteria Decision Making: a unified framework

The model proposed considers a Supernetwork composed of three levels. The first one is represented by service providers, that offer basic services, whereas the second one is composed by the producers of combined services, that receive at least one basic services from the nodes of the first level. The third level contains end users that can decide to buy composed services from the second level or basic services from the first level. The supply chain is represented by a graph $G = [N, L]$, where $N$ denotes the set of nodes and $L$ the set of links, as reported in Figure 5.3

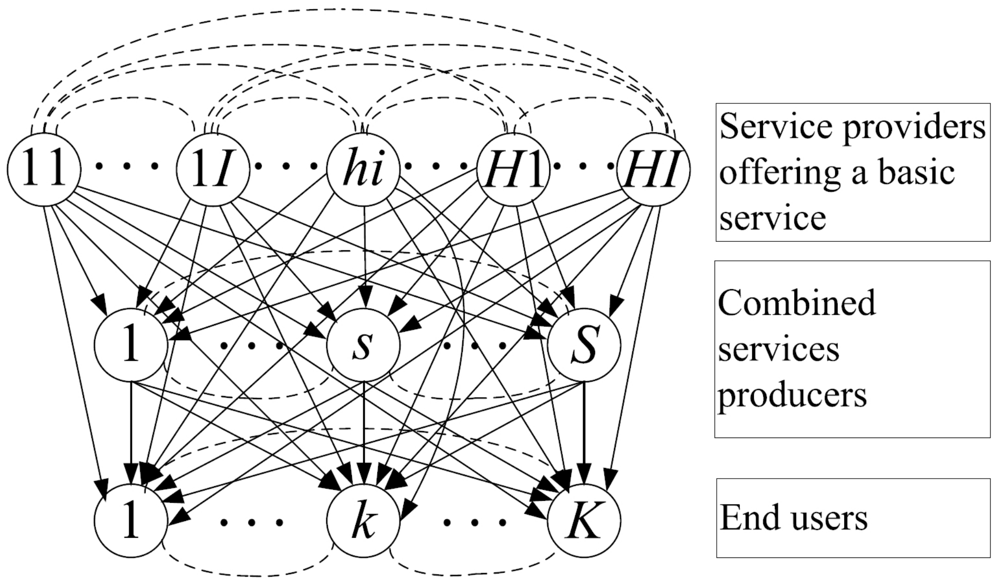



Figure 5.3: Supply Chain Network

The supply chain model considered is composed, on the first level, of $I$ basic services and $H$ distinct service providers, each of which offers a specific basic service denoted by $hi$; the second level is formed by $S$ distinct producers, whose service producer is denoted by $s$. The third level is composed by $K$ end users with typical end user denoted by $k$.

The operation scheme of the supply chain proposed in this model is explained below. The $H$ service providers could offer and deliver at least one basic services or directly to the end users or, alternatively, to intermediate producers. The nodes of the second level, after receiving basic services from level one, build more sophisticated services that they can offer and deliver to the end users. At the end of the supply chain the role of end users is to decide if it is more convenient to buy basic or combined services deciding, furthermore, from which provider or producer.

The models already available in the scientific literature that joints the concepts of supernetwork and social network, do not consider the powerful instruments provided by the social network analysis (Nagurney et al., 2007), (Cruz et al., 2006), (Nagurney et al., 2006). In fact, in these cases the flows among the different level are considered as the social links between different entities, neglecting the concept of social influence and its significant importance and role in each single transaction involving a decision making process. Furthermore, context and then context-awareness are not considered and do not play any role in the decision dynamics. Context is considered fixed and does not dynamically change at the variation to which the process and its components are subjected.

The main novelty introduced in this model, and graphically it is visible from the red links present in Figure 5.4(simplification of a supply chain), is the influence that each decision maker exercises on its neighbors at the same level. The concept of influence is borrowed from social network analysis and it is important to consider it, as highlighted in the previous chapter, in the decision making process.

As for the definition given before, the alternatives of the decision making process, represented by the nodes to which sell or to buy from, are evaluated on the basis of two criteria: cost and quality of service. The objective of the model is to identify optimal values that allow each decision maker to maximise its utility, minimising, at the same time, fruition costs and guaranteeing an adequate level of quality of service.

After its definition, the problem is analysed, identifying resources involved in each state of the process and select the criteria to be optimised. To each link that connects two nodes it is associated a specific costs as well as specific risk and time. Each

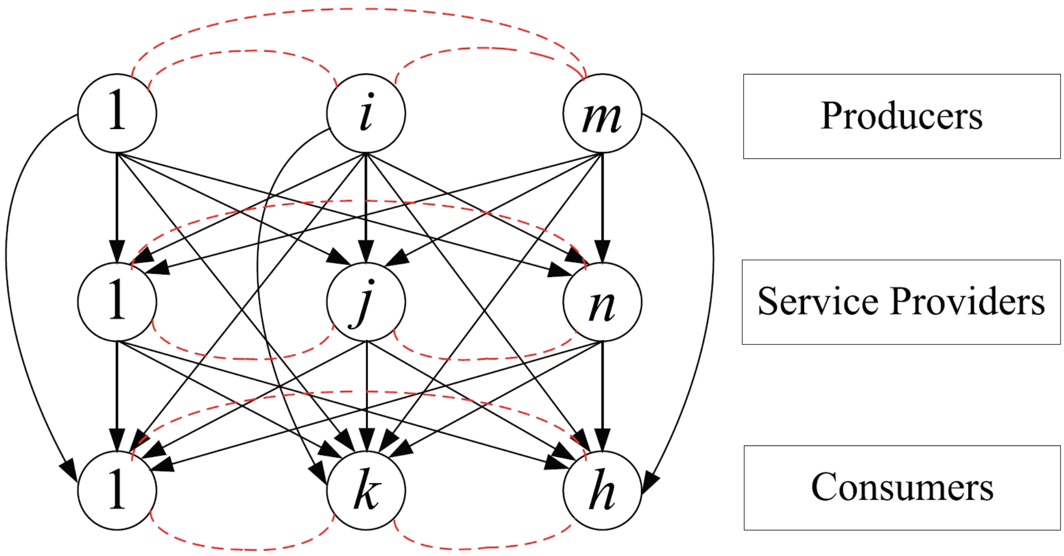



Figure 5.4: Supply Chain Network

decision maker associates a weight to each decision criterion, weight that represents the importance of that criterion in the process. The combination of all cost functions associated to a link (adequately weighted) represents the objective function for that specific link. Combining all the individual objective functions, the objective function of the whole process is obtained, which can be subject to constraints. After applying the optimization process to the objective function the solution to the problem is found that indicates the optimal value for the involved resources, i.e. the value that optimises (i.e. minimises or maximises depending on the objectives) services offered, improving of the whole process. In this model, the objective function is represented by the utility function, i.e. the objective of each decision maker is to choose the alternative(s) that maximise its utility.

## 5.3 Model Description

In the description of the model, it is used the following notation:

- $l$ which indicates the number of levels of the network. In this case it is set to 3;
- $N^a = n_{1_a}, \ldots, n_{s_a}, \ldots, n_{z_a}$ representing the finite set of nodes of level $a$, where $a = 1, \ldots, l$; in this case the set $N^a$ become:
  - $N^1 = \{n_{1_1}, \ldots, n_{i_1}, \ldots, n_{m_1}\}$;
  - $N^2 = \{n_{1_2}, \ldots, n_{j_2}, \ldots, n_{n_2}\}$;
  - $N^3 = \{n_{1_3}, \ldots, n_{k_3}, \ldots, n_{h_3}\}$;
- $A^a = \bigcup_{b=1;b\neq a}^{l} N^b$ represents the set of alternatives for each decision maker $n_{s_a}$ and it is composed by nodes that belong to the level $b \neq a$.
- $G = \{g^1, g^2\}$ is the set of decision criteria that are price and quality of service;
- $R^a = \{r_{1_a}, \ldots, r_{s_a}, \ldots, r_{z_a}\}$ is the set of the resources' vector involved in the transition for each node $n_{s_a}$. In the considered case the vector $r_{s_a}$ is composed by:
  $r_{s_a} = (x_{s_a 1_b}, y_{s_a 1_b}, \ldots, x_{s_a i_b}, y_{s_a i_b}, \ldots, x_{s_a z_b}, y_{s_a z_b})$ where $b = 1, \ldots, l$ and $b \neq a$. In particular, $x_{s_a v_b}$ represents the order quantity that node $n_{s_a}$ places with the supplier $n_{i_b}$ and $y_{s_a v_b}$ is the unitary price for order that node $n_{s_a}$ places with the supplier $n_{i_b}$.
- $Up\_bound^a = ub_{1_a}, \ldots, ub_{s_a}, \ldots, ub_{z_a}$ represents the maximum value that the resources' vector can assume. Each element $ub_{s_a} = (ub\_x_{s_a}, ub\_y_{s_a})$ where $ub\_x_{s_a}$ and $ub\_y_{s_a}$ represent the maximum value of order quantity and unitary

price. These values change along time due to the continuous variation of market. In particular, the variations of quantity and price are described by the following laws:

$$ub\_x_{s_a}(t) = \delta_{s_a} ub\_x_{s_a}(t-1) + (1-\delta_{s_a}) \frac{\sum_{h \neq s,a} ub\_x_{h_a}(t-1) \cdot f(d_{s_a h_a}(t-1)) \cdot a_{s_a h_a}}{\sum_{h \neq s,a} f(d_{s_a h_a}(t-1)) \cdot a_{s_a h_a}} \tag{5.1}$$

$$ub\_y_{s_a}(t) = \delta_{s_a} ub\_y_{s_a}(t-1) + (1-\delta_{s_a}) \frac{\sum_{h \neq s,a} ub\_y_{h_a}(t-1) \cdot f(d_{s_a h_a}(t-1)) \cdot a_{s_a h_a}}{\sum_{h \neq s,a} f(d_{s_a h_a}(t-1)) \cdot a_{s_a h_a}} \tag{5.2}$$

where:

- $\delta_{s_a}$ represents the inclination of node $n_{s_a}$ to be influenced to the nodes to which it is connected, in particular $\delta_{s_a} \in [0, 1]$. There are two limit cases $\delta_{s_a} = 0$ and $\delta_{s_a} = 1$. In the first case, node $n_{s_a}$ is totally influenced by its neighborhood and its preferences depend at all by the nodes to which it is connected. Instead, in the second case, node $n_{s_a}$ will not take into account and then it will not be affected by the preferences of the other nodes.
- $a_{s_a h_a}$ represents an element of the adjacency matrix of the network. If $a_{s_a h_a} = 0$ nodes $n_{s_a}$ and $n_{h_a}$ are not connected and they do not influence each other in the decision making process; instead, if $a_{s_a h_a} = 1$ nodes $n_{s_a}$ and $n_{h_a}$ are connected and they influence reciprocally.
- $f(d_{s_a h_a}(t-1))$ is the importance given by node $n_{s_a}$ to the decisions taken by node $n_{h_a}$ at the time instant $(t-1)$, and it is assumed that it is a non

decreasing function of the distance between $n_{s_a}$ and $n_{h_a}$ as follows:

$$f(d_{s_a h_a}(t-1)) = \frac{1}{d^2_{s_a h_a}(t-1))} \tag{5.3}$$

where:

$$d_{s_a h_a}(t) = [ub\_x_{s_a}(t-1) - ub\_x_{h_a}(t-1)] + x_{s_a h_a}(t) \tag{5.4}$$

The first term of Eq. 5.4 indicates the distance in terms of preferences between nodes $n_{s_a}$ and $n_{h_a}$. The second term, instead, represents the number of times that the two nodes have taken different decisions. As it is possible to see, $x_{s_a h_a}(t)$ is a measure time-dependent and it means that the more recent the decisions are, the more importance they have in the calculation of the distance between the two nodes. It is expressed as:

$$x_{s_a h_a}(t) = \sum_{i=1}^{\#PT} \beta_{s_a h_a}(i) \cdot \gamma^{i-1} \tag{5.5}$$

* $\#PT$ is the number of previous time instants considered by the model and represents the memory owned by the system. If $\#PT = 0$ then the system can be considered as memory less and therefore the output of the system will be based only on current system state, without taking into account its previous history. On the other hand, if $\#PT \neq 0$, it means the system keeps memory of previous history, as it happen in this model; in this case, the output is not dependent on the current state only but it takes into account also what

happened in previous time instants. This can be considered an important property of the system, as the memory of what happened in previous instants of time can significantly influence the behaviour of each single node, contributing to increase or decrease the distance between two nodes in the network;

  * $\beta_{s_a h_a}(i)$ is a coefficient, whose values belong to the set $[0, 1]$, which takes into account how much decisions taken by nodes $n_{s_a}$ and $n_{h_a}$ match with regards to the alternative set $A^a$. In particular, $\beta_{s_a h_a}(i)$ is 0 if $n_{s_a}$ and $n_{h_a}$ have taken, at the considered instant of time, the same decision, 1 if they have taken a different decision, any value between 0 and 1 if there is a partial match only between decisions taken by the two nodes;
  * $\gamma \in ]0, 1[$ is a damping coefficient which is used to weight decisions taken at different instants of time.

- $Low\_bound^a = \{lb_{1_a}, \ldots, lb_{s_a}, \ldots, lb_{z_a}\}$ represents the minimum value that the resources'vector can assume. As for the Up_bound, each vector of low_bound is composed as follows:
  $lb_{s_a} = (lb\_x_{s_a}, lb\_y_{s_a})$ where $lb\_x_{s_a}$ and $lb\_y_{s_a}$ represent the minimum value of order quantity and unitary price. These values do not change along time and are fixed.

- $g^p(n_{v_b}, t)$ is the evaluation of alternative $n_{v_b}$ with regards to criterion $p$ at time $t$.

As expressed above, the criteria considered in this case are two: cost and quality of service. Due to the continuous change of the network and market conditions, the evaluation of each alternative with respect to the two criteria has to be updated at each time instant, following these laws:

$$g_a^1(n_{v_b}, t) = \frac{\sum_{s_a=1}^{|N^a|} \frac{x_{s_a v_b} y_{s_a v_b}}{min(ub\_x_{v_b}, ub\_x_{s_a}) \cdot min(ub\_y_{v_b}, ub\_y_{s_a})}}{|N^a|} \tag{5.6}$$

$$g_a^2(n_{v_b}, t) = \frac{\sum_{i_a=1}^{|N^a|} x_{i_a v_b}(t-1)}{\sum_{i_a=1}^{|N^a|} ub\_x_{i_a}(t-1)} \tag{5.7}$$

Equation 5.6 says that the evaluation of each alternative with respect to the price criterion is updated as the average of products of order quantity and unitary price involved in transitions between all decision nodes of a given network level $a \neq b$ and the node $n_{v_b}$ normalised with regards to upper bounds. Instead, Equation 5.7 expresses that the evaluation of each alternative, with respect to the quality criterion, is updated depending on how much decision nodes from the other levels have exchanged with that alternative at time $t-1$. It is important to highlight that Equation 5.7 regards the producers, because it means that the greater the quantity the alternative received by all decision nodes in the level at time $t-1$ is (the maximum value is when upper level value is reached), the greater the quality of the alternative is, as the alternative is globally recognised as a good buyer. For the consumer the evaluation is updated according to:

$$g_a^2(n_{v_b}, t) = 1 - \left( \frac{\sum_{i_a=1}^{|N^a|} x_{i_a v_b}(t-1)}{\sum_{i_a=1}^{|N^a|} ub\_x_{i_a}(t-1)} \right) \tag{5.8}$$

In this case, the greater the quantity that all the other decision makers nodes in the level received by the alternative at time $t-1$ (the maximum value is when upper level value is reached), the lesser is the quality of goods provided the alternative, as the quality could be degrading with excessive use.

In this model, until now, it has not been taking into account the importance of the variability of the context. Due to the importance that it has in the decision making process, context has been introduced as the third criterion of the model. Hence the set $G$, earlier defined, become:

$G = \{g^1, g^2, g^3\}$

that, in order, represent cost,quality of service and context, respectively.

To each decision criteria, each node assigns a weight, that represents the importance the node $n_{s_a}$ gives to the specific criterion and it is represented by the vector $w_{s_a} = (w^1_{s_a}, \ldots, w^p_{s_a}, w^{p+1}_{s_a})$, that in this model it is represent by $w_{s_a} = (w^1_{s_a}, w^2_{s_a}, w^3_{s_a})$. These weights change dynamically, according to what happen to the network and in the network and this is expressed by the following law:

$$w^j_{s_a}(t) = \delta_{s_a} w^j_{s_a}(t-1) + (1 - \delta_{s_a}) \frac{\sum_{h \neq s,a} w^j_{h_a}(t-1) \cdot f(\overline{d}_{s_a h_a}(t-1)) \cdot a_{s_a h_a}}{\sum_{h \neq s,a} f(\overline{d}_{s_a h_a}(t-1)) \cdot a_{s_a h_a}} \tag{5.9}$$

Whilst the meaning and the values of the parameters $\delta_{s_a}$ and $a_{s_a h_a}$ are the same as before, the function $f(\overline{d}_{s_a h_a}(t-1))$ expresses the importance given by node $n_{s_a}$ to the preferences of node $n_{h_a}$ and it is a non decreasing function of distance as follows:

$$f(\overline{d}_{s_a h_a}(t-1)) = \frac{1}{\overline{d}^2_{s_a h_a}(t-1))} \tag{5.10}$$

where:

$$\overline{d}_{s_a h_a}(t) = \sqrt{\sum_{j=1}^{p} \left[ w^j_{s_a}(t-1) - w^j_{h_a}(t-1) \right]^2 + \overline{x}_{s_a h_a}(t)} \tag{5.11}$$

The first part of Equation 5.11 represents the Euclidean distance between the weight vectors of $n_{s_a}$ and $n_{h_a}$ (belonging to the same level in the network), and indicates the distance between the preferences of the two nodes. The second part, on the other hand, is a measure of the number of times nodes $n_{s_a}$ and $n_{h_a}$ have taken different decisions in previously considered time instants. Furthermore,the second part is strictly dependent from time, and in particular, the more recent decisions are, the more importance they have in the calculation of the distance between the two nodes, as it possible to see from Equation 5.12:

$$\overline{x}_{s_a h_a}(t) = \sum_{i=1}^{\#MEM} \beta_{s_a h_a}(i) \cdot \gamma^{i-1} \tag{5.12}$$

where:

- $\#MEM$ is the number of previous time instants considered by the model and represents the memory owned by the system. If $\#MEM = 0$ then the system can be considered as memory less and therefore the output of the system will be based only on current system state, without taking into account its previous history. On the other hand, if $\#MEM \neq 0$, it means the system keeps memory of previous history, as it happen in this model; in this case, the output is not dependent on the current state only but it takes into account also what happened in previous time instants. This can be considered an important property of the system, as the memory of what happened in previous instants of time can significantly influence the behaviour of each single node, contributing to increase or decrease the distance between two nodes in the network;

- $\beta_{s_a h_a}(i)$ is a coefficient, whose values belong to the set $[0, 1]$, which takes into account how much decisions taken by nodes $n_{s_a}$ and $n_{h_a}$ match with regards to the alternative set $A^a$. In particular, $\beta_{s_a h_a}(i)$ is 0 if $n_{s_a}$ and $n_{h_a}$ have taken, at the considered instant of time, the same decision, 1 if they have taken a different decision, any value between 0 and 1 if there is a partial match only between decisions taken by the two nodes;

- $\gamma \in ]0,1[$ is a damping coefficient which is used to weight decisions taken at different instants of time.

The two terms of Equation 5.11 are not expressed on the same scale. In order to make them commensurable, because the Euclidean distance can assume values in the interval $\left[0;\sqrt{2}\right]$ and the second term assumes the following values $\left\{0, \beta_{s_a h_a}(i)\gamma, \ldots, \beta_{s_a h_a}(\#MEM-1)\gamma^{\#MEM-1}\right\}$, the normalization is $\overline{x}_{s_a h_a}(t) \cdot \frac{\sqrt{2}(1-\gamma)}{1-\gamma^{\#MEM}}$.
The preferences of each node in the network continuously change, strictly depending both on decisional context and on the other nodes in the network. Therefore it is possible to assume the evaluation of an alternative $n_{v_b}$ at time $t$ and in a given context, depends on the evaluation of the alternative at time $t-1$ and on how much decisions taken by other nodes in the network with regards to that alternative differ, both in terms of quantity and cost, at instants $t-1$ and $t-2$, respectively. Also, the evaluation of an alternative, with regards to context criterion, change depending on the role the node has in the transition, that can be producer or consumer. In particular, producer decision maker nodes have to evaluate an alternative taking into account the order quantity and the unitary cost of all the other nodes (from all levels indiscriminately) have exchanged with that alternative: the greater the orders exchanged are, the more reliable it is supposed that alternative to be.
The mathematical formulation expressing the evaluation of an alternative with regards to a given context from the producer point of view is reported below:

$$\begin{aligned} g^3(n_{v_b}, t) = g^3(n_{v_b}, t-1) + \alpha \frac{\sum_{s_a=1}^{|N^a|} \sum_{z_a=1, z_a \neq s_a}^{|N^a|} a_{s_a z_a} \cdot x_{z_a v_b}(t-1) \cdot y_{z_a v_a}(t-1)}{\sum_{s_a=1}^{|N^a|} \sum_{z_a=1, z_a \neq s_a}^{|N^a|} a_{s_a z_a} min(ub\_x_{z_a}, ub\_x_{v_b})} + \\ - \frac{\sum_{s_a=1}^{|N^a|} \sum_{z_a=1, z_a \neq s_a}^{|N^a|} a_{s_a z_a} \cdot x_{z_a v_b}(t-2) \cdot y_{z_a v_a}(t-2)}{\sum_{s_a=1}^{|N^a|} \sum_{z_a=1, z_a \neq s_a}^{|N^a|} a_{s_a z_a} min(ub\_x_{z_a}, ub\_x_{v_b})} \end{aligned} \tag{5.13}$$

where $\alpha$ is a coefficient representing the importance given to the context variability.

On the other hand, it is not enough for the consumer or buyer decision maker nodes to evaluate an alternative taking into account the order quantity and the unitary cost all the other nodes have exchanged with that alternative only. Another important factor to be considered for them is the quality of service provided, which, in this model, it is assumed to be inversely proportional to the order quantity exchanged by the alternative with the other nodes in the network: the greater is the quantity received by other nodes from the evaluated alternative, the slower the service provided is. Then, the evaluation of an alternative with regards to a given context from the consumer point of view can be expressed by following equation:

$$
\begin{aligned}
g^3(n_{v_b},t) = g^3(n_{v_b},t-1) + \frac{\alpha}{2}\frac{\sum_{s_a=1}^{|N^a|}\sum_{z_a=1,z_a\neq s_a}^{|N^a|} a_{s_a z_a}\cdot x_{z_a v_b}(t-1)\cdot y_{z_a v_a}(t-1)}{\sum_{s_a=1}^{|N^a|}\sum_{z_a=1,z_a\neq s_a}^{|N^a|} a_{s_a z_a} min(ub\_x_{z_a}, ub\_x_{v_b})} + \\
-\frac{\alpha}{2}\frac{\sum_{s_a=1}^{|N^a|}\sum_{z_a=1,z_a\neq s_a}^{|N^a|} a_{s_a z_a}\cdot x_{z_a v_b}(t-2)\cdot y_{z_a v_a}(t-2)}{\sum_{s_a=1}^{|N^a|}\sum_{z_a=1,z_a\neq s_a}^{|N^a|} a_{s_a z_a} min(ub\_x_{z_a}, ub\_x_{v_b})} + \\
+\frac{\alpha}{2}\cdot\frac{\left(\sum_{s_a=1}^{|N^a|} x_{s_a v_b}(t-1) + \sum_{s_c=1,c\neq a,b}^{|N^c|} x_{s_c v_b}(t-1)\right)}{\sum_{s_a=1}^{|N^a|} min(ub\_x_{s_a}, ub\_x_{v_b}) + \sum_{s_c=1,c\neq a,b}^{|N^a|} min(ub\_x_{s_c}, ub\_x_{v_b})} + \\
-\frac{\alpha}{2}\frac{\left(\sum_{s_a=1}^{|N^a|} x_{s_a v_b}(t-2) + \sum_{s_c=1,c\neq a,b}^{|N^c|} x_{s_c v_b}(t-2)\right)}{\sum_{s_a=1}^{|N^a|} min(ub\_x_{s_a}, ub\_x_{v_b}) + \sum_{s_c=1,c\neq a,b}^{|N^a|} min(ub\_x_{s_c}, ub\_x_{v_b})}
\end{aligned}
\tag{5.14}
$$

As above mentioned, the aim of each decision maker is to maximise its utility, finding the optimal values of order quantity and price. The utility function can be calculated as the weighted average of evaluations of all the alternatives with regards to each criterion multiplied by the weight that each decision maker gives to that criterion. Formally, if node $n_{s_a}$ is a producer from the alternative $n_{v_b}$ it could gain:

$$
U_{s_a}(n_{v_b},t) = \sum_{j=1}^{2}\left[w_{s_a}^j \cdot g^j(n_{v_b},t)\right] + w_{s_a}^3 \cdot \exp^{-g^3(n_{v_b},t)} \tag{5.15}
$$

Instead, if node $n_{s_a}$ is a consumer:

$$U_{s_a}(n_{v_b}, t) = \sum_{j=1}^{2} \left[ w_{s_a}^{j} \cdot g^{j}(n_{v_b}, t) \right] + w_{s_a}^{3} \cdot \exp^{g^{3}(n_{v_b}, t)} \tag{5.16}$$

Considering the consumer or the buyer node, the context variability affects negatively the utility and it is possible to see in which manner: if many people choose a given alternative, the preference of the node towards that alternative decreases. This fact can cause a degradation in services quality. On the contrary, it can be observed that the context variability affects positively the utility for producer nodes: in fact, if many people choose the services provided by a given alternative, such alternative is assumed to be reliable. The constraints related to utility and other parameters cited above change on the basis of the different roles that nodes can have in the transitions and of the level taken into account. From these statements, three different optimization problems can be recognised according to the reference level, as reported in the following:

- Nodes of level 1 represent the producers selling their products to nodes of levels 2 and to nodes of level 3. Considering the consumers, at each instant of time, the sum of quantities of the alternatives received at the previous instant of time determines, in a proportional way, the quality of what such consumers have bought. The optimization problem is:

$$\begin{cases} \max U_{s_a}(n_{v_b}, t) = \sum_{j=1}^{2} \left[ w_{s_a}^{j} \cdot g^{j}(n_{v_b}, t) \right] + w_{s_a}^{3} \cdot \exp^{g^{3}(n_{v_b}, t)} \\ (x_{s_a v_b}, y_{s_a v_b}) \in \\ b \cdot (x_{s_a v_b})^{A} \leq ub\_x_{s_a} \\ \max(lb\_xv_b, lb\_xs_a) \leq x_{s_a v_b} \leq \min(ub\_x_{s_a}, ub\_x_{v_b}) \\ y_{s_a v_b} \leq \min(ub\_y_{s_a}, ub\_y_{v_b}) \\ x_{s_a v_b} \leq \frac{\sum_{z_a=1, z_a \neq s_a}^{|N^a|} a_{s_a z_a} \cdot x_{z_a v_b}}{\sum_{z_a=1, z_a \neq s_a}^{|N^a|} a_{s_a z_a} \cdot up\_x_{z_a v_b}} \end{cases}$$

The third condition is the Cobb-Douglas production function: the produced quantity must be less than the maximum capacity of the producer. In such a case, b is a positive constant which represents the total factor productivity while A is the return to scale i.e. the relation between the output modification and the change in levels of inputs used in production. In this model $A = 0.75$ represents a decreasing return to scale: in other words, an increase of 1% in input leads to a 0.75% increase in output. Condition 4 and condition 5 come from the following assumptions:

- The order quantity which is transferred from a node $n_{s_a}$ (in level 1) to an alternative $n_{v_b}$ (in level 2 or 3) has to be greater than the lower bound of the alternative while has to be lower than its buying demand (represented by its upper bound);
- The order quantity which is transferred from a node $n_{s_a}$ (in level 1) to an alternative $n_{v_b}$ (in level 2 or 3) has to be greater than the lower bound of the node while has to be lower than its maximum capacity (represented by its upper bound);
- Typically, the node involves some constraints also on price. Therefore, the unitary price has to be lower than the maximum price imposed by

the node (upper bound of the price of the node);

- The unitary price has to be lower than the maximum price related to the alternative (upper bound of the price of the alternative).

The 6-th condition represents the reliability can be associated to the alternatives. Considering a certain level, such reliability is influenced by the decisions of the other nodes in the same level. This fact can be modeled by taking into account the constraint that each node imposes on the order quantity delivered to a given alternative, which depends on the quantities delivered by the other nodes of the same level. Specifically, the amount of order quantity transferred from the node to an alternative, in percentage with respect to the total capacity of the node, cannot be greater than the percentage of order quantity transferred to the same alternative from other nodes of the same level.

- Nodes of level 2 represent the consumers receiving goods from level 1 as well as the producers which resell to level 3 the goods coming from level 1. Considering the consumers and referring to the alternatives in level 1, the quality of each alternative is inversely proportional to the quantities received by the nodes in level 2 by the specific alternative. Indeed, the greater is the total quantity transferred by the alternative, the lower is its quality. Considering instead the producers and referring to the alternatives in level 3, the quality of each alternative is directly proportional to the total quantity received by the alternative from all nodes in the level. In case of transition from level 1 to 2, the optimization problem is:

$$\begin{cases} \max U_{s_a}(n_{v_b}, t) = \sum_{j=1}^{2} \left[ w_{s_a}^j \cdot g^j(n_{v_b}, t) \right] + w_{s_a}^3 \cdot \exp^{-g^3(n_{v_b}, t)} \\ (x_{s_a v_b}, y_{s_a v_b}) \in \\ \max(lb\_xv_b, lb\_xs_a) \leq x_{s_a v_b} \leq \min(ub\_x_{s_a}, ub\_x_{v_b}) \\ y_{s_a v_b} \leq \min(ub\_y_{s_a}, ub\_y_{v_b}) \\ \sum_{v_b=1}^{\left|N^b\right|} x_{s_a v_b} \leq ub\_x_{s_a} \\ y_{s_a v_b} \geq y_{v_b s_a} \\ x_{s_a v_b} \leq \frac{\sum_{z_a=1, z_a \neq s_a}^{\left|N^a\right|} a_{s_a z_a} \cdot x_{z_a v_b}}{\sum_{z_a=1, z_a \neq s_a}^{\left|N^a\right|} a_{s_a z_a} \cdot up\_x_{z_a v_b}} \end{cases}$$

Conditions 3 and 4 establishes that the quantity ordered by a node $n_{s_a}$ in level 2, coming from an alternative $n_{v_b}$ in level 1 has to be greater than the lower bound of the alternative while has to be lower than its capacity (the latter being its upper bound). Condition 5 expresses that the total amount of product received by a node of level 2 from all alternatives of level 1 cannot exceed its maximum capacity in term of buyer demand, represented by its quantity upper bound in the model. Considering condition 6, it can be observed that each node of level 1 (i.e. each alternative for nodes in level 2) has already identified its own optimal choices as quantity and price to be submitted to alternatives in level 2 and 3. Such values can be assumed as having function of constraints when focusing on optimal values for nodes in level 2. Specifically, for each instant of time, the unitary price assigned to products bought by a node $n_{s_a}$ in level 2, coming from an alternative $n_{v_b}$ in level 1, must be greater than the optimal price of the alternative. The 7-th condition points out that in the evaluation of reliability of alternatives, each node is influenced by the decisions of the other nodes of the same level. This fact can be modeled by taking into account the constraint that each node imposes on the order quantity received a given alternative, which depends on the quantities received by the other nodes from the same alternative. Specifically, the amount of order quantity received by the node from an alternative, in percentage with respect to the total capacity of the node, cannot be greater than the percentage of order quantity transferred from the same alternative to the other nodes of the same level.

In case of transition from level 2 to 3, the optimization problem is:

$$\begin{cases} \max U_{s_a}(n_{v_b}, t) = \sum_{j=1}^{2} \left[ w_{s_a}^{j} \cdot g^{j}(n_{v_b}, t) \right] + w_{s_a}^{3} \cdot \exp^{g^{3}(n_{v_b}, t)} \\ (x_{s_a v_b}, y_{s_a v_b}) \in \\ \max(lb\_xv_b, lb\_xs_a) \leq x_{s_a v_b} \leq \min(ub\_x_{s_a}, ub\_x_{v_b}) \\ y_{s_a v_b} \leq \min(ub\_y_{s_a}, ub\_y_{v_b}) \\ b \cdot (x_{s_a v_b})^{A} \leq ub\_x_{s_a} \\ x_{s_a v_b} \leq \sum_{z_c=1, c<a}^{|N^c|} x_{s_a z_c} \\ y_{s_a v_b} \geq median(y_{v_b z_c}), \\ z_c = 1, \ldots, |N^c|, c < a \\ x_{s_a v_b} \leq \frac{\sum_{z_a=1, z_a \neq s_a}^{|N^a|} a_{s_a z_a} \cdot x_{z_a v_b}}{\sum_{z_a=1, z_a \neq s_a}^{|N^a|} a_{s_a z_a} \cdot up\_x_{z_a v_b}} \end{cases}$$

Condition 3 and condition 4 come from the following assumptions:

- The order quantity which is transferred from a node $n_{s_a}$ (in level 2) to an alternative $n_{v_b}$ (in level 3) has to be greater than the lower bound of the alternative while has to be lower than its buying demand (represented by its upper bound);
- The order quantity which is transferred from a node $n_{s_a}$ (in level 1) to an alternative $n_{v_b}$ (in level 2 or 3) has to be greater than the lower bound of

the node while has to be lower than its maximum capacity (represented by its upper bound);

The 5-th condition is the Cobb-Douglas production function, as mentioned above. According to this function, the produced quantity must be less than the maximum capacity of the producer. In such a case, $b$ is a positive constant which represents the total factor productivity i.e. an index of the efficiency of the available technology. $A$ is the return to scale i.e. the relation between the output modification and the change in levels of inputs used in production. In this model $A = 0.75$ represents a decreasing return to scale: in other words, an increase of 1% in input leads to a 0.75% increase in output. The 6-th condition deals with the assumption to have already solved the optimization problem related to transactions from nodes in level 2 towards alternatives in level 1. Each node of level 2 has already identified the optimal value in terms of quantities of orders to be submitted to alternatives of levels 1. The sum of such quantities determines the current capacity of the producer node in level 2. As each node cannot sell more than what it has bought, a constraint can be detected in terms of limits on the quantity delivered by the node $n_{s_a}$ in node 2 to each alternative $n_{v_b}$ in level 3. Such quantity must be lower than the current capacity of the node, which is calculated, for each instant of time, by the sum of all the optimal quantities transferred from alternatives of level 1 to the node in level 2. Condition 7 implicates that the unitary price for orders transferred to alternatives of level 3 must be greater than the median of unitary prices involved in transactions done towards nodes in level 1. The 8-th condition points out that in the evaluation of reliability of alternatives, each node is influenced by the decisions of the other nodes of the same level. This fact can be modeled by taking into account the constraint that each node imposes on the order quantity transferred to a given alternative, which depends on the quantities transmitted by the other nodes to the same alternative. Specifically, the amount of order quantity transferred by the node to an alternative, in percentage with respect to the total capacity of the node, cannot be greater than the percentage of order quantity transferred to the same alternative from the other nodes of the same level.

- Nodes of level 3 represent the consumers evaluating the quality of potential producers in levels 1 and 2. The quality for each alternative in level 3 is inversely proportional to the quantities received by nodes in level 3 from the specific alternative. The optimization problem is:

$$\begin{cases} \max U_{s_a}(n_{v_b}, t) = \sum_{j=1}^{2} \left[ w_{s_a}^{j} \cdot g^{j}(n_{v_b}, t) \right] + w_{s_a}^{3} \cdot \exp^{-g^{3}(n_{v_b}, t)} \\ (x_{s_a v_b}, y_{s_a v_b}) \in \\ \max(lb\_xv_b, lb\_xs_a) \leq x_{s_a v_b} \leq \min(ub\_x_{s_a}, ub\_x_{v_b}) \\ y_{s_a v_b} \leq \min(ub\_y_{s_a}, ub\_y_{v_b}) \\ y_{s_a v_b} \geq y_{s_a v_b} \\ \sum_{v_b=1}^{\left|N^b\right|} x_{s_a v_b} \leq ub\_x_{s_a} \\ x_{s_a v_b} \leq \frac{\sum_{z_a=1, z_a \neq s_a}^{|N^a|} a_{s_a z_a} \cdot x_{z_a v_b}}{\sum_{z_a=1, z_a \neq s_a}^{|N^a|} a_{s_a z_a} \cdot up\_x_{z_a v_b}} \end{cases}$$

  Condition 3 and condition 4 come from the following assumptions:

  - The order quantity which is transferred from an alternative $n_{v_b}$ (in level 1 or 2) to a generic node $n_{s_a}$ (in level 3) has to be greater than the lower bound of the alternative while has to be lower than its capacity (represented by its upper bound);
  - The order quantity which is transferred from an alternative $n_{v_b}$ (in level

1 or 2) to a generic node $n_{s_a}$ (in level 3) has to be greater than the lower bound of the node while has to be lower than its maximum capacity (represented by its upper bound);

– The unitary price has to be lower than the maximum price imposed by the node (upper bound of the price of the node).

The 5-th condition deals with the assumption to have already solved the optimization problem related to transactions from nodes in level 1 and level 2. Each node of level 1 and 2 has already identified the optimal value in terms of quantities and price to be submitted to alternatives of levels 3. Such values should be considered as constraints, when calculating optimal values for nodes in level 3. Specifically, for each instant of time, the unitary price for products bought by a node in level 3, from an alternative in level 1 or 2 must be greater than the optimal value of the price of the alternative. The 6-th condition points out that the total amount of quantities received by a node of level 3 from all alternatives of level 1 or 2 cannot exceed its maximum capacity in term of buyer demand, represented by its quantity upper bound in the model. The 7-th condition points out that in the evaluation of reliability of alternatives, each node is influenced by the decisions of the other nodes of the same level. This fact can be modeled by taking into account the constraint that each node imposes on the order quantity to be received by a given alternative, which depends on the quantities received by the other nodes from the same alternative. Specifically, the amount of order quantity received by the node from an alternative, in percentage with respect to the total capacity of the node, cannot be greater than the percentage of order quantity received from the same alternative by the other nodes of the same level.

## 5.4 Model Perfomance

In this section, the performance of the model has been analysed using a supplier selection problem, even if the model perfomance are still under investigation. In particular, the model is characterised by the following parameters:

- $l = 3$ the number of levels;
- $N^1 = 10$, the number of producers (nodes of the first level);
- $N^2 = 5$, the number of service producers (nodes of the second level);
- $N^3 = 8$, the number of consumers (nodes of the third level);
- each network level follows the Erdös-Rényi model (Erdös and Rényi, 1959) with a probability $p = 0.5$ of having a connection between two nodes of the same network level;
- $\alpha = 1$, the importance given to the context variability according to Equations 5.13 and 5.14;
- $\gamma = 0.75$, the damping coefficient used to weight decisions taken at different time instants, according to Equation 5.12;
- $\#PT = 1$, the number of previous time instants considered by the model when updating upper bounds;
- $\#MEM = 5$, the number of previous time instants considered by the model when evaluating weight, representing the system memory;
- the number of criteria is equal to 3, that are cost, quality of service and context;

- the number of iterations is set equal to 30, that are the number of iterations the simulation run before collecting results.

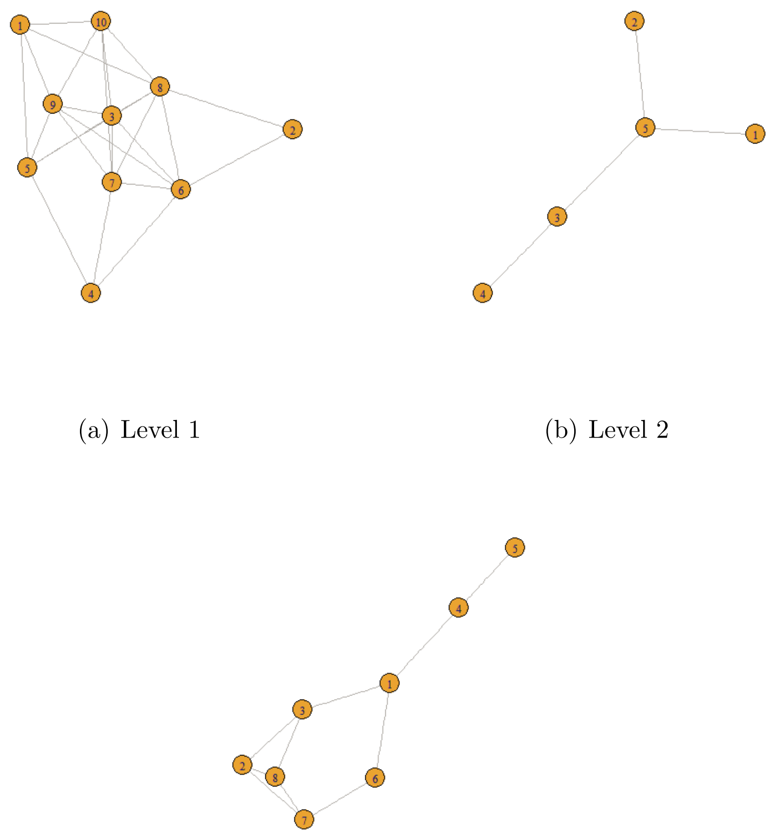
It is also assumed that the network conguration does not change over time, i.e. the numbers of nodes in the network, as well as the mutual connections between them do not change over the time and the upper bounds are not fixed, as they can vary over the time, depending on the sales'trend. The objective of the model is to determine the optimal values of $x_{s_a v_b}$ and $y_{s_a v_b}$, that are the order quantity that node $n_{s_a}$ places with the supplier $n_{v_b}$ and the unitary price for order that node $n_{s_a}$ places with the supplier $n_{v_b}$ after the whole number of iterations.
In Figure 5.5, the configuration of each single level of the supernetwork is reported.

In Table 5.1 the values of $\delta_{s_a}$ are shown:

Table 5.1: Values of $\delta_{s_a}$ for the three network levels

| | Level 1 | Level 2 | Level 3 |
|---|---|---|---|
| $node_1$ | 0.3404 | 0.6855 | 0.1416 |
| $node_2$ | 0.8595 | 0.8485 | 0.1595 |
| $node_3$ | 0.9261 | 0.9941 | 0.0768 |
| $node_4$ | 0.4761 | 0.1824 | 0.5244 |
| $node_5$ | 0.5485 | 0.7532 | 0.2389 |
| $node_6$ | 0.0116 | | 0.7606 |
| $node_7$ | 0.1178 | | 0.2209 |
| $node_8$ | 0.0174 | | 0.9132 |
| $node_9$ | 0.4607 | | |
| $node_{10}$ | 0.6761 | | |

As it is possible to see from the analytical model presented above, a central role in the optimisation process is played by the distance, as expressed from Equation



(a) Level 1

(b) Level 2

(c) Level 3

Figure 5.5: Network Configuration of each single level of the supernetwork

5.11. Focusing, at the moment, on the nodes of level 1 and in the transaction with nodes of level 2, in Table 5.2, it is reported the values of distance at the first time instant considered. As it is possible to see, there are nodes that are closer than others because they attribute similar weights to the decision criteria and they have similar decision at the previous time instant, as for Equation 5.11.

Table 5.2: Values of distance of nodes of level 1 at the first time instant

| Node | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 1.24054 | 0.6749718 | 1.30899 | 1.267738 | 0.6206993 | 0.9713021 | 1.173467 | 1.387209 | 1.025032 |
| 2 | 1.24054 | 0 | 1.160094 | 0.9727207 | 1.020966 | 1.285829 | 1.210442 | 0.7100727 | 0.655453 | 1.125473 |
| 3 | 0.6749718 | 1.160094 | 0 | 1.121376 | 0.9290651 | 0.8274152 | 0.4408313 | 0.7840344 | 1.273789 | 0.8143048 |
| 4 | 1.30899 | 0.9727207 | 1.121376 | 0 | 0.1895117 | 1.430761 | 0.92565 | 0.4547227 | 0.8966495 | 0.8370998 |
| 5 | 1.267738 | 1.020966 | 0.9290651 | 0.1895117 | 0 | 1.397337 | 0.8501497 | 0.5279076 | 0.9655578 | 0.7652098 |
| 6 | 0.6206993 | 1.285829 | 0.8274152 | 1.430761 | 1.397337 | 0 | 1.161181 | 1.262961 | 1.451201 | 0.9590901 |
| 7 | 0.9713021 | 1.210442 | 0.4408313 | 0.92565 | 0.8501497 | 1.161181 | 0 | 0.9935836 | 1.246772 | 0.5578671 |
| 8 | 1.173467 | 0.7100727 | 0.7840344 | 0.4547227 | 0.5279076 | 1.262961 | 0.9935836 | 0 | 0.7197188 | 0.9023894 |
| 9 | 1.387209 | 0.655453 | 1.273789 | 0.8966495 | 0.9655578 | 1.451201 | 1.246772 | 0.7197188 | 0 | 1.154072 |
| 10 | 1.025032 | 1.125473 | 0.8143048 | 0.8370998 | 0.7652098 | 0.9590901 | 0.5578671 | 0.9023894 | 1.154072 | 0 |

After completing all the 30 iterations, in most cases the distance values have changed according to the dynamics of the network, due to the variability of the preferences and of the context. Hence, after the selected simulation interval has run out, the new distance values are reported in Table 5.3. It is observable that the closer two nodes are in terms of distance the more similar are their optimal values of $x_{s_a v_b}$ and $y_{s_a v_b}$, giving rise to clusters of nodes, formed on the basis of their distance, and then on the decisions taken at the previous time instants.

Table 5.3: Values of distance of nodes of level 1 at the time instant 30

| Node | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 1.425069 | 1.261559 | 1.361509 | 1.40536 | 1.118815 | 1.169347 | 1.057354 | 1.332275 | 1.414797 |
| 2 | 1.425069 | 0 | 1.430072 | 1.424432 | 1.413692 | 1.330389 | 1.413303 | 1.427588 | 1.221282 | 1.424983 |
| 3 | 1.261559 | 1.430072 | 0 | 0.6888702 | 0.3296515 | 0.7860841 | 1.0643 | 0.3504881 | 1.263556 | 1.420406 |
| 4 | 1.361509 | 1.424432 | 0.6888702 | 0 | 0.2767589 | 1.408236 | 1.419299 | 0.5078736 | 1.418611 | 1.419685 |
| 5 | 1.40536 | 1.413692 | 0.3296515 | 0.2767589 | 0 | 1.034831 | 1.322879 | 0.8058399 | 1.119167 | 1.417201 |
| 6 | 1.118815 | 1.330389 | 0.7860841 | 1.408236 | 1.034831 | 0 | 0.4638859 | 1.376642 | 0.8121496 | 1.042882 |
| 7 | 1.169347 | 1.413303 | 1.0643 | 1.419299 | 1.322879 | 0.4638859 | 0 | 1.418065 | 0.8119163 | 1.153946 |
| 8 | 1.057354 | 1.427588 | 0.3504881 | 0.5078736 | 0.8058399 | 1.376642 | 1.418065 | 0 | 1.418701 | 1.417589 |
| 9 | 1.332275 | 1.221282 | 1.263556 | 1.418611 | 1.119167 | 0.8121496 | 0.8119163 | 1.418701 | 0 | 1.415337 |
| 10 | 1.414797 | 1.424983 | 1.420406 | 1.419685 | 1.417201 | 1.042882 | 1.153946 | 1.417589 | 1.415337 | 0 |

With respect to the model presented in the previous chapter, in this case it can not happen that two nodes have the same values of $x_{s_a v_b}$ and $y_{s_a v_b}$, but their values can be similar. This explains why, to form clusters it has been set two tolerance thresholds for $x_{s_a v_b}$ and $y_{s_a v_b}$ that are 3 and 0.75. If the optimal values of two nodes differ for a quantity less than the tolerance threshold, they belong to the same cluster. In the considered case, there are 5 clusters, as it follows:

Table 5.4: Clusters of level 1

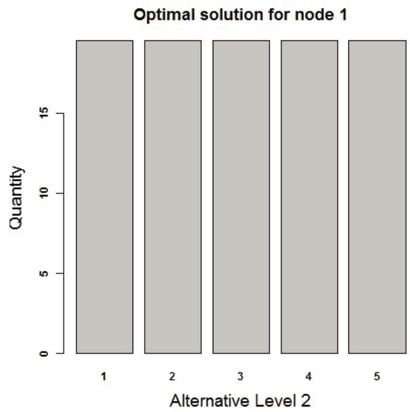
| Cluster | Node |
|---|---|
| $c_1$ | 1,2 |
| $c_2$ | 3,8 |
| $c_3$ | 4,5,6,7 |
| $c_4$ | 9 |
| $c_5$ | 10 |

Analysing in deepen, even if nodes 1 and 2 are not directly connected and $c_1$ can not be considered a "real" cluster, node 1 is influenced in its decisions by node 2 due to its high value of $\delta_{s_a}$ and this influence is transmitted by its common neighbor, node 8.

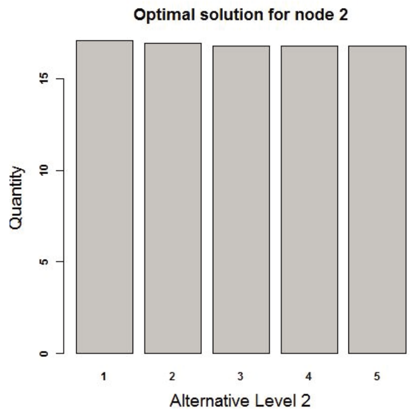
Clusters $c_2$ and $c_3$ are formed by nodes with the smaller distance, as reported in Table 5.3. Instead, clusters $c_4$ and $c_5$ are composed of only one node because its preferences in the previous time instants are far from the ones expressed by the other nodes of the network.

In Figures 5.6, 5.7, 5.8, 5.9 and 5.10 are shown the optimal values of the nodes of the first level, grouped in clusters.

Comparing these results with the ones of the model presented in the previous chapter, in this case $\delta_{s_a}$ has not a direct influence on the optimisation process and

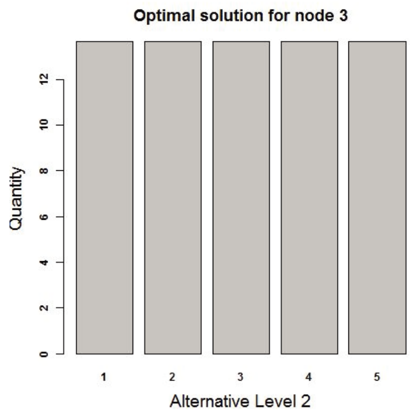



(a) Node 1

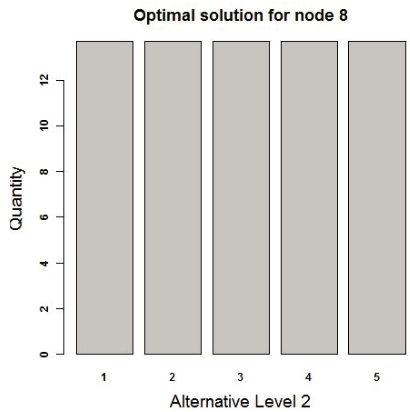



(b) Node 2

Figure 5.6: Cluster $c_1$, composed of nodes 1 and 2



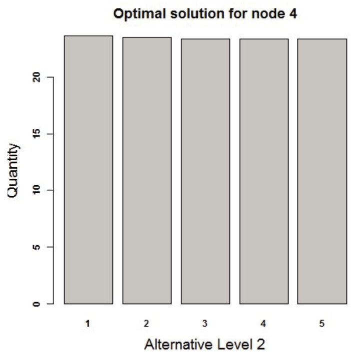

(a) Node 3



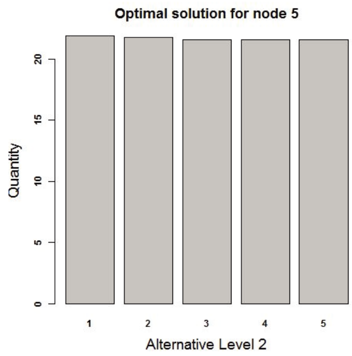

(b) Node 8

Figure 5.7: Cluster $c_2$, composed of nodes 3 and 8

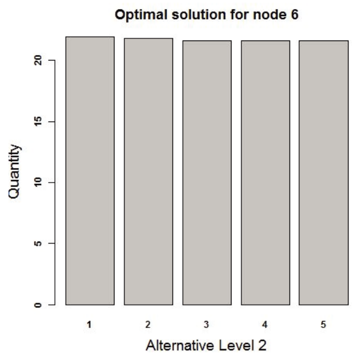
then on the decisions taken by nodes. What plays a central role in this model is the distance between a couple of nodes. The less is the distance the more similar are the decisions taken by nodes. Despite this it is possible to state that the influence of $\delta_{s_a}$



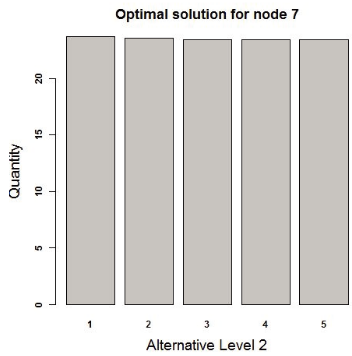

(a) Node 4




(b) Node 5




(c) Node 6




(d) Node 7

Figure 5.8: Cluster $c_3$, composed of nodes 4, 5, 6 and 7

is not so directly visible but it is present. In fact, each node follows who is connected to it and has a greater value of $\delta_{s_a}$ with respect to it, that it is not necessarily the greatest of those connected to it.

For the transaction between nodes of level 1 and 3 it is possible to state the same as what has been said until now.

For the following levels 2 and 3, for the sake of simplicity, it is reported only the values of distance obtained at the last iteration and the optimal values of $x_{s_a v_b}$ and

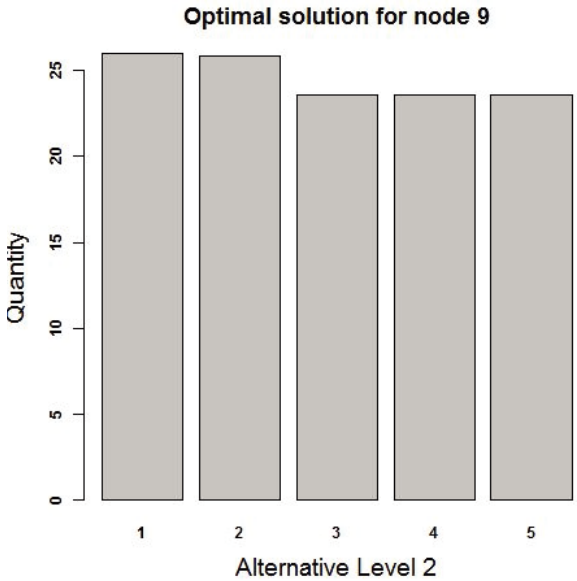



Figure 5.9: Cluster $c_4$, composed of node 9

$y_{s_a v_b}$, grouped as the clusters collected during simulation. For level 2 Table 5.5 shows the distance values at $30^{th}$ iteration, while Figures 5.11, 5.12, 5.13 and 5.14 show the optimal values for each node of the four clusters.

Table 5.5: Values of distance of nodes of level 2 at the time instant 30

| Node | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0 | 0.2512896 | 0.9930421 | 0.920659 | 0.8812107 |
| 2 | 0.2512896 | 0 | 1.066269 | 0.940367 | 0.9459873 |
| 3 | 0.9930421 | 1.066269 | 0 | 0.5781873 | 0.6612002 |
| 4 | 0.920659 | 0.940367 | 0.5781873 | 0 | 0.517191 |
| 5 | 0.8812107 | 0.9459873 | 0.6612002 | 0.517191 | 0 |

For level 3 Table 5.6 shows the distance values at $30^{th}$ iteration, while Figures 5.15, 5.16, 5.17 and 5.18 show the optimal values for each node of the four clusters.

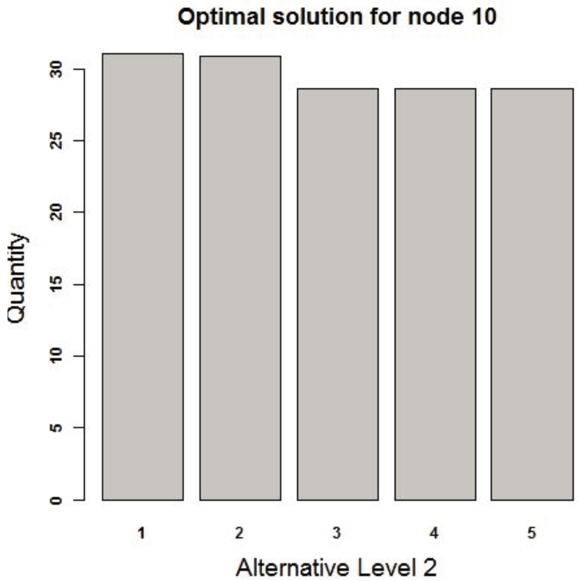



Figure 5.10: Cluster $c_5$, composed of node 10

Table 5.6: Values of distance of nodes of level 3 at the time instant 30

| Node | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0.8932003 | 0.6017647 | 0.2944246 | 0.7025607 | 0.4490914 | 0.8129531 | 0.9260177 |
| 2 | 0.8932003 | 0 | 0.7284354 | 0.8347655 | 0.7810337 | 0.7209168 | 0.6308429 | 0.5054292 |
| 3 | 0.6017647 | 0.7284354 | 0 | 0.5323445 | 0.8099842 | 0.6615017 | 0.6175259 | 0.7443524 |
| 4 | 0.2944246 | 0.8347655 | 0.5323445 | 0 | 0.6971354 | 0.5238086 | 0.8095658 | 0.9406174 |
| 5 | 0.7025607 | 0.7810337 | 0.8099842 | 0.6971354 | 0 | 0.623389 | 0.5792234 | 0.7821023 |
| 6 | 0.4490914 | 0.7209168 | 0.6615017 | 0.5238086 | 0.623389 | 0 | 0.6942684 | 0.872521 |
| 7 | 0.8129531 | 0.6308429 | 0.6175259 | 0.8095658 | 0.5792234 | 0.6942684 | 0 | 0.6092165 |
| 8 | 0.9260177 | 0.5054292 | 0.7443524 | 0.9406174 | 0.7821023 | 0.872521 | 0.6092165 | 0 |

In this case it is important to note that in cluster $c_2$ node 2 can be considered as a sort of bridge between nodes 7 and 8, that among them differ of quantities greater than the thresholds set above, but both differ less than the threshold from node 2. This phenomenon may be due to the high value of $\delta_{s_a}$ of node 8, that has a great

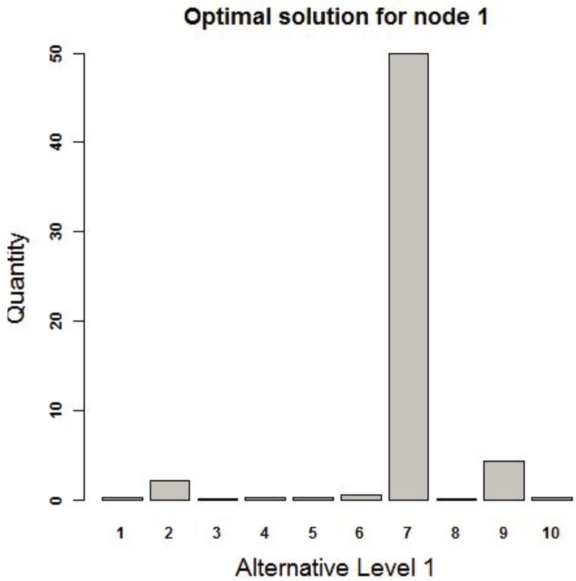
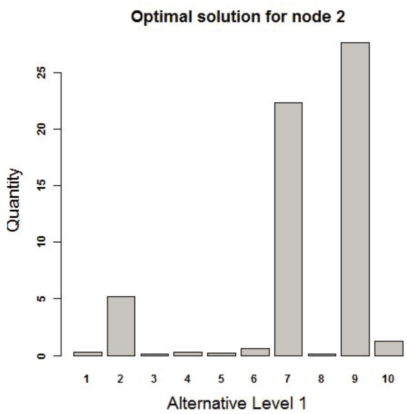
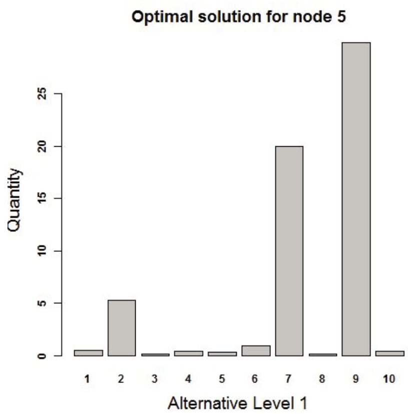



Figure 5.11: Cluster $c_1$, composed of node 1




(a) Node 2




(b) Node 5

Figure 5.12: Cluster $c_2$, composed of nodes 2 and 5

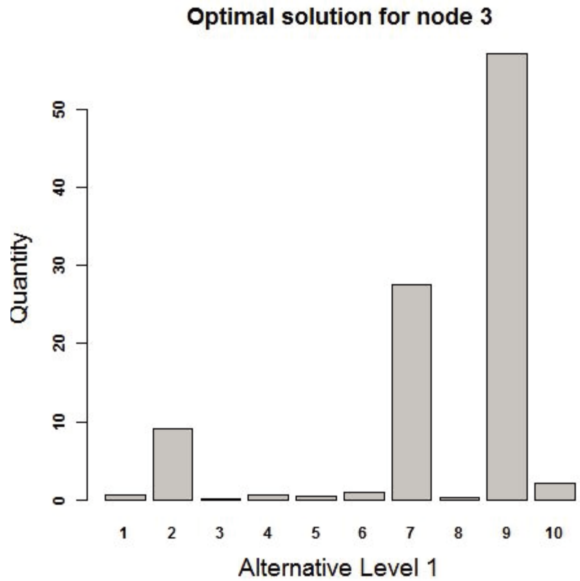



Figure 5.13: Cluster $c_3$, composed of node 3

influence on node 2 and then on node 7. [1]

## 5.5 Summary remarks

In our society, most processes, more or less, can be considered as decision making processes. In particular, they are individual decision making process, where the decision makers decide also under the influence of other individuals composing their network.

In this chapter, the model proposed in chapter 4 has been fit and customised for a

[1] The Figures of the optimal values of $x_{s_a v_b}$ and $y_{s_a v_b}$ are referred to the transitions from level 1 to level 2, from level 2 to level 1 and from level 3 to level 1. For the sake of simplicity the other transactions have been not reported, even if the clustering effect is the same as for the other transactions.

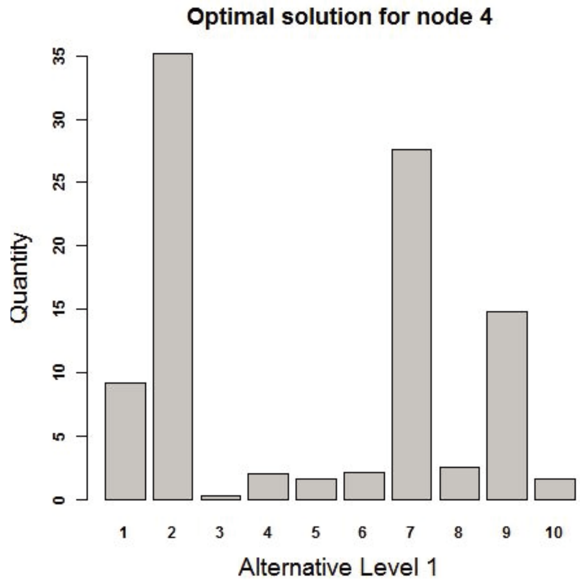



Figure 5.14: Cluster $c_4$, composed of node 4

supply chain problem, which is a multiple criteria decision making problem. Otherwise, in this case a model of influence has been introduced, making more complex the problem itself. In addition to the variation of preferences and of the context, in this case the theory of supernetwork is introduced to solve the supply chain problem. In this way, every node of the network, solving the optimisation problem, finds the best solution for it in order to have the maximum utility, i.e. the greater gain. The first result obtained shows that, despite the previous model, other parameters have a greater influence on the model dynamics, leading to a different network behaviour. In fact, if on the previous model the inclination to be influenced had a great impact on the decisions of the node, in this case the importance of this inclination is not so direct and, hence, its effect can be seen not directly.

This work can be considered as a first attempt to join a context-aware and social multiple criteria decision making with the theory of supernetwork.

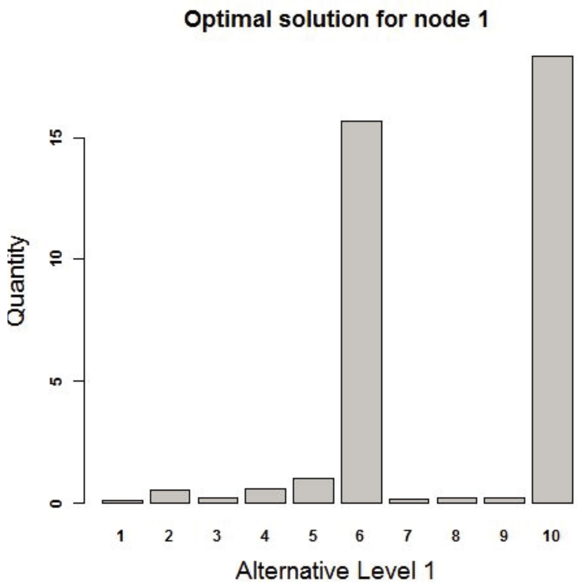
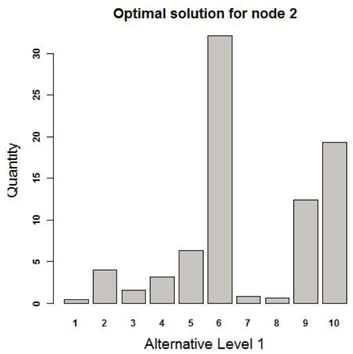
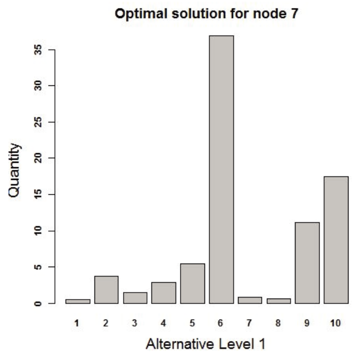


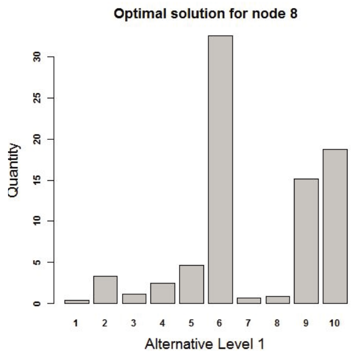

Figure 5.15: Cluster $c_1$, composed of node 1




(a) Node 2




(b) Node 7




(c) Node 8

Figure 5.16: Cluster $c_2$, composed of nodes 2, 7 and 8

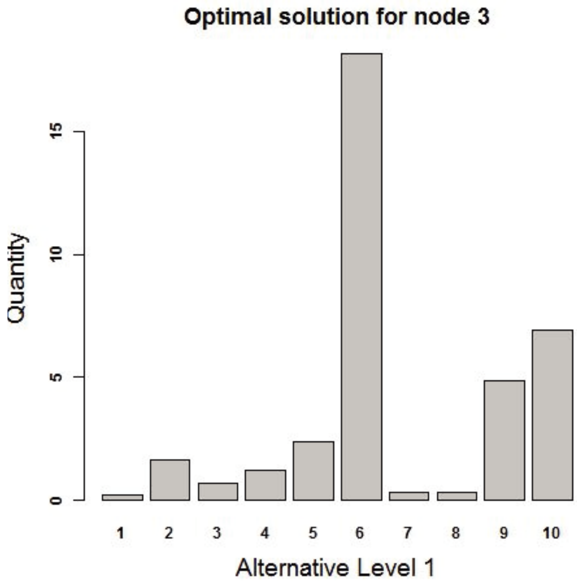
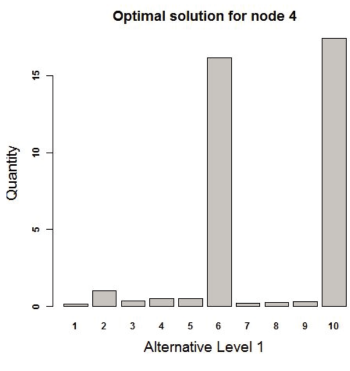
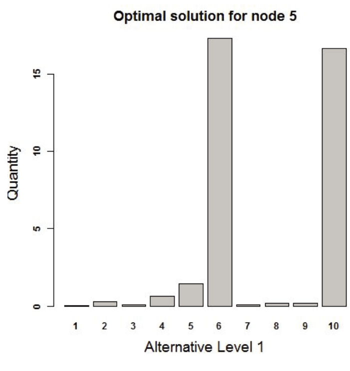



Figure 5.17: Cluster $c_3$, composed of node 3




(a) Node 4




(b) Node 5

Figure 5.18: Cluster $c_4$, composed of nodes 4 and 5

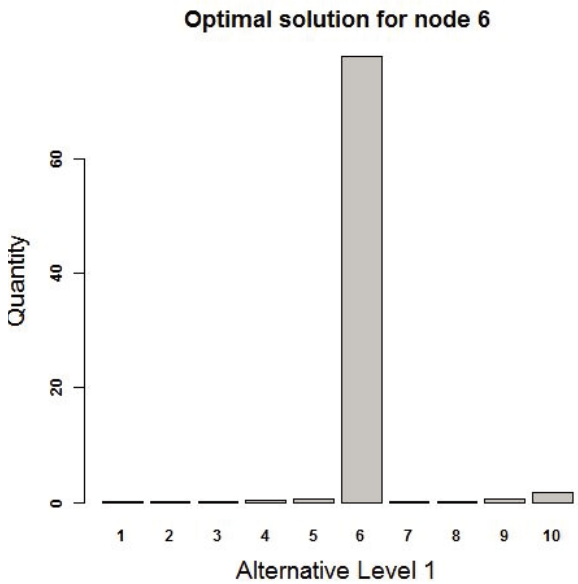



Figure 5.19: Cluster $c_5$, composed of node 6

# Chapter 6

# A dynamic and context-aware model of knowledge transfer and learning using a decision making perspective

## 6.1 Introduction

In the era of innovation and technology advance data, information and knowledge play a central role in any process regarding the development and the progress level of a society. The main aim for all the countries is to become "knowledge societies" in continuous development thanks to the limitless knowledge growth which generate incommensurable value (Fedoroff, 2012). Furthermore, thanks to the evolution of the Information and Communication Technology (ICT) there are no limits on when, where and how knowledge has to be transferred among individuals. Individuals create knowledge and thanks to the "communities of interaction", that can have the boundaries of an organization or can spread through the Internet, new knowledge is generated and improved, starting from the individual one. In fact, the innovation of an organization can be considered as a process through which a problem is defined and solved by the creation of new knowledge (Nonaka, 1994). Looking much more in detail, each individual decides (Guy et al., 2015) and acts within a social network, characterised by a dynamic, ubiquitous, complex and context-dependent nature. For each entity (Cioffi-Revilla, 2013), representing the network node, the consideration of who is connected to whom as well as the structure of the network have an important effect on the type of information passed, on its quantity and on the efficiency of the process itself (Cowan and Jonard, 2004). Furthermore, by taking into account the role of the context, the importance of each single relation (Barrat et al., 2004) and the structure of the network itself can vary depending on the considered context. In fact it is different the level of awareness held by the single node.

In this chapter a process of knowledge transfer is considered using a context-aware decision making perspective in which, before accepting or rejecting knowledge from one of its neighbors, a network node judges if its evaluation satisfies some criteria, i.e. knowledge distance and confidence, and, after that, it decides what to do. If the process takes place and the receiver node accepts the transfer, it will perform a control on what it has just accepted on the basis of three parameters. If the control result is positive, the receiver node will increase its confidence in the sender node. On the contrary case it will decrease its confidence and it will learn only a percentage of the received knowledge.

## 6.2 Knowledge and its Processes within a Network

Taking into consideration a small network or a big one, knowledge guides every process in it and this is the reason why it is subject of interest of many research fields. From Plato to Locke and Kant it is an evidence of the endless research for the definition of knowledge, whose notion and meaning has not a single interpretation. Two of the main feature of the processes that involve knowledge are complexity, due to the complexity that characterises the definition of knowledge itself, and dynamism, caused by the continuous change of knowledge characteristic. Both these features are also related to the property of the environment itself in which the processes are considered.

Between the definition of knowledge and information there is a substantial difference, even if in some cases they are used indifferently. Information is compared to a "flow of messages" (Machlup, 1983; Nonaka, 1994) that can contribute to shape an individual outlook or insight (Davenport and Prusak, 1998). Knowledge instead is based on information and it includes know-how (Zander and Kogut, 1995). More specifically "*knowledge is a fluid mix of framed experience, values, contextual information, and expert insight that provides a framework for evaluating and incorporating new experiences and information*" (Davenport and Prusak, 1998). Information, then, is necessary to create knowledge both from a "syntactic" (the volume of information) and "semantic" (the meaning of information) point of view (Shannon and Weaver, 1949),(Dretske, 1981).

In the Knowledge Management field it is also important to distinguish between two categories of knowledge: tacit and explicit. Tacit knowledge was firstly introduced in 1967 (Polanyi, 1967) and its meaning is well summarised from the sentence "*We*

*can know more than we can tell"*. In fact, it refers to knowledge that is difficult to express and transmit because it depends on human and personal qualities of the individual, that make it not easily transferable among individuals (Nonaka, 1994). It involves both cognitive (mental models, through which represent by the means of analogies and originating perspectives of the world (Johnson-Laird, 1983)) and technical (know-how and skills) elements. Explicit knowledge is easily formalised, codified, transmitted in a formal and a systematic language (Nonaka, 1994),(Brown and Duguid, 1991). It can be found in databases, manuals and documents.
Making a comparison with the theory of signals, tacit knowledge can be considered as an analog signal and explicit knowledge to a digital one. A process that involves the share of tacit knowledge uses a "parallel processing" to resolve the complex aspects of the problem and all the different dimensions of the process are processed simultaneously. Instead explicit knowledge is formalised in databases, archives expressed as a sequence (Bateson, 1972).
As previously said, individuals create knowledge and what is at the basis of this process is commitment (Polanyi, 1967), which is originated from three factors: intention, autonomy and environmental fluctuation. Intention represents the vision of the surrounding environment through the individual approach. Autonomy is the self-motivation to create new knowledge, based for example on deep emotions. Fluctuations represents, instead, events that create discontinuity and chaos with respect to previous environment state. These breakdowns lead individuals to reconsider their previous perspectives, allowing them to adjusting their thoughts and commitments (Winograd and Flores, 1986). Social interactions represent the principal key factor to create new knowledge and, furthermore, allowing the process of conversion from tacit to explicit knowledge and viceversa. Taking as a reference point the ACT model (Adaptive Control of Thought) developed by Anderson (Anderson,

2013), Nonaka (Nonaka, 1994) identified four modes of knowledge creation, as it is represented in Figure 6.1.

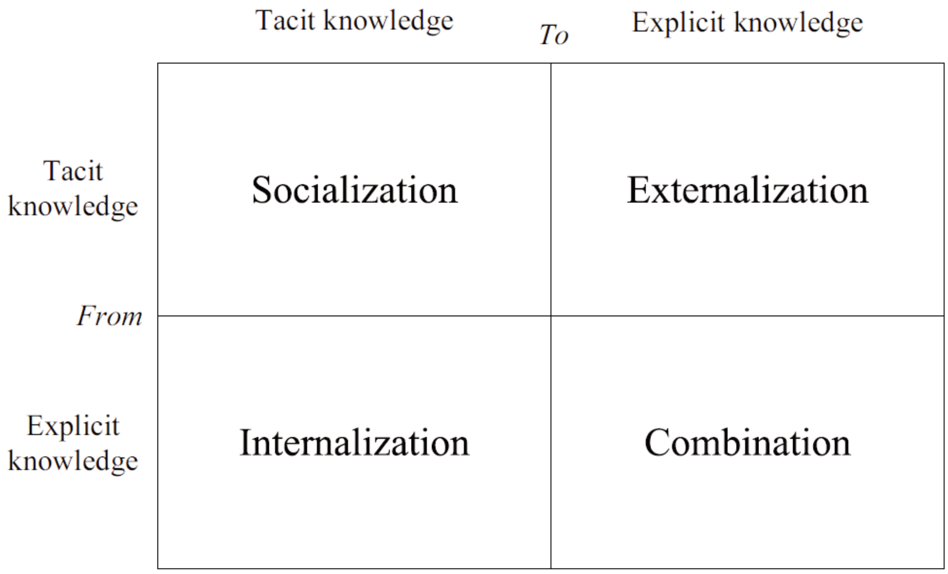



Figure 6.1: The four different modes of Knowledge Creation

The first one represent the process of "SOCIALIZATION". Individuals can get tacit knowledge through interactions among them, without the need to express knowledge through language. Then, sharing experience has a central role for the success of this process.

The second one is the process of "COMBINATION". The conversion of explicit knowledge in explicit knowledge among individuals is possible exchanging knowledge and who receives new knowledge will sorting, adding, recategorizing and recontextualizing it.

The third and forth processes are called "INTERNALIZATION" (from explicit to tacit knowledge, through a process of learning and assimilation) and "EXTERNALIZATION" (from tacit to explicit knowledge, through a process of concept formalisation). The basic idea that underlies both these processes is that tacit and explicit knowledge are complementary and increase their value over time through a process of mutual interaction. These four modes expressed above can be represented by a spiral process, that expresses graphically the fundamental concepts of social interactions that underlies all the processes of conversion, as reported in Figure 6.2.

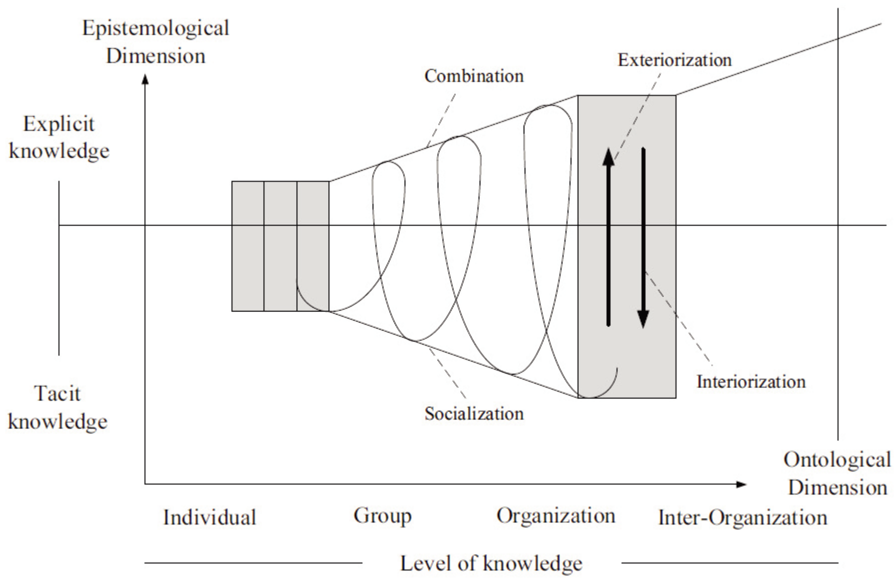



Figure 6.2: The Spiral of Knowledge

Knowledge increase its value along two dimensions: epistemological and ontological. The epistemological dimension represents the conversion from tacit to explicit on the basis of the four modes expressed before; the ontological one, instead, indicates the transfer of knowledge from the single individual to organizations. Hence, in a network as well as among different individuals, knowledge can be shared, transferred and exchanged (Graham et al., 2006). Knowledge sharing corresponds to the provision of information and know-how of a task among individuals inside and outside a group (Cummings, 2004). Knowledge transfer includes two phases: the sharing of knowledge from a source and its acquisition from a recipient. Knowledge exchange involves both knowledge sharing through which a source provides knowledge and knowledge seeking, where a receiver searches knowledge from sources (Wang and Noe, 2010). Several works have analysed the processes involving knowledge in a network by using different perspectives (Lambiotte and Panzarasa, 2009),(Tasselli, 2015),(Hatak and Roessl, 2015).

## 6.3 Model Description

A model of knowledge transfer is characterised by three main features i. e. the dynamism, the complexity and the context-dependence. The model presented in this chapter looks at the knowledge transfer process as a decision making one, taking as reference points two models reported in the scientific literature (Cowan and Jonard, 2004; Luo et al., 2015). These two existing models do not consider the process of knowledge transfer and learning as a decision making one and, hence, they do not investigate the factors and parameters that are at the basis of the exchange process, e. g. the decision criteria. The previous works analyse in depth the effect of the transfer process on the network parameters and structures, given a detailed analysis on them. Instead, in this work it is assumed that the process of knowledge transfer is an individual decision making process where each node, part of a network, is involved in a process of knowledge transfer. In particular, it has to decide whether to accept or not knowledge coming from its neighboring nodes which represent the set of alternatives. In a first stage, it has been chosen to take into account only a single process regarding explicit knowledge, due to its unambiguous and clear characteristics of easy codification and transmission. For the description of the model the following notation has been used:

- $N = \{n_1, \ldots, n_i, \ldots, n_m\}$, a finite set of nodes;
- $K = \{K_1, \ldots, K_k, \ldots, K_p\}$, a finite set of contexts;
- $v_i^{K_k}(t) = \left\{v_{i,1}^{K_k}(t), \ldots, v_{i,l}^{K_k}(t), \ldots, v_{i,q}^{K_k}(t)\right\}$, the knowledge vector of the node $n_i$ with respect to the $q$ categories and the context $K_k$ at time $t$;
- $A_{ij}^{K_k} = \left\{a_{ij}^{K_k}\right\}$, the adjacency matrix representing the network in the context $K_k$. $a_{ij}^{K_k} = \{0, 1\}$ is each single element which identifies if the link between nodes $n_i$ and $n_j$ is present or not;
- $N_i^{K_k} = \left\{n_j \in N : a_{i,j}^{K_k} = 1\right\}$, the set of nodes linked to node $n_i$ in the context $K_k$. It represents the set of alternatives for node $n_i$.

As explained in Section 6.1 one of the context roles is to characterise and differentiate the strength of each node's connection and the structure of the network itself. In order to do so, it has been considered the vector of weights $w_i^{K_k} = \left(w_{i,1}^{K_k}, \ldots, w_{i,j}^{K_k}, w_{i,m}^{K_k}\right)$, where each element $w_{i,j}^{K_k}$ represents the strength of the relation between node $n_i$ and node $n_j$ in the context $K_k$. $w_{i,j}^{K_k}$ can be different from $w_{j,i}^{K_k}$ ($w_{i,j}^{K_k} \neq w_{j,i}^{K_k}$). With respect to the previous models, the decision whether to accept or not the knowledge offered from another node in the network is based on two criteria i. e. knowledge distance and confidence. Each alternative $n_j \in N_i^{K_k}$

has an evaluation on each of the two criteria. The first criterion is defined as:

$$d_{ij,l}^{K_k}(t) = v_{j,l}^{K_k}(t) - v_{i,l}^{K_k}(t) \tag{6.1}$$

This distance represents the quantity of knowledge that node $n_i$ could receive from node $n_j$ in the category $l$ within the considered context. The knowledge distance can be considered the expression of the knowledge heterogeneity of the two nodes involved in the process. If there is a high knowledge gap between two network nodes (high heterogeneity), node $n_i$ could have no gain from the knowledge received from node $n_j$ (Luo et al., 2015).

The second criterion is represented by the confidence. In particular, at the moment, it has been supposed that the confidence $c_{i,j}^{K_k}$ that the node $n_i$ has in node $n_j$ in the context $K_k$ is defined as:

$$c_{i,j}^{K_k}(t) = \frac{w_{i,j}^{K_k}(t) + J_{i,j}^{K_k}}{2} \tag{6.2}$$

where $w_{i,j}^{K_k}(t)$ is the weight that node $n_i$ gives to the link with node $n_j$. $J_{i,j}^{K_k}$ is the Jaccard similarity (Jaccard, 1901) i. e. an expression of the concept of homophily (Lazarsfeld et al., 1954; Di Stefano et al., 2015), calculated as the ratio of the common neighbors of the nodes $n_i$ and $n_j$ to the number of nodes that are neighbors of at least one between $n_i$ and $n_j$. The greater the confidence that $n_i$ has in $n_j$, the more susceptible node $n_i$ is to learn from node $n_j$ (Pentland, 2014). In order to ensure that the knowledge transfer process to take place, the evaluation of alternative $n_j$ belonging to the set $N_i^{K_k}$ in each of the two decision criteria has to satisfy at the same time this two condition:

- $d_{ij,l}^{K_k}(t) \leq d$, that is the knowledge distance has to be under a knowledge distance threshold;
- $c_{i,j}^{K_k}(t) \geq c$, that is the confidence has to be over a certain confidence threshold.

Among the set of nodes satisfying at the same time both conditions related to the two criteria, node $n_i$ for each knowledge category will accept knowledge from the one that can give it the greatest amount of knowledge. The knowledge level of node $n_i$ in the category $l$ in the context $K_k$ will become:

$$v_{i,l}^{K_k}(t+1) = v_{i,l}^{K_k}(t) + max_{n_j \in N_i^{K_k}}((\lambda_{ij,l}^{K_k}(v_{j,l}^{K_k}(t) - v_{i,l}^{K_k}(t))) \tag{6.3}$$

where:

- $v_{i,l}^{K_k}(t)$ ($v_{j,l}^{K_k}(t)$) represents the knowledge level of node $n_i$ ($n_j$) in category $l$ in the context $K_k$ at time $t$;
- $\lambda_{ij,l}^{K_k}$ represents the absorptive capacity of node $n_i$ with respect to the knowledge received from node $n_j$ in the category $l$. In this model, it has been assumed that the value of $\lambda_{ij,l}^{K_k}$ is strictly related to the risk attitude of node $n_i$ (Kahneman and Tversky, 1979). As shown in Figure 6.3, it has been assumed that the process of knowledge transfer is located into the region identified by the red box i. e. the greater the amount of knowledge received by node $n_i$ ($x_1 < x_2$) the greater its utility is ($u(x_1) < u(x_2)$) but the greater its risk aversion is with the increasing quantity of knowledge that a node $n_j$ wants to transfer to node $n_i$ (Binswanger, 1980; Holt and Laury, 2002), in order, for example, not to imperil its security (La Corte et al., 2011). Hence, the value of $\lambda_{ij,l}^{K_k}$ will be a function of the knowledge distance and it can be expressed as:

$$\lambda_{ij,l}^{K_k}(t) = \frac{1}{\exp^{d_{ij,l}^{K_k}}(t)} \tag{6.4}$$

  According to this formulation, the values that $\lambda_{ij,l}^{K_k}(t)$ can assume are included in the set $\left[\frac{1}{\exp^d}; 1\right]$. In such a way if the values are closer to $\frac{1}{\exp^d}$ it means that node $n_i$ is more risk averse and then it assimilates less knowledge, than a node that has a value of $\lambda_{ij,c}^{K_k}(t)$ near to 1 that it assimilates more knowledge.

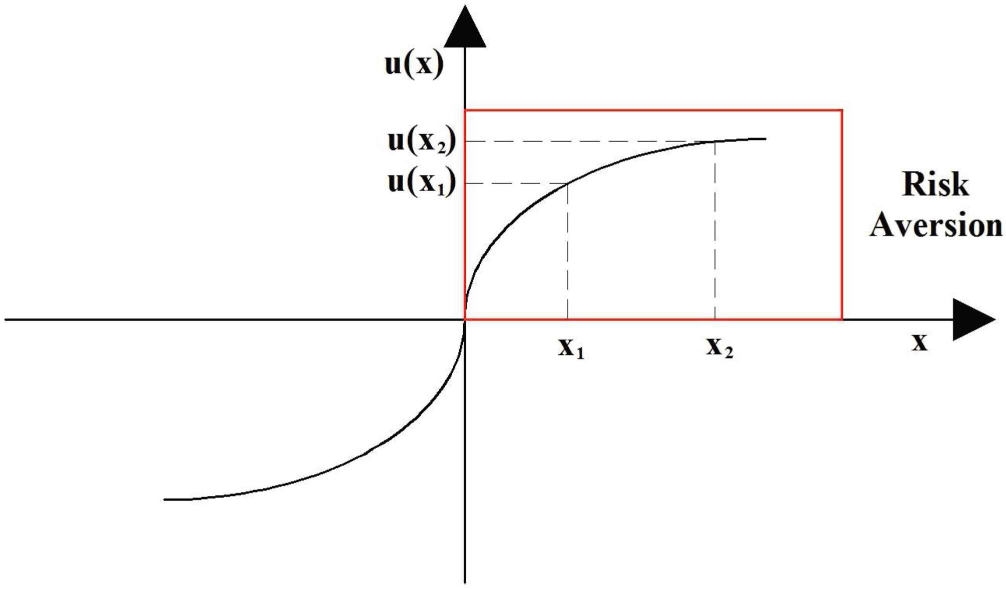



Figure 6.3: Utility function in the prospect theory

After that, node $n_i$ will make a control on the received knowledge before learning it, that is the evaluation of its quality on the basis of three criteria (Bukowitz and Williams, 2000; Suwa et al., 1982):

- Accessibility, defined as the capability for the receiver node to easily access to the whole knowledge that it has received;
- Guidance, defined as the knowledge property to be divided into topics or domain in order to avoid an information overload;
- Completeness, defined as the knowledge property to contain all the information requested by the receiver node

If the evaluation of the received knowledge exceeds the quality threshold in at least two of the three criteria, node $n_i$ will learn and assimilate knowledge at all. Furthermore, it will increase the weight and then the confidence in node $n_j$. On the contrary, node $n_i$ will learn only the 20% of the received knowledge and its confidence in node $n_j$ will decrease. In particular, the weights will increase or decrease as follows:

$$w_{i,j}^{K_k}(t+1) = w_{i,j}^{K_k}(t) \pm \sum_{l=1}^{q} \frac{d_{ij,l}^{K_k}}{100} \tag{6.5}$$

In the proposed model every network node thinks, acts and decides in several and different contexts that are related each other, modifying the measures that characterise the network. In order to calculate and analyse this correlation, we consider each context as a plane of the space and, taking one as a reference plane, the greater the cosine of the angle between two planes is the more similar they are, on the contrary they are less similar. In Figure 6.4 the correlation among contexts is shown. Furthermore, its dynamic nature is shown, because the reference context and the position of each plane in the space can vary at different time instants.

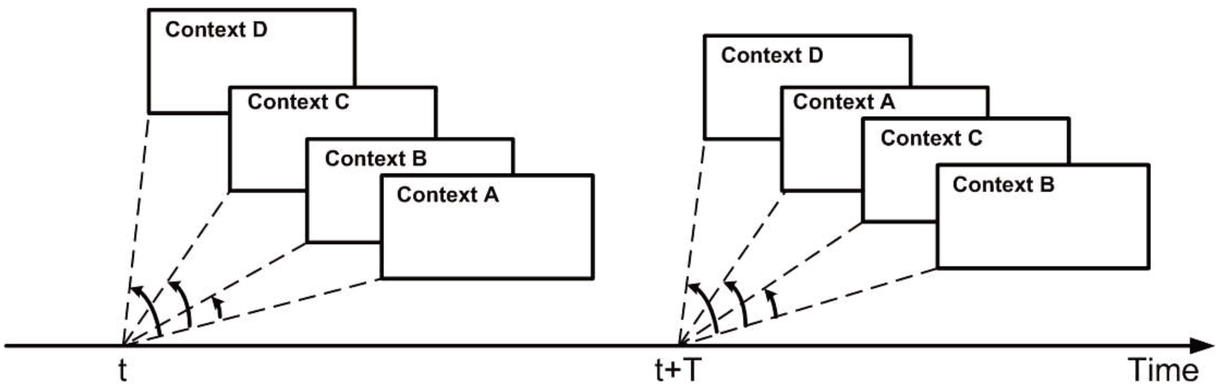



Figure 6.4: Contexts correlation in the space

## 6.4 Results and Discussion

In this section, the model performance under different simulation hypothesis have been analysed, considering for example a scenario in which network nodes have to accept knowledge, defined in Section 6.2, from their neighbors through emails, social networks or via a face-to-face contacts. The results of two networks that follow the first one the Erdös-Rényi model (Erdös and Rényi, 1959) and the second one the Barabási-Albert model (Barabási and Albert, 1999) have been compared. Both networks are characterised by the following parameters:

- $m = 500$, the number of nodes composing the network;
- the number of categories $q$ is set to 5;
- the distance threshold is set to 0.2;
- the confidence threshold is set to 0.4;
- the quality threshold is set to 0.5;
- each knowledge category has a fixed evaluation on each single quality parameter i.e. accessibility, guidance and completeness;
- the network configuration does not change over time i.e. the number and the mutual connections do not change;
- only one context $K_k$ has been considered;
- for the Erdös-Rényi model, a probability $p = 0.3$has been considered, where $p$ represents the probability of having a connection between two nodes;
- the Barabási-Albert model follows a law of linear preferential attachment.

In the two cases, two measures have been used in order to evaluate in which manner the two network models perform. The two measures are:

- the knowledge percentage held by node $n_i$ at time $t+T$ in the context $K_k$:

$$v_i^{K_k}(t+T) = \frac{\sum_{l=1}^{q}(v_{i,l}^{K_k}(t+T) - v_{i,l}^{K_k}(t))}{q \cdot 100} \tag{6.6}$$

- the confidence value of each node at time $t+T$ in the context $K_k$:

$$c_i^{K_k}(t+T) = \frac{\sum_{i \neq j}(c_{i,j}^{K_k}(t+T) - c_{i,j}^{K_k}(t))}{|N|} \tag{6.7}$$

In order to show the dynamism of the proposed model, considering the Erdös-Rényi network configuration, in Figures 6.5 and 6.6, the knowledge level for each node of the network in all the categories $q$ and the confidence level at time $t$ have been reported, respectively. The first value is calculated as the ratio of the sum of the knowledge level held by node $n_i$ in all the categories to the number of categories. Instead, the second one is calculated as the ratio of the sum of the confidence of all the relations of node $n_i$ to the total number of nodes of set $N$. Each node is colored according to the knowledge and confidence level held at time $t$ and the colors association is shown in Table 6.1 and in Table 6.2.

Considering Equation 6.6, in order to track the dynamics of the knowledge transfer process, it has been taken into account 3 time instants, that are $t+5$, $t+10$ and $t+15$. In Figure 6.7, each node is colored according to the percentage of increased knowledge that it holds after each $t+T$ time instants, and, in particular, the colors associated to each percentage interval are shown in Table 6.3. As it is possible to see by observing Figure 6.7 and considering different time instants, the level of knowledge of each node changes dynamically. In particular it increases, but due to the static nature of the network, that is no nodes are added or removed, after a

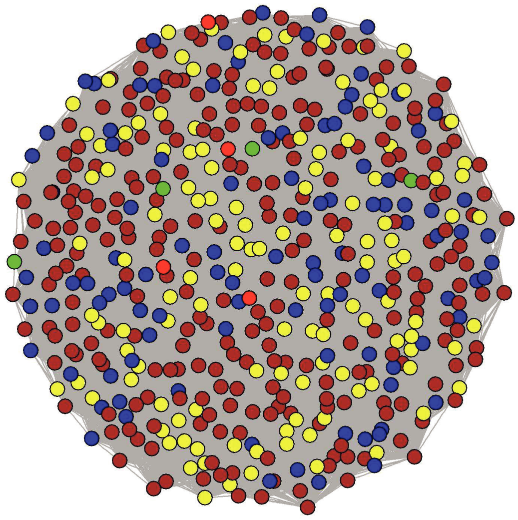


Figure 6.5: Starting Knowledge Level for the network nodes

Table 6.1: Colors associated to the nodes depending on the knowledge level held at time $t$

| Starting Knowledge Level (z) | Color |
|---|---|
| $0 \leq z \leq 0.2$ | Red |
| $0.2 < z \leq 0.4$ | Yellow |
| $0.4 < z \leq 0.6$ | Brown |
| $0.6 < z \leq 0.8$ | Blue |
| $0.8 < z \leq 1$ | Green |

certain time instant the process of knowledge transfer will stop. What it is possible to highlight is the progressive development of the knowledge level in the network, due both to the risk aversion of each node, through which the more it receives the more it is adverse to assimilate, and the quality control of the received knowledge introduced in this model. Considering Equation 6.7 and the time instants $t+5$, $t+10$

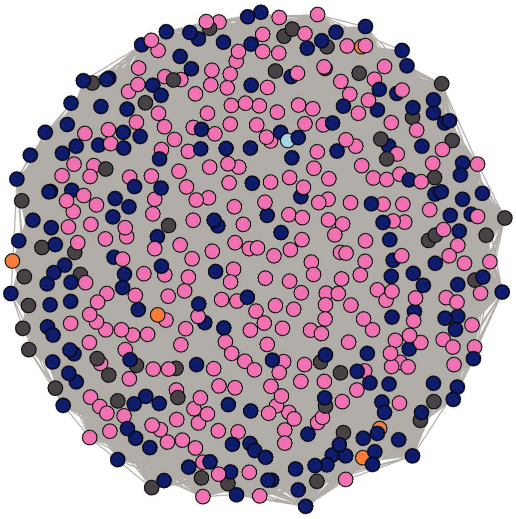


Figure 6.6: Starting Confidence Level for the network nodes

Table 6.2: Colors associated to the nodes depending on the confidence level that it is associated for each node at time $t$

| Starting Confidence Level (s) | Color |
|---|---|
| $s \leq 0.07$ | Light Blue |
| $0.07 < s \leq 0.08$ | Orange |
| $0.08 < s \leq 0.09$ | Grey |
| $0.09 < s \leq 0.1$ | Blue |
| $s > 0.1$ | Pink |

and $t + 15$, Figure 6.8 reports how dynamically the confidence level changes over time. Each node is colored according to its increasing or decreasing value of confidence with respect to the other network nodes. The colors are associated as shown in Table 6.4. The reason of the dynamical behaviour of the increasing/decreasing confidence level is that, the knowledge of the categories that they transferred in a

Table 6.3: Colors associated to the nodes depending on the knowledge percentage held

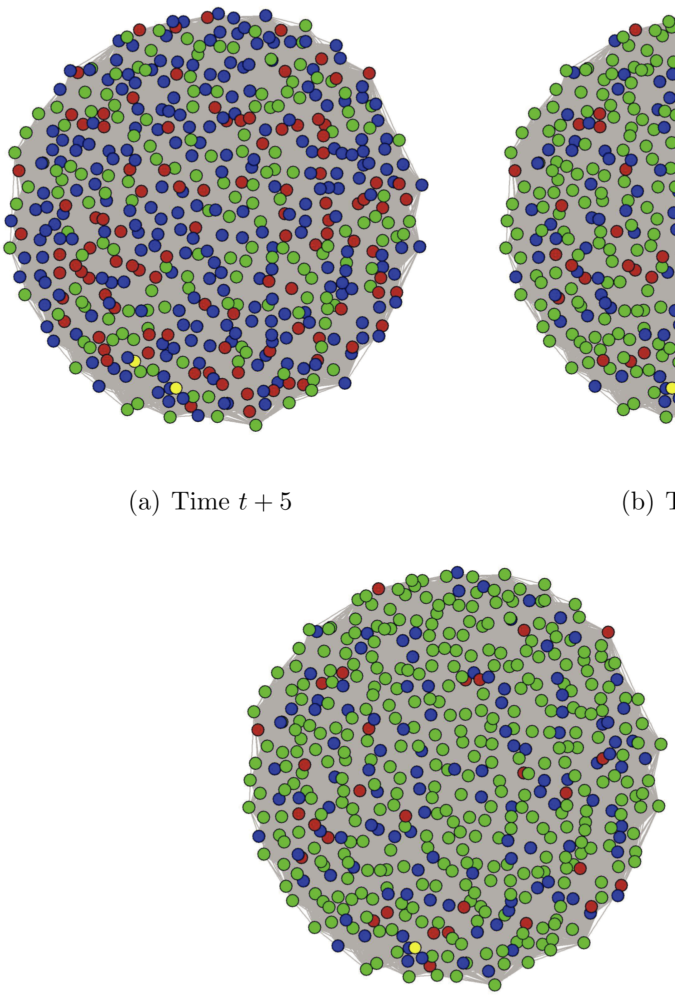
| Knowledge Percentage ($v_i^{K_k}(t+T)$) | Color |
|---|---|
| $v_i^{K_k}(t+T) = 0$ | Red |
| $0 < v_i^{K_k}(t+T) \leq 0.016$ | Yellow |
| $0.016 < v_i^{K_k}(t+T) \leq 0.036$ | Brown |
| $0.036 < v_i^{K_k}(t+T) \leq 0.06$ | Blue |
| $0.06 < v_i^{K_k}(t+T) \leq 1$ | Green |

Table 6.4: Colors associated to the nodes depending on their confidence values

| Confidence Value ($c_{i,j}^{K_k}(t+T)$) | Color |
|---|---|
| $s = 0$ | Light Blue |
| $s > 0$ | Orange |
| $s < 0$ | Grey |

first period was not of a good quality, but after a certain time interval they start to transfer knowledge in other categories whose quality is good, or viceversa.

As for the Erdös-Rényi model, now, using a Barabási-Albert model, it will be shown how the network structure will affect the knowledge dynamics. In Figure 6.9 and 6.10 it is reported the knowledge and confidence level for the network at time $t$ and each node is colored according to Tables 6.1 and 6.2. Due to the fact that not all the nodes are connected to each other and there are nodes with a very few number of links, the starting confidence level is really low, compared to the previous model, in fact for all the nodes it is under the value of 0.07. At the same time instants, the dynamics of the two network models are different because the level of knowledge increases slower than the previous case, as shown in Figure 6.11.



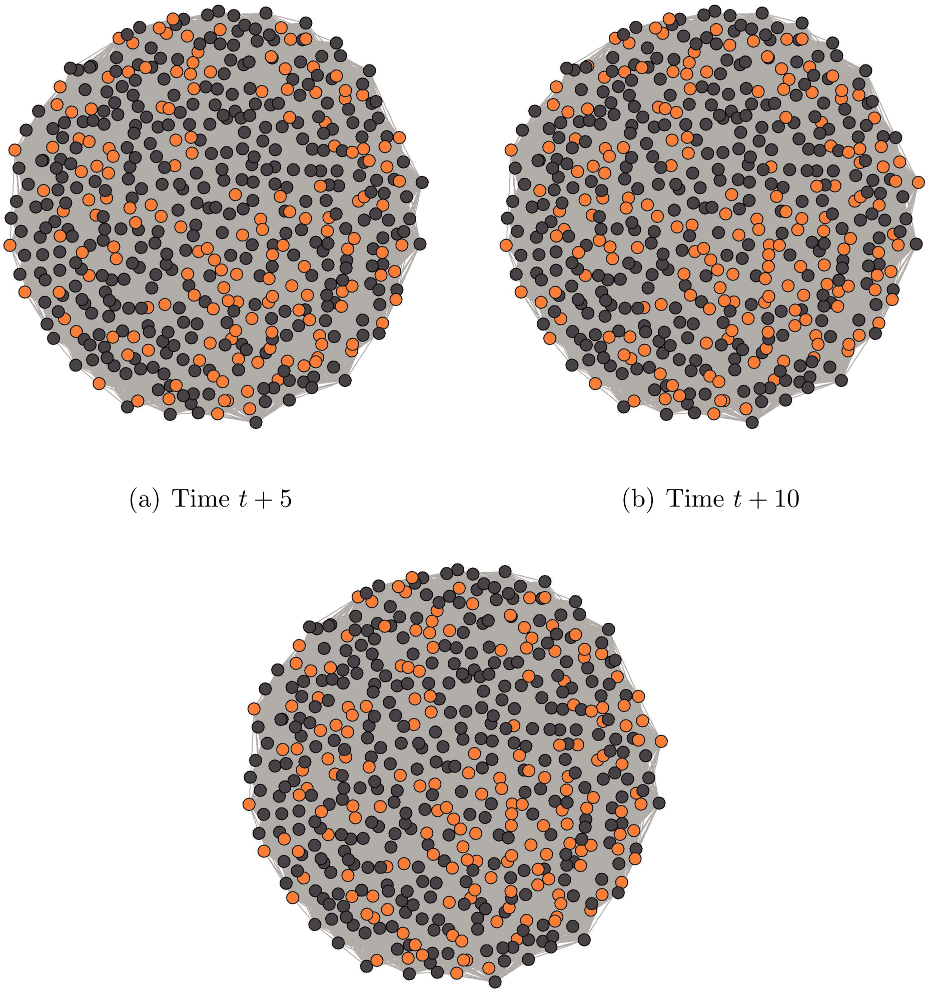
(a) Time $t + 5$

(b) Time $t + 10$

(c) Time $t + 15$

Figure 6.7: Dynamic of the knowledge transfer process for the Erdös-Rényi model

This is due to the structure of the network itself. In fact, in this case the colors associated are different, because in order to appreciate the knowledge increasing we have to change the scale (The higher increasing percentage is 0.00001%). Similarly to what happens for the knowledge, the mechanism of increasing/decreasing of the



(a) Time $t + 5$

(b) Time $t + 10$

(c) Time $t + 15$

Figure 6.8: Dynamic of the confidence level for each node in the network following the Erdös-Rényi model

confidence level is not so evident due to the high centrality held by a little percentage of nodes. From these results, it is observable that in a more distributed network configuration the dynamics of knowledge diffusion and of the confidence level are

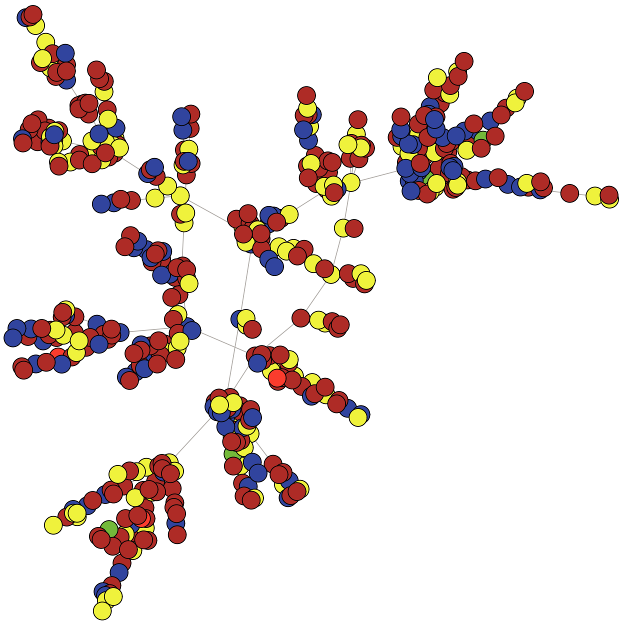


Figure 6.9: Level of Knowledge for the network nodes

observable much more than a centralised structure.

## 6.5 Summary remarks

Nowadays, data, information and knowledge represent the core part of the network. The analysis of their diffusion's patterns could be helpful to predict and study phenomena and node's behaviour within the network itself. Furthermore, by considering the context as a variable that affects the network structure and the knowledge held by the single node, adds further complexity and dynamism to a process that already has these features. Compared to the previous works, the main aim of the model presented in this chapter is to understand why a node, part of a network and considered as a decision maker, decides whether to accept or not knowledge from

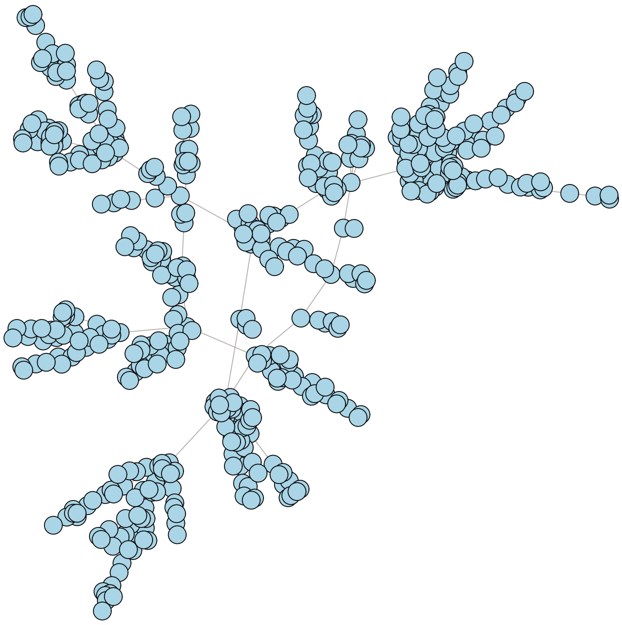
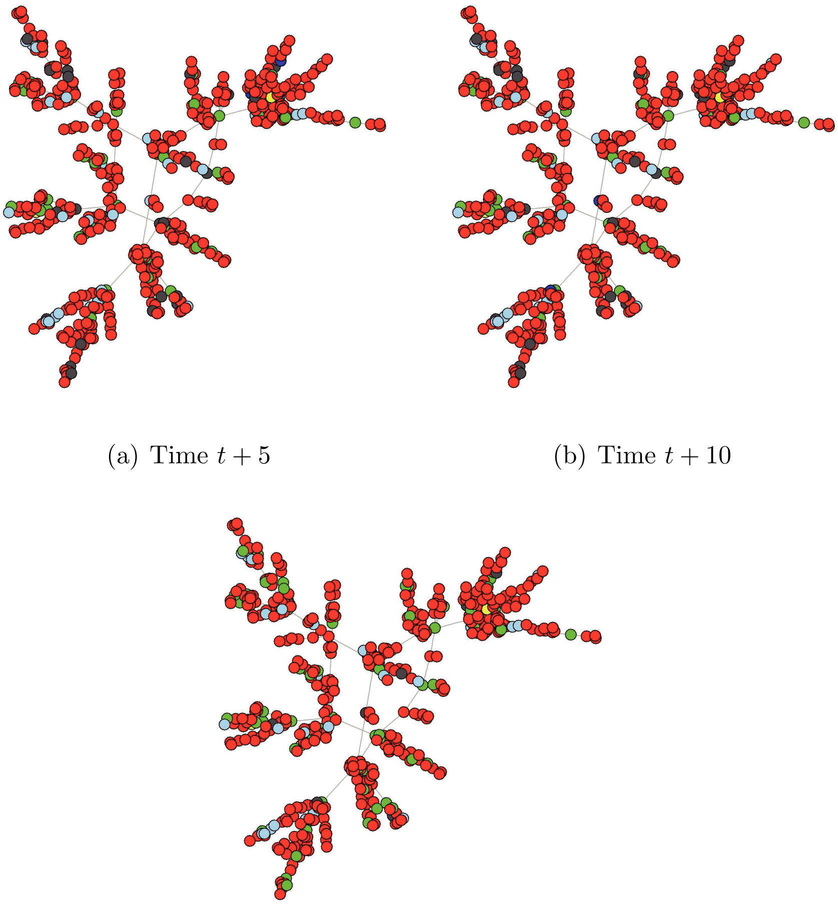


Figure 6.10: Level of Confidence for the network nodes

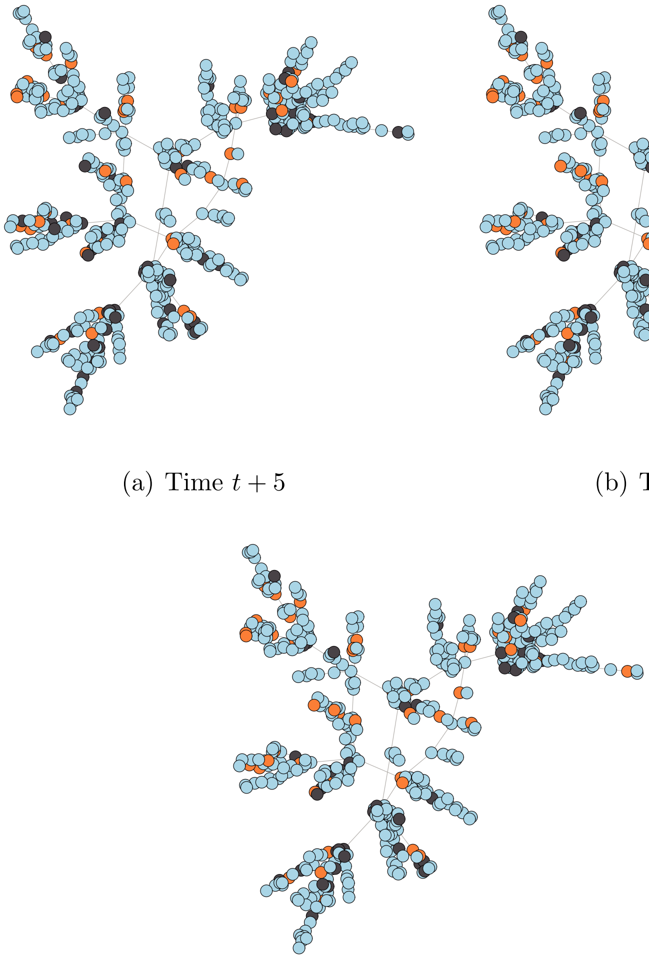
its neighboring nodes that represent the set of alternatives. The decision is based on the evaluation of each alternative based on two decision criteria, the knowledge distance and the confidence. In such a way, the structure of the network and, in particular, the typology of the node's connections, both depending on the context, affect the node's decision. This process is also characterised by a mechanism of confidence increasing and decreasing, that occurs after the evaluation of the quality of the knowledge received at each time instant and which adds dynamism to the model. In this sense, this work is a first attempt to investigate how the introduction of a context-aware decision making perspective in the processes involving knowledge may vary its diffusion's pattern.



(a) Time $t+5$ (b) Time $t+10$

(c) Time $t+15$

Figure 6.11: Dynamic of the knowledge transfer process for the Barabási-Albert model



(a) Time $t+5$

(b) Time $t+10$

(c) Time $t+15$

Figure 6.12: Dynamic of the confidence level for each node in the network following the Barabási-Albert model

# Chapter 7

# Final remarks

Nowadays, due to the continuous growth and diffusion of social networks, individuals are flooded with information, right or wrong as well. Understanding in order to extract much more important information is a complex and difficult activity, but it is necessary for each individual to acquire knowledge of good quality that it can use to perform each of its activities. Most of them can be modeled using a decision making perspective, where a set of alternatives or actions are evaluated using a set of criteria. But *decision making process is characterised by a complex nature and it has a lot of facets that can not be caught if it is observed and analysed only superficially. Representing the decision making process as a flowchart where actions or alternatives, state of being and consequences follow each other not emphasise the importance of who, when, where and how it takes place and its impact on the process dynamics. In fact, each decision maker has sets of values and objectives that most of time are in conflict each other, expression of the non linear and complex nature of the decision maker and of the process itself. In this way, reaching an optimal final decision is not always a simply task, particularly as the decision makers are not fully rational. But, what it is important to highlight is that each decision has an* *effect, more or less significant, on the decision maker(s).* Assuming that *"individual decisions are often influenced by the decisions of other individuals"* (López-Pintado, 2008), the main guideline, that was the basis of the research activities developed in the three years of this Ph. D. course, has been to look at the decision making process not only from a mathematical point of view but also the consideration of all the constraints that characterise a realistic process. In fact an interdisciplinary research activity had a central role in the Ph. D. studies. On one hand, multiple criteria decision analysis provides the mathematical instruments that are appropriate to build the analytical model of the decision making process. On the other hand, looking at the decision making process with a more engineered perspective, the features of sociality, context-dependance and dynamism are highlighted and their importance is considered in the process. To build as a more realistic model the union between the two points of view becomes necessary. In this way, the model is enriched of a lot of facets not visible applying only one perspective. Hence, the main contributions proposed in this Ph. D. dissertation are that answer to the question of Section 1.2:

- The identification of the four main features that characterise a decision making process: context-awareness, social influence, dynamism and multiple criteria.
- The construction of an analytical model which is expression of the interdisciplinary perspective of the decision making process, characterised by a context-aware, social, dynamic and multiple criteria nature. Hence, the model representing an individual decision making process taking place in a social network, has to main novelties that are the variability of the preferences of the decision makers (preferences that are influenced by the other decision makers) and the variability of the context;

- To tailor the theory of supernetworks to the social and context-aware decision making process. Applying the obtained model to a supply chain network in order to analyse the impact of the introduction of the features of context-awareness, social influence and dynamism on the process;
- To apply the decision making perspective to the processes of knowledge transfer and learning taking place in a social network. In this model the importance of each single connection, context and the quality of knowledge transferred have a central role in the dynamic of the process itself.

Considering the theory and the model presented in this Ph. D. dissertation a first attempt to analyse in deepen decision making processes, the future research directions will be addressed by the same aim.
The perfomance of the models presented in this Ph. D. thesis have been analysed in a limited number of applications. Using a social and context-aware decision making perspective, several processes can be analysed and modeled using this framework. The model presented in chapter 4 could be applied to various socio-economic contexts, such as fashion economy, housing location and viral marketing, where the role of the process of social influence is crucial. In fact, the single user behaviour when he has to choose a neighborhood to buy a house or a dress to wear for example, is influenced by his network and, in particular, by the behaviour of his members (Leskovec et al., 2007), (Chen et al., 2010), (McCormick and Livett, 2012), (Allenby et al., 1996).
The model presented in chapter 5, can be studied for different supply chain problems, from transportation to electric and electronic problems (Ramadurai and Ukkusuri, 2010), (Xuan et al., 2011).
Instead, the last model presented in chapter 6 is suitable to model problems referring to the sharing of files through the Internet or also the network of collaboration

among scientists. More generally the model can be used to represent processes where knowledge has a central role and it is expected to be shared among different entities (Chiu et al., 2006), (Leonardi, 2017).

Furthermore, the common guidelines will be to improve these models in order to represent the wide variety of processes listed above, making as much realistic as possible the analytical model. This will be possible, introducing more decision's criteria, a stronger behavioural component and a much more complex network structure and evolution.

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