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Renewable resources and waste recycling

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In this paper we consider an endogenous growth model involving, among other inputs, a renewable resource and secondary materials. Using this analytical framework we explain the effects of waste recycling on the growth rate of the economy, that we take into account. The effects of secondary materials production on the utility and dynamics of renewable resources are also studied. Furthermore, we consider how taxes and subsidies, levied on natural resources and granted for secondary materials, influence the dynamics of the economy during the transitional phase and the stationary growth path. Finally, the validity of Hotelling's rule and the effects of waste recycling on labor productivity are the conclusive topics of our research.

Keywords: endogenous growth, renewable resources, secondary materials, waste recycling

1. Introduction

Over the last decade the opinion that waste management is one of the main problems of environmental economics has been widespread [12,16]. At the beginning the issue was studied jointly with the behavior of consumers, firms and local public authorities. It was seen as a spatially circumscribed problem of limited relevance, with no consequences for the economy as a whole. The abundance of natural resources for productive aims, and of landfill areas for waste disposal purposes, reinforced this conviction. The first papers regarding this issue in fact consider it from a microeconomic point of view [15,17,20,28]. More in general, we can say that in the seventies the general belief was that waste management should be considered with a disaggregated approach. Recently, two facts have been observed with regard to the problem we are handling; Firstly, waste production is increasing in the world as a whole; and secondly, recycling may affect the macroeconomic figures of the economy. Those arguments could explain why some economic institutions are beginning to consider this topic in the context of the economic system [11,25,35]. The recent interest that economic literature has devoted to the aggregate effects of waste recycling may be justified with several considerations. The growing needs of the world population increases both the demand for natural resources and the quantity of waste produced, making it advisable to use more renewable inputs and secondary materials and to move towards more sustainable environmental behavior, through the saving of exhaustible resources. On the other hand, the use of waste recycling as a pollution abatement technology allows us to alleviate the pressure on natural resources and, more in general, on the environment. Finally, waste recycling can help us to reduce the damage caused by the harvesting and extraction of inputs from the earth's crust, at the same time diminishing the quantity of waste discharged into the environment and saving energy [16].

The growing economic interest in this topic has stimulated further research that considers secondary materials production in a macroeconomic analysis, in static and dynamic frameworks, with regard to the domestic and international aspects of this phenomenon [7–10,13,16,22,26,30,32].

In literature there are many articles studying the environment by means of an endogenous growth framework, but there are only a few that consider natural resources and waste recycling together [9,10]. In particular, only exhaustible resources are taken into consideration, not replenishable ones.

Huhtala [16] studies a similar problem, in a dynamic framework, but there are no implications regarding the growth path of the economy in her paper.

To investigate the long-run links between renewable resources and waste recycling an endogenous growth model was built, consisting of three sectors. The first is devoted to producing a final output, the second regards the accumulation of human capital and the last is the waste recycling industry. A standard Cobb–Douglas production function is considered, with constant returns to scale, in which five inputs are taken into account. The law of motion of capital depends on the difference between total output and consumption, while human capital accumulation is similar to that expressed by Lucas [19]. The dynamics of renewable resources is given by a natural reproductive function less harvest flow. The sector of secondary materials production depends on the quantity of labor allotted to it and on the amount of flow and stock of waste. Renewable resources and secondary materials are considered as perfect substitutes for each other, but cases in which they are imperfect substitutes could also be considered (for a similar problem, regarding exhaustible resources, see [10]). Fixed labor time, not employed in human capital accumulation, is allocated between total output and secondary materials production. The utility function is additively separable in consumption and waste stock, as in Keeler et al. [17].

Using the welfare function that we want to maximize, under the constraints considered, we obtain our Hamiltonian and thus derive the first order conditions.

Our theoretical framework allows us to consider the effects of waste recycling on the growth rate of the economy and the impact of secondary materials production on utility. Under the assumption that renewable resources and secondary materials are perfect substitutes in the production function, we will show what happens if the amount of waste recycling, or the price of secondary materials, changes. The paper continues considering the effects of taxes and subsidies on the prices of natural resources and secondary materials respectively, and on the utility function during transitional dynamics and in the stationary growth path. The validity of Hotelling's rule, with regard to renewable resources and recycled waste, is then examined. Finally, we take a look at the relationship existing between the marginal productivity of labor and secondary materials production.

The remainder of the paper is as follows. After the description of the model, we derive the first order conditions. Section 3 is devoted to showing the main results of our study. In section 4 we discuss the workability of our model, offering some examples of its functioning. Conclusive remarks and implications for environmental political economy are the subject of the last section.

We confine the mathematical details and some proofs of propositions to the appendix.

2. The model

The final output Y is a function of five inputs: physical capital K, human capital h, total workers L (in our model it is constant), renewable resource E, and flow of recycled waste M ('secondary materials').

The assumptions regarding human capital accumulation are similar to those made by Lucas [19, p. 17]. Here we also suppose that L and h have elasticity of substitution equal to unity, v is the labor time not destined to human capital formation.

In the specification of the production function, we assume that renewable resources and secondary materials are perfect substitutes.

$$Y = f(K, h, L, E, M) = K^{\alpha_1}(hL\omega_1 v)^{\alpha_2}(E + M)^{\alpha_3},$$
$$\sum_{i=1}^{3} \alpha_i = 1, \tag{1}$$

where $0 < \omega_1 < 1$, is the amount of v devoted to total output production. The investment in physical capital is as in

$$\dot{K} = Y - C, \quad \text{where } K(0) = K_0 \text{ and } K(t) \geqslant 0. \tag{2}$$

We assume that there is no depreciation in physical capital. Aggregate consumption is denoted by $C = xY$, with $0 < x < 1$. Per capita consumption is represented by $c = C/L$. Aggregate saving is $S = sY = \dot{K}$, where $0 < s < 1$, and $s = (1 - x)$.

$$\dot{h} = B(1 - v)h, \quad \text{where } B > 0,\ h(0) = h_0,\ \text{and } h(t) \geqslant 0. \tag{3}$$

Equation (3) is the law of motion of the per capita human capital stock (it is like equation (13) in [19]. B is a parameter of productivity of human capital accumulation activity.

$$\dot{R} = f(R) - rR = \sigma\left(1 - \frac{R}{\tau}\right)R - E,$$
$$\text{where } R(0) = R_0,\ R(t) \geqslant 0. \tag{4}$$

Equation (4) expresses the dynamics of renewable resource stock. We assume that $f(R)$ is the growth function, with properties $f(R) \geqslant 0$, for $0 \leqslant R \leqslant \tau$, $f'(R) > 0$ for $0 \leqslant R \leqslant \bar{R}$, $f'(\bar{R}) = 0$ and $f'(R) < 0$ for $\bar{R} \leqslant R \leqslant \tau$, where $\bar{R}$ is the maximum sustainable yield stock level of our renewable resource, and τ is the ecological carrying capacity [14,18]. We denote the intrinsic growth rate of renewable resources with σ, while E, equal to rR, is the harvest flow of renewable resource, r being the rate of use of the renewable resource (where $0 \leqslant r \leqslant 1$). The assumption with regard to the first derivative of the natural production function $f'(R)$, is justified by the fact that this kind of resource has some maximum and then decreases to zero. Thus there is a maximum sustainable yield, that in equilibrium should be equal to the highest possible harvest rate [5].

$$\dot{J} = \gamma D - M, \quad J(0) = J_0,\ J(t) \geqslant 0 \text{ and } 0 \leqslant \gamma \leqslant 1. \tag{5}$$

The waste stock J moves during time according to (5). It depends on the waste flow D, secondary materials production and the capacity for waste assimilation of the environment, denoted by γ. We assume that the waste flow $D = zY$ $(0 < z \leqslant 1)$, is a constant fraction of the total output (see [4,6]).

$$M = n\omega_2 v(D + J), \quad \text{and} \quad M \gtreqless D. \tag{6}$$

The secondary materials production function is expressed by (6), in which we consider that the amount of M produced depends on $0 < \omega_2 < 1$, the fraction of v used in this activity. We suppose that $\omega_1 + \omega_2 = 1$, i.e. the labor time not utilized in human capital formation is allocated between total output and secondary materials production, n is a strictly positive parameter of productivity. The inputs to the waste recycling industry could be the flow of waste D as well the stock J. This functional form for secondary materials production allows us to reduce the waste stock of the economy, if during transitional dynamics it is greater than its optimal value.

The utility depends on the flow of consumption c and the stock of waste J. The utility function is

$$u = u(c, J). \tag{7}$$

It is additively separable, such that $u_{cJ} = 0$, and has continuous first and second partial derivatives, with $u_c > 0$,

$u_J < 0$, $u_{cc} < 0$, $u_{JJ} < 0$. It is assumed that for $c \to 0$, $u_c \to +\infty$, and $u_{\bar{J}} = 0$ [17].[1]

The total welfare W associated with any particular time path for c and J comes from summing the discounted flow, with the social discount rate $\delta > 0$. The social welfare is

$$W = \int_0^\infty u(c, J) L \mathrm{e}^{-\delta t} \, \mathrm{d}t. \tag{8}$$

We assume that live agents in our economy, in their decisions on consumption and production, consider the welfare and availability of resources of their present or prospective descendants.

In formal terms we want to maximize (8), subject to (1)–(5). The current-value Hamiltonian for the problem is

$$\begin{aligned} \aleph = {} & u(c, J)L + \lambda_1 \big\{ \big[K^{\alpha_1} (hL\omega_1 v)^{\alpha_2} (E+M)^{\alpha_3} \big] - C \big\} \\ & + \lambda_2 \big[B(1-v)h \big] + \lambda_3 \left[\sigma \left(1 - \frac{R}{\tau} \right) R - E \right] \\ & + \lambda_4 (\gamma D - M), \end{aligned} \tag{9}$$

where λ_i, $i = 1, 2, 3, 4$, are the current-value Lagrange multipliers.

The first order and transversality conditions are set out in appendix A. They are necessary and sufficient for the optimal control problem. The proof that the model describes a stable saddle point equilibrium path is given in appendix B.

3. Results of the model

The first order conditions that we have derived allow us to highlight a lot of theoretical issues, such as, for example, the effects of waste recycling on the growth rate of the economy and utility level. Moreover, we can study how the dynamics of natural resources change if secondary materials, and the effects of taxation on natural resources, are taken into account. Finally, we show how labor productivity and Hotelling's rule are influenced by the waste recycling process.

Further, we follow the same order.

The main question is whether or not the growth rate of economy that we have depicted in our model is greater in cases in which waste recycling is considered. If so, the *a priori* information is the existence of a trade-off if we consider secondary materials production, because in this hypothesis a part of the labor time is used in the waste recycling industry and not in the total output sector.

Proposition 1. Given the values of parameters, assuming $g_M > 0$, the growth rate of total output is greater in cases in which secondary materials are considered.

Proof. See appendix C. □

[1] Here $\bar{J}$ is the value that waste stock assumes in the optimal stationary growth path. In this case the first derivative will be zero, because there is no possibility of increasing welfare by means of a change in J.

Therefore, if we have two economies with the same parameters, including labor time devoted to human capital accumulation, the growth rate of the economy will be greater in cases in which waste recycling is taken into account.

There are several reasons for this. Essentially, we should consider that there is a positive macroeconomic externality that emerges from the waste recycling process. To understand this, consider that without this activity there is some positive fraction of total labor, not devoted to human capital accumulation or final output production, that we use to collect and discharge waste. If we now imagine that we use the same amount of labor time to get the same result, but besides we also obtain secondary materials, that increase the output of our economy, then this result holds (for similar outcomes, in a static environment, see [8,26,33]).

This positive macroeconomic externality is alone enough to justify our result, but we can make some further considerations. If we recycle more waste, we diminish the risk of overexploiting natural resources, bringing the system towards a sustainable path.

In the model we have made some assumptions with regard to the effects of waste stock on the utility function, but it is not immediately clear how the marginal disutility of waste stock changes as a consequence of secondary material production.

Proposition 2. The marginal disutility of waste stock u_J falls as a consequence of secondary materials production.

Proof. Using equations (A.8) and (A.4), and differentiating u_J with respect to M, it follows directly that $\partial u_J / \partial M = -\lambda_1 \alpha_3 Y / (E+M)^2 \, L < 0$. □

The intuition behind this outcome is simple. An increase in the quantity of secondary materials produced reduces the stock of waste in the economy that we are considering, such that the marginal disutility of J decreases. This is a direct effect, but there is also an indirect one. Whenever we use more secondary materials to produce goods, this implies a reduction of the negative externality on the environment associated with products that are natural resource intensive, in terms of derivative demand for environmental services, like natural resources and landfill areas to discharge waste.

Another issue that is worth considering, is the effect of waste recycling on the dynamics of renewable natural resources.

Proposition 3. An increase of secondary materials production raises the accumulation of renewable resources stock, while a growth in the shadow price of secondary materials reduces the accrual of renewable resources stock.

Proof. Putting in evidence E in equation (A.4) and substituting in (4), we can calculate the partial derivative of $\dot{R}$ with respect to M and λ_4, to get $\partial \dot{R} / \partial M = 1 > 0$ and $\partial \dot{R} / \partial \lambda_4 = -\lambda_1 \alpha_3 Y / \lambda_4^2 < 0$, such that our result holds. □

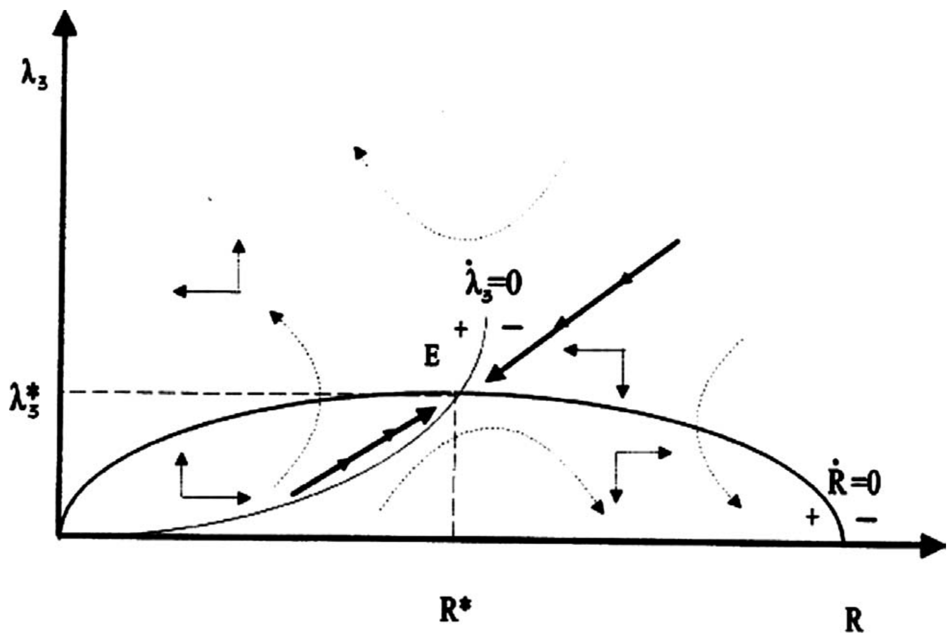

Figure 1.

The first result is intuitive. If we can use more secondary materials, this allows us to reduce the harvest of natural resources. In particular, we can note that $\partial\dot{R}/\partial M = 1$; this means that an additional unit of secondary materials produced allows us to raise the stock of renewable resources by the same amount. It is evident that this result is obtained only in cases where first and secondary inputs are perfect substitutes for each other, as we assume in our model. On the other hand, a change in the price of secondary materials causes income and substitution effects that work in the same direction (inferior inputs are not considered here). In this way if the price of secondary materials increases, the two effects work to reduce the demand for this input, and vice versa if λ_4 decreases.

We can reproduce the equilibrium path of natural renewable resources and their shadow price in a phase diagram. To this aim we use the equations

$$\dot{R} = \sigma\left(1 - \frac{R}{\tau}\right)R - rR \tag{4}$$

and

$$\dot{\lambda}_3 = \delta\lambda_3 - \lambda_3\left[\sigma\left(1 - \frac{2R}{\tau}\right) - r\right]. \tag{A.7}$$

The phase diagram path is drawn in figure 1, in an (R, λ_3) space.

Letting $\dot{R} = 0$ and $\dot{\lambda}_3 = 0$, we have a system of two equations in two unknown (R, λ_3), that we can solve mathematically. From them we obtain two pairs of equilibrium values for natural renewable resources stock and its shadow price, namely $(R^* = \lambda_3^* = 0)$ and $(R^* = \tau - r\tau/\sigma; \lambda_3^* = (\delta - 1)[\sigma(\tau - 2) + r(1 - \tau)]/\tau)$. In the first case, if the stock of natural resources that we are considering is zero, its price will be the same. In the other case, for a positive stock of natural resource, $R = \tau - r\tau/\sigma$, the shadow price will be greater than zero, for a social discount rate higher than 1.

We can form the Jacobian matrix to find that the eigenvalues of the determinant have opposite signs and the trace of determinant is positive; this implies that we have a locally stable saddle point equilibrium (more analytical details are given in appendix D).

The phase diagram confirms the analytical findings because we have a saddle path equilibrium. There are two regions in which the system does not converge to its equilibrium (or is unstable). This happens when a low level of resource stock is associated with a shadow price of renewable resources higher than its equilibrium value. In this case too much of the resource will be harvested such that it will be overexploited, until it is exhausted. Another region in which the system shows unstable dynamics is that in which the stock of natural resource is higher than its optimal value and the shadow price is lower than its equilibrium. This means that the demand for natural resource is too low and the system does not converge to its stationary growth path.

To analyze how the equilibrium changes as a consequence of waste recycling, we can use equation (A.4) and substitute in (4) for E, such that we can write

$$\dot{R} = \sigma\left(1 - \frac{R}{\tau}\right)R - \frac{\lambda_1\alpha_3 Y}{\lambda_4} + M. \tag{4'}$$

In this way it is clear that there is a positive correlation between secondary materials production and the variation over time in the stock of natural resources; in our diagram we therefore obtain a more concave curve for locus $\dot{R} = 0$, such that the price of natural resources will be lower than without secondary materials production and the optimal stock of natural resources will be higher.

One aspect that has been neglected in previous literature on endogenous growth models with renewable natural resources, is how the results of the model change if the policy maker levies a tax on virgin ores or subsidizes secondary materials production. We can analyze this kind of problem in two different environments, in transitional dynamics or in the stationary growth path. It is more interesting to consider what happens in the first case, because during transitional dynamics there is no reason why the primary and secondary inputs should have the same price. To this aim, we can use the first order conditions reported in appendix A; in particular, considering equations (A.3) and (A.1), deriving u_c with respect to the total output, we obtain

$$\frac{\partial u_c}{\partial Y} = -\frac{\lambda_3(E+M)}{\alpha_3 Y^2}, \tag{10}$$

such that we can say, in the case of a tax on a renewable natural resource (that increases λ_3), that the marginal utility of consumption will be lower than without taxation. If however we consider equations (A.4) and (A.1) and take the partial derivative of u_c with respect to Y, the result is

$$\frac{\partial u_c}{\partial Y} = -\frac{\lambda_4(E+M)}{\alpha_3 Y^2} \tag{11}$$

for which if we are given a subsidy on secondary materials prices (such that λ_4 decreases), the marginal utility of consumption will be higher than without the subsidy. These two simple observations allow us to say that, during transitional dynamics, taxes and subsidies, on renewable natural resources and secondary materials respectively, will have asymmetric effects on the marginal utility of consumption. These two measures have the same direct effect of pushing firms to use more secondary materials and less natural resources, but the indirect effects are radically different, because in the first case we reduce the marginal utility of consumption and in the second case we raise it.[2]

We can also consider the effects of a subsidy for secondary materials on the natural resource stock, during transitional dynamics, using equations (A.11) and (A.12), such that putting in evidence R, and taking the partial derivative with respect to the price of M, we obtain

$$\frac{\partial R}{\partial \lambda_4} = \frac{u_J L}{\lambda_4^2}\frac{\tau}{2\sigma}. \tag{12}$$

To interpret this result it is worth remembering that $u_J < 0$. In this way it is evident that a subsidy granted on secondary materials raises the renewable natural resource stock.

In the stationary growth path we know that, under the hypothesis of perfect substitutability between natural resources and secondary materials, the two shadow prices should converge to an identical value, such that the effects of one measure or another will be the same, because taxes and subsidies on renewable resources and secondary materials respectively increase or reduce both prices by the same amount. There is thus no sense in considering the effects of tax and subsidy further in the long-run equilibrium.

There is another interesting issue that we can also investigate, namely the validity of Hotelling's rule for renewable natural resources and secondary materials, along the stationary growth path of the economy that we are considering.

Proposition 4. Along the optimal stationary growth path, the growth rate of shadow prices of renewable resources and secondary materials are both equal to the social discount rate.

Proof. Using (A.11) we can say that $g_{\lambda_3} = \delta$, if and only if $\sigma = r$. Substituting in (A.11) for R, its possible equilibrium values $(0; \tau - r\tau/\sigma)$ this result holds. From equation (A.12) it is immediately possible to conclude, if $u_{\bar{J}} = 0$, that $g\lambda_4 = \delta$. □

This means that Hotelling's rule is satisfied for both inputs considered here (for a discussion of this issue with regard to renewable resources, see [24, p. 178]).

The result that $g_{\lambda_3} = \delta$ means that, in the long-run equilibrium, the growth rate of renewable natural resource stock, given by $\sigma(1 - 2R/\tau)$, should be equal to the renewable resource rate of use r. This implies that in the stationary growth path the renewable resource achieves its maximum sustainable level, because the same amount of resources produced will be harvested. The outcome for which $g_{\lambda_4} = \delta$ confirms that in the steady state, the waste stock is at its optimal level, such that it is not possible to increase secondary materials production.

In economic literature there is a considerable line of thought that points out the effects of environmental quality on labor productivity (see, recently, [34]). From this point of view, it could be interesting to consider how pollution abatement technology, in the form of secondary materials production, influences labor productivity. To this aim we can use (1), to get

$$\frac{\partial Y}{\partial \omega_1} = \alpha_2 \frac{Y}{\omega_1}. \tag{13}$$

Using (A.4) we can substitute in (13) the equilibrium value of total output, to obtain

$$\frac{\partial Y}{\partial \omega_1} = \alpha_2 \frac{\lambda_4(E+M)}{\alpha_3 \lambda_1 \omega_1}, \tag{14}$$

thus we can derive (14) with respect to M, getting

$$\frac{\partial Y}{\partial \omega_1 \partial M} = \alpha_2 \frac{\lambda_4}{\alpha_3 \lambda_1 \omega_1} > 0. \tag{15}$$

This result implies two things: (i) that from a production function point of view the two inputs are complementary [23]; (ii) that a reduction of waste discharged into the environment, by means of secondary materials production, increases the marginal productivity of labor.

[2] Huhtala [16] considers the same problem in a different theoretical framework, concluding that subsidies and taxes have asymmetric effects.

4. Is the theoretical model workable?

After showing how the model works and its main implications, the question is: do worldwide economies really run as the theoretical model suggests? This is an empirical matter in which econometric analyses are necessary.

The problem is that there are no databases available regarding the prices of secondary materials and the waste recycling of output produced by means of renewable resources considered as a whole, although in some cases there is data relative to a single resource, like paper, wood, etc. The scarcity of data, especially in aggregate form, is a serious obstacle to verify whether our model fits with the behavior of real economies.

In economic literature there are few empirical studies, performed without a formal theoretical scheme, that deal with the convergence of prices of secondary materials to those of primary resources [3] and with the relationship between the growth rate of the economy and waste recycling, for some kinds of waste derived from commodities produced by means of renewable resources [2,31]. In particular, the main results of those econometric analyses are: (i) a positive relationship exists between the growth rate of income and recycling of some kind of waste (i.e. paper); (ii) the main cost of waste recycling is the wage of workers employed to produce secondary materials; (iii) waste recycling is mainly driven by market forces; (iv) the prices of secondary materials show convergence to the prices of primary ones.

Of course, the results of the few econometric analyses available are consistent with one of the most interesting outcomes of our analysis, that is the positive correlation between the growth rate of the economy and waste recycling.

Thus we may verify the practical utility of our model, making a numerical example to shed light on the relationship between growth rate of the economy and waste recycling, taking as given the values of some parameters and trying to verify whether proposition one is right. To make a comparative analysis between the growth rate of the economy with or without secondary material production, we assume that we are in the optimal stationary growth path. For the sake of simplicity only two periods of time, t and $t + 1$, are considered. Here we consider that the percentage of the recovered paper data is a good proxy of the recycled waste derived from renewable resources as a whole. The recovery rate of waste is assumed to be forty percent [2] such that for $E^t = 4$, this implies that $M^t = 1.6$. The exponents of our Cobb–Douglas production function are $\alpha_1 = 0.5$, $\alpha_2 = 0.4$ (approximately as in [21]), and then $\alpha_3 = 0.1$. Finally, regarding the labor force, we consider that $L = 1$, $K^t = 900$, $hv^t = 300$ and $\omega_1 = 0.95$, $\omega_2 = 0.015$ and $1 - v = 0.035$. The values of parameters regarding the allocation of the labor force, among the three sectors considered in the model, reflect those of real economies (in particular with regard to the percentage of workers needed to achieve a forty percent waste recovery see [8]). To perform this exercise of comparative statics, it is assumed that in cases where recycling does not occur the labor force previously devoted to this aim is now utilized to produce total output. Now we are able to calculate the output level with and without the secondary materials production at times t, Y^t and Y_p^t respectively. Applying (1), we find that total output is equal to $Y^t = 341.88$ if waste recycling is considered, and $Y_p^t = 332.65$ in the other case. This result is consistent with the findings of Berglund and Söderholm [2], for which the total output is greater if waste is recycled. For simplicity, letting $g_K = g_h = g_E = g_M = 0.02$ (thus the algebra is easier, but this assumption is not crucial to the following results), such that, in the period $t + 1$, $K^{t+1} = 918$, $vh^{t+1} = 306$, $M^{t+1} = 4.08$, and $E^{t+1} = 1.632$. Using again (1), we get the output level at time $t + 1$, in both cases considered, $Y^{t+1} = 348.72$ and $Y_p^{t+1} = 339.3$. Thus we are able to calculate the growth rate of total output for Y and Y_p, respectively equal to $g_Y = 2.0007\%$ and $g_{Y_p} = 1.9999\%$, such that the first proposition holds. It is worth highlighting that if the rate of waste recycling increase by 19% the growth rate of our economy will increase by 1.164%. This result is perfectly consistent with the econometric findings of Berglund and Söderholm [2], relative to forty-nine countries, for the period 1990–1997.

5. Final remarks

What can we say about waste recycling, from a macroeconomic point of view? This process has a lot of positive effects on the economy as a whole. In particular, we have shown that the growth rate of total output will be higher in countries that recycle more waste. The marginal disutility from waste decreases if we increase the quantity of waste recycled. The production of secondary materials allows us to reduce the quantity of renewable natural resources harvested, driving the economic system towards more sustainable paths. Taxes levied or subsidies granted, on renewable natural resources and secondary materials respectively, will have an asymmetric effect on the marginal utility of consumption, encouraging more waste recycling. Finally, labor productivity increases as a consequence of a cleaner environment.

Our findings are not fully known in economic literature. There is only a small stream of economic theory that considers the effects of waste recycling on the growth rate of total output (see, for example, [8–10,26]), but many problems considered here have been neglected in previous studies. The clear implication for the policy maker is the opportunity to support the waste recycling process, to bring the economic system towards a higher welfare level. In the fourth paragraph we have supplied a numerical example to show how the model works, demonstrating that the results of our exercise are consistent with the findings of econometric analyses available at present. Obviously this is just a first step to verify the workability of the model, in particular we need more statistical information than is at hand for the moment. Further empirical studies are necessary to verify the ability of our model to give a good representation of the real world and for prediction purposes. We think that this could be an argument for further interesting research.

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Appendix A. First order and transversality conditions

The first order conditions are

$$\frac{\partial \aleph}{\partial c} = u_c L - \lambda_1 L = 0, \quad \text{or} \quad \lambda_1 = u_c, \tag{A.1}$$

$$\frac{\partial \aleph}{\partial v} = \lambda_1 \alpha_2 \frac{Y}{v} - \lambda_2 B h = 0, \quad \text{or} \quad \lambda_2 = \frac{\lambda_1 \alpha_2 Y}{vBh}, \tag{A.2}$$

$$\frac{\partial \aleph}{\partial E} = \lambda_1 \alpha_3 \frac{Y}{E+M} - \lambda_3 = 0,$$

$$\lambda_3 = \lambda_1 \alpha_3 \frac{Y}{E+M}, \tag{A.3}$$

$$\frac{\partial \aleph}{\partial M} = \lambda_1 \alpha_4 \frac{Y}{E+M} - \lambda_4 = 0,$$

$$\lambda_4 = \lambda_1 \alpha_3 \frac{Y}{E+M}, \tag{A.4}$$

$$\dot{\lambda}_1 = \delta \lambda_1 - \frac{\partial \aleph}{\partial K} = \delta \lambda_1 - \lambda_1 \alpha_1 \frac{Y}{K}, \tag{A.5}$$

$$\dot{\lambda}_2 = \delta \lambda_2 - \frac{\partial \aleph}{\partial h}$$

$$= \delta \lambda_2 - \lambda_1 \alpha_2 \frac{Y}{h} - \lambda_2 \big[B(1-v)\big], \tag{A.6}$$

$$\dot{\lambda}_3 = \delta \lambda_3 - \frac{\partial \aleph}{\partial R}$$

$$= \delta \lambda_3 - \lambda_3 \left[\sigma\left(1 - \frac{2R}{\tau}\right) - r\right], \tag{A.7}$$

$$\dot{\lambda}_4 = \delta \lambda_4 - \frac{\partial \aleph}{\partial J} = \delta \lambda_4 - u_J L. \tag{A.8}$$

The growth rates of the dynamic multiplier are

$$g_{\lambda_1} = \frac{\dot{\lambda}_1}{\lambda_1} = \delta - \alpha_1 \frac{Y}{K}, \tag{A.9}$$

$$g_{\lambda_2} = \frac{\dot{\lambda}_2}{\lambda_2} = \delta - \alpha_2 \frac{\lambda_1 Y}{\lambda_2 h} - B(1-v), \tag{A.10}$$

$$g_{\lambda_3} = \frac{\dot{\lambda}_3}{\lambda_3} = \delta - \left[\sigma\left(1 - \frac{2R}{\tau}\right) - r\right], \tag{A.11}$$

$$g_{\lambda_4} = \frac{\dot{\lambda}_4}{\lambda_4} = \delta - \frac{u_J L}{\lambda_4}. \tag{A.12}$$

Differentiating equations (A.1)–(A.4) logarithmically, the result will be

$$g_{\lambda_1} = g_{u_c}, \tag{A.13}$$

$$g_{\lambda_2} = g_{\lambda_1} + g_Y - g_h, \tag{A.14}$$

$$g_{\lambda_3} = g_{\lambda_1} + g_Y - g_{E+M}, \tag{A.15}$$

$$g_{\lambda_4} = g_{\lambda_1} + g_Y - g_{E+M}. \tag{A.16}$$

The transversality conditions are

$$\lim_{t\to\infty} \mathrm{e}^{-\delta t} \aleph(t) = 0, \tag{A.17}$$

$$\lim_{t\to\infty} \big[\mathrm{e}^{-\delta t} \lambda_1(t) K(t)\big] = 0, \tag{A.18}$$

$$\lim_{t\to\infty} \big[\mathrm{e}^{-\delta t} \lambda_2(t) h(t)\big] = 0, \tag{A.19}$$

$$\lim_{t\to\infty} \big[\mathrm{e}^{-\delta t} \lambda_3(t) R(t)\big] = 0, \tag{A.20}$$

$$\lim_{t\to\infty} \big[\mathrm{e}^{-\delta t} \lambda_4(t) J(t)\big] = 0. \tag{A.21}$$

Appendix B. Proof that the optimal growth path is locally a stable saddle point

We define the endogenous stationary growth path equilibrium that in which the growth rates of $Y/K = \beta$, and $C/K = \beta\chi$, will be equal. This implies that the growth rates of total output, capital and consumption will be the same in the optimum [1,27,29].

To demonstrate that we have a locally stable saddle path, we define the variables that will be constant in the long run equilibrium.

$$\beta = \frac{Y}{K}, \tag{B.1}$$

$$\beta_\chi = \frac{C}{K}, \tag{B.2}$$

$$\theta = \frac{E}{R}, \tag{B.3}$$

$$\varphi = \frac{M}{J}. \tag{B.4}$$

Then

$$g_K = \beta - \beta\varkappa, \tag{B.5}$$

$$g_\beta = g_Y - \beta + \beta\varkappa, \tag{B.6}$$

$$g_{\beta\varkappa} = g_C - \beta + \beta\varkappa, \tag{B.7}$$

$$g_\theta = g_E - \theta, \tag{B.8}$$

$$g_\varphi = g_M - g_J. \tag{B.9}$$

Using the first order conditions, and after a little algebra, we can define the dynamic system $(\beta, \beta\varkappa, \theta, \varphi)$ in terms of the following equations.

$$g_\beta = B(1-v) - \left(1 + \frac{\alpha_1\alpha_3}{\alpha_2}\right)\beta + \beta\varkappa, \quad \text{(B.10)}$$

$$g_{\beta\varkappa} = B(1-v) - \left(1 + \frac{\alpha_1\alpha_3}{\alpha_2}\right)\beta + \beta\varkappa, \quad \text{(B.11)}$$

$$g_\theta = -\theta, \quad \text{(B.12)}$$

$$g_\varphi = B(1-v) - \left[1 + \frac{\alpha_3}{\alpha_2}\right]\alpha_1\beta + \varphi\left(1 - \frac{\gamma}{n\omega_2 v}\right) + \gamma. \quad \text{(B.13)}$$

In the stationary growth path we assume that $g_\beta = g_{\beta\varkappa} = g_\theta = g_\varphi = 0$, such that $\beta = \bar{\beta}$ (where the bar denotes the optimal value of the variable), etc. Thus we can write the Jacobian that we evaluate at the steady state.

$$Jac = \begin{bmatrix} \frac{\partial\dot{\beta}}{\partial\beta} & \frac{\partial\dot{\beta}}{\partial\beta\varkappa} & \frac{\partial\dot{\beta}}{\partial\theta} & \frac{\partial\dot{\beta}}{\partial\varphi} \\ \frac{\partial\dot{\beta}\varkappa}{\partial\beta} & \frac{\partial\dot{\beta}\varkappa}{\partial\beta\varkappa} & \frac{\partial\dot{\beta}\varkappa}{\partial\theta} & \frac{\partial\dot{\beta}\varkappa}{\partial\varphi} \\ \frac{\partial\dot{\theta}}{\partial\beta} & \frac{\partial\dot{\theta}}{\partial\beta\varkappa} & \frac{\partial\dot{\theta}}{\partial\theta} & \frac{\partial\dot{\theta}}{\partial\varphi} \\ \frac{\partial\dot{\varphi}}{\partial\beta} & \frac{\partial\dot{\varphi}}{\partial\beta\varkappa} & \frac{\partial\dot{\varphi}}{\partial\theta} & \frac{\partial\dot{\varphi}}{\partial\varphi} \end{bmatrix}$$

$$= \begin{bmatrix} -\pi - \bar{\beta}\varkappa & \frac{\bar{\beta}\varkappa + \pi}{\eta} & 0 & 0 \\ -\eta^2\left[\bar{\beta} - \frac{\pi}{\eta}\right] & \eta\bar{\beta} - \pi & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \mu\alpha_1\left(\frac{\pi}{(\frac{\gamma}{n\omega_2 v} - 1)} - \frac{\mu\alpha_1}{(\frac{\gamma}{n\omega_2 v} - 1)}\bar{\beta} - \gamma\right) & 0 & 0 & \mu\alpha_1\bar{\beta} - \pi + \gamma \end{bmatrix}.$$

To simplify the symbols of the above Jacobian we put: $B(1-v) = \pi$, $(1 + \frac{\alpha_1\alpha_3}{\alpha_2}) = \eta$ and $(1 + \frac{\alpha_3}{\alpha_2}) = \mu$. After some little algebra, we may check that the determinant of the Jacobian Matrix is negative, and that

$$TrJac = -\pi - \bar{\beta}\varkappa + \eta\bar{\beta} - \pi + \mu\alpha_1\bar{\beta} - \pi + \gamma > 0, \quad \text{(B.14)}$$

this implies that we have a locally stable saddle point equilibrium.[3]

Appendix C. Proof of proposition 1

To check the result shown in proposition 1, we use the assumptions made in appendix B. If v is held constant, it immediately follows that its growth rate will be equal to zero (see [27] among others for the same assumption).

Differentiating equation (1) logarithmically we obtain

$$g_Y = \alpha_1 g_K + \alpha_2 g_h + \alpha_3 g_{E+M}, \quad \text{(C.1)}$$

where $g_{E+M} = g_E + g_M$, such that we can rewrite the equation (C.1) as

$$g_Y = \alpha_1 g_Y + \alpha_2 g_h + \alpha_3 g_E + \alpha_3 g_M. \quad \text{(C.2)}$$

(Remember that in the optimal stationary equilibrium path $g_Y = g_K = g_C$, and $g_\theta = 0$, i.e. in equilibrium θ is constant and its growth rate is zero.) Knowing that $E = rR$ such that from (B.8), we will find that in the optimal stationary growth path $g_E = r$. From (3), it follows that $g_h = B(1-v)$, such that

$$g_Y = \frac{\alpha_2[B(1-v)] + \alpha_3(r + g_M)}{1 - \alpha_1}. \quad \text{(C.3)}$$

Equation (C.3) represents the growth rate of the economy in cases where the waste recycling process is considered.

To derive the growth rate of total output in cases where we do not take waste recycling into account Y_p, we just set up the relative production function, that will be

$$Y_p = f(K, h, L, v, E) = K^{\alpha_1}(hL\omega_1 v)^{\alpha_2}E^{\alpha_3}, \quad \sum_{i=1}^{3}\alpha_i = 1. \quad \text{(C.4)}$$

Differentiating (4) logarithmically the result is

$$g_{Y_p} = \alpha_1 g_K + \alpha_2 g_h + \alpha_3 g_E. \quad \text{(C.5)}$$

After some little algebra we obtain that

$$g_{Y_P} = \frac{\alpha_2[B(1-v)] + \alpha_3 r}{1 - \alpha_1}, \quad \text{(C.6)}$$

such that for the same values of parameters and of v, with $g_M > 0$, the result in proposition 1 claims.

Appendix D. Proof that we have a locally stable saddle point equilibrium in an R, λ_3 space

Using equations (4) and (A.7) and setting $\dot{R} = 0$ and $\lambda_3 = 0$, we obtain these two equations

$$\sigma\left(1 - \frac{R}{\tau}\right)R - rR = 0, \quad \text{(D.1)}$$

and

$$\delta\lambda_3 - \lambda_3\left[\sigma\left(1 - \frac{2R}{\tau}\right) - r\right] = 0. \quad \text{(D.2)}$$

Such that we can form the Jacobian matrix

$$Jac = \begin{bmatrix} \frac{\partial\dot{R}}{\partial R} & \frac{\partial\dot{R}}{\partial\lambda_3} \\ \frac{\partial\dot{\lambda}_3}{\partial R} & \frac{\partial\dot{\lambda}_3}{\partial\lambda_3} \end{bmatrix} = \begin{bmatrix} \sigma\left(1 - \frac{2R}{\tau}\right) - r & 0 \\ \frac{2\sigma\lambda_3}{\tau} & \delta - \left[\sigma\left(1 - \frac{2R}{\tau}\right) - r\right] \end{bmatrix}.$$

The determinant of the Jacobian matrix is

$$det = \sigma\left(1 - \frac{2R}{\tau}\right)\delta - \left[\sigma\left(1 - \frac{2R}{\tau}\right)\right]^2 - r\delta - r^2,$$

[3] For a similar explanation of saddle point existence see [27].

such that our second order equation is

$$-\left[\sigma\left(1-\frac{2R}{\tau}\right)+r\right]^2-\delta\left[\sigma\left(1-\frac{2R}{\tau}\right)\delta+r\right]=0.$$

The two rows ($\delta/2$; $-\delta^3/2$) have opposite signs; this implies that we have a saddle path equilibrium.

We can also calculate the trace of determinants

$$TrJac=\sigma\left(1-\frac{2R}{\tau}\right)-r+\delta-\sigma\left(1-\frac{2R}{\tau}\right)+r=\delta>0.$$

These results mean that we have a locally stable saddle point equilibrium.

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