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Extraction of the Dominant Features of Complex Dynamics in Experimental Air-Water Two-Phase Flows

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Abstract: This paper proposes an innovative approach for the characterisation of the experimental dynamics of two phase flows. These class of systems can express a great variety of different flow patterns, whose characterisation and classification strongly depends on the approach used for feature extraction. Phase space analysis in a traditional delayed embedding has allowed for the observation of the complex dynamics of the system. Nonetheless, the attractors obtained in a delayed embedding, though characterised by a regular complex structure, appear partly folded and are affected by noisy hydrodynamic high order dynamics.
The present paper proposes an application to a case study, represented by an experimental air-water two-phase flow in upward motion inside a vertical pipe, of a Singular Value Decomposition (SVD) approach with the aim of assessing a more appropriate embedding into the phase space spanned by the principal vectors. Reported results demonstrate the ability of the of the proposed methodology to separate the dominant features of the system dynamics from noise-like dynamics, leading to obtain efficaciously unfolded and noise-free versions of the system attractors.

Keywords: Feature extraction, Two-phase flows, Experimental nonlinear dynamics, SVD analysis.

1. Introduction

Several basic industrial processes, ranging from power generation, chemical and processing plants to oil pipelines, present heat and mass transfer applications of two phase flows. When two phase flows occur, very different flow patterns can be observed as well as transitions from a flow pattern to another. Indeed, the dynamical behaviours associated to the various types of flow pattern established in the system represent critical factors for the performances of such industrial systems. This explains the great efforts that have been and are still devoted to flow patterns identification, which represents a fundamental basis for appropriate characterisation of two phase flow systems.
The dynamics of two phase flows are typically of highly complex pulsating nature, under the effect of several nonlinearities deriving from the strong coupling of different mechanisms and of the dependence on various factors.

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Among the others, the most important factors are the differential action of gravity on the two phases and the effect of shear and surface tension forces at their interface. As a consequence, several different flow patterns can be identified, each of which can be characterised in terms of the dynamical behaviour of the void fraction time series.

Among the other, two phase flows of air-water mixtures are often theoretical and experimental analysed with the aim of achieving a reference perspective on the general dynamical behaviours, often valid also for more complex flows, such as those arising in presence of phase changes. In particular, the present study aims at analysing the behaviour of ascending air-water two phase flows in vertical pipes. For this kind of flows heat transfer phenomena connected to phase change are not involved, so that the flow pattern established in the system mainly depends on the mass flow rates of the two phases. By varying the mass flow rate of the two phases, in fact, bubbly, slug, churn and annular flows can be identified as the main flow patterns typical of several classifications [1-3].

The bubbly flow exists for low values of the gas mass flow rate and consists in the motion of dispersed and small gas bubbles in the liquid phase. Coalescence phenomena are at the basis of the transition from bubbly to slug flow, which can be observed by increasing the gas mass flow rate. Slug flow is characterised by gas bubbles, namely Taylor bubbles, enveloped by a liquid film separating them from the pipe walls, alternated to liquid slugs. In the class of slug flow, it is possible to distinguish between: *cap flow*, with short air bubbles (with the head approximately connected to the tail) separated by long liquid slugs; *plug flow*, with gas bubbles and liquid slugs of comparable length; proper *slug flow*, characterised by elongated gas bubbles separated from relatively short liquid slugs, often aerated for the presence of small dispersed air bubbles.

For growing gas mass flow rate, bubble coalescence and increasing aeration of the liquid slug leads to a highly unstable flow pattern addressed as *churn flow*, characterised by waves propagating through the liquid film enveloping the bubbles and occasionally falling within the tube, so to form a short, unstable and highly aerated liquid slug. Finally, the annular flow consists of a thin annular liquid film at the tube wall on which small ripples, interspersed occasionally with large disturbance waves, flow in a regular manner up the tube.

It is usual practice to perform flow pattern identification on the basis of the differences of the dynamical behaviour of the time series of the local void fraction. Therefore, the reliability of the identification approach is highly dependent on the accuracy of the technique adopted to measure the void fraction. Several techniques have been proposed [4-9] and impedance measurements seem to be recognized as the most reliable [6]. At the same time, the performances of flow patter identification approaches depend also on the techniques adopted for time series analysis and feature extraction. Statistical [1, 2, 6] or spectral [9-12] techniques indeed represent the typical approach for flow patterns identification on the basis of the analysis of the experimental void fraction time series. Nonlinear techniques have been also adopted, among the others see [10, 13-16], but a main drawback has been represented by the relatively poor spatial and temporal resolution of the experimental time series.

In order to address this problem, the experimental time series considered in the present study have been detected by means of a resistive probe characterised by high temporal and spatial resolution, which has been appositely set-up as described in [17]. The preliminary analysis in a delayed embedding of the void fraction time series detected by means of this sensor has shown the existence of strange attractors of interesting morphology for the various flow patterns [18].
Nonetheless, attractors obtained in this way are somewhat noisy as a consequence of the superposition of high order dynamics to the dominant dynamics characterizing the flow pattern. Among the others, the most important high order "noisy" dynamics are those of hydrodynamic nature associated to small diameter bubbles dispersed in the liquid slugs and to disturbances on the liquid film enveloping the Taylor bubbles.
Therefore, the present study aims at extracting the dominant features of the flow dynamics under various flow pattern conditions so to separate the dominant features of the system dynamics from noise-like dynamics. The proposed approach is analogous to that proposed in [19] and is based on the calculation of the singular vectors of a n-dimensional delayed embedding, through the application of the technique known as Singular Value Decomposition (SVD) [20], and in the analysis of the restricted portion of the dynamics that is obtained by projecting the attractor onto the phase space spanned by the singular vectors corresponding to the three highest singular values.
Reported results, show that the attractors described in the new embedding present a well defined and regular structure, indicating the existence of a low order source of the system dynamics, which will be analysed in future studies.

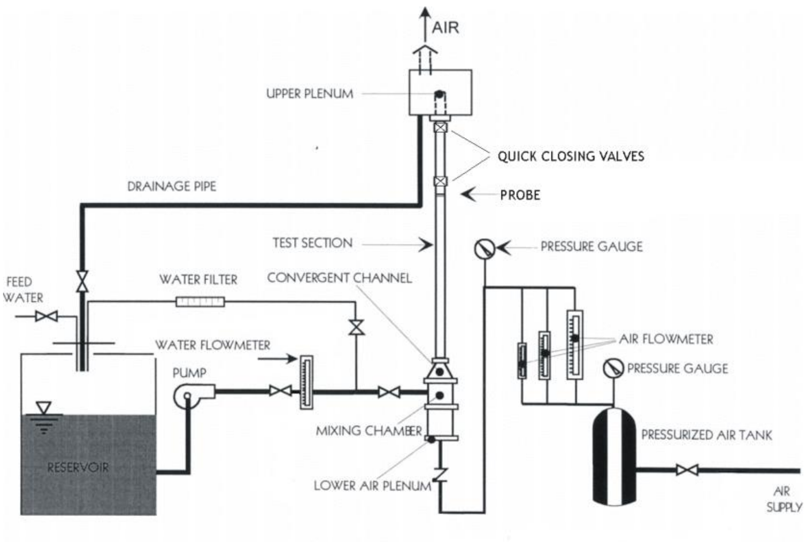

Fig.2 Experimental apparatus

2. Experimental Apparatus

The experimental apparatus reported in Fig. 1 has been built and tested in order to study the dynamics of two-phase flow in vertical pipes. The test section is a vertical pipe of diameter 0.026 m diameter and length 3 m. The apparatus is equipped by an electromagnetic flowmeter and three air flow metres, respectively used for the measure of the water velocity and mass flow rate and for the regulation of the air flow rate in the range between 10 and 210 l/min. The air is supplied to the mixing section by a pressurised tank fed by a compressor, whereas the water flow rate can be varied in the range 0-150 l/min by means of a series of valves and bypasses placed at the pump outlet.

A resistive probe for the measure of the void fraction is placed at a distance of over 100 times the diameter of the pipe from the mixing section, i.e. over the required entry region for two phase flows, in order to ensure a well established flow regime. In particular, the void fraction probe has been designed and realised for the experimental campaign and operates in the resistive range (carrier frequency of 20 kHz). The sampling frequency was set at 1 kHz with a cut-off frequency of 200 Hz. A detailed description of the experimental probe and on the wide set of experimental tests performed is reported in [17].

3. Dynamical Feature Extraction

The results of preliminary linear analyses of the experimental time series have been shown to be unable to deal with the intrinsic complexity of two phase flows dynamics. Hence, in [18] a morphological analysis of the three-dimensional attractors has been proposed in a classical Takens' delayed embedding of the experimental void fraction time series [21]. In particular, it has been observed that the attractors obtained for some of the flow patterns are characterised by a regular fractal structure, which is indeed one of the most important evidences of deterministic chaotic behaviour.

In the present study, the aim is to improve the dynamical representation by adopting a new embedding, derived through the application of *Singular Value Decomposition* technique, *SVD* [20], to the classical delayed embedding based on Takens' theorem, similarly to the approach proposed in [19]. The new representation is characterised by a drastic reduction of noisy dynamics and, above all, a sensitive improvement of the attractor unfolding, so that the dominant morphological characteristic can be fully exploited.

As a first step, the phase space reconstruction consists in the creation of a $n \times w$ matrix, S, where n is the length of a window moving through the data and w are the independent variables defining the phase space, i.e. delayed version of the experimental void fraction time series $s(t)=(s_0, s_1, s_2, \ldots, s_i, \ldots)$, with each column delayed τ time steps from the previous. The condition $w>2d+1$ for an appropriate embedding is implicitly respected if w is set much greater than the unknown fractal dimension d on the basis of a preliminary estimation.

The second step consists in the application of the *SVD* approach to matrix *S*. This is done through the calculation of a new diagonal matrix, equivalent to the original one, i.e. with identical singular values but in decreasing order. In particular, S is factorized into its singular values according to equation:

$$\Lambda = M^T S C \qquad (1)$$

In (1) Λ is the diagonal matrix containing the w singular values λ_i of S in decreasing order and M and C are the matrices of the singular vectors associated with Λ. Details on the factorization can be found in [20]; what is interesting for the scopes of the present study is that the high level singular values in Λ are associated to the dominant singular vectors, i.e. those representing the dominant features of the system dynamics, whereas the low level ones correspond to local behaviors or noise-like components. Therefore, the system can be virtually partitioned into two subsystems: the first deriving from noise free data (i.e. the main features and the relevant details) and the second from noisy dynamical behaviours, which can be considered superimposed and then eliminated.

In order to choose how many singular vectors are needed to accurately describe the dominant dynamics of the system, it is possible to analyse, under the various possible flow patterns, the distribution of the spectrum of the normalized singular values $(\lambda_i)_n$, obtained by dividing the singular value λ_i for its maximum $(\lambda_i)_{max}$ under the given flow condition. By the analysis of the spectrum of the singular values reported in Fig. 3 for some cases representative of the typical flow patterns, it is possible to observe that only the three highest singular values are relevant in the spectrum and can therefore be chosen to describe the dominant dynamics of the system. It is worth observing that even for the flow patterns that seem to require the consideration of a higher number of singular values, it is possible to claim that only the three highest are actually relevant. The rising of higher order singular values in the *bubbly flow* and *cap flow* spectrum is due, in fact, to the comparatively lower amplitude of the void fraction oscillations under these flow conditions, which determines a greater relevance of noisy dynamics of hydrodynamic origin.

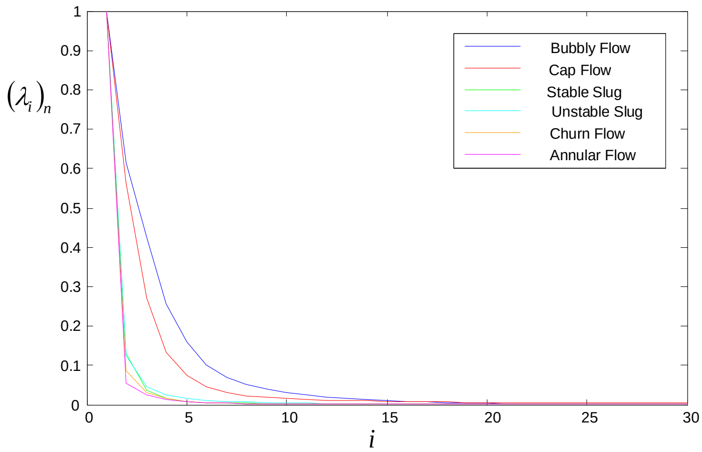

Fig.3 Spectrum of the normalised singular values under various flow patterns.

4. Results and Discussion

The described approach has been used in the present study in order to obtain a denoised and unfolded representation of the experimental dynamics. The *SVD* technique has been applied to the delayed embedding S of the experimental void fraction time series, created considering $\tau=1$ and $w=40$ in order to ensure that w is sufficiently greater than m, i.e. greater than the (unknown) system dimension. The length n of the observation window has been set at 10000 data samples in order to be wide enough to obtain a well defined attractor in phase space, i.e. an attractor whose morphology does not change if further data samples are added.

The claimed advantages of the proposed methodology can be observed in the results reported in the Fig. 4 to 9, which report the attractors of the same operating condition in two different embeddings. In particular, the phase space adopted for the plots on the left hand side of each figure is the basic three dimensional Takens' delayed embedding, whereas the projections on the pseudo-phase space spanned by the three dominant principal vectors of the improved embedding obtained through application of *SVD* are those reported on the right hand side of each figure. It is worth observing that, as discussed on the basis of previous observations on the spectrum of the singular values, the three-dimensional pseudo-phase space can indeed be considered an appropriate embedding for the dominant dynamical behaviour under the various flow patterns.

By comparing the two methods of representation it is possible to observe that the attractors in the delayed phase space are in all cases sensibly affected by a higher noise level and are not sufficiently unfolded with respect to the corresponding attractors in the principal component embeddings, the last being characterised by a very low level of noise and a satisfactory unfolding. It is worth to remind that, even if the two attractors of each flow pattern appear different, they are, nonetheless, expressions of the same dynamical behaviour. In fact, they are morphologically equivalent and, therefore, characterised by the same *invariants of the dynamics*, such as fractal dimension and Lyapunov exponents [22-25].

The successful unfolding contributes to the achievement of a clear and well defined morphology of the attractors. This is a main advantage for the distinction of different flow patterns through a comparison of the representation of their dynamics in the phase space spanned by the principal components.

Moreover, in some cases the proposed embedding amplifies important characteristics of the system dynamics. For example, the right hand cap flow attractor in Fig. 5 shows a clear distribution of the trajectory in alternated bands, which is a hint of the fractal (i.e. chaotic) nature of the system dynamics.

Finally, the representation in the principal component phase space is very effective in underlining the differences between the various flow patterns.

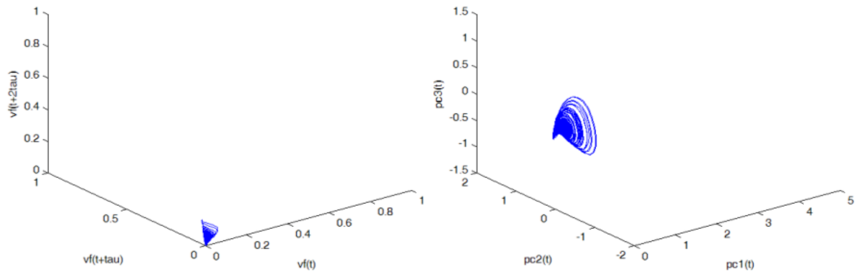

Fig.4 Attractors in the delayed and principal component embeddings for the bubbly flow; air flow rate 2 lit/min - water flow rate 32.4 lit/min.

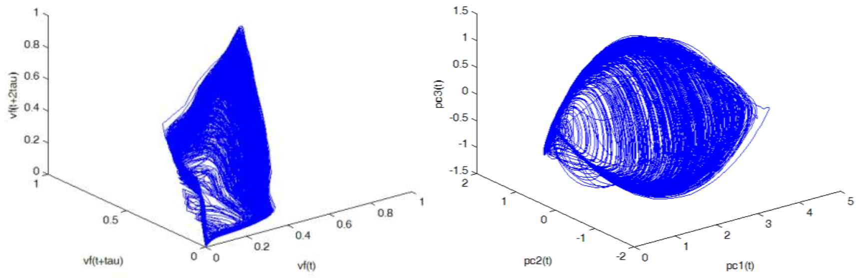

Fig.5 Attractors in the delayed and principal component embeddings for the cap flow; air flow rate 5 lit/min - water flow rate 20.28 lit/min.

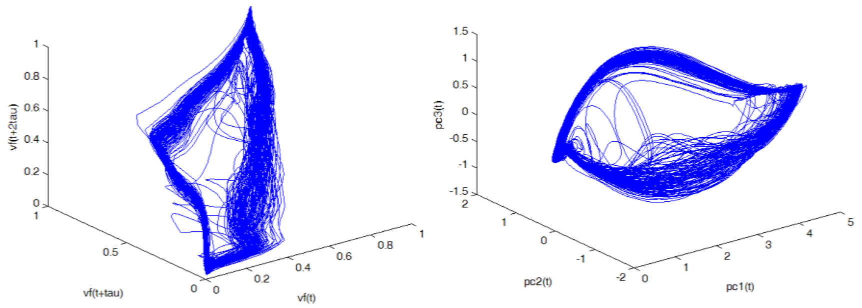

Fig.6 Attractors in the delayed and principal component embeddings for the plug flow; air flow rate 10 lit/min - water flow rate 9.06 lit/min.

Each type of flow pattern is, in fact, characterised by a specific morphology, sufficiently different from that of the other flow patterns.

In particular:

- each flow pattern attractor occupies a different phase space region;
- each attractor differently "fills" its own region of phase space; for example, the cap flow attractor (properly 3-D) has a higher filling rate than that of the plug flow (which moves around a sort of 2-D limit cycle);

- the attracting region is progressively shifted, with a continuous trend from bubbly to annular flow, with respect to the first principal component.

These differences are very important as the morphological considerations drawn insofar are related to the fractal nature and to the stretch and folding behaviour of the attractors [24], which can be considered as the topological expressions of the mentioned invariants of the dynamics, whose calculation is behind the scope of the present study and will be the object of future studies.

5. Conclusions

This study proposes a phase space approach for the description of typical complex dynamics of two-phase flow. At first the singular vectors of the classical delayed embedding are calculated and the attractors of the system dynamics are projected on the state space spanned by these eigenvectors. In this way the dominant feature of the dynamics, corresponding to a subset of the highest singular values, are separated from noisy dynamics in the time series, corresponding to the remaining lower singular values. The morphology of the attractors in the obtained unfolded and noise-free representation is analysed. Reported results demonstrate that the proposed approach represent a powerful tool for the identification of two-phase flow patterns.

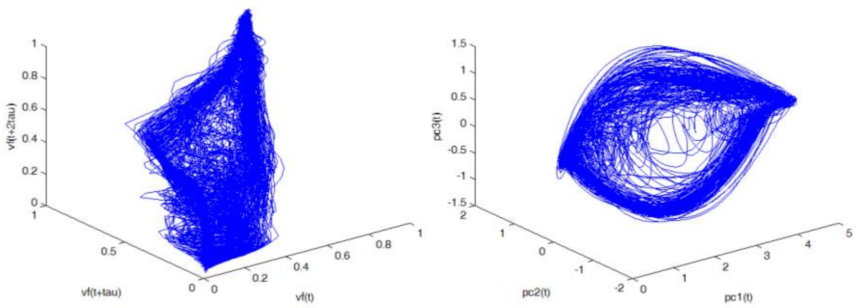

Fig.7 Attractors in the delayed and principal component embeddings for the slug flow; air flow rate 40 lit/min - water flow rate 16.80 lit/min.

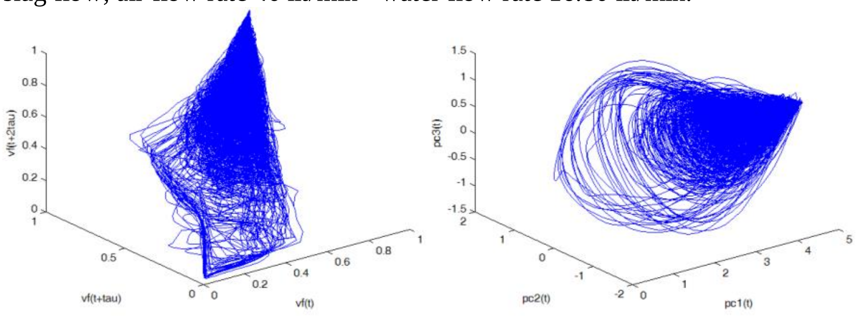

Fig.8 Attractors in the delayed and principal component embeddings for the churn flow; air flow rate 80 lit/min - water flow rate 9.01 lit/min.

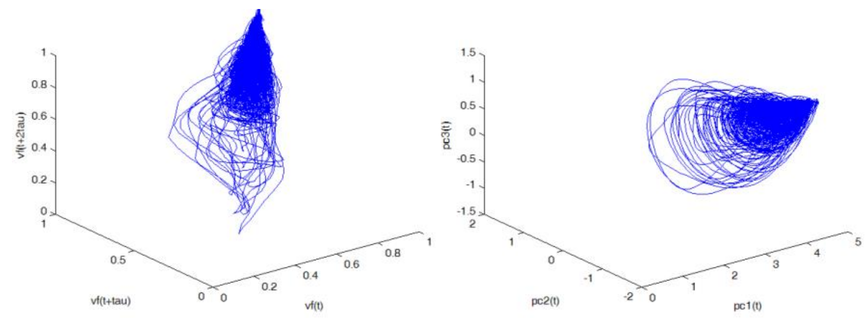

Fig.9 Attractors in the delayed and principal component embeddings for the annular flow; air flow rate 80 lit/min - water flow rate 5.58 lit/min.

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