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Natural resources dynamics: Exhaustible and renewable resources, and the rate of technical substitution

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Abstract

In this paper we study both exhaustible and renewable resources in an endogenous growth model. In particular, we consider the hypotheses in which the rate of technical substitution (RTS) between those two inputs is or is not equal to one. Moreover, we depart from a basic theoretical framework to account for the negative externality constituted by waste accumulation. Finally, a comparative analysis is made between Pigouvian tax and waste recycling, as an environmental policy to correct market failure represented by refuse accumulation.

JEL classification: Q32; O11

Keywords: Economic growth; Endogenous technological progress; Exhaustible resources; Pigouvian taxes; Renewable natural inputs; Rate of technical substitution (RTS); Waste recycling

Introduction

In the mists of time our forefathers basically used just renewable resources, like water, wind, and wood, to produce energy. It was only with the industrial revolution and the invention of the internal-combustion engine that we increased the use of non-renewable resources like oil, coke, etc. The cohabitation of both kinds of natural resources has thus become the rule in economic systems all over the world. The increasing use of exhaustible resources tends to result in, first of all, rising prices of this type of input as a consequence of its increasing scarcity and, secondly, a negative externality in the form of pollution discharged into the environment. To offset the constraint to growth represented by the presence of non-renewable inputs, economic systems have been forced, among other things, to resort to the so-called ‘‘backstop technologies’’ phenomenon, that is the employment of the same renewable sources of energy as previously, but utilizing new processes (Nordhaus, 1974).

In the real world economic interest in energy technology development has been so generalized over the last few years that venture capital disbursements have risen sharply (The Economist, 2001, p. 64). Future scientific improvements will probably allow us to achieve the optimal growth path of the economy: ethanol, for example, derived from a renewable resource (sugarcane), is (or could become) a good substitute for gasoline (Goldember et al., 2001; Anderson, 2001). In the United States the Energy Policy Act of 2003 requires states to use 5 billion gallons of ethanol per year by 2012. Roughly speaking, a lot of attention is being paid to technological discoveries regarding the increase in renewable input utilization, such as energy sources and ways to avoid the depletion of non-replenishable inputs. This dynamic substitution process may help to lead economic systems towards a more sustainable development.

After the seminal paper of Hotelling (1931), many theoretical studies on exhaustible resources have been carried out from the late nineteen-sixties on, such as those of Anderson (1972), Barbier (1999), Cummings (1969), Di Vita (2006), Hoel (1978), Kamien and Schwartz (1978), Krautkraemer (1989), Schou (2000), Solow (1974a), Stiglitz

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(1974), Tahvonen (1997), Vousden (1973).[1] Each of these papers has placed emphasis on some aspects of the issue: common access to the resource studied, the role of the discount rate, ore quality selection, technological change, the environmental costs of mining exhaustible resources, etc.; none of them, however, takes into account the presence of renewable resources.

Another stream of economic literature deals with renewable natural resources: this includes the works of Plourde (1970), Huhtala (1999), Di Vita (2004), Mendelssohn and Sobel (1980), Olson and Roy (1996, 2000), Tahvonen and Kuuluvainen (1993). This current is less substantial, because for this kind of input the main problem is just to ensure its optimal harvest rate. Finally, a few studies only consider renewable and non-replenishable natural resources in the same model (see D'Arge and Kogiku, 1973; Dasgupta and Heal, 1974; Kemp and Van Long, 1980; Smith, 1974; Tahvonen and Salo, 2001). In particular, all these works of research consider replenishable natural sources of energy such as eolian, solar, geothermal, etc.; they do not entertain the hypothesis of deriving a good substitute for exhaustible resources from agriculture (like ethanol from sugarcane).

The latter studies focus on the relationship between the two kinds of natural resources mentioned here, but they do not take into consideration at least three problems. Firstly, a reproductive function for renewable resources is not explicitly introduced. Secondly, the problem of substitutability between exhaustible and renewable resources is viewed in terms of costs only and not from the angle of technological possibility. Thirdly, the negative externalities associated with the presence of a non-renewable resource and with waste accumulation are not taken into account. Our paper will try to bridge this gap in economic literature.

The article attempts to answer the following questions: how can the introduction of a parameter representing the degree of technical substitutability between exhaustible and renewable resources change the known results in this field? Can technological change explain the process of internalization of negative externality represented by refuse accumulation? In order to reduce scrap flow to its optimal level, is it better to introduce a Pigouvian tax or to promote waste recycling?

Our results show that if the rate of technical substitution (RTS for short) between the two kinds of natural production factors considered here is different from one, the economic system cannot achieve the optimal growth path. This is the reason why technological change plays a fundamental role in pushing the economic system towards a more sustainable path. In particular, new scientific discoveries have three kinds of spillover: they eliminate the obstacles preventing the use of renewable natural inputs instead of non-replenishable ones, thus alleviating the burden of exhaustible resources on the economic system; they increase the amount of waste that can be recycled; and they reduce the price of reproducible resources. Finally, while Pigouvian tax and waste recycling have the same effects when the static first order conditions are considered, secondary materials production causes a positive externality in terms of dynamic efficiency, making it possible to reduce the amount of exhaustible resources mined.

The problem of imperfect substitutability between exhaustible and renewable natural input is well known in economic literature (Dasgupta and Heal, 1974), but has not been fully explored. Solow (1974b) affirms: "... *As you would expect, the degree of substitutability is also a key factor. If it is very easy to substitute other factors for natural resources, then there is in principle 'no problem'*...". Under the hypothesis of technical perfect substitutability (i.e. $RTS = 1$) between natural inputs, there is some 'back-stop technology', such that production is not constrained by the exhaustible resource; if there is no substitute for the non-replenishable input, on the other hand, catastrophe is unavoidable. Solow suggests that there is a wide range of cases in between, in which the question is real, interesting and not foreclosed; he thus emphasizes the importance of the elasticity of substitution between the two inputs.

In this paper we show that even when the elasticity of substitution between two kinds of natural resources is infinite, the RTS may represent a constraint to growth if it is less than one; in other words, renewable and exhaustible inputs may or may not be perfect technical substitutes for each other. The negative externality represented by waste accumulation is accounted for informal analysis. Finally, we try to find solutions to the problem of choosing between waste recycling and Pigouvian tax to correct the cause of market failure represented by waste accumulation. These are problems that have been insufficiently dealt with in previous research.

Our endogenous growth model consists basically of two sectors. The final output is produced in the first, while knowledge is accumulated in the second. When the waste recycling process is considered, there are three industries involved. A standard Cobb–Douglas production function is assumed. Among other inputs, we consider two natural resources, one exhaustible and the other replenishable. Physical capital is accrued by means of net investment. The dynamics of technology stock is as in Romer (1990). New scientific discoveries are neither exogenous nor costless, but depend on the labor time allotted to this aim (Kamien and Schwartz, 1978). The amount of exhaustible resources available declines at the same rate as that of extraction. A logistic function describes the behavior over time of the renewable natural resources stock. The utility level depends on consumption and on the amount of waste accumulated.

The optimal growth path is constrained by the amount of labor time available in our economy, that we assume constant for the sake of simplicity.

[1]For a detailed survey of this field of literature, see Krautkraemer (1998).

In third section, we consider the possibility that the waste derived from exhaustible resources can be recycled: this reduces not only the amount of litter dumped on the environment but also the flow of this kind of natural input extracted from the world's crust. For renewable resources, the main problem is finding the optimal harvest rate, thus the problem of recycling the waste derived from this kind of input may be ignored.

To simplify the reading of the paper we denote the parameters with greek letters: lambda represents the dynamic multipliers, the capital letters are the stock variables and the small letters are the flow variables; finally, asterisks indicate the optimal values.

The remainder of the paper is as follows. After the description of the model, we derive the first order conditions. In third section the analytical framework is extended and some comparative analysis is made. Conclusive remarks and implications for environmental political economy are the subject of the last section.

We confine the mathematical details to the appendix.

A growth model with exhaustible and renewable resources

We start by presenting a simple theoretical framework which considers both kinds of natural resource. The production function of the final output y depends on five inputs: physical capital K, stock of technology A, labor time devoted to final output production l_1, rate of use of exhaustible resources e, amount of renewable resources harvested r. The total number of skilled workers N is assumed constant and normalized to one. Labor may be used not only to produce the final output but also in R&D activity $l_2 = 1 - l_1$. In what follows we are interested only in internal solutions, in which $l_i > 0$, for $i = 1, 2$. Technology (or knowledge) is accumulated as a consequence of the labor time allotted to this purpose (Romer, 1990). In other words, technological change is costly and does not occur by chance: it depends rather on the effort devoted to it and is an outcome of one sector of our economy (Kamien and Schwartz, 1978; Romer, 1990). All the variables considered in the model are functions of the time t; to make the paper easier to read we have suppressed the subscript (t) in the ensuing discussion.

The production function is

$$y = f(K, A, l_1, e, r) = K^{\alpha_1}(Al_1)^{\alpha_2}(e + \psi r)^{\alpha_3}, \quad \sum_{i=1}^{3} \alpha_i = 1, \tag{1}$$

where ψ is a parameter describing the kind of technological substitutability between exhaustible and renewable resources. In this way, if $\psi = 1$, we may use e or r indiscriminately. If this condition is not satisfied (i.e. $0 < \psi < 1$) we are in a case of imperfect technical substitutability, for which the marginal productivity will not be the same for e and r, and the RTS between those two inputs will be lower than in the hypothesis in which $\psi = 1$.[2]

The law motion of physical capital is

$$\dot{K} = y - c \quad \text{where } K(0) = K_0 \text{ and } K(t) \geqslant 0. \tag{2}$$

Eq. (2) is a constraint which considers the change of physical capital over time. For the sake of simplicity, depreciation in K is not considered here. Aggregate consumption is denoted by $c = xy$, with $0 < x \leqslant 1$. Per capita consumption is represented by $c = c/N$. Aggregate saving is $s = zy = \dot{K}$, where $0 \leqslant z < 1$, and $z = (1 - x)$.

$$\dot{A} = \xi(1 - l_1)A \quad \text{where } A(0) = A_0 > 0 \text{ and } A(t) \geqslant 0. \tag{3}$$

Eq. (3), which is of an endogenous nature, describes the stock of technology accumulated, where $\xi > 0$ is a productive parameter of scientific research (Romer, 1990).

$$\dot{E} = -e \quad \text{where } E(0) = E_0 > 0, \ E(t) \geqslant 0. \tag{4}$$

$\dot{E}$ considers the dynamics of the exhaustible resource stock E as a function of the flow of virgin ores extracted from the world's crust (for a similar specification see Vousden, 1973).

$$\dot{R} = f(R) - \rho R = \sigma\left(1 - \frac{R}{\pi}\right)R - r \quad \text{where}$$
$$R(0) = R_0, \ R(t) \geqslant 0. \tag{5}$$

Eq. (5) expresses the behavior over time of the renewable resource stock R. We assume that $f(R)$ is the growth function, with properties $f(R) \geqslant 0$, for $0 \leqslant R \leqslant \pi$, $f'(R) > 0$ for $0 \leqslant R \leqslant \bar{R}$, $f'(\bar{R}) = 0$ and $f'(R) < 0$ for $\bar{R} \leqslant R \leqslant \pi$, where $\bar{R}$ is the maximum sustainable yield stock level of our renewable resource, and π is the ecological carrying capacity (Hanley et al., 1997; Li and Löfgren, 2000). σ denotes the intrinsic growth rate of renewable resources, while r, equal to ρR, is the harvest flow of this natural input, ρ being the rate of use of R (where $0 \leqslant \rho \leqslant 1$). The assumption regarding the first derivative of the natural production function $f'(R)$, is justified by the fact that this kind of resource has a maximum point and then decreases to zero. Thus, there is a maximum sustainable yield, that in equilibrium should be equal to the highest possible harvest rate (Clark, 1999).

$$\dot{J} = d - \gamma J = (e + r) - \gamma J, \quad J(0) = J_0, \ J(t) \geqslant 0. \tag{6}$$

[2]Some technical observations are necessary here. In a space (e, r) along the isoquant, inputs have a constant technical substitutability, but for $\psi < 1$ the resources are imperfect technical substitutes (because the isoquant has a slope greater than one). Letting the other inputs be constant, in (1) we may calculate the RTS of e for r, to find $RTS = (\partial y/\partial e)/(\partial y/\partial r) = 1/\psi$. This means that for $\psi < 1$ the RTS is greater than one and the two inputs are imperfect technical substitutes (for the significance of the TRS between two inputs, in cases with $n > 2$ production factors, see Varian, 1992). Remembering that ψ is a constant and for $[\partial(e/r)/(e/r)] \neq 0$, the elasticity of substitution between exhaustible and renewable resources (given by $\partial(e/r)/(e/r)/\partial RTS/RTS$) *seems* to be infinite, but in a recent contribution Blackorby and Russell (1989) have shown that in cases with more than two inputs the so-called Allen Elasticity (or "partial elasticity of substitution", see Allen, 1938) is completely uninformative about the curvature of the isoquant, although this concept is widely used in empirical studies.

This is the motion equation of the waste stock J (Smith, 1972); it is a function of the litter flow d produced in the economy (as a by-product of the transformation of inputs into outputs), minus the amount of J that the ecosystem is capable of absorbing by biodegradation. $0 \leqslant \gamma < 1$ is a parameter representing the capacity of the environment to assimilate waste (Huhtala, 1999; Plourde, 1972). As the forerunners of waste management analysis observed, the conservation of matter principle makes it possible to affirm that, in a closed system, the tonnage of raw materials utilized by an economy is approximately equal to the weight of waste generated (D'Arge and Kogiku, 1973). In formal terms $d = e + r$. This specification of waste stock dynamics reflects the materials balance approach (Ayres, 1999; Huhtala, 1999; Radetzki and Van Duyne, 1985).

The welfare of society, at any point in time, is a function of the per capita flow of consumption and the stock of waste (Aronsson and Löfgren, 1999; D'Arge and Kogiku, 1973; Hoel, 1978; Huhtala, 1999; Keeler et al., 1971; Lusky, 1976; Plourde, 1972; Smith, 1972; Tahvonen, 1997). The inclusion of J in our utility function indicates that we will pay a price to reduce the amount of residuals accumulated. The instantaneous utility function can be indicated by

$$u = u(c, J) = \frac{c^{1-\theta} - 1}{1 - \theta} - \frac{J^{1+\omega} - 1}{1 + \omega} \quad \text{with } \theta, \gamma, \omega > 0. \tag{7}$$

Eq. (7) has continuous first and second partial derivatives, with $u_c > 0$, $u_{cc} < 0$, $u_J < 0$, $u_{JJ} < 0$ and $U_{cJ} = 0$. It is assumed that for $c \to 0$, $u_c \to +\infty$, where θ and ω are two more parameters, representing the elasticity of marginal utility with respect to consumption and waste stock.

The exogenous social discount rate $\delta > 0$ is applied to the flow of utility. The total welfare W associated with any particular time path for c and J is derived by summing the discounted flow. The problem of social planner is to choose c and J, such to maximize

$$W = \int_0^\infty u(c, J) \mathrm{e}^{-\delta t} \, \mathrm{d}t; \tag{8}$$

it reflects that current agents in this economy take into account the welfare and resources of their present and prospective descendants. Assuming that the case of $\psi = 1$ prevails, the current-value Hamiltonian $\aleph$ for the problem is

$$\begin{aligned} \aleph = {} & \frac{c^{1-\theta} - 1}{1 - \theta} - \frac{J^{1+\omega} - 1}{1 + \omega} + \lambda_1 \{ [K^{\alpha_1} (Al_1)^{\alpha_2} (e + r)^{\alpha_3}] - C \} \\ & + \lambda_2 [\xi (1 - l_1) A] - \lambda_3 e + \lambda_4 \left[\sigma \left(1 - \frac{R}{\pi} \right) R - r \right] \\ & - \lambda_5 [(e + r) - \gamma J], \end{aligned} \tag{9}$$

where λ_i, $i = 1, 2, 3, 4, 5$, are the dynamic multipliers of the stock variables. Note that we consider the shadow price of waste λ_5 to be negative, because that flow generates disutility.

First order conditions for an optimal solution, together with the usual transversality constraints, are confined to Appendix A.[3] Here it is assumed that all the conditions in Theorem 9 of Seierstad and Sydæter (1997, p. 217), together with the concavity on the maximized Hamiltonian, are satisfied.

Investigating the model behavior under different hypotheses

In what follows we shall use the theoretical framework outlined in the previous paragraph. First of all, we assume $\psi = 1$ and then introduce some complication into the model, to take into account the hypothesis of $\psi < 1$. Further on, we extend the model to consider what happens when waste recycling occurs, in cases where it is not indifferent whether secondary materials are used instead of one or both of the natural resources considered. Finally, we evaluate the use of Pigouvian tax to internalize the negative externality represented by waste accumulation. The paragraph concludes with a comparison between waste recycling and Pigouvian tax, as instruments to increase welfare and achieve the optimal amount of scrap discharged into the environment.

The basic model

The first question that we should tackle is: how does the presence of a renewable natural input influence the dynamics of the exhaustible resources stock? To this aim we may use (A.3); after putting in evidence e, and substituting in [4], the result is

$$\dot{E} = \frac{\lambda_1 \alpha_3 y}{\lambda_4 + \lambda_5} - r. \tag{10}$$

From the above equation we can see that an increase in the rate of use of renewable resources allows us to reduce the extraction of e by the same amount, and that a negative relationship exists between $\dot{E}$ and λ_4. In other words, under the condition $RTS = 1$ between the two natural resources, the first step to offset the constraint to growth due to use of exhaustible resources is to increase the employment of renewable natural input.

The problem is now to define the role played by technological change in alleviating the constraint to growth represented by non-replenishable resources. To answer this question we use (A.2) and (3).[4]

After some algebra, and calculating the implicit derivative of e^* with respect to ξ, we obtain

$$\frac{\partial e^*}{\partial \xi} = -\frac{\alpha_2 (e + r)}{\xi (\xi - 1)(\alpha_3 - 1)} < 0. \tag{11}$$

[3]For expositional reasons, the demonstration that the model exhibits a unique optimal saddle point is omitted. It was derived following a standard procedure similar to that of Schou (2000). The formal proof is available, upon request, from the author.

[4]By means of first order conditions evaluated in the optimal stationary growth path and reported in the Appendix, we can derive some *qualitative* information by taking the partial derivative of some endogenous variable with respect to a parameter. For this kind of analysis see Seierstad and Sydæter (1997, p. 219).

We may thus say that an inverse relationship exists between the rate of use of the exhaustible resource and changes in knowledge accumulation, represented by the parameter of productivity in this sector of our economy. An increase in knowledge allows us to reduce the pressure on the environment in different ways: improving the efficiency of the use of inputs through new production processes; increasing the possibility of using other inputs instead of the exhaustible ones; reducing the quantity of e extracted from the earth's crust.

Starting from (A.13), putting in evidence δ and substituting in (A.12), we obtain

$$g_{\lambda_3} = g_{\lambda_4} + \sigma(1 - 2R/\pi) - \rho, \tag{12}$$

such that with an increase in the rate of use of the renewable resource ρ, the growth rate of the shadow price of exhaustible resource decreases. In the optimal stationary growth path, we know that $g_{\lambda_3} = g_{\lambda_4}$, such that $\sigma\left(1 - 2R/\pi\right) - \rho = 0$.[5] As a result, Hotelling's rule is verified also for a reproducible natural input.

During transitional dynamics, an increased use of renewable resources allows us to reduce pressure on the exhaustible natural input and its price.

To investigate how technology influences the dynamics of λ_3, we may put in evidence δ in (A.10) and then substitute in (A.12), to find

$$g_{\lambda_3} = g_{\lambda_1} + \alpha_3 K^{\alpha_1 - 1}\left(A - \frac{\dot{A}}{\xi}\right)^{\alpha_2}(e + r)^{\alpha_3}. \tag{13}$$

From (13) it is clear that the greater the changes in technological stock over time, the lower g_{λ_3} will be. An increase in the absolute accumulation rate of knowledge diminishes the demand for exhaustible resources and mitigates the dynamics of λ_3.

Imperfect technical substitutability between exhaustible and renewable resources

The assumption $\psi = 1$ employed in the previous section makes analysis simple, but may not always be realistic. It is thus worth highlighting what happens if $\psi < 1$. We assume this condition because historically, while exhaustible resources generally offer a greater productivity than renewable ones, the latter do not represent a constraint to growth, because the main problem for them is merely to harvest them at the optimal rate.

Assuming $\psi < 1$ in our production function and Hamiltonian, only two first order conditions change with respect to the case in which $\psi = 1$, more precisely the partial derivatives of $\aleph$ with respect to e and r. Using $\partial\aleph/\partial e$, and (3) we obtain

$$\dot{E} = \frac{\lambda_1 \alpha_3 y}{\lambda_3 + \lambda_5} - \psi r. \tag{14}$$

Comparing (10) with the equation above, it is evident that in cases of imperfect substitutability we will extract more exhaustible resources from the world's crust. When $\psi = 1$, in fact, one more unit of renewable natural input allows us to save the same amount of non-replenishable ones. Thus, we can affirm that the hypothesis of perfect technical substitutability between the two natural production factors is better for the environment.

It is interesting to see whether the effects of technological change on the rate of use of exhaustible resources, in the case $\psi < 1$, are the same as in the case $\psi = 1$. To this aim we again use $\partial\aleph/\partial e$ and (3). After some simple algebra and evaluating the implicit derivative of e^* with respect to ξ, the result is

$$\frac{\partial e^*}{\partial \xi} = -\frac{\alpha_2(e + \psi r)}{\xi(\xi - 1)(\alpha_3 - 1)} < 0. \tag{15}$$

Making a simple comparison between (15) and (11), it is easy to see that in this case, under *ceteris paribus* conditions, the reduction of the rate of use of non-replenishable natural inputs will be smaller than in the case where $\psi = 1$.

For $\psi < 1$, the shadow prices of exhaustible and renewable resources will be different in the stationary growth path. This implies that Hotelling's rule will also be satisfied if and only if $\sigma(1 - 2R/\pi) - \rho = 0$. Unlike the results obtained under the assumption $\psi = 1$, however, this is not an implicit result of the model, because (A.17) does not hold if $\psi < 1$.[6] Thus, combining (A.12) with (A.13), we may obtain three possible results

$$g_{\lambda_3} \begin{cases} > g_{\lambda_4} & \text{if } \sigma(1 - 2R/\pi) > \rho, \\ = g_{\lambda_4} & \text{if } \sigma(1 - 2R/\pi) = \rho, \\ < g_{\lambda_4} & \text{if } \sigma(1 - 2R/\pi) < \rho. \end{cases} \tag{16}$$

In the case where $g_{\lambda_3} = g_{\lambda_4}$, corresponding to the middle row of (15), the system is at its optimal growth path; in the other hypotheses the economic system runs the risk of diverging from its optimal growth path. If we are under-utilizing renewable resources (i.e. $\sigma(1 - 2R/\pi) > \rho$), the growth rate of λ_3 will be higher than its optimal value, thus over-exploiting the exhaustible natural input. Vice versa, when the growth rate of the shadow price of the exhaustible resource is lower than its optimal value (i.e. $\sigma(1 - 2R/\pi) < \rho$) we are oversweating the renewable natural input. In the first case there is a risk of depleting exhaustible resources, while in the second hypothesis the danger is of using up the renewable one. In both cases the economic system is moving in the direction of an unsustainable growth path.

[5] From Eqs. (A.2) and (A.3), and taking the logarithmic derivative, we show that $g_{\lambda_3} = g_{\lambda_4}$. Thus, the last result holds, from (A.12) and (A.13), if and only if $\sigma(1 - 2R/\pi) = \rho$.

[6] Under the hypothesis $\psi < 1$ we have: [A2′] $\partial\aleph/\partial e = \lambda_1 \alpha_3 Y/(e + \psi r) - \lambda_3 - \lambda_5 = 0$, and [A.3′] $\partial\aleph/\partial r = \lambda_1 \alpha_3 \psi Y/(e + \psi r) - \lambda_4 - \lambda_5 = 0$. Such that e and r therefore have different marginal productivity and shadow prices.

Waste recycling under imperfect substitutability between natural resources

In the simple version of the model there are two forces at work that may offset the constraint to growth represented by exhaustible resources: technological progress and the increasing use of renewable natural inputs. The results of the previous section show that when natural resources are not perfect technical substitutes of each other, the scarcity of non-replenishable resources may bring the system to exploit one or both natural inputs. This suggests that we should look for another channel to bring the economic system towards more environmentally friendly behavior. We want therefore to extend our model further, to include the recycling of waste derived from non-renewable resources, and to see what happens in this hypothesis.

The process of transforming litter into secondary input is considered both an effective pollution abatement measure and a strategy to offset the scarcity of exhaustible resources (Di Vita, 2004; Huhtala, 1999; Lusky, 1976; Smith, 1974). In this case we assume that there is another production process inside the economic system, transforming waste into a substitute input by using the refuse derived from exhaustible resources. We do not consider here the possibility of transforming scrap from reproducible natural input into secondary materials, because for this kind of input the main problem is merely to achieve an optimal harvest rate. Indeed, the environment has a greater capacity to absorb waste derived from renewable resources.

To produce the secondary material m it will be necessary to devote some labor time to the third sector of our economy l_3. In cases where waste is recycled, the constraint on labor time will be $l_1 + l_2 + l_3 = 1$.

The Cobb–Douglas production function of secondary materials is

$$m = f(e, l_3) = e^{\mu_1} l_3^{\mu_2} \quad \text{with } \mu_1 + \mu_2 = 1, \tag{17}$$

that exhibits constant returns to scale. In continuous time we may overlook the temporal lag between the extraction, use and recycling of the exhaustible resource, because the differences are not so important as with respect to the flows in the previous period (Hoel, 1978). In particular, with regard to the process of transformation of refuse into input, the most relevant phenomenon in magnitude is the so-called ‘‘closed-loop’’ (i.e. the recycling of new scrap inside the production process). For example, pyro-processing R&D was part of the U.S. Integrated Fast Reactor development program, which proposed that a reprocessing and fuel-recycling plant be integrated into each reactor complex (van Hippel, 2001). This means that transformation of residuals into input has more and more frequently become another stage in the production process, increasing the effective long-term supply of the resources that are being reused (Solow, 1974b; Tietenberg, 1992; Weinstein and Zeckhauser, 1974). Finally, it is worth noting that it is cheaper to recycle the current flow of scrap than previously-stored residuals.

The motion equation of waste stock becomes

$$\dot{J} = d - m - \gamma J = (e + r) - e^{\mu_1} l_3^{\mu_2} - \gamma J \quad \text{where}$$
$$J(0) = J_0 \text{ and } J(t) \geqslant 0, \tag{18}$$

where the accumulation of scrap is reduced as an effect of the recycling process. It is interesting to note that if $l_3 = 0$, (18) is the same as (6). The dynamic multiplier of secondary materials will be the same as that of an exhaustible resource, because in our model they are perfect substitutes of each other. In this manner m does not enter explicitly into the production function, because when recycling takes place, it is indifferent whether a virgin exhaustible resource is used or a secondary material derived from it.

Under these new conditions and using the constraints expressed in Eqs. (2)–(5), the appropriate Hamiltonian expressed at current values is

$$\begin{aligned} \aleph = {} & \frac{c^{1-\theta} - 1}{1 - \theta} - \frac{J^{1+\omega} - 1}{1 + \omega} + \lambda_1 \left\{ [K^{\alpha_1} (A l_1)^{\alpha_2} (e + \psi r)^{\alpha_3}] - C \right\} \\ & + \lambda_2 [\xi (1 - l_1 - l_3) A] - \lambda_3 e + \lambda_4 \left[\sigma \left(1 - \frac{R}{\pi} \right) R - r \right] \\ & - \lambda_5 [(e + r) - e^{\mu_1} l_3^{\mu_2} - \gamma J]. \end{aligned} \tag{19}$$

In this case few first order conditions change with respect to the case where waste recycling is not considered. In particular, $\partial \aleph / \partial e$ becomes

$$\frac{\partial \aleph}{\partial e} = \frac{\lambda_1 \alpha_3 y}{e + \psi r} - \lambda_3 - \lambda_5 \left[1 - \mu_1 \frac{m}{e} \right] = 0, \tag{20}$$

and we have to consider the marginal productivity of labor employed in the third sector of our economy (i.e. secondary materials production).

$$\frac{\partial \aleph}{\partial l_3} = -\lambda_2 \xi A + \lambda_5 \mu_2 \frac{e^{\mu_1} l_3^{\mu_2}}{l_3} = 0. \tag{21}$$

The above equation is the standard first order condition for optimal employment of labor in transforming litter into substitute inputs. The other first order and dynamic conditions are the same as in the previous version of the model.

In (20) the addend $\lambda_5 \mu_1 (m/e)$ represents the positive effects of waste recycling, measured by the marginal productivity in value of scrap derived from exhaustible resources transformed into substitute input.

To understand the effect on the rate of exhaustible resources extraction, of transforming waste (from a non-reproducible natural input) into secondary materials, we may make a comparison between (A.2) and (20), to see that under the hypotheses taken into account ($\psi < 1$ and waste recycling runs), λ_3 will be greater if a scrap recovery process takes place. This implies that fewer natural resources will be extracted from the earth's crust.

Using $\partial \aleph / \partial r$, in cases where $\psi < 1$, and (20), after some little algebra we get

$$\frac{\lambda_1 \alpha_3 y}{e + \psi r} (1 - \psi) + \lambda_5 \mu_1 \frac{m}{e} = \lambda_3 - \lambda_4. \tag{22}$$

The above equation implies that the two shadow prices will never be the same under the assumption $\psi < 1$, such that λ_3 will be always greater than λ_4. While the first addend on the left-hand side of the equation measures the loss of marginal productivity due to imperfect substitutability, the second addend accounts for the positive effects of waste recycling. In other words, the scrap recovery process increases the difference between λ_3 and λ_4, making it more profitable to use a renewable natural input.

Putting in evidence λ_4 in (22) and calculating the partial derivative of the shadow price of renewable resources with respect to μ_1, we obtain

$$\frac{\partial \lambda_4}{\partial \mu_1} = -\lambda_5 \frac{m}{e}(1 + \mu_1 \ln e) < 0, \quad (23)$$

such that we may affirm that a rise in productivity of waste recycling allows us to reduce the shadow price of the renewable resource.

Starting from (20), under the assumption for which $A = \dot{A}/(1 - l_1 - l_3)\xi$, we may calculate the partial derivative of m with respect to ξ, to investigate the effects of an increase in the efficiency of knowledge accumulation on the process of transforming refuse into substitute inputs:

$$\frac{\partial m}{\partial \xi} = \frac{\mu_1 \alpha_2 \alpha_3 \lambda_1 \lambda_5 y \dot{A}}{(e + \psi r) e \xi [A(1 - l_3) - \dot{A}]} > 0. \quad (24)$$

We may thus affirm that technological improvement allows us to increase the amount of secondary materials produced.

To examine how secondary materials production modifies the dynamics of the exhaustible resources' shadow price, we put in evidence λ_3 in (20) and substituting in (A.7), deriving with respect to μ_1, we obtain

$$\frac{\partial \dot{\lambda}_3}{\partial \mu_1} = \delta \lambda_5 e^{\mu_1 - 1} l_3^{\mu_2} (1 + \ln e) > 0. \quad (25)$$

This means that an improvement in waste recycling efficiency raises the rate of change during time of λ_3, such that this process sends the right message to the economic system, diminishing the opportunity cost of using renewable resources instead of non-replenishable ones.

To take into account the effects of secondary materials production upon the dynamics of λ_4, we combine (20) with $\partial \aleph / \partial r$ in cases where $\psi < 1$; substituting in (A.8) and partially deriving with respect to μ_1 we obtain

$$\frac{\partial \dot{\lambda}_4}{\partial \mu_1} = -\left[\delta \lambda_5 e^{\mu_1 - 1} l_3^{\mu_2} (1 + \ln e)\right] \left[\delta + \left(\frac{2\sigma R}{\tau} - \sigma + \rho\right)\right] < 0. \quad (26)$$

This means that the production of secondary materials reduces the dynamics of the shadow prices of renewable natural resources, facilitating the use of this kind of natural input.

Moreover, it is worth considering waste recycling in our analysis because this process not only increases the efficiency of exhaustible resource use (Weinstein and Zeckhauser, 1974), but also improves the standard of welfare. It is possible to ignore the refuse discharge problem as long as J has no major effects on social welfare, but when, as a result of an increase in income, the stock of scrap goes beyond a certain threshold level (i.e. $d > \gamma J$), we can affirm that accumulation creates a loss of utility, measured by λ_5; this is the negative shadow price of waste, and represents an externality cost of the use of both kinds of natural inputs considered in the model. Without an environmental policy, therefore, the dynamic multipliers of e and r will be higher than their optimal values, in cases where the externality is fully internalized. This is why the economic system tends to over-use both kinds of natural resources.

By means of (A.2), calculating the partial derivative of λ_5 with respect to α_3, we are able to show that the negative externality represented by waste discharged into the environment is increasing with natural resource use

$$\frac{\partial \lambda_5}{\partial \alpha_3} = \frac{Y}{e + \psi r}[\lambda_1 + \alpha_3 \ln(e + \psi r)] > 0. \quad (27)$$

If no pollution abatement policy is adopted, growth increases consumption but also reduces welfare, as a consequence of waste accumulation. The net result on the welfare level of those two opposite effects is not immediately evident: at the moment, by using $\partial \aleph / \partial e$ for $\psi < 1$ and (A.1), and knowing that $u_J = \lambda_5$ (derived from $\partial \aleph / \partial J = 0$), it is just possible to affirm that the marginal utility of consumption is sub-optimal high, because the negative effects of waste accumulation are not taken into account.[7]

$$u_C = \frac{(\lambda_3 + \lambda_5)(e + \psi r)}{\alpha_3 y}. \quad (28)$$

From the point of view of the benevolent social planner, it is necessary to follow some environmental policy to internalize the negative externality. Here we consider two possible options: the so-called "mixed regime" in which a Pigouvian tax τ (assumed equal to the social cost of waste accumulation represented by λ_5) is levied on λ_4 and waste derived from e is recycled; and the "pure-regime" in which τ is imposed on both shadow prices of natural resources. In the following part of this section we shall deal with the first.

The best way to ensure that the optimal amount of waste is recycled is to make the marginal disutility of waste $\partial U / \partial J$ equal to the value of an increase in secondary materials production, due to the recycling of one more unit of refuse derived from the exhaustible resource $\lambda_5 \partial m / \partial e$.[8]

[7] Note that at an individual level the utility function is $u(c)$, because consumption is rivalrous and excludible, while the negative effects of waste stock accumulation are a public bad, such that consumers do not take the latter into consideration in their decisions (for the difference between centralized and market choices in a similar environment, see Lusky, 1975).

[8] In other words, $\lambda_5(\partial m / \partial e)$ is none other than the value of the reduction in loss of total utility, in one more unit of waste not discharged into the environment.

Remember that we have assumed secondary materials to be perfect substitutes of exhaustible resources, such that the condition regarding the optimal amount of waste recycled implies that the shadow price of e should be equal to its marginal productivity.

Calculating the first order condition of the Hamiltonian with respect to the waste stock, we get

$$\partial\aleph/\partial J = -\gamma J^{\omega} + \gamma\lambda_5 = 0 \quad \text{or} \quad J^{\omega} = \lambda_5, \tag{29}$$

where $u_J = -\gamma J^{\omega}$. Considering that $\lambda_5 \partial m/\partial e = \lambda_5 \mu_1(m/e)$, using the condition for the optimal level of waste recycling $u_J = \lambda_5 \partial m/\partial e$, and rewriting (19) as

$$\lambda_5\left[1 - \mu_1\frac{m}{e}\right] = \frac{\lambda_1\alpha_3 y}{e + \psi r} - \lambda_3, \tag{30}$$

we observe that the optimal level of refuse recycled ensures that the value of the marginal productivity of exhaustible resources is equal to its own shadow price, thus fully internalizing the negative externality represented by refuse production. It is worth noting that the result above holds if and only if $\lambda_5 = 0$.

Now we are able to take into account the problem regarding the net effect of growth on welfare w, by means of its net changes Δw

$$\Delta w = \partial u/\partial c - \partial u/\partial J, \tag{31}$$

by using $\partial\aleph/\partial e$ for $\psi < 1$ and (A.1), and knowing that $u_J = \lambda_5$, the result without the waste recycling process is

$$\Delta w = \partial u/\partial c - \partial u/\partial J = \frac{\lambda_3(e + \psi r)}{\alpha_3 y} + \lambda_5\left[\frac{(e + \psi r)}{\alpha_3 y} - 1\right]. \tag{32}$$

We may compare this with the net effects on welfare when secondary materials production is at its optimal level Δw^*

$$\Delta w^* = \frac{\lambda_3(e + \psi r)}{\alpha_3 y}. \tag{33}$$

It is immediately clear that $\Delta w^* > \Delta w$, for $(e + \psi r)/\alpha_3 y < 1$ (this result comes from $\partial\aleph/\partial e$, when $\psi < 1$). In other words the optimal amount of waste recycling ensures the maximum level of welfare.

Pigouvian taxes as an instrument to achieve the maximum level of welfare

As an alternative to the waste recycling process we may internalize the negative externality represented by scrap accumulation by imposing a Pigouvian tax τ on both the shadow prices of natural resources. In cases of perfect and imperfect technical substitutability between exhaustible and renewable resources, τ^* should be equal to λ_5. In the latter hypothesis, using (A.2) and (A.3), the shadow prices of resources become

$$\lambda_3 = \lambda_1\alpha_3\frac{y}{e + r}, \tag{34}$$

and

$$\lambda_4 = \lambda_1\alpha_3\frac{y}{e + r}. \tag{35}$$

λ_3 and λ_4 will therefore be equal to their marginal productivity (this is also true for $\psi < 1$) and the negative externality upon the utility function will be fully internalized.

To account for the effect of Pigouvian tax on the changes over time of λ_3 and λ_4, we may use (A.7) and (A.8) and substituting, respectively, for both dynamic multipliers (A.2) and (A.3), we get

$$\dot{\lambda}_3 = \delta\frac{\lambda_1\alpha_3 y}{e + \psi r}, \tag{36}$$

and

$$\dot{\lambda}_4 = \left[\frac{\psi\lambda_1\alpha_3 y}{e + \psi r}\right]\left[\delta + \left(\frac{2\sigma R}{\tau} - \sigma + \rho\right)\right]. \tag{37}$$

In this case there are no effects on the behavior over time of both shadow prices of natural inputs.

By means of [31] we may see that when a Pigouvian is levied, the net effect of growth on welfare, by means of net changes Δw^{T}, is

$$\Delta w^{\mathrm{T}} = \partial u/\partial c - \partial u/\partial J = \frac{\lambda_3(e + \psi r)}{\alpha_3 y}, \tag{38}$$

that is the same as in the case of waste recycling.

Now, what can we say about the question whether the mixed regime (waste recycling and Pigouvian tax) is better than the pure regime in which τ^* is levied on both prices of natural resources? From static first order conditions, it is just possible to say that the performances of the economy represented in our model are the same; no differences emerge. The problem is the dynamics: in cases where only Pigouvian tax is applied there are no effects on technology accumulation, and on the behavior over time of both prices of natural resources. The previous results regarding the dynamic effects of waste recycling could be questionable because referred to the single differential equation. Thus, it could be interesting to make a sensitivity analysis on the Hamiltonian as a whole, to derive more generalised insights.

A comparison between waste recycling and Pigouvian taxes (sensitivity analysis)

In waste recycling under imperfect substituability between natural resources section we were dealing with a model in which waste recycling was considered: we may conclude that this process offsets the constraint to growth represented by exhaustible resources and reduces the environmental burden. On the other hand, we know that to produce secondary materials we have to divert the labor force from other sectors to recycling, and that we may also internalize the externality represented by the sub-optimal accumulation of waste by introducing a Pigouvian tax.

To make a comparison between the mixed regime and that in which we use only a fiscal measure, we make a sensitivity analysis on the Hamiltonian as a whole (Kamien and Schwartz, 1991; Malanowski, 1984; Seierstad and Sydæter, 1997), calculating its partial derivative with respect to a parameter, representing an improvement in the waste recycling process, or the Pigouvian tax, respectively. Here we are interested only in a qualitative analysis because there are too many variables and parameters to make a numerical calculus. Using $\aleph$ from (20) and calculating its partial derivative at the optimal level $\aleph^*$ with respect to μ_1, we obtain

$$\frac{\partial \aleph^*}{\partial \mu_1} = \lambda_5^* e^{*\mu_1} (\ln e^*) l_3^{*\mu_2} > 0. \tag{39}$$

This means that an improvement in the secondary materials production process increases the value $\aleph^*$, that is none other than the constrained optimal welfare level (for an economic interpretation of the Hamiltonian see Dorfman, 1969).

In the Hamiltonian there is no parameter accounting for a change in Pigouvian tax, thus we may substitute τ in (20) for λ_5 when $l_3 = 0$ (i.e. no waste recycling occurs), and calculating the partial derivative, the result is

$$\frac{\partial \aleph^*}{\partial \tau} = -[(e^* + r^*) - \gamma J^*] < 0. \tag{40}$$

In this case it is evident that an increase in Pigouvian tax reduces the value of $\aleph^*$, such that efficient management of refuse shows a better outcome in dynamics than τ. This result confirms our findings in the previous paragraph.

Final remarks

A few words are necessary to conclude our paper. The economy shows a worse performance in cases of imperfect technical substitutability between exhaustible and renewable resources than in cases where it is possible to use one natural input instead of another indifferently in the production function. Technological progress allows us to offset the constraint to growth represented by exhaustible resources by means of four different channels: firstly, by improving technical substitutability between the resources; secondly, by reducing the amount of exhaustible natural input drawn from the earth's crust; thirdly, by increasing the amount of secondary materials that we are able to produce; fourthly, by sending the right message to the system regarding the behavior over time of the prices of both kinds of natural resource. Finally, the waste recycling process permits us to achieve two results at the same time: to improve the marginal productivity of exhaustible resources and to fully internalize the negative externality represented by waste stock accumulation. Using the first order conditions, the same results are shown by the economy in both regimes (so-called "mixed" and "pure", as defined before), but in the dynamics the combined use of waste recycling and Pigouvian tax ensures greater positive effects on the constrained optimal level of welfare.

These are the results of our theoretical framework; at the moment no empirical analyses have been carried out on this issue. We think that it might be a good topic for further applied studies.

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Appendix

First order and transversality conditions

The first order conditions are:

$$\frac{\partial \aleph}{\partial c} = c^{-\theta} - \lambda_1 = 0 \quad \text{or} \quad \lambda_1 = c^{-\theta}, \tag{A.1}$$

$$\frac{\partial \aleph}{\partial e} = \lambda_1 \alpha_3 \frac{y}{e+r} - \lambda_3 - \lambda_5 = 0 \quad \text{or}$$
$$\lambda_3 = \lambda_1 \alpha_3 \frac{y}{e+r} - \lambda_5, \tag{A.2}$$

$$\frac{\partial \aleph}{\partial r} = \lambda_1 \alpha_3 \frac{y}{e+r} - \lambda_4 - \lambda_5 = 0 \quad \text{or}$$
$$\lambda_4 = \lambda_1 \alpha_3 \frac{y}{e+r} - \lambda_5, \tag{A.3}$$

$$\frac{\partial \aleph}{\partial l_1} = \alpha_2 \lambda_1 \frac{y}{l_1} - \lambda_2 \xi A = 0 \quad \text{or} \quad \lambda_2 \xi A = \alpha_2 \lambda_1 \frac{y}{l_1}, \tag{A.4}$$

$$\dot{\lambda}_1 = \delta \lambda_1 - \frac{\partial \aleph}{\partial K} = \delta \lambda_1 - \lambda_1 \alpha_1 \frac{y}{K}, \tag{A.5}$$

$$\dot{\lambda}_2 = \delta \lambda_2 - \frac{\partial \aleph}{\partial A} = \delta \lambda_2 - \lambda_1 \alpha_2 \frac{y}{A} - \lambda_2 \xi (1 - l_1), \tag{A.6}$$

$$\dot{\lambda}_3 = \delta \lambda_3 - \frac{\partial \aleph}{\partial E} = \delta \lambda_3, \tag{A.7}$$

$$\dot{\lambda}_4 = \delta \lambda_4 - \frac{\partial \aleph}{\partial R} = \delta \lambda_4 + \lambda_4 \left(\frac{2\sigma R}{\pi} - \sigma + \rho \right), \tag{A.8}$$

$$\dot{\lambda}_5 = \delta \lambda_5 - \frac{\partial \aleph}{\partial J} = \delta \lambda_5 + J^{\omega} - \gamma \lambda_5. \tag{A.9}$$

The growth rates of the dynamic multiplier are:

$$g_{\lambda_1} = \frac{\dot{\lambda}_1}{\lambda_1} = \delta - \alpha_1 \frac{y}{K}, \tag{A.10}$$

$$g_{\lambda_2} = \frac{\dot{\lambda}_2}{\lambda_2} = \delta - \lambda_1 \alpha_2 \frac{y}{\lambda_2 A} - \xi (1 - l_1), \tag{A.11}$$

$$g_{\lambda_3} = \frac{\dot{\lambda}_3}{\lambda_3} = \delta, \tag{A.12}$$

$$g_{\lambda_4} = \frac{\dot{\lambda}_4}{\lambda_4} = \delta + \sigma\left(\frac{2R}{\pi} - 1\right) + \rho, \tag{A.13}$$

$$g_{\lambda_5} = \frac{\dot{\lambda}_5}{\lambda_5} = \delta + \frac{\gamma J^{\omega}}{\lambda_5} - \gamma. \tag{A.14}$$

Differentiating Eqs. (A.1) and (A.4) logarithmically, the results will be

$$g_{\lambda_1} = g_{u_c}, \tag{A.15}$$

$$g_{\lambda_2} = g_{\lambda_1} + g_y - g_A. \tag{A.16}$$

Using (A.2) and (A.3), deriving logarithmically, we may also write

$$g_{\lambda_3} = g_{\lambda_4}. \tag{A.17}$$

The transversality conditions are:

$$\lim_{t\to\infty} \mathrm{e}^{-\delta t}\aleph(t) = 0, \tag{A.18}$$

$$\lim_{t\to\infty} [\mathrm{e}^{-\delta t}\lambda_1(t)K(t)] = 0, \tag{A.19}$$

$$\lim_{t\to\infty} [\mathrm{e}^{-\delta t}\lambda_2(t)A(t)] = 0, \tag{A.20}$$

$$\lim_{t\to\infty} [\mathrm{e}^{-\delta t}\lambda_3(t)E(t)] = 0, \tag{A.21}$$

$$\lim_{t\to\infty} [\mathrm{e}^{-\delta t}\lambda_4(t)R(t)] = 0, \tag{A.22}$$

$$\lim_{t\to\infty} [\mathrm{e}^{-\delta t}\lambda_5(t)J(t)] = 0. \tag{A.23}$$

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