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Investment in Tourism Market: A Dynamic Model of Differentiated Oligopoly

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Abstract. We present a theoretical dynamic model in tourism economics, assuming that the market for tourism is an oligopoly with differentiated products. Destinations can invest in order to improve their stock of physical, natural or cultural resources. Tourism flows yield current revenues, but they are usually detrimental for the stock of resources. We find the solution of the dynamic model, and in particular we find the open-loop Nash equilibrium of the game among destinations, under alternative settings, depending on whether the degree of differentiation among destinations is exogenous or endogenous. In particular, under the latter case, an increase of the number of destinations leads to a higher degree of product differentiation in steady state.

Key words: differential games, reservation price, tourism

JEL classification: D43, D92, L83

1. Introduction

In this paper we take a microeconomic perspective in order to study the optimal behavior of destinations as concerns investment and tourism flow regulation over time, and their effects on the stock of physical, natural or cultural resources. When we use the word "destination" we do not intend to necessarily refer to specific local sites, but countries, or regions, could be the appropriate level of analysis as well.

We assume that the market of tourism is an oligopoly, where differentiated products are supplied.

The fact that the tourist goods are differentiated is, to some extent, obvious: not only different types of tourism do exist (e.g., sea-side or mountain resort tourism, cultural tourism,...), but tourism products are clearly differentiated even within the same type: the sea-side resort tourism in Bali is different from the sea-side resort tourism in Italy, and the sea-side resorts are different even across the regions of the same country. Thus, our model can apply both to the tourism industry as a whole and to a specific type of tourism.

It is more important to discuss why we believe that this market is an oligopoly. Consider that the available tourism destinations are a given (though large) number, and the entry of new "suppliers" is costly. More important, interaction among destinations is present, and the choice of each destination concerning its tourism product clearly affects the optimal behavior of any other destinations in that field over the world, so that strategic interdependency among destinations does exist. Moreover, the number of the intermediary organizers of tourism flows (like tour operators) is limited and the markets are highly concentrated, with each specific destination usually served by a very small number of operators (see Cooper et al., 1993). Generally, a correspondence between (a set of) tour-operators and (a set of) destinations can be identified. Thus, the differentiated oligopoly model is the appropriate theoretical tool to investigate the behavior of destinations in tourism market.

However – to the best of our knowledge – the available literature on oligopoly competition in tourism markets is quite restricted, and it does not take into explicit consideration the role of product differentiation and its relationship with the stock of resources and its dynamics.[1] We believe that the literature developed by industrial organization about the optimal behavior of suppliers and policy-makers in markets with differentiated products can provide useful insights for tourism economics.

Of course, tourism products present some specificity to be taken into account. In particular, dynamic phenomena affecting the stock of resources are particularly relevant in the case of tourism. In fact, tourist flows are necessary to give revenue; however, they usually have detrimental effects on the environment of the destinations, that is, on natural and physical resources, as well as on cultural heritage. The stock and the state of resources, in turn, affect the consumer reservation price: the larger (or better) the stock, the higher is the consumer reservation price, ceteris paribus.[2] Moreover, the stock of resources can be improved through appropriate investments: more precisely, costly appropriate investments can be useful to contrast the depletion entailed by tourism flows over time. Thus, we believe that a *dynamic approach* is necessary.

In particular, we take a differential game approach to study the investment efforts over time, made by tourism destinations. We propose two different models, according to whether the degree of substitutability between different tourism goods is exogenous or it can be influenced by destinations. We find the open-loop Nash equilibrium of the differential game among the destinations, and we focus on the steady-state allocations.

Our model permits to study the optimal path of tourist presence over time, for a given destination. Moreover, it permits to study the convenience for a system of destinations to introduce new products or to increase product differentiation. Eventually, it permits to study the relationship between the product differentiation and the number of available destinations. This last point is perhaps worth stressing. In fact, it can be argued that the diffusion of information and communication technology (ICT), and broadly speaking the economic globalization, have led to an increase of the number of available goods in general, and the number of tourism destinations in particular. However, several opinions exist concerning the degree of differentiation among goods (and among tourism destinations in particular). According to some authors (see, e.g. Throsby 2001), the increased number of available goods associates with a lower differentiation among them, while the opposite view is supported by, e.g., Varian (2003) or Varian and Shapiro (1999) according to whom ICT permits a deeper degree of product differentiation. The conclusion of our model is that both results are possible, according to the parameter configuration, but for a number of destinations sufficiently high, the increase of the destinations' number associates with a *higher* degree of product differentiation in steady state, provided that product differentiation is attainable through appropriate investments.

The paper is structured as follows. Section 2 illustrates the basic setup, and in particular the demand side. Section 3 presents the model in which each destination chooses the amount of tourists' presence and the amount of investment aimed at enhancing its natural and cultural stock. A short digression focuses on the effect of the number of competitors upon the individual and aggregate profits. In Section 4 we discuss the case in which investments aimed at affecting the degree of substitutability among tourism goods are possible. Section 5 gathers the main conclusions.

2. The Basics of the Model

We consider the tourism market as an oligopoly under full information condition. At any time $t \in (0, +\infty)$, each destination $i (i = 1, 2, \ldots, n)$ offers a tourism product, which is differentiated with respect to the production of any other different destinations. Let P_i denote the price of the product supplied by destination i. Let $x_i(t)$ be the tourists' presence in destination i at time t. The demand side is represented by the following inverse demand function:

$$P_i(t) = A_i(t) - Bx_i(t) - D \sum_{j \neq i} x_j(t). \qquad (1)$$

Equation (1), firstly introduced by Bowley (1924), is widely used in industrial-organization theory: see, e.g., Spence (1976), Dixit (1979), Singh and Vives

(1984), Vives (2000). Parameter $B > 0$ captures the sensitivity of the price of variety i to the quantity i. Parameter $D, 0 \leq D \leq B$, captures the sensitivity of price of variety i to the quantity of goods of different variety; this means that parameter D captures the degree of substitutability between any pair of varieties: the lower D, the less substitutable (i.e., more differentiated) are goods; in other words, D reflects the differences in the levels of characteristics across the different tourism destinations: the more marked the differences, the lower is D. In the limiting case $D = B$ the varieties are perfectly substitutable (i.e., goods are homogeneous), and the homogeneous oligopoly case establishes; in the opposite limiting case, $D = 0$, the differentiation is the largest, products are totally independent, and each supplier behaves as a monopolist: thus, the monopolistic competition turns out to be a particular case of this general model.

In most industrial-organization models, A is parameter capturing the market size or the highest reservation price. Here, we consider $A_i(t)$ as a variable rather than as a parameter. We assume that the highest reservation price (or market size) for variety i is directly linked to the stock of its physical, natural, and/or cultural resources.[3] This stock varies over time, for three reasons: (i) the tourism flows, i.e., the tourists' presence, x, may have a significant impact on the stock of resources; (ii) the amount of investment aimed at protecting environment (or heritage), k, may have a positive effect; (iii) a proportional natural depreciation (or even a proportional natural regeneration) may occur at the rate δ. Hence, we assume that the dynamics of variable $A_i(t)$ is described by the following equation:

$$\frac{dA_i(t)}{dt} \equiv \dot{A}_i(t) = -\alpha x_i(t) + k_i(t) - \delta A_i(t). \qquad (2)$$

Three points are worth stressing as far as Equation (2) is concerned. First, notice that the tourism flow x may have a negative, null or positive effect on the product quality and hence on the stock of resources, according to whether $\alpha > 0$, $\alpha = 0$ or $\alpha < 0$, respectively. In fact, since the seminal contribution of Budowsky (1976), a large debate in tourism economics has been developing on whether tourists' flows lead to degradation or enhancement of natural and cultural capital stocks (or neither of them). Using the labels of Budowsky, three types of inter-relation between tourism and environment can exist: conflict (which corresponds to the case $\alpha > 0$ in the present model); coexistence ($\alpha = 0$); symbiosis ($\alpha < 0$). In the remainder of the paper we assume $\alpha = 1$, so that the case of the detrimental effect is posited; however the model can be easily used to study also the different cases.

Secondly, notice that if $\delta > 0$, a depreciation occurs to the stock of resources (this could well be the case of cultural heritage); if $\delta < 0$ the stock grows naturally (like in the case of environmental regeneration or resilience).

In the remainder of the paper we assume $\delta > 0$, but the model can be easily discussed under the alternative hypothesis.

Third, it is possible to enhance the stock of environmental or cultural resources through appropriate investment taking place over time, $k_i(t)$.

Just to give an example regarding the dynamics described by Equation (2), let us think of the sand of a beach: its stock may vary for three reasons: (i) for the presence of tourists carrying the sand in their shoes (according to the term $-\alpha\, x$), (ii) for the effect of sea (according to $-\delta A$), and (iii) for investment of destination, carrying new sand on the beach (the term $+k$).

We assume that the investment aimed at enhancing the stock of resources entails a quadratic cost, according to the following function φ :

$$\varphi(k_i(t)) = z \cdot [k_i(t)]^2/2, \quad z > 0 \tag{3}$$

which means that the marginal productivity of investment k_i is decreasing. We also assume that the tourists' presence in destination i, x_i, entails a production cost for tourism service, according to the generic function $c(x_i(t))$, with $dc(\cdot)/dx \equiv c'(x) > 0$ Hence, the instantaneous profit for destination i at time t is:

$$\pi_i(t) = P_i(t) \cdot x_i(t) - c(x_i(t)) - z \cdot [k_i(t)]^2/2. \tag{4}$$

We assume that the objective of each destination is to achieve the maximum present value of the flows of its profits over time:

$$\mathrm{Max}\Pi_i = \int_0^{+\infty} \pi_i(t) e^{-\rho t} dt, \tag{5}$$

where $\rho > 0$ is the discounting rate, assumed to be constant and equal across the destinations' population. The dynamic problem is subject to the constraint (2) and to the initial conditions $\{A_i(0) = A_{i0}\}_{i=1}^{n}$. We solve the problem in two different settings.

First (in Section 3), we assume that each destination can control its tourists' presence and the investment aimed at enhancing its stock of resources. The problem faced by destination i has two control variables, $x_i(t)$ and $k_i(t)$, and one relevant state variable, $A_i(t)$. Provided that the tourists' presence in destination j affect the profit – and hence the optimal choice – of destination i, strategic interaction among destinations is present, and a differential game has to be solved.

Secondly (in Section 4), we assume that the degree of differentiation, D, is a variable rather than a parameter, and it is possible to affect its value through costly investment, h_i, decided by the destinations. In this case, a differential game arises, in which each destination faces a problem with three control variables, $k_i(t)$, $x_i(t)$, $h_i(t)$, and two state variables, $A_i(t)$ and $D(t)$.

In both games we adopt the open-loop Nash equilibrium as the solution concept.

3. The Optimal Plans When Tourism Flows are Endogenously Set by Destinations

Destinations can choose different policies as concerns the size of tourism, as well as the effort in investment aimed at enhancing their stock of (natural or cultural) resources. Formally, this means that $x_i(t)$ and $k_i(t)$ can be interpreted as choice variables set by destination i. The problem faced by destination i is:

$$\mathrm{Max}\Pi_i = \int_0^{+\infty} \left\{ \left[A_i(t) - Bx_i(t) - D\sum_{j\neq i} x_j(t) \right] x_i(t) - c[x_i(t)] - z \cdot [k_i(t)]^2/2 \right\} e^{-\rho t} dt \tag{6}$$

$$\text{s.t.: } \frac{dA_i(t)}{dt} \equiv \dot{A}_i(t) = -x_i(t) + k_i(t) - \delta A_i(t); \quad A_i(0) = A_{i0},$$

where $A_i(t)$ is the state variable.[4] The corresponding Hamiltonian function is

$$H_i = \left\{ \left[A_i(t) - Bx_i(t) - D\sum_{j\neq i} x_j(t) \right] x_i(t) - c[x_i(t)] - \frac{z[k_i(t)]^2}{2} + \lambda_i(t) \left[-x_i(t) + k_i(t) - \delta A_i(t) \right] \right\} e^{-\rho t}, \tag{7}$$

where $\lambda_i(t)$ is the current-value co-state variable associated to the state variable $A_i(t)$.

Different solution concepts for differential games exist. We focus on the open-loop Nash equilibrium. Under this solution concept, players precommit their decisions on the control variables to a given time path: they design the optimal plan at the initial date ($t = 0$) and then stick to it forever. Differently, under the closed-loop solution concept, players do not precommit on any path and their decisions at any instant t depend on all the preceding history, and specifically on the observable value of the state variable at that instant. The closed-loop solution is strongly time-consistent and therefore sub-game perfect, while the open-loop solution is in general only weakly time-consistent.[5] In the case of the present section, however, the open-loop

Nash equilibrium coincides with the closed-loop Nash equilibrium and it is therefore strongly time-consistent (see also below, note 7).

The first order conditions and the adjoint equations pertaining to player i are:

$$\begin{cases} \frac{\partial H_i(t)}{\partial x_i(t)} = 0 \\ \frac{\partial H_i(t)}{\partial k_i(t)} = 0 \\ -\frac{\partial H_i(t)}{\partial A_i(t)} = \frac{d\lambda_i(t)}{dt} - \rho\lambda_i(t) \end{cases} \tag{8}$$

to be considered along with the dynamic constraint, the initial conditions and the transversality conditions $\lim_{t\to+\infty} A_i(t)\lambda_i(t)e^{-\rho t} = 0$

The three conditions of system (8) imply, respectively:

$$A_i(t) - 2Bx_i(t) - D\sum_{j\neq i} x_j(t) - c'(x_i(t)) - \lambda_i(t) = 0 \tag{9}$$

$$k_i(t) = \lambda_i(t)/z \tag{10}$$

$$d\lambda_i(t)/dt \equiv \dot{\lambda}_i(t) = -x_i(t) + (\rho+\delta)\lambda_i(t) \tag{11}$$

From Equation (9) we can obtain the reaction curve, which links x_i with the sum of x_j. Then, we impose the symmetry condition $x_i = x_j = x\,\forall i,j$, so that $\sum_{j\neq i} x_j = (n-1)x$. Similarly, we assume $A_i = A_j = A$, $k_i = k_j = k$, $\forall i,j$. Thus, the equilibrium under symmetry conditions implies:

$$A(t) - [2B + D(n-1)]x(t) = c'(x(t)) + zk(t). \tag{12}$$

Intuitively, the left-hand side of Equation (12) can be interpreted as the marginal revenue from tourists' presence, while the right-hand side represents the marginal cost, taking into account that the tourism flows generate damages to the natural resources that must be paid according to addendum $zk(t)$.

Differentiating (10) w.r.t. time we obtain $\dot{k}_i(t) = \dot{\lambda}_i(t)/z$. Differentiating (12) w.r.t. time, and considering them along with (11) and (2), we obtain a dynamic system, in the relevant variables $x(t)$, $k(t)$, $A(t)$; its representation in matrix form is provided below by (13):

$$\begin{bmatrix} \dot{A}_i(t) \\ \dot{k}_i(t) \\ \dot{x}_i(t) \end{bmatrix} = \begin{bmatrix} -\delta & +1 & -1 \\ 0 & \rho+\delta & -1/z \\ -\beta\delta & \beta(1 - z(\rho+\delta)) & 0 \end{bmatrix} \begin{bmatrix} A_i(t) \\ k_i(t) \\ x_i(t) \end{bmatrix} \tag{13}$$

where $\beta = 1/\left[2B + D(n-1) + c''_x\right]$. The 3×3 matrix in system (13) is the Jacobian associated to this dynamic system.

It is easy to find the steady state. Condition $\dot{A}(t) = 0$ implies $A = (k - x)/\delta$, and condition $\dot{k}(t) = 0$ implies $k = x/[z(\rho + \delta)]$. Moreover, $\dot{A}(t) = \dot{k}(t) = 0$ imply $\dot{x}(t) = 0$. Substituting these values into Equation (12) we obtain:

$$x \cdot \left[\frac{1 - z(\rho + \delta)}{\delta z(\rho + \delta)} - (2B + D(n-1))\right] = c'(x) + \frac{x}{(\rho + \delta)}. \tag{14}$$

Equation (14) gives the steady-state value of variable x. By simple substitutions it is immediate to find the steady state values of k and A.

As to the dynamic property of the steady state, it is immediate to realize that it is stable in the saddle sense, since the Jacobian matrix of system (13) presents only one (out of 3) eigen-value with a negative real part.

An intuitive explanation for the optimality condition (14) can be provided: it requires to equate the marginal cost of tourism flow (the right-hand side) with the marginal revenue from tourism (the left-hand side). However, a relevant problem is present in the steady-state solution of the problem at hand, as compared to a standard static problem: a larger tourism flow, x, requires a larger k in steady state (ceteris paribus); this, in turn, may imply a larger market size, A. The steady-state market size, hence, is positively related to the steady-state level of x. Consequently, the marginal revenue is not necessarily a decreasing function of x. Put differently, the first order condition (14) is not necessarily associated with a maximum point, but it could be associated with a minimum point, the maximum being a corner solution (if it exists). This issue is well-known in similar problems in environmental economics.[6] A complete study of the second order condition is required. Alternatively, we can compare the marginal revenue function with the marginal cost function. The intersection is a maximum – i.e., condition (14) identifies the tourist flow associated with the maximum profit – if and only if the slope of the marginal revenue function is algebraically smaller than the slope of the marginal cost function at the point of their intersection. The complete discussion of different cases concerning the critical point identified by (14) is provided by Appendix A. Here we focus on the case in which (14) identifies an interior maximum; such a case establishes when condition (15) holds (as it is proved in Appendix 1):

$$\frac{1 - z(\rho + 2\delta)}{\delta z(\rho + \delta)} - c''(x) < [2B + D(n-1)] < \frac{1 - z(\rho + \delta)}{\delta z(\rho + \delta)}. \tag{15}$$

Focusing on such a case, we develop some comparative statics considerations on the equilibrium steady state. To this end, consider first order condition (14) as an implicit function $g(\cdot) = 0$:

$$g(x, B, D, n, z, \rho, \delta) = x \cdot \left[\frac{1 - z(\rho + \delta)}{\delta z(\rho + \delta)} - (2B + D(n-1)) \right] - c'(x) - \frac{x}{(\rho + \delta)} = 0. \quad (16)$$

We can apply to (16) the implicit function theorem, to study how the steady-state value of x reacts to the parameters. Provided that $0 < \delta < 1$, we obtain:

$$\frac{\partial x}{\partial B} = -\frac{\partial g(\cdot)/\partial B}{\partial g(\cdot)/\partial x} < 0; \quad \frac{\partial x}{\partial D} = -\frac{\partial g(\cdot)/\partial D}{\partial g(\cdot)/\partial x} < 0;$$
$$\frac{\partial x}{\partial n} = -\frac{\partial g(\cdot)/\partial n}{\partial g(\cdot)/\partial x} < 0; \quad \frac{\partial x}{\partial z} = -\frac{\partial g(\cdot)/\partial z}{\partial g(\cdot)/\partial x} < 0;$$
$$\frac{\partial x}{\partial \rho} = -\frac{\partial g(\cdot)/\partial \rho}{\partial g(\cdot)/\partial x} < 0; \quad \frac{\partial x}{\partial \delta} = -\frac{\partial g(\cdot)/\partial \delta}{\partial g(\cdot)/\partial x} < 0.$$

In order to have an explicit solution, consider the particular case where the marginal cost is the increasing and convex function $c'(x) = cx^2,\ c > 0$, so that the cost function (apart from fixed costs) is $c(x) = cx^3/3$. In this case, the steady-state solution for the tourists' presence, x^* is:

$$x^* = \frac{1}{c} \cdot \left[\frac{1 - z(\rho + 2\delta)}{\delta z(\rho + \delta)} - (2B + D(n-1)) \right]. \quad (17)$$

It is immediate to verify that the comparative statics properties outlined above hold in the particular case of (17); moreover, in this case, $\partial x / \partial c < 0$.

The economic meaning of these properties is easily explained. (i) The larger the number of destinations competing with substitutable products, n, the smaller is the steady-state individual production of each destination in the symmetric Nash equilibrium. (ii) The larger is B (ceteris paribus), the smaller the marginal revenue, and hence the smaller the optimal amount of sold product. (iii) The larger is D – i.e., the more similar are the products – the smaller the optimal amount of sold product: price competition among destinations is harsher and the marginal revenues are lower, leading to a smaller optimal production. (iv) The higher is the investment cost in protecting the natural stock (connected with parameter z), the smaller the optimal scale of tourism.

In correspondence to x^* given by (17), it is immediate to find the steady state levels of investment aimed at (maintaining) the stock of resources, and the stock of resources itself, as captured by market variable A:

$$k^* = \frac{1}{z(\rho + \delta)} x^*; \quad A^* = \frac{1 - z(\rho + \delta)}{z\delta(\rho + \delta)} x^* \quad (18)$$

Hence, in this specific version of the model, both the investment and the stock of resources have to be proportional to the scale of tourism, in steady state.

As a last remark, we note that, in the case of the present section, a closed-loop Nash equilibrium – namely, the memoryless closed-loop equilibrium – collapses into the open-loop one: the latter is therefore strongly time-consistent. This is due to the fact that there is no feedback from the current value of state variables to the current value of control variables, so that the possibility of changing the choice during the time in which the game takes place is pointless.[7]

3.1. A DIGRESSION: THE EFFECT OF THE NUMBER OF PRODUCTS

It is easy to check that the maximum profit (in steady state) for the aggregate system of the n destinations *is not* a monotonic function of the number of products. Indeed, focusing on the steady-state profit (under the hypothesis of an interior solution), it is immediate to find that the aggregate profits, $\Pi = n\pi$ is a function of degree 4 in n. In fact, n affects (negatively) the individual optimal production, x, and consequently the production cost $c(x)$ as well as the investment efforts in enhancing the stock of resources, k, and the reservation price A: the effect on price is not clear-cut as long as steady-state levels of both A and x depend negatively on n. Hence, it is not surprising that the aggregate profits are not necessarily increasing in the number of products. Put differently, an increase of the number of products, even if it is costless, could lead to a smaller aggregate profit.[8] The economic reason is simple: a larger number of destinations means a larger number of products, but also higher (costly) efforts of investment aimed at enhancing the specific stocks of resource.

In the specific case of tourism markets, there is a large consensus on the fact that the recent diffusion of information and communication technology, and economic globalization, have led to an increase in the number of destinations: the effect on aggregate profits, ceteris paribus, is not clear-cut. As an example drawn from the real world, let us think of the current debate among Italian and Croatian Adriatic destinations concerning the convenience of offering a “new package” (i.e., a new destination) consisting of an integrated stay in Italy and Croatia. The introduction of such a new product within the Adriatic tourism, affects the equilibrium values of investment, the reservation prices of tourists, the optimal sizes of tourism flows, and prices and the profit of destinations; the dimension of such effects are rather complicate to compute, so that no simple recommendation is possible. Moreover, as a note of caution, remember that in this model the focus is only on the steady state of the symmetric equilibrium.

4. Investing in Product Differentiation

Now we sketch the optimal plan of destinations in the case in which they can invest in order to increase product differentiation. To this end, remember that, up to now, we considered D as a parameter, capturing the degree of differentiation between any pair of tourism products. Strictly speaking, D is a parameter connected with the consumer preferences, but we can guess that it reflects the fact that destinations are objectively differentiated, thanks to differences in natural resources, history, tradition, and so on.

To some extent, however, the differentiation may be modified, by appropriate investment efforts by part of destinations. In this respect, the degree of differentiation becomes a state variable moving over time, which is affected – at least in part – by appropriate investment. Note, however, that D is common to all destinations, since D denotes the *symmetric* degree of differentiation among products. In this respect, D is a public good. In other words, given the symmetric nature of product differentiation in this model, there exists a complete spillover effect in investment process.[9]

We assume that, at the initial instant $t=0$, it is $D(0)=D_0$, with $0<D_0 \leq B$ (if $D_0=B$ destinations offer the same homogeneous good). Product differentiation may increase, i.e., parameter D may decrease, through appropriate investments, h_i, aimed at product differentiation.

In order to capture the relationship between the investments aimed at product differentiation on the one side, and the degree of substitutability among goods, on the other side, we have to look for a function exhibiting the following properties:

(i) $\dot{D}(t)$ has to depend negatively on the sum of investments in product differentiation made by all destinations, $\sum_{l=1}^{n} h_l(t)$;
(ii) $D(t)$ has to remain constant if destinations do not invest in product differentiation, that is, $\dot{D}(t) = 0$ for $\sum_{l=1}^{n} h_l(t) = 0$;
(iii) if products are independent, it is pointless any further investment for product differentiation, that is, for $D=0$, it has to be $\dot{D}(t) = 0$;
(iv) with infinite investment effort in differentiation, the differentiation becomes instantaneously the largest, i.e., products become independent; this means that $\dot{D}(t) = -D(t)$ for $\sum_{l=1}^{n} h_i(t) \to +\infty$.

A function exhibiting all these properties can be the following one, borrowed from Cellini and Lambertini (2002):

$$\frac{dD(t)}{dt} \equiv \dot{D}(t) = -\frac{\sum_{i=1}^{n} h_i(t)}{1+\sum_{i=1}^{n} h_i(t)} \cdot D(t) \tag{19}$$

Equation (19) can be interpreted as a production function whose output is a decrease in $D(t)$, obtained through appropriate investments. It is immediate to check that this technology exhibits decreasing returns w.r.t. aggregate investment – an easily tenable assumption.

We assume that the cost of the investment effort in differentiation obeys the linear equation $w(h_i(t)) = w \cdot h_i(t)$, $w > 0$.[10]

The individual problem faced by destination i is:

$$\text{Max}\Pi_i = \int_0^{+\infty} \left\{ \left[A_i(t) - Bx_i(t) - D(t)\sum_{j\neq i} x_j(t) \right] x_i(t) - c(x_i(t)) - z\cdot[k_i(t)]^2/2 - wh_i(t) \right\} e^{-\rho t} dt \tag{20}$$

$$\text{s.t.: } \dot{A}_i(t) = -x_i(t) + k_i(t) - \delta A_i(t); \quad A_i(0) = A_0 > 0;$$

$$\dot{D}(t) = -D(t)\sum_{l=1}^{n} h_l(t) \Big/ \left[1 + \sum_{l=1}^{n} h_l(t) \right]; \quad 0 < D(0) = D_0 \leq B.$$

The control variables in this problem are $x_i(t)$, $k_i(t)$, $h_i(t)$, while $A_i(t)$ and $D(t)$, are the state variables. Let $\eta(t)$ be the current-value co-state variable associated with $D(t)$. The corresponding Hamiltonian function H_i is:

$$H_i = \left\{ \left[A_i(t) - Bx_i(t) - D(t)\sum_{j\neq i} x_j(t) \right] x_i(t) - c(x_i(t)) - z\cdot[k_i(t)]^2/2 - wh_i(t) + \lambda_i(t)\left[-x_i(t) + k_i(t) - \delta A_i(t) \right] + \eta(t)\left[-D(t)\sum_{l=1}^{n} h_l(t) \Big/ \left(1 + \sum_{l=1}^{n} h_l(t) \right) \right] \right\} \cdot e^{-\rho t}. \tag{21}$$

Remember that $D(t)$ is common to all players, and the effort for investment in differentiation made by j-site *directly* affects the objective function of the i-site through this state-variable. For this reason, the open-loop Nash equilibrium does not coincide with the closed-loop one, in this case: the control variable of player j directly affects the state variable pertaining to different players, that – in turn – affects the optimal choice regarding their control variables. We present here only the open-loop solution, that is, we assume that each destination chooses the plan of its actions at the initial time, and then sticks to it forever. This solution is only weakly time consistent, because each destination would find it optimal to change its plan, if the implementation of the plans by part of competing destinations were observed over time. However, the analytical closed-loop solution is, in this case, rather difficult to find, and it requires strict conditions on parameters in order to exist.[11] Moreover, it seems to be quite tenable – in the present case – that destinations set their plans at a given time and then stick to them, at least for a period of time, without reacting in any instant to the observed actions of the opponents.

The first order conditions and adjoint equations pertaining to $x_i(t)$, $k_i(t)$ and $A_i(t)$ are the same as (9), (10) and (11), apart from the fact that D has to be interpreted now as a variable moving over time rather than as a parameter; moreover two further conditions have to be considered:

$$\frac{\partial H_i(t)}{\partial h_i(t)} = 0 \Rightarrow -\frac{\eta(t)D(t)}{\left(1 + \sum_{i=1}^{n} h_i(t)\right)^2} = w, \tag{22}$$

$$-\frac{\partial H_i(t)}{\partial D(t)} = \dot{\eta}(t) - \rho\eta(t). \tag{23}$$

First order conditions and adjoint equations have to be considered along with the usual transversality conditions.

Equation (22) can be solved under the symmetry assumption $h_i(t) = h_j(t) = h$, $\forall i,j$, (along with similar symmetry assumptions concerning x and k) and a function linking $h(t)$ to $D(t)$ obtains:

$$h(t) = \frac{1}{n}\left[\sqrt{\frac{-\eta(t)D(t)}{w}} - 1\right]. \tag{24}$$

Differentiation w.r.t. time and appropriate substitutions lead to

$$\dot{h}(t) = \begin{cases} \dfrac{D(t)}{2nw(1+nh(t))}\cdot\left[(n-1)x^2 - \dfrac{(1+nh(t))^2\rho w}{D(t)}\right] & \text{for } D(t) > 0 \\ 0 & \text{for } D(t) = 0 \end{cases} \tag{25}$$

Hence, steady state $\dot{h}(t) = \dot{D}(t) = 0$ establishes for $D = 0$ or for $D = \rho w/[(n-1)x^2]$, with $h = 0$.

The case $D(t) = 0$ describes the situation where products have become completely independent, so that it is pointless to invest any further in product differentiation. The case where $D = \rho w/[(n-1)x^2]$ describes a steady state, in which a certain degree of substitutability among products is present. Note that this solution is incomplete, as long as x itself has to be interpreted as the steady-state value of x, which depends on the parameters, and on variable D itself, according to Equation (9). Moreover, appropriate parameter conditions have to be posed, in order to guarantee that the solution is economically meaningful. However, the positive direct effect of w

on D is rather obvious: the higher the investment cost, the higher is the optimal level of D, that is, the lower the optimal effort is for investment in differentiation.[12]

It is interesting to focus on the relationship between the number of destinations, n, and the optimal degree of product differentiation in steady sate. The following relationship among steady state variables must hold:

$$D - \frac{\rho w}{(n-1)[x(n,D)]^2} = 0 \tag{26}$$

with $x_n = \partial x / \partial n < 0$, $x_D = \partial x / \partial D < 0$. Applying the implicit function theorem to (26), one can obtain:

$$\frac{\partial D}{\partial n} = -\frac{x^2 + 2x \cdot (n-1)x_n}{\rho w / D^2 + 2x \cdot (n-1)x_D} \tag{27}$$

The sign of (27) is in general ambiguous, but for n sufficiently high, it is for sure negative. This means that, beyond a threshold level of n, specifically, for $n > \max\{1 + \rho w / (2D^2 x |x_D|), 1 + x/(2|x_n|)\}$, i.e., for a number of existing destinations sufficiently high, an higher number of destinations associates with a larger level of steady state product differentiation, that is, a lower level of D.

This point can be interpreted taking into account the current debate on the effects of the increased number of tourism destinations upon their differentiation. As already mentioned, it has been argued that the diffusion of digital information and communication and the drop in transport costs lead to a higher number of possible tourism destinations. This is true for any type of tourism and hence for the industry of tourism as a whole. At the same time, it has been suggested that these phenomena lead to lower product differentiation and, more generally to a limitation of cultural diversities.[13] The present model shows that this can not be the case: the higher the number of possible destinations – at least when the number of destinations is sufficiently high – the higher the degree of product differentiation at the steady state equilibrium. The *economic* reason is simple: a higher number of competitors means harsher market competition; as a consequence, investment in product differentiation become more and more convenient, resulting in an increased product differentiation. The valorization of the idiosyncratic elements of the respective stock of natural or cultural resources by part of destinations can be example of investment aimed at product differentiation in the real world.

A last point is worth mentioning. Each destination compares the costly efforts of investing in differentiation with the benefits from product differentiation, and – under the Nash equilibrium solution concept – it chooses the optimal amount of effort, given the efforts of its opponents. However, because of the complete externality from individual efforts to the degree of differentiation, the individual effort in product-differentiation is generally under-sized as compared to a cooperative solution. This result is common to the models on R&D investment (when investment have positive spillovers for the rivals) or to the models on advertising (when advertisement of a firm benefits all the competing firms as well). The effort in differentiation carried out by a single destination is lower than the level which would be optimal for the set of destinations as a whole. As an immediate corollary of this point, we can state that some forms of inter-destination coordination are necessary to overcome the market inefficiency implied by the public-good nature of product differentiation. This point is in our future research agenda.

5. Conclusions

In this paper we have proposed a differential game model of oligopoly with differentiated goods to study the development strategies of competing tourism destinations. As a matter of fact, tourism goods are differentiated in the real world; dynamic plans are necessary to enhance the stock of physical, natural and cultural resources over time; more importantly, competition among different destinations takes place over time, and strategic interdependency among them exits.

Our model permits to deal with some specific points: the determination of the time path of optimal investment in enhancing the stock of resources, and its relationship with the scale of tourism flows; the relationship between the scale of tourism and investment effort, on the one side, and product differentiation and the number of available products on the other side; the optimal efforts of investment in product differentiation.

In general, a larger scale of tourism has to be associated with larger investment for protecting resources and/or for increasing product differentiation. Of course, different costs and benefits associated to different types of investment lead to different optimal plans, and hence to different optimal choice concerning the scale of tourism.

We have put particular stress on the effect of the number of destinations upon the individual efforts in product differentiation and hence in the resulting level of product substitutability and profits in steady state. Specifically, we have shown that a non-monotonic relationship holds between the number of destinations, on the one side, and the steady state optimal degree of endogenous product differentiation and aggregate profits on the other side. However, if the number is sufficiently high, an even higher number of destination associates with a larger level of product differentiation in steady state. Nevertheless, the individually optimal degree of differentiation is lower than the socially optimal level, given that product differentiation has a public good nature.

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Notes

1. Among recent studies on oligopoly competition in tourism markets see Davies and Downward (1998), Mazzeo (2002); Piga (2003) provides a dynamic model.
2. This simply derives from the usual properties according to which the "quality" of the tourism product is proportional to the ratio of tourists over the stock of resources, and the price is directly linked with the quality, as suggested, e.g., by Lanza and Pigliaru (1994).
3. A similar assumption is made by Piga (2003).
4. In principle, we should consider the dynamic equation and the initial condition concerning each state variable $A_j, j = 1, 2, \ldots, n$. However, this is not necessary here since the problem at hand presents separate dynamics, that is, the dynamic equation of each state variable is directly influenced only by the choice variable pertaining to the player to whom the considered state variable is associated. Put differently, we do not insert the dynamics of state variables $A_j, j \neq i$, in the problem of player i, since they are is immaterial to its solution.
5. Another strongly time-consistent (and therefore subgame perfect) solution concept is the feedback one, based on the Bellman equation. For a clear exposition of the difference among these equilibrium solutions see Basar and Olsder (1982, pp. 318–327, and ch. 6).
6. See, for instance, the antipollution policy problem by Forster (1980), as it is presented in Chiang (1992).
7. The reason why the memoryless closed-loop solution coincides with the open-loop one, rests on the fact that, in this case, the adjoint equation corresponding to the two different solution concepts coincide. This is due to the property that the game has separate dynamics and the state variable pertaining to any given player does not affect the optimal choice of different players. There exist several classes of games where the closed-loop and the open-loop solutions coincide: see Mehlmann (1988, ch. 4), Reinganum (1982), Fershtman (1987), Fershtman et al. (1992), Dockner et al. (2000, ch. 7).
8. Just to give a numerical example, if we set $z = 0.1$, $B = 1$, $D = 0.5$, $\rho = \delta = 0.02$, $c = 1$, the steady state optimal production is $x_i = (24847 - n)/2$ and the aggregate profits are $n\pi_i = n(n - 24,847)^2(15,550 - n)/12$ which are positive and increasing in n over the interval $1 < n$ and decreasing over $5083 < n$.
9. Just to give a trivial example, when Las Vegas invests in order to offer a more and more differentiated product, any other site over the world becomes more and more differentiated with respect to Las Vegas! Note that the externality effect under consideration entails that the outcome of investment activity is public domain via the demand function. On the contrary, the externality effects usually considered in the literature are of a technological nature, and are associated with information leakage or transmission.
10. Of course, we are aware that it is questionable that investment k_i affecting resources, on the one side, and investment h_i affecting differentiation among destinations, on the other side, are different variables: in the real world, it is likely that the investments affecting the natural resources stock also affect the perceived product differentiation.
11. See Cellini and Lambertini (2004) for the closed-loop solution of a similar (but simpler) problem, and the comparison with the open-loop solution. See also Dockner et al. (2000)

for a discussion about pros and cons of open-loop vs. closed-loop solutions of differential games.

12. In order to find a solution for the steady-state configuration it is not sufficient to postulate a quadratic marginal cost function $c(x)$: in this case, a cubic equation has to be solved, and we need further numerical constraint to find the solution analytically. Also in this case, however, the steady state is stable in the saddle sense, since the (5×5) Jacobian matrix associated to the dynamic system presents 2 eigen-values out of 5 with a negative real part.
13. See, e.g., Throsby (2001), p. 70 and pp. 155–156; see also various contributions in Cooper and Wahab (2001).

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Appendix A

First order condition (14) in Text establishes the equality between the marginal revenue and the marginal cost (revenue and cost include investment in enhancing the stock of resources, along with the effect of tourists' presence). Different cases must be considered.

$$\text{If } [2\mathrm{B} + \mathrm{D}(\mathrm{n}-1)] \geq \frac{1-\mathrm{z}(\rho+\delta)}{\delta \mathrm{z}(\rho+\delta)}, \tag{A}$$

the marginal revenue is decreasing *and non-positive* for any positive value of x. Consequently, the corner solution $x=0$ is the optimum, i.e., the allocation associated to the maximum profit.

$$\text{If } [2B + D(n-1)] < \frac{1-z(\rho+\delta)}{\delta z(\rho+\delta)}, \tag{B}$$

the marginal revenue is positive (so that both the marginal revenue and the marginal cost are positively sloped), and the intersection between the marginal revenue curve and the marginal cost curve represents the maxim profit point if and only if the marginal cost curve intersect the marginal revenue curve from below. Hence, we distinguish three sub-cases:

$$\text{If } \frac{1-z(\rho+\delta)}{\delta z(\rho+\delta)} - [2B + D(n-1)] > c''(x) + \frac{1}{(\rho+\delta)}, \tag{B.1}$$

the optimum is $x \to +\infty$ (This is due to the fact that the marginal revenue increases at a speeder pace than the marginal cost, as x increases. Unless some capacity constraint on the tourism flows is operative, like $x \leqslant x^{\wedge}$, there is no finite solution for x);

$$\text{If } \frac{1-z(\rho+\delta)}{\delta z(\rho+\delta)} - [2B + D(n-1)] = c''(x) + \frac{1}{(\rho+\delta)} \tag{B.2}$$

the optimum is indeterminate (marginal cost and marginal revenue coincide);

$$\frac{1-z(\rho+\delta)}{\delta z(\rho+\delta)} - [2B + D(n-1)] < c''(x) + \frac{1}{(\rho+\delta)} \tag{B.3}$$

the internal critical point denotes the maximum profit.

In sum, the dynamic problem can lead to a steady state with a positive and finite value for x, only under condition $[2B + D(n-1)] < [1 - z(\rho+\delta)]/[\delta z(\rho+\delta)]$ in case (B.3). The system of these two disequations condition coincide with the condition given by (15) in Text.