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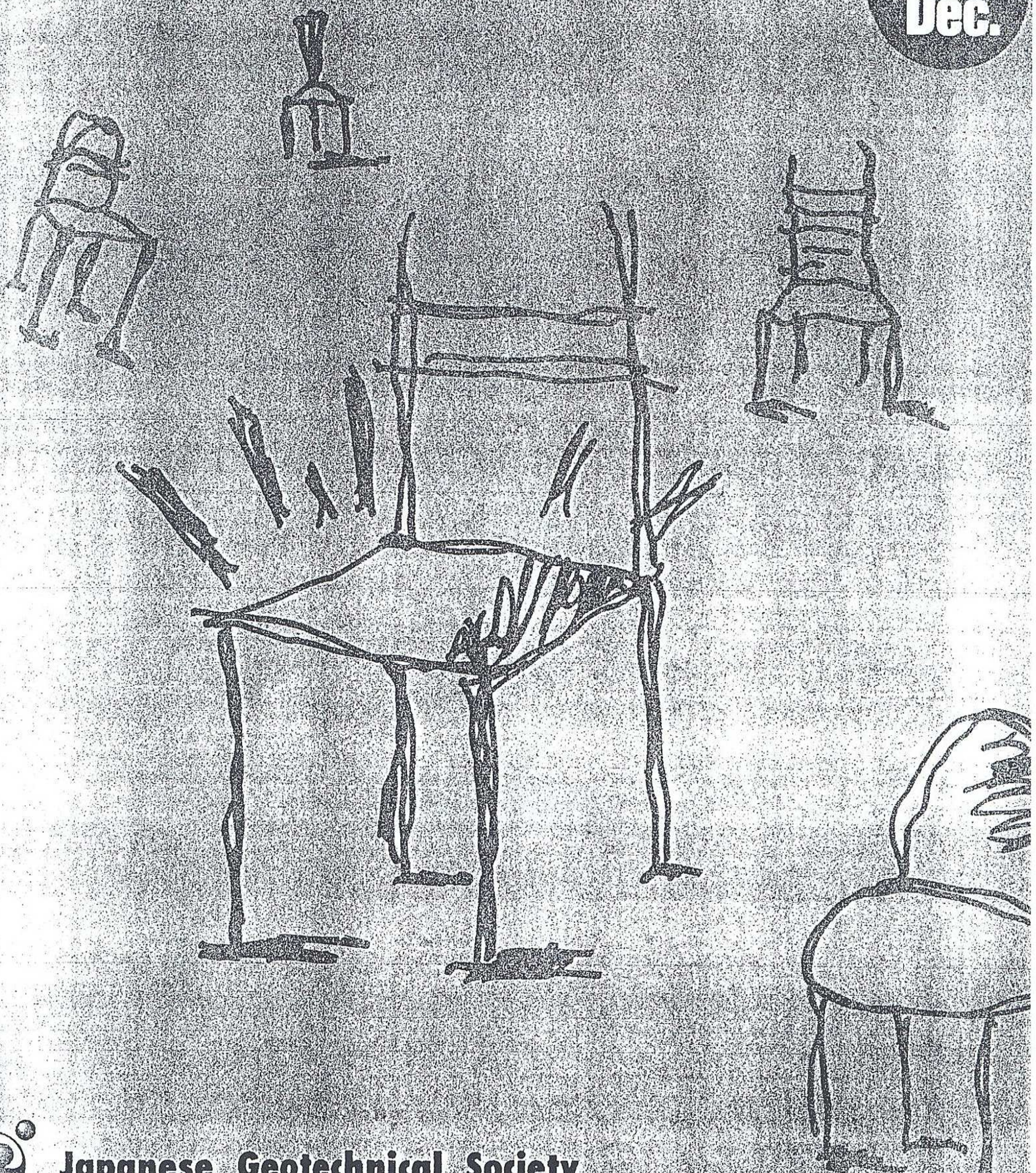

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EARTH PRESSURE ON REINFORCED EARTH WALLS UNDER GENERAL LOADING

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ABSTRACT

The limit equilibrium analysis applied to a plane failure surface is utilized for the evaluation of the active earth pressure on reinforced-earth walls under different loading conditions, such as seismic loading, pore water pressures into the fill, vertical and horizontal loads acting on the top at some distance. Due to the complexity of boundary and loading conditions considered, a limit equilibrium method with a plane failure surface has been used because this allows to deduce closed form solutions that could be quite helpful in design reinforced-earth vertical walls. The analysis presents two different loading conditions on a reinforced earthfill and two solutions in terms of earth pressure coefficient are given. Earth pressure coefficients are a function of non-dimensional parameters such the ratio λ between the distance d of the applied surcharge and the height of the wall H, or the ratio m between the horizontal and the vertical surcharge q_h and q_v respectively. Pore water pressure effects on earth pressure have been taken into account by means of the pore pressure ratio $r_u = u/\gamma H$. Seismic loading have been taken into account in a pseudo-static way, in terms of horizontal and vertical seismic coefficients. In a conventional design procedure, the analysis allows to define spacing or number of reinforcement as well as their length according to the failure wedge predicted.

Key words: active earth pressure, design, earthfill, horizontal load, inclined load (IGC: E5/H2)

INTRODUCTION

A design procedure for reinforced-earth structures involves mainly two aspects: 1) evaluation of the required reinforcement force to ensure equilibrium for an assigned loading condition; 2) definition of appropriate spacing and bond lengths of reinforcement based on reinforcement tensile strength and the failure wedge respectively. This paper is focused mainly on the first point and is devoted to the evaluation of the earth pressure at failure for the case of vertical reinforced-earth walls under different loading conditions, such as seismic loading, vertical and horizontal surcharge acting on the top at some distance and pore water pressures in the fill.

Some of these loading conditions have been treated in the literature, however the combined effects on earth pressure have not been investigated. The effect of a surcharge, located at a certain distance from the wall, for example, is usually evaluated assuming elastic behavior of the soil (Laba et al., 1984; Kennedy et al., 1980). Elastic solutions, that are quite useful in many cases, however do not take into account the shear strength of the soil or its bulk unit weight; the principle of superposition is applied, distinguishing the effect of the soil weight from that of the surcharge and this appears to be an unrealistic assumption because the failure wedge is influenced by both unit weight and surcharge. When a rigid plastic behavior of the soil is utilized, the earth pressure coefficient due to the surcharge may be quite different from that due to the weight of the soil. It has been shown that, if the surcharge is applied at a certain distance from the wall, the earth pressure coefficient due to the surcharge is somewhat less than that due to the unit weight of the soil (Motta, 1994). Several solutions for the earth pressure coefficient are presented in the literature, as design charts, for vertical walls or steep slopes, taking into account simple loading conditions as the effect of self weight and pore pressure only. Solutions are based on the two wedge mechanism of failure or the logarithmic spiral mechanism (Jewell et al., 1984; Jewell, 1990). Recently, a comparison between the continuum and the structural approach has been given by Michalowski and Zhao (1995) utilizing both the homogenization method and the limit analysis method. Closed form solutions which take into account more specific loading conditions however are not available.

In the cases that have been studied, due to the complexity of boundary and loading conditions considered, a

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limit equilibrium method with a plane failure surface is more convenient because this allows for the deduction of closed form solutions that could be quite helpful in the design of reinforced-earth vertical walls. The assumption of a plane failure surface is reasonable, if the wall is vertical, because the solutions deduced by assuming a plane failure surface are practically coincident with those obtained using more sophisticated failure curves, such as the logarithmic spiral arc. If the slope of the wall is less than 90° however the failure plane approach may result in an unconservative design.

ANALYSIS

In the following presentation it will be assumed that the soil is cohesionless, dry or saturated, and that the limit active state has been reached in the soil. It will be also assumed that the reinforcement is able to resist only the axial stress.

With these assumptions, the total required force R_a of the reinforcement can be written in the following conventional form:

$$R_a=\frac{1}{2}\gamma H^2 K_a \quad (1)$$

where:

$R_a=\Sigma T_i$=total required force of reinforcement to ensure equilibrium

T_i=force of the reinforcement

γ=bulk unit weight of the embankment

K_a=earth pressure coefficient

H=height of the wall

For the design of reinforcement, therefore the coefficient K_a should be evaluated based on the boundary and loading conditions. As stated above, two boundary and loading conditions were analysed:

Case a)
Reinforced-Earth Wall Subjected to Seismic Loading, Distanced Surcharge and Pore Water Pressure

It is considered a reinforced-earth vertical wall, as shown in Fig. 1, on which a uniform vertical surcharge of intensity q is applied at a certain distance d from the wall and indefinitely extended behind the wall, so that the failure wedge intersects it at the top level. More generally, it was assumed that seismic horizontal and vertical inertia forces are applied into the sliding mass to simulate an earthquake loading. The surcharge is assumed with mass so that the inertia forces are also applied on it. According to Bishop and Morgenstern (1960), pore pressure will be taken into consideration using the pore pressure ratio r_u defined as the ratio between the pore pressure u and the overburden pressure, that is:

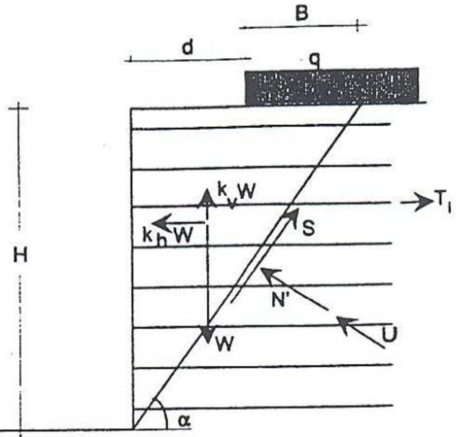


Fig. 1. Scheme for the evaluation of the earth pressure coefficient for case a) loading condition

$$r_u=\frac{u}{\gamma h} \quad (2)$$

Referring to Fig. 1, the total force R_a, required for a unit width, of the soil reinforcement can be evaluated considering the two equations in the horizontal and vertical direction, respectively:

$$R_a+S\cos\alpha-N'\sin\alpha-U\sin\alpha-k_h W-k_h qB=0 \quad (3')$$

$$W-k_v W+qB-qk_v B-S\sin\alpha-N'\cos\alpha-U\cos\alpha=0 \quad (3'')$$

where:

$$S=N'\tan\phi' \quad (4')$$

is the soil shear strength along the failure surface and:

$$B=H/\tan\alpha-d \quad (4'')$$

$$U=r_u W/\cos\alpha \quad (4''')$$

it follows:

$$R_a=\frac{1}{2}\gamma H^2(1-k_v-r_u)\tan(\alpha-\phi')/\tan\alpha+qH(1-k_v)\times\tan(\alpha-\phi')(1/\tan\alpha-\lambda)+\frac{1}{2}k_h\gamma H^2/\tan\alpha+qk_h H(1/\tan\alpha-\lambda)+\frac{1}{2}\gamma H^2 r_u \quad (5)$$

where λ is the ratio between the distance at which the surcharge is applied and the wall height; that is:

$$\lambda=d/H \quad (6)$$

Assuming:

$$\theta=\tan^{-1}[k_h/(1-k_v)] \quad (7)$$

$$\theta_1=\tan^{-1}[k_h/(1-k_v-r_u)] \quad (8)$$

the critical angle α_c that gives the maximum earth pressure is found by derivating (5) and it, after some manipulations, is given by the following expression:

$$\tan\alpha_c=\frac{1+A-\cos(2\phi-\theta_1)/\cos\theta_1-A\cos(2\phi-\theta)/\cos\theta+\Delta^{1/2}}{2[A\lambda+\cos(\phi-\theta_1)\sin\phi/\cos\theta_1+A\cos(\phi-\theta)\sin\phi/\cos\theta]} \quad (9)$$

where:

$$\Delta = 2(1+A)^2 - 2A(1+A)[\cos(2\phi-\theta_1)/\cos\theta_1 + A\cos(2\phi-\theta)/\cos\theta] + 4A\lambda[\sin(\phi-\theta_1)\cos\phi/\cos\theta_1 + A\sin(\phi-\theta)\cos\phi/\cos\theta] \quad (10)$$

and

$$A = n_q(1-k_v)/(1-k_v-r_u) \quad (11)$$

$$n_q = 2q/\gamma H \quad (12)$$

Utilizing (9), the earth pressure coefficient K_a in (1) is given by:

$$K_a = (1-k_v-r_u) \times \frac{\cos\theta[\cos(\phi-\theta_1) - \sin(\phi-\theta_1)/\tan\alpha_c] + A\cos\theta_1[(1-\lambda\tan\alpha_c)\cos(\phi-\theta) - (1/\tan\alpha_c - \lambda)\sin(\phi-\theta)]}{\cos\theta\cos\theta_1(\cos\phi + \sin\phi\tan\alpha_c)} + r_u \quad (13)$$

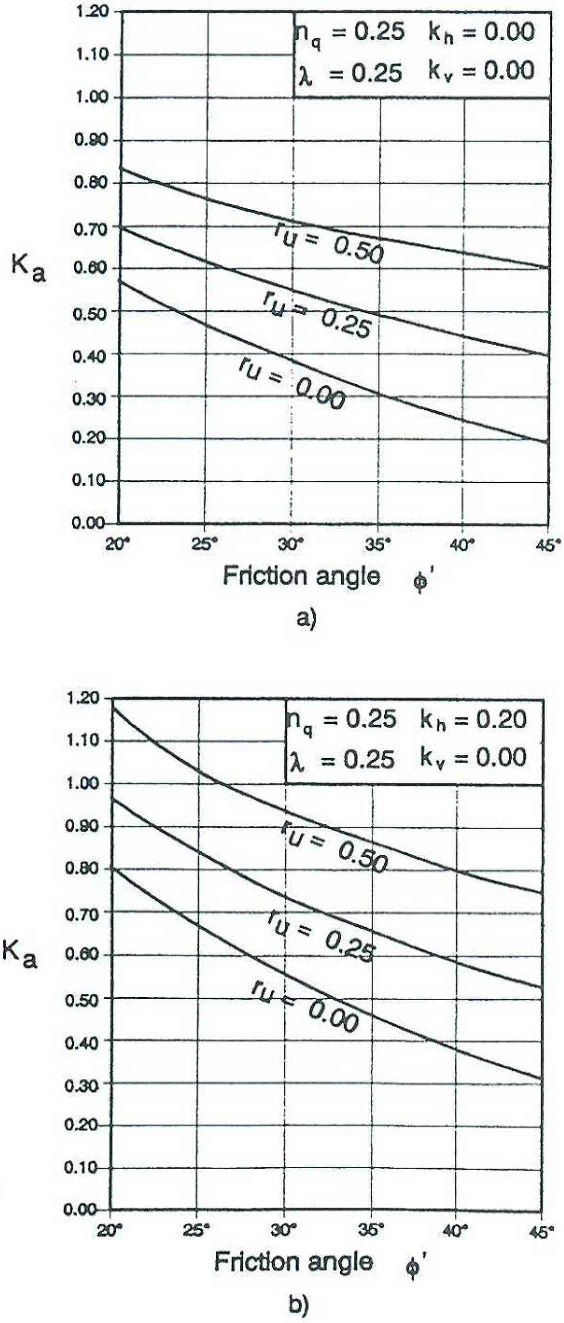


Fig. 2. Earth pressure coefficient versus friction angle for case a) loading condition: a) no seismic loading, b) seismic coefficient $k_h = 0.20$

The coefficient K_a, so defined, takes into account horizontal and vertical seismic coefficient, pore water pressure and distanced surcharge together. The solution is similar to that proposed by Motta (1994); here, in addition, the influence of pore water pressure has been taken into account. It is possible to show that when $n_q = 0$ ($q = 0$), and $r_u = 0$, (13) reduces to the Mononobe-Okabe equation for a smooth vertical wall.

As an example, in Fig. 2 values of K_a are shown for pore pressure ratio $r_u = 0$, 0.25 and 0.50 and for $\lambda = 0.25$ and $n_q = 0.25$. Figure 2(a) concerns K_a values for no seismic loading, while Fig. 2(b) concerns a seismic horizontal coefficient $k_h = 0.20$.

Case b)
Reinforced-Earth Wall Subjected to Distanced Vertical and Horizontal Surcharge

In Fig. 3 a vertical earth reinforced wall is subjected to vertical and horizontal surcharge applied at a certain distance from the wall. The soil is assumed to be cohesionless and dry. No seismic loading was considered.

As in the case a), for both horizontal and vertical equilibrium, the following equation for the magnitude of

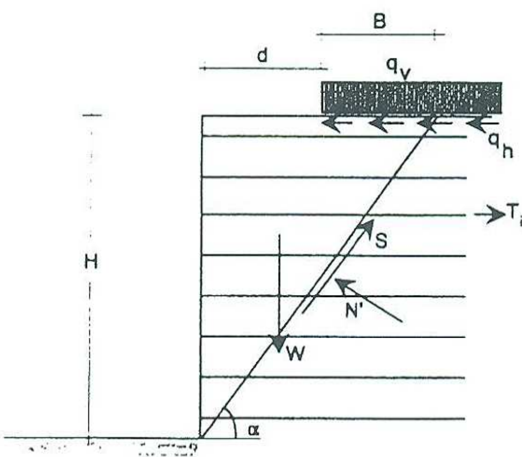


Fig. 3. Scheme for the evaluation of the earth pressure coefficient for case b) loading condition

the required reinforcement force for unit width can be determined:

$$R_a=\frac{1}{2}\gamma H^2[(1+n_q)\tan(\alpha-\phi)/\tan\alpha + mn_q/\tan\alpha-\lambda n_q(\tan(\alpha-\phi)+m)] \quad (14)$$

where:

$$m=\frac{q_h}{q_v} \quad (15)$$

$$n_q=\frac{2q_v}{\gamma H} \quad (16)$$

and λ is given by (6). Derivating (14) with respect to the angle α of the failure plane, the following expression for the critical angle α_c is given:

$$\tan\alpha_c=\frac{\sin^2\phi-C\sin\phi\cos\phi+[\sin^2\phi-(C-D)\sin\phi\cos\phi-CD\cos^2\phi]^{1/2}}{\sin\phi\cos\phi+C\sin^2\phi+D} \quad (17)$$

where:

$$C=\frac{mn_q}{(1+n_q)} \quad (18)$$

$$D=\frac{\lambda n_q}{(1+n_q)} \quad (19)$$

Utilizing (17), the earth pressure coefficient is given by:

$$K_a=\frac{(1+n_q)(\tan\alpha_c-\tan\phi)+n_q[m(1+\tan\alpha_c\tan\phi)-\lambda\tan\alpha_c(\tan\alpha_c-\tan\phi)]}{\tan\alpha_c(1+\tan\alpha_c\tan\phi)}-\lambda mn_q \quad (20)$$

The influence of the horizontal surcharge on the earth pressure coefficient is shown in Fig. 4 for a value of $n_q=0.50$ and for $\lambda=0.50$. The ratio $m=q_h/q_v$ has been established varying in the range 0-0.4. The analysis shows that the effect of the horizontal surcharge reduces with increasing the angle of internal friction of the soil.

ALLOWABLE BOUNDARY CONDITIONS AND STRESS DISTRIBUTION

In the present analysis it has been assumed that the failure wedge intersects the surcharge applied on the top for a certain width B given by (4″), as shown in Fig. 1 or in Fig. 3. This should be verified when the critical angle α_c is found by means of (9) or (17). For some loading conditions, indeed, it may occur that the surcharge does not affect the failure wedge. This is likely when the load is far from the wall. Because B should be greater than zero, the following expression must be satisfied:

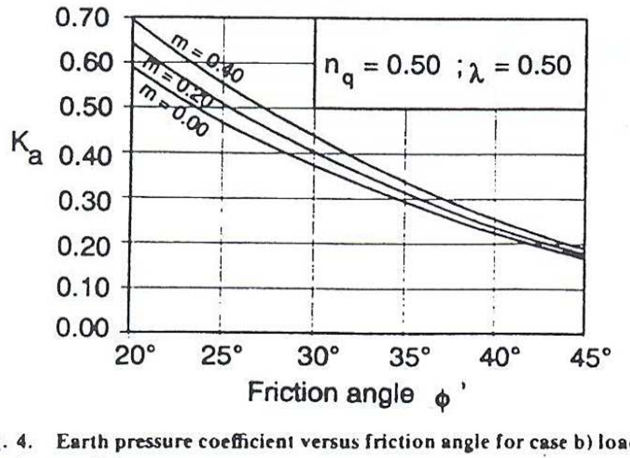


Fig. 4. Earth pressure coefficient versus friction angle for case b) loading condition

$$\tan\alpha_c<1/\lambda \quad (21)$$

Otherwise the failure wedge does not intersect the surcharge and the earth pressure coefficient will be determined only by the weight of the soil. As an example, in Fig. 5 the horizontal stress distribution that should be supported by reinforcement is shown for a soil with a friction angle $\phi'=30°$ and for the following parameters: $n_q=0.50$; $\lambda=0.25$ and 0.50; $m=q_h/q_v=0$, 0.20. The values of the horizontal stress σ_h have been scaled with respect to the value of the overburden vertical pressure $\sigma_v=\gamma H$ at the bottom of the wall. The horizontal stress distribution has been obtained by using a numerical derivation of (14) with respect to the depth z below the top of the wall and utilizing the same procedure suggested by Stenfelt and Hansen (1983), i.e.:

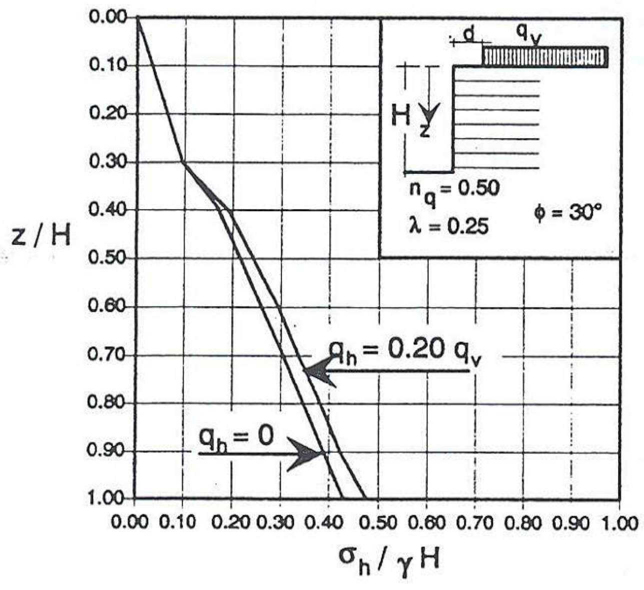
$$\sigma_h(z)=dR_a(z)/dz \quad (22)$$

Since the values of n_q and λ vary with the depth z, it follows that the angle α_c of the critical failure wedge will also vary with depth. According to the analysis, if the surcharge is applied at a certain distance, as in this example, the contribution to the required axial force of reinforcement in the upper part of the fill, will be given only by the soil weight, so that the horizontal stress distribution will be linear up to a certain depth below the ground surface; afterwhich, for greater depths, the surcharge will also contribute. In this particular case, due to the large extension of the applied load, the contribution of the surcharge to stress distribution is roughly constant with the depth in the lower part of the fill.

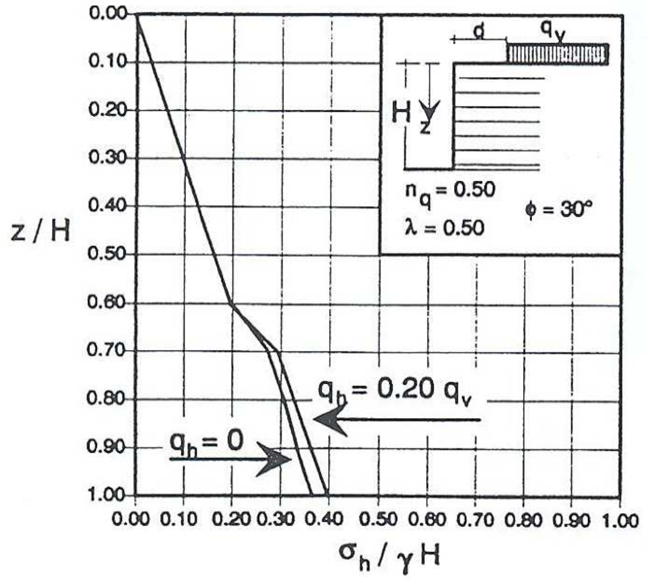
The traction force F_t acting on a reinforcing element at the depth z can be evaluated as:

$$F_t(z)=\sigma_h(z)\Delta h\Delta s$$

where Δh and Δs are the vertical and horizontal spacing between reinforcing elements. The analysis shows that the contribution to the total required force due to the vertical and horizontal surcharge decreases with increasing distance of the load from the face of the fill; the greater the distance of the loading from the face, the lower will be the zone of the fill in which the surcharge will affect the reinforcement strips. In the case $\lambda=0.25$ of Fig. 5(a), for example, the surcharge affects the earth pressure below the depth $z/H=0.30$, while for the case $\lambda=0.50$ of Fig. 5(b) the effect of the surcharge begins at about the depth $z/H=0.60$. It should be kept in mind however that the solutions presented assume that the surcharge is indefinitely extended behind the wall; obviously, if the width of the loading is small compared to the wall height, the stress distribution could be quite different.



a)



b)

Fig. 5. Horizontal stress versus depth for case b) loading condition; $n_q=0.50$

CONCLUSIONS

Utilizing the limit equilibrium method applied to a plane failure surface, two closed form solutions for a reinforced earth wall have been derived that permit the evaluation of the force reinforcement for unit width for the case of a dry or saturated fill with a distanced surcharge and subjected to seismic loading, or for the case of a dry soil with vertical and horizontal surcharge applied at some distance from the face.

For a lone earth pressure coefficient that is a function of the boundary and loading conditions defined in the analysis, the evaluation of the minimum required force of the reinforcement to ensure equilibrium has been made very simple. In a conventional design procedure, the analysis allows to define spacing or number of reinforcements as well as their length according to the failure wedge predicted.

Some indications on the stress distribution with depth has also been given in order to assess the zone in which a stronger concentration of stress may be expected in the reinforcement.

It is pointed out however that solutions apply only if the boundary conditions assumed in the analysis are verified so that this control must be made for a correct evaluation of earth pressure.

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