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Micromagnetic analysis of statistical switching in perpendicular STT-MRAM with interfacial Dzyaloshinskii–Moriya Interaction

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The technological implementation of STT-MRAMs in real devices needs a complete description of the statistical switching behavior at room temperature. In this work, we investigate, by means of a full micromagnetic model, the effect of the interfacial Dzyaloshinskii–Moriya Interaction (IDMI) on the critical current density and probability density function (PDF) of the switching time in perpendicular STT-MRAMs. We show that, for large enough values of the symmetric exchange interaction, the negative effect of the IDMI on the critical current is strongly reduced. In addition, a comprehensive analysis of the four main statistical moments (mean, standard deviation, skewness and kurtosis) points out that to fit the switching time PDF in order to maintain a quadratic error at least one order of magnitude smaller than Skew Normal PDF, a Pearson Type IV PDF has to be used also in presence of the IDMI.

***Index Terms*—Spin-Transfer Torque, Dzyaloshinskii–Moriya Interaction, STT-MRAM, Write Error Rate, skewness, kurtosis**

I. Introduction

THE spin-transfer torque (STT)-driven magnetization switching is a fundamental ingredient for the design of an emerging storage technology, i.e. the STT-MRAM [1]–[4]. Since the introduction of materials with perpendicular anisotropy for the fabrication of magnetic tunnel junctions (MTJs) [5], [6], this technology have shown great improvements in terms of reduction of critical current density (below 10^6 A/cm^2) and power consumption by maintaining at the same time high thermal stability. These features, together with scalability and integration with complementary metal-oxide semiconductor (CMOS) process and technology, make STT-MRAMs a promising candidate for the next generation of memories [7]–[10].

Recently, Liu *et al.* [11] have proposed to use a hybrid free layer coupled with a seed layer to maintain a large TMR while scaling the MTJ cell size down to 20nm. In their experiment, the best TMR was achieved in presence of a Pt seed layer. However, the adoption of Pt layer can introduce an additional degree of freedom in the energy landscape of the free layer, i.e. the interfacial Dzyaloshinskii-Moriya Interaction (IDMI) [12], [13], due to a large spin-orbit coupling between the free layer ferromagnet and the Pt layers. Lately, micromagnetic simulations [14], [15] have shown that the IDMI affects the MTJ device performance in two ways, by decreasing thermal stability and increasing switching times for a fixed current.

In this work, the stochastic switching in a perpendicular MTJ was investigated by means of a full micromagnetic model. We studied the effect of the interfacial IDMI on the critical current density, showing that, if the free layer is properly designed in order to have a large enough exchange constant, the negative effect due the IDMI is reduced. An analytical PDF involving four statistical moments (mean, standard deviation (STD), skewness and kurtosis) is necessary to describe appropriately the PDF as computed from micromagnetic results, differently from what found in Ref. [16]. To this aim, we considered the Pearson Type IV PDF which takes into account all the four main statistical moments [17]–[19]. Eventually, the Write Error Rate (WER) [7], [16], [20]–[22], one of the most important parameter for the design of an efficient memory cell in terms of energy dissipation and switching time, is computed and is lowered in the absence of IDMI.

The paper is organized as follows: Section II describes the full micromagnetic model and device. Section III deals with the key passages for the development of the Pearson Type IV PDF. The discussion of the results is presented in Sections IV,

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whereas conclusions are summarized in Section V.

II. Full Micromagnetic Model and Device

The micromagnetic computations are performed by means of a state-of-the-art micromagnetic solver which numerically integrates the Landau-Lifshitz-Gilbert-Slonczewski (LLGS) equation by applying the time solver described in [23], [24]. The effective magnetic field includes the exchange, magnetostatic, anisotropy and external fields, as well as the interfacial DMI and the thermal field [25]. The IDMI contribution $\mathbf{h}_{\mathrm{IDMI}}$ is obtained from the functional derivative of the DMI energy density $\varepsilon_{\mathrm{IDMI}} = D\left[m_z\nabla\cdot\mathbf{m} - (\mathbf{m}\cdot\nabla)m_z\right]$ under the hypothesis of thin film $(\partial\mathbf{m}/\partial z = 0)$ [23], [26]:

$$\mathbf{h}_{\mathrm{IDMI}} = -\frac{1}{\mu_0 M_s}\frac{\delta\varepsilon_{\mathrm{IDMI}}}{\partial\mathbf{m}} = -\frac{2D}{\mu_0 M_s}\left[(\nabla\cdot\mathbf{m})\hat{z} - \nabla m_z\right] \quad (1)$$

where μ_0 is the vacuum permeability, M_s is the saturation magnetization, $\mathbf{m}$ is the normalized magnetization of the free layer, D is the parameter taking into account the intensity of the IDMI, m_z is the out-of-plane component of the normalized magnetization, and $\hat{z}$ the unit vector along the out-of-plane direction. The IDMI affects the boundary conditions of the ferromagnetic sample following the expression $\frac{d\mathbf{m}}{dn} = \frac{D}{2A}\left[(\hat{z}\times\mathbf{n})\times\mathbf{m}\right]$ [23], [26], where $\mathbf{n}$ is the unit vector normal to the edge, and A is the exchange constant.

The thermal effects are considered as a stochastic term $\mathbf{h}_{\mathrm{th}}$ added to the deterministic effective magnetic field in each computational cell assumed to be uncorrelated for each computational cell [27], [28]. The discretization cell used is 1.0x1.0x1.0 nm^3.

The device under investigation is a state-of-the-art MTJ with a CoFeB (1 nm) free layer and a circular cross section with diameter of 30 nm, coupled to a Platinum heavy metal in order to achieve a large enough IDMI. The physical parameters used in this study are: M_S=1000 kA/m [5], perpendicular anisotropy constant k_u=0.80 MJ/m^3, Gilbert damping α =0.03 [5], and spin-polarization factor η =0.66.

III. Pearson type IV PDF

The switching process is nonlinear, thus the PDF of the switching time t_s is expected to be non-Gaussian. While in Ref. [16], the skew normal distribution [29] was used to describe the asymmetry of the PDF of t_s, here we show that the kurtosis is also necessary to close fit the micromagnetic PDFs of t_s. Among the Pearson distribution function types (I–XII), we find that the Pearson Type IV function [17]–[19] is a suitable option. By considering the variable of interest (in our case $x = t_s$) and the first four statistical moments, mean μ, STD σ, skewness γ and kurtosis β_2, the PDF $f(x)$ can be written as:

$$f\left(x\big|_{\mu,\sigma,\gamma,\beta_2}\right) = \frac{k}{\sigma}\mathrm{e}^{\left(-\upsilon\tan^{-1}\left(\frac{\hat{x}-\lambda}{a}\right)\right)}\left[1+\left(\frac{\hat{x}-\lambda}{a}\right)^2\right]^{-m} \quad (2)$$

where $m > 1/2$, $\hat{x} = \frac{x-\mu}{\sigma}$, $\lambda = \frac{a\upsilon}{b}, \upsilon = \frac{2c_1(1-m)}{\sqrt{4c_0c_2 - c_1^2}}$,

whereas $a = \sqrt{\frac{b^2+(b-1)}{b^2+\upsilon^2}}$, b = 2m-1, m = 1/2c_2, β_1=γ^2 and $c_0 = (4\beta_2 - 3\beta_1)/D$, $c_1 = (\gamma\cdot(\beta_2+3))/D$, $c_2 = (\gamma/D)\cdot(2\beta_2 - 3\beta_1 - 6)$, $D = (10\beta_2 - 12\beta_1 - 18)$.

In Eq. (2) m, v, a, and λ are real-valued parameters, and $-\infty < x < \infty$. k is a normalization constant that depends on m, v, a, and can be expressed by:

$$k = \left|\frac{\Gamma(m + (\nu/2)\cdot 1i)}{\Gamma(m)}\right|^2 \Big/ a\beta\left(m-\frac{1}{2},\frac{1}{2}\right) \quad (3)$$

Here, $\Gamma(m)$ and $\beta(z,w)$ are the gamma and beta functions as defined in Ref. [30]. A detailed description of Pearson function is provided in Ref. [19].

IV. Results

Recent micromagnetic simulations have explored how the IDMI affects the MTJ device performances, finding that it decreases the thermal stability while increasing both the critical switching current and the switching times for a fixed current [14], [15]. Such results suggest that IDMI should be minimized for efficient STT memory applications. However, we will show that, by properly tuning the intensity of the symmetric exchange interaction, the IDMI negative effect on the critical switching current J_{Crit} can be compensated.

We carried out a systematic study of the switching properties within a simulation time of 20 ns. We considered a parallel initial state of the magnetization, where the polarizer has a uniform magnetization with direction along the positive out-of-plane direction. The parallel to anti-parallel switching time $t_{s,P\rightarrow AP}$ was considered as the time interval between the application of the current (the initial z-component of the average magnetization on the whole structure is $<m_{z0}>$), and the time instant where the z-component of the average magnetization is equal to $<m_{zf}>$ = -0.9. The applied current densities J_{MTJ} have negative sign [19], but we consider the magnitude in the following pages, for the sake of simplicity. All routines and studies were performed by means of a parallel processing framework which was developed for accelerating algorithms computation and demonstrated successfully results [31]–[33] in different research fields.

First of all, we study at zero temperature the effect of the IDMI on the critical current density for three values of A. Differently from recent micromagnetic simulations [14], [15] where the IDMI increases switching times, our results point out that, if the free layer is properly designed in order to have a larger exchange constant (A=20 pJ/m [34], in those works A was 10 pJ/m [14], [15]), the negative effect of the IDMI on the critical current density can be highly reduced (see Fig. 1).

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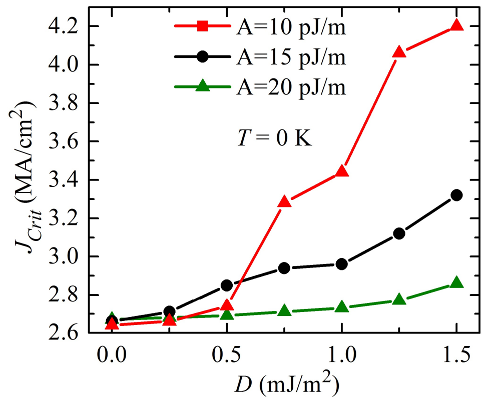

Fig. 1. Critical switching current density as a function of the IDMI parameter for three typical values of the exchange constant A, as indicated in the legend.

Then, we fix A=20 pJ/m [34] and evaluate the four statistical moments of t_s as a function of the IDMI (we have considered a maximum value for D=1.5mJ/m^2 in agreement with recent experimental findings [35], [36]), by performing 1000 realizations at T=300 K. The current is applied after 2 ns in order to allow for a free evolution of the magnetization thus ensuring the stochasticity of the free layer initial state.

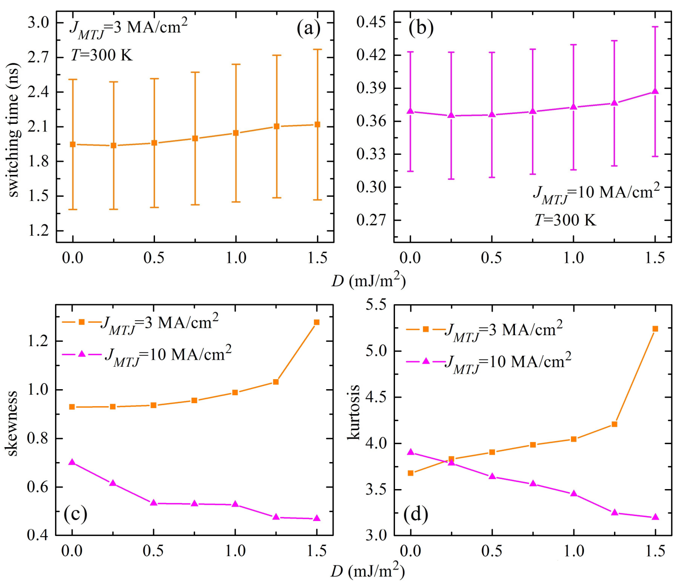

Fig. 2. (a) and (b) mean switching time as a function of the IDMI parameter for J_{MTJ}=3 MA/cm^2 (orange line) and 10 MA/cm^2 (magenta line), respectively, where the standard deviation is indicated as the width of the error bar. (c) Skewness and (d) kurtosis as a function of the IDMI parameter for two values of the current density, as reported in the legend.

Fig. 2 (a) and (b) show the mean switching time for J_{MTJ}=3 MA/cm^2 (near the critical current), and 10 MA/cm^2 (much larger than the critical current), respectively, where the error bar represents the STD. For both current values, the IDMI leads to a negligible increase of the mean switching time. The skewness and kurtosis are depicted in Fig. 2 (c) and (d), respectively, for the same abovementioned values of J_{MTJ}. Interestingly, these two higher order moments exhibit an opposite behavior. Specifically, for J_{MTJ}=3 MA/cm^2 the IDMI leads to a less Gaussian-like and more asymmetrical shape of the PDF since both skewness (solid line orange with squares in Fig. 2(c)) and kurtosis (solid line orange with squares in Fig. 2(d)) increase with D. For currents well above the critical value (J_{MTJ}=10 MA/cm^2), our data show an opposite trend, i.e. the IDMI reduces the skewness (solid line magenta with triangles in Fig. 2(c) and kurtosis (solid line magenta with triangles in Fig. 2(d)), resulting into a PDF characterized by a larger Gaussianity and symmetry. The complex shape of the switching PDFs can be ascribed to two main competitive aspects: the thermal agitation during the switching process and the initial magnetization trajectory dispersion [16].

Now, we wish to compare the statistical switching behavior with and without the IDMI. For the case with IDMI, a typical value of D is about 1.5 mJ/m^2 [35], [36] for the considered structure, therefore, hereafter, we use that value. Fig. 3 (a) and (b) depict a comparison at T=300 K between the PDFs as computed using Pearson Type IV formulation [18] for the reference values D=0.0mJ/m^2 and 1.5 mJ/m^2 and the micromagnetic results as obtained with 1000 simulations, for J_{MTJ}=3 MA/cm^2 and 10 MA/cm^2, respectively. As we can see, the Pearson Type IV PDF excellently fits the micromagnetic results. Moreover, close to the critical current, the PDF is strongly asymmetric being right-skewed (Fig. 3(a)), whereas, as the current increases, a Gaussian-like shape is achieved (Fig. 3(b)), confirming what comes out from the analysis of the four statistical moments (Fig. 2).

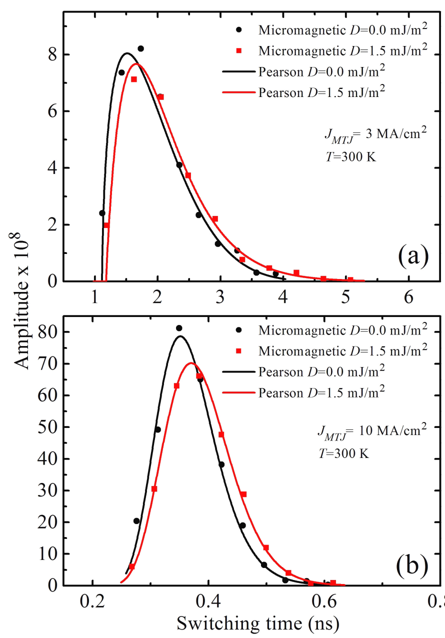

Fig. 3. A comparison at T=300 K between the Pearson PDFs for two values of D=0.0mJ/m^2 (solid black line) and 1.5 mJ/m^2 (solid red line) and the micromagnetic results as obtained with 1000 simulations (black circles and red squares, respectively), for (a) J_{MTJ}=3 MA/cm^2 and 10 MA/cm^2 (b). The Pearson PDFs are obtained from Eq. (2) by considering, in (a) for D=0.0mJ/m^2, a mean μ=1.95 ns, a STD σ=0.57 ns, a skewness γ=0.93 and a kurtosis β_2=3.68, and for D=1.5 mJ/m^2, μ=2.12 ns, σ=0.65 ns, γ=1.28 and β_2=5.24, whereas in (b) for D=0.0mJ/m^2, μ=0.37 ns, σ=0.05 ns, γ=0.70 and β_2=3.90, and for D=1.5 mJ/m^2, μ=0.39 ns, σ=0.06 ns, γ=0.47 and β_2=3.2.

Figs. 4 (a) and (b) show the mean switching time as a function of the current density with and without IDMI at T=0 K and T=300 K, respectively. In both cases, the IDMI slightly affects the switching time. However, the main difference is observed for current densities near the critical value, where the IDMI contribution leads to a higher switching time. We explain this effect because, without IDMI, the magnetization switching is uniform (see Movie1), while it is non-uniform in presence of the IDMI. In the latter, we observe that the precessional state of the magnetization before its reversal is given by a counter-clockwise magnetization rotation of 360° with the successive nucleation of a magnetic domain. When the *z*-component of the magnetization is negative, the rotation changes from counter-clockwise to clockwise with the successive expulsion of the domain and the full switching occurs. This non-uniform behavior delays the accomplishment of the magnetization reversal process (see Movie2). Fig. 4 (c) and (d) show the WER [7], [16], [20] as a function of the current pulse duration for two different pulse amplitudes. The WER can be computed as the one's complement of the cumulative distribution function (CDF), $F(T)$, and it has to be less than 0.1% to be compatible with commercial CMOS technology. In the discrete case, from Eq. (2), the WER can be written as:

$$\mathrm{WER}(T) = 1 - F(T) = 1 - \sum_{t_s \le T} f(t_s) \qquad (4)$$

The WER decreases rapidly with the current pulse width with similar qualitative trend for both cases, with and without IDMI. However, the presence of the IDMI slightly deteriorates the bit write efficiency, since it leads to higher WER for lower current pulse width. Similar results are obtained if the anti-parallel to parallel switching is investigated.

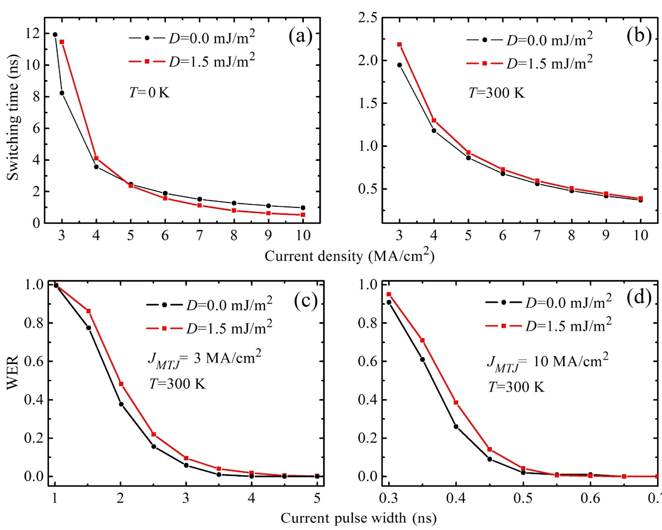

Fig. 4. Switching time as a function of the current density for D=0.0 and D=1.5 mJ/m^2 at (a) T=0 K and (b) T=300 K. Write error rate (WER) as a function of the current pulse width at room temperature for D=0 and D=1.5 mJ/m^2 at (c) J_{MTJ}=3 MA/cm^2, and (d) J_{MTJ}=10 MA/cm^2.

V. Conclusion

In summary, the STT-driven magnetization switching has been micromagnetically studied in a perpendicular MTJ in presence of the IDMI. At T=0K, the negative effect of the IDMI on the critical current density can be reduced by properly designing the MTJ free layer in order to have a large enough symmetric exchange constant. On the other hand, the statistical analysis of the switching time performed at room temperature T=300 K has shown that the mean and STD are not affected by the IDMI, whereas the higher order moments skewness and kurtosis show monotonic behaviors with the IDMI. In addition, we have found that micromagnetic PDF of the switching time is well fitted by a Pearson Type IV PDF which provides a quadratic fitting error one order of magnitude smaller than skew normal PDF as demonstrated in Ref. [19]. In the industrial field, the use of skewness and kurtosis can open the path for more accurate prediction of the switching PDFs with beneficial effects on both design and manufacturing processes for the minimization of the memory failure probability. Finally, while the influence of the IDMI on the switching time-current relation is quite negligible, it slightly increases the WER with respect to the case without IDMI.

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