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## Improved Energy Harvesting from Wideband Vibrations by Nonlinear Piezoelectric Converters

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### Abstract

Vibration harvesters typically are linear mass-spring devices working at resonance. A different approach is here proposed based on nonlinear converters that exploit stochastic resonance with white-noise excitation. A piezoelectric beam converter is coupled to permanent magnets creating a bistable system bouncing between two stable states in response to random excitation. Under proper conditions, this significantly improves energy harvesting from wide-spectrum vibrations. A cantilever beam with added nonlinearity has been simulated by using a MATLAB<sup>®</sup> Stochastic Differential Equation (SDE) Toolbox with a white-noise vibration. A nonlinear converter was then realized by screen printing low-curing-temperature lead zirconate titanate (PZT) films on steel cantilevers and excited with white-noise vibrations. Experimental results show that the performances of the converter in terms of output voltage at parity of mechanical excitation are markedly improved.

*Keywords:* Energy harvesting, piezoelectric converter, wide-spectrum vibrations, nonlinear bistable system, stochastic resonance.

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### 1. Introduction

In energy harvesting from vibrations, mechano-electrical converters are typically realized as linear mechanical resonators. The effectiveness of such converters is maximum when operated at resonance, but is considerably suboptimal with frequency-varying and wideband vibrations. Lowering the converter quality factor increases the bandwidth, but worsens the peak response. To overcome that limitation, multiple converters with different frequency responses can be combined in arrays [1-3]. This work proposes a novel approach based on a stochastic nonlinear oscillator [4-6].

### 2. Nonlinear oscillator

Fig. 1a shows an equivalent mechanical model for an undamped nonlinear system, where nonlinearity is introduced in the original linear system, formed by mass  $m$  and spring with stiffness  $k$ , by the element with nonlinear

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mechanical stiffness  $k_{NL}$  and a nonlinear associated force  $F_{NL}$ . The coefficients  $\alpha$  and  $\beta$  are positive real numbers. Fig. 1b plots the potential  $U(x)$ , obtained by integrating the total force function, for different values of  $(k - \alpha)$  and  $\beta$ . For  $(k - \alpha) \geq 0$ , the potential  $U(x)$  has only one stable equilibrium point in the origin and the system has a linear behaviour. When  $(k - \alpha) < 0$ ,  $U(x)$  shows a symmetric double well with two stable equilibrium points, implying a bistable behavior.

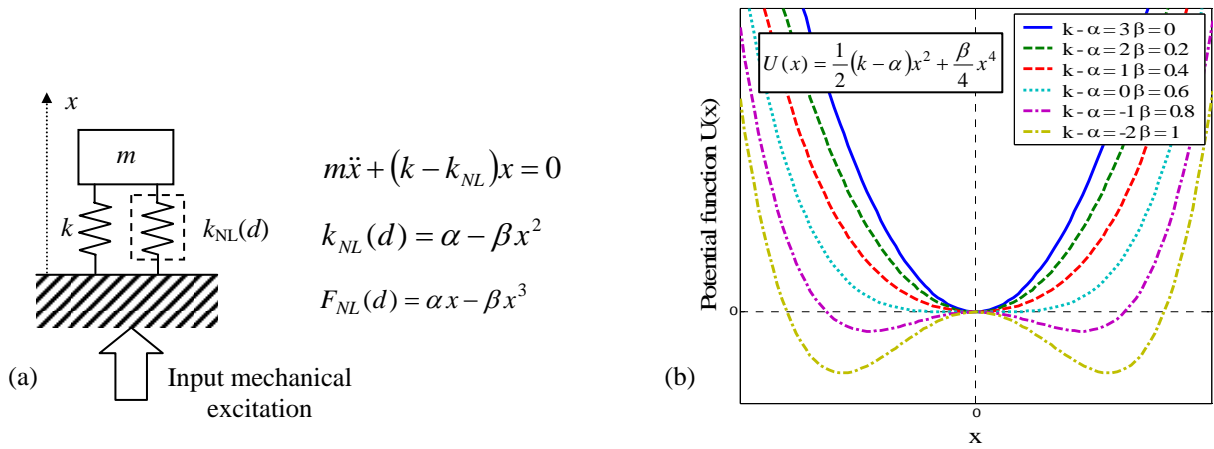


Fig. 1. (a) Equivalent mechanical model for a nonlinear undamped system. Nonlinearity is introduced by an additional nonlinear spring with mechanical stiffness  $k_{NL}$ . The term  $F_{NL}$  is the associated nonlinear force. (b) Potential function  $U(x)$  for different values of  $(k - \alpha)$  and  $\beta$ .

Considering a cantilever beam, it is possible to create bistability by positioning two permanent magnets with opposite polarities respectively on the cantilever tip and at a distance  $d$  along the beam axis, as shown in Fig. 2. The vertical component of the repulsive force between the magnets is a nonlinear odd function of the tip displacement  $x$ , here named  $F_{NL}$ . The coefficients  $\alpha$  and  $\beta$  result to be decreasing functions of the distance  $d$ .

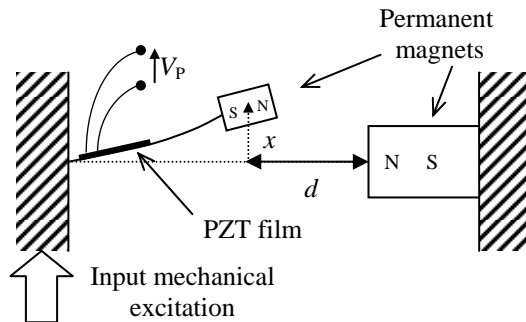


Fig. 2. Bistable system formed by a piezoelectric cantilever beam and two permanent magnets. When distance  $d$  is low enough,  $(k - \alpha)$  becomes negative and the cantilever bounces between two stable states.

### 3. Analysis of the principle

The nonlinear switching mechanism model has been expressed as one-dimension Itô form [7] and simulated by using MATLAB® SDE (Stochastic Differential Equation) Toolbox [8]. The Eulero-Maruyama method has been used and a white noise vibration source has been applied at the cantilever base. Fig. 3a shows the qualitative deflection of the cantilever beam in presence of the nonlinearity and in the linear case, corresponding respectively to finite and exceedingly large distance  $d$  between the cantilever and the external magnet.

Moreover Fig. 3b reports the responses in the frequency domain. The nonlinear case provides a wider spectrum compared to resonant behaviour of the linear case. This is expected to produce a improved effectiveness in converting wide-spectrum vibrations.

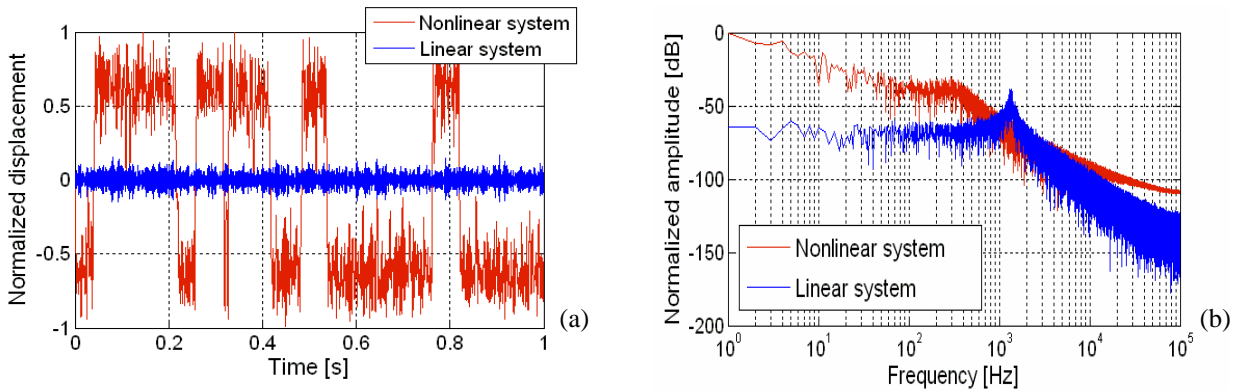


Fig. 3. Simulation results: (a) deflection of the cantilever in the time (a) and frequency (b) domains. The signals are normalized to the maximum value of amplitude. The nonlinear evolution represents the stochastic resonance condition.

#### 4. Experimental results

To experimentally evaluate the proposed approach, piezoelectric bimorph converters were realized by screen printing low-curing-temperature PZT films on steel cantilevers [9]. A permanent magnet was fixed on the cantilever tip while an external permanent magnet, with opposite polarity, was fixed on a micrometric stage. The converter was excited at the base by mechanical vibrations with white acceleration spectrum using an electrodynamic shaker, as shown in Fig. 4, varying the distance  $d$  between the cantilever tip and the fixed permanent magnet.

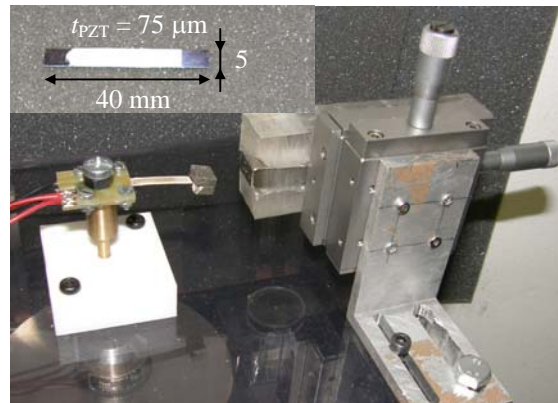


Fig. 4. Experimental setup for bistable system characterization. In the top-left inset: low-curing-temperature piezoelectric converter and its physical dimensions.

Fig. 5a shows the measured open-circuit output voltage  $V_p$  of the converter under these excitation conditions ( $a_{\text{peak}} = 1g$ ). As shown in the frequency spectra of the measured signals of Fig. 5b, when the distance  $d$  is decreased the system initially maintains a linear behaviour with a decrease in the resonance frequency until the equivalent stiffness ( $k - \alpha$ ) is positive. At the same time,  $V_p$  increases due to the increased equivalent compliance. When the magnets are adequately close (bottom plot,  $d = 10.5$  mm), the equivalent stiffness ( $k - \alpha$ ) becomes negative, thereby making the system bistable. In this condition, switching between the two stable states occurs and  $V_p$  significantly increases, together with converted power. This is demonstrated by the computed rms values of the voltage  $V_p$  that result respectively  $V_{d=25.0\text{mm}} = 0.78V$ ,  $V_{d=12.0\text{mm}} = 1.68V$ , and  $V_{d=10.5\text{mm}} = 3.33V$ .

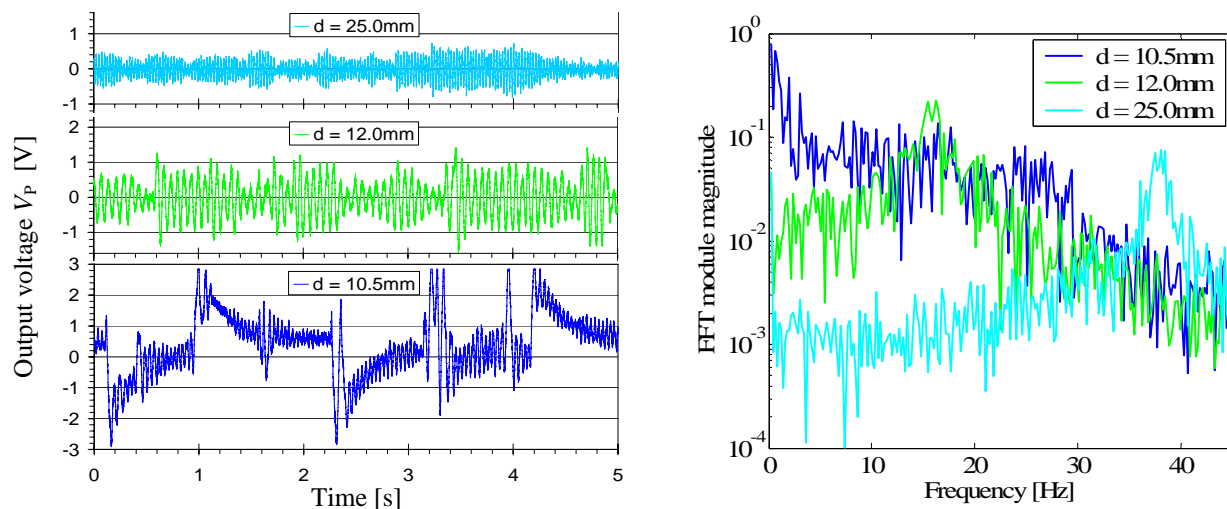


Fig. 5. (a) Output voltage  $V_p$  of the cantilever measured with a custom-made high-input-impedance buffer for different values of the distance  $d$  between the two magnets with a white input spectrum vibrations ( $a_{\text{peak}} = 1\text{g}$ ). (b) Frequency amplitude spectra of the output voltage  $V_p$ , measured for different values of the distance  $d$ .

Considering the intrinsic high-pass function between the output voltage  $V_p$  of the piezoelectric converter and the tip deflection  $x$ , the experimental results of the bottom plot of the fig 5a show the same bistable behavior of the simulation results of Fig. 3a (nonlinear system). Again, comparing the frequency amplitude spectra of the experimental results shown in Fig. 5b with the simulation results of Fig. 3b, and considering the high-pass function of the piezoelectric converter, it can be observed a very good agreement, evidencing a wider spectrum for the bistable system with respect to the linear case.

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