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Phil. Trans. R. Soc. A 2008 **366**, 569-577
doi: 10.1098/rsta.2007.2111

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Phil. Trans. R. Soc. A (2008) **366**, 569–577
doi:10.1098/rsta.2007.2111
Published online 13 August 2007

Experimental separation of chaotic signals through synchronization

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In this paper using a negative feedback scheme we study the problem of synchronizing two systems (each of them made of n independent chaotic circuits) through the transmission of a unique signal (i.e. a scalar variable). To find the appropriate values of the feedback gains, an approach based on the design of an asymptotic observer leading to a set of linear matrix inequalities is used for piecewise linear systems, while for systems with continuous nonlinearities a master stability function approach is adopted. Numerical results showing the suitability of the approach are reported. Furthermore, the experiment showing separation and synchronization of two pairs of chaotic circuits is discussed. Despite the presence of parameter mismatches, separation and synchronization of the two systems can be achieved. This is an experimental demonstration of the successful possibility of multiplexing two (or more) chaotic signals in the same channel.

Keywords: synchronization; chaos; master stability function; linear matrix inequalities

1. Introduction

Synchronization of two or more chaotic systems is one of the most important topics of nonlinear dynamics and chaos (Pecora & Carroll 1990; Boccaletti *et al.* 2002), and many schemes have been proposed to achieve complete synchronization between two or more chaotic units (Boccaletti *et al.* 2002).

In this paper we study the case of the simultaneous synchronization of two groups of n chaotic systems in a negative feedback scheme. The master and slave systems are each formed by n independent chaotic systems (i.e. n different systems which do not interact with each other). In general, the synchronization of two groups of such chaotic systems requires n independent feedback signals. In our case, instead, it is investigated if and under which conditions synchronization can be achieved using only one feedback signal which depends on the chaotic systems of the master (i.e. it is, for instance, a linear combination of the state variables of the master chaotic systems). This problem is referred to as separation and synchronization of chaotic signals. Such problem is similar to that investigated by Tsimring & Sushchik (1996) and Carroll & Pecora (1999) regarding multiplexed chaotic systems. However, in neither case, synchronization of continuous time

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One contribution of 14 to a Theme Issue 'Experimental chaos II'.

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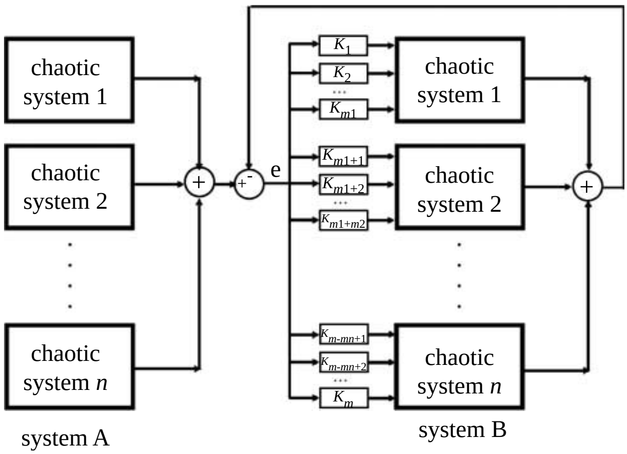

Figure 1. Separation and synchronization scheme. 1-1′, 2-2′ and n-n' are identical systems starting from different initial conditions; the error is the difference between a linear combination of the state variables of the master and slave systems.

flows is shown. In our paper, this is achieved with a new technique and experimentally demonstrated with a circuit implementation of one of the examples shown.

In particular, two different strategies have been developed in order to solve the problem of separation and synchronization: the first, based on linear matrix inequalities (LMI; Boyd *et al.* 1994), is suitable for piecewise linear (PWL) systems; the second, based on a master stability function (MSF) approach, is designed for systems with continuous nonlinearities.

The approaches are first described for the general case of master–slave systems made of n chaotic subsystems. Numerical and experimental results concerning the case of $n=2$ are then shown.

2. Separation and synchronization of PWL chaotic systems

We first describe how the scalar signal is used to connect two multiplexed systems. The scheme proposed is based on negative feedback. Let us consider two multiplexed systems as shown in figure 1, where a directional coupling from the multiplexed system A to B is taken into account. The two multiplexed systems are considered identical, i.e. they are formed by the same n independent subsystems with equal parameters. A linear combination of the state variables of system A is sent to system B, where an error signal is built by comparing the received signal with the same linear combination of the corresponding state variables of system B.

The error signal is weighted by suitable gains and added to the dynamic equation of each state variable of system B as in the negative feedback scheme for two chaotic systems (Kapitaniak 1994).

Therefore, the equations of the master are given by $\dot{\boldsymbol{X}}_{\rm m}=f(\boldsymbol{X}_{\rm m})$ and those of the slave are given by $\dot{\boldsymbol{X}}_{\rm s}=f(\boldsymbol{X}_{\rm s})+\boldsymbol{K}e$, where $\boldsymbol{K}$ is the gains vector and e is the (scalar) error signal. Assuming that the master is composed by n systems of order $m_1, m_2, \ldots, m_n$, then $\boldsymbol{X}_{\rm m}\in\mathbb{R}^m$ with $m=m_1+m_2+\cdots+m_n$, $\boldsymbol{X}_{\rm s}\in\mathbb{R}^m$ and $\boldsymbol{K}\in\mathbb{R}^m$. This scheme is summarized in figure 1. In order to synchronize the

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master and slave systems, the error has to asymptotically converge to zero. The slave system can be thus considered as an observer of the master system, so that the problem of separation and synchronization is equivalent to the design of an asymptotic observer in which the gains have to be calculated in order to ensure the stability of the error system.

First, chaotic systems characterized by PWL nonlinearities are considered. In this case, a nonlinear observer designed through the definition of an LMI problem is used. In each region of the PWL, the systems of this class assume a different linear behaviour switching through the PWL regions. Therefore, a PWL system is characterized by the set of its possible linearizations. Since, in each region, each linear system can be observed using the classical linear control techniques, our idea is to design an observer which simultaneously guarantees asymptotically stable error dynamics in each of these regions. Therefore, to solve the problem of separation and synchronization, the observer should be designed by solving a simultaneous stability problem. A suitable technique to solve a simultaneous stability problem is based on LMIs and is briefly described in the following.

Let us define as $\boldsymbol{e_X}=\boldsymbol{X}_{\rm m}-\boldsymbol{X}_{\rm s}$ the state estimation error. In general, the equation that describes the error system dynamics for PWL systems is

$$\dot{\boldsymbol{e}}_{\boldsymbol{X}} = A_i\boldsymbol{X}_{\rm m} - A_j\boldsymbol{X}_{\rm s} - \boldsymbol{K}C(\boldsymbol{X}_{\rm m}-\boldsymbol{X}_{\rm s}), \tag{2.1}$$

where A_i and A_j respectively represent the linearization of the observed system (the master) and of the observer (the slave) in ith or jth region of the PWL nonlinearity. The two matrices A_i and A_j are different when the two systems work in different regions of the PWL nonlinearity. Otherwise (i.e. when the observer works in the same region of the observed system), the matrices A_i and A_j are equal and the error system dynamic reduces to

$$\dot{\boldsymbol{e}}_{\boldsymbol{X}} = (A_i - \boldsymbol{K}C)\boldsymbol{e}_{\boldsymbol{X}}. \tag{2.2}$$

In this situation, the observer can be designed to be stable by solving the following LMI problem

$$\begin{cases} A_i^{\rm T}P - C^{\rm T}Q^{\rm T} + PA_i - QC < 0; \quad i=1,\ldots,q \\ P>0 \end{cases} \tag{2.3}$$

where q is the number of regions of the considered PWL nonlinearity. Equation (2.3) gives the LMI constraints which have to be fulfilled to solve the problem of separation and synchronization. If the overall LMI problem is feasible, its solution leads to a gain vector $\boldsymbol{K}$ able to stabilize all the possible error dynamics.

This procedure permits the design of the observer able to reconstruct the dynamics of the observed system. A necessary condition to the stability of the error dynamics is imposed. In fact, the error system is imposed to be stable only if observed system and observer are in the same PWL region at the same time. Otherwise, when the two systems are in different regions, the error dynamics is given by equation (2.1).

Numerical simulations and experimental results, reported in the following sections, show that, provided that the eigenvalues of the error system (i.e. the eigenvalues of $(A_i-\boldsymbol{K}C)$ for $i=1,\cdots,q$) as designed by solving the stability problem are sufficiently fast, the necessary condition is sufficient for the synchronization.

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3. Master stability function for multiplexed systems

The approach is now extended to multiplexed systems with continuous nonlinearities. The synchronization properties of such multiplexed systems can be studied with the MSF. The approach, introduced by Pecora & Carroll (1998), considers N identical oscillators coupled with the same function of the components from each oscillator to the other oscillators into an arbitrary network which admits the synchronization manifold as an invariant manifold. The approach is based on the linearization of the network dynamics around the synchronization manifold.

In Pecora & Carroll (1998), the dynamics of each node of the network is modelled as $\dot{\boldsymbol{x}}^i = F(\boldsymbol{x}^i) + \sigma\sum_j G_{ij}H(\boldsymbol{x}^j)$ where $\dot{\boldsymbol{x}}^i = F(\boldsymbol{x}^i)$ represents the dynamics of each isolated node, σ is the coupling strength, $H : \mathbb{R}^m \to \mathbb{R}^m$ the coupling function and $G = [G_{ij}]$ is a zero-row sum matrix modelling network connections. The synchronization properties of this network are studied by calculating the maximum Lyapunov exponent $\lambda_{\max}$ of the generic variational equation

$$\dot{\zeta} = [DF + (\alpha + \mathrm{i}\beta)DH]\zeta, \qquad (3.1)$$

as a function of α and β, where DF and DH represent the Jacobian of $F(\boldsymbol{x}^i)$ and $H(\boldsymbol{x}^j)$ computed around the synchronous state. Once obtained $\lambda_{\max}$, which does not depend on the connection network, the stability of the synchronization manifold in a given network can be evaluated by computing the eigenvalues γ_h (with $h=2, \dots, N$) of the matrix G. If all eigenmodes with $h=2, \dots, N$ are stable, then the synchronous state is stable at the given coupling strength. In fact, we recall that, since G is zero-row sum, the first eigenvalue is $\gamma_1=0$ and represents the variational equation of the synchronization manifold.

In particular, if G has real eigenvalues the MSF can be computed only as function of α. In the following we will restrict our analysis to this case. The functional dependence of $\lambda_{\max}$ on α can give rise to three different cases (Boccaletti *et al.* 2006). The first case, denominated as type I, is the case in which network nodes cannot be synchronized. In the second case (type II), increasing the coupling coefficient σ always leads to a stable synchronous state. In the third case (type III), network nodes can be synchronized only if $\sigma\gamma_h$ for $h=2, \dots, N$ lie in the interval with negative values of $\lambda_{\max}$.

Referring to the formulation of the synchronization problem of multiplexed systems the matrix DH becomes

$$DH = \begin{bmatrix} k_1b_1 & k_1b_2 & \dots & k_1b_m \\ k_2b_1 & k_2b_2 & \dots & k_2b_m \\ \vdots & \vdots & \ddots & \vdots \\ k_mb_1 & k_mb_2 & \dots & k_mb_m \end{bmatrix}, \qquad (3.2)$$

where $k_1, k_2, \dots, k_m$ are the gains ($K = [k_1 \quad k_2 \quad \dots \quad k_m]$) and $b_1, b_2, \dots, b_m$ with $b_i = \{0, 1\}$ specify which state variables are selected to build the error signal e. Let us define $B = [b_1 \quad b_2 \quad \dots \quad b_m]$.

The master stability equation generally depends on K and B. At this point, the problem of synchronizing multiplexed systems can be translated into the problem of the existence of suitable values of K and B for which the MSF is either type II or III.

To solve this problem, an approach based on genetic algorithms can be used. Genetic algorithms are an optimization procedure based on the evolution of a population of individuals coding the possible solutions to the problem

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(Goldberg 1989). In our case, the problem is to obtain an MSF with $\lambda_{\max}<0$. To this aim, the fitness function can be defined as follows:

$$f = \min \lambda_{\max}. \tag{3.3}$$

Given this fitness function, the genetic algorithms are used to search K and B which minimize the maximum Lyapunov exponent of equation (3.1). If the optimum value is such that $\lambda_{\max}<0$, then the problem of synchronizing multiplexed systems has a solution. Of course, the existence of this solution is not related to the stability of the synchronization manifold in a given complex network, but $\lambda_{\max}<0$ ensures that there exist synchronizable networks with multiplexed systems as nodes.

The MSF approach is totally general, and it does not depend on the network structure. When applied to the case of master–slave synchronization, one obtains

$$G = \begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix}$$

and $\gamma_1=0$, $\gamma_2=-1$. $\lambda_{\max}<0$ ensures that there exists a suitable value of the coupling strength ($\bar{\sigma}$ such that $\lambda(\bar{\alpha})<0$ with $\bar{\alpha}=\bar{\sigma}\gamma_2=-\bar{\sigma}$) for which the two systems will synchronize.

4. Numerical results

In this section, two numerical examples with $n=2$ are considered. The first case takes into account two PWL multiplexed systems. The example deals with the double-scroll like chaotic oscillator described in Elwakil *et al.* (2000), called in the following a DSLC oscillator. The two DSLC oscillators forming the multiplexed systems are characterized by two different values of the parameter a so that the following equations are used for the master system:

$$\begin{aligned}
\dot{x}_{1\mathrm{m}} &= y_{1\mathrm{m}}, \\
\dot{y}_{1\mathrm{m}} &= z_{1\mathrm{m}}, \\
\dot{z}_{1\mathrm{m}} &= -a_1(x_{1\mathrm{m}} + y_{1\mathrm{m}} + z_{1\mathrm{m}} - \mathrm{sgn}(x_{1\mathrm{m}})), \\
\dot{x}_{2\mathrm{m}} &= y_{2\mathrm{m}}, \\
\dot{y}_{2\mathrm{m}} &= z_{2\mathrm{m}} \quad \text{and} \\
\dot{z}_{2\mathrm{m}} &= -a_2(x_{2\mathrm{m}} + y_{2\mathrm{m}} + z_{2\mathrm{m}} - \mathrm{sgn}(x_{2\mathrm{m}})).
\end{aligned} \tag{4.1}$$

The slave dynamics are designed as described in §3:

$$\begin{aligned}
\dot{x}_{1\mathrm{s}} &= y_{1\mathrm{s}} + k_1 e, \\
\dot{y}_{1\mathrm{s}} &= z_{1\mathrm{s}} + k_2 e, \\
\dot{z}_{1\mathrm{s}} &= -a_1(x_{1\mathrm{s}} + y_{1\mathrm{s}} + z_{1\mathrm{s}} - \mathrm{sgn}(x_{1\mathrm{s}})) + k_3 e, \\
\dot{x}_{2\mathrm{s}} &= y_{2\mathrm{s}} + k_4 e, \\
\dot{y}_{2\mathrm{s}} &= z_{2\mathrm{s}} + k_5 e \quad \text{and} \\
\dot{z}_{2\mathrm{s}} &= -a_2(x_{2\mathrm{s}} + y_{2\mathrm{s}} + z_{2\mathrm{s}} - \mathrm{sgn}(x_{2\mathrm{s}})) + k_6 e,
\end{aligned} \tag{4.2}$$

where $e = C(\boldsymbol{X}_{\mathrm{m}} - \boldsymbol{X}_{\mathrm{s}}) = x_{1\mathrm{m}} + y_{1\mathrm{m}} + z_{1\mathrm{m}} + x_{2\mathrm{m}} + y_{2\mathrm{m}} + z_{2\mathrm{m}} - (x_{1\mathrm{s}} + y_{1\mathrm{s}} + z_{1\mathrm{s}} + x_{2\mathrm{s}} + y_{2\mathrm{s}} + z_{2\mathrm{s}})$ and $a_1=0.8$, $a_2=0.6$.

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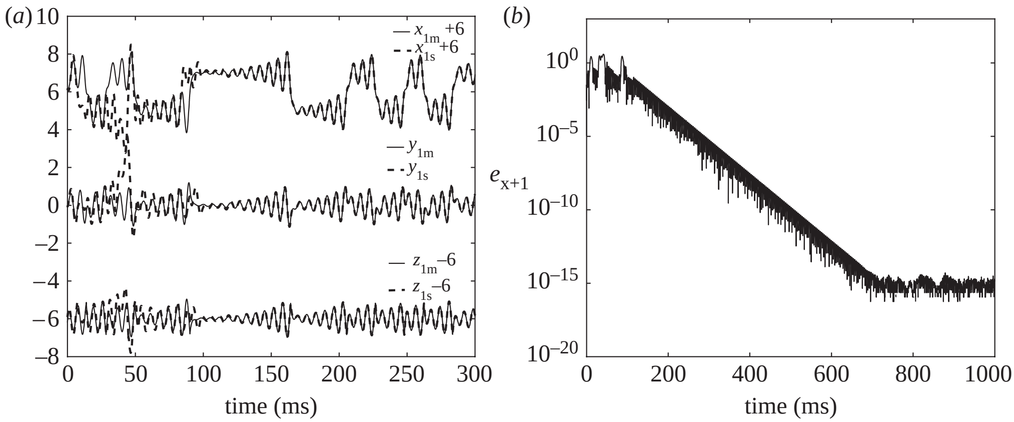

Figure 2. Separation and synchronization of two DSLC oscillators. (*a*) Trend of $x_{1m}+6$, y_{1m} and $z_{1m}-6$ (continuous lines) compared with $x_{1s}+6$, y_{1s} and $z_{1s}-6$ (dotted lines). (*b*) Semilogarithmic plot of the absolute error for x_1.

The LMI problem for the simultaneous stability is feasible and leads to the gains vector $\boldsymbol{K}=[-1.5710 \quad -0.4173 \quad 1.0232 \quad 1.6569 \quad 1.6126 \quad -0.7322]^{\mathrm{T}}$ which stabilizes the error system. Figure 2*a* shows the temporal evolution of three of the six state variables of the master compared with the corresponding variables of the slave. The semilogarithmic plot of the absolute error for x_1 is shown in figure 2*b*.

The second numerical example reported deals with the case of systems with continuous nonlinearities, on which the MSF approach has been applied. Several multiplexed systems obtained by pairing two of three well-known chaotic systems described earlier (i.e. Lorenz system, Chua's circuit and Rössler oscillator) have been investigated. Moreover, these multiplexed systems differ from the way in which the two subsystems are coupled. In other words, depending on the definition of the error scalar signal adopted, different multiplexed systems are obtained: in general, they have different synchronization properties. In most cases, the resulting multiplexed system has MSF of type I and therefore synchronization cannot be achieved, but this can be due to the choice of the coupling parameters (i.e. K). For this reason, when the MSF of the system is type I, we investigated the possibility of finding a coupling vector such that its MSF is type II or III by applying genetic algorithms with the fitness function defined as in (3.3). In several cases, this approach has been shown to be effective to find suitable parameters.

For instance, in the case of a multiplexed system made of a Lorenz system and a Rössler oscillator, coupled through $e=x_{\mathrm{Lm}}-x_{\mathrm{Ls}}+y_{\mathrm{Lm}}-y_{\mathrm{Ls}}+x_{\mathrm{Rm}}-x_{\mathrm{Rs}}+y_{\mathrm{Rm}}-y_{\mathrm{Rs}}$ (which is type I for $K=[1\ 1\ 1\ 1\ 1\ 1]$), we were able to find a suitable coupling vector $K=\bar{K}_{\mathrm{LR}}$ such that the multiplexed system has MSF of type III. Genetic algorithms (GAs) with cross-over probability $p_{\mathrm{c}}=0.7$, mutation probability $p_{\mathrm{m}}=0.7$ and generation gap $g_{\mathrm{gap}}=0.9$ were used. The elements of vector K were searched for in the range $[-2;2]$. Chromosomes are represented with 16 bit precision, thus it is possible to select 2^{16} values inside the fixed range.

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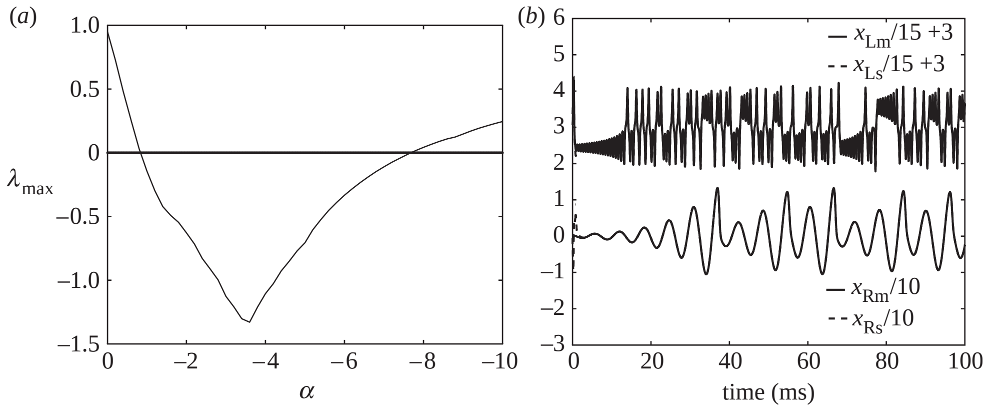

Figure 3. (*a*) MSF for a multiplexed system formed by a Lorenz system and Rössler oscillator. (*b*) Trend of x_{Lm} and x_{Rm} (continuous lines) compared with x_{Ls} and x_{Rs} (dotted lines). Signals are normalized for better visualization.

After 30 generations (with 20 individuals), the following vector has been found to minimize the fitness function (3.3):

$$\bar{K}_{\mathrm{LR}} = [1.4346 \quad 1.2546 \quad -0.1287 \quad 0.9619 \quad 0.1043 \quad -0.0445].$$

The resulting MSF is shown in figure 3*a* and, as it can be noted, it is type III. This means that using $K = \bar{K}_{\mathrm{LR}}$ the considered multiplexed system can be synchronized by setting the eigenvalues $\sigma\gamma_h$ of connection matrix G inside the range of α corresponding to the negative values of the MSF. According to figure 3*a*, we fix $\sigma=4$ so that the eigenvalue of σG is $\sigma\gamma_2 = -\sigma = -4$. Figure 3*b* shows the trends of the state variables x_{Lm} and x_{Rm} compared with the correspondent slave variables.

5. Experimental results

A physical implementation of the system has been then realized. Field programmable analogue array (FPAA) boards have been used for the implementation of the four chaotic systems and the circuitry needed to calculate the error signal. FPAAs are programmable analogue circuits which allow simple nonlinear circuits to be implemented in an efficient and fast approach. In Caponetto *et al.* (2005), the use of FPAA boards to implement chaotic circuits is described. In particular, the AN221E04 Anadigm board has been used.

Following the guidelines described in Caponetto *et al.* (2005), the boards can be suitably programmed to obtain the dynamics of the systems described in §4, i.e. the two DSLC oscillators. The experimental chaotic attractors match the simulated ones.

Two FPAA-based systems have been implemented and connected to experimentally investigate the problem of separation and synchronization. The experimental results obtained agree with the numerical simulations carried out, showing the real possibility of separating and synchronizing two pairs of chaotic circuits through a unique feedback signal.

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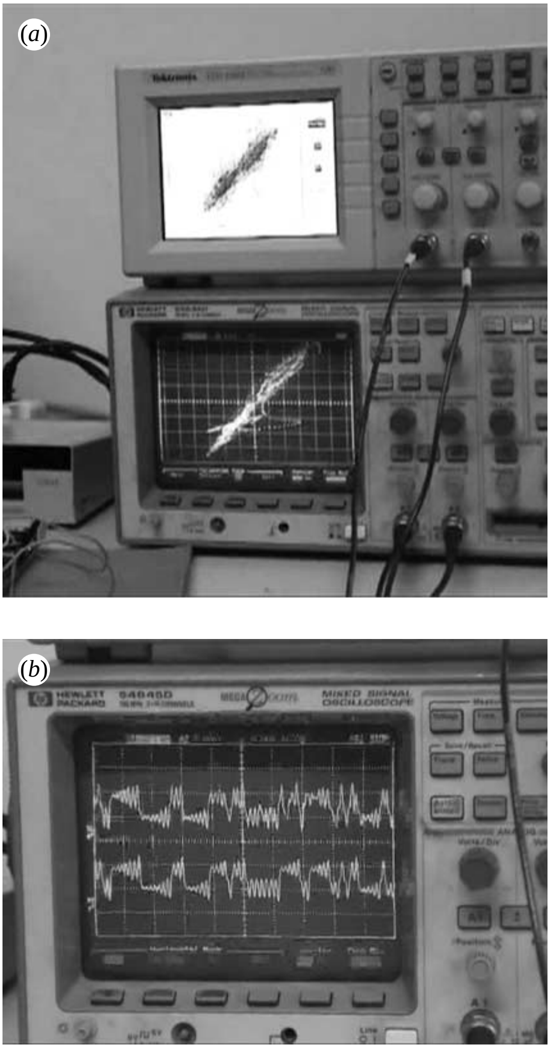

Figure 4. Separation and synchronization of a pair of DSLC oscillators. (*a*) Synchronization plot: x_{1m} versus x_{1s} on the upper oscilloscope and x_{2m} versus x_{2s} on the lower oscilloscope. With feedback, separation and synchronization are achieved. (*b*) Trends of x_{2m} versus x_{2s} when the feedback is switched on. The two systems are synchronized except for some mismatches due to circuital parameter tolerances.

Figure 4*a* shows the oscilloscope traces of the two synchronization plots x_{1m} versus x_{1s} and x_{2m} versus x_{2s}. When the error signal is not fed back into the slave system, the two systems are not synchronized; switching on the error feedback, as shown in figure 4*a*, the slave system follows the master dynamics. The synchronization is furthermore stressed in figure 4*b* where the trends of x_{2m} and x_{2s} are reported. The mismatches visible in figure 4*b* are due to the fact that a real case with circuits which necessarily have slightly different parameters is considered.

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6. Conclusions

In this paper, starting from the scheme of negative feedback (Kapitaniak 1994), a new synchronization scheme for chaotic systems is investigated. In this scheme, master and slave systems are constituted by n independent chaotic systems and a unique scalar variable is transmitted. For PWL systems, suitable values of the feedback gains are found using an approach based on the design of an asymptotic observer through the solution of an LMI problem. For systems with continuous nonlinearities, the problem of separation and synchronization is solved by adopting an approach based on MSF and GA.

Several numerical examples have been reported to show that two pairs of chaotic signals can be effectively separated and synchronized. Furthermore, an experimental investigation of one of the reported case studies has been carried out using a circuit implementation based on programmable nonlinear analogue boards. The experimental results discussed confirm the suitability of the approach even in the real case, when non-identical systems are necessarily considered.

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