The Value-Analytic Hierarchy Process: a Lean Multi Criteria Decision Support Method

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The Analytic Hierarchy Process uses pairwise comparison in order to define priorities for criteria and for alternatives, obtaining an overall ranking which represents a "rational decision".

In literature there are many examples of AHP applications where, in order to operate the ranking of alternatives according to qualitative and quantitative criteria, authors assign an utility value to the judgments on qualitative criteria but also to the values assumed on the basis of quantitative criteria.

A modified approach, that was called Value-Analytic Hierarchy Process (V-AHP), was developed as a combination of traditional AHP rating on qualitative criteria and a "lean" rating on quantitative criteria. Such procedure allows to limit the use of the traditional AHP method only to criteria expressed by qualitative judgments and/or by scales different from ratio ones. A numerical example was carried out, according to the introduced methodology, in order to rank 15 potential simulated industrial investments evaluated by qualitative and quantitative criteria. The V-AHP can be defined a "lean" Multi Criteria Decision Support Method potentially applicable to any multi-criteria decision-making context.

Keywords: Value-AHP, lean approach, Multi Criteria Decision Support Method, Investments

1. INTRODUCTION

Advantages and disadvantages, costs and benefits which characterize all decisions depend on multiple, often conflicting, points of view or criteria used in decision making activity.

For several years, the mathematics applied to decision support, in particular to operations research, has been directed towards the study of objective functions able to condense into a single scale of measure various aspects belonging to the real world. Since Seventies, Multiple Criteria Decision Analysis (MCDA) is a mathematical discipline which offers a realistic and naturally multidimensional approach to decision theory (Bouyssou, et al., 2000) (Figueira et al., 2005), generating a considerable interest among scientists.

In late Seventies, Prof. T.L. Saaty developed the Analytic Hierarchy Process MCDA methodology based on pairwise comparisons of criteria and of alternatives as well, in order to obtain an overall ranking able to represent a "rational decision".

According to Saaty, the modern cartesian mathematical thinking is based on the implementation of measures made only on scales provided of a "zero" point and measurement units; the basic concept is that no measure has a meaning, except the one assigned by those who have to interpret it (Saaty, 2008). According to Saaty, we can all agree on the numerical value read on a scale but not on the utility that can be attributed to it, which is mainly subjective.

The creator of AHP also shows that in mathematics there are two basic topologies: cardinal and ordinal. Cardinal topology allows the quantification of a phenomenon by means of scales provided of a "zero" point and measurement units originally chosen arbitrarily but applied in a uniform manner in the next. The result of this operation is called "measurement" (Stevens, 1946). Ordinal topology allows the dominance definition of one element over others, according to a common criteria or point of view: the result of this operation is defined "priority" (Saaty, 2006). In ranking alternatives, scientific community is often oriented to use the cardinal topology instead of the ordinal topology; but the priority definition needs to use ordinal topologies and invariant absolute scales to uniquely identify a quantity as a multiple of another one (Saaty, 2008).

The pairwise comparison or the definition of a relative importance between entities, according to a criterion, allows the priorities definition for intangible entities, which are free of scales of measurement by definition, but also for tangible entities evaluable on scales with "zero" point and measurement units (Aczel & Saaty, 1983).

In literature, there are many examples of AHP application in which, in order to operate the ranking of an indefinite number of alternatives based on qualitative and quantitative criteria, the authors assign utility to judgment, based on qualitative criteria, but also to performances, based on quantitative criteria, read on a scale having a "zero" point and measurement units (Saaty, 1986), (Saaty, 1990), (Saaty, et al., 1991), (Tripodo & Dazzi, 1995), (D'Urso, et al., 2011), (Bruno, et al., 2012).

The traditional AHP methodology is very profitable for achieving a rational ranking with qualitative criteria, but the same procedure applied to quantitative criteria frankly appears unnecessarily complex, for ranking purpose.

A "lean" AHP procedure, called Value-Analytic Hierarchy Process (V-AHP), which combine the traditional AHP procedure for the rating of qualitative criteria and a simpler rating procedure for quantitative criteria, is then suggested. In order to clarify what the simplified procedure is, some basic concepts of traditional AHP will be presented, to better highlight the differences with the V-AHP; finally a simple application will be carried out about capital investment decisions.

2. THE TRADITIONAL ANALYTIC HIERARCHY PROCESS (AHP)

2.1 The Analytic Hierarchy Process for relative comparisons (AHP-R)

The first step in the application of AHP method in order to rank, by pairwise comparisons, a limited number of alternatives, is the definition of the hierarchy in at least 3 levels. Figure 1 shows a three levels AHP hierarchy and the connections among objective, criteria and alternatives.

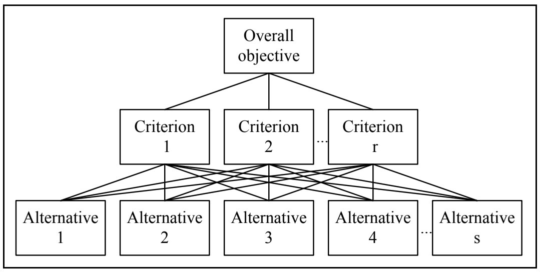

Figure 1. Three levels AHP-R hierarchy

The next step in the application of the AHP method is the construction of pairwise comparison matrices where all the elements belonging to the same hierarchical level, subordinate to a higher hierarchical level element, are compared. One matrix of pairwise comparison between r criteria and r matrices of pairwise comparison between s alternatives are required. The elements of each pair are compared in order to decide which of them is more important in relation to a considered element of higher level and to what extent.

The result of each comparison is a coefficient a_{ij}, said coefficient of dominance, which represents an estimate of the dominance of the element *i-th* with respect to the element *j-th;* each coefficient a_{ij} is defined using the Saaty fundamental scale (see Table 1) which correlates the first nine integers with the judgments concerning the comparison.

Coefficients a_{ij} must should be equal to one on the main diagonal, reciprocal and positive to ensure the symmetry of the judgments of dominance. The coefficients of dominance define a square, positive and reciprocal matrix $\boldsymbol{A} = (\boldsymbol{a_{ij}})$ of pairwise comparisons. Matrix $\boldsymbol{A}$ should be as consistent as possible. If matrix $\boldsymbol{A}$ is completely consistent, then its main eigenvector is equal to n.

Coefficient a_{ij}	Comparison judgment
1	Equal importance between *i* and *j*
3	Weak prevalence of *i* versus *j*
5	High prevalence of *i* versus *j*
7	Demonstrated prevalence of *i* versus *j*
9	Absolute prevalence of *i* versus *j*
2, 4, 6, 8	Intermediate values

Table 1. Saaty fundamental scale

Next step is to define the local weights which measure the importance of the elements belonging to the same hierarchical level, subordinate to a higher hierarchical level element. Supposing as known the array W of local weights of the n elements, it is possible to calculate the coefficient of dominance of each pair of elements as the ratio of their respective weights:

$$\boldsymbol{a_{ij}} = \frac{\boldsymbol{w_i}}{\boldsymbol{w_j}} \tag{1}$$

where:

$$\boldsymbol{W} = \begin{matrix} \boldsymbol{w_1} \\ \vdots \\ \boldsymbol{w_n} \end{matrix} \tag{2}$$

with:

$$\sum_{i=1}^{n} \boldsymbol{w_i} = \boldsymbol{1} \tag{3}$$

Multiplying the matrix A by the array W we obtain:

$$\boldsymbol{AW} = \boldsymbol{nW} \tag{4}$$

i.e. for the matrix algebra, W is an eigenvector of the matrix A with eigenvalue equal to n.

Assuming as impossible to define the local weights in (2), an estimate of them can be entrusted to the judgment by one or more experts who, not disposing of instruments and/or measurement scales, will not be able to directly determine exact weights. On the contrary, they will provide local rough estimates of their relationships, by relying on personal experience and invoking the aid of the Saaty fundamental scale.

Matrix A, in general, will not be consistent because of the difficulties, for the experts, in maintaining the consistency of judgment in all pairwise comparisons (De Felice & Petrillo, 2010); usually (1) is not valid and, in order to determine the weight, array relevant properties of the matrices theory must be used (Saaty, 1986).

To solve such problem we just calculate the array W' which satisfies the equation:

$$\boldsymbol{AW'} = \boldsymbol{\lambda_{max}W'} \tag{5}$$

i.e. we simply determine the eigenvector associated to the principal eigenvalue λ_{max} and then we normalize W' on the sum of its elements by calculating the local weights, which are the components of the main eigenvector normalized on unit:

$$w_i = \frac{w_i'}{\sum_{i=1}^{n} w_i'} \quad (6)$$

It must ensure that the local weights, obtained by the resolution of (6), reflect the judgments of experts; in other words, it is necessary to examine to what extent the ratio w_i/w_j differs from the estimate a_{ij} provided by the expert. For this purpose are defined the Consistency Index *CI* and Consistency Ratio *CR*. If the consistency ratio *CR* exceeds a threshold value of 10%, experts must increase the consistency of their judgments by changing the coefficients a_{ij}.

The final step to rank a limited number of alternatives is the definition of global weights of the alternatives by applying the principle of hierarchical composition to determine the importance of the basic hierarchical elements with reference to the primary objective. The local weights of each element are multiplied by local weights of the corresponding superordinate elements and the products thus obtained are summed. Going on hierarchically, the local weights of all the elements are so transformed progressively into global weights. The global weights of the elements at the base of the hierarchy are the main result of the evaluation. The definition of the global weights is performed by processing the product between the $s \times r$ matrix consisting of *r* arrays of *s* alternatives local weights and of the $r \times 1$ array of the *r* local criteria weights; then it is obtained the $s \times 1$ array of global weights of the *s* alternatives, where $s \in [2, 7]$ (Saaty 2003). The global weights allow the general ranking.

2.2 The Analytic Hierarchy Process for absolute comparisons (AHP-A)

The Analytic Hierarchy Process permits, by absolute comparisons, the ranking of an unlimited number of alternatives. Cognitive psychology demonstrated that it is possible to operate two types of comparisons: absolute and relative. In absolute comparison we tend to compare alternatives with a standard, in the memory of the decision maker, developed through experience and knowledge of the phenomenon; relative comparisons are made by pairwise comparisons between options having a common attribute. Absolute comparisons are thus useful for the classification "one at time" of alternatives using "intensity" rating for each criterion, and this intensity rating can be expressed in quantitative or qualitative terms.

The AHP-A method needs a hierarchy consisting of at least 4 levels (see Figure 2).

In this case we obtain a $r \times 1$ array with local weights of *r* criteria and a $n \times 1$ array with local weight of *n* intensity.

Associating the *i-th* alternative ($i = 1, 2 \ldots s$) and the *j-th* intensity ($j = 1, 2 \ldots n$) relatively to the *k-th* criterion ($k = 1, 2 \ldots r$), it determines the $1 \times r$ array with local weights of the *i-th* alternative. The final step for the ranking of an unlimited number of alternatives is the definition of the global weight of the *i-th* alternative by applying the principle of hierarchical composition, i.e. processing the product of the $r \times 1$ array with local weights of the *r* criteria and the $1 \times r$ array with local weights of the *i-th* alternative. Iterating the above final step for each alternative we obtain the $s \times 1$ array with global weights of the *s* alternatives, where *s* is unlimited. The global weights allow the ranking of alternatives.

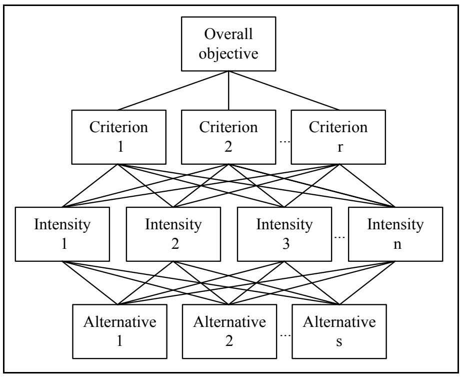

Figure 2. Four levels AHP-A hierarchy

3. THE VALUE - ANALYTIC HIERARCHY PROCESS (V-AHP)

This research work simplifies the traditional AHP methodology introducing the V–AHP version.

Studies on V-AHP arise from the need, highlighted by Saaty (Saaty, 1986), to set priorities by pairwise comparisons for both entities: intangible, by definition without scales, and tangible, evaluable on scales provided of "zero" point and measurement units. According to the creator of the AHP, a number has no meaning except the one assigned to it by those who are called upon to interpret it: we can all agree on a numeric value detected or calculated, but not on the utility that the value can have in each. Traditional AHP realizes ranking of alternatives giving *utility* to *judgments* - but also to *values* - related to the alternatives and measured respectively on qualitative and quantitative criteria. But in the ranking of alternatives on quantitative criteria, values are read on scale provided of "zero" point and measurement units; each ratio of this values is an absolute numbers and according to Saaty, absolute numbers are not subject to interpretation. Our vision is based on inter-criteria weighing and not on intra-criteria weighing.

The Value-Analytic Hierarchy Process allows the ranking of alternatives; it is the combination of traditional AHP rating on qualitative criteria and "lean" rating on quantitative criteria. The latter is obtained by the ratio between the value of performance related to the *i-th* alternative and the sum of performance values related to all the alternatives under consideration.

For the analytical discussion of the V-AHP we consider the array *L* having, as components, *n* quantitative performance values on ratio scale:

$$L = \begin{bmatrix} l_1 \\ \vdots \\ l_n \end{bmatrix} \quad (7)$$

The pairwise comparison, operated by the ratio of the *n* quantitative performance values, gives a matrix *B* having size $n \times n$, rank 1 and principal eigenvalue $\lambda_{max} = n$; columns of the matrix B are the linear combinations of n quantitative performance values.

$$\boldsymbol{B} = \begin{bmatrix} \frac{l_1}{l_1} & \cdots & \frac{l_1}{l_n} \\ \vdots & \ddots & \vdots \\ \frac{l_n}{l_1} & \cdots & \frac{l_n}{l_{nj}} \end{bmatrix} \quad (8)$$

It is therefore valid the following equation:

$$\boldsymbol{BL} = \boldsymbol{nL} \quad (9)$$

which can be expressed in a matrix form:

$$\begin{bmatrix} \frac{l_1}{l_1} & \cdots & \frac{l_1}{l_n} \\ \vdots & \ddots & \vdots \\ \frac{l_n}{l_1} & \cdots & \frac{l_n}{l_{nj}} \end{bmatrix} \begin{bmatrix} \boldsymbol{l_1} \\ \vdots \\ \boldsymbol{l_n} \end{bmatrix} = \boldsymbol{n} \begin{bmatrix} \boldsymbol{l_1} \\ \vdots \\ \boldsymbol{l_n} \end{bmatrix} \quad (10)$$

Array L, having as components the n quantitative performance values, is then, for the matrix B, the principal eigenvector associated with the principal eigenvalue $\lambda_{max} = n$.

Array W, with the local weights of the n quantitative performance values, normalized on unit according to (3), can be obtained by ratio of quantitative performance values, without the implementation of the traditional Saaty AHP procedure. In particular, the distributive rating is realized by the ratio between the value of performance related to the *i-th* alternative and the sum of performance values related to all the alternatives under consideration (11); the ideal rating is then obtained by operating the ratio between the quantitative performance value related to the *i-th* alternative and the maximum quantitative performance value among the n alternatives under consideration (12).

$$\boldsymbol{w_i} = \frac{\boldsymbol{l_i}}{\sum_{i=1}^{n} \boldsymbol{l_i}} \quad (11)$$

$$\boldsymbol{w_i^I} = \frac{\boldsymbol{l_i}}{\boldsymbol{max_{i=1}^{n}\{l_i\}}} \quad (12)$$

V-AHP operates according to the procedure in Figure 3.

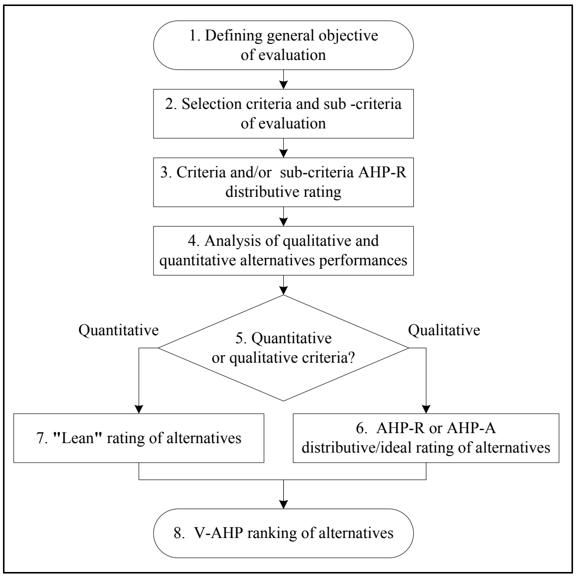

Figure 3. V-AHP procedure

Step 1 define the general objective of evaluation. In step 2 are defined qualitative and quantitative alternatives evaluation criteria and sub-criteria. Criteria distributive rating are calculated in step 3 by AHP-R i.e. resolving equation 5. Step 4 analyzes alternative performances on quantitative and qualitative criteria. Step 5 distinguishes between quantitative and qualitative criteria. If the criterion is qualitative then, in step 6, is calculated the distributive or ideal rating of alternatives by described AHP-R or AHP-A. Otherwise, if the criterion is quantitative, in step 7, is calculated the rating lean of alternatives by equation 11 or 12. Step 8 defines the V-AHP combining rating lean on quantitative criteria and distributive or ideal rating on quantitative criteria to obtain the alternatives global rating and ranking.

4. NUMERICAL EXAMPLE

A numerical example concerning Value-Analytic Hierarchy Process was performed according to the procedure described in Figure 3:

1. general objective of evaluation is the ranking of 15 potential simulated capital investment solutions;
2. criteria selected, in order to evaluate the capital investments solutions, are:
   - payback time (PBT);
   - discounted cash flow rate of return (DCFRR);
   - net present value (NPV);
   - entrepreneur strategic judgment (ESJ);

   PBT, DCFRR, NPV are criteria evaluable on ratio scales, provided of “zero” point and measurement units, where it is possible to read quantitative performance values. These criteria can be classified as “quantitative”; ESJ is a criterion evaluable on a non ratio scale where it is possible to define priorities assigning utility to expert judgments. This criterion can be classified as “qualitative”;
3. the distributive AHP-R rating of criteria was performed by traditional Analytic Hierarchy Process for relative comparisons, as shown in Table 2 and Table 3.

i/j	PBT	DCFRR	NPV	ESJ
PBT	1	1/3	1/5	1/7
DCFRR	3	1	1/3	1/5
NPV	5	3	1	1/3
ESJ	7	5	3	1

Table 2. Criteria pairwise comparisons matrix

Criterion [-]	Criterion type [-]	Distributive AHP-R rating [%]
PBT	Quantitative	5,53%
DCFRR	Quantitative	11,75%
NPV	Quantitative	26,22%
ESJ	Qualitative	56,50%

Table 3. Criteria name, type and AHP-R rating

4. results of qualitative and quantitative performances analysis is shown in Table 4 where alternatives are identified by ID number. Quantitative performance values are read on PBT, DCFRR, NPV criteria while qualitative judgments are read as level on ESJ criterion;

ID	PBT [yr]	DCFRR [%]	NPV [k€]	ESJ [-]
1	5,90	22,55%	2.342,61	High
2	6,81	16,03%	1.052,69	Low
3	6,14	20,45%	2.117,70	High
4	7,12	14,86%	796,38	Low
5	6,08	20,81%	2.113,48	Medium
6	5,52	26,37%	2.997,25	Medium
7	6,22	20,14%	1.978,51	Low
8	7,03	15,22%	897,48	Medium
9	5,96	21,70%	2.301,36	High
10	5,47	26,52%	2.972,36	High
11	7,99	12,64%	191,79	Medium
12	6,25	19,32%	1.854,70	High
13	5,48	26,81%	3.215,38	Medium
14	6,16	20,15%	2.007,25	Medium
15	5,61	25,41%	2.963,75	Medium

Table 4. Performance of alternatives

5. for PBT, DCFRR and NPV the V-AHP procedure will be used, for ESJ the traditional AHP will be used;

6. alternative ideal AHP-A rating is performed for absolute comparisons. First step of this phase, shown in Table 5, is the pairwise comparisons of three levels judgement (low, high, medium) on ESJ criterion. Second step of this phase, shown in Table 6, concerns the definition of AHP-R ideal rating intensity by traditional AHP-R procedure applied to mentioned three levels. Third step of this phase is shown in Table 7 and realizes the association between judgment and intensity AHP-R ideal rating, for each ID alternative.

i/j	Low	Medium	High
Low	1	1/5	1/9
Medium	5	1	1/5
High	9	5	1

Table 5. Intensity pairwise comparisons matrix

ESJ [-]	AHP-A ideal rating [-]
Low	0,0790
Medium	0,2811
High	1,0000

Table 6. Intensity AHP-A ideal rating

ID	AHP-A ideal rating on ESJ [-]
1	1,00000
2	0,07904
3	1,00000
4	0,07904
5	0,28115
6	0,28115
7	0,07904
8	0,28115
9	1,00000
10	1,00000
11	0,28115
12	1,00000
13	0,28115
14	0,28115
15	0,28115

Table 7. AHP-A alternative ideal rating

7. alternative ideal "lean" rating is calculated as described in §3. Maximum values of performance for NPV and DCFRR are required; thus, ideal "lean" rating was obtained by the ratio between the quantitative performance value related to *i-th* alternative and the maximum quantitative performance value among the 15 alternatives; for PBT the minimum performance value is required; then the same procedure was applied but on the inverse performances values. Results are shown in Table 8.

ID	Lean ideal rating on PBT [-]	Lean ideal rating on DCFRR [-]	Lean ideal rating on NPV [-]
1	0,92715	0,84126	0,72856
2	0,80318	0,59815	0,32739
3	0,89037	0,76285	0,65862
4	0,76839	0,55422	0,24768
5	0,89962	0,77630	0,65730
6	0,99135	0,98365	0,93216
7	0,87950	0,75134	0,61533
8	0,77785	0,56764	0,27912
9	0,91717	0,80967	0,71574
10	1,00000	0,98935	0,92442
11	0,68484	0,47167	0,05965
12	0,87536	0,72070	0,57682
13	0,99882	1,00000	1,00000
14	0,88790	0,75165	0,62427
15	0,97500	0,94806	0,92174

Table 8. Lean alternative ideal rating

8. alternative ideal V-AHP rating is the combination of "lean" ideal rating on quantitative criteria (Table 8) and AHP-A ideal rating on qualitative criteria (Table 7). V-AHP ideal rating is obtained by hierarchical composition between ideal rating of alternatives and distributive AHP-R rating of criteria, i.e., for each ID alternative, product by 1×4 array of alternative ideal rating and 4×1 array of criteria distributive rating. Table 9 summarizes the V-AHP ranking of alternatives, based on decreasing V-AHP ideal rating.

Rank	ID	V-AHP ideal rating [-]
1	10	0,97893
2	1	0,90615
3	9	0,89852
4	3	0,87656
5	12	0,84933
6	13	0,59377

7	6	0,57365
8	15	0,56583
9	5	0,47215
10	14	0,45994
11	7	0,34291
12	8	0,34174
13	11	0,26777
14	2	0,24519
15	4	0,21721

Table 9. V-AHP ranking of alternatives

5. CONCLUSIONS

The Value-Analytic Hierarchy Process simplifies the traditional Multiple Criteria Decision Analysis Analytic Hierarchy Process methodology.

As it was said in literature, there are many examples of application of AHP method in which, in order to operate the ranking of alternatives evaluated on qualitative and quantitative criteria, authors assign utility also to values on quantitative criteria. The Value-Analytic Hierarchy Process allows the decision analyst to limit the use of the traditional AHP procedure to cases in which it is really necessary, that is when performances of the alternatives are expressed by means of qualitative judgments and/or different scales from quantitative.

The numerical example, carried out in order to rank 15 potential simulated capital investment solutions evaluated on qualitative and quantitative criteria, is just a typical example of MCDA application, but the Value-Analytic Hierarchy Process is potentially applicable to any multi-criteria decision-making context (B2C, B2B, B2G), strategic but also operational, (think about managerial implications in terms of market attractiveness, suppliers selection, make or buy decision, procurement decision, resources allocation, etc.). As demonstrated in numerical example, it is easily implementable using common spreadsheet and does not require dedicated software.

For all the mentioned reasons, V-AHP can be defined a "Lean" Multi Criteria Decision Support Method.

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