Journal of Dynamical and Control Systems, Vol. 10, No. 3, July 2004, 303–314 (©2004)

DYNAMIC OLIGOPOLY WITH STICKY PRICES: CLOSED-LOOP, FEEDBACK, AND OPEN-LOOP SOLUTIONS

R. CELLINI and L. LAMBERTINI

Abstract. We investigate a dynamic oligopoly game with price adjustments. We show that the subgame perfect equilibria are characterized by larger output and lower price levels than the open-loop solution. The individual (and industry) output at the closed-loop equilibrium is larger than its counterpart at the feedback equilibrium. Therefore, firms prefer the open-loop equilibrium to the feedback equilibrium, and the latter to the closed-loop equilibrium. The opposite applies to consumers.

1. Introduction

The aim of this note consists in assessing comparatively the properties of open-loop, feedback, and closed-loop memoryless equilibria in a dynamic oligopoly model with price dynamics first introduced by Simaan and Takayama [16] and then extended by Fershtman and Kamien [8].

Broadly speaking, the main difference between the open-loop equilibrium on one hand and the feedback and closed-loop equilibria on the other is that the former does not take into account strategic interaction between players through the evolution of state variables over time and the associated adjustment in controls. Under the open-loop rule, players choose their respective plans at the initial date and commit to them forever. Therefore, in general, open-loop equilibria are not subgame perfect, in that they are only weakly time consistent since players make their action 'by the clock' only. For an exhaustive discussion of this issue, see [1, Chap. 6]. However, there are classes of games where open-loop equilibria are subgame perfect (see [2, 5, 7, 12, 14]). For a survey, see [10, 11].

A further distinction can be made between the closed-loop equilibrium and the feedback equilibrium, which are both strongly time consistent and, therefore, subgame perfect since, at any date τ, players decide 'by the stock'

2000 *Mathematics Subject Classification.* 49N70, 91A23, 91B24.

Key words and phrases. Pricing, differential games, loop decision rules, subgame perfection.

1079-2724/04/0700-0303/0

of all state variables. However, while the closed-loop memoryless equilibrium takes into account the initial and current levels of all state variables,[1] the feedback equilibrium accounts for the accumulated stock of each state variable at the current date. If one player decides according to the feedback rule, then it is optimal for the others to do so as well. Hence, the feedback equilibrium is a closed-loop equilibrium, while the opposite is not true in general.[2]

We extend the analysis of [8] to investigate the open-loop, closed-loop memoryless, and feedback equilibria of an industry with more than two players. Then, we characterize the closed-loop equilibrium for this market, to show the following results:

(i) both subgame perfect equilibria involve a larger production and a lower price as compared with the open-loop solution;
(ii) the steady state price and output levels are, respectively, higher and lower in the closed-loop equilibrium than in the feedback equilibrium.

Property (i) can be reformulated by saying that if firms are unable to initially commit to a given output plan for the whole time horizon, then subgame perfection entails overproduction (for analogous results see [15, 17]). Property (ii) suggests that the feedback rule allows firms to reduce overproduction as compared with the closed-loop rule, precisely because according to the feedback rule they look exclusively at the current level of the state variable.

The remainder of the paper is organized as follows. The model is laid out in Sec. 2. Sections 3 and 4, which illustrate the open-loop and feedback equilibria, are simply the extension of analysis made in [8] to the oligopoly case. The closed-loop equilibrium is analyzed in Sec. 5. A comparative assessment of steady states is given in Sec. 6. Section 7 concludes the paper.

2. Set-up

Probably the simplest way to think about the dynamics of market interaction consists in assuming that prices evolve over time according to some acceptable rules. That is, it consists in taking price as the state variable.

---

[1] The information set associated with the closed-loop decision rule can take several forms. One consists in the level of the state variable(s) at the intial and current dates. This is usually defined as the closed-loop memoryless decision rule. Another consists in the whole path of state variable(s) from the initial date to the present time. This is defined as a closed-loop perfect state information rule.

[2] For an exhaustive exposition of the difference among these equilibrium solutions, see [1, pp. 318–327, Chap. 6, Proposition 6.1].

This is the problem analyzed in [8, 16].[3] Here, we present a generalization of Fershtman and Kamien's set-up to the case of N firms.[4]

Consider an oligopoly where, at any $t \in [0, \infty)$, N firms produce quantities $q_i(t)$, $i \in \{1, 2, \ldots, N\}$, of the same homogeneous good at a total cost $C_i(t) = cq_i(t) - \frac{1}{2}[q_i(t)]^2$, $c > 0$.

In each period, market demand determines the price level

$$\widehat{p}(t) = A - \sum_{i=1}^{N} q_i(t).$$

However, in general, $\widehat{p}(t)$ will differ from the current price level $p(t)$, since there is price stickness, and price moves according to the following equation:

$$\frac{dp(t)}{dt} \equiv \dot{p}(t) = s\left\{\widehat{p}(t) - p(t)\right\}. \tag{1}$$

Note that the dynamics described by (1) establishes that price adjusts proportionally to the difference between the price level given by the inverse demand function and the current price level, where the speed of adjustment is determined by the constant s with $0 < s < 1$. This amounts to saying that the price mechanism is sticky, i.e., firms face menu costs in adjusting their price to the demand conditions deriving from consumers' preferences: they may not (and, in general, they will not) choose outputs so that the price reaches immediately $\widehat{p}(t)$.

The instantaneous profit function of the firm i is

$$\pi_i(t) = q_i(t) \cdot \left[p(t) - c - \frac{1}{2}q_i(t)\right]. \tag{2}$$

Hence, the problem of the firm i is

$$\max_{q_i(t)} J_i = \int_0^\infty e^{-\rho t} q_i(t) \cdot \left[p(t) - c - \frac{1}{2}q_i(t)\right] dt \tag{3}$$

subject to (1) and to the conditions $p(0) = p_0$ and $p(t) \geq 0$ for all $t \in [0, \infty]$.

We solve the problem by considering, in turn, the open-loop solution, the feedback solution, and the closed-loop memoryless solution.

3. Open-loop solution

Here we look for the open-loop Nash equilibrium, i.e., we examine a situation where firms commit to a production plan at $t = 0$ and stick to that plan forever.

---

[3]See also [11, Chap. 5] for an exhaustive exposition of both contributions, and [9, 19] for further results on the same model, in the case of a finite horizon.

[4]An interesting application of this model to the analysis of adverting strategies is in [13]. Trade policy issues are investigated by Dockner and Haugh [3, 4].

The Hamiltonian function is

$$\begin{aligned}\mathcal{H}_i(t) = e^{-\rho t} \cdot \Bigg\{ q_i(t) \cdot \left[p(t) - c - \frac{1}{2} q_i(t) \right] \\ + \lambda_i(t) s \left[A - \sum_{i=1}^{N} q_i(t) - p(t) \right] \Bigg\},\end{aligned} \tag{4}$$

where $\lambda_i(t) = \mu_i(t) e^{\rho t}$ and $\mu_i(t)$ is the co-state variable associated with $p(t)$. The supplementary variable $\lambda_i(t)$ is introduced to ease calculations as well as the remainder of the exposition. In the remainder of the paper, the superscript OL indicates the *open-loop* equilibrium level of a variable. The outcome of the open-loop game is summarized as follows.

Proposition 1. *At the open-loop Nash equilibrium, the steady state levels of the price and the individual output are*

$$p^{OL} = A - Nq^{OL}, \quad q^{OL} = \frac{(A-c)(s+\rho)}{(s+\rho)(1+N)+s}.$$

The pair $\{p^{OL}, q^{OL}\}$ *is a saddle point.*

Proof. Consider the first-order condition (FOC) w.r.t. $q_i(t)$, calculated using (4):

$$\frac{\partial \mathcal{H}_i(t)}{\partial q_i(t)} = p(t) - c - q_i(t) - \lambda_i(t) s = 0. \tag{5}$$

This yields the optimal open-loop output for the firm i as follows:[5]

$$q_i(t) = \begin{cases} p(t) - c - \lambda_i(t) s & \text{if } p(t) > c + \lambda_i(t) s, \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

The adjoint conditions for the optimum are:

$$\begin{aligned} -\frac{\partial \mathcal{H}_i(t)}{\partial p(t)} = -q_i(t) + \lambda_i(t) s = \frac{\partial \mu_i(t)}{\partial t} \quad \Rightarrow \\ \frac{\partial \lambda_i(t)}{\partial t} = \lambda_i(t)(s+\rho) - q_i(t),\end{aligned} \tag{7}$$

$$\lim_{t\to\infty} \mu_i(t) \cdot p(t) = 0. \tag{8}$$

Differentiating (6) and using (7), we obtain

$$\frac{dq_i(t)}{dt} \equiv \dot{q}_i(t) = \frac{dp(t)}{dt} - s\left[(\rho+s)\lambda_i(t) - q_i(t)\right]. \tag{9}$$

Now substitute into (9)

$$\frac{dp}{dt} = s\left\{\widehat{p}(t) - p(t)\right\}$$

[5]In the remainder, we consider the positive solution. Obviously, the derivation of the steady state entails nonnegativity constraints on the price and quantity that we assume to be satisfied.

with $\widehat{p}(t) = A - Nq(t)$, where a symmetry assumption is introduced for an individual firm output, and

$$s\lambda(t) = p(t) - c - q(t)$$

from (6). This yields

$$\frac{dq(t)}{dt} = sA + (s+\rho)c - (2s+\rho)p(t) + [s(1-N) + s + \rho]\, q(t). \qquad (10)$$

Note that $\frac{dq(t)}{dt} = 0$ is a linear relationship between $p(t)$ and $q(t)$. This, together with $\frac{dp(t)}{dt} = 0$, also a linear function, fully characterizes the steady state of the system. The dynamic system can be immediately rewritten in the matrix form as follows:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -s & -sN \\ -(2s+\rho) & s+\rho-s(N-1) \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} + \begin{bmatrix} sA \\ sA + (s+\rho)c \end{bmatrix}. \qquad (11)$$

Since the determinant of the above (2×2)-matrix is negative, the equilibrium point is a saddle, with

$$q^{OL} = \frac{(A-c)(s+\rho)}{(s+\rho)(1+N)+s}, \quad p^{OL} = A - Nq^{OL}. \qquad (12)$$

This concludes the proof. □

As in the duopoly case described by [8, pp. 1159–1161], here the static Cournot–Nash equilibrium price and the output $\{p^{CN}, q^{CN}\}$ are also obtained from (12), in the limit as $\rho \to 0$ or $s \to \infty$. For all positive levels of the discount rate and any finite speed of adjustment, the static Cournot price (output) is higher (lower) than the open-loop equilibrium price (output).

4. Feedback solution

In this section, we extend the analysis of the feedback solution made in [8] to the case of N firms. Using Bellman's value function approach, the feedback solution must satisfy the following set of Hamilton–Bellman–Jacobi equations (see [18]):

$$\begin{aligned} \rho V_i(p(t)) = \max_{q_i(t)} \Bigg\{ & q_i(t) \cdot \left[p(t) - c - \frac{1}{2} q_i(t) \right] \\ & + \frac{\partial V_i(p(t))}{\partial p(t)} s \left[A - \sum_{i=1}^{N} q_i(t) - p(t) \right] \Bigg\}, \end{aligned} \qquad (13)$$

where $V_i(p(t))$ is the value function for the firm i. In the sequel, the indication of time will be omitted to ease the exposition. Given the linear-quadratic form of the maximand, we follow [8], and propose the quadratic-value function:

$$V_i(p) = \frac{k_i p^2}{2} + h_i p + g_i \tag{14}$$

so that

$$\frac{\partial V_i(p)}{\partial p} = k_i p + h_i. \tag{15}$$

In the sequel, the superscript F stands for the *feedback*. The outcome of the game is summarized as follows.

Proposition 2. *At the feedback Nash equilibrium, the steady state levels of the price and individual output are*

$$p^F = \frac{A + N\left(c - \overline{h}s\right)}{N\left(1 - \overline{k}s\right) + 1},$$

$$q^F = \begin{cases} p^F\left(1 - s\overline{k}\right) + \overline{h}s - c & \textit{if } p^F > \dfrac{c - \overline{h}s}{1 - s\overline{k}}, \\ 0 & \textit{otherwise}, \end{cases}$$

where

$$\overline{h} = \frac{c - s(A - Nc)\overline{k}}{\rho + s\left(\overline{k}s - 2N\overline{k}s + N + 1\right)},$$

$$\overline{k} = \frac{\rho + 2s(N+1) - \sqrt{\rho^2 + 4s\left(\rho + N\rho + 2s + sN^2\right)}}{2(2N-1)s^2}.$$

Proof. Taking the FOC w.r.t. q_i, we obtain:

$$q_i^F = p - c - s\frac{\partial V_i(p)}{\partial p} = p - c - s(k_i p + h_i), \tag{16}$$

where we invoke the symmetry conditions $g_i = g$, $k_i = k$, and $h_i = h$, so that $q_i = q$ for all i. On the basis of (16) and (1), we find

$$p^F = \frac{A + N(c - hs)}{N(1 - ks) + 1}, \tag{17}$$

where h and k can be calculated by the following procedure. We can rewrite (13) as

$$\pi V(p) - \max\left\{\pi + \frac{\partial V(p)}{\partial p} s \frac{dp}{dt}\right\} = 0, \tag{18}$$

i.e.,

$$\beta_1 p^2 + \beta_2 p + \beta_3 = 0, \tag{19}$$

where

$$\beta_1 = \frac{k\left[\rho + s\left(2 + 2N + ks - 2ksN\right)\right] - 1}{2}, \tag{20}$$

$$\beta_2 = c - h(\rho + s + sN) - ks\left(A + Nc + hs - 2hsN\right), \tag{21}$$

$$\beta_3 = \frac{2g\rho - c^2 + hs\left(2A + 2Nc + hs - 2hsN\right)}{2}. \tag{22}$$

Equation (19) is satisfied if expressions (20)–(22), i.e., coefficients β_1, β_2, and β_3, are simultaneously zero. This gives a system of three equations in three variables, $\{g, h, k\}$, with the following solutions:

$$g = \frac{c^2 - hs\left(2A + 2Nc + hs - 2hsN\right)}{2\rho}, \tag{23}$$

$$h = \frac{c - s\left(a - Nc\right)\overline{k}}{\rho + s\left(\overline{k}s - 2N\overline{k}s + N + 1\right)} \equiv \overline{h}, \tag{24}$$

$$k = \frac{\rho + 2s\left(N + 1\right) \pm \sqrt{\rho^2 + 4s\left(\rho + N\rho + 2s + sN^2\right)}}{2\left(2N - 1\right)s^2}. \tag{25}$$

We must choose the smaller solution for k in (25): the larger solution is inconsistent with the stability of the steady state (see also [8, p. 1164]). For $N = 2$, expression (25) coincides with expression (3.2) obtained by Fershtman and Kamien [8, Theorem 2, p. 1157]. This concludes the proof. □

5. Closed-loop solution

It remains to investigate the closed-loop memoryless solution. We use the superscript CL to denote the *closed-loop* equilibrium levels of the relevant variables. The Hamiltonian of the firm i is given by (4), and the outcome is summarized by the following proposition.

Proposition 3. *At the closed-loop Nash equilibrium, the steady state levels of the price and the individual output are*

$$p^{CL} = A - Nq^{CL},$$
$$q^{CL} = \frac{\left(A - c\right)\left(\rho + sN\right)}{\left(N + 1\right)\rho + \left(N^2 + N + 1\right)s}.$$

The pair $\left\{p^{CL}, q^{CL}\right\}$ *is a saddle point.*

Proof. The first-order condition w.r.t. q_i, calculated using (4), obviously coincides with condition (5) calculated in the open-loop case:

$$\frac{\partial \mathcal{H}_i}{\partial q_i} = p - c - q_i - \lambda_i s = 0. \tag{26}$$

This yields the closed-loop output for the firm i as follows (again, in the remainder we will consider only the positive solution):

$$q_i^{CL} = \begin{cases} p - c - \lambda_i s & \text{if } \ p > c + \lambda_i s, \\ 0 & \text{otherwise.} \end{cases} \tag{27}$$

The adjoint conditions for the optimum are

$$-\frac{\partial \mathcal{H}_i}{\partial p} - \sum_{j \neq i} \frac{\partial \mathcal{H}_i}{\partial q_j} \frac{\partial q_j^{CL}}{\partial p} = \frac{\partial \lambda_i}{\partial t} + \rho \lambda_i. \tag{28}$$

Now assume that

$$\frac{\partial \mathcal{H}_i}{\partial q_j} = -\lambda_j s, \quad \frac{\partial q_j^{CL}}{\partial p} = 1. \tag{29}$$

Therefore,

$$\sum_{j \neq i} \frac{\partial \mathcal{H}_i}{\partial q_j} \frac{\partial q_j^{CL}}{\partial p} = -\sum_{j \neq i} \lambda_j s \tag{30}$$

is the additional term in the co-state equation, characterizing the strategic interaction among firms, which is not considered by definition in the open-loop solution (see, e.g., [6]). Equation (28) can be rewritten as

$$-q_i + \lambda_i s + \sum_{j \neq i} \lambda_j s = \frac{\partial \lambda_i}{\partial t} + \rho \lambda_i,$$

and, invoking the symmetry, we obtain

$$\frac{\partial \lambda}{\partial t} = -q + \lambda \left(\rho + N s \right). \tag{31}$$

Then we have the transversality condition

$$\lim_{t \to \infty} \mu_i \cdot p = 0. \tag{32}$$

Differentiating (27) w.r.t. time and using (31), we obtain:

$$\frac{dq_i}{dt} = \frac{dp}{dt} - s \left[(\rho + w) \lambda_i - q_i \right]. \tag{33}$$

Now substitute into (33) the expressions

$$\frac{dp}{dt} = s \left\{ \widehat{p} - p \right\}$$

with $\widehat{p}(t) = A - Nq$, where a symmetry assumption is introduced for an individual firm output, and

$$s\lambda = p - c - q,$$

which follows from (26). This yields

$$\frac{dq}{dt} = \rho \left(c - p + q \right) + s \left[A - p + q - N \left(p - c \right) \right]. \tag{34}$$

As in the open-loop case, $dq/dt = 0$ is a linear relationship between p and q. This, together with $dp/dt = 0$, which is also a linear function, yields

$$p^{CL} = A - \frac{N(A-c)(\rho + sN)}{(N+1)\rho + (N^2+N+1)s},$$
$$q^{CL} = \frac{(A-c)(\rho + sN)}{(N+1)\rho + (N^2+N+1)s} \tag{35}$$

as the unique steady state of the system.[6] We can immediately rewrite the dynamic system in the matrix form to verify that the pair $\{p^{CL}, q^{CL}\}$ is stable in the saddle sense. The proof of this is omitted for the sake of brevity. □

6. Comparative assessment of steady states

Now we can compare the steady state levels of price and individual output in the three cases analyzed above, as well as in the static case. We have the following result.

Proposition 4. *For all $s \in [0,1]$ and all $N \in [1, \infty)$, we have*

$$q^{CL} > q^{F} > q^{OL} > q^{CN},$$
$$p^{CN} > p^{OL} > p^{F} > p^{CL}.$$

The proof is straightforward. Confining our attention to the equilibria of the dynamic setting, all subgame perfect equilibria entail a higher (individual and industry) output and a lower market price than the open-loop equilibrium (which is not subgame perfect). Therefore, Proposition 4 produces a relevant consequence.

Corollary 5. *From the firms' viewpoint, the open-loop equilibrium is preferred to both the feedback equilibrium and closed-loop memoryless equilibrium. On the contrary, the closed-loop memoryless equilibrium is socially preferred to the feedback and open-loop equilibria.*

Fershtman and Kamien [8, pp. 1159–1161] also investigated the properties of the *limit* games, where the speed of adjustment s tends to infinity or ρ becomes zero. They established that, in such cases, the open-loop equilibrium coincides with the Nash equilibrium of the static game. However, considering an infinitely high speed of the price adjustment seems more a mathematical *curiosum* than a theoretically relevant case, where for $s > 1$, the instantaneous change in price is larger than the error $\widehat{p}(t) - p(t)$. If we confine to $s \in [0,1]$, then we obtain the following consequence of Proposition 1.

---

[6] Of course, we mean by "unique" the only steady state with positive price and output.

Corollary 6. *For $s = 1$,*

$$q^{OL} = \frac{(A-c)(1+\rho)}{(N+1)(1+\rho)+1},$$

which is greater than the static Cournot–Nash output

$$q^{CN} = \frac{A-c}{N+2}$$

for all $N \geq 1$ and all $\rho > 0$.

In the limit, $q^{OL} \to q^{CN}$ as $\rho \to 0$ for all admissible N and s.

Finally, one can verify what happens to the steady state levels of output as N tends to infinity, to verify the following consequence of Proposition 4.

Corollary 7. *As the number of firms becomes infinitely large, optimal individual output tends to zero independently of the solution concept.*

Therefore, if the market becomes perfectly competitive, open-loop, closed-loop, and feedback solutions coincide with the static Cournot–Nash solution, which itself reproduces the perfectly competitive outcome.

7. Concluding remarks

We have investigated the properties of a dynamic oligopoly game with sticky prices. Specifically, we have made two contributions with respect to the available literature. First, we have generalized the analysis presented in [8] to the case of N, rather than two, firms. Second, we have analyzed the case of the closed-loop (memoryless) solution concept, in addition to the open-loop and feedback solutions.

The foregoing analysis shows that the subgame perfection always entails larger output and lower price levels in steady state, as compared with the weakly time consistent open-loop solution. In particular, the individual and industry output associated with the closed-loop equilibrium is larger than its counterpart at the feedback equilibrium.

We make two further and related remarks. First, the foregoing analysis highlights that the larger the relevant information set, the larger the overproduction compared with the (commitment) open-loop equilibrium. The second is that among the subgame perfect solution concepts, the feedback rule turns out to be able to minimize the overproduction. Accordingly, while firms would prefer the open-loop equilibrium to the feedback equilibrium, and the latter to the closed-loop equilibrium, the opposite holds from the social-welfare standpoint.

References

1. T. Basar and G. J. Olsder, Dynamic noncooperative game theory. *Academic Press, San Diego* (1982); second edition (1995).
2. S. Clemhout and H. Y. Wan, Jr., A class of trilinear differential games. *J. Optim. Theory Appl.* **14** (1974), 419–424.
3. E. J. Dockner and A. A. Haug, Tariffs and quotas under dynamic duopolistic competition. *J. Int. Economics* **29** (1990), 147–159.
4. ———, The closed loop motive for voluntary export restraints. *Canad. J. Economics* **3** (1991), 679–685.
5. E. J. Dockner, G. Feichtinger, and S. Jørgensen, Tractable classes of nonzero-sum open-loop Nash differential games: Theory and examples. *J. Optim. Theory Appl.* **45** (1985), 179–197.
6. R. Driskill and S. McCafferty, Dynamic duopoly with adjustment costs: A differential game approach. *J. Econom. Theory* **69** (1989), 324–338.
7. C. Fershtman, Identification of classes of differential games for which the open-loop is a degenerated feedback Nash equilibrium. *J. Optim. Theory Appl.* **55** (1987), 217–231.
8. C. Fershtman and M. I. Kamien, Dynamic duopolistic competition with sticky prices. *Econometrica* **55** (1987), 1151–1164.
9. ———, Turnpike properties in a finite-horizon differential game: Dynamic duopoly with sticky prices. *Int. Econom. Review* **31** (1990), 49–60.
10. C. Fershtman, M. Kamien, and E. Muller, Integral games: Theory and applications. In: *Dynamic Economic Models and Optimal Control* (G. Feichtinger, ed.), Amsterdam, North-Holland (1992), pp. 297–311.
11. A. Mehlmann, Applied Differential Games. *Plenum Press, New York* (1988).
12. A. Mehlmann and R. Willing, On nonunique closed-loop Nash equilibria for a class of differential games with a unique and degenerate feedback solution. *J. Optim. Theory Appl.* **41** (1983), 463–472.
13. C. Piga, Competition in a duopoly with sticky price and advertising. *Int. J. Industrial Organization* **18** (2000), 595–614.
14. J. Reinganum, A class of differential games for which the closed loop and open loop Nash equilibria coincide. *J. Optim. Theory Appl.* **36** (1982), 253–262.
15. S. S. Reynolds, Capacity investment, preemption and commitment in an infinite horizon model. *Int. Econom. Review* **28** (1987), 69–88.
16. M. Simaan and T. Takayama, Game theory applied to dynamic duopoly problems with production constraints. *Automatica* **14** (1978), 161–166.
17. A. M. Spence, Investment strategy and growth in a new market. *Bell J. Economics* **10** (1979), 1–19.
18. A. W. Starr and Y. C. Ho, Nonzero-sum differential games. *J. Optim. Theory Appl.* **3** (1969), 184–208.

19. S. Tsutsui and K. Mino, Nonlinear strategies in dynamic duopolistic competition with sticky prices. *J. Econom. Theory* **52** (1990), 136–161.

(Received April 15 2003)

Authors' addresses:
R. Cellini
Università di Catania
Dipartimento di Economia e Metodi Quantitativi
Corso Italia 55, 95129 Catania, Italy
E-mail: cellini@unict.it

L. Lambertini
Università degli Studi di Bologna
Dipartimento di Scienze Economiche
Strada Maggiore 45 40125 Bologna, Italy
E-mail: lamberti@spbo.unibo.it