

## Research Article

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# A precise mass function in the excursion set approach

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**Abstract:** In the present paper, using previous results from Del Popolo papers, we show how the mass function evolution can be obtained in the framework of a spherical collapse model, which has been modified to take account of dynamical friction, the cosmological constant, and angular momentum which proto-structures acquire through tidal interaction with neighbouring ones. We found an improved barrier which is in excellent agreement with simulations. The quoted barrier is used to calculate the mass function. In the case of the  $\Lambda$ CDM paradigm, our mass function is in good agreement (within some %) with the mass function of Klypin's Bolshoi simulation for the virial mass range  $5 \times 10^9 - 5 \times 10^{14} h^{-1} M_{\odot}$ , and  $0 \lesssim z \lesssim 10$ . Similar agreement is obtained with Tinker's mass function, and Castorina's simulations.

**Keywords:** cosmology: theory - large scale structure of Universe - galaxies: formation

## 1 Introduction

The  $\Lambda$ CDM model has been proven to be very successful in fitting a large variety of data (Spergel 2003; Del Popolo 2007; Komatsu *et al.* 2011; Del Popolo 2013, 2014; Ade *et al.* 2015). At kpc-scales, the model is suffering problems like the cusp/core problem (Flores & Primack 1994; Cardone *et al.* 2011a,b; Cardone & Del Popolo 2012; Del Popolo 2012a,b; Del Popolo *et al.* 2013; Del Popolo & Hioteles 2014; Del Popolo *et al.* 2014 or the missing satellite problem (Moore *et al.* 1999; Hioteles & del Popolo 2013) (see Del Popolo & Le Delliou 2017, for a review). At large scales, the model is afflicted by the fine-tuning problem (Weinberg 1989; Astashenok & Del Popolo 2012) and the cosmic coincidence problem (*e.g.*, Velten *et al.* 2014). Another fundamental test of the  $\Lambda$ CDM model is the accurate prediction of the number density of dark matter halos per mass interval (see Del Popolo & Yesilyurt 2007; Hioteles & Del Popolo 2006; Hioteles & del Popolo 2013), dubbed mass function (MF). At low redshifts ( $z \leq 2$ ), the high mass end of the MF is very sensitive to cosmological parameters like  $\sigma_8^1$ , the

Universe dark matter (DM), dark energy (DE) content, (*i.e.*,  $\Omega_m$ , and  $\Omega_{\Lambda}$ ), the equation of state parameter  $w$ , and its evolution (Holder *et al.* 2001; Haiman *et al.* 2001; Weller *et al.* 2001; Majumdar & Mohr 2002). At redshifts larger than the one previously indicated, the MF is an important probe of the Universe reionization history (*e.g.*, Furlanetto *et al.* 2006), quasar abundance (*e.g.*, Haiman & Loeb 2001).

A simple model to get the MF is that of Press & Schechter (1974) (PS). In this model, initial fluctuations are spherical, and with Gaussian distribution, and their evolution is followed from the linear phase until collapse, using a spherical collapse model (SCM) (Del Popolo & Gambera 1998, 1999, 2000; Del Popolo *et al.* 2000; Del Popolo 2010, 2011; Del Popolo *et al.* 2013a,b,c). Since the density field is Gaussian one can calculate the probability that the overdensity  $\delta = \frac{\rho - \rho_b}{\rho_b}$  on a given scale exceeds the critical threshold in the spherical collapse,  $\delta_c$ . This constant quantity is proportional to the number of cosmic structures characterized by a density perturbation  $> \delta_c$ . Unfortunately, in the PS theory, the number of objects in the high mass tail is underpredicted, and overpredicted for objects in the low mass tail of the MF (*e.g.* Efstathiou *et al.* 1988; Gross *et al.* 1998; Jenkins *et al.* 2001; White 2002).

As shown in Del Popolo & Gambera (1998, Eq. 28, Figure 6), and Sheth *et al.* (2001) (SMT) the tidal field and shear modifies the collapse of a given region making  $\delta_c$  dependent on mass, and this change the mass function (Del Popolo & Gambera 1999, 2000) (SMT). A deeper analysis of the previous, semi-analytic models for the mass function

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**1**  $\sigma_8$  represents the linear power spectrum amplitude on a scale of  $8 h^{-1}$  Mpc.



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showed some problems: the PS MF, as already reported, overpredicts the MF at all masses except the low and high mass extremes (Reed *et al.* 2003), and even the ST MF overpredicts the haloes number at large masses (Lukic *et al.* 2007). In the case of the Bolshoi simulation (Klypin *et al.* 2011) (K11), the discrepancy is  $< 10\%$  at  $z = 0$ , in the mass range  $5 \times 10^9 - 5 \times 10^{14} M_\odot$  (K11), while at  $z = 10$  the ST MF gives  $\simeq 10$  times more haloes than simulations (K11).

Another important issue is that of the universality of the MF, namely, its independence on redshift and cosmology (Tinker *et al.* 2008; Crocce *et al.* 2010; Bhattacharya *et al.* 2011; Watson *et al.* 2013). In order to discuss this issue, we have to recall that halo identification is performed mainly by means of the Friends of Friends (FoF) algorithm which identifies halos by a percolation technique, connecting particles, within a certain distance (the linking length  $b = 0.15 - 0.2$ ) to each other, in the same halo, or the SO method. The SO method first finds the halo's centre from potential minimum or most bound particle to identify haloes with spheres reaching a threshold density, given with respect to either critical or background density. SO-based halo mass functions are not universal especially at higher  $z$ , while the FoF MF scales very closely to the universal behaviour (K11).

More recent simulations showed a non-universal behaviour also in the case of FoF MF when one requests a high precision between simulations and fitting formulas. The FOF MF of the Millennium simulation increases by 20% for the redshift range  $z = 0 - 10$  (Reed *et al.* 2007), and when corrected for "spurious FOF linking between haloes", it shows the same evolution (20%) in the range  $z = 0 - 1$  (Fakhouri & Ma 2008). Further studies on the FOF MF showed evidence of non-universality coming from redshift dependence (Bhattacharya *et al.* 2011; Crocce *et al.* 2010), or cosmology dependence (Courtin *et al.* 2011), and other showed an almost universal behaviour (Watson *et al.* 2013). This lead many authors to propose a redshift dependent FOF MF (*e.g.*, Bhattacharya *et al.* 2011; Crocce *et al.* 2010; Watson *et al.* 2013)

In this paper, we will use the model of Del Popolo & Gambera (1998, 1999, 2000), Del Popolo (2002a), and Del Popolo (2006a,b) to show how the evolution of the mass function can be obtained in a modified spherical collapse model, taking into account angular momentum acquired by proto-structures as a result of tidal interaction with neighboring ones, dynamical friction, and the cosmological constant. In Section 2, we discuss the barrier, in Section 3 the MF, and Sections 4, and 5, are devoted to results and conclusions.

## 2 Barrier

An improvement of the PS formalism to model the halo formation statistics is the "excursion set formalism" (ESF) (*e.g.*, Bond *et al.* 1991; Lacey & Cole 1993). The halo statistics is obtained from the average overdensity,  $\bar{\delta}(R_f)$ , within a window of radius  $R_f$ . If density perturbations are represented by a Gaussian density field  $\bar{\delta}(R_f)$  vs  $R_f$ , in a hierarchical universe, is a random walk (see Del Popolo & Yesilyurt 2007). In the ESF, a halo forms when the random walk crosses a threshold value, or barrier,  $\delta_c$ , or equivalently the mass variance  $S$ . The PS MF is reobtained in the ESF by studying random walks and flat barriers. In order to improve on the PS formalism, several years ago, random walks were considered with a non-flat barrier, usually dubbed moving barrier, since the barrier is changing (moving) with  $S$ . As shown by Del Popolo & Gambera (1998, 1999, 2000), in such barriers, the collapse threshold becomes mass dependent. Del Popolo & Gambera (1999), and Del Popolo & Gambera (2000) showed that the mass dependence solves the problems of the PS MF, suppressing the abundance of haloes of low mass, and increasing that of massive ones (with respect the PS MF). The new threshold (barrier) gave rise to a MF in good agreement with observations (see Del Popolo 2006a; Zhang *et al.* 2008). As already reported SMT found a similar barrier.

The Del Popolo's barrier and that of SMT are respectively given by

$$B(M) = \sqrt{a} \delta_c(z) \left( 1 + \frac{\beta}{a v^\alpha} \right) \quad (1)$$

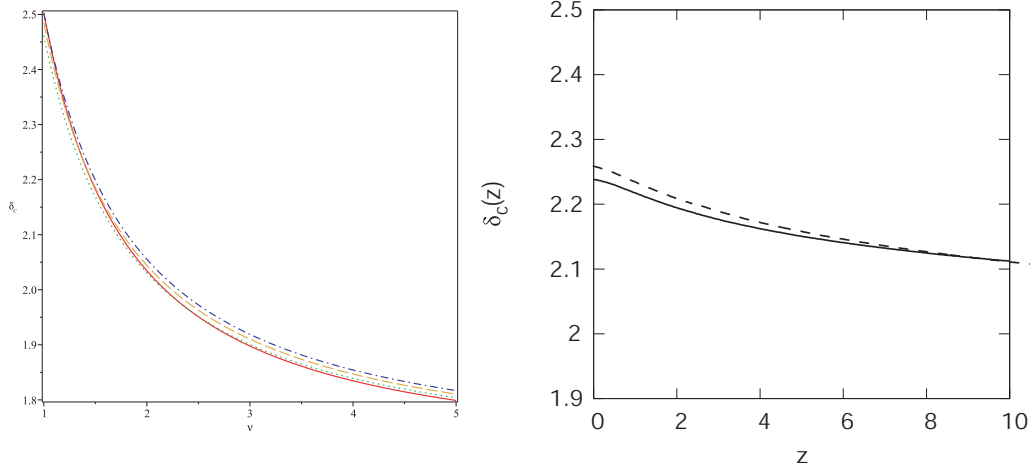
and

$$B(M)_{ST} = \sqrt{a_1} \delta_c(z) \left( 1 + \frac{\beta_1}{a_1 v^{\alpha_1}} \right) \quad (2)$$

where  $v = (\frac{\delta_c}{\sigma})^2$ ,  $\delta_c$  is the critical overdensity needed for collapse in the spherical model, and  $\sigma(M)$  is the r.m.s. density fluctuation, on a comoving scale including a mass,  $a = 0.67$ , and,  $a_1 = 0.707$ ,  $\alpha = 0.585$ ,  $\alpha_1 = 0.615$ ,  $\beta = 0.46$ , and  $\beta_1 = 0.485$ .

As shown by SMT, by changing the shape of the barrier, going from a flat barrier to a barrier increasing with  $S$ , allows to incorporate several physical effects, like fragmentation and mergers (SMT), apart the effects of tidal torques (Del Popolo & Gambera 1998), cosmological constant (Del Popolo 2006a), dynamical friction (see the next sections). The barrier given by Eq. (1) can be generalized to consider the effect of a cosmological constant and dynamical friction (DF) as was done in Del Popolo (2006b), and is given by

$$\delta_c = \delta_{co} \quad (3)$$



**Figure 1.** The collapse threshold  $\delta_c(v)$  vs  $v$ . Left panel: the solid line is the shows the SMT result, the short-dashed line that of Del Popolo & Gambera (1998), taking into account the effect of the tidal field, the long-dashed the result of Del Popolo (2006a,b) taking into account the effect of the tidal field, and the cosmological constant, and the dot-dashed line is  $\delta_c(v)$  taking into account the effect of the tidal field, the cosmological constant, and DF (Del Popolo 2006b). Right panel: the collapse threshold in terms of redshift when angular momentum and  $\Lambda$  is taken into account (solid line) for a mass  $M = 10^{11} h^{-1} M_\odot$ . The dashed line represents the same quantity when dynamical friction is taken into account.

$$\left[ 1 + \int_{r_i}^{r_{\text{ta}}} \frac{r_{\text{ta}} L^2 \cdot dr}{GM^3 r^3} + \frac{\lambda_o}{1 - \mu(\delta)} + \Lambda \frac{r_{\text{ta}} r^2}{6GM} \right] \simeq \delta_{\text{co}} \left[ 1 + \frac{\beta}{v^\alpha} + \frac{\Omega_\Lambda \beta_2}{v^{\alpha_2}} + \frac{\beta_3}{v^{\alpha_3}} \right]$$

where  $\alpha_2 = 0.4$ ,  $\beta_2 = 0.02$ ,  $\alpha_3 = 0.45$ ,  $\beta_3 = 0.07$ ,  $\Omega_\Lambda$  is the contribution to the density parameter coming from  $\Lambda$ ,  $\mu(\delta)$  is given in Colafrancesco *et al.* (1995, (Eq. 29)),  $\lambda_o = \epsilon_o T_{\text{co}}$ , and  $\epsilon_o$ , is proportional to the coefficient of DF  $\eta$  (see Eq. 23 of Antonuccio-Delogu & Colafrancesco 1994), and  $T_{\text{co}}$  is the collapse time of a perturbation without DF (see Eq. 24 of Antonuccio-Delogu & Colafrancesco 1994). The angular momentum,  $L$ , is calculated as shown in Del Popolo (2006b, 2009); Del Popolo & Kroupa (2009), while the term related to DF is obtained in Antonuccio-Delogu & Colafrancesco (1994).

Figure 1 (left panel) compares  $\delta_c(v)$  obtained by SMT by means of an ellipsoidal collapse model with the modified collapse thresholds obtained by Del Popolo. In the plot, the solid line represents the SMT result, Eq. (2) (with  $\alpha_1 = 1$ ), the dashed line Eq. (1) (with  $a = 1$ ) obtained by Del Popolo & Gambera (1998), the dashed line the improvement of Eq. (1) taking into account  $\Lambda$ , and the dot dashed line the improvement of Eq. (3) taking into account also the effect of DF.

All  $\delta_c(v)$  are monotonically decreasing function of  $v$ , and mass  $M$ , and monotonically increasing function of  $S$ . Their behaviours tend to the typical value of the spherical collapse ( $\delta_c = 1.686$ ) for large  $v$ . This imply that smaller objects are less likely to form structures than higher one,

since, in order less massive, "peaks" (in the initial random field) form structure, have to cross a higher threshold, in comparison with more massive ones (Peebles 1990; Del Popolo & Gambera 1996; Audit 1997; Del Popolo *et al.* 2001; Del Popolo 2002b,c; Del Popolo *et al.* 2013a,b; Pace *et al.* 2014)

The effect of a non-zero cosmological constant is similar to that of angular momentum, slowing down especially the collapse of structure of large mass. The role of the cosmological constant tends to vanish with increasing redshift,  $z$ . Then the final result is that the behaviour of the moving barrier reduces the small haloes abundance, in comparison with a flat barrier (PS mass function), and increases that of massive haloes. In the case of the ellipsoidal collapse of ST, this is due to the fact that smaller haloes are characterized by larger ellipticity, and consequently by a larger collapse time (see Zhang *et al.* 2008).

In addition to the  $\delta_c$  dependence on mass, one needs to take into account its time evolution. In Figure 1 (right panel), we plot  $\delta_c(z)$ , following Del Popolo *et al.* (2013a) for a  $\Lambda$ CDM model, taking into account the angular momentum and  $\Lambda$  for a mass  $M = 10^{11} h^{-1} M_\odot$  (for a generalization to DE models, see Del Del Popolo *et al.* 2013b; Pace *et al.* 2014). The dashed line adds the effect of dynamical friction. Angular momentum causes  $\delta_c$  to be a monotonic decreasing function of  $z$  (while already is the case for mass  $M$ ):  $\delta_c(z)$  is larger than  $\delta_c$  at all values of  $z$ . Thus, structure formation is "suppressed" at high  $z$  by angular momentum. This explains why our MF predicts a smaller

abundance than that of Sheth & Tormen (2002) (ST1) with increasing  $z$ .

### 3 The mass function

In ESF, the unconditional mass function,  $n(M, z)$ , namely, the average comoving number density of haloes in a mass range  $M, M + dM$  is given by:

$$n(M, z) = \frac{\bar{\rho}}{M^2} \frac{d \log v}{d \log M} v f(v) \quad (4)$$

(Bond *et al.* 1991), where  $\bar{\rho}$  is the background density. The quantity  $f(v)$  is the so-called "multiplicity function" or the distribution of the first crossings.

As shown by Sheth & Tormen (2002) (ST1), for a large range of moving barriers, one can approximate the first crossing distribution using the formula

$$f(S) dS = |T(S)| \exp\left(-\frac{B(S)^2}{2S}\right) \frac{dS/S}{\sqrt{2\pi S}} \quad (5)$$

where  $S \equiv S_*(\frac{\sigma}{\sigma_*})^2 = \frac{S_*}{v}$ , being  $\sigma_* = \sqrt{S_*}$ , and  $T(S)$  can be obtained using a Taylor expansion of  $B(S)$ :

$$T(S) = \sum_{n=0}^5 \frac{(-S)^n}{n!} \frac{\partial^n B(S)}{\partial S^n} \quad (6)$$

The multiplicity function, which is, as previously reported, the distribution of first crossings of a barrier  $B(v)$  by independent uncorrelated Brownian random walks (Bond *et al.* 1991), is given by  $v f(v) = 2S f(S, t)$ .

Then, knowing the barrier (e.g., Eq. (2)), the multiplicity function,  $v f(v)$ , can be obtained following ST1, namely using their equations Eqs. (5) and (6), truncated at fifth order  $n = 5$ , as by them shown:

$$v f(v) = A \left( 1 + \frac{\beta g(\alpha)}{(av)^\alpha} \right) \sqrt{\frac{av}{2\pi}} \exp \left\{ \frac{-av}{2} \left[ 1 + \frac{\beta}{(av)^\alpha} \right]^2 \right\} \quad (7)$$

where

$$g(\alpha) = \left| 1 - \alpha + \frac{\alpha(\alpha-1)}{2!} - \dots - \frac{\alpha(\alpha-1) \cdots (\alpha-4)}{5!} \right| \quad (8)$$

and  $\alpha = 0.6, \beta = 0.46, a = 0.707, A \simeq 1$ .

$$v f(v) \simeq \left( 1 + \frac{0.094}{(av)^{0.6}} \right) \sqrt{\frac{av}{2\pi}} \exp \left\{ -\frac{1}{2} av \left[ 1 + \frac{0.5}{(av)^{0.6}} \right]^2 \right\}, \quad (9)$$

which is a good approximation to the first crossing distribution of the ellipsoidal barrier obtained through the simulations of unconstrained, independent random walks and that fitting the GIF simulations (see Eq. 2 of Sheth & Tormen (2002), ST1)

$$v f(v) \simeq A \left( 1 + \frac{1}{(av)^{0.3}} \right) \sqrt{\frac{2av}{\pi}} \exp(-av/2), \quad (10)$$

where  $A = 0.322$ .

Applying the previous methods to the barrier given by Eq. (3), we get

$$v f(v) = A_1 \left( 1 + \frac{\beta_1 g(\alpha_1)}{(av)^{\alpha_1}} + \frac{\beta_2 g(\alpha_2)}{(av)^{\alpha_2}} + \frac{\beta_3 g(\alpha_3)}{(av)^{\alpha_3}} \right) \sqrt{\frac{av}{2\pi}} \exp \left\{ \frac{-av}{2} \left[ 1 + \frac{\beta_1}{(av)^{\alpha_1}} + \frac{\beta_2}{(av)^{\alpha_2}} + \frac{\beta_3}{(av)^{\alpha_3}} \right]^2 \right\} \quad (11)$$

which writing explicitly the constants  $\alpha_1, \alpha_2$ , and  $\alpha_3$  becomes

$$v f(v) \simeq A_1 \sqrt{\frac{2av}{\pi}} \left( 1 + \frac{0.1218}{(av)^{0.585}} + \frac{0.0079}{(av)^{0.4}} + \frac{0.1}{(av)^{0.45}} \right) \times \exp \left\{ -0.4019 a \sqrt{v}^{2.12} \left[ 1 + \frac{0.5526}{(av)^{0.585}} + \frac{0.02}{(av)^{0.4}} + \frac{0.07}{(av)^{0.45}} \right]^2 \right\}, \quad (12)$$

where  $A_1 = 0.93702$  and  $a=0.707$ . The CDM spectrum used in the present paper is that of Bardeen *et al.* (1986) (equation (G3)). It is important to stress that all numerical constants (except  $a$ ) derive from barrier calculations. The condition

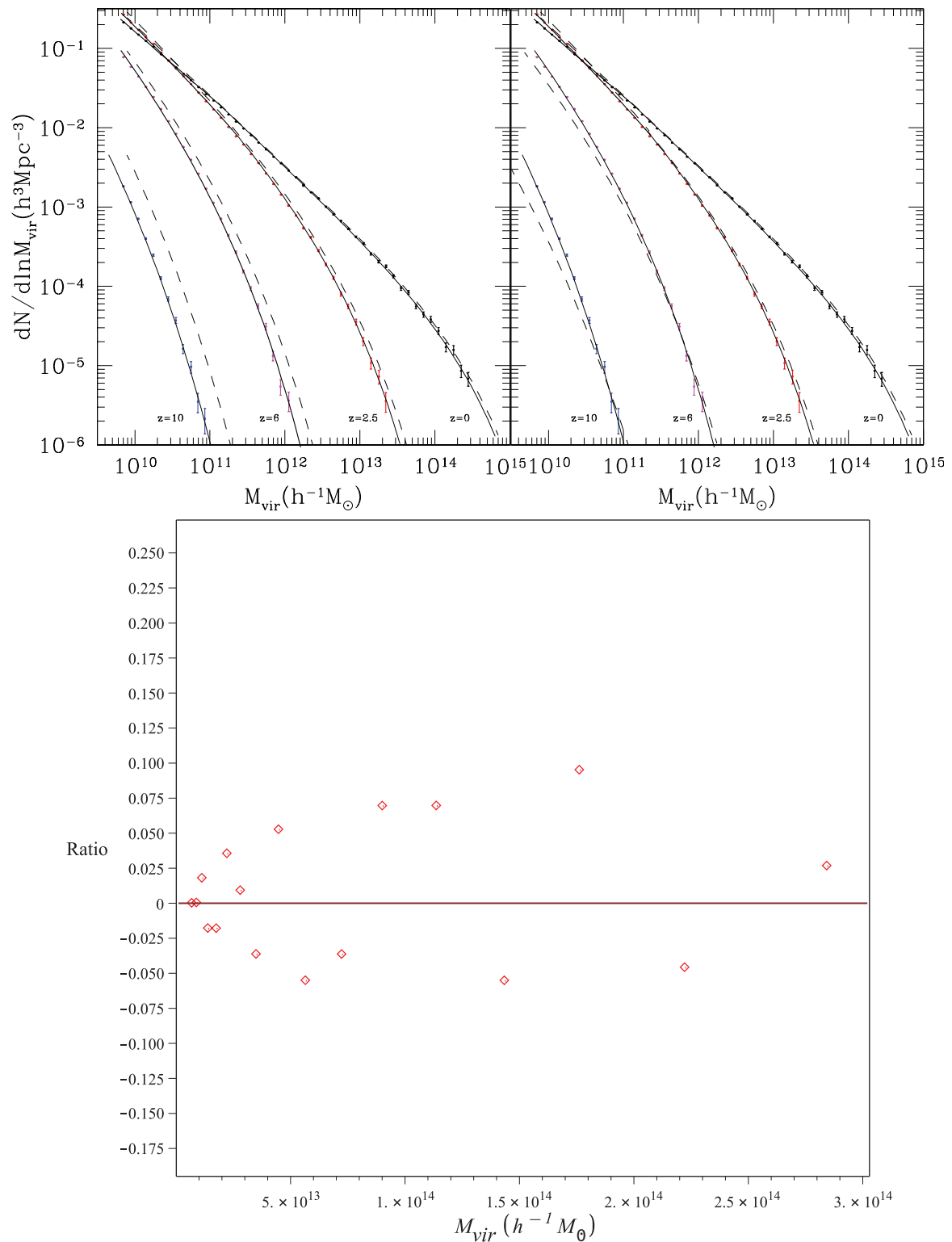
$$\int_0^\infty f(v) dv = 1. \quad (13)$$

gives the normalization constant  $A$ . Although the parameter  $a$ , which gives the number of high mass haloes, could also be obtained by the excursion set theory with a diffusing barrier, as shown by Maggiore & Riotto (2010), it was determined as a fit to the massive haloes number in the simulations of ST.

## 4 Results

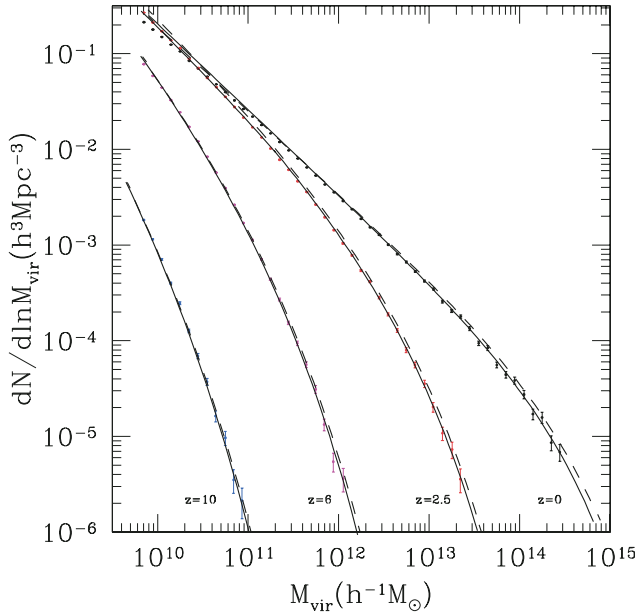
In the present section, we will compare our mass function with the result of K11, in the redshift range  $0 < z < 10$ , and with Tinker *et al.* (2008) (T08).

In Figure 2 (left panel), the solid line plots our MF given by Eq. (12), the dashed line Eq. (10), namely the ST1



**Figure 2.** Comparison of the mass functions of ST1 and this work with the Bolshoi MF. Left panel: the dashed line represents the ST1 MF, while the solid line ours. Diamonds represent the Bolshoi MF. From left to right  $z$  receives the values 10, 6, 2.5, 0. Right panel: Curves and symbols are as in the left panel, except that now the dashed line represents the correction by Klypin to the ST MF. Bottom panel: the ratio between the Klypin's data and our mass function.





**Figure 3.** Same as in Figure 2 but for Tinker's MF.

MF, and the diamonds with error-bars represent the MF obtained in the Bolshoi simulation by K11. The  $z$  dependence in the MF of this paper, comes just from  $\delta_c(z)$ . At  $z = 0$ , rightmost curves, the ST1 MF deviates from the simulation data in less than 10% in the mass range  $M_{\text{vir}} \simeq 5 \times 10^9 - 5 \times 10^{14} h^{-1} M_{\odot}$ . Going to higher redshifts ST1 overpredicts the simulation results, and the overprediction increases with increasing redshift. At  $z = 6$ , and for masses  $M_{\text{vir}} \simeq 1 - 10 \times 10^{11} h^{-1} M_{\odot}$ , ST1 predicts 1.5 more halos than simulations, and the situation is much worse at  $z = 10$  since the ST1 MF predicts 10 times more haloes than simulations. Our mass function (solid line), is in good agreement with the simulations with maximum deviations of  $\simeq 3\%$  calculated by average on each curve from the central values, intersecting all the errorbars.

The reason our MF is in better agreement with simulations than the ST1 mass function is given by the following reasons. ST1 introduced the effects of asphericity considering an intuitive parameterization of an elliptical collapse without considering the interaction with neighbors (isolated spheroid), and not the effect of  $\Lambda$ . So, while their model is an improvement on the PS model based on spherical collapse, the improvement is partial, and the MF shows the quoted discrepancies with simulations. In our model, angular momentum acquisition through tidal torques, and  $\Lambda$  is taken into account, together with the effects of dynamical friction. Moreover, ST1 did not consider the  $z$  dependence of  $\delta_c$  (as done in many other recent papers). The previous improvements give rise to a MF in good agreement with simulations, as we will detail in the fol-

lowing paragraphs. Moreover, barriers like ours, increasing with  $S$  allow mergers, and fragmentation. So, the precise MF is related to the barrier, whose shape depends on the effects of dynamical friction, those of the cosmological constant, and those due to the angular momentum.

Klypin, as previously done by several authors (e.g., Reed *et al.* 2003), proposed an improvement to the ST mass function, multiplying it for a correction factor

$$F(\delta) = \frac{(5.501\delta)^4}{1 + (5.500\delta)^4} \quad (14)$$

According to K11, the corrected mass function gives a MF having deviations smaller than 10 % from simulations, in the mass range  $5 \times 10^9 - 5 \times 10^{14} h^{-1} M_{\odot}$ . This is shown in Figure 2 (right panel), by plotting the comparison of the Bolshoi data with the K11 correction (dashed line), and the result of our model (solid line). As it is evident, our MF gives a much better result than the K11 correction. The bottom panel of Figure 2 shows the ratio between K11 data and our mass function.

As already reported, our result is in agreement with K11 simulation with T08, of Cohn & White (2008). Our result, similarly to that of the previous ones, shows an increasing overprediction of the ST1 MF going to larger masses and redshifts, together with a steepening of the MF with mass.

T08 proposed a new SO MF, and discussed its non-universality. The Tinker  $f(\sigma)$  function is given by

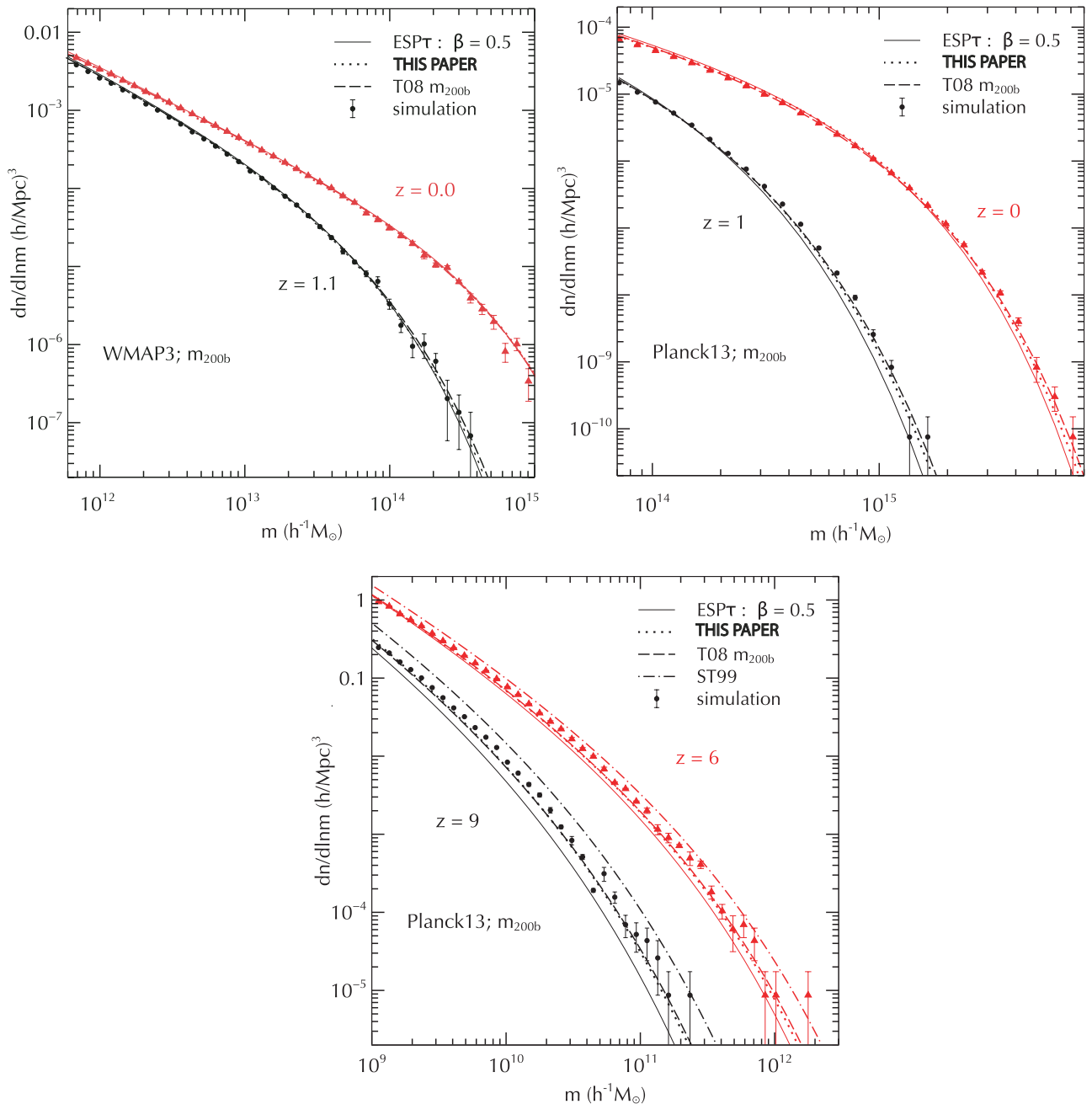
$$f(\sigma) = \frac{A}{(1+z)^{0.14}} \left[ \left( \frac{b}{(1+z)^a} \frac{a}{\sigma} \right)^{\frac{a}{(1+z)^{0.06}}} + 1 \right] e^{-c/\sigma^2} \quad (15)$$

where for  $\Delta = 200^2$   $A = 0.186$ ,  $a = 1.47$ ,  $b = 2.57$ ,  $c = 1.19$ , and  $\alpha = 0.0106$ .

Figure 3 shows the comparison of T08 mass function (dashed line) with the Bolshoi data, and our result (solid line). The previous result shows that the MF generated from our barrier is in good agreement both with simulations at  $z = 0$ , and with its redshift evolution, with a precision of the order of 3%.

Finally, in Figure 4, we compare the mass function of this paper with recent results of Castorina *et al.* (2016). In the top panel, the solid line represents Eq. 15 in Castorina *et al.* (2016) (ESP $\tau$  model), the points with errorbars shows the simulation in the same paper for WMAP3 (left panel) and Planck13 (right panel) cosmologies. The dashed line is T08, and the dotted line the MF of this paper. The comparison is made for two redshifts  $z = 0$ , and  $z = 1.1$ . The

**2**  $\Delta$  represents the overdensity in a sphere of a given radius,  $R_{\Delta}$  at a given epoch,  $\Delta = \frac{M_{\Delta}}{\frac{4\pi}{3} R_{\Delta}^3 \bar{\rho}}$



**Figure 4.** Comparison of the mass function of this paper with other mass functions at  $z = 0$ , and  $z = 1.1$ . The solid line represents Eq. 15 in Castorina *et al.* (2016) (ESP $\tau$  model), the points with errorbars the simulation in the same paper for WMAP3 (left panel) and Planck13 (right panel) cosmologies. The dashed line is T08, and the dotted line the MF of this paper. The bottom plot represents the same quantities for  $z = 6$ , and  $z = 9$ . The dot-dashed line represents the ST1 MF.

plot in the bottom panel represents the same quantities for  $z = 6$ , and  $z = 9$ . The dot-dashed line represents the ST1 MF. Figure 4, confirms the previous results. ST mass function overpredicts the simulations MF, while T08 is in good agreement with them. Castorina *et al.* (2016) MF underpredicts the halo abundance, and the problem increases with

$z$ . Our MF is in good agreement with the simulations data and T08.

Multiplicity functions like that of Klypin, and Tinker, or MF data in Castorina *et al.* (2016), differently from ours (Eq. 12), are produced by high resolution N-body simulations fits, similar in functional forms to SMT, Jenkins *et al.* (2001). These fits have no theoretical foundations, reveal-

ing the importance of obtaining a realistic analytical form from first principles. Such form is both able to better "describe" simulations and physically motivated. The MF (Eq. 12) obtained in this paper does provide an excellent prediction of high resolution simulations, and at the same time, derives from solid physical and theoretical arguments.

On top of this theoretical advantage, our approach can very accurately predict the dark matter halo distribution at much lower computational cost than high resolution simulations. This is because we can derive its functional form without having to rely on numerical results: it follows directly by using an improved barrier.

In conclusion, the excursion set approach, with a structure formation physics motivated barrier, produces an excellent approximation to the numerical multiplicity function: improving the barrier form (with more and more physical effects: angular momentum acquisition, non-zero cosmological constant, etc.) and increases the approximation accuracy. Moreover, this method displays a remarkable versatility: any effect, such as the presence of a non-zero cosmological constant, is very easy to take into account by embedding it in the barrier.

The role of angular momentum in shaping the MF was discussed in section 2, where we showed that it reduces or prevents structure formation, especially at small scales.

The term involving the cosmological constant has the same effect as those involving angular momentum and DF, namely, slowing down the collapse (Del Popolo & Gambera 1997, 1998, 1999; Del Popolo 2002a). This gives rise to a delay in large-scale structure formation, reducing their abundance and steepening the MF. This would also produce a larger proportion of high- $z$  haloes, that would be smaller than the resolution of simulations Feyereisen (2015). At the same time the cosmological constant clearly decreases the number of halos in the high-mass tail, (Feyereisen private communication and paper in preparation).

Dynamical friction also slows down the collapse, similarly to the cosmological constant. Of the three effects taken into account, angular momentum is the strongest in slowing down the collapse, followed by dynamical friction.<sup>3</sup>

Before concluding, we want to point out that the agreement between our MF and the Bolshoi simulation data could be further improved assuming a slight redshift dependence of  $A_2$  and  $\alpha$ , as done in many of the papers cited in the Introduction.

<sup>3</sup> For an alternative description of the effects of tidal shear and angular momentum for the  $\Lambda$ CDM and dark energy models, we refer to Reischke *et al.* 2016a,b; Pace *et al.* 2014.

## 5 Discussion

In the present paper, we selected physical effects (tidal field, cosmological constant, and dynamical friction) known to play an important role in the shaping of the MF and explained why the PS MF gives bad fits to the observed MF, and why the ST MF has problems in reproducing it at high  $z$ . At the same time, the approach leaves us with a semi-analytical form of the MF in very good agreement with simulations. In this sense our result is much more physical than that of simulations in general and gives a fit to their results.

The paper shows that the introduction of a moving barrier makes the collapse threshold mass-dependent, contrary to the standard spherical collapse model, but in parallel with extended models where shear, tidal fields and/or angular momentum are taken into account.

An interesting feature of a moving barrier is the possibility to introduce effects such as mergers, tidal torques, dynamical friction, and cosmological effects such as the cosmological constant (note that, as it is also pointed out by Murray *et al.* (2013), ST, SMT, and ST1, Spedicato *et al.* 2003 fitted their mass function with an EdS model).

The effect of introducing the cosmological constant remains minor compared with other effects such as tidal fields and angular momentum, but both the cosmological constant and the angular momentum slow down the collapse.

The positive consequence of these aspects is to solve the PS approach problems, in particular to reduce (increase) the number of objects at low (high) mass.

A similar result has been found for the ellipsoidal collapse in SMT.

The barrier for the first crossing shapes the mass function and modify its functional form with respect to the simple PS formulation. The improved mass function yields results in very good agreement with N-body simulations (T08; K11; Castorina *et al.* 2016 simulation), within 3% level at  $z = 0$  and during its time evolution.

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