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Procedia Engineering 199 (2017) 900–905

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

Crack localization in beams by frequency shifts due to roving mass with rotary inertia

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Abstract

The problem of crack localization has been tackled in the past by detecting frequency changes due to the application of a mass appended to the structure. However the effect of the rotary inertia of the mass on the natural frequencies of a cracked beam has not been deeply investigated. In this paper a novel explicit closed form solution of the governing equation of a beam with a concentrated mass, with rotary inertia, in the presence of multiple cracks is proposed. Furthermore, an analytical proof to show that the natural frequencies of a cracked beam with a roving body with a rotary inertia will generally change abruptly as the body passes over a crack, provided that the crack permits differential flexural rotations, is presented. Numerical results in terms of natural frequencies are provided and the procedure to exploit the occurrence of frequency shifts to detect and locate each crack is described.

Keywords: Crack localization; Concentrated mass; Rotary inertia; Generalised functions; Inverse problems.

1. Introduction

The problem of damage identification has been efficaciously addressed in the past by using vibrational measurements as described in the comprehensive review by Doebling et al. [1]. Mainly natural frequencies are monitored and precisely changes of frequency are usually understood as damage indicators. Unfortunately measurable frequency changes from the healthy to the damaged structure occur only in presence of significant

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1877-7058 © 2017 The Authors. Published by Elsevier Ltd.
Peer-review under responsibility of the organizing committee of EURODYN 2017.
10.1016/j.proeng.2017.09.229

damage and they are strongly affected by environmental noise and experimental uncertainties. With regard to beam-like structures, mode shape measurements have been proposed as additional data for crack identification in view of their capability to locate discontinuities at the cracked cross sections [2,3]. Moreover, mode shape changes induced by the cracks have been enhanced by the use of an appended roving mass in conjunction with the wavelet coefficients when applied to the differences between the undamaged and the damaged beams [4-7]. However, the adoption of mode shape measurements implies both analytical and experimental complications.

Another major issue related to the identification of concentrated cracks on beams is the a priori knowledge of the total number required by the available procedures. Hence a methodology able to deliver the exact number of concentrated cracks, as usually required by real applications, is desirable and presented in this work.

Precisely, the basic idea of this paper is considering a roving mass (at different positions at each test and not allowed to move axially during the frequency measurement) by considering the effect of its rotational inertia on frequency changes on a cracked beam. In fact, since a concentrated crack produces a rotation discontinuity at the cracked cross section, the rotational inertia of the concentrated mass induces a shift of the natural frequencies as the mass passes from one side to another of the crack. In order to prove the latter statement the exact solution of a beam with multiple cracks in presence of a concentrated mass (the translational and rotational inertia terms are both included) is presented. The latter solution is obtained by modelling each crack as an internal elastic rotational spring and making use of the distribution theory to deal with the presence of discontinuities [8,9].

A crack detection procedure, relying on the above mentioned frequency shifts, without any a priori knowledge of the number of cracks, is hence proposed. The procedure does not require any test execution on the healthy beam as well as evaluation of its dynamic properties.

A numerical investigation to show the performance of the proposed procedure based on the occurrence of frequency shifts to detect and locate each crack is finally presented.

2. A multi-cracked beam with a roving mass with rotary inertia

The use of generalised functions, already adopted by some of the authors for cracked beams with no additional masses [8,9], has been proved to be an efficient approach to deal with along axis singularities in beams. In this section the generalized function approach is extended to account for the presence of a roving mass with translational and rotary inertia on beams with multiple cracks. The solution, proposed in explicit closed form, is unique and is provided in terms of four (boundary conditions dependent) integration constants only, that does not require any additional node, or continuity condition, at the cracked sections and at the mass location.

The proposed explicit closed form solutions allows to prove that frequency shifts occur at cracked cross sections induced by a concentrated roving mass with rotary inertia.

Let us consider an Euler-Bernoulli beam, referred to a spatial abscissa x spanning from 0 to the length L, with flexural stiffness EI and distributed mass m, in presence of multiple cracks at $x_i, i=1,\ldots,n$, and an additional concentrated mass M_o with an eccentricity e at x_o. The presence of a concentrated mass M_o along a beam axis, besides inducing a translational inertial force, activates also a rotational contribution accounting for the rotary inertia effect thus providing an inertia of $M_o e^2$.

The free vibration mode $\phi(\xi)$ of the beam is governed by the following fourth order differential equation:

$$\phi^{IV}(\xi)-\alpha^4\phi(\xi)=\alpha^4 m_0\phi(\xi_0)\delta(\xi-\xi_0)-\alpha^4 m_0\varepsilon^2\delta'(\xi-\xi_o)\phi'(\xi_o)+\sum_{i=1}^{n}\lambda_i\phi''(\xi_i)\delta''(\xi-\xi_i) \tag{1}$$

where $\delta(\bullet)$ is the well known Dirac's delta distribution and the apex indicates spatial differentiation with respect to the normalised abscissa $\xi = x/L$. Furthermore, in Eq.(1), $\alpha^4=\omega^2 mL^4/EI$ is the frequency parameter with ω circular frequency, while $m_o = M_o/(mL)$ represents the concentrated mass normalised with respect to the beam mass density and the beam length; finally $\varepsilon=e/L$ is the eccentricity of the mass normalised with respect to the beam length.

Eq.(1) has been specially deviced to account for the influence of rotary inertia and translational inertia of the mass M_o by means of the terms $-\alpha^4 m_0\varepsilon^2\delta'(\xi-\xi_o)\phi'(\xi_o)$ and $\alpha^4 m_0\phi(\xi_0)\delta(\xi-\xi_0)$, respectively. Furthermore, the summation term on the right side of Eq.(1) is due to the influence of the unknown rotational discontinuities $\Delta\phi'(\xi_i) = \lambda_i \phi''(\xi_i)$ at the cracked cross sections with $\lambda_i = EI/(K_i L)$ being the normalised crack compliance that accounts for the spring rotational stiffness K_i equivalent to the *i*-th crack severity.

Eq.(1) can be straightforwardly and efficaciously integrated by employing the Laplace transform procedure to deal with the terms containing generalised functions. The solution is provided in an explicit form as follows:

$$
\begin{aligned}
&\phi(\xi) = C_1 f_1(\xi,\alpha) + C_2 f_2(\xi,\alpha) + C_3 f_3(\xi,\alpha) + C_4 f_4(\xi,\alpha) \\
&\underset{j=1,\dots 4}{f_j(\xi,\alpha)} = h_j(\xi,\alpha) + \bar{h}_o(\xi,\alpha)\alpha^4 m_o f_j(\xi_o,\alpha) + \bar{h}_R(\xi,\alpha)\alpha^4 m_0 \varepsilon^2 f_j'(\xi_o,\alpha) + \sum_{k=1}^{n} \bar{h}_k(\xi,\alpha)\lambda_k f_j''(\xi_k,\alpha) \\
&h_1(\xi,\alpha) = \sin\alpha\xi; \quad h_2(\xi,\alpha) = \cos\alpha\xi; \quad h_3(\xi,\alpha) = \sinh\alpha\xi; \quad h_4(\xi,\alpha) = \cosh\alpha\xi; \\
&\bar{h}_o(\xi,\alpha) = \frac{1}{2\alpha^3}\left[-\sin\alpha(\xi-\xi_o) + \sinh\alpha(\xi-\xi_o)\right]U(\xi-\xi_o) \\
&\bar{h}_R(\xi,\alpha) = \frac{1}{2\alpha^2}\left[\cos\alpha(\xi-\xi_o) - \cosh\alpha(\xi-\xi_o)\right]U(\xi-\xi_o) \\
&\bar{h}_k(\xi,\alpha) = \frac{1}{2\alpha}\left[\sin\alpha(\xi-\xi_k) + \sinh\alpha(\xi-\xi_k)\right]U(\xi-\xi_k)
\end{aligned}
\tag{2}
$$

In particular, the functions $h_1(\xi,\alpha), h_2(\xi,\alpha), h_3(\xi,\alpha), h_4(\xi,\alpha)$, represent the solution of the homogeneous beam, while $\bar{h}_o(\xi,\alpha)$, $\bar{h}_R(\xi,\alpha)$, $\bar{h}_k(\xi,\alpha)$, $k = 1,\dots,n$, represent additional functions able to account for the presence of the translational mass, the rotary inertia, and the cracks, respectively.

The integration constants C_1, C_2, C_3, C_4 appearing in the solution in Eq.(1) can be obtained by imposing the relevant boundary conditions. In particular, for the case of a clamped-clamped beam, fully analyzed in the following applications, the following frequency characteristic equation is derived:

$$
\left[f_3(1,\alpha) - f_1(1,\alpha)\right]\left[f_4'(1) - f_2'(1,\alpha)\right] - \left[f_4(1,\alpha) - f_2(1,\alpha)\right]\left[f_3'(1,\alpha) - f_1'(1,\alpha)\right] = 0 \tag{3}
$$

Eq.(3) can be solved numerically in terms of the frequency parameter α. The roots of the equation provide the fundamental frequencies of the beam, and allow to compute the integration constants and to derive the corresponding mode shapes.

Based on Eq.(3), it can be proved that the frequency equation of a cracked beam undergoes an abrupt change when a concentrated mass with rotary inertia crosses a crack, that implies the occurrence of a frequency jump.

In fact, for the case of a clamped-clamped beam with a single crack ($n = 1$) at ξ_1, the $f_j(1,\alpha), j = 1,\dots,4,$ functions and their first derivatives appearing in Eq.(3) can be written as:

$$
\begin{aligned}
f_j(1,\alpha) &= h_j(1,\alpha) + \bar{h}_o(1,\alpha)\alpha^4 m_o f_j(\xi_o,\alpha) + \bar{h}_R(1,\alpha)\alpha^4 m_0 \varepsilon^2 f_j'(\xi_o,\alpha) + \bar{h}_1(1,\alpha)\lambda_1 f_j''(\xi_1,\alpha) \\
f_j'(1,\alpha) &= h_j'(1,\alpha) + \bar{h}_o'(1,\alpha)\alpha^4 m_o f_j(\xi_o,\alpha) + \bar{h}_R'(1,\alpha)\alpha^4 m_0 \varepsilon^2 f_j'(\xi_o,\alpha) + \bar{h}_1'(1,\alpha)\lambda_1 f_j''(\xi_1,\alpha)
\end{aligned}
\tag{4}
$$

where $f_j(\xi_o,\alpha), f_j'(\xi_o,\alpha), f_j''(\xi_o,\alpha)$ depend on whether the mass is located on the left (i.e. $\xi_o = \xi_1^-$) or on the right (i.e. $\xi_o = \xi_1^+$) of the crack. In particular the latter terms take the following form for $\xi_o = \xi_1^-$:

$$
f_j(\xi_o,\alpha) = h_j(\xi_o,\alpha) \quad , \quad f_j'(\xi_o,\alpha) = h_j'(\xi_o,\alpha) \quad , \quad f_j''(\xi_1,\alpha) = h_j''(\xi_1,\alpha) + \bar{h}_R''(\xi_1,\alpha)\alpha^4 m_0 \varepsilon^2 f_j'(\xi_o,\alpha) \tag{5}
$$

while they take the following form for $\xi_o = \xi_1^+$:

$$
f_j(\xi_o,\alpha) = h_j(\xi_o,\alpha) \quad , \quad f_j'(\xi_o,\alpha) = h_j'(\xi_o,\alpha) + \bar{h}_1'(\xi_o,\alpha)\lambda_1 f_j''(\xi_1,\alpha) \quad , \quad f_j''(\xi_1,\alpha) = h_j''(\xi_1,\alpha) \tag{6}
$$

Comparing Eqs.(5) and (6), accounting for the definitions in Eq.(2), reveals that the functions $f_j'(\xi_o,\alpha), f_j''(\xi_1,\alpha)$ change in view of additional terms, depending on whether the mass is on the left or the right side of the crack. Hence, the discontinuous form of the frequency equation (3), when the mass crosses a crack position, is strictly due to the presence of the rotary inertia and leads to the occurrence of frequency shifts.
Without loss of generality, the boundary conditions relative to clamped-clamped beams are those employed in this paper; however the achieved conclusions do not depend on the considered boundary conditions.

3. Numerical evidence for crack localization

The frequency shift due to the presence of a concentrated rotary inertia, as shown in the previous section, applies to any natural frequency of the damaged beam. However it is reasonable to state that the frequency shift amplitude varies with the monitored frequency. In this section the explicit expression of the characteristic equation (3) is adopted to provide numerical evidence that frequencies higher than the first enhance the presence of cracks since they are affected by higher frequency shift amplitudes. The numerical results presented in this section provide the basis for a crack localization procedure by detecting the existence of jumps of natural frequencies to be measured by free vibration tests as the concentrated mass with rotary inertia changes its position along the beam span.

3.1. Single crack

The case of a clamped-clamped beam with a single crack, with compliance parameter (severity) $\lambda_1 = 1$, located at $\xi_1 = 0.7$ is presented. The appended concentrated mass $m_o = 0.05$ with eccentricity $\varepsilon = 0.1$, is considered. In Fig.1 the first three fundamental frequencies (in terms of the frequency parameter α^2) are plotted vs the roving mass position ξ_o. Three different cases are considered: 1. beam without any additional roving body (dash-dot line); 2. beam with an additional roving body with rotary inertia effect neglected (dashed line); 3. beam with an additional roving body accounting for the rotary inertia effect (continuous line).

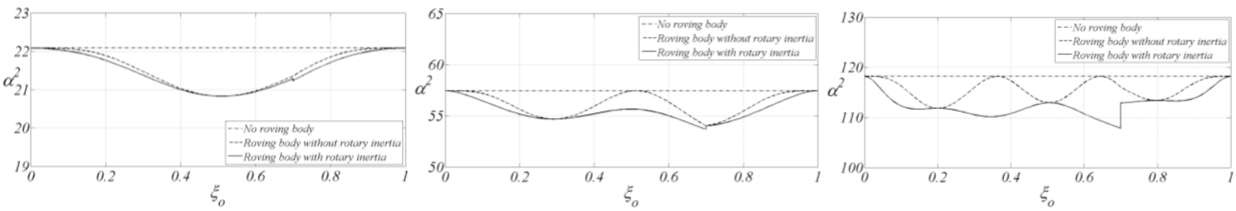

Fig. 1. Single-cracked beam: frequency parameter α^2 vs mass position for the first three frequencies.

From Fig.1 it may be noted that the crack has caused a reduction in the natural frequencies for the considered modes, independently of the mass location. Furthermore, the frequency curve obtained when the rotary inertia effect of the roving mass is neglected is bounded by the cases with no additional mass and that when the rotary inertia of the roving mass is included.

In particular, the presence of the translational mass with rotary inertia neglected causes a first derivative discontinuity in the frequency curve as the mass position coincides with the crack location. However, the latter property is hardly recognizable and is not definitely suggested for crack location purposes unless wavelet transform is applied to the measured signal to enhance any sort of irregularity [5, 10].

The novel results shown in Fig.1 is that there is a sudden jump in the first three frequencies as the mass possessing rotary inertia passes the crack.

In order to gain more information, the above mentioned frequency jump is plotted as function of various parameters in Fig.2 to assess their influence on the dynamic behaviour of the beam under study. In particular, by considering the *i*-th frequency parameter at the left $(\alpha_i^-)^2$ and right $(\alpha_i^+)^2$ sides of the crack (located at ξ_1), and the corresponding frequency parameter $(\alpha_i^o)^2$ in absence of any additional concentrated mass, the following normalised frequency jump $\Delta\alpha_i^2 = [(\alpha_i^+)^2 - (\alpha_i^-)^2]/(\alpha_i^o)^2$ is plotted in Fig.2.

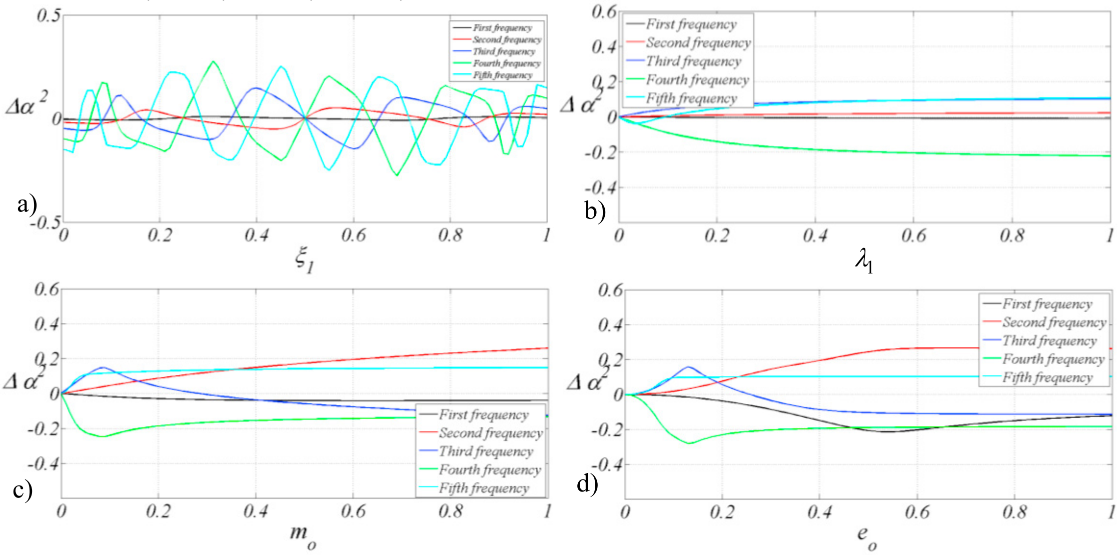

Fig. 2. Single cracked clamped-clamped beam ($\lambda_1 = 1$), relative frequency jumps for the first five frequencies vs: (a) crack position, (b) crack compliance, (c) mass entity and (d) eccentricity sensitivities.

In Fig.2a the normalized frequency jump is shown as a function of the crack location ξ_1. Generally speaking the maximum value of the frequency shift increases as higher frequencies are considered. Furthermore, the normalized frequency jump crosses the zero axis for all the frequencies, for symmetric reasons, when the crack is located at $\xi_1 = 0.5$.

In Fig.2b the crack compliance has been considered variable in the range $0 \le \lambda_1 \le 1$. It is worth noting that, as the crack compliance increases, the frequency jump increases in magnitude except in a restricted region involving small cracks for the case of the fifth frequency.

In Fig.2c the dependence of the normalised frequency jump with respect to the mass ($0 \le m_o \le 1$) is considered. Even in this case the trend in the frequency jumps as the mass increases is not always regular, as the third and fourth frequencies demonstrate.

Finally, in Fig.2d the effect of the eccentricity ($0 \le \varepsilon \le 1$) is assessed. Again, the trend of the frequency jump is not always monotonic.

3.2. Multiple cracks

Finally, in order show how the proposed procedure is effective even in presence of a large number of cracks, a clamped-clamped beam with $n = 10$ equally spaced cracks is investigated. All the cracks have the same severity ($\lambda_i = 0.1$), while the mass and eccentricity are $m_o = 0.05$ and $\varepsilon = 0.1$, respectively.

In Fig.3 the first three fundamental frequencies (in terms of the frequency parameter α^2) are shown as a function of the mass position ξ_o for the cracked beam. The same three different cases as in the previous applications are considered.

The results in Fig.3 show how, even in presence of a considerable number of cracks, the abrupt jump due to the rotary inertia effect can be easily noticed at each passage of the mass over each crack.

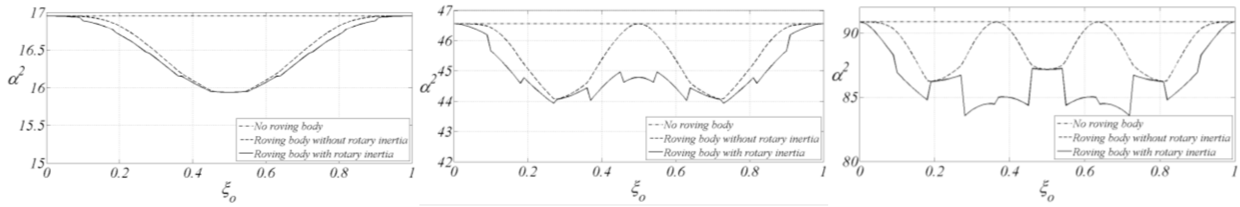

Fig. 3. Multi-cracked beam: frequency parameter α^2 vs mass position for the first three frequencies.

4. Conclusions

A procedure for crack localization in beams based on the use of the rotary inertia of a roving body on the natural frequencies of a beam has been presented in this study. Precisely, it has been shown that jumps of the vibration frequencies appear as a concentrated mass with rotary inertia passes a crack. The proof of such a property has been based on the formulation the explicit solution of the governing equation of the Euler-Bernoulli beam in presence of multiple cracks and a concentrated mass by accounting for the effect of the rotary inertia of the appended mass.

The proposed methodology does not require any a priori knowledge of the number of cracks and is able to deliver the exact number of concentrated cracks, as usually required by real applications. Furthermore, the procedure does not require any test execution on the healthy beam as well as evaluation of its dynamic properties but it relies exclusively on free vibration tests on the cracked beam.

A parametric analysis conducted on a cracked beam showed the influence of both crack and mass parameters on the shift of the first few frequencies.

The presented study can be considered as the theoretical basis for the future development of a proper damage detection technique.

Acknowledgements

This work is part of the National Research Project ''Advanced mechanical modeling of new materials and structures for the solution of 2020 Horizon challenges'' (2017–2020), supported by MIUR, Grant No. 2015JW9NJT

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