



A first approach to the experimental study of fracture parameters in opening and mixed mode by caustics

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Abstract

Among the experimental methodologies used to determine the fracture parameters, considering the up-and-coming possibility of the caustics technique, the authors have started a search path to provide a setup and reliable test procedures for determining the K parameter in mode I and mixed mode. This work shows the set-up used and the first results. Firstly, the caustics for K_I were obtained using a specific loading system to be used on an optical bench; then, the measurement system was transferred on a material testing machine in order to apply loads of greater magnitude. The results obtained are in agreement with the theoretical estimates in the opening mode and they may be considered promising for the analysis of mixed mode, although still substantially different from the values observed in the literature.

Keywords: Type your keywords here, separated by semicolons ;

1. Introduction

Many methodologies were used to investigate the fracture parameters and, since the sixties the caustics method was used to analyze many parameters in the field of elasticity, fracture mechanics, elastodynamics, contact [1-20]. Basing on the envelop of the ray reflected or refracted, in fact, it is possible to examine several aspects of the stress-strain state, as well as the variation of the material behavior. The method is flexible and reliable and the setup is not too complicated: these advantages lead us to consider it particularly suitable for the analysis of parameters of fracture mechanics. Let me remember some discussions with my colleague and friend Prof. Paolo Lazzarin, about the

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possibility of monitoring the parameters of the fracture, and in particular the K , through optical methods. In order to verify experimentally some studies conducted by him in theory [21-23], he suggested the method of caustics as an optimal method for analyzing the parameter K in mixed mode. The present paper is our first approach to measure the K parameter either in mode I or in mixed mode.

The tests were performed first on an optical bench, on which is mounted the system load, so we can use the facilities of the laboratory of optics, for that reason has been realized a system designed for small loads. In a second time, to use higher loads and to try the first approach to mixed mode, it is preferred to mount the optical systems on a testing machine.

2. Description of the investigation

The tests were performed on PMMA specimens in form of compact tensile (CT) test realized by waterjet cutting as shown in Figure 1. The waterjet technique assures the working of the polymer without leaving considerable residual stresses, as put in evidence submitting the specimens to a preliminary photoelastic analysis. The thickness (t) was chosen at 3 mm in order to avoid large stress. The material was firstly tested to determine the elastic parameters by traditional tests. The measure of the Young's and the Poisson's moduli was performed by longitudinal and transversal strain gauges and their values were $E=3375$ MPa and $\nu=0.39$ respectively.

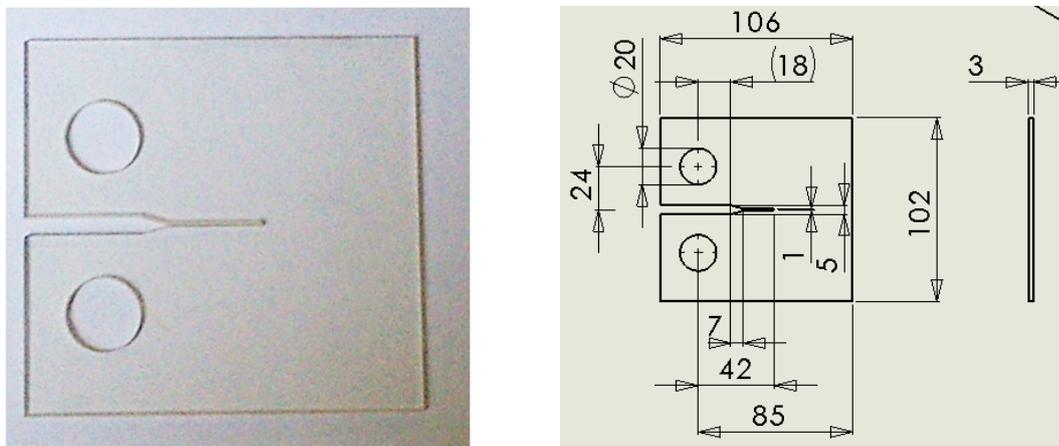


Fig. 1. The specimens used for tests.

2.1. Loading setups

The first series of tests was carried out on a specific testing machine on purpose realized (Figure 2) to perform simple opening tests on the CT specimen under displacement control at low loading level. The loading device was built up on an aluminum frame by assembling an electrostatic actuator M 230-25 Physik Instrumente on line with the specimen and a load cell Tekkal L2320/50LBS. A controller Mercury II C-862 handled the displacement. In Table 1 the main characteristics of the devices were shown. The specimens were connected to the loading chain by simple grips realized by U-shaped steel frames, bolted to the CT holes.

The data were acquired by the Spider8 acquisition system. The actuator displacements and the loading data were controlled by specific LabVIEW software. A scheme of the block diagram and front panel are shown in Figure 3.

The second series of tests was carried out on the same kind of specimens mounted on a Zwick-Roell Z100 static testing machine by a gripping system able to modify the notch orientation in order to simulate the mixed modes from

mode I to mode II. Following the literature [24], the system of Figure 4 was adopted, considering some variations to fit the grips to the possibility of transmitting the laser light. The perimeter holes number was modified to six at about 26 degrees and the hole diameter was enlarged to 20 mm. The grips were carried out in AISI 304 steel by waterjet cutting. The assembly was performed so that, after the specimen mounting, the line joining the holes connected to the grips of the testing machine pass through the notch apex. The first and last holes allow the application loading in the mode I and II respectively.

Table 1. Devices characteristics.

Actuator		Loading cell		Controller	
Model	PI M 230-25	Model	Tekkal L2320/50LBS	Model	Mercury II C-862
Min.incr. motion	0.05 μm	Sensistivity	2 mV/V	Type	Stand-alone DC-servo-motor
Velocity	0.8 mm/s	Range	0-150 N	Servo	31-bit velocity 16-bit PID
Force	70 N	Bridge	4x350 Ω	Encoder	A/B TTL signals 10 counts/s
Motor type	DC motor gearhead	Linearity	0.1% f.s.	Command set	40 high level



Fig. 2. The testing machine for the first series of tests.

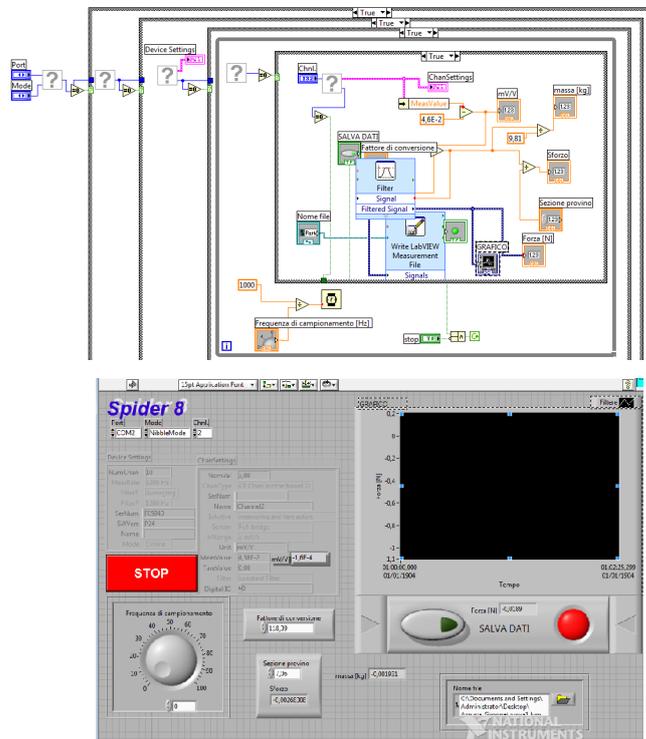


Fig. 3. Block diagram and front panel for the acquisition system.

2.2. Testing setups

The first series of tests was performed on an optical bench as shown in Figure 5. The laser source Coherent DPSS-532 was mounted on the Newport holographic bench and a magnifier lens 10 x expanded the green laser beam. The

expanded laser beam, after hitting the specimen notch apex is shared in the transmitted and reflected beams. Two ground glass screens, placed at the same distance (0.5 m or 1 m) from the specimen and normal to the beams allow the vision of the caustics. Behind each of them, a camera recorded the image. Because the beam expanded, the magnification factor is greater than 1.

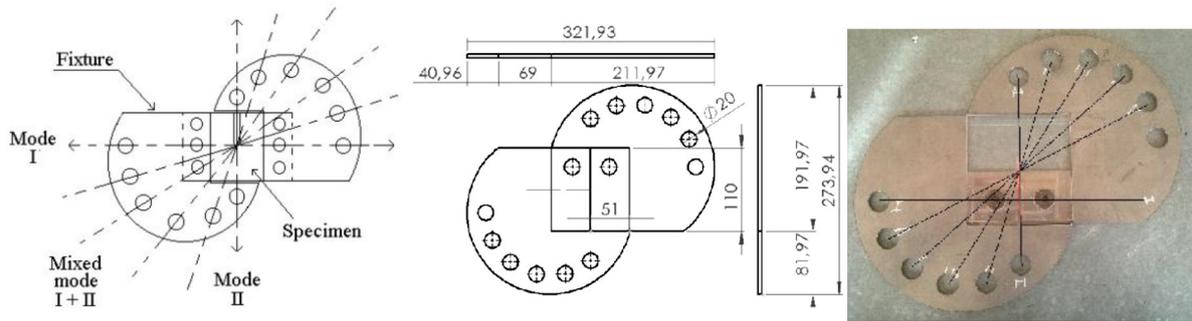


Fig. 4. Grips adopted for the mixed mode.

For the second series, instead, the laser source, the beam expander and the screens were placed directly on tripods in front of the testing machine and the images were acquired by a couple of b/w video cameras DMK 23G445 with a resolution of 1280x960 pixels at 30 frames/s. On the video camera body, two lenses (HF35HA-1B, 1:1.6/35 mm and HF75HA-1B, 1:2.8/75 mm) were mounted at the two screen distances. Because of the need to synchronize the images with the applied loads, a specific routine in LabVIEW was coded.

The diameters of the caustics were measured by counting the pixels and processing it using Matlab, scaling the measure of the caustics diameter on the basis of a known size in the same image (Figure 6).

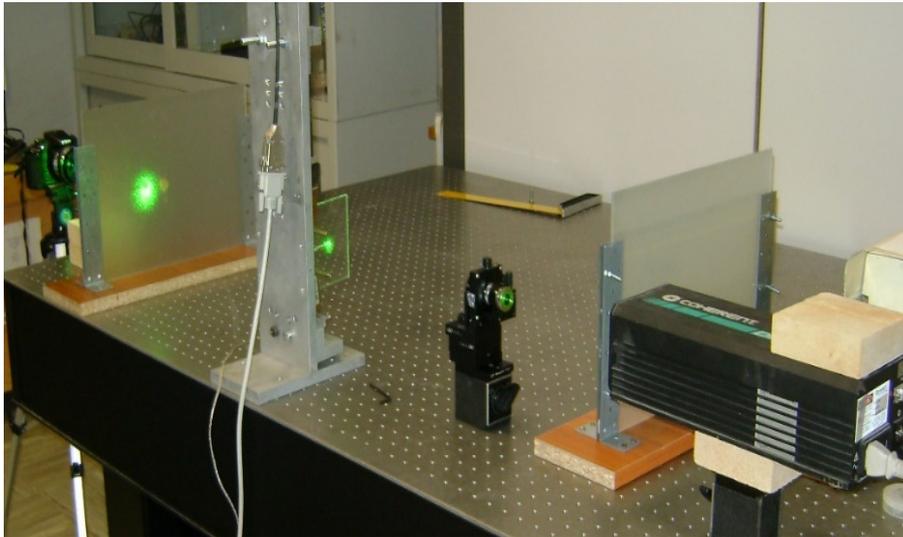


Fig. 5. The experimental setup for the first series of tests.

2.3. FEM analysis

In order to have some more accurate information about the load to apply and the answer in terms of stress-strain status, a numerical simulation was carried out. The mesh was realized using 9278 triangular elements with 17181 nodes. The specimen was placed with the notch horizontal in mode I and vertical in mode II. The constraints were mounted inside the lower hole, to simulate the contact between the coaxial cylinder and the hole itself. The force was applied in the vertical direction, acting on the inner surface of the top hole, with intensity of 160 N. Figure 7 shows the Von Mises stresses obtained for the two modes. The maximum stress obtained is about 50 MPa, with maximum displacement of about 0.8 mm, for mode I and 24.5 MPa, with maximum displacement of about 0.6 mm, for mode II.

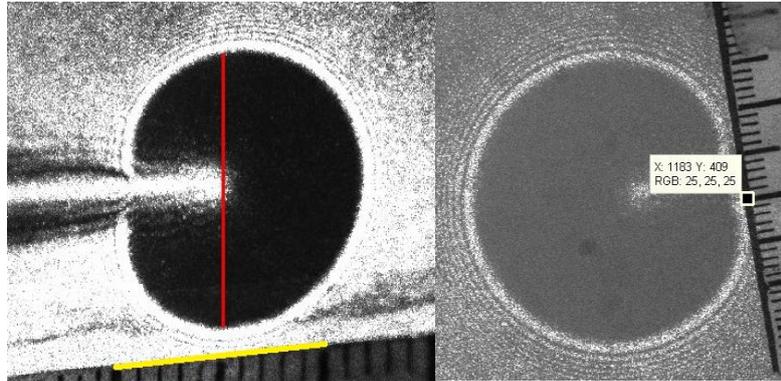


Fig. 6. Measure of the diameter for transmitted (a) and reflected (b) caustics.

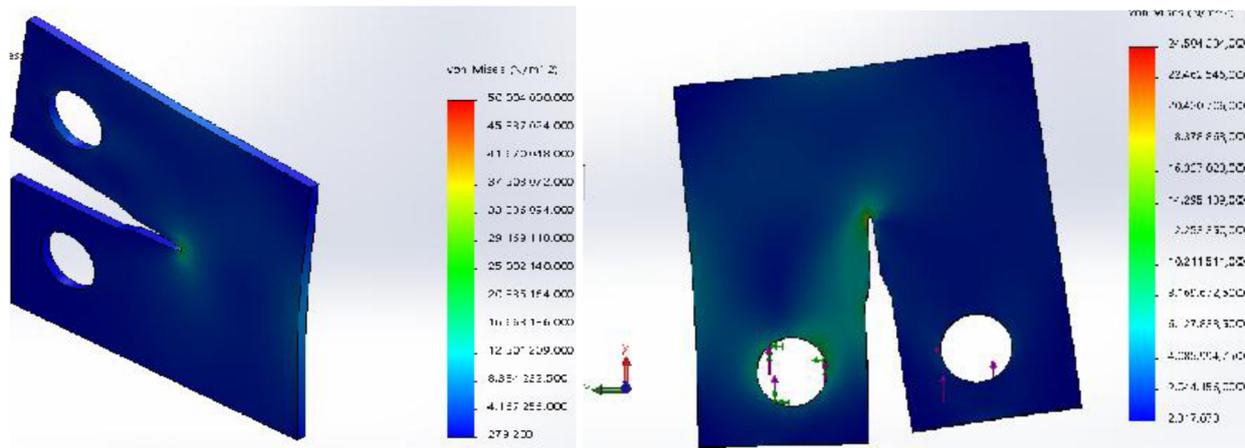


Fig. 7. Von Mises stresses for (a) mode I; (b) mode II.

2.4. Tests procedure

As well known by literature, near the crack, the state passes from plain strain to plain stress state. To be sure that the measure is performed in plane stress, the caustics radius r_0 must be greater than the half thickness of the specimen. In this case, it is possible to use the Williams-Cawood formula [5]:

$$K_I = \frac{Y(A)P}{BW^{\frac{1}{2}}} \quad Y(A) = \frac{(2+A)}{(1-A)^{\frac{3}{2}}} (0.886 + 4.64A - 13.32A^2 + 14.72A^3 - 5.6A^4) \quad (1)$$

where $A=a/W$ is the ratio between the crack length and the width.

In order to assure the minimum radius, you must increase the applied load or the distance of the screens z_0 .

Several tests were performed in mode I on different specimens varying the distance z_0 (0.5 m and 1 m) and the applied load. The appearance of the caustic curves was noted for loads higher than 70 N. Figure 8 shows, as an example, the transmitted and reflected caustics at 70 N and at a distance 1 m. All the tests allow measuring the caustics diameter D and the evaluation of the radius r_0 on the base of the formula:

$$r_0 = \frac{D}{fm} \quad (2)$$

with $f = 3.17$ and $m_t = 2.1$ for the transmitted beam and $m_r = 3.2$ for the reflected beam.

The first tests did not assure that the curve was totally in the area of plane stress, being $r_0 < t/2$ either for the transmitted or for the reflected beam. In order to respect this limitation, the load and the distance were modified.

The experimental values for K_I were calculated by:

$$K_I = \frac{2^{\frac{3}{2}} \pi^{\frac{1}{2}} E}{3\nu B z_0} r_0^{\frac{5}{2}} \quad (3)$$

Finally, tests were performed in mixed mode, modifying the angle of the support (Figure 4) respect the load: approaching to 90 degrees, mode I becomes predominant; the influence of mode II increases close to 0 degrees. The tests were carried out at 0, 22.53, 41.11, 58.02 and 74.17 degrees with a 160 N load at the two screens distances of 0.5 m and 1 m. Further tests were performed on specimens with the same plane geometry but 6 mm thickness at 200 N and 240 N with the same sequence of angles.

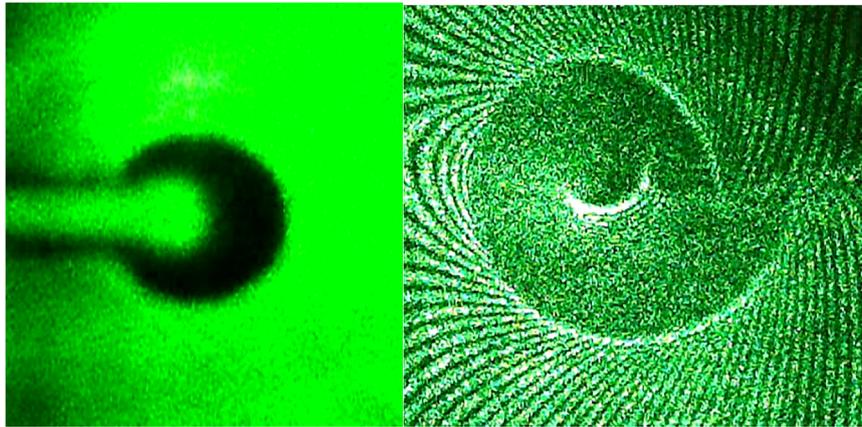


Fig. 8. transmitted (a) and reflected (b) caustics at 70 N and screens distance 1 m.

3. Results

The results of the series of test carried out by the equipment specially designed and the first experimental setup (Figures 2 and 5) are shown in Table 2, for the screens distance of 0.5 m and 1 m, respectively. The diameters of the transmitted and reflected caustics and the corresponding radii are reported. The theoretical values of K_I are derived by Eq. (1), the experimental ones (K_I^t and K_I^r) are derived by Eq. (3) calculating the radii (r_0^t and r_0^r) by Eq. (2), considering $m_t = 2.1$ for the transmitted beam and $m_r = 3.2$ for the reflected beam. In the same table, the error in transmitted and reflected results are reported, defined as the percentage difference between the theoretical and the experimental (transmitted e_t or reflected e_r) evaluation related to the theoretical one:

$$e_t = (K_I - K_I^t)/K_I \qquad e_r = (K_I - K_I^r)/K_I \qquad (4)$$

Table 2. Radii and K_I for the first series.

Distance [m]	Load [N]	D_t [mm]	D_r [mm]	r_0^t [mm]	r_0^r [mm]	K_I [N m ^{-3/2}]	K_I^t [N m ^{-3/2}]	K_I^r [N m ^{-3/2}]	e_t %	e_r %
0.5	70	8.2	13.0	1.23	1.28	550442	513238	566651	6.7	-2.9
0.5	80	8.5	14.0	1.27	1.38	629077	561477	681989	10.75	-8.41
1	70	13.0	19.5	1.95	1.92	761723	812312	776953	-6.6	-2.0
1	70	11.0	17.5	1.65	1.72	550443	534855	595693	2.8	-8.2
1	75	13.9	20.2	2.09	1.99	816045	960288	851880	-17.7	-4.4
1	81.91	13.8	21.0	2.08	2.07	891336	948244	942149	-6.4	-5.7
1	83.23	14.1	20.6	2.11	2.03	905711	988161	892531	-9.1	1.5
1	83.44	13.9	21.2	2.10	2.09	908040	968947	963584	-6.7	-6.1
1	86.06	14.1	20.2	2.11	1.99	936485	991679	857164	-5.9	8.5
1	87.2	15.1	22.8	2.26	2.25	948694	1173348	1153184	-23.7	-21.5
1	89.5	15.0	23.4	2.25	2.31	973428	1157828	1231905	-18.9	-26.5

The second series, performed on the static testing machine in mode I, reveals the results reported in Table 3.

Table 3. Radii and K_I for the second series.

Distance [m]	Load [N]	D_t [mm]	D_r [mm]	r_0^t [mm]	r_0^r [mm]	K_I [N m ^{-3/2}]	K_I^t [N m ^{-3/2}]	K_I^r [N m ^{-3/2}]	e_t %	e_r %
0.5	160	9.7	13.2	2.04	2.08	1735435	1811769	1906609	4.2	9.0
1	160	13.5	24.1	2.85	2.85	1735435	2087715	2357106	16.9	26.37

From both tables it is possible to verify a good match between the measured values and the theoretical values expected from the literature for polymers; the errors are in many cases content within acceptable values and become elevated only for larger loads and to a greater extent for larger distances of the screens.

Finally, Figure 9 shows the measures of the diameter of the transmitted caustics for the different gripping position. The angles vary from 0 degrees (mode II) to 90 degrees (mode I) showing a substantial increase of D_t from mode II to mode I.

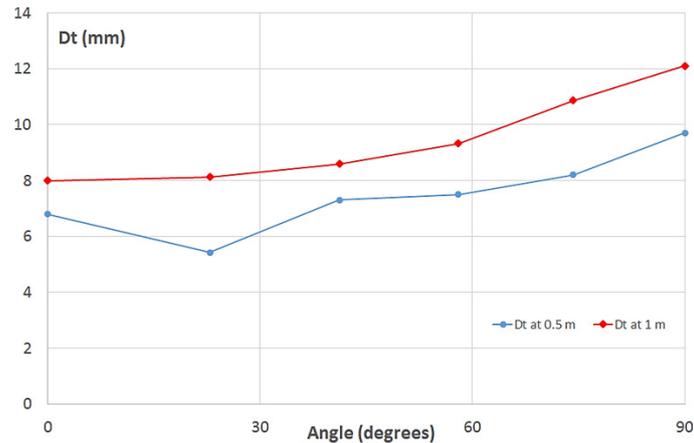


Fig. 9. Diameters of the transmitted caustics in mixed mode.

4. Conclusions

The authors have performed their first approach to the methodology of caustics in order to evaluate experimentally the parameter K . A series of tests was carried out by a setup mounted on an optical bench, using a purposely built loading system. A second series, instead, was tested mounting the optical system on the testing machine in order to apply larger loads and to mount a specific system for gripping the specimen, able to vary the load angle and produce the mixed mode.

The tests in mode I show a good agreement with the theoretical ones, mainly for lower loading. The first tests carried out in mixed mode allow to measure the diameters of caustics but are yet to be correlated with the correct theoretical models.

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