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The Station Location Problem on Two Intersecting Lines

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Abstract

The station location problem consists of placing new stations along the railway tracks of an existing network in order to increase the number of users. In this paper we consider the problem for a railway network consisting of two intersecting lines forming an angle α. An approach for solving the problem in polynomial time for sufficiently large angles α is presented.

Keywords: station location, covering by discs, facility location

1 Introduction

The station location problem consists of placing some new stations along the tracks of an existing railway network in order to increase the number of users.

The motivation for studying this problem is mainly to increase the attractiveness of train travel for local traffic in an existing railway network. But new stations are costly for the company and new stops will result in a longer travelling time for those people that are already using the network. Therefore our goal is to minimize the number of new stations. Actually, this research was motivated by a collaboration with with the Deutsche Bahn about exactly this

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question. However, in this paper we look at a somewhat simplified scenario, where we ignore for example actual costs of building stations or travel time.

The problem of locating stops in a transportation network has been studied in several papers, analyzing bus transportation networks as well as railway networks. The problem of locating stops in a bus network is considered in [5,3,4], all dealing with the problem of opening new stops out of a set of given candidates.

The station location problem was first introduced and modelled in [1], where it is shown to be NP-complete. Schöbel et al. [6] describe a reduction from the station location problem to a discrete set covering problem. In particular the covering matrix is analyzed, and an efficient solution method is presented for the special case when the railway network consists of one straight line only. Kranakis et al. [2] study the "MAX gain" problem of finding the placement of a fixed number of k stations such that the number of covered settlements is maximized. In [7] the problem of covering population areas is presented. While in the previous papers settlements are represented as points, in [7] demand regions are considered, and an efficient algorithm for finding an optimal solution in the case when the railway network consists of one straight line is given.

While the station location problem for one line segment is solvable in polynomial time, in general the problem becomes NP-hard for two or more line segments [1]. However, a heuristic approach to solve the station location problem in practice consists of a decomposition of the railway network into independent line segments. Then, for a good solution of the problem the common area between line segments needs to be considered in more details. The problem considered in this paper may be viewed as a first step towards this approach.

The paper is organized as follows. In Section 2 the station location problem is modelled as a minimization problem and the variant of the problem studied in this paper is defined. In Section 3, we define the problem of covering a common region of two lines by discs and characterize the solutions to this problem depending on the angle between the two lines. In Section 4 we approach the station location problem on two intersecting lines and apply results of the covering by discs problem to obtain optimal solutions for the station location problem for sufficiently large values of the angle α formed by the two lines.

2 Problem definition

A railway network and a set $\mathcal{P}$ of settlements are given. A settlement $P_i \in \mathcal{P}$ is covered by the railway network if there is a station at distance less than or equal to a fixed radius R from P_i. We assume that none of the settlements in $\mathcal{P}$ is currently covered by an existing station, that each settlement in $\mathcal{P}$ can be covered by a new station, and that stations can be placed anywhere along the tracks. The goal is to select as few new stations as possible to cover all the settlements. Precisely, given a set $\mathcal{S}$ of points in the plane, define $cover(\mathcal{S}) = \{x \in \mathbb{R}^2 : d(x, \mathcal{S}) \leq R\}$ as the points at distance less than or equal to R from $\mathcal{S}$, the distance d being the Euclidian distance. Then the station location problem can be modelled as the following minimization problem:

Station Location: Given a geometric graph $G = (V, E)$, i.e. a set V of vertices in the plane and a set E of edges represented as straight line segments, a set $\mathcal{P}$ of points in the plane and a fixed radius R. Find a minimum number of vertices $\mathcal{S}$ on the edges such that $\mathcal{P} \subseteq cover(\mathcal{S})$.

The NP-hardness of the station location problem has been shown [1] by a transformation from the geometric covering problem. The geometric covering by discs is defined as follows: Given a set $\mathcal{P}$ of points in the plane, find the minimum number of discs D that cover $\mathcal{P}$, where a point P_i in the plane is *covered* by a disc $D = (c, R)$ with center in c and radius R, if $d(P_i, c) \leq R$.

In this paper we focus on the station location problem when the railway network consists of two straight lines with a common end point O forming an angle α, and the set $\mathcal{P}$ of settlements is placed in the common area of the two lines. More precisely, for a given radius R and two lines l_1, l_2 we call the area in the plane that can be covered on one hand by discs of radius R having centers on l_1, and on the other hand by discs of radius R having centers on l_1 the *common area of* l_1 *and* l_2. See Figure 1. Then define:

Station Location on Two intersecting Lines (*SLTL*): Given a geometric graph $G = (V, E)$ that consists of two adjacent edges represented as straight line segments, a set $\mathcal{P}$ of points in the common area of the two lines, and a fixed radius R. Find a minimum number of vertices $\mathcal{S}$ on the edges with $O \in \mathcal{S}$ such that $\mathcal{P} \subseteq cover(\mathcal{S})$.

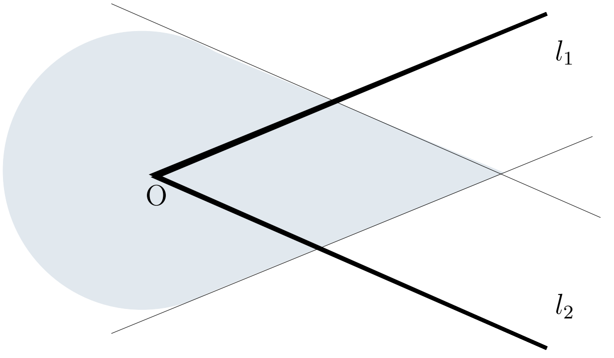

Fig. 1. The grey area is the common area of the lines l_1 and l_2.

3 Covering by Discs

In this section a variant of the geometric covering by discs problem is studied. Given two line segments, l_1 and l_2, with a common end point O, we are interested in covering the entire common area of the lines l_1 and l_2 by discs centered on either line. Precisely:

Covering by Discs on Two Lines ($CDTL$): Given two straight line segments with a common end point O forming an angle $\alpha < 180°$. Find the minimum number of discs centered on either line that cover the common area of the two lines.

Obviously, an optimum solution of the $CDTL$ problem is at least a feasible solution of the $SLTL$ problem.

3.1 Covering by Discs on Two Lines

In this paragraph we consider $CDTL$ for the cases $\alpha \geq 60°$ and $41.4° \leq \alpha < 60°$. Let us introduce some notation that we use in the paper. Figure 2 illustrates the notation. We assume $R = 1$.

- l_1 and l_2 are the two lines of the $CDTL$ problem;
- O is the common end point of the lines l_1 and l_2;
- α is the angle formed by the lines l_1 and l_2;
- $\overrightarrow{xOy}$ is the orthogonal coordinate system in the plane having origin in O, positive x-axis in the bisector of the angle α;
- t_1 and t_2 are the two lines at distance 1 from l_1 and l_2 respectively, which intersect each other in the positive x-axis;

- C is the disc centered in O with radius 1;
- O_1 is the intersection point of the lines t_1 and t_2;
- C_1 and C_1' are the two unique discs centered respectively on l_1 and l_2 passing trough O_1;
- $O^- = (-1, 0)$ and $O^+ = (1, 0)$;
- A is the intersection point of the line t_2 and the disc C;
- B is the intersection point of the line t_1 and the disc C.

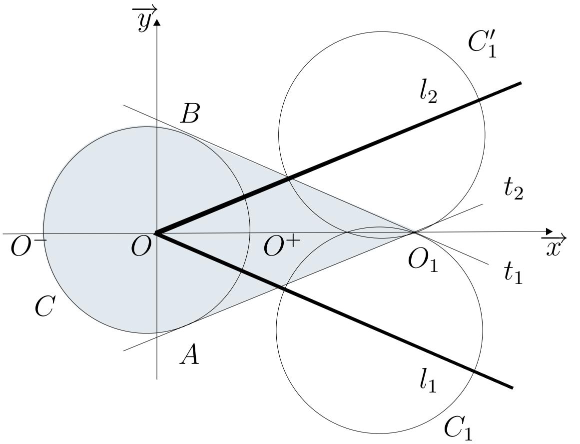

Fig. 2. The notation.

Lemma 3.1 *Any optimal solution for the* CDTL *contains at least three discs.*

Proof. The disc C centered in O is the only disc that covers the point O^-. The discs C_1 and C_1' are the only two discs that cover the point O_1 and those points infinitely close to it on the lines t_1 and t_2. □

Therefore, without loss of generality, we can assume that the discs C, C_1 and C_1' are contained in any optimal solution to the *CDTL* problem. The next step is to investigate the number of discs that are needed to cover the remaining part of the common area. (See Figure 2.) More precisely, we consider the following problem: Given two straight line segments l_1 and l_2 with a common end point O, the discs C, C_1 and C_1'. Find the minimum set of discs including C, C_1 and C_1' that cover the common area.

Lemma 3.2 *The minimum number of discs that cover the common area is three when* $60° \leq \alpha$.

Proof. We prove that the discs C, C_1 and C_1' cover the common area if the angle α formed by the lines l_1 and l_2 is greater than or equal to $60°$. According to the orthogonal coordinate system $\overrightarrow{xOy}$, if $m = \tan(\frac{\alpha}{2})$, $m > 0$ then:

l_1: $y = -mx$	l_2: $y = mx$
t_1: $y = -mx + \sqrt{1+m^2}$	t_2: $y = mx - \sqrt{1+m^2}$
O: $(0,0)$	O_1: $\left(\frac{\sqrt{1+m^2}}{m}, 0\right)$
C: $x^2 + y^2 = 1$	C_1: $\left(x - \frac{1}{m\sqrt{1+m^2}}\right)^2 + \left(y + \frac{1}{\sqrt{1+m^2}}\right)^2 = 1.$

The three discs cover the entire common area if the disc C_1 passes through or contains the point $A = \left(\frac{m}{\sqrt{1+m^2}}, \frac{-1}{\sqrt{1+m^2}}\right)$. C_1 passes through the point A, if

$$\left(\frac{m}{\sqrt{1+m^2}} - \frac{1}{m\sqrt{1+m^2}}\right)^2 = 1$$

which leads to $m = \pm\frac{1}{\sqrt{3}}$. Since $m > 0$, the only acceptable solution is $m = \frac{1}{\sqrt{3}}$, which corresponds to $\alpha = 60°$. □

Note that when the angle α equals 60° the discs C_1 and C_1' pass through the point O^+ and the points of the segment $\overline{O^+O_1}$ are all contained in the two discs. See Figure 3.

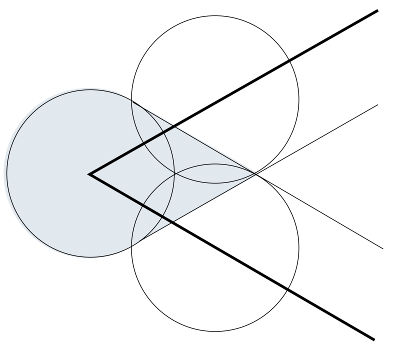

Fig. 3. The case $\alpha = 60°$.

Lemma 3.3 *The minimum number of discs that cover the common area is five when* $41.4° \leq \alpha < 60°$.

Proof. Let $O_2 = \left(\frac{1-m^2}{m\sqrt{1+m^2}}, 0\right)$ be the intersection point of C_1, C_1' and the x-axis, other than O_1. When the angle α is smaller than 60°, the points A and B are not covered by the discs C_1 and C_1', as well as those infinitely close to them on the lines AO_1, and BO_1 and the ones on the segment O_2O^+. See Figure 4.

C_2 and C_2' are the discs through O_2 with centers on l_1 and l_2. The five discs C, C_1, C_1', C_2 and C_2', cover the common area if the disc C_2 passes through or

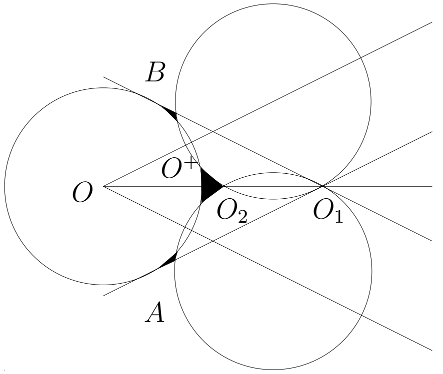

Fig. 4. The uncovered area for $\alpha < 60°$ and three discs.

contains the point $A = \left(\frac{m}{\sqrt{1+m^2}}, \frac{-1}{\sqrt{1+m^2}}\right)$. C_2 passes through the point A, if

$$\left(\frac{m}{\sqrt{1+m^2}} - \frac{1-3m^2}{m(1+m^2)\sqrt{1+m^2}}\right)^2 + \left(\frac{-1}{\sqrt{1+m^2}} + \frac{1-3m^2}{(1+m^2)\sqrt{1+m^2}}\right)^2 = 1$$

which leads to $m = \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{7}}$. The only acceptable solution is $m = \frac{1}{\sqrt{7}}$, which corresponds to $\alpha = 41.4°$. Moreover, the four discs C, C_1, C_1' and C_2 do not cover the common area because the disc C_2 does not contain the points of the common area on the segment $\overline{BT}$, where T is the intersection point of the line t_1 and the disc C_2. See Figure 5 $\square$

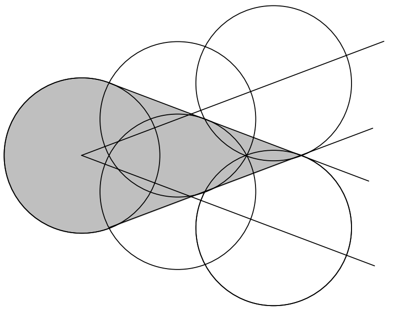

Fig. 5. The case $\alpha = 41.4°$.

3.2 *The* CDTL *Algorithm*

The approach used in the previous section leads to the covering by discs algorithm shown in this section. For the cases $60° \leq \alpha$ and $41.4° \leq \alpha < 60°$ we use Lemma 3.2 and 3.3 respectively. For $\alpha < 41.4°$, the algorithm successively places discs on the two lines according to the proof of Lemma 3.3. However, this yields a solution that is not necessarily optimal. Altogether, the algorithm gives an optimal solution for the *CDTL* problem when the angle α is greater than or equal to 41.4° and only a feasible solution otherwise. It could be improved to an optimal solution for $\alpha < 41.4°$. But this would require an elaborate case distinction.

Algorithm 1 *CDTL*

Input

- two lines l_1 and l_2 having a common point O forming an angle α less than 180°;
- c, disc centered in O;
- bis, the bisector of l_1 and l_2.

Output collection D of discs that cover the common area.
The algorithm uses the following notation.

- c^1_X is out of the two discs through the point X with center on line l_1, the one that is not already in D;
- c^2_X is out of the two discs through the point X with center on line l_2 the one that is not already in D;
- $l^{\perp}_{1,X_1}$ is the perpendicular line to l_1 passing trough the point X_1;
- $Z = d \cap t_2$ is, out of the two intersection points of the disc d with the line t_2, the one that is closer to A;
- $Z = d \cap bis$ is, out of the two intersection points of the disc d with the line bis, the one that is closer to O.

$X := t_1 \cap t_2$
compute c^1_X and c^2_X
$D := \{c, c^1_X, c^2_X\}$
while A is not covered **do**
    $X_1 := c^1_X \cap t_2$
    $X := c^1_X \cap bis$
    $X_2 := l^{\perp}_{1,X_1} \cap bis$
    if $d(O, X_2) \leq d(O, X)$ **then**
        compute c^1_X and c^2_X
        $D := D \cup \{c^1_X, c^2_X\}$
    else
        compute $c^1_{X_1}$ and $c^2_{X_1}$
        $D := D \cup \{c^1_{X_1}, c^2_{X_1}\}$
        $c^1_X := c^1_{X_1}$
        $c^2_X := c^2_{X_1}$

The loop in lines 4–15 builds the collection of discs and the algorithm stops when the point A is contained in a disc. The points X and X_2, constructed in lines 6 and 7, are those we refer to in order to find the next discs. The algorithm runs in time linear in the number of discs.

Lemma 3.4 *If α is smaller than or equal to $60°$ then any disc centered on line l_1 covering the point A also covers the point O^+.*

Proof. Let C_A and C_{O^+} be the circles with radius 1, centered in A and O^+ respectively, refer to Figure 6.

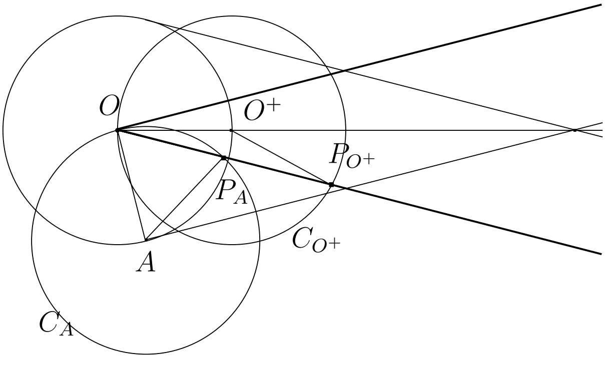

Fig. 6. The discs C_A and C_{O^+}

The discs centered in points on l_1 contained in the intersection of the two discs are the only ones covering both A and O^+. Let $P_A = \{C_A \cap l_1\} \setminus O$, $P_{O^+} = \{C_{O^+} \cap l_1\} \setminus O$, $\psi := \angle O^+OP_{O^+}$ and $\varphi := \angle AOP_A$. It is sufficient to show that $\overline{OP_A} \subset \overline{OP_{O^+}}$. Since the triangles $OO^+P_{O^+}$ and OAP_A are isosceles, then $\overline{OP_A} \subset \overline{OP_{O^+}}$ if and only if $\psi \leq \varphi$. It is $\psi = \frac{\alpha}{2}$ and $\varphi = 90° - \alpha$, then $\psi \leq \varphi$ if and only if $\alpha \leq 60°$. □

Lemma 3.5 *If $d(O, X_2) \leq d(O, X)$ then the disc C^1_X contains the point I_1, and if $d(O, I_2) > d(O, I)$ then the disc $C^1_{X_1}$ contains the point X.*

Proof. This can be easily proved applying the pythagorean theorem. □

Theorem 3.6 *The collection D of discs produced by Algorithm 1 covers the entire common area of the two given lines.*

Proof. Let P be a point in the common area. If P is at distance less than or equal to 1 from O, then it is covered by the disc c. So let P be a point in the triangle O_1O^+A and $D_{l_1} = \{D_i\}$ be the collection of discs with center on l_1 produced by the algorithm. We will show that P is covered by at least one disc in D_{l_1}. Consider the intervals $I_i^{bis} = D_i \cap bis$ and $I_i^{t_2} = D_i \cap t_2$ and let ℓ_i^{bis} and $\ell_i^{t_2}$ be the points in I_i^{bis} and $I_i^{t_2}$ respectively farthest from O_1. Note that $\overline{O^+O_1} \subset \bigcup_i I_i^{bis}$, $\overline{AO_1} \subset \bigcup_i I_i^{t_2}$ and that the left points ℓ_i^{bis} and $\ell_i^{t_2}$ of the intervals appear along $\overline{O_1O^+}$ and $\overline{O_1A}$ respectively in increasing order of their indices.

For $i > 1$ let Pol_i be the polygon with vertices $\ell_i^{bis}, \ell_{i-1}^{bis}, \ell_{i-1}^{t_2}, \ell_i^{t_2}$ and Pol_1 the triangle $\ell_1^{bis}O_1\ell_1^{t_2}$. See Figure 7. Note that, by Lemma 3.4 and Lemma 3.5, the union of the polygons Pol_i $(1 \le i \le |D_{l_1}|)$ contains the triangle OO^+A, so the point P is contained in at least one polygon Pol_i. Therefore P is contained in the disc D_i because it is convex and covers all vertices of Pol_i. □

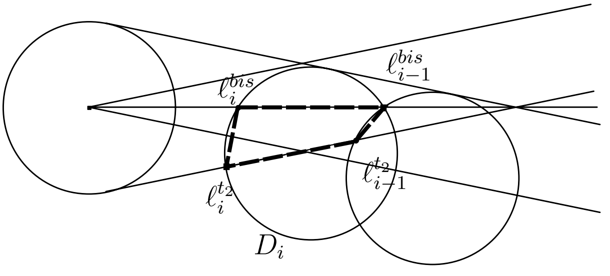

Fig. 7. The polygon Pol_i

4 The Station Location Problem

As explained before, given an existing railway network and a set $\mathcal{P}$ of n settlements the problem is to cover the settlements placing as few new stations as possible. We represent:

(i) The railway network as a graph in the plane where the nodes are the stations and the edges are straight line segments representing the tracks between stations;

(ii) the set $\mathcal{P}$ as n points in the plane.

We consider the *SLTL* problem, where the network consists of two line segments, l_1 and l_2 forming an angle α and a set $\mathcal{P}$ of points all lying in the common area of the two lines is given.

Theorem 4.1 *Three stations cover all the settlements in $\mathcal{P}$ if $\alpha \ge 60°$.*

Proof. According to Lemma 3.2, two discs, plus one centered on the common point, are enough to cover the common area between two lines having a common point, when the angle α formed by the two lines is greater than or equal to $60°$. □

Theorem 4.2 *Five stations cover all the settlements in $\mathcal{P}$ if $41.4° \leq \alpha < 60°$.*

Proof. According to Lemma 3.3, four discs, plus one centered on the common point, are enough to cover the common area between two lines having a common point, when the angle α formed by the two lines is greater than or equal to $41.4°$ and smaller than $60°$. □

We now want to generate a minimum set of points on the lines l_1 and l_2 that cover a given set of settlements $\mathcal{P}$. Let $P_i \in \mathcal{P}$. We call C_i the circle with center in P_i and radius R. Since each settlement in $\mathcal{P}$ can be covered by a new station along the tracks, $J^i := C_i \cap \{l_1 \cup l_2\}$ is not empty. Let J_1^i be the interval induced by point P_i on the line l_1 and J_2^i the interval induced by point P_i on l_2. Then the settlement P_i is covered if and only if there is a station placed in $J_i = J_1^i \cup J_2^i$.

Let $\mathcal{I}_1 = \{I_1^l\}$ and $\mathcal{I}_2 = \{I_2^l\}$ be the sets of subintervals induced by all points in $\mathcal{P}$ on the lines l_1 and l_2 respectively. More precisely, if $J_1^i = [a_i, b_i]$ and $J_1^j = [a_j, b_j]$ are intervals induced on l_1 by P_i and P_j, such that $J_1^i \cap J_1^j \neq \emptyset$ with $a_i \leq a_j$, then they detect the subintervals:

$$\begin{array}{ll} [a_i, a_j],\ [a_j, b_i],\ [b_i, b_j] & \text{if } b_i \leq b_j \text{ or} \\ [a_i, a_j],\ [a_j, b_j],\ [b_j, b_i] & \text{if } b_i \geq b_j. \end{array}$$

This can be extended to any number of intervals. Note that the resulting number of sub-intervals on each line is at most $2n - 1$.

Theorem 4.3 *An $\mathcal{O}(n)$ time algorithm optimally solves the* SLTL *problem if $\alpha \geq 60°$.*

Proof. Let $\mathcal{P} = \{P_i : 1 \leq i \leq n\}$. For all $P_i \in \mathcal{P}$ compute the intervals J_1^i and J_2^i. Let $I_1^i := I_1^{i-1} \cap J_1^i$ and $I_2^i := I_2^{i-1} \cap J_2^i$. Note that I_1^i and I_2^i are the intervals on l_1 and l_2 that cover all the points up to P_i. If $I_2^n = I_1^n = \emptyset$ we need two stations else we need only one in addition to O. Computing the intervals I_j^i and J_j^i takes constant time for every i. □

Theorem 4.4 *An $\mathcal{O}(n^3)$ time algorithm optimally solves the* SLTL *problem if $41.4° \leq \alpha < 60°$.*

Proof. For a given set $\mathcal{P}$, Algorithm 2 successively checks if $\mathcal{P}$ can be covered by a set of two, three or four stations on the two lines. In case not all points in $\mathcal{P}$ can be covered by four stations, the result is a covering of the entire common area of the two lines with five stations. Obviously, the running time is in $\mathcal{O}(n^3)$.

□

Algorithm 2 *SLTL*

Input two lines l_1 and l_2 having a common point O, set of points $\mathcal{P}$ in the common area of l_1 and l_2;
Output collection of one, two or three intervals on l_1 and l_2, such that O together with one arbitray point from each interval of this collection cover $\mathcal{P}$; if no such collection exists five points covering the entire common area.

$I_1^0 := l_1$, $I_2^0 := l_2$
for $P_i \in \mathcal{P}$ **do**
$I_1^i := I_1^{i-1} \cap J_1^i$
$I_2^i := I_2^{i-1} \cap J_2^i$
if $I_1^n \neq \emptyset$ or $I_2^n \neq \emptyset$ **then**
output O and I_1^n resp. I_2^n
else
for every subinterval $I \in \mathcal{I}_1 \cup \in \mathcal{I}_2$ **do**
$I_1^0 := l_1$, $I_2^0 := l_2$
for $P_i \in \mathcal{P}$ **do**
if P_i covered by I **then**
$I_1^i := I_1^{i-1}$
$I_2^i := I_2^{i-1}$
else
$I_1^i := I_1^{i-1} \cap J_1^i$
$I_2^i := I_2^{i-1} \cap J_2^i$
if $I_1^n \neq \emptyset$ or $I_2^n \neq \emptyset$ **then**
output O, I and I_1^n resp. I_2^n
if $I_1^n = \emptyset = I_2^n$ for all $I \in \mathcal{I}_1 \cup \in \mathcal{I}_2$ **then**
for every pair of subintervals $I, I' \in \mathcal{I}_1 \cup \in \mathcal{I}_2$ **do**
$I_1^0 := l_1$, $I_2^0 := l_2$
for $P_i \in \mathcal{P}$ **do**
if P_i is covered by I or by I' **then**
$I_1^i := I_1^{i-1}$
$I_2^i := I_2^{i-1}$
else
$I_1^i := I_1^{i-1} \cap J_1^i$
$I_2^i := I_2^{i-1} \cap J_2^i$
if $I_1^n \neq \emptyset$ or $I_2^n \neq \emptyset$ **then**
output O, I, I' and I_1^n resp. I_2^n
if $I_1^n = \emptyset = I_2^n$ for all pairs of subintervals I, I' **then**
output five points on l_1 and l_2 covering of the entire common area

Remark If we do not assume that there is necessarily a station in the common point O of the two lines, then an $\mathcal{O}(n^2)$ algorithm solves the problem when the angle $\alpha \geq 60°$ and an $\mathcal{O}(n^4)$ algorithm solves the problem for $41.4° \leq \alpha < 60°$ using the same approach used in Theorem 4.3 and Theorem 4.4.

5 Conclusion

We considered the problem of covering the common area between two intersecting straight line segments. In general this problem is NP-hard. A characterization of solutions to this problem depending on the angle between the line segments is given. This leads to an algorithm with running time linear in the number of discs to find a small, though in general not minimum, number of discs covering the common area. The result is then applied to the station location problem for a railway network that consists of two intersecting tracks forming a sufficiently large angle.

From our collaboration with the Deutsche Bahn, we know that railway networks are to some extent decomposable in large regions where only one track is relevant for placing new stations. For future work it would be interesting to apply a decomposition technique to the railway network, where our result can be used as a subroutine for regions where tracks meet.

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