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LIMIT ANALYSIS OF PLANAR FRAME STRUCTURES UNDER SEISMIC LOAD THROUGH IMMUNE ALGORITHMS

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Abstract. *In this paper an original strategy for the effective evaluation of the ultimate resistance and the corresponding failure mechanism of planar frame structures subjected to seismic loads is presented. The methodology is based on the generation of the so called elementary collapse mechanisms to be combined following a collapse load factor minimization criterion. When a large number of possible mechanisms has to be investigated a prompt procedure able to quickly converge to the actual collapse load factor and to jump out of local minima is needed. For this reason a procedure which makes use of evolutionary algorithms based on natural computation, in particular immune algorithms, is here adopted and a dedicated user-friendly software is developed in the NetLogo programming environment. The presence of increasing lateral forces and permanent distributed loads acting on beams, which affects the occurrence and the location of along-axis plastic hinges, is here evaluated by means of an exact formulation. Validations of the proposed procedure are reported together with some analyses on general trends in the seismic behaviour of planar frames. The results demonstrate the reliability of the procedure and can provide useful information also in view of seismic design optimization strategies.*

1 INTRODUCTION

Limit analysis represents a widely adopted strategy for the assessment of the bearing capacity and collapse mechanism of frame structures since it is a reliable method, and is faster with respect to nonlinear static and dynamic analyses. It is directly based on the kinematic theorem of limit analysis; by analyzing all the possible collapse mechanisms of a structure and the related collapse loads, the correct ultimate load is determined seeking the absolute lowest value among the considered mechanisms. This method therefore does not require the direct computation of stiffness matrix and it is not necessary to apply the complete history of loading.

In this approach, one of the most frequently used methods is that first developed by Neal and Symonds [1,2] in which only the elementary mechanisms are analyzed and these are combined to obtain a final collapse mechanism whose load factor is lower than all the possible combinations. The mechanism associated to the lowest load factor represents the real failure mechanism of the structure.

In the literature significant contributions to the automatic computation of the mechanisms for limit analysis of frames have been given by Watwood [3], Gorman [4] and Deeks [5].

The main limitation of plastic analysis and design of frames using a combination of elementary mechanisms is the tedious work of combining them to find the true collapse mechanism. Since both steps of generating the elementary mechanisms and combining them are time-consuming, it is therefore important to develop a methodology capable of finding an approximate collapse load factor and the corresponding mechanism as fast and accurate as possible. To this purpose, very interesting approaches may be found in heuristic algorithms based on natural computation, which have the capability to converge on a good solution independently of the specific search space to which they are applied [6,7]. Among them, many studies have been dedicated to the use of genetic algorithms for engineering purposes when a functional to be maximized can be defined and when the configuration of the system is particularly suitable to be described by means of arrays of integer numbers, which represent the "chromosomes" [8-12].

The present work focuses on the strategy of seeking the collapse load by means of an immune algorithm (IA), which is an optimization approach inspired by the "clonal selection" process of the biological immune system. This algorithm is presented and described in the paper.

The applications have been developed to frame structures subjected to seismic load scenarios in which only the horizontal forces acting at the floor levels are incremented while permanent vertical loads on the beams remain constant. The presence of permanent loads on beams affects the location of plastic hinges when an incremental horizontal load distribution is considered [13]. For this reason neglecting them may lead to coarse errors in the seismic collapse prediction of frame structures. In the applications reported in the paper the exact location of the plastic hinges on the beam, and the related collapse mechanisms are taken into account.

Each elementary collapse mechanism of planar frames, together with arbitrary combinations are built and analyzed by means of an original software code in the agent-based programming language NetLogo [14].

Several applications have been performed in which the values of the collapse load, obtained by means of the proposed method for seismic applications, have been compared to the correspondent results provided by nonlinear push over analysis showing a very good correspondence. The achieved results, show some general trends in the seismic behavior of planar frames and therefore not only may provide significant information on the seismic performance of frame structures, but also represent a useful tool in their optimal design.

2 ELEMENTARY COLLAPSE MECHANISMS FOR FRAME STRUCTURES

In the present study planar regular frames, whose columns at the ground level are assumed to be clamped, are considered. These are characterized by the number of floors N_f and the number of columns N_c.

By considering the *i*-th floor and the *j*-th column of the frame, the plastic moments of the structural members are assumed to be $M_{b,ij}$ for beams and $M_{c,ij}$ for columns.

The frame may be loaded, at each floor, by concentrated horizontal forces F_i and permanent vertical distributed loads q_{ij}. Plastic hinges in the collapse mechanism can therefore be located in s "critical sections" correspondent to each joint and to a certain section of each beam which can coincide with its middle, in case of concentrated forces, or it can vary along the span in case of distributed loading. It is worth to point out that in the case of more than two members converging in a joint, a different critical section must be considered for each member (Figure 1).

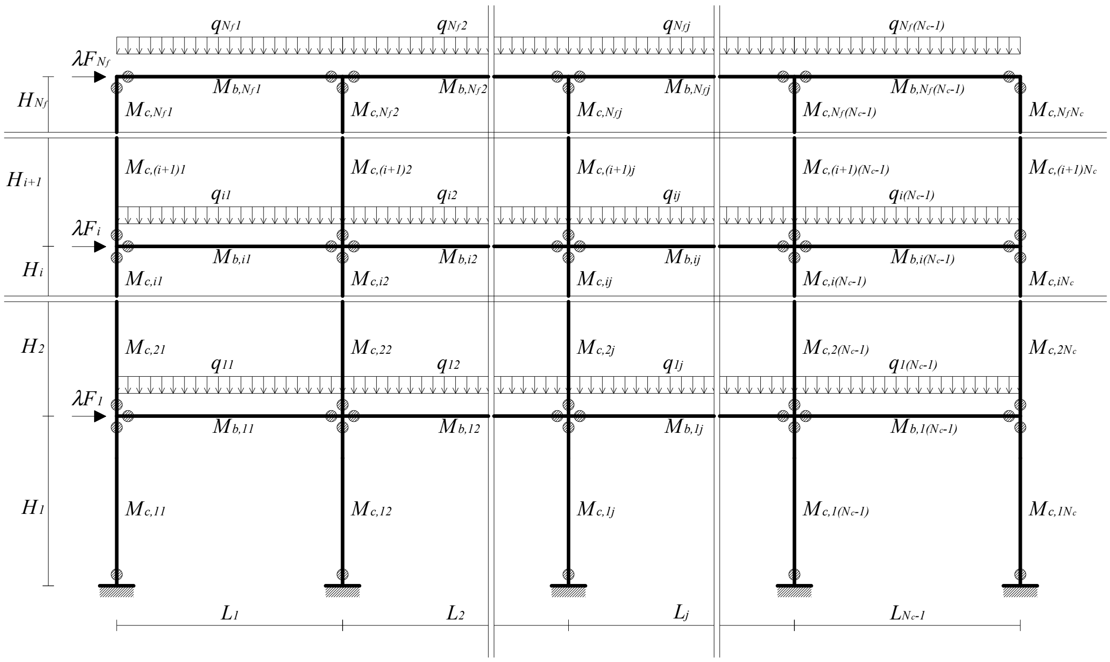


Figure 1. Layout of a generic planar frame.

In a kinematic plastic analysis approach the total number of possible collapse mechanism should be considered. Nevertheless, as introduced by Neal and Symonds [1,2] only a small number of independent elementary collapse mechanisms can be taken into account and they must be combined each other in order to give the lowest collapse load. In the present paper, three different elementary collapse mechanisms are considered: floor, beam and node mechanisms.

3 SEISMIC COLLAPSE LOAD

In the case in which only the horizontal forces are considered variable, as in the case of seismic analysis of structures, while the vertical loads are assumed to be distributed and of constant value, for each collapse mechanism the virtual work theorem states:

$$\lambda_c W_{ext} + W_{extV} = W_{\text{int}} \tag{1}$$

Where W_{extV} represents the work done by the vertical permanent load, which are not magnified. Therefore the value of the multiplier λ_c in this case is given by:

$$\lambda_c = \frac{W_{\text{int}} - W_{extV}}{W_{ext}} \quad (2)$$

With reference to the external work, different distributions of the horizontal forces $F_{h,i}$ considered applied at each floor level can be selected according to a fixed shape, while distributed constant vertical loads q_{ij} act on each of the beams.

The external work done by the permanent vertical load for each beam is related to the beam mechanism shown in Figure 2 where a plastic hinge can be opened at a position x_{ij} from the left end of the beam dependent on the magnitude of the uniform load acting on the beam

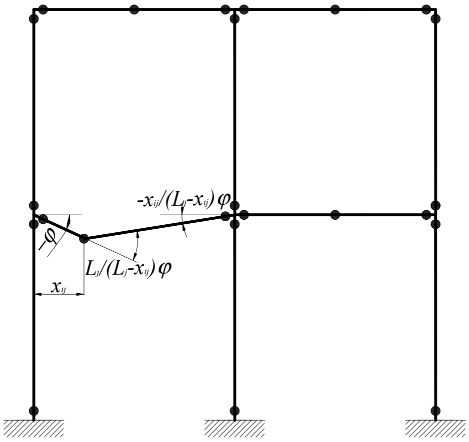


Figure 2. Beam mechanism in case of permanent vertical loads

In fact, at any loading stage the bending moment in each beam is the superposition of the one due to the uniformly distributed vertical loads and that due to horizontal forces (Figure 3). Therefore, increasing the horizontal forces the first plastic hinge opens at the beam end opposite to the horizontal force, with plastic moment $M_{b,ij}$, while the second hinge forms at the first end when the distributed load is equal to the limit value [13]:

$$q_{\lim,ij} = \frac{4M_{b,ij}}{L_j^2} \quad (3)$$

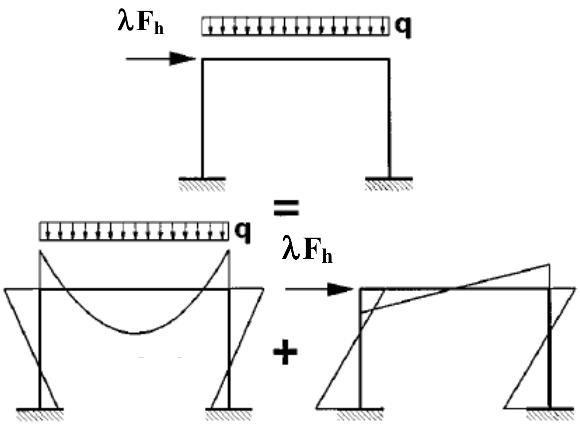


Figure 3. Location of plastic hinge on beams.

When the vertical load q_{ij} exceeds the limit value provided in (3) a hinge can open along the beam at the abscissa

$$x_{ij} = L_j - 2\sqrt{\frac{M_{b,ij}}{q_{ij}}} \tag{4}$$

Therefore, the external work for each beam and floor mechanism is:

$$W_{ext}(beam\ mech)_{ij} = 0.5 q_{ij} x_{ij} L_j \varphi$$

$$W_{ext}(floor\ mech)_i = \left(\sum_{k=i}^{N_f} F_{h,k} \right) d_{h,i} = \left(\sum_{k=i}^{N_f} F_{h,k} \right) H_i \varphi \tag{5}$$

For the internal work related to floor and node elementary mechanisms the same expressions of the case of proportional load hold, while for beam mechanisms it is (Figure 2):

$$W_{\text{int}}(beam\ mech)_{ij} = 2M_{b,ij} \left(1 + \frac{x_{ij}}{L_j - x_{ij}} \right) \varphi \tag{6}$$

When the elementary mechanisms are combined, the rotations related to each critical section are obtained adding all the relevant values. The external work in a combined mechanism is the sum of the elementary ones while the internal work is computed by multiplying each total rotation for the plastic moment of the related element. Therefore, when elementary mechanisms with opposite rotations in a critical section are added, the total rotation may turn out to be zero thus providing a smaller dissipated energy. Of course in order to obtain the lowest value of λ_c, the dissipated energy must be as smaller as possible, and this is fulfilled by means of the optimization procedure.

4 OPTIMIZATION PROCEDURE THROUGH IMMUNE ALGORITHM

Analyzing all the possible combinations of elementary mechanisms, the minimum value of λ_c must be sought in order to obtain the real collapse load.

Many studies have been presented in the literature dealing with the method of combining elementary mechanisms proposed by Neal and Symonds [1]. In this paragraph the optimization procedure adopted in the present paper, which makes use of immune algorithms, is described.

Artificial immune systems are inspired by principles and mechanisms of theoretical immunology and have been applied over the years to solve various complicated optimization problems (see ref. [15] for an overview on the field). The role of the immune systems is that of protecting the body from disease-causing agents (pathogens), and eliminate malfunctioning cells. When foreign pathogens (antigens) invade an organism, the latter stimulates an immune response: first of all the antigens are recognized by immune cell receptors, then a “clonal selection” process causes a proliferation of these cells and the secretion of antibodies.

In an artificial immune algorithm, the antigen represents the configuration of variables in the optimal solution of the optimization problem while the antibody represents a trial solution for the same variables. Once defined a fitness function as a measure of the affinity of a given antibody with the antigen, the algorithm can act over a population of P potential solutions (antibodies) by applying, iteratively, the "survival of the fittest" principle of the Darwinian evolution: in such a way, a sequence of new generations of antibodies is produced and evolves towards a stationary population where the large majority of surviving solutions with high fitness – i.e. with high affinity with the antigen – do coincide and approach as much as possible the real solution of the problem.

In order to translate into this scenario the original problem of finding the linear combination of elementary collapse mechanisms with the minimum value of λ_c, corresponding to the true collapse load factor, each antibody of the population is coded as a string of integer numbers, where each number represents how many times a given elementary mechanism enters in the combination. Therefore, if there are in total $N = N_{fm} + N_{bm} + N_{nm}$ elementary mechanisms, a generic antibody A_i of the population (i = 1, …, P) represents a generic weighted combination of those mechanisms and can be coded in the following string of elements (molecules):

$$A_i \equiv (c_1, c_2, c_3, \dots c_k, \dots, c_N) \tag{7}$$

with $c_k \in [0,c_{max}]$, being c_{max} the maximum number of times the k-th mechanism is involved in the combination (c_k = 0 means that the mechanism is not involved at all). Given the string, it is possible to calculate the load factor λ_i of the corresponding combination (antibody). The overall number of possible different antibodies is thus $P_{max} = (c_{max} + 1)^N$, a quantity which rapidly increases with N even for small values of c_{max} (for example, if c_{max} = 2 and N = 22, one obtains $P_{max} \approx 31\text{x}10^9$). Task of the immune algorithm is that of exploring the space of all the possible antibodies, in search of the combination of mechanisms which minimizes λ_i or, that is the same, which maximizes the corresponding fitness function, here defined as

$$f(A_\mathrm{i}) = K - \lambda_\mathrm{i} \tag{8}$$

where K is a constant greater enough to ensure positive values for the fitness. In the following, without loss of generality, we set K = 100. Starting from an initial population of P antibodies, randomly chosen among the P_{max}, at each time step a new generation is created from the old one, where individuals with a higher fitness score are more likely to be selected than those that have low fitness scores. The clonal selection method adopted in this paper is based on the so called "tournament selection", with a tournament size of three: this means that, iteratively, groups of 3 antibodies are drawn randomly (with a uniform probability distribution) from the old generation, and each time that one with the highest fitness, say A^*, is chosen within each group.

Depending on the value of an opportune control parameter, the new antibodies can either be simply clones of the original one or can be derived from the latter through a mechanism, called "hypermutation" [16], which randomly changes the value of each entry of each original antibody string (choosing in the interval $[0,c_{max}]$) with a probability $p_H = 1 - f(A^*) / K$, where $f(A^*)$ is the fitness of A^*. Once the new generation is created, there is also a chance (tuned by the parameter "error rate") that further random mutations, or errors in the cloning process, will occur at level of each molecule of the newly generated antibodies. In Figure 4 a sketch of these operations for a generic antibody is summarized**.**

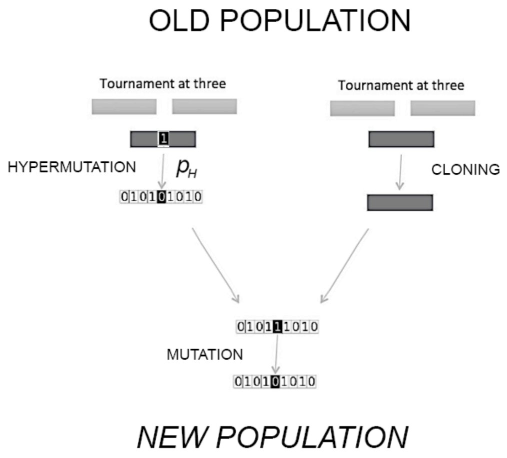


Figure 4. Natural selection process through tournament with size of three for a generic antibody. Boxes in dark grey represent antibodies with higher fitness, i.e. with higher affinity with the antigen corresponding with the optimal solution. In this example $c_{max} = 1$.

By iteratively repeating this process several times, antibodies with the highest fitness will be progressively selected in the space of all the possible combinations and will quickly spread among the population reducing the diversity of the individuals, until (almost) only one of them will survive: hopefully, that one with the maximum fitness (and, correspondingly, with the minimum load factor), i.e. with the highest affinity with the antigen corresponding with the optimal solution. Of course, it is frequent for the dynamics to remain trapped into local maxima of fitness (minima of λ_i), therefore it is convenient to launch the algorithm many times (events), each time starting from a different initial population, in order to gain more chances to reach the global maximum of fitness, corresponding to the true collapse load factor of the structure.

5 APPLICATIONS

In this section some applications will be presented, aiming at validating the proposed approach and deducing how some geometric parameters affect the collapse load and mechanisms of frame structures.

All the applications have developed by means of an original code which allows both to calculate all the elementary collapse mechanisms of a given planar frame and to implement the genetic algorithm for the determination of their combination with the minimum collapse load factor. For this purpose, a very powerful software has been adopted, that is NetLogo [14], which is a freeware multiplatform environment with an owner high level programming language and with a very ductile and versatile user interface (Figure 5)

Two frames have been taken into account representative of high and low structures. Both these frames have floor height H and bay length L equal to 3 m and 4 m respectively. The first one, F1, has six storeys and three bays while the second, F2, has three storeys and three bays (Figure 6). All the beams have the same plastic moment (60 kN*m) and the same permanent vertical load acting on them (20 kN/m), the plastic moments of the columns is equal to 100 kN*m at the top floor and increases of 50 kN*m for each underlying floor.

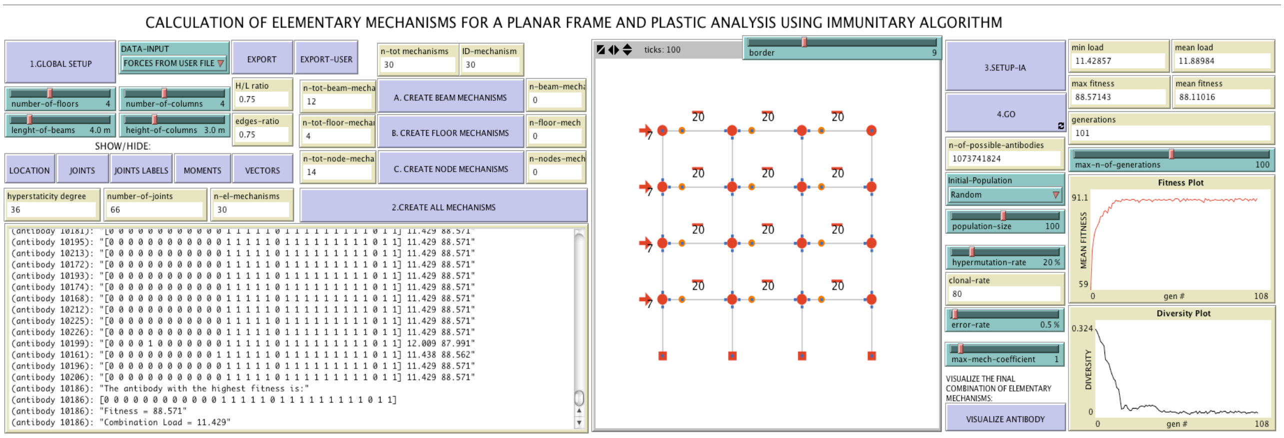


Figure 5 Graphical aspect of the user interface realized in the NetLogo environment.

As already described, the location the potential along span plastic hinge depends on the length and on the plastic moment of the beam, as well as on the intensity of the permanent vertical load acting on it. In particular, when the acting load is lower than a limit value (provided in Eq. (3)), no along span plastic hinge can occur (i.e. plastic hinges can occur only at the two ends of the beam), while when the acting load is higher than the limit load the along span plastic hinge can occur at a specific section (see Eq. (4)). These potential plastic hinges may anyway not be involved in the failure mechanism. In both the considered frames, a potential along span plastic hinge located at 0.5359 m from the left end of all the beams can occur.

For both the frames F1 and F2 two different load scenarios have been considered: one with the horizontal loads proportional to the vertical load distribution (mass proportional distribution) denoted in the following as LC1, and one with the horizontal loads proportional to the product between the vertical load intensity and the height (inverse triangular distribution) denoted as LC2.

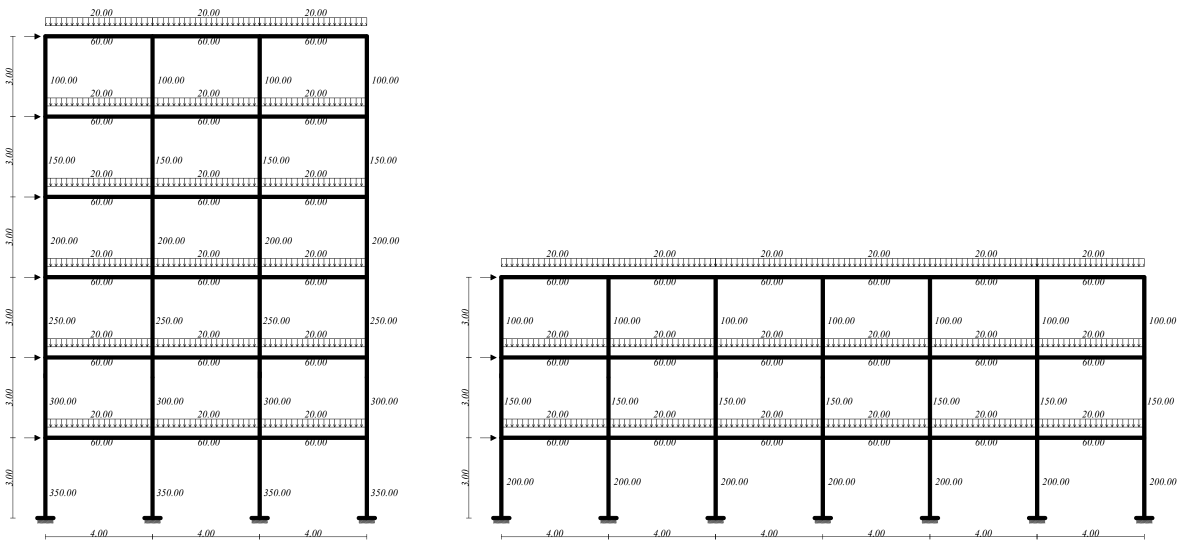


Figure 6 Geometric and mechanical layout of frames F1 and F2

Due to the different number of storeys and bays in the two frames, the number of elementary mechanisms is also different for the two frames. In particular, one finds 46 elementary mechanisms for F1 and 40 for F2, respectively. Consequently, from the point of view of the proposed optimization approach based on immune algorithms, in the first case each antibody will have 46 molecules (therefore, for c_{max}=1, the total number of possible antibodies will be $P_{\max}$~7x10^{13}), while in the second case each antibody will have 40 molecules (therefore $P_{\max}$~1x10^{12}).

The setup of the simulation parameters for all the applications will be always the same: the size of the initial population of antibodies, randomly chosen among the $P_{\max}$, is fixed at P = 100 individuals; the hypermutation rate at 20% (therefore the cloning rate at 80%) and the error rate at 2%; finally, the number of generations has been set to 100, enough to reach a stationary state for the average fitness of the antibodies.

All the results obtained through the proposed procedure have been also compared and validated by means of a classic pushover approach in terms of ultimate load. A numerical model was implemented in the well known FEM software SAP2000 [17]. The considered model employs lumped nonlinearities, simulated by means of perfectly plastic hinges. Since in a pushover analysis the location of plastic hinges has to be set 'a priori', in the beams the critical sections have been set every 10 cm.

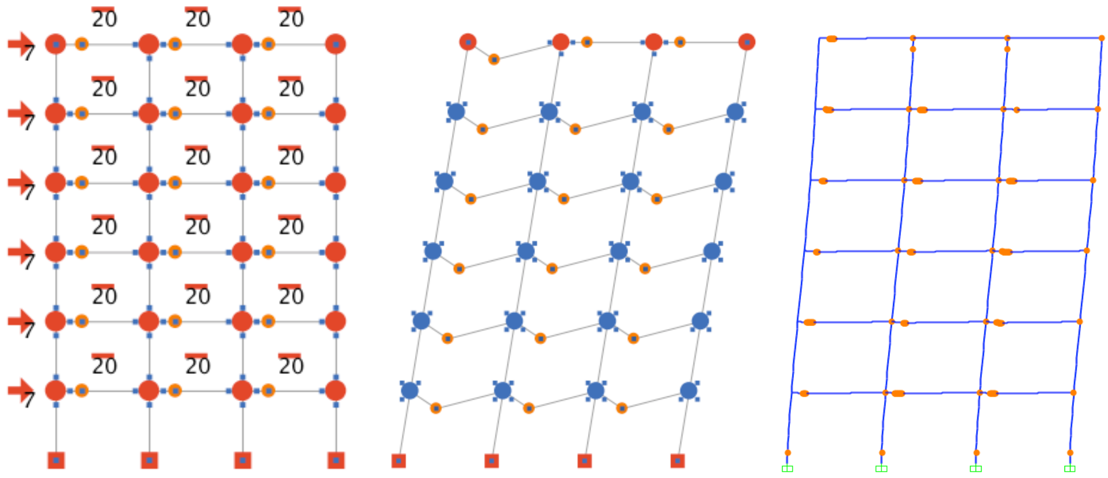
For the six storeys frame F1 in the LC1 scenario, the immune algorithm finds a collapse multiplier λ_c=7.88. The corresponding antibody with the maximum fitness presents a combination where all the elementary mechanisms are involved except the two beam mechanisms of the right and central spam of the last floor and the two node mechanisms of the same floor. In other words, the selected string of molecules has 42 unitary values and 4 zeros.

For the same scenario, the FEM approach leads to a collapse multiplier of the horizontal loads equal to λ_{SAP}=7.808. The very close results allow to assess the validity of the proposed procedure. It is worth to notice that, since the forces are gradually incremented in a pushover analysis, the actual collapse multiplier of a structure represents an upper bound for such an approach. Furthermore, due to the kinematic theorem of plastic analysis represents a lower bound for a limit analysis approach; therefore it has to be $\lambda_{c,SAP} \leq \lambda_u \leq \lambda_c$.

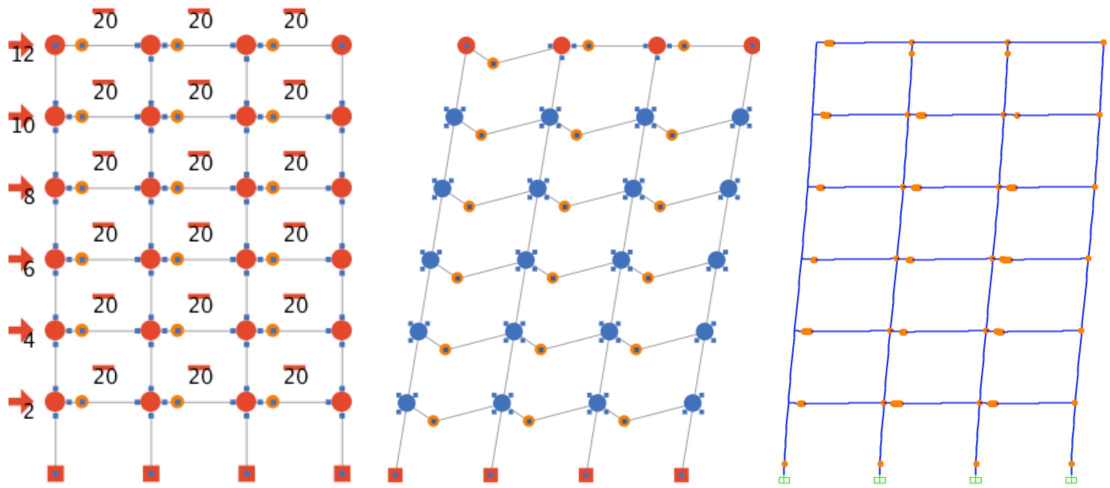
Considering again the frame F1, but in the LC2 scenario, the corresponding results are λ_c=6.36 and λ_{SAP}=6.306. It is worth to notice that the collapse load related to the inverse triangular distribution of horizontal loads is smaller than the correspondent value for mass proportional distribution since such distribution tends to involve in the collapse mechanisms the highest storeys which are usually weaker than the lower ones.

In Figure 7 the collapse mechanisms of the frame F1 in both the load conditions, calculated by means of the proposed NetLogo software and SAP2000 have been reported showing a very good correspondence. In particular in the upper left part of the figure the values of the horizontal loads proportional to the vertical load distribution (LC1) are reported. The collapse mechanism shown in the upper central part of the figure is obtained by means of the proposed NetLogo code while the correspondent mechanism provided by SAP 2000 is reported in the upper right part. The lower part of Figure 7 refers to horizontal loads proportional to the product between the vertical load intensity and the height (LC2) and shows the values of the forces and the calculated collapse mechanisms in the same order than the upper part.

As it can be easily noticed the shape of the collapse mechanisms is the same for both the load conditions.



(a)



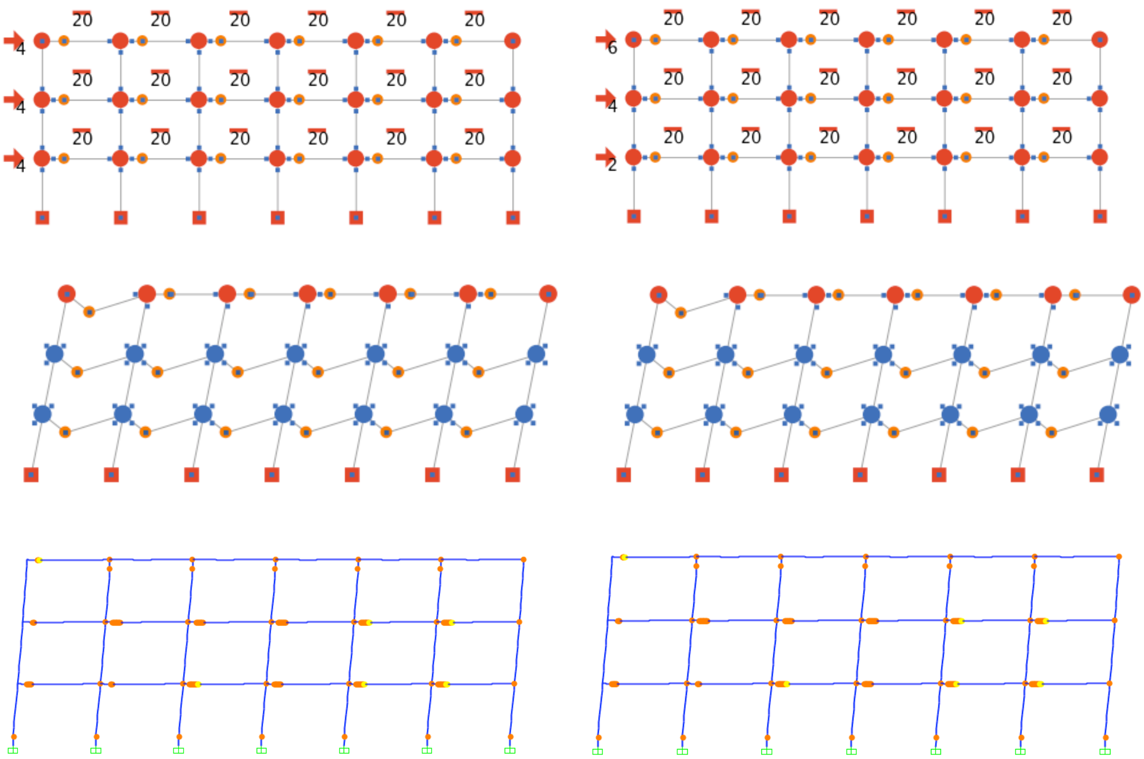
(b)

Figure 7 Collapse mechanisms for the six storey frame: (a) LC1, (b) LC2

Concerning the analysis of the three storey frame F2, the immune algorithm gives, for the LC1 load condition, the collapse load λ_c=47.54. The corresponding value calculated by means of the pushover analysis is λ_{SAP}=47.200. On the other hand, for the case of load condition LC2 the collapse loads evaluated by means of the two procedures are, respectively, λ_c=40.75 and λ_{SAP}=40.452.

Once again, as expected, the collapse load related to the inverse triangular distribution of horizontal loads is smaller than the correspondent value for mass proportional distribution since the former is typically heavier.

Figure 8 shows for the frame F2, here in the left and right part, the values of the horizontal loads in LC1 and LC2 and the collapse mechanisms, calculated by means of the proposed NetLogo software and SAP2000. Also in this case the shape of the collapse mechanisms is the same for both the load conditions



(a) (b)

Figure 8 Collapse mechanisms for the three storey frame: (a) LC1, (b) LC2

The two chosen example may be considered representative of tall and low-rise buildings, which might be easily studied with the proposed approach. In order to provide general consideration on buildings, in the future, parametric analyses also accounting for vertical and horizontal irregularities will be performed.

6 CONCLUSIONS

An automatic approach for the evaluation of plastic loads and failure modes of planar frames, based on the generation of elementary collapse mechanisms and on their linear combination, has been presented. The proposed approach represents an extension of the method of combination of elementary mechanisms, and accounts for permanent distributed vertical loads in addition to horizontal concentrated increasing forces. The procedure makes use of an original software developed in the agent-based programming language NetLogo, which is able to automatically determine and visualize all the elementary mechanisms in planar frames. Then, by means of an optimization procedure based on immune algorithms, the code calculates, with great accuracy and in a very short computing time, the collapse load and the related mechanism. Some applications with a seismic point of view considering a system of horizontal forces whose magnitude increases while the vertical loads are assumed to be constant have been performed. The results, either in terms of collapse load or mechanism, have been compared with the correspondent provided by non linear pushover analysis showing a very good correspondence in a significantly shorter computing time. It has been shown that the collapse multiplier provided by SAP2000 is always slightly lower than that obtained with the proposed methodology. The seismic applications represent an original contribution towards the limit behaviour of structures under earthquake excitations, since some general trends of seismic behaviour of planar frames can be deduced from the obtained results.

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