J. Math. Anal. Appl. 427 (2015) 962–976

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Nonexistence of certain universal polynomials between Banach spaces ☆

Raffaella Cilia [a], Joaquín M. Gutiérrez [b,*]

[a] *Dipartimento di Matematica, Università di Catania, Viale Andrea Doria 6, 95125 Catania, Italy*
[b] *Departamento de Matemáticas del Área Industrial, ETS de Ingenieros Industriales, Universidad Politécnica de Madrid, C. José Gutiérrez Abascal 2, 28006 Madrid, Spain*

ARTICLE INFO

Article history:
Received 18 October 2014
Available online 25 February 2015
Submitted by Richard M. Aron

Keywords:
Universal non-compact polynomial
Universal non-unconditionally converging polynomial

ABSTRACT

A well-known result due to W.B. Johnson (1971) asserts that the formal identity operator from ℓ_1 into ℓ_∞ is universal for the class of non-compact operators between Banach spaces. We show that there is neither a universal non-compact polynomial nor a universal non-unconditionally converging polynomial between Banach spaces.

1. Introduction

J. Lindenstrauss and A. Pełczyński [20, Theorem 8.1] proved that the *sum* operator $\sigma : \ell_1 \to \ell_\infty$ defined by

$$\sigma(x) := \left(\sum_{i=1}^{n} x_i \right)_{n=1}^{\infty} \qquad \text{for } x = (x_n)_{n=1}^{\infty} \in \ell_1$$

is universal for the class of non-weakly compact operators; that is, an operator $T \in \mathcal{L}(X, Y)$ is not weakly compact if and only if there exist operators $A \in \mathcal{L}(\ell_1, X)$ and $B \in \mathcal{L}(Y, \ell_\infty)$ such that the following diagram commutes:

$$\begin{array}{ccc} X & \xrightarrow{T} & Y \\ A\uparrow & & \downarrow B \\ \ell_1 & \xrightarrow[\sigma]{} & \ell_\infty \end{array}$$

☆ Both authors were supported in part by Dirección General de Investigación, MTM 2012-34341 (Spain).
* Corresponding author.
E-mail addresses: cilia@dmi.unict.it (R. Cilia), jgutierrez@etsii.upm.es (J.M. Gutiérrez).

http://dx.doi.org/10.1016/j.jmaa.2015.02.072
0022-247X/

W.B. Johnson [19] showed that the formal identity operator $\ell_1 \to \ell_\infty$ is universal for the class of non-compact operators.

The existence of universal operators may be useful when we try to prove that certain operators belong to a given operator ideal. The universal non-weakly compact operator was used for instance to prove that every polynomially continuous operator is weakly compact [16, Theorem 3]. The universal non-compact operator has recently been used in [18, Remark 3.3].

A. Pietsch proposed in 1983 [27] to design a theory of ideals of multilinear functionals somehow parallel to the theory of operator ideals. Since then, a large amount of work has been done in order to prove that certain properties of linear operators are true or false in the nonlinear case (polynomials, multilinear mappings, differentiable or holomorphic mappings, etc.). In this paper we give examples of properties that cannot be transferred from the linear to the nonlinear situation. Concretely, we show that certain classes of polynomials do not admit a universal polynomial.

Before giving an idea about the problem and the difficulties encountered, we introduce some definitions and notation.

Given an integer $k \geq 1$, we extend to the polynomial setting notions established in [12] for the case of (linear bounded) operators. Suppose that $\mathcal{Q}$ is a class of k-homogeneous (continuous) polynomials between Banach spaces so that a polynomial P is in $\mathcal{Q}$ whenever there exist operators A, B so that BPA is in $\mathcal{Q}$. The natural examples of such classes are all the polynomials that do not belong to a given ideal of k-homogeneous polynomials. A polynomial P_0 of such a class $\mathcal{Q}$ is said to be *universal for* $\mathcal{Q}$ provided for each P in $\mathcal{Q}$, P_0 factors through P, that is, there exist operators A and B so that $BPA = P_0$.

The notion of *ideal of k-homogeneous polynomials* is well known and has been widely studied in the literature (see, for instance, [6, §3] or the recent paper [24]).

Throughout, E, F, X, and Y denote Banach spaces, E^* is the dual space of E, B_E stands for its closed unit ball, and S_E is the unit sphere of E. The closed unit ball B_{E^*} of the dual space will always be endowed with the weak-star topology. By $\mathbb{N}$ we represent the set of all natural numbers, and by $\mathbb{K}$ the scalar field (real or complex). We denote by $\mathcal{L}(E, F)$ the space of all (linear bounded) operators from E into F endowed with the operator norm, and by $\mathcal{K}(E, F)$ the subspace of compact operators. The symbol I_E stands for the identity map on E. An operator $h \in \mathcal{L}(E, F)$ is an *embedding* if it is an isomorphism onto its image $h(E) \subseteq F$.

The unit vector basis of c_0 will be denoted by $(e_n)_{n=1}^\infty$ and the unit vector basis of ℓ_1 by $(e_n^*)_{n=1}^\infty$.

Given $k \in \mathbb{N}$, we denote by $\mathcal{P}(^kE, F)$ the space of all k-homogeneous (continuous) polynomials from E into F endowed with the supremum norm. For the general theory of polynomials on Banach spaces, we refer the reader to [8] and [21].

Recall that with each $P \in \mathcal{P}(^kX, Y)$ we can associate a unique symmetric k-linear (continuous) mapping $\widehat{P} : X \times \overset{(k)}{\cdots} \times X \to Y$ so that

$$P(x) = \widehat{P}\left(x, \overset{(k)}{\ldots}, x\right) \qquad (x \in X)\,.$$

With each $P \in \mathcal{P}(^kX, Y)$ we can also associate a unique operator

$$\overline{P} : \widehat{\otimes}^k_{\pi_s,s} X \longrightarrow Y$$

which is called the *linearization* of P and is given by

$$\overline{P}\left(\sum_{i=1}^n \lambda_i x_i \otimes \overset{(k)}{\cdots} \otimes x_i\right) = \sum_{i=1}^n \lambda_i P(x_i)\,, \text{ for } \lambda_i \in \mathbb{K} \text{ and } x_i \in X\,,$$

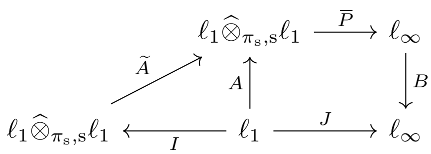
where $\widehat{\otimes}^k_{\pi_s,s} X$ denotes the k-fold symmetric tensor product of X [11]. It is well known that P is compact (see definition in Section 2) if and only if $\overline{P}$ is compact [28, Lemma 4.1].

For non-explained notation and terminology on Banach spaces, the reader is referred to [7]. For operator ideals, see [26].

We now include a few words about the origin of our work and about how we tried to tackle it.

For some time, we tried to find a universal non-compact k-homogeneous polynomial for a fixed integer $k \geq 2$. In fact, we prove that such a polynomial does not exist. Similarly, there is no universal non-unconditionally converging polynomial (see definition in Section 3).

For simplicity, we only consider in this short overview the case of 2-homogeneous polynomials. If J is the formal identity operator from ℓ_1 into ℓ_∞ and $P \in \mathcal{P}(^2\ell_1, \ell_\infty)$ is the non-compact polynomial given by $P(x) := \left(x_n^2\right)_{n=1}^{\infty}$ for $x = (x_n)_{n=1}^{\infty} \in \ell_1$, then there are operators A and B such that the right-hand side of the following diagram commutes:



Denote by $I : \ell_1 \to \ell_1\widehat{\otimes}_{\pi_s,s}\ell_1$ an onto isomorphism (for the nonsymmetric tensor product, see [29, Exercise 2.6]; the symmetric case may be viewed as an application of the Pełczyński decomposition technique [1, Theorem 2.2.3]). Then A induces an operator $\widetilde{A} : \ell_1\widehat{\otimes}_{\pi_s,s}\ell_1 \to \ell_1\widehat{\otimes}_{\pi_s,s}\ell_1$ so that $\widetilde{A} \circ I = A$. The operator $J \circ I^{-1}$ defines a non-compact polynomial $P_0 \in \mathcal{P}(^2\ell_1, \ell_\infty)$ which should be a natural candidate to be universal. Now, for P_0 to be universal, we would need to be able to write $\widetilde{A}$ in the form $U \otimes U$, with $U \in \mathcal{L}(\ell_1, \ell_1)$. The nonexistence of a universal non-compact polynomial implies that $\widetilde{A}$ cannot be written as a tensor product of operators. This is a by-product of our result which may be of independent interest.

2. Polynomials which do not belong to a surjective-type ideal

Recall that an operator ideal $\mathcal{U}$ is said to be *injective* [26, 4.6.9] if, given an operator $T \in \mathcal{L}(E, F)$ and an into isomorphism $i : F \to G$, we have that $T \in \mathcal{U}$ whenever $iT \in \mathcal{U}$. The ideal $\mathcal{U}$ is *surjective* [26, 4.7.9] if, given $T \in \mathcal{L}(E, F)$ and a surjective operator $q : G \to E$, we have that $T \in \mathcal{U}$ whenever $Tq \in \mathcal{U}$. We say that $\mathcal{U}$ is *closed* [26, 4.2.4] if for all E and F, the space $\mathcal{U}(E, F) := \{T \in \mathcal{L}(E, F) : T \in \mathcal{U}\}$ is closed in $\mathcal{L}(E, F)$.

A list of injective and surjective operator ideals may be seen in [14]. In particular, the ideal of compact operators is closed, injective, and surjective.

The following result is well known. We include it here for the sake of clarity.

Proposition 2.1. *Let X and Y be Banach spaces. The following assertions are equivalent:*

(a) *There are operators*

$$X \xrightarrow{j} Y \xrightarrow{\pi} X$$

such that $\pi \circ j = I_X$; in other words, X is isomorphic to a complemented subspace of Y;

(b) *There are operators*

$$X \xrightarrow{j} Y \xrightarrow{\pi'} Y$$

with j injective, π' a projection (that is, $\pi' \circ \pi' = \pi'$), and $\pi'(Y) = j(X)$.

Sketch of proof. (a) $\Rightarrow$ (b). Define $\pi' := j \circ \pi$.

(b) $\Rightarrow$ (a). Given $y \in Y$, since $\pi'(y) \in j(X)$ and using the injectivity of j, there is a unique $x \in X$ so that $\pi'(y) = j(x)$. Define $\pi(y) := x$. $\square$

Next, we recall a result from [4, Proposition 5]. However, we prefer to sketch a different proof whose ingredients will be needed later on.

Proposition 2.2. *Given Banach spaces X and Y, and $k \in \mathbb{N}$, the space $\mathcal{P}(^kX, Y)$ is isomorphic to a complemented subspace of $\mathcal{P}(^{k+1}X, Y)$.*

Proof. The proof of [2, Proposition 5.3] is given for $Y = \mathbb{K}$, but it also works in the vector-valued case. We include the details which will be needed later on. Given $\psi \in S_{X^*}$, choose $x_0 \in X$ so that $\psi(x_0) = 1$. Define the operators

$$\mathcal{P}(^kX, Y) \xrightarrow{j} \mathcal{P}(^{k+1}X, Y) \xrightarrow{\pi} \mathcal{P}(^kX, Y)$$

by $j(R)(x) := \psi(x)R(x)$ for all $R \in \mathcal{P}(^kX, Y)$ and $x \in X$, and

$$\pi(P)(x) := \sum_{i=1}^{k+1} \binom{k+1}{i} \psi(x)^{i-1}(-1)^{i-1}\widehat{P}\left(x_0^i, x^{k+1-i}\right) =: Q(x)$$

for $P \in \mathcal{P}(^{k+1}X, Y)$ and $x \in X$, where $\widehat{P}\left(x_0^i, x^{k+1-i}\right)$ means that $\widehat{P}$ is applied to the vector x_0 taken i times and to x taken $k+1-i$ times.

As in [2, Proposition 5.3], define the operator

$$\mathcal{P}(^{k+1}X, Y) \xrightarrow{\pi'} \mathcal{P}(^{k+1}X, Y)$$

by

$$\pi'(P)(x) := P(x) - P(x - \psi(x)x_0)\,, \quad \text{for } P \in \mathcal{P}(^{k+1}X, Y) \text{ and } x \in X\,.$$

It is shown in the proof of [2, Proposition 5.3] that

$$P(x) - P(x - \psi(x)x_0) = \psi(x)Q(x) \quad \text{for all } x \in X\,,$$

and that j and π' satisfy condition (b) of Proposition 2.1. We shall give in detail the proof of $\pi'\left(\mathcal{P}(^{k+1}X, Y)\right) = j\left(\mathcal{P}(^kX, Y)\right)$. Indeed, for all $P \in \mathcal{P}(^{k+1}X, Y)$, we have $\pi'(P) = \psi Q = j(Q)$, so $\pi'\left(\mathcal{P}(^{k+1}X, Y)\right) \subseteq j\left(\mathcal{P}(^kX, Y)\right)$. For the reverse inclusion, let $R \in \mathcal{P}(^kX, Y)$. Since $\pi'(\psi R) = \psi R$ (see the proof of [2, Proposition 5.3]), we have $j(R) = \psi R = \pi'(\psi R)$ which implies $j\left(\mathcal{P}(^kX, Y)\right) \subseteq \pi'\left(\mathcal{P}(^{k+1}X, Y)\right)$, and so

$$\pi'\left(\mathcal{P}(^{k+1}X, Y)\right) = j\left(\mathcal{P}(^kX, Y)\right)\,.$$

By (b) $\Rightarrow$ (a) of Proposition 2.1 above, we obtain $\pi \circ j = I$, where I is the identity map on $\mathcal{P}(^kX, Y)$, and the proof is finished. □

Let $\mathcal{U}$ be a closed surjective operator ideal. As in [14, p. 472], we denote by $\mathcal{C}_{\mathcal{U}}(E)$ the collection of all sets $A \subset E$ so that $A \subseteq T(B_Z)$ for some Banach space Z and some operator $T \in \mathcal{U}(Z, E)$.

Definition 2.3. Given a closed surjective operator ideal $\mathcal{U}$ and an integer $k \geq 1$, let

$$\mathcal{P}_{\mathcal{U}}(^kX, Y) := \left\{P \in \mathcal{P}(^kX, Y) : \, P(B_X) \in \mathcal{C}_{\mathcal{U}}(Y)\right\}\,.$$

The space $\mathcal{P}_{\mathcal{U}}(^kX, Y)$ will be endowed with the supremum norm.

For instance, $\mathcal{P}_{\mathcal{K}}(^kX,Y)$ will be the space of compact polynomials; that is, $P \in \mathcal{P}_{\mathcal{K}}(^kX,Y)$ if and only if $P(B_X)$ is relatively compact in Y.

Proposition 2.4. *If $\mathcal{U}$ is a closed surjective operator ideal, the space $\mathcal{P}_{\mathcal{U}}(^kX,Y)$ is closed in $\mathcal{P}(^kX,Y)$.*

Proof. Let $(P_n) \subset \mathcal{P}_{\mathcal{U}}(^kX,Y)$ be a Cauchy sequence. Then (P_n) has a limit $P \in \mathcal{P}(^kX,Y)$. So, for every $\epsilon > 0$ there is $n_0 \in \mathbb{N}$ such that

$$n \geq n_0 \quad \Rightarrow \quad \|P - P_n\| < \epsilon\,.$$

Hence, for $n \geq n_0$,

$$P(B_X) \subseteq P_n(B_X) + \epsilon B_{\mathcal{P}(^kX,Y)}\,.$$

Since $P_n(B_X) \in \mathcal{C}_{\mathcal{U}}(Y)$, we have by [14, Proposition 3] that $P(B_X) \in \mathcal{C}_{\mathcal{U}}(Y)$, and so $P \in \mathcal{P}_{\mathcal{U}}(^kX,Y)$. □

Proposition 2.5. *Given $\psi \in S_{X^*}$, choose $x_0 \in X$ so that $\psi(x_0) = 1$. Consider the operators*

$$\mathcal{P}(^kX,Y) \xrightarrow{j} \mathcal{P}(^{k+1}X,Y) \xrightarrow{\pi} \mathcal{P}(^kX,Y)$$

constructed in the proof of Proposition 2.2 above. If $\mathcal{U}$ is a closed surjective operator ideal, restricting j and π to the spaces $\mathcal{P}_{\mathcal{U}}$, we have

$$\mathcal{P}_{\mathcal{U}}(^kX,Y) \xrightarrow{j} \mathcal{P}_{\mathcal{U}}(^{k+1}X,Y) \xrightarrow{\pi} \mathcal{P}_{\mathcal{U}}(^kX,Y)\,,$$

that is, j and π take polynomials in $\mathcal{P}_{\mathcal{U}}$ into polynomials in $\mathcal{P}_{\mathcal{U}}$.

Proof. If $Q \in \mathcal{P}_{\mathcal{U}}(^kX,Y)$, then $j(Q) = \psi Q$. If $\mathbb{D} := \{\alpha \in \mathbb{K} : |\alpha| \leq 1\}$, there are a Banach space Z and an operator $T \in \mathcal{U}(Z,Y)$ such that

$$j(Q)(B_X) \subseteq \psi(B_X)Q(B_X) \subseteq \mathbb{D}Q(B_X) \subseteq \mathbb{D}T(B_Z) = T(\mathbb{D}B_Z) = T(B_Z)\,.$$

Therefore, $j(Q) \in \mathcal{P}_{\mathcal{U}}(^{k+1}X,Y)$.

Let now $P \in \mathcal{P}_{\mathcal{U}}(^{k+1}X,Y)$. Using the above argument, the polarization formula [21, Theorem 1.10], and the fact that multiplication by scalars preserves sets in $\mathcal{C}_{\mathcal{U}}(Y)$, we have that

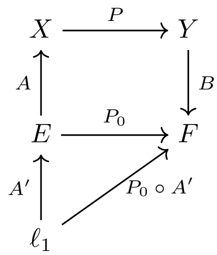
$$\left\{\psi(x)^{i-1}(-1)^{i-1}\widehat{P}\left(x_0^i, x^{k+1-i}\right) : x \in B_X\right\} \in \mathcal{C}_{\mathcal{U}}(Y)\,.$$

The sum of sets in $\mathcal{C}_{\mathcal{U}}(Y)$ is in $\mathcal{C}_{\mathcal{U}}(Y)$ [14, Proposition 3]. Therefore, $\pi(P)(B_X) \in \mathcal{C}_{\mathcal{U}}(Y)$. □

We now focus our attention on non-compact polynomials.

Lemma 2.6. *Given $k \in \mathbb{N}$ ($k \geq 1$), if there is a universal k-homogeneous non-compact polynomial, then there is a universal k-homogeneous non-compact polynomial defined on ℓ_1.*

Proof. If $P_0 \in \mathcal{P}(^kE,F)\backslash\mathcal{P}_{\mathcal{K}}(^kE,F)$ is universal non-compact, there are a sequence $(x_n) \subset S_E$ and $\delta > 0$ such that the sequence $(P_0(x_n))$ is δ-separated. Define $A' \in \mathcal{L}(\ell_1, E)$ by $A'(e_n) := x_n$. Given a polynomial $P \in \mathcal{P}(^kX,Y)\backslash\mathcal{P}_{\mathcal{K}}(^kX,Y)$, we have a factorization

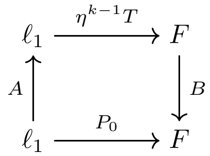


Therefore, the polynomial $P_0 \circ A' \in \mathcal{P}(^k\ell_1, F)$ is universal for the class of non-compact polynomials. □

From now on, let $J \in \mathcal{L}(\ell_1, \ell_\infty)$ be the formal identity operator.

Proposition 2.7. *Let $k \in \mathbb{N}$ $(k \geq 2)$. If there is a universal k-homogeneous non-compact polynomial $P_0 \in \mathcal{P}(^k\ell_1, F)$, it can be taken of the form $P_0 = \xi^{k-1}J$, where $\xi \in \ell_\infty$.*

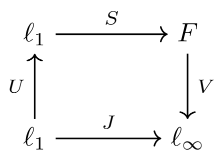
Proof. Let $T \in \mathcal{L}(\ell_1, F)\backslash\mathcal{K}(\ell_1, F)$ and $0 \neq \eta \in \ell_\infty$. Define $P \in \mathcal{P}(^k\ell_1, F)$ by $P(x^*) := \langle x^*, \eta\rangle^{k-1}T(x^*)$ for all $x^* \in \ell_1$. By Proposition 2.5, P is not compact. Therefore, there are operators A and B so that P_0 factors in the form



For $x^* \in \ell_1$, we have

$$P_0(x^*) = B \circ (\eta^{k-1}T) \circ A(x^*) = (\eta \circ A)(x^*)^{k-1} B \circ T \circ A(x^*) = \psi(x^*)^{k-1}S(x^*)$$

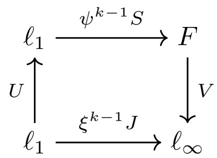
where $\psi := \eta \circ A \in \ell_\infty$ and $S := B \circ T \circ A \in \mathcal{L}(\ell_1, F)$. By Proposition 2.5, since P_0 is not compact, S is not compact. Hence, we can factor J in the form



For all $x^* \in \ell_1$, we have

$$V \circ (\psi^{k-1}S) \circ U(x^*) = \psi(U(x^*))^{k-1}V \circ S \circ U(x^*) = \xi(x^*)^{k-1}J(x^*)$$

for $\xi := \psi \circ U \in \ell_\infty$, so the following diagram commutes:



that is, $V \circ P_0 \circ U = V \circ (\psi^{k-1}S) \circ U = \xi^{k-1}J$, so $\xi^{k-1}J$ is universal non-compact. □

Theorem 2.8. *Let $k \geq 2$ be an integer. Then there is no universal k-homogeneous non-compact polynomial.*

Proof. By Proposition 2.7, if a universal k-homogeneous non-compact polynomial exists, it has the form $\xi^{k-1}J \in \mathcal{P}({}^k\ell_1, \ell_\infty)$. Let $P \in \mathcal{P}({}^k\ell_1, c_0) \setminus \mathcal{P}_{\mathcal{K}}({}^k\ell_1, c_0)$ be given by

$$P(\phi) := \left(\phi_1^{k-2}\phi_n^2\right)_{n=1}^{\infty} \quad \text{for } \phi = (\phi_n)_{n=1}^{\infty} \in \ell_1 .$$

Then we have

$$\begin{array}{ccc} \ell_1 & \xrightarrow{\ P\ } & c_0 \\ {\scriptstyle A}\uparrow & & \downarrow{\scriptstyle B} \\ \ell_1 & \xrightarrow{\ \xi^{k-1}J\ } & \ell_\infty \end{array}$$

Choose $x_0^* \in \ell_1$ such that $\langle x_0^*, \xi\rangle \neq 0$. Note that $A(x_0^*) \neq 0$. We can assume $\|A(x_0^*)\| = 1$. Let $C := |\langle x_0^*, \xi\rangle^{k-1}|$. Recall that $\ker\xi$ is isomorphic to ℓ_1 [15, Proposition I.5.4]. Since $A|_{\ker\xi} : \ker\xi \to \ell_1$ is not compact, it preserves a copy of ℓ_1. By [3, Part 2, Chapter 4, §1, Lemma 4], we can find a sequence $(x_n^*) \subset \ker\xi$ such that $(A(x_n^*))_{n=1}^{\infty}$ is a normalized basic sequence equivalent to a normalized block-basis $(u_n)_{n=1}^{\infty}$ of $(e_n^*)_{n=1}^{\infty}$ and $\|A(x_n^*) - u_n\| \underset{n}{\to} 0$.

Let $H \in \mathcal{L}(\ell_1, \ker\xi)$ be the into isomorphism given by $H(e_n^*) := x_n^*$. Assume $\|J(x_n^*)\| \underset{n}{\to} 0$. Then $\|J \circ H(e_n^*)\| \underset{n}{\to} 0$ which implies that J is compact [7, Exercise VII.5], which is impossible. Hence, by passing to a subsequence, if necessary, we can assume that $\|J(x_n^*)\| > \delta$ $(n \in \mathbb{N})$ for some $\delta > 0$.

There is $j_1 \in \mathbb{N}$ such that

$$|\langle A(x_0^*), e_j\rangle| < \frac{\delta C}{\|B\| 2^k} \qquad (j \geq j_1).$$

We can find $n_0 \in \mathbb{N}$ so that

$$n \geq n_0 \quad \Rightarrow \quad \|A(x_n^*) - u_n\| < \frac{\delta C}{\|B\| 2^k} .$$

Choose $j_2 \in \mathbb{N}$ $(j_2 \geq n_0)$ such that $\langle u_{j_2}, e_j\rangle = 0$ for all $j < j_1$. Consider the vectors

$$v_1 := x_0^* - x_{j_2}^* , \quad \text{and} \quad v_2 := x_0^* + x_{j_2}^* .$$

Then

$$\left\|\left(\xi^{k-1}J\right)(v_2) - \left(\xi^{k-1}J\right)(v_1)\right\| = 2\left\|J\left(x_{j_2}^*\right)\right\| \left|\langle x_0^*, \xi\rangle^{k-1}\right| > 2\delta C . \tag{1}$$

To simplify notation, let

$$p := A(x_0^*) = (p_j)_{j_1}^{\infty} \in S_{\ell_1} , \quad \text{and} \quad q := A\left(x_{j_2}^*\right) = (q_j)_{j=1}^{\infty} \in S_{\ell_1} .$$

We have

$$\begin{aligned} &\|PA(v_2) - PA(v_1)\| \\ &\quad = \Bigg\| \left(\left\langle A\left(x_0^* + x_{j_2}^*\right), e_1\right\rangle^{k-2} \left\langle A\left(x_0^* + x_{j_2}^*\right), e_j\right\rangle^2\right)_{j=1}^{\infty} \\ &\qquad - \left(\left\langle A\left(x_0^* - x_{j_2}^*\right), e_1\right\rangle^{k-2} \left\langle A\left(x_0^* - x_{j_2}^*\right), e_j\right\rangle^2\right)_{j=1}^{\infty} \Bigg\| \end{aligned}$$

$$
\begin{aligned}
&= \sup_{j\in\mathbb{N}} \left|\left(p_j^2+q_j^2+2p_jq_j\right)(p_1+q_1)^{k-2} - \left(p_j^2+q_j^2-2p_jq_j\right)(p_1-q_1)^{k-2}\right| \\
&= \sup_{j\in\mathbb{N}} \Bigg| \left(p_j^2+q_j^2+2p_jq_j\right)\left[\binom{k-2}{0}p_1^{k-2}+\binom{k-2}{1}p_1^{k-3}q_1+\cdots+\binom{k-2}{k-2}q_1^{k-2}\right] \\
&\quad - \left(p_j^2+q_j^2-2p_jq_j\right) \\
&\quad \times \left[\binom{k-2}{0}p_1^{k-2}-\binom{k-2}{1}p_1^{k-3}q_1+\cdots+(-1)^{k-2}\binom{k-2}{k-2}q_1^{k-2}\right]\Bigg| \\
&= \sup_{j\in\mathbb{N}} 2\Bigg|\binom{k-2}{1}p_j^2p_1^{k-3}q_1+\binom{k-2}{3}p_j^2p_1^{k-5}q_1^3+\cdots \\
&\quad +\binom{k-2}{1}p_1^{k-3}q_j^2q_1+\binom{k-2}{3}p_1^{k-5}q_j^2q_1^3+\cdots \\
&\quad +2p_jq_j\left[\binom{k-2}{0}p_1^{k-2}+\binom{k-2}{2}p_1^{k-4}q_1^2+\cdots\right]\Bigg| \\
&< 4\,\frac{\delta C}{\|B\|2^k}\,2^{k-2} = \frac{\delta C}{\|B\|}\,,
\end{aligned}
$$

since

$$
\begin{aligned}
&|p_j| < \frac{\delta C}{\|B\|2^k} \qquad (j \ge j_1) \\
&|q_j| = \left|\left\langle A\left(x_{j_2}^*\right), e_j\right\rangle\right| = \left|\left\langle A\left(x_{j_2}^*\right) - u_{j_2}, e_j\right\rangle\right| < \frac{\delta C}{\|B\|2^k} \qquad (j < j_1)\,.
\end{aligned}
$$

Therefore,

$$
\|BPA(v_2) - BPA(v_1)\| < \|B\|\,\frac{\delta C}{\|B\|} = \delta C\,.
$$

Combining this with formula (1) yields

$$
2\delta C < \delta C\,,
$$

a contradiction. □

3. Polynomials which do not belong to an injective-type ideal

The ideal of unconditionally converging operators is closed and injective, but not surjective [14]. In this section we deal with non-unconditionally converging polynomials. The reason why we have chosen this class is that the techniques needed in the proofs are similar to those used in the non-compact case. Other classes are left for future work.

An operator $T \in \mathcal{L}(E,F)$ is *unconditionally converging* if, for every weakly unconditionally Cauchy series $\sum_{i=1}^{\infty} x_i$ in E, the series $\sum_{i=1}^{\infty} T(x_i)$ is unconditionally convergent in F [22, Definition 1].

Following M. Fernández-Unzueta [10, Definition (1.3)], we say that a polynomial $P \in \mathcal{P}(^kE,F)$ is *unconditionally converging* if, for every weakly unconditionally Cauchy series $\sum_{i=1}^{\infty} x_i$ in E, the sequence

$$
\left(P\left(\sum_{i=1}^{n} x_i\right)\right)_{n=1}^{\infty} \subset F
$$

is norm convergent. The space of all unconditionally converging k-homogeneous polynomials from E into F is denoted by $\mathcal{P}_{\mathrm{uc}}({}^kE, F)$.

A weaker notion of unconditionally converging polynomials had been introduced earlier by M. González and the second named author in [13, Definition 3] but we shall use the one given in [10] which seems to have nicer properties [17,25].

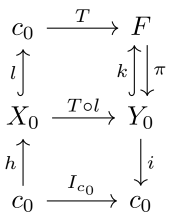
We believe that the following two results are well known but we have not found them in the literature written in the form that suits our purposes.

Proposition 3.1. *Let $T \in \mathcal{L}(c_0, F)\backslash\mathcal{K}(c_0, F)$. Then there are operators $A \in \mathcal{L}(c_0, c_0)$ and $B \in \mathcal{L}(F, \ell_\infty)$ such that $I = B \circ T \circ A$, where $I : c_0 \hookrightarrow \ell_\infty$ is the natural embedding. If F is separable, I may be taken to be the identity map on c_0.*

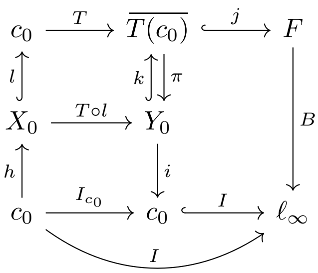
Proof. Since T is not compact, there are a subspace X_0 of c_0 isomorphic to c_0 and an isomorphic embedding $l : X_0 \hookrightarrow c_0$ such that $T \circ l$ is an into isomorphism [23, Lemma I.3.1]. Let $h : c_0 \to X_0$ be the onto isomorphism provided by the just mentioned result. Let $x_n := h(e_n)$ and $Y_0 := T \circ l(X_0) \subseteq F$, with natural embedding $k : Y_0 \hookrightarrow F$. If F is separable, since Y_0 is isomorphic to c_0 and c_0 is separably injective [9, Theorem 5.14], there is a surjective operator $\pi : F \to Y_0$ such that $\pi \circ k = I_{Y_0}$. Let $i : Y_0 \to c_0$ be given by $i \circ T \circ l(x_n) := e_n$. Then

$$i \circ \pi \circ T \circ l \circ h(e_n) = i \circ \pi \circ T \circ l(x_n) = i \circ \underbrace{\pi \circ k}_{=I_{Y_0}} \circ T \circ l(x_n) = i \circ T \circ l(x_n) = e_n = I_{c_0}(e_n)\,,$$

and the result is proven by setting $A := l \circ h$ and $B := i \circ \pi$ with $I := I_{c_0}$.



If F is not separable, replace F by $\overline{T(c_0)}$, letting $j : \overline{T(c_0)} \hookrightarrow F$ be the natural embedding.



By the injectivity of ℓ_∞ [9, Proposition 5.13], the operator $I \circ i \circ \pi : \overline{T(c_0)} \to \ell_\infty$ is extendible to an operator $B : F \to \ell_\infty$. Then

$$B \circ j \circ T \circ l \circ h(e_n) = I \circ i \circ \pi \circ T \circ l \circ h(e_n) = I \circ I_{c_0}(e_n) = I(e_n)\,,$$

and the proof is finished. $\square$

Corollary 3.2. *The natural embedding $I \in \mathcal{L}(c_0, \ell_\infty)$ is universal for the class of non-unconditionally converging operators.*

Proof. Let $T \in \mathcal{L}(E, F)$ be non-unconditionally converging. Then there is an embedding $j : c_0 \hookrightarrow E$ such that the sequence $(T \circ j(e_n))_{n=0}^{\infty}$ is equivalent to the c_0-basis [13, Lemma 4]. By Proposition 3.1, we can find operators $A \in \mathcal{L}(c_0, c_0)$ and $B \in \mathcal{L}(F, \ell_\infty)$ such that the following diagram commutes:

$$\begin{array}{ccccc} c_0 & \xhookrightarrow{j} & E & \xrightarrow{T} & F \\ {\scriptstyle A}\big\uparrow & & & & \big\downarrow{\scriptstyle B} \\ c_0 & & \xrightarrow{\quad I \quad} & & \ell_\infty \end{array}$$

and the proof is finished. □

Of course, if F is separable, ℓ_∞ may be replaced by c_0.

We need a lemma which, as one of the referees has kindly pointed out, is contained in the proof of [10, Theorem 1.14].

Lemma 3.3. *Given a polynomial $P \in \mathcal{P}(^k E, F) \backslash \mathcal{P}_{\mathrm{uc}}(^k E, F)$, where $k \geq 1$ is an integer, there is an operator $j : c_0 \to E$ such that $P \circ j \in \mathcal{P}(^k c_0, F) \backslash \mathcal{P}_{\mathrm{uc}}(^k c_0, F)$.*

Therefore, if there is a universal non-unconditionally converging polynomial, we can assume that it is defined on c_0.

The following well-known result will be used at several places.

Lemma 3.4. *(See [5, Corollaries 2.4 and 2.5].) For every integer $k \geq 1$ and every Banach space F, we have*

$$\mathcal{P}_{\mathrm{uc}}(^k c_0, F) = \mathcal{P}_{\mathcal{K}}(^k c_0, F) \,.$$

Proposition 3.5. *Given an integer $k \geq 2$, if there is a universal non-unconditionally converging k-homogeneous polynomial P_0, it may be taken of the form $P_0 := \xi^{k-1} I$, where $\xi \in \ell_1$ and $I \in \mathcal{L}(c_0, \ell_\infty)$ is the natural embedding.*

Proof. Using Lemma 3.4 and the comment after Lemma 3.3, we need to look for universal non-compact k-homogeneous polynomials on c_0. Suppose there is a universal non-unconditionally converging k-homogeneous polynomial $P_0 \in \mathcal{P}(^k c_0, F)$. Let $T \in \mathcal{L}(c_0, Y) \backslash \mathcal{K}(c_0, Y)$ and $\phi \in \ell_1$. Then there are operators A and B such that, for all $x \in c_0$, we have

$$P_0(x) = B \circ \left(\phi^{k-1} T\right) \circ A(x) = (\phi \circ A)(x)^{k-1} B \circ T \circ A(x) \,,$$

so $P_0 = \psi^{k-1} S$ for some $\psi \in \ell_1$ and $S \in \mathcal{L}(c_0, F) \backslash \mathcal{K}(c_0, F)$.

$$\begin{array}{ccc} c_0 & \xrightarrow{\phi^{k-1} T} & Y \\ {\scriptstyle A}\big\uparrow & & \big\downarrow{\scriptstyle B} \\ c_0 & \xrightarrow{\quad P_0 \quad} & F \end{array}$$

By Corollary 3.2, there are operators U and V such that the following diagram commutes:

$$\begin{array}{ccc} c_0 & \xrightarrow{S} & F \\ {\scriptstyle U}\uparrow & & \downarrow{\scriptstyle V} \\ c_0 & \overset{I}{\hookrightarrow} & \ell_\infty \end{array}$$

Let $\xi := \psi \circ U \in \ell_1$. For all $x \in c_0$, we have

$$V \circ \left(\psi^{k-1} S\right) \circ U(x) = (\psi \circ U)(x)^{k-1} V \circ S \circ U(x) = \xi(x)^{k-1} I(x)\,,$$

so $V \circ \left(\psi^{k-1} S\right) \circ U = \xi^{k-1} I$ and the following diagram commutes:

$$\begin{array}{ccc} c_0 & \xrightarrow{\psi^{k-1}S} & F \\ {\scriptstyle U}\uparrow & & \downarrow{\scriptstyle V} \\ c_0 & \xrightarrow{\xi^{k-1}I} & \ell_\infty \end{array}$$

and this finishes the proof. □

Given a vector $x \in X$, denote by $[x]$ the linear span of x.

Proposition 3.6. *Given an integer $k \geq 2$, if there is a universal non-unconditionally converging k-homogeneous polynomial, it may be chosen of the form $(e_1^*)^{k-1} I$, where $I \in \mathcal{L}(c_0, \ell_\infty)$ is the natural embedding.*

Proof. By Proposition 3.5, if such a universal polynomial exists, we may assume that it has the form $\xi^{k-1} I$ with $\xi \in \ell_1$. Choose $x_0 \in c_0$ with $\xi(x_0) = 1$. Since $\ker e_1^*$ and $\ker \xi$ are both subspaces of c_0 of codimension one, they are isomorphic to c_0 [15, Proposition I.5.4,2]. Let

$$U : \ker e_1^* \longrightarrow \ker \xi$$

be an onto isomorphism.

Define

$$A : c_0 = [e_1] \oplus \ker e_1^* \longrightarrow [x_0] \oplus \ker \xi$$

by $A(\lambda e_1 + e) := \lambda x_0 + U(e)$, for $e \in \ker e_1^*$. Let

$$B : [x_0] \oplus \ker \xi \longrightarrow \ell_\infty$$

be given by $B(\lambda x_0 + y) := \lambda e_1 + U^{-1}(y)$, for $y \in \ker \xi$. By the injectivity of ℓ_∞ [9, Proposition 5.13], B has an extension to an operator $\ell_\infty \to \ell_\infty$ that we still denote by B. Then, for $e \in \ker e_1^*$, we have

$$\left[(e_1^*)^{k-1} I\right](\lambda e_1 + e) = \lambda^{k-1}(\lambda e_1 + e)$$

and

$$\left(\xi^{k-1} I\right)[A(\lambda e_1 + e)] = \left(\xi^{k-1} I\right)[\lambda x_0 + U(e)] = \lambda^{k-1}[\lambda x_0 + U(e)]$$

since $U(e) \in \ker \xi$, so

$$B \circ \left(\xi^{k-1} I\right) \circ A(\lambda e_1 + e) = \lambda^{k-1} B\left[\lambda x_0 + U(e)\right] = \lambda^{k-1}(\lambda e_1 + e)\,,$$

and

$$B \circ \left(\xi^{k-1} I\right) \circ A = \left(e_1^*\right)^{k-1} I$$

which makes the following diagram commutative:

$$\begin{array}{ccc} c_0 & \xrightarrow{\ \xi^{k-1} I\ } & \ell_\infty \\ A \big\uparrow & & \big\downarrow B \\ c_0 & \xrightarrow{\ (e_1^*)^{k-1} I\ } & \ell_\infty \end{array}$$

and this finishes the proof. □

Proposition 3.7. *Given an integer $k \geq 2$ and $P \in \mathcal{P}({}^k c_0, c_0) \backslash \mathcal{P}_{\mathrm{uc}}({}^k c_0, c_0)$, suppose there exist operators $A \in \mathcal{L}(c_0, c_0)$ and $B \in \mathcal{L}(c_0, \ell_\infty)$ such that $\left(e_1^*\right)^{k-1} I = BPA$. Then A is an into isomorphism.*

Proof. Let $(x_n) \subset \ker e_1^*$ be a sequence such that $\|A(x_n)\| \to 0$. Then

$$\|BPA(x_n)\| \to 0\,.$$

We have $\left[\left(e_1^*\right)^{k-1} I\right](e_1 + x_n) = e_1 + x_n$, so

$$\begin{aligned} \|x_n\| &= \|e_1 + x_n - e_1\| \\ &= \left\| \left[\left(e_1^*\right)^{k-1} I\right](e_1 + x_n) - \left[\left(e_1^*\right)^{k-1} I\right](e_1) \right\| \\ &= \|BPA(e_1 + x_n) - BPA(e_1)\| \\ &= \left\| B\widehat{P}(Ae_1 + Ax_n, \overset{(k)}{\ldots}, Ae_1 + Ax_n) - BPA(e_1) \right\| \\ &= \left\| \binom{k}{1} B\widehat{P}(Ae_1, \ldots, Ae_1, Ax_n) + \binom{k}{2} B\widehat{P}(Ae_1, \ldots, Ae_1, Ax_n, Ax_n) \right. \\ &\qquad \left. + \cdots + BPA(x_n) \right\| \longrightarrow 0 \quad \text{for } n \to \infty\,, \end{aligned}$$

and $A|_{\ker e_1^*}$ is an into isomorphism.

Since $c_0 = [e_1] \oplus \ker e_1^*$, a generic sequence of c_0 has the form $(\lambda_n e_1 + x_n)_{n=1}^\infty$ with $(x_n) \subset \ker e_1^*$. Suppose that

$$\|A(\lambda_n e_1 + x_n)\| \underset{n}{\longrightarrow} 0\,.$$

Then $\|BPA(\lambda_n e_1 + x_n)\| \to 0$. If $\lambda_n \to 0$, then

$$\|A(x_n)\| = \|A(\lambda_n e_1 + x_n) - A(\lambda_n e_1)\| \longrightarrow 0$$

which implies $\|x_n\| \to 0$, and $\|\lambda_n e_1 + x_n\| \to 0$. If $\lambda_n \nrightarrow 0$, by passing to a subsequence, we can assume $|\lambda_n| > \delta$ $(n \in \mathbb{N})$ for some $\delta > 0$. Then

$$\delta^{k-1}\|\lambda_n e_1 + x_n\| < |\lambda_n|^{k-1}\|\lambda_n e_1 + x_n\| = \left\| \left[(e_1^*)^{k-1} I \right] (\lambda_n e_1 + x_n) \right\|$$
$$= \|BPA(\lambda_n e_1 + x_n)\| \longrightarrow 0$$

so $\|\lambda_n e_1 + x_n\| \to 0$. Since $\lambda_n e_1$ and x_n have disjoint support, this implies $\lambda_n \to 0$, a contradiction. Therefore,

$$\|A(\lambda_n e_1 + x_n)\| \underset{n}{\longrightarrow} 0 \quad \Longrightarrow \quad \lambda_n e_1 + x_n \underset{n}{\longrightarrow} 0 \,,$$

and A is an into isomorphism. □

Therefore, there is $m > 0$ such that

$$m\|x\| \leq \|A(x)\| \leq \|A\|\|x\| \qquad (x \in c_0)\,.$$

Let

$$\mu_n := \frac{1}{\|A(e_n)\|} \qquad (n \in \mathbb{N})\,.$$

Then

$$\frac{1}{\|A\|} \leq \mu_n \leq \frac{1}{m} \qquad (n \in \mathbb{N})\,,$$

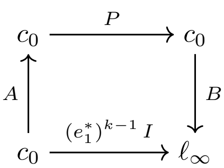
and $\|A(\mu_n e_n)\| = \mu_n\|A(e_n)\| = 1$ for all $n \in \mathbb{N}$.

Theorem 3.8. *Given an integer $k \geq 2$, there is no universal non-unconditionally converging k-homogeneous polynomial.*

Proof. Assume there is a universal non-unconditionally converging k-homogeneous polynomial which, by Proposition 3.6, may be taken of the form $(e_1^*)^{k-1} I$. Let $P \in \mathcal{P}(^k c_0, c_0)$ be the polynomial given by

$$P(y) := \left(y_1^{k-2} y_n^2\right)_{n=1}^{\infty} \quad \text{for } y = (y_n)_{n=1}^{\infty} \in c_0.$$

There are operators A and B such that the following diagram commutes:



There is $j_1 \in \mathbb{N}$ such that

$$\left|\left\langle A(\mu_1 e_1), e_j^* \right\rangle\right| < \frac{1}{2^k \|A\|^k \|B\|} \qquad (j \geq j_1)\,. \tag{2}$$

As in the proof of [15, Proposition I.5.4], there is a subsequence $(n_j)_{j=1}^{\infty}$ of $\mathbb{N}$ and a normalized block-basis $(u_j)_{j=1}^{\infty}$ of $(e_n)_{n=1}^{\infty}$ such that

$$\left\|A\left(\mu_{n_j} e_{n_j}\right) - u_j\right\| < \frac{1}{2^k \|A\|^k \|B\|} \qquad (j \in \mathbb{N})\,.$$

Choose $j_2 \in \mathbb{N}$ such that $\langle u_{j_2}, e_j^* \rangle = 0$ for all $j < j_1$. Consider the vectors

$$v_1 := \mu_1 e_1 - \mu_{n_{j_2}} e_{n_{j_2}} \quad \text{and} \quad v_2 := \mu_1 e_1 + \mu_{n_{j_2}} e_{n_{j_2}} .$$

Then

$$\left\| \left[(e_1^*)^{k-1} I \right] (v_2) - \left[(e_1^*)^{k-1} I \right] (v_1) \right\| = \left\| 2\mu_1^{k-1} \mu_{n_{j_2}} e_{n_{j_2}} \right\| = 2\mu_1^{k-1} \mu_{n_{j_2}} \geq \frac{2}{\|A\|^k} . \tag{3}$$

To simplify notation, let

$$p := A(\mu_1 e_1) = (p_j)_{j=1}^{\infty} \in B_{c_0} \quad \text{and} \quad q := A(\mu_{n_{j_2}} e_{n_{j_2}}) = (q_j)_{j=1}^{\infty} \in B_{c_0} .$$

We have

$$\begin{aligned}
&\|PA(v_2) - PA(v_1)\| \\
&= \sup_{j \in \mathbb{N}} \Big| \left\langle A\left(\mu_1 e_1 + \mu_{n_{j_2}} e_{n_{j_2}}\right), e_j^* \right\rangle^2 \left\langle A\left(\mu_1 e_1 + \mu_{n_{j_2}} e_{n_{j_2}}\right), e_1^* \right\rangle^{k-2} \\
&\quad - \left\langle A\left(\mu_1 e_1 - \mu_{n_{j_2}} e_{n_{j_2}}\right), e_j^* \right\rangle^2 \left\langle A\left(\mu_1 e_1 - \mu_{n_{j_2}} e_{n_{j_2}}\right), e_1^* \right\rangle^{k-2} \Big| \\
&= \sup_{j \in \mathbb{N}} \left| \left(p_j^2 + q_j^2 + 2p_j q_j\right) (p_1 + q_1)^{k-2} - \left(p_j^2 + q_j^2 - 2p_j q_j\right) (p_1 - q_1)^{k-2} \right| .
\end{aligned}$$

The calculations follow as in the proof of Theorem 2.8 to finish with

$$\|PA(v_2) - PA(v_1)\| < 2 \frac{1}{2^k \|A\|^k \|B\|} (2^{k-2} + 2^{k-2}) = \frac{1}{\|A\|^k \|B\|} ,$$

since

$$\begin{aligned}
|p_j| &< \frac{1}{2^k \|A\|^k \|B\|} \qquad (j \geq j_1) \\
|q_j| &= \left| \left\langle A\left(\mu_{n_{j_2}} e_{n_{j_2}}\right), e_j^* \right\rangle \right| = \left| \left\langle A\left(\mu_{n_{j_2}} e_{n_{j_2}}\right) - u_{j_2}, e_j^* \right\rangle \right| \\
&< \frac{1}{2^k \|A\|^k \|B\|} \qquad (j < j_1) .
\end{aligned}$$

Therefore,

$$\|BPA(v_2) - BPA(v_1)\| < \frac{\|B\|}{\|A\|^k \|B\|} = \frac{1}{\|A\|^k} .$$

Combining this with formula (3) yields:

$$\frac{2}{\|A\|^k} < \frac{1}{\|A\|^k} ,$$

a contradiction. □

References

[1] F. Albiac, N.J. Kalton, Topics in Banach Space Theory, Grad. Texts in Math., vol. 233, Springer, New York, 2006.

[2] R.M. Aron, M. Schottenloher, Compact holomorphic mappings on Banach spaces and the approximation property, J. Funct. Anal. 21 (1976) 7–30.

[3] B. Beauzamy, Introduction to Banach Spaces and Their Geometry, North-Holl. Math. Stud., vol. 68, North-Holland, Amsterdam, 1982.
[4] F. Blasco, Complementation in spaces of symmetric tensor products and polynomials, Studia Math. 123 (1997) 165–173.
[5] F. Bombal, M. Fernández-Unzueta, I. Villanueva, Unconditionally converging multilinear operators, Math. Nachr. 226 (2001) 5–15.
[6] R. Cilia, J.M. Gutiérrez, Ideals of integral and r-factorable polynomials, Bol. Soc. Mat. Mexicana 14 (2008) 95–124.
[7] J. Diestel, Sequences and Series in Banach Spaces, Grad. Texts in Math., vol. 92, Springer, Berlin, 1984.
[8] S. Dineen, Complex Analysis on Infinite Dimensional Spaces, Springer Monogr. Math., Springer, Berlin, 1999.
[9] M. Fabian, P. Habala, P. Hájek, V. Montesinos, J. Pelant, V. Zizler, Functional Analysis and Infinite-Dimensional Geometry, CMS Books Math., Springer, New York, 2001.
[10] M. Fernández-Unzueta, A new approach to unconditionality for polynomials on Banach spaces, Bol. Soc. Mat. Mexicana (3) 8 (2002) 37–48.
[11] K. Floret, Natural norms on symmetric tensor products of normed spaces, Note Mat. 17 (1997) 153–188.
[12] M. Girardi, W.B. Johnson, Universal non-completely-continuous operators, Israel J. Math. 99 (1997) 207–219.
[13] M. González, J.M. Gutiérrez, Unconditionally converging polynomials on Banach spaces, Math. Proc. Cambridge Philos. Soc. 117 (1995) 321–331.
[14] M. González, J.M. Gutiérrez, Surjective factorization of holomorphic mappings, Comment. Math. Univ. Carolin. 41 (2000) 469–476.
[15] S. Guerre-Delabrière, Classical Sequences in Banach Spaces, Monogr. Textb. Pure Appl. Math., vol. 166, Dekker, New York, 1992.
[16] J.M. Gutiérrez, J.G. Llavona, Polynomially continuous operators, Israel J. Math. 102 (1997) 179–187.
[17] J.M. Gutiérrez, I. Villanueva, Extensions of multilinear operators and Banach space properties, Proc. Roy. Soc. Edinburgh Sect. A 133 (2003) 549–566. See Corrigendum in [25, §2].
[18] A. Hinrichs, E. Novak, H. Woźnickowski, Discontinuous information in the worst case and randomized settings, Math. Nachr. 286 (2013) 679–690.
[19] W.B. Johnson, A universal non-compact operator, Colloq. Math. 23 (1971) 267–268.
[20] J. Lindenstrauss, A. Pełczyński, Absolutely summing operators in $\mathcal{L}_p$-spaces and their applications, Studia Math. 29 (1968) 275–326.
[21] J. Mujica, Complex Analysis in Banach Spaces, Math. Stud., vol. 120, North-Holland, Amsterdam, 1986.
[22] A. Pełczyński, Banach spaces on which every unconditionally converging operator is weakly compact, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astron. Phys. 10 (1962) 641–648.
[23] A. Pełczyński, On strictly singular and strictly cosingular operators. I. Strictly singular and strictly cosingular operators in $C(S)$-spaces, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astron. Phys. 13 (1965) 31–36.
[24] D. Pellegrino, J. Ribeiro, On multi-ideals and polynomial ideals of Banach spaces: a new approach to coherence and compatiblity, Monatsh. Math. 173 (2014) 379–415.
[25] A.M. Peralta, I. Villanueva, J.D. Maitland Wright, K. Ylinen, Quasi-completely continuous multilinear operators, Proc. Roy. Soc. Edinburgh Sect. A 140 (2010) 635–649.
[26] A. Pietsch, Operator Ideals, North-Holl. Math. Library, vol. 20, North-Holland, Amsterdam, 1980.
[27] A. Pietsch, Ideals of multilinear functionals (designs of a theory), in: H. Baumgärtel, et al. (Eds.), Proc. of the Second International Conference on Operator Algebras, Ideals, and Their Applications in Theoretical Physics, Leipzig, 1983, in: Teubner-Texte Math., vol. 67, Teubner, Leipzig, 1984, pp. 185–199.
[28] R.A. Ryan, Applications of topological tensor products to infinite dimensional holomorphy, PhD Thesis, Trinity College, Dublin, 1980.
[29] R.A. Ryan, Introduction to Tensor Products of Banach Spaces, Springer Monogr. Math., Springer, London, 2002.