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Precoat Filtration with Body-feed and Variable Pressure. Part I: Mathematical Modelling

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Abstract

In order to set up a subsequent optimization of the precoat filtration with body-feed cycles by supplying the slurry with a centrifugal pump, a fluid-dynamic analysis of the filter-pump system with variable supply pressure was performed. Consequently an ordinary differential equation was formulated. Then a closed-form solution was found to obtain an equation useful for both the optimization of the filtration and the filter design. The application of the new mathematical modelling was compared with the application of Carman equation, classically used in filtration, which assumes a constant pressure filtration. The simulation showed that, due to different permeability of the filter, some conditions of filtration can make unrealistic the results of the classic Carman equation. Therefore, in filtration practice it is confirmed the opportunity to have a more accurate equation for both the design and the optimization.

Keywords: Precoat filtration with body-feed; Mathematical modelling; ODE; Optimization; Filterability; Agricultural and food engineering

1. Introduction

The precoat filtration with body-feed is an unit operation of agricultural and food engineering. Mostly it is implemented by using centrifugal pump, which pump curve has a partial horizontal trend. Classically, in filtration theory, this prerogative of the centrifugal pumps leads to the simplifying assumption that filtration occurs with constant pressure. Because of this, it is easy to integrate the Darcy's differential equation [1, 2 and 3] for the precoat filtration with body-feed, obtaining the well known Carman equation [4]. This is the equation which relates the filtration time with the filtrate volume, the operating pressure, the filter area, and the solid-liquid suspension characteristics. The Carman equation is the start point for the subsequent optimization of the filtration cycles, e.g. by establishing the relationship between the filtration time and the filter cleaning time [5].
A better optimization of the precoat filtration with body-feed could be obtain, with some economic benefits, if an integration of the Darcy ODE was developed starting from actual trend of the pressure produced by the centrifugal pump, that is if a variable pressure was considered, as expected from the pump curve. In this sense a proposal was done by Tiller and Crump [6] many years ago in accordance with a graphic method of integration of the Darcy ODE. However the graphic procedure is tedious since it is iterative and not computerizable.
For this reason the aim of this work was to find an analytical solution to the Darcy ODE for the filtration with variable pressure in order to obtain a quick and easy-to-use equation for the subsequent optimization calculations of filtration cycles, even if more complex of the Carman equation.

2. The mathematical problem

2.1 Darcy ODE and Carman solution

Darcy proposed the following equation to correlate the flow rate of filtrate to the geometric parameters (filter area and filtering layer thickness), functional parameters (operating pressure) and material parameters (filtrate viscosity and permeability of the filtering layer):

$$\dot{V} = \frac{K \cdot A \cdot \Delta p}{\mu \cdot l} = \frac{A \cdot \Delta p}{\mu \cdot R} \tag{1}$$

where: $\dot{V} = dV/dt$ is the instant flow rate; A is the area of the filter; Δp is the pressure difference across the filtering layer; μ is the viscosity of the filtrate; l is the thickness, increasing with the time, of the filtering layer; K is the specific permeability of the filtering layer (m^2); $R=l/K$ (m^{-1}) is the total resistance of the filtering layer.

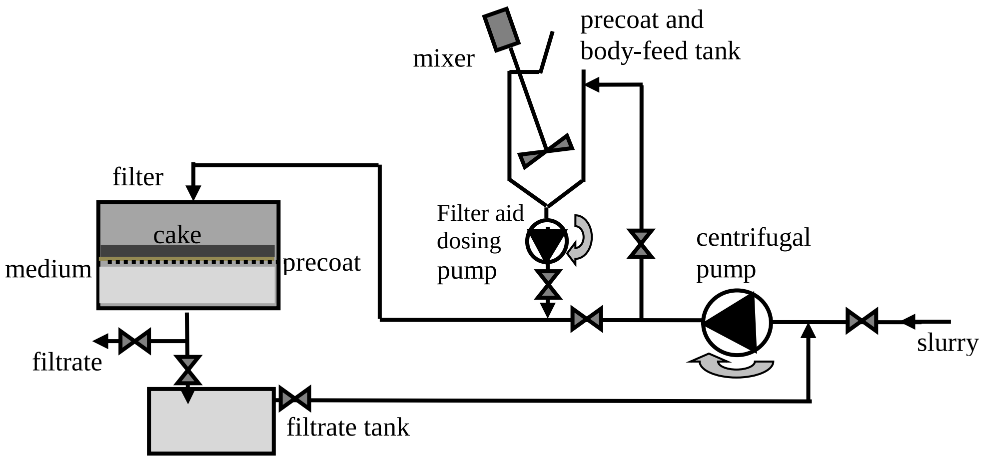

Fig. 1 - Filtration layout of the precoat and body-feed. The filtration layer is consisting of the medium (woven fabrics), the precoat and the cake

If the resistance of the medium (woven fabrics) and the precoat is neglected and a mass balance between the solids (including filter aid) of the slurry and the solids of the filter cake is considered, the Darcy ODE is obtained:

$$\dot{V} = \frac{A^2 \cdot \Delta p}{\mu \cdot c \cdot V \cdot \alpha} \tag{2}$$

where: V (m^3) is the volume of the filtrate; c is the concentration of the solids in the slurry mixed with the filter aid (kg/m^3); α is the specific resistance (m/kg) that is considered constant due to the body-feed of the filter aid.

If the pressure difference across the filtering layer Δp is constant, because a perfectly horizontal curve pump is considered, it is easy to integrate the ODE (2) to obtain the Carman equation:

$$\frac{V^2}{2} = \frac{A^2 \cdot \Delta p}{\mu \cdot c \cdot \alpha} t \tag{3}$$

where: t is the time (s). The quantity $\frac{\mu \cdot c \cdot \alpha}{\Delta p} = F_k$ is known as filterability. Therefore it is easy to find in scientific literature [5] the tern $\mu \cdot c \cdot \alpha$ indicated as the product: $F_k \cdot \Delta p$. Then the (3) becomes: $\frac{V^2}{2} = \frac{A^2}{F_k} t$. Clearly if the filter is to body-feed with a correct amount of filter aid, then $\mu \cdot c \cdot \alpha$ is constant and if Δp is constant too during the filtration, even the filterability F_k will be constant.

2.2 Mathematical modelling of centrifugal pump curve

In practice the flow rate-pressure curves ($\dot{V}$ -P_p) of the centrifugal pumps used in the filters have often experimental values such as those shown in the figure 2.
They are values of the classic monoimpeller centrifugal pump with exit blade angles typically between 30° and 40°.
We imagined at this point to be able to represent the experimental data of figure 2 with a parabola characterized by only two constants $P_{p\max}$ and B:

$$P_p = P_{p\max} - B \cdot \dot{V}^2 \tag{4}$$

In fact, it is missing the linear term of the flow rate $\dot{V}$ because, as it can be seen in the experimental values, the tangent in the point $\dot{V}=0$ is horizontal.
To confirm the correctness of the choice represented by the (4) in the figure 2, even the corresponding curve was drawn, which is practically overlapped to the experimental values (R^2=0.999). When $\dot{V}=0$, the constant $P_{p\max}$ is equal to P_p and the constant B is obtained imposing for $\dot{V}_{\max}$ the $P_p=0$: $B = \frac{P_{p\max}}{\dot{V}^2_{\max}}$.

In order to determine the equation (4) of the curve of the commercial centrifugal pumps used coupled with the filter, it is sufficient to know the $P_{p\max}$ and the $\dot{V}_{\max}$.
During filtration the pressure drop in the pipeline can be considered negligible compared to the pressure difference across the filtering layer Δp. Then we can say that the pressure produced by the pump P_p is equal to the pressure difference across the filtering layer Δp, ($P_p \approx \Delta p$).

Consequently, at the beginning of the filtration (t=0), when the filtering layer consists only of the medium and the precoat, the reduced pressure drop produced by the latter (Δp_0 equal to few tenths of bar) moves the operating point of the pump curve far to the right (fig. 2), that is in an area where the efficiency is low and there is a high risk of cavitation.

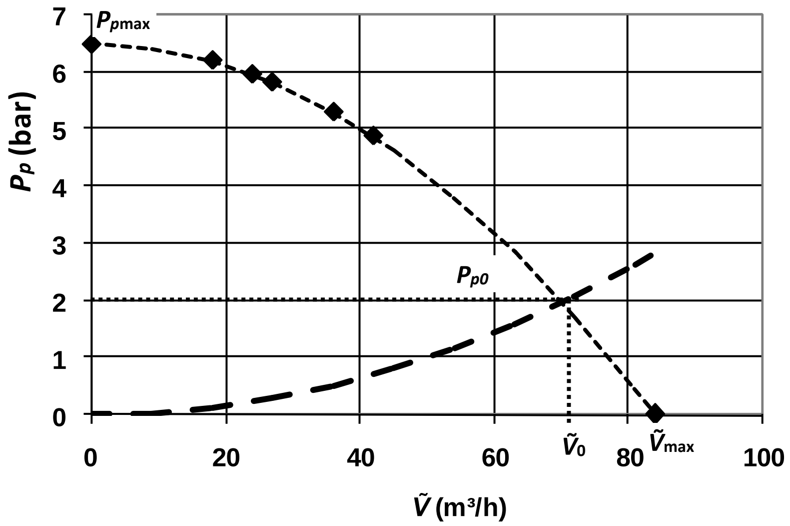

Fig. 2 – Experimental characteristic points (■) $\dot{V}$ -P_p of a commercial monoimpeller centrifugal pump (Lowara). Pump curve (------) as parabola with only two constants. Load losses curve (- - - -) of the valve located in the pipe between the pump and the filtering layer. It is adjusted so that when t=0, the pressure of the pump P_{p0} is about 2 bar for a specific flow rate $\beta_0=\dot{V}_0/A\approx 2$ m^3/(h·m^2)

To avoid these negative effects, the filters are equipped by a valve located in the pipe between the pump and the filtering layer, in order to create an additional load loss to that of the precoat. The valve must be regulated, at the beginning of the filtration, by closing the shutter, until the pressure of about 2÷2,5 bar, measured before the valve, is reached, value equal to the pressure drop of the valve P_{Rv}.
In this new situation, always at the beginning of the filtration, it must be written: $P_{p0} \cong \Delta p_0 + (2 \div 2.5) \cong 2 \div 2.5$ bar. The operating point of the pump is no longer dangerously right, but approximately near to the design conditions (fig. 2). More broadly, that is with reference to any instant of filtration, it must be written that the pressure produced by the pump has to be equal to the sum of the pressure drop induced by the filtering layer Δp (increasing with the filtration time) and the pressure drop produced by the valve P_{Rv}:

$$P_p = P_{p\max} - B \cdot \dot{V}^2 = \Delta p + P_{Rv} \tag{5}$$

As regards the load losses produced by the valve, being able to consider the turbulent flow, we can write: $P_{Rv} = k \cdot \dot{V}^2$.
By introducing the last one in the (5), gathering in a single constant $B_k = B + k$, and pointing out the pressure drop of the filtering layer, Δp, we obtain:

$$\Delta p = P_{p\max} - B_k \cdot \dot{V}^2 \tag{6}$$

In the practice of the design of the most diffused filters (pressure chamber), $P_{p\max}$ ranges between 6 and 7 bar, while the constant B_k is determined by the (6) at the initial time t=0, that is when the filter pressure drop $\Delta p = \Delta p_0 \approx 0$, and by considering the rule of thumb, that is the initial flow rate of the filter area unit β_0 ranges between 1,5 and 2,5 $\frac{\text{m}^3}{\text{h} \cdot \text{m}^2}$, with a better value of: $\beta_0 = \frac{\dot{V}_0}{A} = 2 \frac{\text{m}^3}{\text{h} \cdot \text{m}^2}$:

$$B_k = \frac{P_{p\max}}{\beta_0^2 \cdot A^2} \tag{7}$$

In conclusion the (6) becomes:

$$\Delta p = P_{p\max}\left(1 - \frac{\dot{V}^2}{\beta_0^2 \cdot A^2}\right) \tag{8}$$

3. Mathematical solution of Darcy ODE with variable pressure

By introducing the Darcy's equation (2) in the (8), we obtain:

$$\left(\frac{A}{\mu c\alpha \cdot V \cdot \beta_0}\right)^2 P_{p\max} \cdot \Delta p^2 + \Delta p - P_{p\max} = 0 \tag{9}$$

By solving the second degree equation (9), we obtain:

$$\Delta p = \frac{1}{2P_{p\max}}\left(\frac{\mu c\alpha \cdot V \cdot \beta_0}{A}\right)^2 \left[-1 + \sqrt{1 + \left(\frac{2P_{p\max}A}{\mu c\alpha \cdot V \cdot \beta_0}\right)^2}\right] \tag{10}$$

By introducing the (10) in the Darcy ODE (2), we obtain:

$$\dot{V} = \frac{\mu c\alpha \cdot V \cdot \beta_0^2}{2P_{p\max}} \left[-1 + \sqrt{1 + \left(\frac{2P_{p\max}A}{\mu c\alpha \cdot V \cdot \beta_0}\right)^2}\right] \tag{11}$$

The quantity $\frac{2P_{p\max}}{\mu c\alpha \cdot \beta_0^2}$, constant during the filtration, has the dimension of a time and for convenience we call it t_0. Instead the quantity $\frac{2P_{p\max}A}{\mu c\alpha \cdot \beta_0}$ has the dimension of a volume and we call it V_0. The (11) becomes:

$$\dot{V} = -\frac{V}{t_0} + \frac{V_0}{t_0}\sqrt{1+\frac{V^2}{V_0^2}} \quad (12)$$

If we apply the (12) to the initial time t=0, where the filtrate volume is still zero (V=0) and the flow rate is the initial one $\dot{V}_0$, it can be simplified: $\dot{V}_0 = \frac{V_0}{t_0}$, by signaling the physical meaning of V_0 and t_0: their rate is equal to the initial flow rate $\dot{V}_0$ and therefore their subscript is explained. The same result can be obtained by dividing the amounts $V_0 = \frac{2P_{p\max}A}{\mu c\alpha \cdot \beta_0}$ and $t_0 = \frac{2P_{p\max}}{\mu c\alpha \cdot \beta_0^2}$. In fact $\beta \cdot A$ is obtained, that is the initial flow rate $\dot{V}_0$.
By integrating [7] the ODE (12), we obtain:

$$\frac{t}{t_0} = \frac{1}{2}\left(\frac{V^2}{V_0^2} + \frac{V}{V_0}\cdot\sqrt{1+\frac{V^2}{V_0^2}}\right) + \frac{1}{2}\ln\left[\frac{V}{V_0} + \sqrt{1+\frac{V^2}{V_0^2}}\right] \quad (13)$$

If we define the dimensionless volume $g = \frac{V}{V_0}$ and the dimensionless time $\tau = \frac{t}{t_0}$, the integral (13) can be rewritten:

$$\tau = \frac{1}{2}g\left(g + \sqrt{1+g^2}\right) + \frac{1}{2}\ln\left[g + \sqrt{1+g^2}\right] \quad (14)$$

With the previous definitions, the Carman solution (3) obtained in par. 2.1 becomes:

$$\frac{t}{t_0} = \frac{V^2}{V_0^2} \Rightarrow \tau = g^2 \quad (15)$$

4. Results and concluding discussion

The figure 3 shows the comparison between the volume of the filtrate calculated with the Carman equation (3), obtained with a constant Δp approximation, and the volume of the filtrate calculated with the equation (13) obtained from the exact solution of the Darcy ODE with a variable Δp. Both equations were applied with $\mu c\alpha = 10^9\ \mathrm{Pa \cdot s/m^2}$, $P_{p\max} = 650{,}000\ \mathrm{Pa}$ and $A = 36\ \mathrm{m^2}$. We can observe that the higher the filtration time the closer are the curves and the lower is the error using the simplified Carman equation (fig. 4). If the values of the filter area A are changed, the result of the figure 4 doesn't change. Instead when a filter aid with a larger particle size – and therefore with a higher permeability - is used, the specific resistance α decreases so much, and also $\mu c\alpha$. As shown in figure 4, in particular with a 10-fold reduction of $\mu c\alpha$ (10^8 Pa·s/m^2), a clear worsening of the error committed using Carman equation occurs, even if, increasing the filtration time, some improvement can be obtained. However, the filtration time cannot be as high as you want. There are two reasons that lead to its limitation. The first one is due to the limit in the available space in the filter above the precoat that can be occupied by the cake, usually about 20 mm.

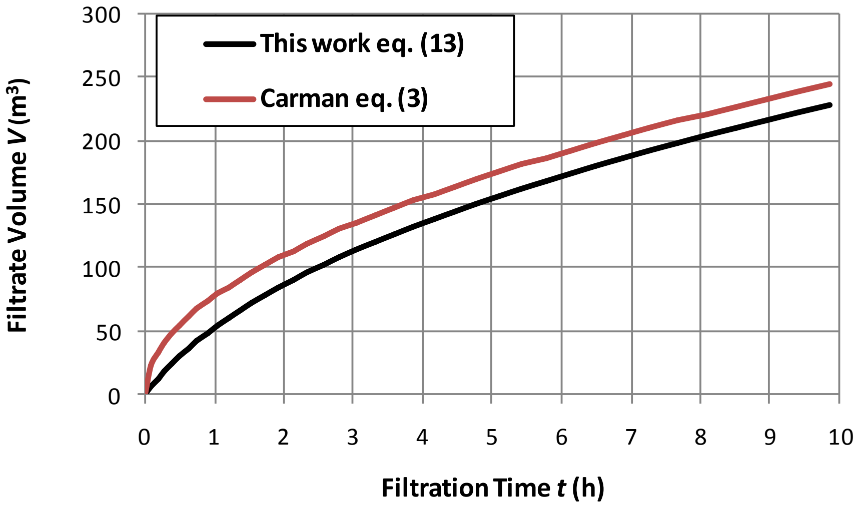

Fig. 3 – Filtrate volume vs. filtration time: comparison between Carman equation (3) and equation presented in this work (13). Both the equations are applied with $\mu c\alpha = 10^9$ Pas/m^2, $P_{p\max} = 650{,}000$ Pa and $A = 36$ m^2

The second one is that a maximization of the average flow rate of the filtrate is needed in order to reduce the costs of the operation. This means we need to optimize the cycles of filtration-cleaning. As you will see in the second part, this can be done finding a relationship between the cleaning time θ and the filtration time t. Since the first one is slightly variable, a quite accurate filtration time will be defined depending on the boundary conditions of the filtration. As you will see, we can easily be in the optimized condition with filtration time of the order of few hours for which, therefore, the use of the Carman equation (3) becomes unacceptable for the big mistake introduced by it.

Finally the figure 5 shows the dimensionless volume vs. dimensionless time as provided by the equations (14) and (15).

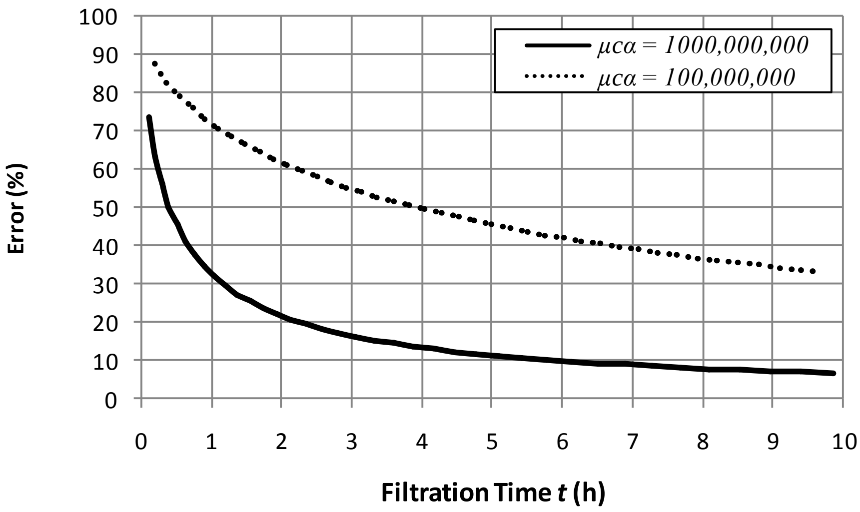

Fig. 4 – Error due to the use of the approximate Carman equation. The quantity $\mu c\alpha$ is expressed in Pa·s/m^2

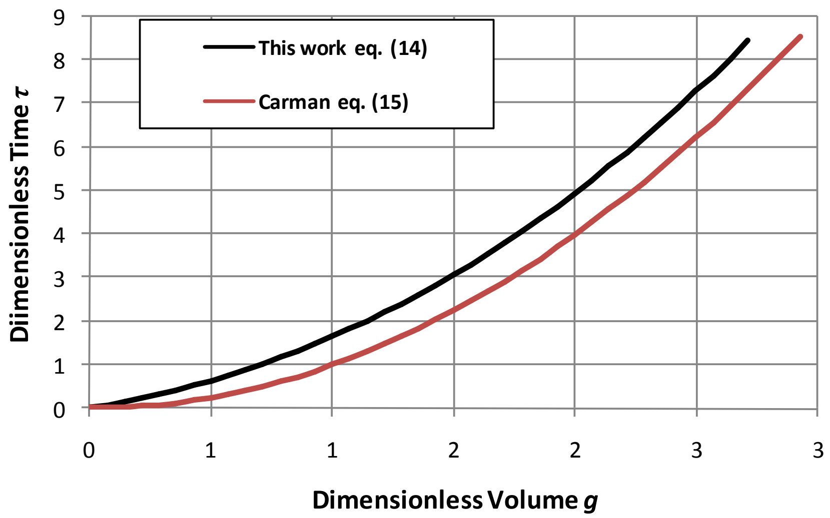

Fig. 5 –Dimensionless τ time vs. dimensionless volume *g*

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