Case Studies on Transport Policy 8 (2020) 188–196

Contents lists available at ScienceDirect

Case Studies on Transport Policy

journal homepage: www.elsevier.com/locate/cstp

A stochastic approach to the benefit cost ratio analysis of safety treatments

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1. Introduction

The decision making process for safety interventions is complex, involving a number of actors (experts, public, politicians, etc.) and issues (environment, economy, mobility) competing for the scarce available resources. The risk of making poor decisions and the cost of making better decisions can be reduced by the use of reliable studies on how effective different safety measures are (OECD, 2012). In this framework, Road Agencies set specific quantitative safety targets and adopt related road safety strategies towards these targets, within the established priorities and the available resources. In particular, benefit-cost analyses are carried out in a more or less systematic way to maximize results within the limited funds that are available at a time of economic crisis.

Benefit-Cost analysis (BCA) aims at comparing the benefits and costs of different policy alternatives, measured in monetary units. Measures for which benefits are greater than costs are called cost-effective, and ranked according to their benefit-cost ratio.

The BCA requires basically 3 different estimates:

1. an estimate of the safety problem, i.e. crash number and severity basing on crash history and/or Safety Performance Function.
2. An estimate of the effectiveness, i.e. Crash Modification Factors (CMFs) of road safety measures identified for solving the safety problem.
3. An estimate of the life cycle cost of each measure.

The most important uncertainties involved in developing such assessment process concern the adoption of appropriate values for the safety effects of road safety measures.

Scientific accuracy is difficult to obtain in the field of CMFs, not only because several assumptions are necessary in the process but also because it is very difficult to separate the safety effect of a measure from the effect of several other microscopic or macroscopic measures and phenomena (including statistical randomness) occurring at the same place. Two main issues affect the reliability of CMFs: accuracy and transferability. The former factor pertains to the data quality, the small sample size, the bias and confounding factors not eliminated. The latter factor has to do with the fact that the CMF estimates come from studies conducted in differing circumstances which were not directly correlated to the CMF value by the way of a function. Hauer et al. (2012) described how important is the site-to-site variability inasmuch, along with the uncertainties inherent in the estimation, the site-to-site variability is able to considerably increase the value of the variance. Moreover, it is necessary to assess whether the studies can be generalized in time and space (external validity of research), e.g. from one country to another or from one decade to another, showing the consistency in time and space of studies that have evaluated the effects of road safety measures (Shadish et al., 2002).

A framework for interpreting road safety evaluation studies in theoretical terms has been proposed by Elvik, 2004. This framework is a conceptual scheme that can be used to develop arguments for or against the general validity of road safety studies. Cumulative meta-analysis is well suited for assessing external validity based on the range of replications (Elvik et al., 2009), but the applicability of the technique is likely to be limited and it can be applied to assess external validity when a large number of studies have been reported during a long period of time.

To make progress towards reducing the uncertainty about CMFs a two-pronged strategy has to be followed. First, the CMF estimates used to produce the probability distributions have to be consistent. Second, the dependence of the CMFs on the relevant circumstances has to be established by the way of a function (OECD, 2012).

Any future improvement in knowledge of the effectiveness of safety measures, i.e. development of quality Crash Modification Functions, will likely have tangible effects on the way safety decisions are made. On the other hand, the development of new reliable CMFs is costly and time consuming. A typical project to develop a reliable CMF related to roadway features in the United States, for example, is estimated to cost about $US 200,000. Therefore, to find a way for using correctly the current available CMFs is important in the short term as well the development of new CMFs in the medium term.

In this context, it is necessary to account for the heterogeneity of study findings by considering that CMF is not a constant but it is instead a random variable. Thinking of a CMF as a random variable allows us to correctly frame the question of accuracy and transferability of existing CMFs.

Whether the decisions we make are right or wrong depends on the

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https://doi.org/10.1016/j.cstp.2018.07.006
Received 25 June 2017; Accepted 12 July 2018
Available online 18 July 2018

size of the standard deviation of the probability distribution of the CMF. The smaller the standard deviation, the larger the probability that decisions we make are correct.

The Highway Safety Manual (HSM) in Part A, Appendix C (HSM, 2010) treats the variance in CMF giving a procedure to evaluate the probability of failure (i.e. what the chance is that implementing the treatment is the wrong decision). The standard error of CMF is used to define a confidence interval that indicates the range of values that contain the true treatment effect with a given level of confidence. The interval limits CMF_α may be easily calculated, with approximation, using the formula that assumes a normal distribution of CMF as suggested in HSM using Eq. (1):

$$CMF_\alpha = E(\theta) \pm Z_\alpha \times \sigma(\theta) \tag{1}$$

where

- θ is the random variable associated to the probability density function of the CMF;
- E(θ) is the expected (mean) value of the CMF;
- σ(Θ)is the standard error of the CMF;
- Z_α is the standardized normal variable with probability 1-α (e.g. Zα = 1.96 for α = 5%);
- 1-α is the confidence level (e.g. α = 5%).

Basing on this approach, it would be reasonable, in the decision making process, to give less consideration to treatments for which the associated CMF has a confidence interval that includes 1.0 that means there is a probability that crashes will remain unchanged or experience a slight increase (i.e. CMF > 1). Furthermore, it may be prudent in some situations to give greater consideration to treatments with smaller confidence intervals because of the greater level of certainty in the results. This procedure is simple to be used and able to take into account the expected value of CMF and its variance as well. However, as will be showed in the following, it is not able to catch the whole variability of the phenomenon which involves also the uncertainty in the estimation of crashes and, as a consequence, the variability in the estimation of both benefits and benefit-cost ratio. Therefore, also the estimation of the future number of crashes plays a fundamental role in BCA.

The best tool for predicting the mean number of crashes and casualties in the years ahead from the implementation of the treatment is the Safety Performance Function (SPF). SPF is a regression equation that related the predicted number of crashes to significant covariates (e.g. AADT, length, cross section, alignment geometry, etc). SPFs modeling, variance and transferability concerns (Borsos et al., 2016) are of the same nature and relevance of CMF (D'Agostino, 2014). Calibration of SPFs using local data and appropriate covariates or even surrogate measure (Cafiso et al., 2016, 2018a), improves the precision of the estimation, but the variability in the predicted number of crashes persists and it is a further causes of randomness to be considered in the BCA. In the present paper both the variance of the CMFs and SPFs is taken into account in a reliability based assessment of safety benefits to catch this variability and to point out as a more rigorous probabilistic approach can lead to different conclusions about the decision to implement or do not implement a treatment. First, the methodology framework is described. The analyzed methodologies include the use of Monte Carlo simulation and the estimation of the B/C mean and variance as moments of random variables. A case study is presented to compare the results with the HSM approach and conclusions are drawn out at the end of the paper.

2. Methodology

Given the benefits in terms of crash reduction and the cost of the treatment the B/C ratio can be defined as follow:

$$\frac{B}{C} = \frac{\text{Expected Benefits}}{\text{Expected Cost}} = \frac{\mu \cdot a \cdot (1-\theta)}{c} \tag{2}$$

where:

- μ = number of target crashes expected without the treatment (considered as a random variable);
- θ = crash modification factors of the treatments to be applied (considered as a random variable);
- a = monetary value of the target crash (considered as a constant); and
- c = cost of the treatment implementation (considered as a constant).

To carry out a BCA using a stochastic approach, Eq. (2) necessities to be evaluated considering both θ and μ as random variables and performing a reliability assessment of the B/C ratio.

By definition, reliability is unity minus failure probability. As a result, central to reliability analysis is the determination of the failure probability. Given the probability density function B/C(z; x_i), where z is the random variable, x_i represents the set of design parameters (a, c, μ, θ), the probability that the failure event F occurs is denoted by P(F|z). The failure probability can be calculated by the following integral (Mordechai, 2011; Shooman, 1968):

$$P(F|z) = 1 - \int_{-\infty}^{F} B/C(z;x)dz \tag{3}$$

where

- B/C(z; x) is the probability density function of B/C;
- F represents the failure event: B/C(z; x) < F.

When the joint distribution B/C(z; x) of the random variables μ and θ is unknown the numerical solution for this integral is infeasible

In the present research work, B/C(z; x) is a continuous random variable in the ± ∞ sample space (i.e. can assume also negative values) and its distribution is a priori unknown. In other terms the solution is unfeasible in a closed form inasmuch the jointly distribution B/C(z; x) is unknown. Despite, the distribution form is unknown, the moments of B/C can be determined in terms of moments of the basic random variables rather easily, especially if the variables are independent and only the first two moments, mean E[B/C] and variance σ^2 [B/C], are of interest:

$$E[B/C] = \frac{a}{c} \cdot E[\mu] \cdot (1 - E[\theta]) \tag{4}$$

$$\sigma^2[B/C] = \left(\frac{a}{c}\right) \cdot [(1-E[\theta])^2 \cdot \sigma^2[\mu] + E[\mu]^2 \cdot \sigma^2[\theta] + \sigma^2[\theta] \cdot \sigma^2[\mu]] \tag{5}$$

where

- E[μ], σ^2[μ] are the mean and variance of μ (number of target crashes on the unit);
- E[θ], σ^2[θ] are the mean and variance of θ (CMF or combination of CMFs).

E[θ], σ^2[θ] are known values if the CMF is estimated with a proper reliable methodology (e.g. Empirical Bayes Before/After, full bayes B/A).

E[μ] and σ^2[μ] can be determined from the over the sites crash frequency distribution. It is assumed to follow a Negative Binomial distribution with mean and standard deviation derived from the SPF calibration.

In other terms let Eq. (6) be the general form of an SPF and k the over-dispersion parameter

$$E[\mu] = e^{\alpha_0} \cdot L \cdot AADT^{\alpha} \cdot e^{\sum_i \beta_i \cdot x_i} \text{[number of Crash/year]} \tag{6}$$

Where:

- $E[\mu]$: predicted crash annual frequency;
- L: length of road segment;
- AADT: average annual daily traffic [veh/day]; and
- α, $\alpha 0$ and β_i: regression terms.

Then $\sigma^2[\mu]$ assumes the following form

$$\sigma^2(\mu) = E(\mu) + E(\mu)^2 \cdot k \tag{7}$$

With an approximation, E[B/C] and σ[B/C] can be used to calculate the confidence limits assuming the Normal distribution for B/C

$$B/C_\alpha = E(B/C) \pm Z_\alpha \times \sigma(B/C) \tag{8}$$

where

- θ is the random variable associated to the probability density function of the CMF;
- E(B/C) is the expected (mean) value of the B/C ratio;
- σ(B/C) is the standard error of the B/C ratio;
- Zα is the standardized normal variable with probability $1-\alpha$.

If the joint distribution of B/C is assumed unknown, the Monte Carlo simulations (MC) can be implemented for the purpose of reliability analyses. The basic idea of MC is very simple and relies on repeated random sampling to obtain numerical results.

For example, it is possible to define the failure probability of the B/C < F (e.g. F = 3 means B/C < 3 is the failure event) by the following Eq. (9):

$$P(F|B/C(z;x) < F) \approx 1/N \sum_i 1(B/C(\theta^{(i)};\mu^{(i)}, a, c) < F) \equiv P_F^{MC} \tag{9}$$

where:

- N is the total number of MC simulation independent samples;
- 1(.) is unity if the statement is true and is zero otherwise;
- θ(i) and μ(i) are the i-th sample of θ and μ, drawn randomly from the distributions of θ and μ respectively (i = 1, …, N);
- a, c are crash cost and treatment cost, respectively, and
- P_F^{MC} is the estimator of P(F|B/C(z; x) < F).

This estimator is unbiased, i.e. the expected value of P_F^{MC} is exactly P(F|B/C(z; x)) with c.o.v. (coefficient of variation) = $\{[1- P(F|B/C(z; x))/N/ P(F|B/C(z; x))\}^{0.5}$ (Mordechai, 2011). Note that the c.o.v. does not depend on the number of design parameters and does not depend on the complexity of the problem, either. This is the key advantage for MC method, especially for engineering problems where nonlinearity and uncertainty dimension is usually high. The only criticism for MC is that it is inefficient for problems with very small failure probabilities or with very high reliabilities. However, this limitation has been gradually removed by the recent advancements in the Monte Carlo based reliability methods and in the possibility to produce easily very large random samples using computerized tools (e.g. SAS, Statgraphics, etc).

The percent error involved with the estimated probability P(F|B/C) has been found to be (Shooman, 1968)

$$\%\ e = 200 \cdot \sqrt{((1-P(F|B/C))/(N \cdot P(F|B/C))} \tag{10}$$

where:

- P(F|B/C) is the failure probability;
- N is the total number of MC independent samples.

This rule can be used to find the sample size (N) required for realizing a specific accuracy in the estimated probability of failure

In the present case the size of the sample (N) was fixed at 100,000. These assures that there is a 95% chance that the estimated probability of failure has an error less than ± 1.5%.

In the following case study, results obtained using a MC simulation are assumed as the more accurate reliability estimation and will be compared with the others obtained applying the simplified hypothesis of normal distribution for B/C and the HSM procedure, to check the level of approximation and the consistency of the results in the decision making process. These comparisons are made for practical application of the methodology. Indeed, using HSM approach or assuming a Normal distribution for B/C it is possible to compute easily the reliability of B/C for different percentile or Eqs. (1) and (8) using e.g. excel sheets.

3. Monte Carlo simulation and case study

For showing a practical example of how the procedure can be applied and what kind of results can be achieved in a reliability based assessment of benefit cost ratio, the methodology was applied to a case study of retrofitting safety barriers in motorways. The data used for this investigation pertain to an Italian rural motorway (double carriageway, access control), the A18 Messina-Catania, which is approximately 76 km long. The cross section is made up of 4 through-traffic lanes (3.75 m wide), 2 in each direction, divided by a median with barriers and an emergency/shoulder right lane (3.0 m wide). The motorway was built in the late 1970 and, a part of normal maintenance of pavement, markings and signs, renewal works were carried out only on a limited extension of the highway, especial for retrofitting the safety barriers to the new EU standards (Cafiso et al., 2017a,b). Retrofitting old guardrails with new ones complying with present EU standards is a key policy for infrastructure safety of Italian motorway agencies. Consequently, the main safety problem to be addressed refers to the ran-off road (RoR) crashes in sharp curves.

The BCA was carried out taking into account a treatment on a curve of 500 m radius. This is the minimum bending radius of 11 curves in the motorway sections with old barriers. Table 1 shown a summary statistics of AADT and curve length (L) in those sites.

In the following the calibration of SPFs is reported as well as all the information about CMFs.

3.1. Probability density function of the number of crashes

The widespread form of regression model with random effects that is considered is the Poisson-Gamma model. The model can be described as follows:

$$Y_{i,t} \sim \text{Poisson}(\varepsilon_i \lambda_{i,t}) \tag{11}$$

where:

- $Y_{i,t}$ are the observed number of crashes at site i in year t,
- $\lambda_{i,t}$ are the predicted number of crashes at site i in year t,
- ε_i is a multiplicative random effect at site i.

For the Poisson-Gamma model, εi is gamma distributed with the mean having a value of 1 and variance equal to k the over-dispersion parameter. When E(ε) = 1 and Var(ε) = k, the Poisson-Gamma function becomes NB distribution (Lord, 2006; Cameron and Trivedi, 1998; Hauer, 1997). In the following the SPF calibration methodology is reported as well as the dataset used for the elaboration.

Table 1
Summary statistics of AADT, Length and observed crashes for the sites in the case study.

	Sites to treat (11 sections)		
	Max	Min	Average
AADT (Veh/day)	9290	22,410	15,503
L (m)	110	810	414

In the selection of the sample for modeling the SPF, to avoid problem related to different roadside condition, only sections of the motorway without retrofitting works were included. The analysis period for model calibration is of the 12 years from 2001 until 2012, during which 441 severe (fatal plus injury) crashes occurred (ISTAT, 2012). Moreover, considering the target crashes of CMFs, the SPF was calibrated on ran-off road crashes only (Cafiso et al., 2018b).

Many studies (Higle and Witkowski, 1988; Miaou and Lord, 2003; Heydecker and Wu, 2001), suggest that the dispersion parameter is not constant for a given data set but actually varies from site to site depending on the length of a roadway segment (Hauer, 2001). The more appropriate varying form (i.e. HSM, 2010) is such that the dispersion parameter for road segments is inversely proportional to segment length ($k = a{\cdot}L^{-1}$) (Cafiso et al., 2017c). For this study, following Cafiso and Silvestro (2010), the equation form for the calibration of the variable dispersion parameter was more flexible including one more regression parameter (b):

$$k = a{\cdot}L^{b} \tag{12}$$

The results of the model calibration are shown in Table 2 for the model form:

$$\lambda_{i,t} = y_i{\cdot}e^{\alpha_0}{\cdot}L{\cdot}AADT^{\alpha}{\cdot}e^{\beta{\cdot}CCR}\ \text{[number of Crash/year]} \tag{13}$$

where:

- $\lambda_{i,t}$: predicted mean annual (fatal plus injury) RoR crash frequency at site i;
- L: length of road segment [m];
- AADT: average annual daily traffic [veh/day];
- α, $\alpha 0$ and β: regression terms;
- CCR: Curvature Change Rate [gon/m] (200 gon = π rad);
- y_i: the time trend coefficient in the year i.

The goodness of fit of the model was investigated using the plot of cumulative residuals (CURE) (Hauer and Banfo, 1997). The CURE plot showed reasonable good fits of the model to the data with values never exceeding the ± 2σ bounds (Fig. 1).

From Table 2 and Eq. (13), considering as expected value the predicted crashes in 2012, for a single segment with an average length of 414 m, an average value of AADT of 15,500 veh/day and a radius of 500 m the average number of RoR crashes in the last year of analysis (2012) is:

$$\lambda_{2012} = 0.76{\cdot}e^{-16.105}{\cdot}L{\cdot}AADT^{0.786}{\cdot}e^{2.486{\cdot}CCR} = 0.087 \tag{14}$$

3.2. Probability density function of crash modification factor

The CMF for retrofitting of motorway barriers with new ones meeting the EU standard was investigated by Cafiso et al. (2017) using

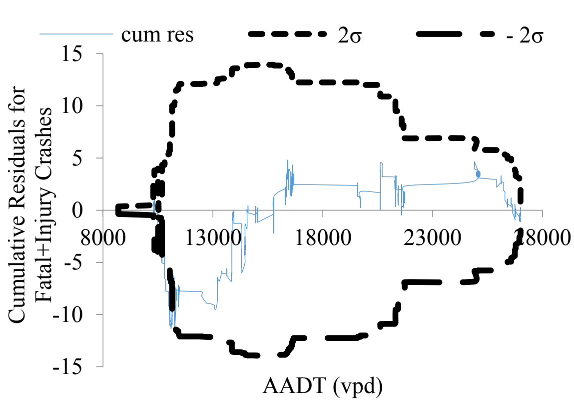

Fig. 1. CURE Plots with ± 2σ for the ran-off-road crashes.

Table 2
Value of regression parameters, (Standard error) and [p-value] for the SPF calibrated.

		Ran-off road crashes
Intercept (α_0)		−16.1053 (2.444) [< .0001]
AADT (α)		0.7862 (0.254) [0.002]
CCR (β)		2.486 (0.0236) [< .0001]
Years (y_i)	2000	1.33
	2001	0.85
	2002	1.15
	2003	1.58
	2004	1.35
	2006	1.47
	2007	1.07
	2008	0.95
	2009	1.03
	2010	1.29
	2011	0.85
	2012	0.76
k (Over-dispersion parameter)		$6.1{\cdot}L^{-0{,}85}$

Table 3
Mean CMFs E(θ) and standard deviation σ [θ] used in the analysis for retrofitting with barriers meeting the EU standard.

	Total		Ran off road		Non-Ran off road	
CMF	0.71		0.28		0.98	
Stdev	0.09		0.07		0.152	
95% interval	0.52	0.79	0.16	0.41	0.68	1.28

Table 4
Present Value (PV) and cost (k€ = Thousand Euro) at the first year after the implementation.

Barrier	
Cost [k€/km] (k$/km)	180 € (243 $)
Serv. life	20
Total PV (20th year) [k€/km] (k$/km)	180 € (243 $)
Crashes	
Cost [€/Crash]	€ 250,000.00 ($ 337,500.00)
Average PV [€/Crash]*	€ 198,500.00 ($ 267,950.00)

* The average value in 20 years was assumed because it is not possible to predict the year of the crash occurrence.

data from the same infrastructure (Table 3). The description of the Empirical Bayes B/A procedure used for the estimation of CMFs and standard deviation is not of interest for the present paper. Therefore, it is not reported here and the interested readers may refer to the reference for more details.

The value of CMF and standard deviation taken into account in the following elaboration were those related to the ran-off road crashes. A Gamma distribution is assumed for the CMF (Hauer, 1997).

3.3. Cost of crashes and treatment

The service life of the treatment represents how long such treatment will continue to deliver safety outcomes without maintenance needs. The entire analysis period was 20 years, which is equal to the service life of the barriers treatment as reported in Table 5. For that reason, both the Present net Value (PV) of costs for crashes and for implementation of the treatment were actualized at the year of construction with the following Eq. (15) and results are reported in Table 4:

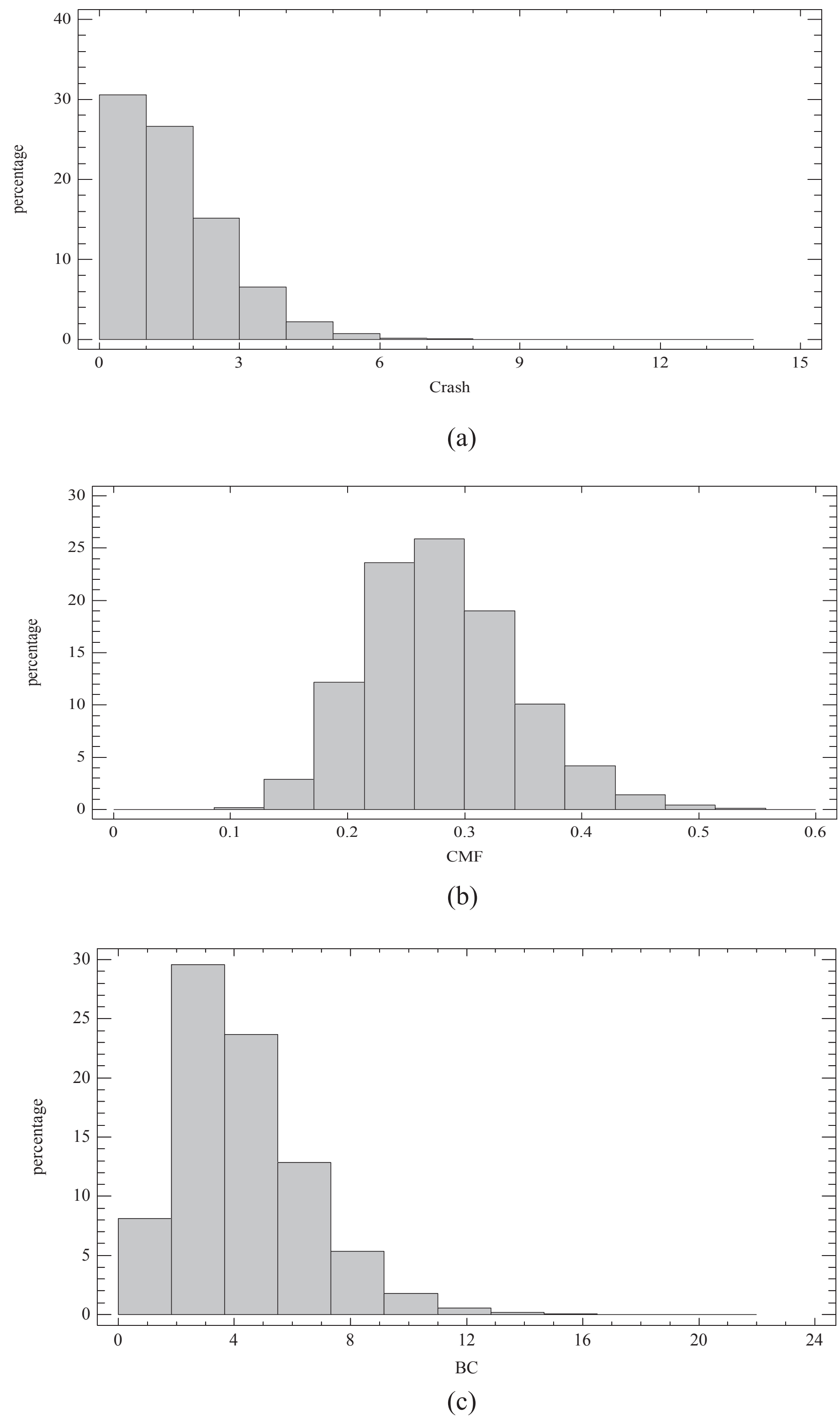

Fig. 2. Frequency histograms of MC sample for Crash number (a), CMF (b) and B/C (c).

$$PV = \sum_{i=0}^{N} \frac{C_i}{(1+r)^i} \tag{15}$$

where,

- PV is the present net value of costs in the service life;
- C_i is the costs of crashes or treatments at year i, i = 1,…,20;
- r is the discount rate, assumed equal to 3%.

3.4. Monte Carlo simulation approach

With aims to take into account the variability in the estimation of the number of crashes and CMF in the sites to be treated as well as to have a reference population of B/C, a MC simulation approach was used. Starting from the results of the average number of crashes predicted by the SPF reported in Table 2, a Monte Carlo simulation was

Table 5
Summary statistics of CMFs, B/C ratio and Benefits.

	B/C	
Deterministic		3.33
Normal Distribution	Mean	3.33
	Variance	6.60
MC Sample	Mean	3.31
	Variance	6.63
	Benefit [k€]	
Deterministic		248.16
HSM	Mean	248.16
	80th percentile	229.3
MCS	Mean	246.97
	80th percentile	91.4
Normal	Mean	248.16
	80th percentile	87.06

*80th percentile refers to one tailed lower limit.

performed to derive a random sample of the predicted number of crashes in the service life period of the treatment. With the same methodology, it was possible to generate random samples of θ as well, and from Eq. (2) the random sample of B/C following the steps reported below (Cafiso and D'Agostino, 2016, 2015):

1. The mean $\lambda_{i,t}$ and variance Var_i of the crash counts for similar sites is derived from Eqs. (7), (12) and (13) for a given value of L, AADT and CCR;
2. a random value of ε_i is generated from the gamma distribution with $E(\varepsilon) = 1$ and $Var(\varepsilon) = k$;
3. 20 random values of crashes $Y_{i,t}$ are generated from the Poisson distribution, from Eq. (11) to calculate the number of crash count in the analysis period;
4. a random value of θ is generated following a gamma distribution θ with $E(\theta) = 0.28$ and $Var(\theta) = 0.0049$; and
5. a random values of B/Ci is calculated by combining all the previous results in step 3 and 4 using Eq. (2).

The steps reported above were repeated several times (100,000 in the case study) in a way to have a simulated population suitable for predicting as close as possible the failure probability of $B/C < F$. Fig. 2 shows the frequency histograms of the samples generated in the MC simulation. As expected, both crash and θ (CMF) variables follow the assumed Poisson and Gamma distribution, respectively. B/C frequency distribution shows skewness and shape which clearly deviate from the Normal distribution. In the present case study, this was confirmed with 99% confidence since the smallest P-value amongst several tests of normality resulted less than 0.01 (i.e. Chi-square, Shapiro-Wilk, Skewness Z-score, Kurtosis Z-score).

3.5. Results and discussion

Table 5 shows mean and variance of the random samples generated in MC simulations for the CMF and B/C ratio. For the same variables, the mean and variance calculated as linear combination of uncorrelated random variables are reported in Table 5, too. Data in Table 5, show good fitting of the output parameters mean and variance of B/C between the MC simulation and the moments calculated using Eqs. (4) and (5). This result confirms the correct application of the numerical simulation.

For comparison the mean and 80th confidence limit of expected benefits and B/C ratios were carried out applying different statistics:

- (Normal) assuming a normal distribution for B/C with mean and standard deviation calculated as combination of uncorrelated random variables (Table 5);
- (MCS) Monte Carlo Simulation, and
- (HSM) using confidence intervals of CMF as suggested in the HSM.

Fig. 3 shows the mean value of B/C and the 80th (lower) and 20th (upper) two-tailed confidence limits obtained with the three different approaches (Normal, MC, HSM).

Results reported in Fig. 3 point out as both Normal and HSM hypothesis gave the same estimation of the mean value of B/C ratios calculated in the MC simulations, but they are unsuccessful in the estimation of the confidence intervals due to the different estimation of the variance and hypothesis of probabilistic distribution of B/C. Specifically, calculating the CMF's confidence intervals, as suggested by HSM, fails to catch the larger variability of B/C missing to consider the crash prediction as a random variable with as consequence an underestimation in the upper percentile. Assuming a normal distribution for B/C improves the estimation of the confidence limits, but with a tendency to overestimate always the lower limit and also underestimate the upper limit, due to the skewness and shape of the actual distribution of B/C which deviate from normality (Fig. 3).

In the decision making process, it is usual to fix a minimum critical value of B/C ratio (e.g. B/C ≥ 3) to take the decision of implement or not implement the treatment. Applying the reliability-based approach it is possible to calculate the probability of failure of such assumption using the MC simulation, the Normal distribution and the HSM approach. With the hypothesis of Normality for B/C the probability of failure is calculated with Eq. (3) and in the MC simulation with Eq. (9). Adopting the HSM approach, the probability of failure is calculated iteratively, as the percentile of the CMF that gives the assumed B/C ratio.

Fig. 4 shows the results of the probability of failure assuming the threshold value of 3.0 (i.e. failure if B/C less than 3).

Again, different methodologies lead to different results. The HSM approach, again produces the largest deviation from the correct value provided by the MC simulation. Even if with improvements in the

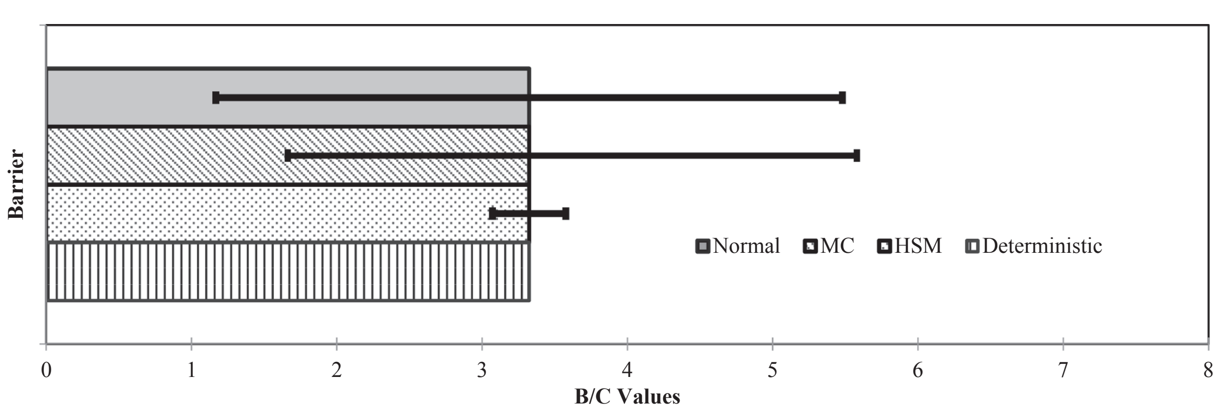

Fig. 3. Mean value of B/C function and the 80th confidence interval for Normal distribution, Monte Carlo simulation (MC), HSM and deterministic approach.

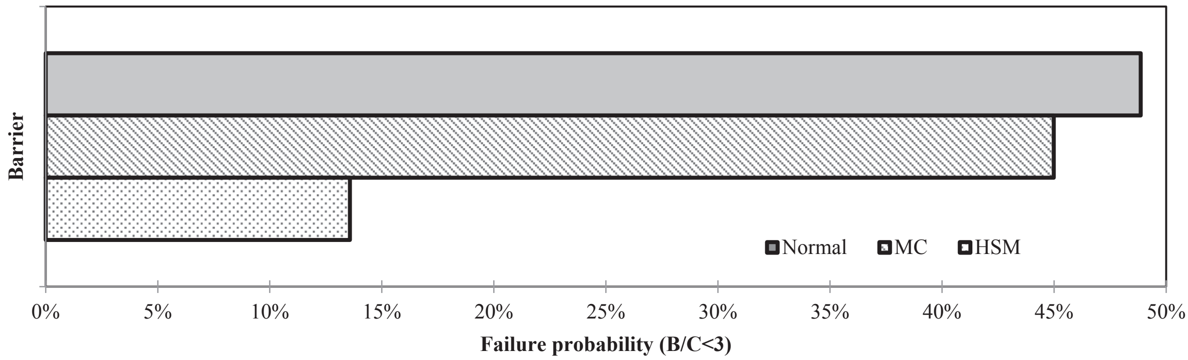

Fig. 4. Failure probability of B/C < 3 with different approach.

Table 6
CMF and cost for installation of Chevron on curves.

	CMF	Service Life	Cost [k€/km] (k $/km)	Total PV (20th year) [k€/km] (k$/km)
E[θ]	0.73	10 years	15 €	26 €
σ [θ]	0.11		(20 $)	(35 $)

estimation, the shape and skewness of the B/C distribution cause bias in the results assuming normality for B/C.

The reliability analysis can be used to evaluate the safety performance of the selected treatment using as criteria the B/C ratio percentile (Fig. 3) and the highest chances to achieve a minimum critical B/C value (Fig. 4). Moreover, the reliability analysis makes possible to compare two different treatments in terms of chance that the B/C ratio of the former is higher than the latter.

For example, in the presented case study, improving curve delineation with chevron signs may be introduced as alternative treatment to the more expansive safety barrier retrofitting.

The CMF for installation of chevron on curve in motorway has been estimated in 0.73 with a standard error of 0.11 (Montella, 2009). This treatment is characterized by higher CMF and standard error, but also by reduced installation and maintenance costs (Table 6).

Applying the procedure previously described, the B/C ratios were calculated and results are reported in Fig. 5. In Fig. 5 also the probability of $B/C_{Barrier} > B/C_{chevron}$ is shown. This probability was calculated with the count of the events $B/C_{Barrier} > B/C_{chevron}$ in the MC samples and by comparison of the two distribution curves in the Normal hypothesis (a similar test cannot be performed in the HSM framework).

Also in this instance, MC simulation delivers different results when compared to the Normal distribution and HSM approaches. The tendency of the approximated methods to over or underestimate the B/C percentiles is confirmed also in this example. Installing new barriers get lower B/C ratio than the delineation treatment when evaluated with the Deterministic, HSM and Normal methods.

Instead the MC simulation highlighted a slight higher 80th percentile of the $B/C_{barrier}$ ratio, but a limited chance (6%) to achieve a better B/C than curve delineation. This result is consistent from a reliability point of view because the greater mean value of $B/C_{chevron}$ is compensated by the higher standard deviation in the estimation of the 80th percentile (left side of the distribution curve).

When the two distributions are compared for estimating the probability of $B/C_{Barrier} > B/C_{chevron}$, the overall curves are considered and the probability to draw values greater than the mean (right side of the curve) is higher in the $B/C_{chevron}$ distribution. The different shape and skewness of the two distribution is evident in Fig. 6 where the cumulative frequency histograms of the MC samples are reported.

Therefore, from a reliability point of view we can assess that chevron delineation has a lower B/C ratio than installing new barriers as worst estimation (80th percentiles comparison), but in multiple treatments we can expect 94% of chances to have an higher B/C with new chevron delineation than installing new safety barriers. This probability is under estimated equal to 74% by the Normal method, while the HSM approach is not suitable to determine this parameter.

4. Conclusions

Due to the variance of CMFs and crash frequency the safety benefits of treatments are uncertain. To deal with the uncertainty inherent in the decision making process, a reliability assessment of Benefit Cost ratio must be performed introducing a stochastic approach. In this framework, the failure probability plays a fundamental role, being itself a fair indicator of the reliability of the decision-making process. Particularly a failure probability that B/C is lower than a target value can give an idea of the chance to take the right decision. If different alternatives have to be compared, only the reliability analysis is able to provide an estimation of the probability to achieve the highest B/C ratio between the two treatments.

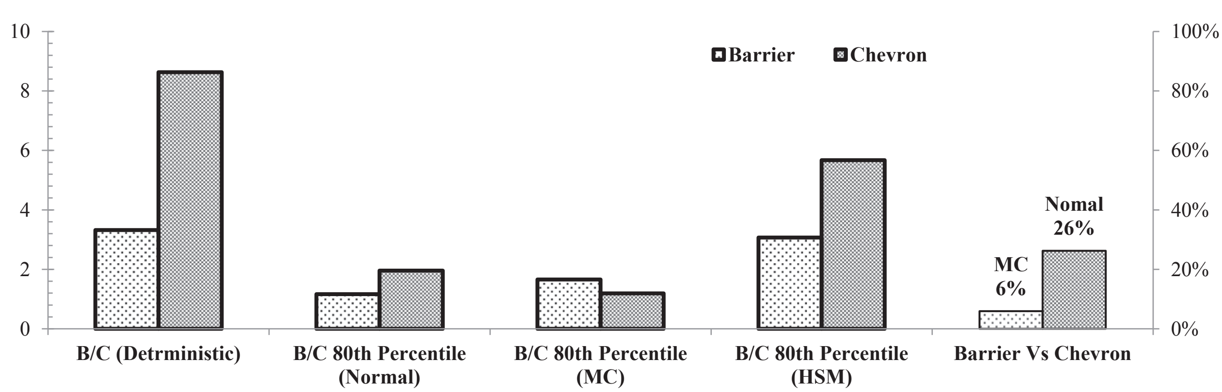

Fig. 5. Comparison of B/C of alternative treatments.

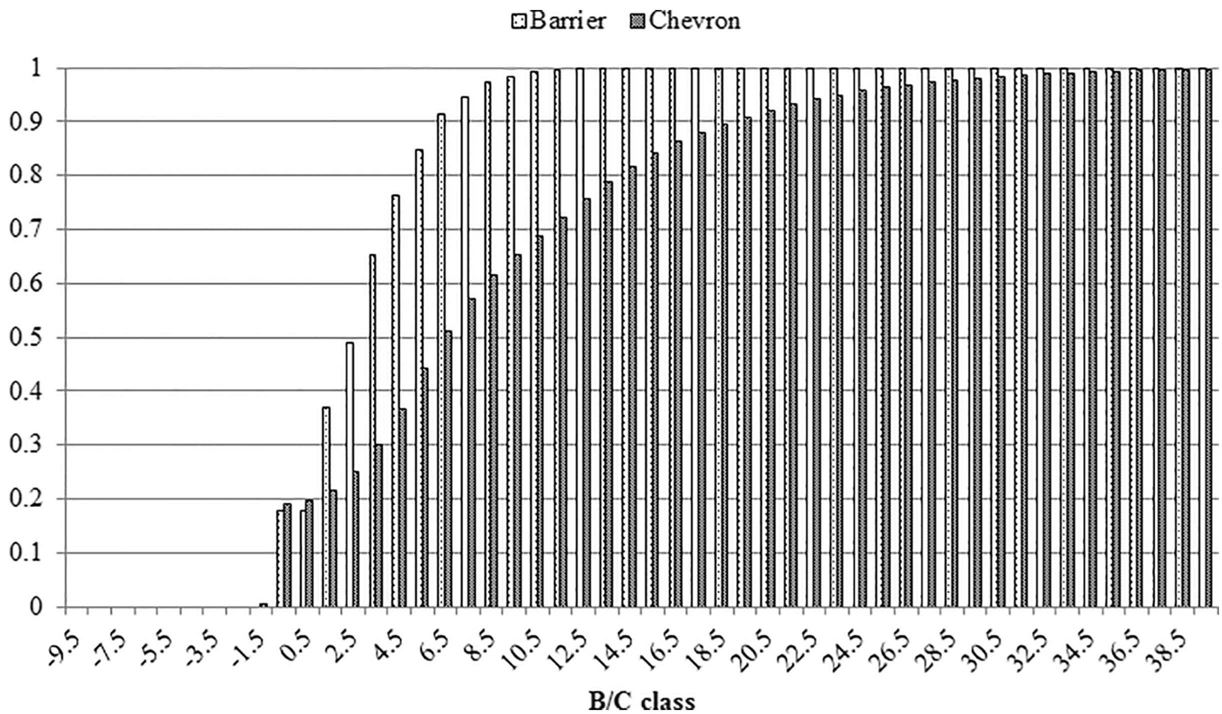

Fig. 6. Cumulative frequency histogram of B/C for alternative treatments.

The proposed methodology applies the Monte Carlo (MC) simulation as the most suitable tool to calculate reliability, as the joint distribution function of the B/C random variable is unknown. For practical application, the confidence interval of B/C can be determined rather easily assuming Normality for the joint distribution or applying the HSM approach, as well. However, the shape and skewness of the actual distribution of B/C cause bias in the results assuming normality for B/C. Despite, the results show that the Normality hypothesis for B/C was not statistically significant, it leads to better results than those provided by using the confidence intervals of CMF to estimate the B/C ratio as suggested in the HSM. MC is relatively easily to carry out with the modern computer programs, but for practical application the hypothesis of normality for B/C can simplify the reliability assessment improving results when compared to the HSM approach.

The case study was effective to show power and issues of the reliability analysis carried out by using different approaches. Moreover, the specific case study of installing new safety barriers with high construction costs is relevant for Motorway Agencies facing the problem to select the treatments able to achieve the best safety benefits with limited budget. In the comparison between the installation of new safety barriers and the delineation of curves with chevron the former in 94% of application will provide higher B/C ratios. Of course the highest benefits are expected with the installation of new barriers even if with higher construction costs.

Despite the theoretical procedure is detailed presented to make possible the use in any circumstance when mean and variance of CMF and Crash frequency are known, results coming from the case study cannot be generalized depending on the characteristics of the probabilistic distributions of CMF and crash frequency. Another issue in the transferability of results is related to the use of monetary valuation of safety benefits. Some international comparisons show that official estimates of the value of a statistical life vary by a factor of almost 60 between the countries with the highest and lowest estimates.

Acknowledgement

An earlier version of this paper was presented in WCTR-2016 at Shanghai and the authors acknowledge the opportunity given by WCTRS in this regard.

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