EPJ Web of Conferences **117**, 03013 (2016) DOI: 10.1051/epjconf/201611703013
NN2015

Shear viscosity to electric conductivity ratio of the QGP

A. Puglisi[1,2], S. Plumari[1,2] and V. Greco[1,2]

[1] Department of Physics and Astronomy, University of Catania, Via S. Sofia 64, I-95125 Catania, Italy
[2] Laboratorio Nazionale del Sud, INFN-LNS, Via S. Sofia 63, I-95125 Catania, Italy

Abstract

The transport coefficients of strongly interacting matter are currently subject of intense theoretical and phenomenological studies due to their relevance for the characterization of the quark-gluon plasma produced in ultra-relativistic heavy-ion collisions (uRHIC). We predict that $(\eta/s)/(\sigma_{el}/T)$, independently on the running coupling $\alpha_s(T)$, should increase up to about ~ 20 for $T \to T_c$, while it goes down to a nearly flat behavior around $\simeq 4$ for $T \geq 4\,T_c$. Therefore we find a stronger T-dependence of σ_{el}/T with respect to η/s that in a quasiparticle approach is constrained by lQCD thermodynamics. A conformal theory, instead, predicts a similar T dependence of η/s and σ_{el}/T.

1 Introduction

Relativistic Heavy Ion Collider (RHIC) at BNL and Large Hadron Collider (LHC) at CERN have produced a very hot and dense system of strongly interacting particles as in the early universe with energy densities and temperatures largely above the transition temperature $T_c \simeq 160$MeV [1] expected for the transition from nuclear matter to the Quark-Gluon Plasma (QGP) [2]. The phenomenological studies by viscous hydrodynamics [3, 4] and parton transport [5–7] of the collective behavior of such a matter has shown that the QGP has a very small shear viscosity to entropy density ratio η/s, quite close to the conjectured lower-bound limit $\eta/s = 1/4\pi$ [8].

Another key transport coefficient of interest is the electric conductivity σ_{el} which represents the linear response of the system to an applied external electric field. Several processes occurring in uRHIC as well as in the Early Universe are regulated by the electric conductivity. Very high electric and magnetic fields ($eE \simeq eB \simeq m_\pi^2$, with m_π the pion mass) are expected to be produced in the very early stage of the collisions [9, 10]. First preliminary studies in lQCD has extracted only few estimates with large uncertainties [11,12] and only recently more safe extrapolation has been developed [13–15]. In this work we emphasize the connection with the η/s [16].

2 Shear viscosity and electric conductivity

In this Section we report the general formulas for shear viscosity and electric conductivity. For a system with different species, shear viscosity can be written as [17]:

$$\frac{\eta}{s} = \frac{1}{15Ts}\left\langle \frac{p^4}{E^2} \right\rangle \left(\tau_q \rho_q^{tot} + \tau_g \rho_g\right) \tag{1}$$

where T is the temperature, $\rho_{q(g)}$ the quark (gluon) density, $\tau_{q(g)}$ relaxation time and $\langle \cdots \rangle$ the thermale average, being E the energy and p the momentum of particles.

Electric conductivity can be written as [18]:

$$\sigma_{el} = \frac{e^2}{3T}\left\langle \frac{\vec{p}^{\,2}}{E^2} \right\rangle \sum_{j=q,\bar{q}} f_j^2\, \tau_j \rho_j = \frac{e_\star^2}{3T}\left\langle \frac{\vec{p}^{\,2}}{E^2} \right\rangle \tau_q \rho_q \tag{2}$$

where $e_\star^2 = e^2 \sum_{j=u,d,s}^{\bar{u},\bar{d},\bar{s}} f_j^2 = 4e^2/3$ with f_j the fractional quark charge.
The thermal average $\langle p^4/E^2 \rangle$ and $\langle p^2/E^2 \rangle$ will be fixed employing a quasi-particle (QP) scheme tuned to reproduce the bulk thermodynamics evaluated by lQCD [19]. This means that thermodynamical terms in Eqs. (1)-(2) are determined by the Lattice QCD thermodynamics and do not rely on the detailed value of $m_{q,g}(T)$ in the QP model. The quark and gluon masses are given by $m_g^2 = 3/4\, g^2 T^2$ and $m_q^2 = 1/3\, g^2 T^2$ in terms of a running coupling $g(T)$ that is determined by a fit to the lattice energy density. In Ref. [19] we have obtained: $g^2_{QP}(T) = 48\pi^2/\,(11N_c - 2N_f)\ln[\lambda \left(\frac{T}{T_c} - \frac{T_s}{T_c}\right)]^2$ with $\lambda = 2.6$, $T_s/T_c = 0.57$, $T_c = 160\,\mathrm{MeV}$. We also notice that a self-consistent dynamical model has been developed in [6,20] and leads to nearly

the same behavior of the strong coupling $g(T)$. For its general interest and asymptotic validity for $T \to \infty$, we also consider the behavior of the pQCD running coupling constant: $g_{pQCD}(T) = \frac{8\pi^2}{9} \ln^{-1}\left(\frac{2\pi T}{\Lambda_{QCD}}\right)$.

The last term to be defined and fixed in Eqs. (1)-(2) are the relaxation times for quarks and gluons:

$$
\begin{aligned}
\tau_q^{-1} &= \sum_{i=q,\bar{q},g} \langle \rho_i v_{rel}^{iq} \sigma_{tr}^{iq} \rangle = \langle \sigma(s)_{tr} v_{rel} \rangle (\rho_q \sum_{i=u,d,s}^{\bar{u},\bar{d},\bar{s}} \beta^{qi} + \rho_g \beta^{qg}) \\
\tau_g^{-1} &= \sum_{i=q,\bar{q},g} \langle \rho_i v_{rel}^{ig} \sigma_{tr}^{ig} \rangle = \langle \sigma(s)_{tr} v_{rel} \rangle \left(\rho_q^{tot} \beta^{qg} + \rho_g \beta^{gg} \right)
\end{aligned}
\tag{3}
$$

where σ_{tr}^{ij} is the transport cross-section, v_{rel}^{ij} is the relative velocity of the two scattering particles. As done within the Hard-Thermal-Loop (HTL) approach, we will consider the total transport cross section regulated by a screening Debye mass $m_D = g(T)T$: $\sigma_{tr}^{ij}(s) = \int \frac{d\sigma}{dt} \sin^2\Theta \, dt = \beta^{ij} \frac{\pi \alpha_s^2}{m_D^2} \frac{s}{s+m_D^2} h(a)$ where $\alpha_s = g^2/4\pi$, $h(a)$ regulates the anisotropy of scatterings (for more details see Ref. [16,21,22]). The coefficient β^{ij} depends on the pair of interacting particles: $\beta^{qq} = 16/9$, $\beta^{qq'} = 8/9$, $\beta^{qg} = 2$, $\beta^{gg} = 9$ which are directly related to the quark and gluon Casimir factor, for example $\beta^{qq}/\beta^{gg} = (C_F/C_A)^2 = (4/9)^2$. It is not obvious that relaxation times are those evaluated with the same coupling $g(T)$ from the QP model. We fix $\tau_{q,g}$ in order to reproduce the minimum $\eta/s = 1/4\pi$. In Fig. 1 we show η/s as a function of T/T_c: red thick line is obtained using g_{QP} rescaled to the minimum $\eta/s = 1/4\pi$, blue dot-dashed line using g_{pQCD} and symbols several lQCD results (open and full circles [23], full squares [24], diamonds and triangles [25]).

We want to stress that the σ_{el}/T predicted, with the same τ_q that reproduces $\eta/s = 1/4\pi$, is in quite good agreement with most of the lQCD data, shown by symbols in Fig. 2 (grey squares [11], triangles [14], circle [15], yellow diamond [12], orange square [26] and red diamonds [13]). Therefore a low σ_{el}/T is obtained at variance with the early lQCD estimate, Ref. [11]. In Fig.2, we also plot the $\mathcal{N} = 4$ Super Yang Mills (green dotted line) electric conductivity [27] that predicts a constant behavior for $\sigma_{el}/T = e^2 N_c^2/(16\pi) \simeq 0.0164$. We note that in our framework one instead expects that the σ_{el} should still have a strong T-dependence: using simple considerations one can obtain $\sigma_{el}/T \simeq \eta/s \, T/m(T)$ [16] which shows that, even if η/s is constant, σ_{el}/T has an extra T-dependence that generates the steep decrease close to T_c.

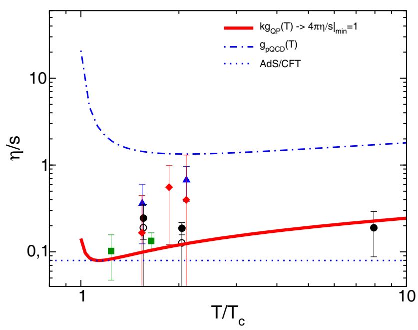

Figure 1: Shear Viscosity as a function of temperature T/T_c. See the text for details.

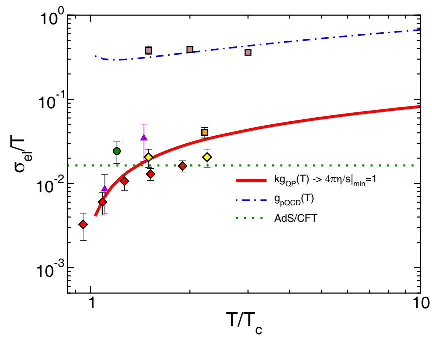

Figure 2: Electric conductivity as a function of temperature T/T_c. See the text for details.

An interesting quantity under our investigation is to consider the ratio between η/s and σ_{el}/T which can be written as:

$$\frac{\eta/s}{\sigma_{el}/T} = \frac{6}{5}\frac{1}{T s\, e_\star^2}\frac{\langle p^4/E^2\rangle}{\langle p^2/E^2\rangle}\left(1+\frac{\tau_g}{\tau_q}\frac{\rho_g}{\rho_q^{tot}}\right), \qquad \frac{\tau_g}{\tau_q} = \frac{C^q+\frac{\rho_g}{\rho_q}}{6+\frac{\rho_g}{\rho_q}C^g}. \tag{4}$$

The previous equation is written in terms of generic relaxation times and we note that the ratio τ_g/τ_q is proportional to the coefficient $C^q = (\beta^{qq}+\beta^{q\bar{q}}+2\beta^{q\bar{q}'}+2\beta^{qq'})/\beta^{qg}$ and $C^g = \beta^{gg}/\beta^{qg}$ which represent the relative magnitude between quark-(anti-)quark and gluon-gluon with respect to gluon-quark scatterings ($C^q|_{pQCD} = \frac{28}{9} \simeq 3.1$ and $C^g|_{pQCD} = \frac{9}{2}$).
In Fig. 3 we show $(\eta/s)/(\sigma_{el}/T)$ as a function of T/T_c: the red solid line is the the prediction for the ratio using $g_{QP}(T)$, but it is clear from the Eq. (4) that the ratio is independent on the running coupling itself (blue dashed line using g_{pQCD}). The ratio is sensitive only to the relative strength of the quark scatterings: as we can see comparing the orange curve (obtained increasing the quark scatterings $C^q = 10\,C^q_{pQCD}$) with the black curve (increasing the gluon scatterings $C^g = 10C^g_{pQCD}$). As $T \to T_c$ a steep increase is predicted that is essentially regulated by $\langle p^2/E^2\rangle$.

In this work we point out the direct relation between the shear viscosity η and the electric conductivity σ_{el}. In particular, we have discussed why most recent lQCD data [13–15] predicting an electric conductivity $\sigma_{el} \simeq 10^{-2}T$ (for $T < 2\,T_c$) , appears to be consistent with a fluid at the minimal conjectured viscosity $4\pi\eta/s \simeq 1$. The increasing behavior of σ_{el}/T as a function of temperature supports AdS/QCD predictions [28] (violet dot-dashed line in Fig. 3).

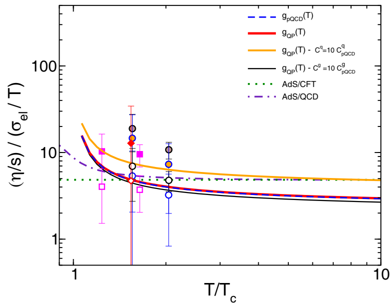

Figure 3: Ratio $(\eta/s)/(\sigma_{el}/T)$ as a function of temperature T/T_c. See the text for details.

We found the ratio $(\eta/s)/(\sigma_{el}/T)$ is independent of the uncertainties of the running coupling $g(T)$ and it is regulated by the relative strength and chemical composition of the QGP through the term $(1 + \tau_g\rho_g/\tau_q\rho_q^{tot})$. Our study provides a criterion to interpret the ratio and understand the relative role of quarks and gluons in the QGP thanks to the developments of lQCD techniques.

V.G. acknowledges the support of the ERC-StG Grant under the QGP-Dyn project.

References

[1] B. V. Jacak and B. Muller, *Science* **337**, 310 (2012).

[2] M. P. Lombardo, PoS LATTICE **2012** (2012) 016

[3] P. Romatschke and U. Romatschke, Phys.Rev.Lett. **99**, 172301 (2007).

[4] H. Niemi, G. S. Denicol, P. Huovinen, E. Molnar and D. H. Rischke, Phys. Rev. Lett. **106** (2011) 212302.

[5] Z. Xu, C. Greiner, and H. Stocker, Phys. Rev. Lett. **101**, 082302 (2008).

[6] W. Cassing and E. Bratkovskaya, Nucl. Phys. **A831**, 215 (2009).

[7] S. Plumari and V. Greco, AIP Conf. Proc. **1422**, 56 (2012).

[8] P. Kovtun, D. Son, and A. Starinets, Phys. Rev. Lett. **94**, 111601 (2005).

[9] K. Tuchin, *Adv.High Energy Phys.* 2013 (2013) 490495.

[10] Y. Hirono, M. Hongo, T. Hirano, *arXiv*:1211.1114.

[11] S. Gupta, *Phys. Lett.* B **597**, 57 (2007).

[12] G. Aarts, C. Allton, J. Foley, S. Hands, and S. Kim, *Phys. Rev. Lett.* **99**, 022002 (2007).

[13] A. Amato, G. Aarts, C. Allton, P. Giudice, S. Hands, and J. Skullerud, *Phys. Rev. Lett.* **111**, 172001 (2013).

[14] H.-T Ding, A. Francis, O. Kaczmarek, F. Karsch, E. Laermann et al., PoS 185 (2012).

[15] B. B. Brandt, A. Francis, H. B. Meyer, and H. Witting, PoS 186 (2012).

[16] A. Puglisi, S. Plumari and V. Greco, arXiv:1407.2559 [hep-ph].

[17] C. Sasaki and K. Redlich, *Phys. Rev. C* **79**, 055207 (2009).

[18] D. Fernandez-Fraile and A. Gomez Nicola, Phys. Rev. D **73** (2006) 045025.

[19] S. Plumari, W. M. Alberico, V. Greco and C. Ratti, Phys. Rev. **D84** (2011) 094004.

[20] R. Marty, E. Bratkovskaya, W. Cassing, J. Aichelin, and H. Berrehrah, Phys. Rev. C **88**, 045204 (2013).

[21] S. Plumari, A. Puglisi, F. Scardina and V. Greco, *Phys. Rev. C.* **86**, 054902 (2012).

[22] A. Puglisi, S. Plumari and V. Greco, Phys. Rev. D **90** (2014) 11, 114009

[23] S. Sakai and A. Nakamura, Lattice calculation of QGP viscosities: Present results and next project Proc. Sci. LAT2007(2007)221.

[24] H. B. Meyer, *Phys. Rev. D* **76** 101701 (2007).

[25] A. Nakamura and S. Sakai, *Phys. Rev. Lett.* **94** 072305 (2005).

[26] P. V. Buividovich, M. N. Chernodub, D. E. Kharzeev, T. Kalaydzhyan, E. V. Luschevskaya and M. I. Polikarpov, *Phys. Rev. Lett.* **105**, 132001 (2010)

[27] S. C. Huot, P. Kovtun, G. D. Moore, A. Starinets and L. G. Yaffe, *JHEP*12 (2006) 015.

[28] S. I. Finazzo and J. Noronha, *Phys.Rev. D* **89** 106008 (2014).