

Convective overshoot at the base of the Sun's convection zone*

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Summary. The non-local form of mixing length theory, as proposed by Shaviv and Salpeter (1973), is applied to the deeper part of the solar convection zone. The result is a layer of convective overshoot (with inward convective heat transport) which extends over a few tenths of a scale height, and has a boundary layer at its base where the temperature gradient suddenly changes from nearly adiabatic to the radiative value.

The overshoot layer is part of a (slightly) sub-adiabatic region, of approximately one scale height thickness. We argue that this region should be capable to store sufficient magnetic flux, in the form of thin toroidal flux tubes, to account for the flux observed at the solar surface.

Key words: convective overshoot

1. Introduction

For several reasons a layer of convective overshoot below the base of the solar convection zone seems to be interesting. One is the low lithium abundance of the Sun's atmosphere, which can be explained if convective mixing reaches down to a temperature of around $2.5 \cdot 10^6$ K where Li is destroyed by nuclear reactions (Böhm, 1963). A second reason, which we shall discuss in Sect. 3 below, is that the overshoot layer is (slightly) subadiabatic and therefore suited for storage of magnetic flux (van Ballegooyen, 1982). A third reason is that theoretical solar models so far have failed to reproduce exactly all eigenfrequencies of the observed p-mode oscillations (Christensen-Dalsgaard, 1984). A solar envelope which includes an overshoot layer possibly could help to resolve this problem (Ulrich and Rhodes, 1984). Finally, attempts to explain the Sun's differential rotation by numerical models often critically depend on the boundary conditions which must be satisfied at the bottom of the model convection zone (e.g. Moss and Vilhu, 1983; Pidotella et al., 1985). Ideally, the radiative interior, or at least its outer part, should be included into the model; convective overshoot should then generally be admitted.

We think that a fast and simple model of a solar convection zone, including an overshoot layer, might be a valuable tool in order to further discuss the above-mentioned problems. In order

to prepare such a model we start from the "Fast model of the Solar Convection Zone" by Belvedere et al. (1980), including however only the ionization of H and He. In the lower part of the convective layer we then replace the ordinary mixing-length theory by the non-local formalism which was applied by Shaviv and Salpeter (1973) to convective stellar cores. In the following section we shall briefly describe this formalism and present a few results of typical model calculations. The final section will contain a discussion of these results in the context of the magnetic flux storage problem.

2. The model

Our model essentially follows the recipe of Shaviv and Salpeter (1973), except that sinking rather than rising bubbles of solar gas will be considered. For a sinking bubble starting at height r_1 , the temperature excess $\delta T(r_2; r_1)$ and the downward velocity $v(r_2; r_1)$ at depth r_2 [$r_2 < r_1$] are formally given by the same expressions [(6) and (7) of Shaviv and Salpeter],

$$\delta T(r_2; r_1) = - \int_{r_1}^{r_2} \left[\frac{dT}{dr} - \left(\frac{dT}{dr} \right)_{ad} \right] dr \quad (1)$$

and

$$v^2(r_2; r_1) = 2 \int_{r_1}^{r_2} \frac{g(r)}{T(r)} \delta T(r; r_1) dr; \quad (2)$$

however, δT is now negative as long as the stratification is super-adiabatic (and even in part of the sub-adiabatic region). The convective heat flux in our case is

$$F_{conv}(r) = -f_d C_p \rho \cdot v(r; r+l) \cdot \delta T(r; r+l), \quad (3)$$

where C_p is the specific heat at constant pressure, ρ is the density, and l is the mixing length. According to Shaviv and Salpeter the unknown parameter f_d (f_u in their expression) should be $\lesssim \frac{1}{2}$ because sinking or rising bubbles should both occupy at most half of any given area. In our treatment we adjust f_d in such a way that the total heat flux is continuous at that level, r_c , within the convection zone where we switch from the local mixing-length formalism to the non-local description, (1) to (3). A typical value of f_d obtained in this way is 0.2, consistent with Shaviv and Salpeter's idea.

The condition that expression (3), plus the radiative flux, give the total solar flux, $L_\odot/4\pi r^2$, and the condition of hydrostatic equilibrium yield the stratification of temperature and pressure

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Table 1. Parameters of 3 solar overshoot models, as functions of the ratio, l/H , of mixing length to scale height. At depths $r_P, r_{\delta T}, r_v$ respectively, the superadiabaticity, the temperature excess of sinking bubbles, and the convection velocity become zero. The depth of the overshoot region is $d = r_{\delta T} - r_v$, and d/H is its ratio to the scale height at $r_{\delta T}$. Z is the total depth, i.e. $r_{\odot} - r_v$, b the thickness of the boundary layer, i.e. the distance of the maximum of F_{rad} from r_v , and f_d is the filling factor of expression (3). All distances are in 10^6 m

l/H	r_P	$r_{\delta T}$	r_v	d	d/H	Z	b	f_d
1.0	591	574	567	7.3	0.19	129	0.05	0.22
1.25	550	522	510	11.6	0.23	186	0.06	0.19
1.50	524	484	468	16.1	0.28	228	0.07	0.16

with depth. As in ordinary mixing length theory, such a calculation contains one free function, the mixing length l , which we specify, as usual, by its ratio, α , to the local pressure scale height, H . Since we do not calculate complete solar models, the parameter α is not adjusted by a calibration of the model's radius. Instead we compute solar envelope models for several values of α , see Table 1.

A more general approach to the problem of convective overshoot at the base of a convection zone is the "plume" model of Schmitt et al. (1984). These authors point out that in the case with no entrainment into the plumes their treatment essentially reduces to the recipe of Shaviv and Salpeter; detailed calculations of this limiting case are however not presented in their paper. We therefore think the present note might still be useful. A particular difficulty of the plumes is their diverging cross section at the depth where the vertical velocity becomes zero. The calculations therefore break down before the thin boundary layer at the bottom of the overshoot region is reached. Our model, with a constant filling factor f_d , does include this boundary layer.

Our simple model is quite capable to produce the essential result of more involved models, such as the plume model or the one of van Ballegoijen (1982): an overshoot layer which is slightly subadiabatic and whose thickness is a few tenths of a pressure scale height. No special assumptions concerning an entrainment function are necessary; and the formalism can easily be incorporated into stellar constitution codes, with $\alpha = l/H$ still the only free parameter.

In Fig. 1 we show the result for the case $\alpha = 1.25$. In this example the overshoot region begins near $r = 5.22 \cdot 10^8$ m, where the sinking parcels start to be warmer than the ambient gas. Convective transport therefore is downwards, and $F_{rad}/F_{tot} > 1$. Since F_{rad} is determined through the diffusion approximation, and since the opacity, κ , varies as $\rho T^{-3.5}$, and $\rho \sim p/T$, F_{rad} continues to rise $\sim p^{1.4}$ as long as the stratification is (almost) adiabatic (cf. Schmitt et al., 1984). As the maximum of F_{rad}/F_{tot} occurs very close to the lower boundary of the overshoot layer, we have a simple relationship between the thickness, d , of this layer and the maximum radiative flux:

$$F_{rad}/F_{tot} = \exp(1.4d/H)$$

For $d = 0.23H$ we obtain 1.38, in perfect agreement with the numerical result shown in Fig. 1. The maximum lies only 60 km above the level where $v = 0$, and most of the transition to $F_{rad} =$

F_{tot} occurs in a still narrower region. Thus the estimated upper bound of ≈ 500 km for the thickness of this region of Schmitt et al. is rather generous; this is a consequence of the neglect of the large term $d \ln F_{rad}/d \ln p$ in their Eq. (3.66).

Van Ballegoijen's (1982) boundary layer treatment seems to be more realistic: He finds a thickness of order Hv/c , where v is the convection velocity within the overshoot layer, and c is the sound velocity (we do not share Schmitt et al.'s criticism of this result since the important term containing S' is not neglected in Eq. (45) of van Ballegoijen; the sharp boundary layer may however be thickened by very small scale turbulence which is not considered here). Using $H = 5 \cdot 10^7$ m, $v = 50$ m/s, and $c = 10^5$ m/s as typical values, we find $Hv/c = 25$ km; an exact evaluation of van Ballegoijen's Eq. (49) yields a value about 3 times

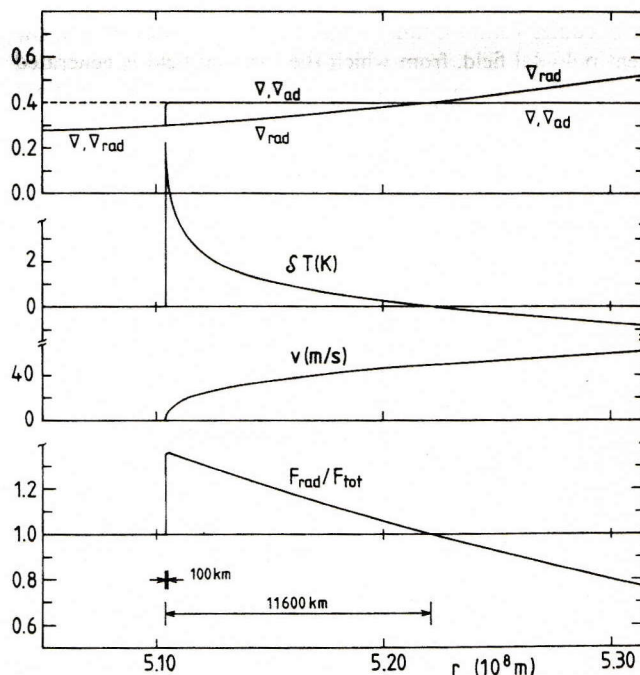


Fig. 1. Temperature gradients, $\nabla \cdot \nabla_{rad}$ and ∇_{rad} , temperature excess of convective "bubbles", convection velocities and the ratio, F_{rad}/F_{tot} , of the radiative and total energy fluxes, as functions of the distance, r , from the Sun's center. Case $l/H = \alpha = 1.25$

larger. This is in good agreement with our numerical results listed in Table 1.

In some of our calculations we found small spatial oscillations of the superadiabatic gradient, a possibility which was also mentioned by Shaviv and Salpeter. We think that these are caused by numerical inaccuracies occurring in the evaluation of the integrals (1) and (2), for which we used a simple trapezium rule. The point r_F where $\nabla - \nabla_{ad}$ changes its sign is not well-defined in such a case, and Table 1 contains an approximate value. The proper overshoot layer, between $r_{\delta T}$ and r_v , is, however, essentially unaffected by such oscillations, in particular the boundary layer where $|\nabla_{ad} - \nabla|$ is no longer small. By successive divisions of the step-width by 2 we cover this latter layer still by several points of integration. – We should finally mention that the entire stratification depends very little on the level r_c where the non-local description of convection sets in: In the case $\alpha = 1.25$, for example, $r_c = 6 \cdot 10^8$ m and $r_c = 6.25 \cdot 10^8$ m give $r_v = 5.10 \cdot 10^8$ m and $r_v = 5.09 \cdot 10^8$ m, respectively.

3. Storage of magnetic flux

Parker (1975) first suggested that the solar dynamo should operate in the deepest part of the Sun's convection zone, and we shall now discuss that the magnetic field, which is predominantly in the toroidal direction, is located in the overshoot region of our model. Several reasons suggest that such a field should exist in the very inhomogeneous form of concentrated flux tubes rather than in a smooth field geometry. Firstly, instabilities can destroy a smooth field (Gilman, 1970; Acheson, 1978; Schmitt and Rosner, 1983), although such instabilities are partly "killed off" (Hughes, 1984) when the thermal diffusivity and the kinematic viscosity assume eddy values. Secondly, the convective time scale at the base of the convective zone is of the order of 1 month; an originally homogeneous field would be expelled from the convective eddies within a time of the same order. And thirdly, the parent poloidal field, from which the toroidal field is generated by a non-uniform angular velocity, has a highly intermittent structure, at least at the solar surface. It is therefore most natural to assume that the toroidal field is equally structured.

For the storage of magnetic flux it is important to notice that not only the proper overshoot layer (where $F_{rad} > F_{\infty}$) may be at our disposal, but the whole sub-adiabatic region between r_F and r_v , covering almost one scale height. We must now estimate the number and strength of flux tubes which can possibly be stored in this region. According to Spruit and van Ballegoijen (1982), the field strength beyond which instability occurs depends on $|\nabla - \nabla_{ad}|$ by the criterion

$$\beta |\nabla - \nabla_{ad}| > \gamma^{-1}, \quad (4)$$

where γ is the ratio of specific heats, and $\beta = 2\mu p/B^2$. Figure 2 shows the result for the three cases $\alpha = 1, 1.25$ and 1.5 . In the case $\alpha = 1.25$, for example, a tube with diameter $5 \cdot 10^6$ m (i.e. about 10% of a scale height, so that the tube may still be considered "thin") could have a field of $5 \cdot 10^4$ gauss, and a total flux of about 10^{22} Mx. Ten such flux tubes, each having one excursion to the solar surface to form a bipolar region, would then provide the total magnetic flux of order 10^{23} Mx seen at the maximum of the activity cycle (Howard and LaBonte, 1981). As the flux-containing layer extends over, say, 25 degrees of latitude, there is enough room for the tubes to be isolated thin flux tubes, em-

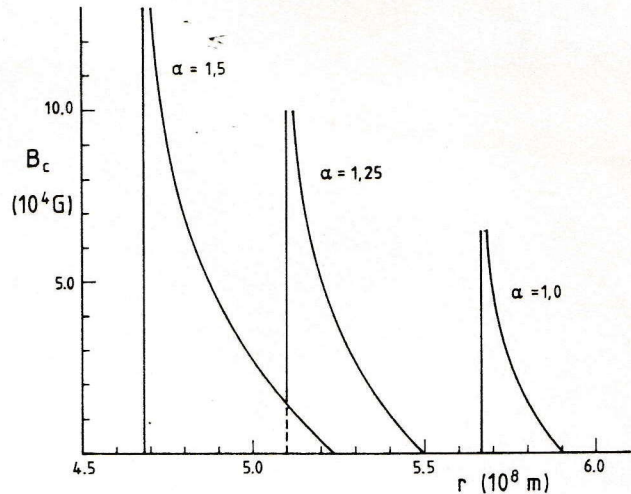


Fig. 2. The maximum field strength of flux tubes, as a function of depth, in the sub-adiabatic parts of three model convection zones, according to criterion (4)

bedded in essentially non-magnetic gas; the application of the stability analysis of Spruit and van Ballegoijen therefore appears justified.

A deeper subadiabatic region, such as our model with $\alpha = 1.5$, is even more capable of storing the necessary magnetic flux. Our main conclusion therefore is that the subadiabatic layer resulting from convective overshoot at the base of the Sun's convection zone is suited for storage of flux tubes of quite sufficient strength and number.

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References

- Acheson, D.J.: 1978, *Phil. Trans. Roy. Soc. London, A* **289**, 459
 Belvedere, G., Paternò, L., Roxburgh, I.W.: 1980, *Astron. Astrophys.* **91**, 356
 Böhm, K.-H.: 1963, *Astrophys. J.* **138**, 297
 Christensen-Dalsgaard, J.: 1984, *Theoretical Problems in Stellar Stability and Oscillations*, A. Noels and M. Gabriel (Eds.), Liège, p. 155
 Gilman, P.A.: 1970 *Astrophys. J.* **162**, 1019
 Howard, R., LaBonte, B.J.: 1981, *Solar Phys.* **74**, 131
 Hughes, D.W.: 1984, *The Hydromagnetics of the Sun*, Proc. 4th Europ. Meeting on Solar Phys., p. 55
 Moss, D., Vilhu, O.: 1983, *Astron. Astrophys.* **119**, 47
 Parker, E.N.: 1975, *Astrophys. J.* **198**, 205
 Pidotella, R.M., Stix, M., Belvedere, G., Paternò, L.: 1985, *Astron. Astrophys.* (in press)
 Schmitt, J.H.M.M., Rosner, R.: 1983, *Astrophys. J.* **265**, 901
 Schmitt, J.H.M.M., Rosner, R., Bohn, H.U.: 1984, *Astrophys. J.* **282**, 316
 Shaviv, G., Salpeter, E.E.: 1973, *Astrophys. J.* **184**, 191
 Spruit, H.C., van Ballegoijen, A.A.: 1982, *Astron. Astrophys.* **106**, 58
 Ulrich, R.K., Rhodes, E.H.: 1984, *Theoretical Problems in Stellar Stability and Oscillations*, A. Noels and M. Gabriel (Eds.), Liège, p. 250
 van Ballegoijen, A.A.: 1982, *Astron. Astrophys.* **113**, 99.