Baltic Astronomy, vol. 24, 263–275, 2015

ON THE PSEUDO PHASE-SPACE DENSITY OF DARK MATTER HALOES AND THE UNIVERSALITY OF DENSITY PROFILES

A. Del Popolo[1,2,3]

[1] *Dipartimento di Fisica e Astronomia, University of Catania, Viale Andrea Doria 6, 95125 Catania, Italy*

[2] *INFN sezione di Catania, Via S. Sofia 64, I-95123 Catania, Italy*

[3] *International Institute of Physics, Universidade Federal do Rio Grande do Norte, 59012-970 Natal, Brazil*

Received: 2015 February 23; accepted: 2015 April 30

Abstract. We examine the radial dependence of the pseudo phase-space density, $\rho(r)/\sigma^3(r)$, for structures on galactic and galaxy cluster scales. We find that it is approximately a power-law only in the range of halo radius resolved by current simulations (down to 0.1% of the virial radius) while it has a non-power law behavior below the quoted scale, with inner profiles changing with mass. The non-power-law behavior is more evident for halos constituted of both dark matter and baryons, while halos constituted just of dark matter are characterized by an approximately power-law behavior. The results argue against universality of the pseudo phase-space density and, as a consequence, argue against universality of density profiles constituted of dark matter and baryons as also discussed in Del Popolo (2009).

Key words: cosmology: theory – cosmology: large-scale structure of universe – cosmology: dark energy – galaxies: formation

1. INTRODUCTION

In the present cosmic concordance ΛCDM model, the universe is composed of two components, a non-relativistic component (baryons + cold dark matter, $\Omega_{\rm nr} = 0.274$), and a vacuum energy density ($\Omega_\Lambda = 0.726$) that fits accurately the current observational data (Komatsu et al. 2011; Allen et al. 2011; Del Popolo 2007; Del Popolo 2013, 2014a).

However, the ΛCDM model suffers from several problems, at small-scales, e.g., the cusp/core problem (Cardone & Del Popolo 2012; Del Popolo et al. 2013a; Saburova & Del Popolo 2014; Del Popolo & Hiotelis 2014), the missing satellite problem, the too-big-to-fail problem (Del Popolo & Gambera 1997; Del Popolo et al. 2014; Del Popolo & Le Delliou 2014), and on larger scales the cosmological constant problem (Weinberg 1989; Astashenok & Del Popolo 2012), and the cosmic coincidence problem.

An important issue in the ΛCDM model is to determine the total mass of virialized halos (Del Popolo & Gambera 1996; Del Popolo 2002; Hiotelis & Del

Popolo 2006, 2013) (galaxy and galaxy clusters), and their density profiles.

The last decade studies of dark matter (DM) halos have converged on a broad consensus about their equilibrium structure. While the work of Navarro et al. (1996, 1997) concluded that spherically-averaged density profile, $\rho(r)$, of equilibrium structure of DM halos is well approximated by a universal profile known as the Navarro-Frenk-White (NFW) profile, more recently, it has been shown that the functional form of the universal profile is better approximated by profiles whose logarithmic slope, $d\ln\rho/d\ln r \propto r^{\alpha}$, becomes increasingly shallower inwards (e.g., Navarro et al. 2004, 2010; Stadel et al. 2009). Moreover, there is a discussion whether the profile is actually universal or not (Moore et al. 1998; Subramanian 2000; Klypin et al. 2001; Ricotti 2003; Ricotti et al. 2007; Fukushige et al. 2004; Cen et al. 2004; Merrit et al. 2006; Graham et al. 2006; Gao et al. 2008; Del Popolo 2010, 2014b; Babyk et al. 2014). Regardless of which density profile functional form proves to best describe N-body halos, the underlying physics that drives the halos to have this shape is not yet fully understood.

In order to have more insights into the quoted problem, Taylor & Navarro (2001, hereafter TN01) and Hansen (2004) considered the radial profile of the space density of N-body halos. They measured the quantity $\rho(r)/\sigma^3(r)$[1], which has the dimensionality of phase-space density. In spherically-symmetric equilibrium halos, $\rho(r)/\sigma^3(r)$ is proportional to the coarse-grained phase-space density, a quantity distinct from the fine-grained phase-space density whose conservation is ensured by the collisionless Boltzman equation (e.g., Dehnen 2005). TN01 identified that the quantity $Q(r) = \rho/\sigma^3(r)$, which has become known as the pseudo phase-space density, behaves as a power law over 2–3 orders of magnitude in radius inside the virial radius. Other studies (e.g., Rasia et al. 2004; Ascasibar et al. 2004) have confirmed the scale-free nature of $Q(r)$, and their results indicate that its slope lies in the range $\alpha = 1.90 \pm 0.05$. This property is remarkable since neither the density $\rho(r)$ nor the velocity dispersion $\sigma(r)$ separately shows a power-law behavior. More recently, Ludlow et al. (2011) calculated that the $Q(r)$ of Einasto halos should be close to power laws over a wide range of radii. However, very close to the center, $Q(r)$ for values of α typical of CDM halos deviate significantly from a power law.

Despite the insights obtained in previous studies (Austin et al. 2005; Barnes et al. 2006; Hoffman et al. 2007), the origin for such universality of $Q(r)$ is not yet understood, and the question of how this quantity relates to the true coarse-grained phase-space density has been investigated by several authors (Hoffman et al. 2007; Vass et al. 2009a,b; Maciejewski et al. 2009; Sharma & Steinmetz 2006).

Moreover, some findings have called into question the universality of $\rho/\sigma(r)^3$. For instance, Schmidt et al. (2008) have shown that simulated halos are better fit by a different power-law relation, and Ma et al. (2009) found that $\rho/\sigma(r)^3$ is approximately a power law only over the limited range of halo radius resolvable by current simulations.

Furthermore, all the previous quoted analyses did not study the possible outcome produced by the presence of baryons on $\rho/\sigma(r)^3$, whose effect is to shallow (El-Zant et al. 2001, 2004; Romano-Diaz et al. 2008, 2009) and to steepen (Blumenthal et al. 1986; Gnedin et al. 2004; Klypin et al. 2002) the dark matter profile. In collisionless N-body simulations, this complicated interplay between baryons and dark matter is not taken into account, because it is very hard to

[1] $\sigma(r)$ is the one-dimensional radial velocity dispersion.

include the effects of baryons in the simulations.

In the present paper, we focus on the study of the phase-space density proxy ρ/σ^3 in the case of dark matter and baryons halos, by using the results of Del Popolo (2009).

The paper is organized as follows. In Section 2, we summarize the Del Popolo (2009) model. In Section 3, we discuss the results. Finally, Section 4 presents the conclusions.

2. SUMMARY OF THE METHOD

In the following, we summarize the method used in the present paper, which is fully described in Del Popolo (2009).

An often used model to study the non-linear evolution of perturbations of dark matter is the standard spherical collapse model (SSCM) introduced by Gunn & Gott (1972) and extended in subsequent papers (Ryden & Gunn 1987; Gurevich & Zybin 1988a,b; White & Zaritsky 1992; Sikivie et al. 1997; Williams et al. 2004; Del Popolo & Kroupa 2009; Cardone et al. 2011a,b; Del Popolo 2012a,b; Del Popolo et al. 2013b,c,d; Pace et al. 2014).

In the quoted model, the evolution of a spherical density perturbation, divided into shells, is evolved from the linear phase to the phase of maximum expansion (named turn-around), and then till the final collapse.

Knowing the initial comoving radius x_i, the mean fractional density excess inside the shell, $\bar{\delta}_i$, and the density parameter Ω_i, it is possible to obtain the final time averaged radius of a given Lagrangian shell (Peebles 1980). If mass is conserved and each shell is kept at its turn-around radius, one can easily obtain the shape of the density profile (Peebles 1980; Hoffman & Shaham 1985; White & Zaritsky 1992) at turn-around. Starting from this, the final density profile is obtained assuming that the potential well near the center varies adiabatically (Gunn 1977; Filmore & Goldreich 1984, hereafter FG84), what means that a shell near the center makes many oscillations before the potential changes significantly (Gunn 1977; FG84; Zaroubi & Hoffman 1993).

The model takes into account angular momentum, dynamical friction, and baryons adiabatic contraction. There are two sources of angular momentum of collisionless dark matter: (a) bulk streaming motions, and (b) random tangential motions. The first one (ordered angular momentum, see Ryden & Gunn 1987, hereafter RG87) arises due to tidal torques experienced by proto-halos. The second one (random angular momentum (RG87)) is connected to random velocities (see RG87). Dynamical friction was calculated dividing the gravitational field into an average component and a random component generated by the clumps constituting hierarchical universes. We took into account dynamical friction by introducing the dynamical friction force in the equation of motion (Eq. A14 in Del Popolo 2009). The shape of the central density profile is influenced by baryonic collapse: baryons drag dark matter in the so-called adiabatic contraction (AC) steepening the dark matter density slope. Blumenthal et al. (1986) described an iterative approximate analytical model to calculate the effects of AC, solved with iterative techniques (Spedicato et al. 2003). More recently, Gnedin et al. (2004) proposed a simple modification of the Blumenthal model, which describes numerical results more accurately. For systems in which angular momentum is exchanged between baryons and dark matter (e.g., through dynamical friction), Klypin et al. (2002)

introduced a modification to Blumenthal's model. The adiabatic contraction was taken into account by means of Gnedin et al. (2004) model and Klypin et al. (2002) model taking also account of exchange of angular momentum between baryons and dark matter.

In order to calculate $\rho(r)/\sigma^3(r)$, as Williams et al. (2004), we determine for different halos their density profiles, $\rho(r)$ and $\sigma(r)$, by means of Del Popolo (2009).

3. RESULTS AND DISCUSSIONS

As previously discussed in the introduction, several studies have found that $\rho(r)/\sigma^3(r)$ behaves as a power law over 2–3 orders of magnitude in radius inside the virial radius. This finding is unexpected because the density profile, $\rho(r)$, undergoes a considerable slope change in the same radial interval. The scale-free nature of the phase-space density implies that the double logarithmic slope of the velocity dispersion changes in such a way so as to offset the change in the density profile slope. A question that could be asked is the following: does the power-law nature found is a real characteristic of all equilibrium N-body halos, at galactic scales or galaxy cluster scales, or this kind of behavior has been observed since $\rho(r)/\sigma^3(r)$ has been studied in a limited range of halo radius and without taking into account the effect of baryons? In order to answer this question and seeing if the same behavior is valid for halos of different masses, galaxies or clusters of galaxies, we shall use the SSCM model summarized in the previous section and fully described in Del Popolo (2009) to calculate $\rho(r)/\sigma^3(r)$ in a wider radial range than that resolved by current simulations. Using Del Popolo (2009), we generate two sets of halos with galaxy and cluster scale masses. Within each of the two sets, we study three different cases: in the case A we take into account all the effects included in Del Popolo (2009), namely, angular momentum, dynamical friction, baryons, adiabatic contraction of dark matter; in the case B there are no baryons and dynamical friction; in the case C, only angular momentum is taken into account, reduced in magnitude as in Del Popolo (2009) in order to reproduce the angular momentum of N-body simulations (dashed line in Fig. 2 of Del Popolo 2009) and a NFW profile (solid histogram in Fig. 2 of Del Popolo 2009).

For what concerns the case C, we recall that in Del Popolo (2009) we performed an experiment similar to that performed by Williams et al. (2004), namely, we reduced the magnitude of the angular momentum, of a factor of 2, and that of the dynamical friction force, of a factor 2.5 with respect to the typical values calculated and used in the model in order to reproduce angular momentum of N-body simulations and the NFW profile.

3.1. *Pseudo-phase-space density for galaxies and clusters*

In Fig. 1a, we plot $\rho(r)/\sigma^3(r)$ with respect to radius, over 9 orders of magnitude, for a galaxy of $10^9 M_\odot$. The solid, dotted and long-dashed lines represent, respectively: (a) the slope in the case A; (b) the slope in the case B; (c) the slope in the case C. The short-dashed line represents the slope -1.9 found in several studies.[2] Fig. 1b shows a zoom-in view of the portion of the left figure that is resolvable by current simulations: $0.003 \leq /r_{-2} \leq 30$[3]. Fig. 1 illustrates that

[2] As reported in the introduction, some studies, e.g. Ascasibar et al. (2004), showed that the slope $\alpha = 1.90 \pm 0.05$, compatible with a value of $\alpha = -1.875$ from TN01.

[3] r_{-2} is the radius at which the logarithmic slope of the space density is –2.

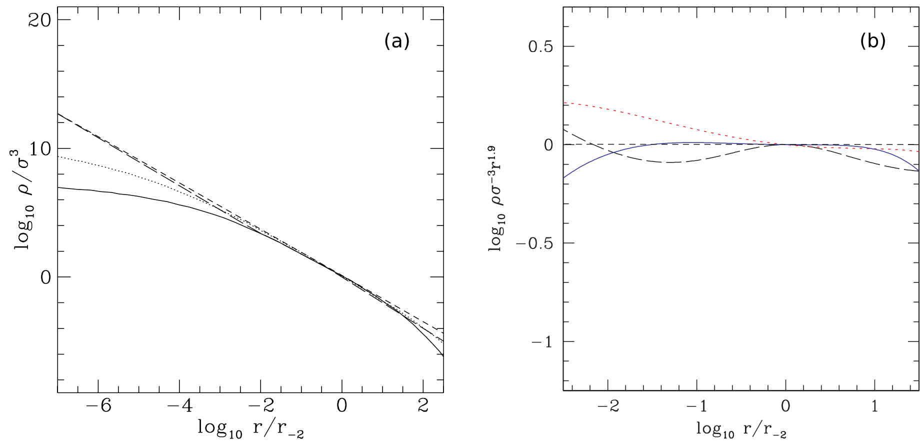

Fig. 1. Radial profiles of the pseudo-phase-space density for a $10^9 M_\odot$ galaxy. Panel (a): solid line, dotted line, and long-dashed line represent, respectively, the pseudo-phase-space density for the cases A, B, and C described in the paper . The short-dashed line represents the slope -1.9 found in several studies (see the introduction). Panel (b) shows a zoom-in view of the portion of the left-hand figure, multiplied by $r^{1.875}$, that is resolvable by current simulations. Symbols are as in panel (a).

$\rho(r)/\sigma^3(r)$ is not a power-law in r for both case A and case B. In the cases A and B, halos deviate strongly from a power-law at small radius. The zoom-in panel shows, however, that the slope of $\rho(r)/\sigma^3(r)$ happens to be quite close to -1.9 over the limited range, $r/r_{-2} \simeq 0.001$ to 10, that is resolvable by current simulations. The deviations are only starting to show up at the smallest radius, $r/r_{-2} \simeq 0.001$, near the simulation resolution limit. It is therefore not surprising that the power-law behavior of $\rho(r)/\sigma^3(r)$ continues to appear to be valid even though the latest simulations find the Einasto form to be a better fit for $\rho(r)$ than GNFW (generalized NFW). If for $r/r_{-2} \geq 0.001$ the $\rho(r)/\sigma^3(r)$ is more or less a power-law, the important point to note is that at radius smaller than ≤ 0.001, ($10^{-7} \leq r/r_{-2} \leq 0.001$), $\rho(r)/\sigma^3(r)$ deviates far away from a pure power law. For our cases A and B, the shape of $\rho(r)/\sigma^3(r)$ flattens continuously towards the halo center. This is not unexpected since the corresponding density profile, as shown in Del Popolo (2009), flattens going to the inner part of the halo, where it has a flat profile. As already reported, the density profile corresponding to the case C is characterized by the fact that we take into account only angular momentum reduced as in Del Popolo (2009) in order to reproduce a NFW profile with inner slope $\rho \propto r^{-1}$.

The corresponding $\rho(r)/\sigma^3(r)$ profile (long-dashed line in Fig. 1a) shows an approximatively power-law behavior with the slope $\rho(r)/\sigma^3(r) \propto r^{-1.9}$, in agreement with several of the results in the literature (e.g., TN01). It is worth noting, however, that even the NFW halo shows wiggles in the corresponding $\rho(r)/\sigma^3(r)$ profile; that is, NFW haloes have not an exact power-law $\rho(r)/\sigma^3(r)$.

At large values of radius, $r/r_{-2} \geq 1$, there is also a deviation of $\rho(r)/\sigma^3(r)$ from a power-law. The reason of this deviation can be explained by means of Jeans equation, as done in Williams et al. (2004), and it is due to the fact that at large radii the equilibrium condition is not well satisfied. This apparently leads to

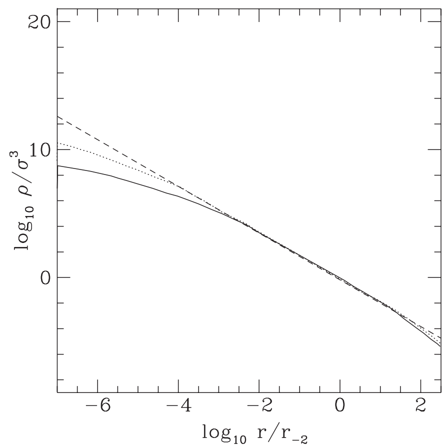

Fig. 2. Radial profiles of the pseudo-phase-space density for a $10^{14} M_{\odot}$ cluster of galaxies. Solid line, dotted line, and long-dashed line represent, respectively, the pseudo-phase-space density for the cases A, B, and C described in the paper.

a break in the scale-free behavior of $\rho(r)/\sigma^3(r)$ at large radii.

In Fig. 2, we plot $\rho(r)/\sigma^3(r)$ with respect to radius, in the case of a cluster of galaxies of $10^{14} M_{\odot}$. Here, the solid line, the dotted line, and the dashed line represent case A, case B, and case C. The behavior of the phase-space proxy is similar to the case of a galaxy. Also, in this case at small and large radii, $\rho(r)/\sigma^3(r)$ is not a power-law, but in a different radius range with respect to galaxies, namely, $10^{-7} \leq r/r_{-2} \leq 0.01$ and $r/r_{-2} \leq 10$.

Moreover, the $\rho(r)/\sigma^3(r)$ slope of both case A and case B, for small radii, is steeper than in the case of a galaxy, and the slope of curve relative to the NFW profile (dashed line) is slightly less steep, namely $\rho/\sigma^3 \propto r^{-0.8}$, in agreement with Williams et al. (2004) results.

In order to explain why the inner $\rho(r)/\sigma^3(r)$ profiles are not power-laws and why the slopes are steeper in the case of a cluster of galaxies, we have to recall how the density profiles, $\rho(r)$, are formed in Del Popolo (2009). The differences in the slopes with mass for the three cases (A, B, and C) plotted can be explained as follows. In case A, baryons, dynamical friction and angular momentum are present. The final density profile and final slope are determined by the interplay of these three factors.

Let us see how each of these factors acts in shaping the profile and how they interplay. The angular momentum sets the shape of the density profile at the inner regions. Particles with larger angular momenta are prevented from coming close to the halo's center and so contributing to the central density, which has the effect of flattening the density profile. The effects of dynamical friction are very similar to changing the magnitude of angular momentum (see Fig. 11 of Del Popolo 2009), with the final result of producing shallower profiles. Baryons have another effect, at an early redshift, the dark matter density experiences the adiabatic contraction by baryons producing a slightly more cuspy profile. This last is overcome from the previous two effects (angular momentum and dynamical friction). As shown by Fig. 11 of Del Popolo (2009), the magnitude of dynamical friction effect is a bit larger than that due to angular momentum and that these two effects add to improve the flattening of the profile.

The quoted effects act in a complicated interplay. Initially, at high redshift (e.g., $z = 50$), the density profile is in the linear regime. The profile evolves to the non-linear regime, and virializes. At an early redshift, (e.g. $z \simeq 5$, for dwarf galaxies), the dark matter density experiences the adiabatic contraction by baryons producing a slightly more cuspy profile. The evolution after virialization is produced by secondary infall, two-body relaxation, dynamical friction and angular momentum. Angular momentum, as described, contributes to reduce the inner slope of the density profiles by preventing particles from reaching halo's center, while dynamical friction dissipates the clumps orbital energy and deposits it in the dark matter with the final effect of erasing the cusp (similarly to El-Zant et al. 2001, 2004; Tonini et al. 2006; Romano-Diaz et al. 2008). The cusp is slowly eliminated and within $\simeq 1$ kpc a core forms, for objects of the mass of dwarf galaxies. It is now clear why, going from a model which takes into account baryons, dynamical friction, and angular momentum (solid line, case A) to one taking account just of angular momentum (dashed line, case B) and to one taking account just of angular momentum reduced to reobtain N-body simulations angular momentum, one obtains so different behavior of the inner density profile slopes.

The main reason of the difference in behavior of the profile slopes between galaxies and galaxy clusters is due to the fact that in the case of clusters the virialization process starts much later with respect to galaxies. In the case of galaxies, the profile strongly evolves after virialization through the processes described previously. In the case of dwarf galaxies of $10^9 M_\odot$, we have shown (Del Popolo 2009) that the profile virializes at $z \simeq 10$ and from this redshift to $z = 0$ its shape continues to evolve, except at $z \simeq 5$ when adiabatic contraction steepens the profile. In the case of a galaxy cluster of $10^{14} M_\odot$, the profile virializes at $z \simeq 0$ (Del Popolo 2009) and, as a consequence, the further evolution, observed in galaxies, cannot be observed in galaxy clusters. Summarizing, Figs. 1 and 2 show that $\rho(r)/\sigma^3(r)$ is not a power-law as shown in some previous studies reported in the introduction. Our results are in line with some recent works that have called into question the universality of $\rho/\sigma(r)^3$. For instance, Schmidt et al. (2008) have advocated that individual simulated haloes are better fit by a generalized power-law relation that is not necessarily $\rho/\sigma(r)^3$:

$$\frac{\rho}{\sigma_D^\epsilon} \propto r^{-\alpha}, \tag{1}$$

where $\sigma_D = \sigma_r \sqrt{1 + D\beta}$, and D parameterizes a generalized σ_D; for instance, $D = 0$ and -1 correspond to $\sigma_D = \sigma(r)$ and σ_t, respectively[4]. Schmidt et al. (2008) have shown that the best-fit values of (D, ϵ, α) differ from halo to halo, and, as a set, they roughly follow the linear relations $\epsilon = 0.97D + 3.15$ and $\alpha = 0.19D{+}1.94$. For $\sigma = \sigma(r)$ (i.e., $D = 0$), the optimal relation is $\rho/\sigma(r)^{3.15} \propto r^{-1.94}$, which is consistent with the reported behavior of $\rho/\sigma(r)^3$ in N-body simulations within error bars. Similarly, Ma et al. (2009) examined the radial dependence of $\rho/\sigma(r)^3$ over 12 orders of magnitude in radius by solving the Jeans equation for a broad range of input ρ and velocity anisotropy β and found that $\rho/\sigma(r)^3$ is

[4] Note that $\sigma(r)$ and σ_t are the one-dimensional radial and tangential velocity dispersions, and are the same used by Schmidt et al. (2008) through simulations.

approximately a power law only over the limited range of halo radius resolvable by current simulations (down to $\sim 0.1\%$ of the virial radius), and $\rho/\sigma(r)^3$ deviates significantly from a power-law below this scale for both the Einasto and NFW density profiles, $\rho(r)$.

3.2. Comparison of our DM density profiles and $\rho(r)/\sigma^3(r)$ with simulations

In order to show how our results for $\rho(r)/\sigma^3(r)$ are in agreement with high-resolution simulations, we compare them, in the radius range studied, with recent results of Ludlow et al. (2011), who calculated the pseudo phase-space density for the Einasto profile.

In Fig. 3, which plots the top panel of Fig. 2 of Ludlow et al. (2011), the mean profiles and one-sigma scatter of Q calculated by Ludlow et al. (2011) are shown as solid red lines with error bars. The dotted curve shows Bertschinger's $r^{-1.875}$ result, while the solid black line, almost indistinguishable from the dotted line, was calculated with the model of the present paper without taking into account baryons, in order to be able to compare the result for Q with those of dissipationless simulations (like those of Ludlow et al. 2011). Fig. 3 shows a very good agreement of the result of the present paper with those of Ludlow et al. (2011) in their studied radius range $\simeq 10^{-2}$–$10^{1.5}$. Note that χ in Fig. 3 is the exponent in

$$Q(r) = \frac{\rho}{\sigma^3} = \frac{\rho_0}{\sigma_0^3}(\frac{r}{r_0})^{-\chi} \tag{2}$$

which, as mentioned previously, was originally reported by TN01.

Fig. 4 shows (in blue) the density profiles for three different values of χ, and compares them to Einasto profiles. The values of α of the three Einasto profiles shown (in red) have been chosen, by Ludlow et al. (2011), to match as closely as possible the profiles corresponding to the pseudo phase-space density models. For $\alpha = 0.1$ and $\alpha = 0.17$ the corresponding pseudo phase-space density profiles are very well approximated by power laws over the whole radial range plotted.

Only for larger values of α, such as 0.3, clear deviations from a power law are noticeable. The thick solid curve in black shows the profile corresponding to the billion-particle Aq-A-1 halo, namely Navarro et al. (2010) highest resolution halo. The green lines, solid and dash-dot, marked DP in Fig. 4, plot the pseudo phase-space density profiles calculated with the model of the present paper for case B and case C, respectively. The green solid line shows that the flattening that we obtain at $r/r_{-2} = 10^{-3.7}$ (smallest value plotted by Ludlow et al. 2011 in their Fig. 5b) is close to the case $\alpha = 0.17$ (typical of ΛCDM models) plotted in Fig. 5b of Ludlow et al. (2011). The flattening in the quoted Ludlow et al. figure for $\alpha = 0.10$ at $r/r_{-2} = 10^{-3.7}$ is almost identical to that of the present paper for case C (green dash-dot line).

Fig. 4 shows a very good agreement of the result of the present paper with those of Ludlow et al. (2011), in their studied radius range $\simeq 10^{-3.7}$–$10^{1.5}$, for an Einasto profile with both $\alpha = 0.17$ and $\alpha = 0.10$. Ludlow et al. (2011) did not probe a pseudo-phase-space density profile at a smaller radius, as done in the present paper. At a radius of $\simeq 10^{-3.7}$, i.e., the minimum radius plotted by Ludlow et al., a small discrepancy from pure power-law behavior of the pseudo-phase-space density profile starts to be seen, and if one goes to a smaller radius (e.g., 10^{-9}, as in the present paper) the flattening should be larger. Moreover, as reported in the conclusion section of Ludlow et al. (2011), "significant differences

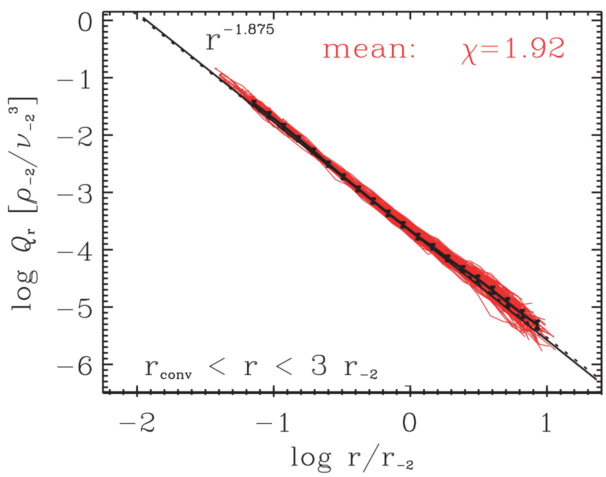

Fig. 3. Comparison of the pseudo-phase-space density profiles of Fig. 2 of Ludlow et al. (2011) with the result of the present paper (solid line).

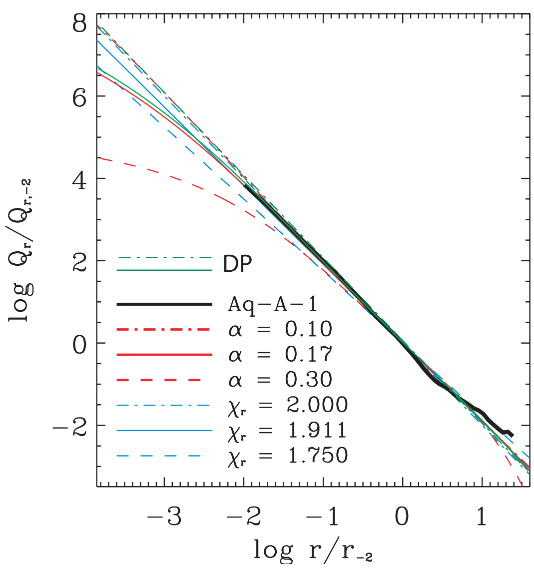

Fig. 4. Comparison of the pseudo-phase-space density profiles of Fig. 5b of Ludlow et al. (2011) with those obtained in the present paper. The result of the present paper (solid green line) for case B is in good agreement with the pseudo-phase-space density profile of Einasto with $\alpha = 0.17$ typical of ΛCDM models and the case $\alpha = 0.10$ is almost identical to that of the present paper for case C (green dash-dot line). DP stands for Del Popolo (i.e., the result of the present paper).

are only expected at radii well inside 1% of the scale radius, r_{-2}, and are therefore beyond the reach of current simulations."

3.3. Universality of the phase-space density profile

Another issue to discuss is the interrelation between the universality of the pseudo-phase-space density and that of the halos density profiles. As previously discussed, there are different methods to analyze the structure of dark matter halos. A standard approach involves investigating the halos density profiles. Few theoretical attempts have been made to understand the origin of this density profile (e.g., Salvador-Solé et al. 2007; Henriksen et al. 2007) with a varying level of success. The scale-free nature of $\rho(r)/\sigma^3(r)$ represents a novel way of looking at the properties of halos. If this property is "universal", it amounts to a hitherto unrecognized constraint on the shape of the density profiles (Austin et al. 2005). Our result confirms this point of view: the $\rho(r)/\sigma^3(r)$ profiles flattening towards the halo center are generated by similar density profiles, which have the logarithmic slope $\alpha \simeq 0$ for $10^8 - 10^9 M_\odot$ and $\alpha \simeq 0.6$ for $M \simeq 10^{14} M_\odot$ (Del Popolo 2009).

At the same time, our main result is that $\rho(r)/\sigma^3(r)$ is not universal if studied in the appropriate radius range, and, similarly, we expect that halos density profiles are not universal, because their inner part should depend on mass, as the $\rho(r)/\sigma^3(r)$ profiles, and should also flatten towards the halo center, showing flat cores in the center of the halo (as shown in Del Popolo 2009). This result is in agreement with several previous ones, described in the reminder of this paper.

Ricotti's (2003) N-body simulations suggested that the density profile of DM halos is not universal (in agreement with the other quoted studies), but that there are instead shallower cores in dwarf galaxies and steeper cores in clusters. This leads to the conclusion that density profiles do not have a universal shape (see also Subramanian et al. 2000; Cen et al. 2004; Graham et al. 2006; Merrit et al. 2005, 2006).

There are also observational evidences of a mass dependence of the dark matter density profile. Simon et al. (2003, 2004ab) removed the contribution of the stellar disk to the rotation curve of five galaxies in order to reveal the rotation curve of their dark matter halo. They found that the galaxies NGC 2976, NGC 6689, NGC 5949, NGC 4605, and NGC 5963 have very different values of the slope: $\alpha \simeq$ 0.01, 0.80, 0.88, 0.88, 1.28, respectively. By using the THINGS sample, de Blok et al. (2008) concluded that galaxies brighter than $M_B > -19$ have profiles that can be equally well described by cored or cuspy profiles, while those with $M_B < -19$ are best fitted by cuspy profiles.

In the case of clusters of galaxies, Host & Hansen (2009) took a sample of 11 highly relaxed clusters and used the measurements of the X-ray emitting gas to infer model-independent mass profiles. They then made comparisons with a number of different models that have been applied as mass profiles in the literature, concluding that there is a strong indication that this inner slope needs to be determined for each cluster individually. This implies that X-ray observations do not support the idea of a universal inner slope, but perhaps show a hint of a dependence with redshift or mass. Similar result comes from the studies of Sand et al. (2002, 2004, 2008) and Newman et al. (2013a,b), which studied the external parts of several clusters of galaxies through weak lensing, and the inner one through strong-lensing, and stellar dynamics.

Before concluding, I would like to mention that the equations in the present paper have similarities to those in the study of Sapar (2014), in which non-relativistic low-velocity massive neutrinos (or a generic weak-interacting particle) have cooled down to very low temperatures and velocities, so that they may affect the evolution of halos. Interestingly, in that scenario, cores are formed in the galactic center, instead of the cusps predicted by simulation.

4. CONCLUSIONS

In the present paper, we checked if the pseudo phase-space density, $\rho(r)/\sigma^3(r)$, behaves as a power law over 2–3 orders of magnitude in radius inside the virial radius by means of the model described in Del Popolo (2009). We find that $\rho(r)/\sigma^3(r)$ is not in general a power-law for the case A (dark matter and baryons) and case B (no baryons) described in the paper. In the radial range probed by current N-body simulations (down to 10^{-3} virial radii), $\rho(r)/\sigma^3(r)$ approximately behaves like a power-law, while for radial scales below the resolution of current simulations, there are significant deviations from a power-law profile. A similar, non power-law behavior is observed at large radii ($>$ 10 virial radii). In the paper, we also set the angular momentum and dynamical friction so that the density profile is approximately an NFW profile (case C). In this case, $\rho(r)/\sigma^3(r)$ is approximately a power law. The pseudo phase-space density was calculated for structures on galactic and cluster of galaxy mass scale. The behavior of $\rho(r)/\sigma^3(r)$ observed was similar, but in the case of galaxy clusters the slope was steeper in both case A and case B. This difference is connected to the different redshift at which the two classes of objects formed, higher for galaxies, lower for galaxy clusters, which implies a longer time at disposal of galaxies to evolve. The results of the quoted study are in agreement with those of Schmidt et al. (2008) and Ma et al. (2009). We conclude that radial profiles of the pseudo phase-space density corresponding to the density profiles which flatten going towards the halo center cannot be power-laws, and the prediction of N-body simulations of a power-law behavior in $\rho(r)/\sigma^3(r)$ is due just to the fact that the pseudo phase-space density is observed only down to a resolution limit of 10^{-3} virial radii.

The results argue against universality of the pseudo phase-space density and, as a consequence, argue against universality of density profiles constituted by dark matter and baryons as also discussed in Del Popolo (2009).

REFERENCES

Allen S. W., Evrard A. E., Mantz A. B. 2011, ARA&A, 49, 409

Ascasibar Y., Yepes G., Gottlober S. 2004, MNRAS 352, 1109A

Astashenok A. V., Del Popolo A. 2012, Class. Quant. Grav., 29, 5014

Austin C. G., Williams L. L. R., Barnes E. I. 2005, ApJ, 634, 756

Babyk Yu. V., Del Popolo A., Vavilova I. B. 2014, Astr. Rep., 58, 587

Barnes E. I., Williams L. L. R., Babul A., Dalcanton J. J. 2006, ApJ, 643, 797

Blumenthal G. R., Faber S. M., Flores R., Primack J. R. 1986, ApJ, 301, 27

Cardone V. F., Del Popolo A. 2012, MNRAS, 427, 3176

Cen R. Y., Dong F., Bode P., Ostriker J. P. 2004, astro-ph/0403352

Cardone V. F., Del Popolo A., Tortora C., Napolitano N. R. 2011a, MNRAS, 416, 1822

Cardone V. F., Leubner M. P., Del Popolo A. 2011b, MNRAS, 414, 2265

de Blok W. J. G., Walter F., Brinks E. et al. 2008, AJ, 136, 2648

Dehnen W. 2005, MNRAS, 360, 892

Del Popolo A. 2002, MNRAS, 336, 81

Del Popolo A. 2007, Astron. Rep., 51, 169
Del Popolo A. 2009, ApJ 698, 2093
Del Popolo A. 2010, MNRAS 408, 1808
Del Popolo A. 2012a, MNRAS, 419, 971
Del Popolo A. 2012b, MNRAS, 424, 38
Del Popolo A. 2013, AIP Conf. Proc., 1548, 2
Del Popolo A. 2014a, IJMPD, 23, 1430005
Del Popolo A. 2014b, JCAP, 07, 019
Del Popolo A., Gambera M. 1996, A&A, 308, 373
Del Popolo A., Gambera M. 1997, A&A, 321, 691
Del Popolo A., Hiotelis N. 2014, JCAP, 01, 047
Del Popolo A., Kroupa P. 2009, A&A, 502, 733
Del Popolo A., Le Delliou M. 2014, JCAP, 12, 051
Del Popolo A., Cardone V. F., Belvedere G. 2013a, MNRAS, 429, 1080
Del Popolo A., Pace F., Lima J. A. S. 2013b, MNRAS, 430, 628
Del Popolo A., Pace F., Lima J. A. S. 2013c, IJMPD, 22, 1350038
Del Popolo A., Pace F., Maydanyuk S. P. et al. 2013d, Phys. Rev. D, 87, 043527
Del Popolo A., Lima J. A. S., Fabris J. C., Rodrigues D. C. 2014, JCAP, 04, 021
El-Zant A., Hoffman Y., Primack J., Combes F., Shlosman I. 2004, ApJ, 607, 75
El-Zant A., Shlosman I., Hoffman Y. 2001, ApJ, 560, 636
Filmore J. A., Goldreich P. 1984, ApJ, 281, 1 (FG84)
Fukushige T., Kawai A., Makino J. 2004, ApJ, 606, 625
Gao L., Navarro J. F., Cole S. et al. 2008, MNRAS, 387, 536
Gnedin O. Y., Kravtsov A. V., Klypin A. A., Nagai D. 2004, ApJ, 616, 16
Graham A. W., Merritt D., Moore B. et al. 2006, AJ, 132, 2701
Gunn J. E. 1977, ApJ, 218, 592
Gunn J. E., Gott J. R. 1972, ApJ, 176, 1
Gurevich A. V., Zybin K. P. 1998a, Zh. Eksp. Teor. Fiz., 94, 3
Gurevich A. V., Zybin K. P. 1998b, Zh. Eksp. Teor. Fiz., 94, 5
Hansen S. H. 2004, MNRAS, 352, L41
Henriksen R. N. 2007, ApJ, 671, 1147
Hiotelis N., Del Popolo A. 2006, Ap&SS, 301, 67
Hiotelis N., Del Popolo A. 2013, MNRAS, 436, 163
Hoffman Y., Shaham J. 1985, ApJ, 297, 16
Hoffman Y., Romano-Diaz E., Shlosman I., Heller C. 2007, ApJ, 671, 1108
Host O., Hansen S. H. 2009, ApJ, 736, 52
Klypin A., Kravtsov A. V., Bullock J. S., Primack J. R. 2001, ApJ, 554, 903
Klypin A., Zhao H., Somerville R. S. 2002, ApJ, 573, 597
Komatsu E., Smith K. M., Dunkley J. et al. 2011, ApJS, 192, 18
Ludlow A. D., Navarro J. F., White S. D. M. et al. 2011, MNRAS, 415, 3895
Ma Ch.-P., Chang P., Zhang J. 2009, arXiv:0907.3144v1
Maciejewski M., Colombi S., Alard C. et al. 2009, MNRAS, 393, 703
Merrit D., Navarro J. F., Ludlow A., Jenkins A. 2005, ApJ, 624, L85
Merritt D., Graham A. W., Moore B. et al. 2006, AJ, 132, 2685
Moore B., Governato F., Quinn T. et al. 1998, ApJ, 499, L5
Navarro J. F., Frenk C. S., White S. D. M. 1996, ApJ, 462, 563

Navarro J. F., Frenk C. S., White S. D. M. 1997, ApJ, 490, 493
Navarro J. F., Hayashi E., Power C. et al. 2004, MNRAS, 349, 1039
Navarro J. F., Ludlow A., Springel V. et al. 2010, MNRAS, 402, 21
Newman A. B., Treu T., Ellis R. S., Sand D. J. 2013a, ApJ, 765, 25
Newman A. B., Treu T., Ellis R. S. et al. 2013b, ApJ, 765, 24
Pace F., Batista R. C., Del Popolo A. 2014, MNRAS, 445, 648
Peebles P. J. E. 1980, *The Large Scale Structure of the Universe*, Princeton University Press, Princeton, N. J.
Rasia E., Tormen G., Moscardini L. 2004, MNRAS, 351, 237
Ricotti M. 2003, MNRAS, 344, 1237
Ricotti M., Pontzen A., Viel M. 2007, ApJ, 663, 53
Romano-Diaz E., Shlosman I., Hoffman Y., Heller C. 2008, ApJ, 685, L105
Romano-Diaz E., Shlosman I., Heller C., Hoffman Y. 2009, ApJ, 702, 1250
Ryden B. S., Gunn J. E. 1987, ApJ, 318, 15 (RG87)
Saburova A., Del Popolo A. 2014, MNRAS, 445, 3512
Salvador-Sole E., Manrique A., Gonza'lez-Casado G., Hansen S. H. 2007, ApJ, 666, 181
Sand D. J., Treu T., Ellis R. S. 2002, ApJ, 574, L129
Sand D. J., Treu T., Smith G. P., Ellis R. S. 2004, ApJ, 604, 88
Sand D. J., Treu T., Ellis R. S. et al. 2008, ApJ, 674, 711
Sapar A. 2014, Baltic Astronomy, 23, 71
Schmidt K. B., Hansen S. H., Maccio A. V. 2008, ApJ, 689, L33
Sharma S., Steinmetz M. 2006, MNRAS, 373, 1293
Sikivie P., Tkachev I. I., Wang Y. 1997, Phys. Rev. D, 56, 1863
Simon J. D., Bolatto A. D., Leroy A., Blitz L. 2003, ApJ, 596, 957
Simon J. D., Bolatto A. D., Leroy A., Blitz L. 2004a, in *Satellites and Tidal Streams*, ASP Conf. Ser., 327, 18
Simon J. D., Bolatto A. D., Leroy A. et al. 2004b, ApJ, 621, 757
Spedicato E., Bodon E., Del Popolo A., Mahdavi-Amiri N. 2003, 4OR, Vol. 1, Issue 1, 51
Stadel J., Potter D., Moore B. et al. 2009, MNRAS, 398, 21
Subramanian K., Cen R., Ostriker J. P. 2000, ApJ, 538, 528
Taylor J. E., Navarro J. F. 2001, ApJ, 563, 483 (TN01)
Tonini C., Lapi A., Salucci P. 2006, ApJ, 649, 591
Vass I. M., Kazantzidis S., Valluri M., Kravtsov A. V. 2009a, ApJ, 698, 1813
Vass I. M., Valluri M., Kravtsov A. V., Katzandis S. 2009b, MNRAS, 395, 1225
Weinberg S. 1989, Reviews of Modern Physics, 61, 1
White S. D. M., Zaritsky D. 1992, ApJ, 394, 1
Williams L. L. R., Babul A., Dalcanton J. J. 2004, ApJ, 604, 18
Zaroubi S., Hoffman Y. 1993, ApJ, 416, 410