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Gray's constant and "swiss cheese" and "sea serpents" in stellar convection zones★

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Summary. Gray (1985), on the basis of Zeeman broadening measurements on a sample of G and K dwarfs found an interesting relation between the average magnetic field strength, B, and the areal coverage factor, A_0: the product A_0B is a constant independent of spectral type and rotational velocity.

Pidatella and Stix (1986) applied a non-local form of the mixing length theory to the lower part of the solar convection zone to estimate the size of the overshoot layer and computed the magnetic field strength beyond which thin toroidal flux tubes, located in the overshoot layer, become unstable.

In the present work we extend the calculations of Pidatella and Stix (1986) to a number of main sequence spectral types, ranging from F5 to K0, to point out the implications that Gray's constant, if it represents a real physical fact, may have on models describing the magnetic flux storage in the overshoot layer with filling factor f (the "swiss cheese" factor) and the subsequent emersion at the surface of flux tubes with n loops (the "sea serpent" factor) to form active regions. We find that the depth of the overshoot layer, its thickness and the marginal instability magnetic field intensity increase with advancing spectral type, while the product nf decreases.

A possible explanation of why the Sun does not fit Gray's law is also proposed.

Key words: stars: late type, structure, atmospheres, magnetic field – Sun: structure, magnetic fields

1. Introduction

Gray (1985), on the basis of measurements of Zeeman broadening made on a sample of G and K dwarfs (Robinson et al., 1980; Marcy, 1984; Gray, 1984) found an interesting relation between B, the average magnetic field strength, and A_0, the relative amplitude of the two symmetrically displaced components of the Zeeman triplet: the product A_0B is approximately a constant, independent of physical parameters such as spectral type and rotational velocity. The factor A_0, which represents the areal coverage factor, can indeed be expressed as:

$$A_0 = 0.5\alpha(1 + \cos^2\langle\psi\rangle)$$

where α is the fraction of the stellar surface covered with fields of average strength B and $\langle\psi\rangle$ is an unknown average inclination angle between the field lines and the line of sight (see Gray, 1984). Therefore, the total number of magnetic lines of force seen in the photosphere of a star of radius R, i.e. the total absolute flux through the stellar surface is expressed by:

$$\phi_s = 4\pi R^2\alpha B = \frac{4\pi R^2 A_0 B}{0.5(1 + \cos^2\langle\psi\rangle)} \tag{1}$$

Gray (1985), assuming $\langle\psi\rangle \sim 34°–45°$, finds a constant value 600 to 700 Gauss for the ratio $A_0B/(0.5(1 + \cos^2\langle\psi\rangle))$, and we shall take a mean value of 650 Gauss in deriving the numbers shown in the table below.

Pidatella and Stix (1986) applied the non-local convection formalism of Shaviv and Salpeter (1973) to the lower part of the Sun's convection zone to estimate the size of the overshoot layer in the radiative core. As is well known, this layer is slightly subadiabatic and may be considered as a stable region suited for storage of magnetic flux. They found that the overshoot layer has a thickness of a few tenths of a scale height, and computed the field strength beyond which thin toroidal flux tubes, located in such an overshoot layer, become unstable according to a criterion derived by Spruit and Van Ballegooijen (1982). In this way it was possible to argue that the overshoot layer can indeed store sufficient magnetic flux to account for the total flux observed at the Sun's surface (see also the approach of Schmitt et al., 1984).

In the present work we essentially reproduce the calculations of Pidatella and Stix (1986), for a number of main sequence spectral types ranging between F5 and K0, to point out the implications that the constancy of the product A_0B (Gray's constant), as shown by observations, may have on models describing the magnetic flux storage in the overshoot layer and the subsequent emersion at the stellar surface to form active regions.

2. The model

We write the total magnetic flux which can be stored in the stellar overshoot layer (OL) in the form:

$$\phi_{OL} = f dR_{\delta T} B_c \tag{2}$$

Here f is that fraction of the cross section, $dR_{\delta T}$, of the overshoot layer which is filled with fields of strength B_c, derived according to the above criterion. We call f the "swiss-cheese

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factor". $R_{\delta T}$ is the distance from the centre where the temperature excess δT of descending parcels of fluid changes sign, and is typically the radius of the overshoot layer shell. Here we assume that this same radius essentially determines the latitude extent of the layer – i.e. $R_{\delta T}/2$ on both sides of the equator. The thickness of the proper overshoot region is $d = R_{\delta T} - R_v$, where R_v is the distance from the centre where the fluid radial velocity v vanishes. It must also be mentioned that B_c depends on the depth within the overshoot layer. However, we take B_c at $r = R_{\delta T}$ as typical. For further details we refer to Pidatella and Stix (1986).

We now assume that each flux tube, which becomes unstable in the overshoot layer, forms n loops when rising toward the surface to generate active regions. We call n the "sea-serpent factor". Hence:

$$\phi_s = 2n\phi_{\rm OL} \tag{3}$$

where the factor 2 takes into account that each loops has two footpoints. Comparing (1), (2) and (3), we get:

$$nf \simeq 0.4 \frac{R^2}{dR_{\delta T}B_c} \tag{4}$$

where the value of Gray's constant $\simeq 650$ gauss has been substituted and B_c is to be taken in tesla (1 tesla = 10^4 gauss).

We repeated the numerical calculations of Pidatella and Stix (1986), based on the convection zone model of Belvedere et al. (1980) and on the non-local formalism of the mixing length theory of Shaviv and Salpeter (1973), for a number of different main sequence spectral types. Mass, luminosity and radius of these stars were taken from Allen (1973), and a ratio of mixing length to scale height of 1.25 was taken. The results are summarized in Table 1.

3. Discussion and conclusions

We note that the depth of the overshoot layer and its thickness increase with advancing spectral type (this being a consequence of the increase of the thickness of the convection zone). The threshold value of the magnetic field B_c, beyond which magnetic buoyancy instability arises, also increases, as a consequence of the marginal instability criterion (Spruit and Van Ballegooijen, 1982) $\beta|\nabla - \nabla_{\rm ad}| \simeq \gamma^{-1}$, where the symbols have the usual meaning and $\beta = 2\mu P/B_c^2$ (cf. Pidatella and Stix, 1986). The increase of B_c is essentially due to the higher value of the gas pressure P in the deeper layers of later type stars.

Table 1. Results obtained for four different spectral types and a ratio of mixing length to scale height of 1.25. R is the stellar radius, $R_{\delta T}$ is the distance from the centre where the temperature excess of sinking bubbles vanishes (the typical radius of the overshoot layer), $d = R_{\delta T} - R_v$ is the thickness of the overshoot layer (R_v being the distance from the centre where the convective velocity goes to zero), B_c is the field strength beyond which thin toroidal flux tubes located in the overshoot layer become unstable (taken at $r = R_{\delta T}$), and nf is the product of the "sea serpent" factor, n, (the number of loops formed by each flux tube when rising toward the surface to generate active regions) and the "swiss cheese" factor, f, (the filling factor of thin toroidal flux tubes of mean magnetic field strength B_c within the cross section of the overshoot layer). Gray's constant implies a decrease of the product nf with the advancing spectral type. Here lengths are in 10^6 m and B_c is in tesla (10^4 gauss)

Type	R	$R_{\delta T}$	d	B_c	nf
F5	837	825	2.0	0.8	212
G0	729	605	10.1	4.2	8.3
G5	650	448	12.5	9.0	3.4
K0	592	280	12.1	46.9	0.9

The main result is that Gray's constant, if it represents a real physical fact, implies, according to (4) and the non-local form of the mixing length theory, a strong decrease of the product nf with advancing spectral type. To be consistent with this, the physical structure of the convection zones of earlier/later spectral types has to be such that a larger/smaller relative number of thin toroidal flux tubes can be allocated in the overshoot layer with lower/higher values of the marginal magnetic field strength and/or a larger/smaller number of loops are formed in the flux tubes when rising toward the surface to erupt and generate active regions. Note that we cannot get separate values of f and n, the theory giving only their product. This is a rather negative result, since we cannot distinguish between the contributions of the two different physical effects (swiss cheese effect and sea serpent effect along the lower main sequence). Understanding the former is easier, on the basis of the overshoot layer properties discussed above, while it is more difficult to explain the latter, i.e. why the number of loops should increase in earlier stars. A suggestion might be that dynamo waves with smaller wave-number are preferentially generated in shallow convection zones, depending also on the rotation rate and the magnetic turbulent diffusivity.

Therefore we may not be led to conclude that Gray's constant is simply the surface manifestation of what occurs in the overshoot layers of stellar convection zones.

What about the Sun? Measurements of photospheric intermittent fields show values of about $1.5\,10^3$ gauss, whereas the top area coverage is about 2% of the surface. Thus, for the Sun, A_0B is about 20 times smaller than Gray's constant, a value clearly too small to fit the correlation shown in Gray's (1985) Fig. 1 and such as to put the Sun itself in the category of non-detection if it were at stellar distances.

According to Gray (1985) "it would be premature to conclude that the Sun, and by implication the other stars in the non-detection category, do not fit the correlation", essentially because of the uncertainties in measuring the complex pattern (north and south polarities mixed, e.g.) of photospheric fields.

However, a different explanation may be suggested: most of the stars for which A_0B has been measured fall in the category of younger and faster rotators, showing a larger Ca II emission flux, but no definite cycles (cf. Vaughan, 1980; Vaughan and Preston, 1980). This is also true for the stars for which Zeeman broadening was not detected, but, as Gray (1985) suggests, large rotation rate or macroturbulence is expected to increase the detection threshold. Therefore it seems to us that Gray's constant applies only to the more active class of stars and not to the slow rotators with low level cyclic activity such as the Sun.

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