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SIMPLE TOOLS FOR UNDERSTANDING THE LIMITATIONS OF ORBITAL ANGULAR MOMENTUM ANTENNAS

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Abstract

A new strategy is proposed for the understanding of the ultimate limitations of Orbital Angular Momentum antennas when applied to enhance the transmission data rate in a wireless communication system. The introduced tools rely on the Hankel transform properties and on the Singular Value Decomposition of the radiation operator at hand. The developed theory is supported by numerical results concerning scenarios and synthesis problems of actual interest.

Index Terms – Antenna theory, Orbital Angular Momentum.

I. INTRODUCTION

Due to the limited radio spectrum, one of the main challenges for modern wireless communication systems is that of enhancing the transmission data-rate. Amongst the novel techniques developed for utilizing the radio spectrum with maximum efficiency, great attention has been recently devoted to Orbital Angular Momentum (OAM) antennas [1]-[5]. Such systems have been in some cases proposed as means to improve almost "indefinitely" [1] the channel capacity in a link amongst two antennas.

Roughly speaking, the idea is to take profit from the fact that an antenna could simultaneously generate different fields each associated to a different amount of 'orbital momentum', i.e., to a different angular variation of the phase such as $e^{j\phi}$, $e^{2j\phi}$, and so on, ϕ denoting the azimuth angle in the observation domain. Then, by associating a different information to each of these patterns, one could eventually enlarge "at his will" [1] the channel capacity.

As the debate is still open (see for example [2]-[4]), we aim at furnishing in the following some additional points of view and tools on the subject. In particular, by considering the case of aperture antennas, we focus on the possibility to get an arbitrary multiplication of channels in the far-field zone. To this aim, we exploit in Sections II and III, respectively, some crucial properties of the Hankel transform and the Singular Value Decomposition (SVD) of the radiation operator relating the 'field vortices' to the corresponding sources.

II. A MATHEMATICAL SETTING IN TERMS OF HANKEL TRANSFORMS

As the far field of an aperture antenna is proportional to the Fourier transform of the aperture field [6], a convenient mathematical setting is furnished by the Hankel transform [7]. In fact, with respect to our specific problem, such a tool can be exploited as follows.

Let $f(\rho',\phi')$ denote the component of interest of the aperture field, ρ' and ϕ' respectively being the radial and angular variables spanning the aperture. By virtue of eq. (9.14) in [7], $f(\rho',\phi')$ can be expanded in a multipole series as:

$$f(\rho',\phi') = \sum_{m=-\infty}^{\infty} f_m(\rho') e^{jm\phi'} \quad with \quad f_m(\rho') = \frac{1}{2\pi}\int_0^{2\pi} f(\rho',\phi') e^{-jm\phi'} d\phi' \tag{1}$$

Then, by exploiting the theory in [7], if k and ϕ respectively denote the radial and azimuth coordinates in the observation domain then the Fourier transform of the source (1) can be written as follows:

$$F(k,\phi) = \sum_{m=-\infty}^{\infty} F_m(k) e^{jm\phi} \quad with \quad F_m(k) = \int_0^{\infty} f_m(\rho') J_m(k\rho')\rho' d\rho' = \mathrm{H}_m[f_m(\rho')] \tag{2}$$

wherein H_m and J_m respectively denote the m-order Hankel transform and the m-order Bessel function of first kind.

It is worth noting that, by denoting with θ the observation elevation angle with respect to boresight and introducing the spectral variables $u=\beta\sin\theta\cos\phi$ and $v=\beta\sin\theta\sin\phi$ (β denoting the wavenumber), it results $k=(u^2+v^2)^{1/2}=\beta\sin\theta$. Therefore, relations (1),(2) entail that:

i. an angular variation of order m of the source in terms of the ϕ' variable corresponds to an angular variation of order m of the far field in terms of the ϕ variable;

ii. for any fixed m order of angular variation, the function $f_m(\rho')$ univocally determines the corresponding function $F_m(k)$ and vice versa. Both the forward and backward relations are ruled by an Hankel transform of order m;

iii. since in (2) no terms having a variable amplitude are present in front of $F_m(k)e^{jm\phi}$, the Hankel transform relationship also determines the power pattern associated to each source component $f_m(\rho')e^{jm\phi'}$.

Point iii above has the following crucial consequence. Since Bessel functions J_m present a m-order zero around the origin [8], whatever the source at hand the corresponding far field will have a null in the boresight direction. Such a hole will have increasing size and depth with m. Under such circumstances, assuming a receiver positioned at the broadside direction is able to detect and understand the (weaker and weaker) signal associated to the m-order vortex, a huge price is paid. In fact, the majority of the power radiated by the transmitting antenna is moved out of the broadside direction, with two related consequences. First, the field level is uselessly large in a number of spatial directions.

Second, spectral resources are wasted, as a number of potential space channels are occupied by the toroidal patterns associated to the vortices. Numerical examples concerning such circumstances will be shown at the Conference.

III. The SVD Analysis of the Radiation Operator

In order to get an even deeper understanding of OAM antennas limitations, we analyzed the SVD of the radiation operator relating, through (2), the 'field vortices' to the corresponding sources. We show in Fig. 1(a) the singular values of a source of radius of 4λ (λ denoting the wavelength in free space) for different values of the *m*. In addition to the step-like behavior typical of compact operators and related to the degrees of freedom of scattered fields [9], it is easy to note that, as *m* increases, the singular values' decay begins for lower and lower values of the *n* index. Since the left-hand and right-hand singular functions of the radiation operator respectively represent a basis for the expansions of the source and the far field [10], and the singular values can be thought as scalar (gain) factors by which each source is multiplied to give the corresponding far field, distribution shown Fig. 1(a) has an important consequence: realizing OAM antennas without incurring into superdirective solutions becomes progressively harder as the requested order of angular variations increases.

By exploiting further tools which will be presented at the Conference, we used the SVD of the radiation operator also to evaluate the aperture power required to generate a far-field distribution guaranteeing at the same time a given *m*-order vortex and a fixed signal amplitude to a receiver positioned in the broadside direction. The results of this study, which are summarized in Fig. 1(b), entail that, whatever the transmitting and receiving antennas' size, producing a small increase of *m* requires a huge increase of the aperture power.

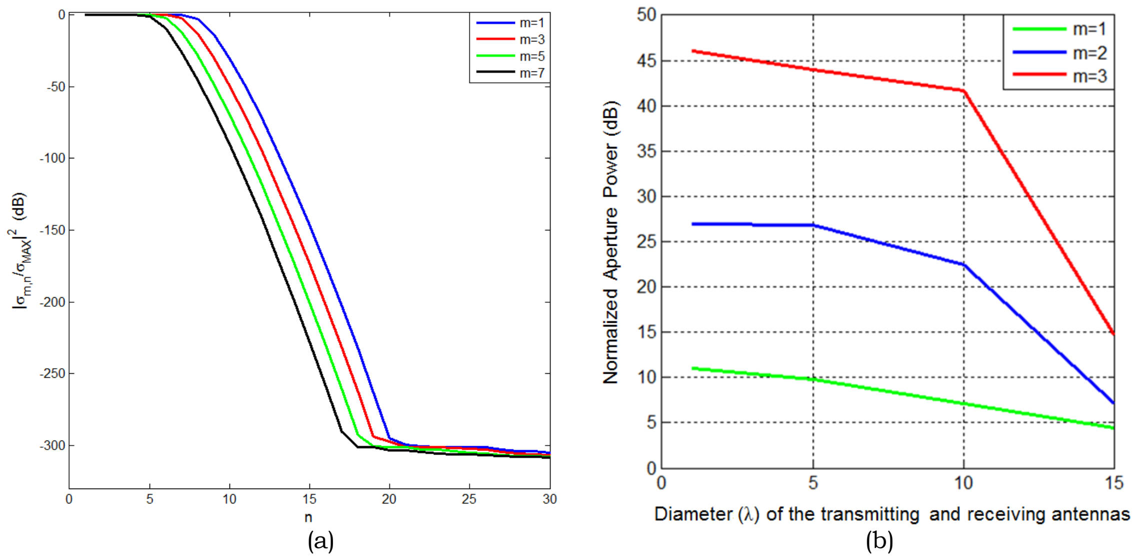

Fig. 1 – Singular values of the radiation operator (a) and outcomes of an ad-hoc synthesis problem aimed at minimizing the input power of a source guaranteeing a fixed level of signal to a receiver located at the Fraunhofer distance and having the same size of the transmitter (b).

IV. CONCLUSION

A deep analysis of OAM antennas' ultimate limitations has been provided with reference to their application to the enhancement of the transmission data rate in a wireless communication system. The provided results confirm the observations made in some notable recent contributions about the OAM antennas' disadvantages when used in long-range communications links.

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