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ITERATED VIRTUAL EXPERIMENTS AND COMPRESSIVE SENSING FOR QUANTITATIVE INVERSE SCATTERING PROBLEMS

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Abstract

In this contribution the solution of non linear inverse scattering problems is tackled by means of a novel iterative approach based on the emerging virtual scattering experiments' paradigm. The proposed method is also enhanced by using the compressive sensing (CS) paradigm, up to now adopted only in linear inverse scattering procedures.

Index Terms – Compressive Sensing, Distorted Born Iterative Method, Linear Sampling Method, Non-linear Inverse Scattering, Virtual Experiments.

I. INTRODUCTION

In inverse scattering problems one amounts to retrieve the location, shape and electromagnetic properties of unknown targets illuminated by means of known incident fields. Although they are relevant for a large number of imaging applications, the development of effective and reliable inversion procedures still represents an open issue, due to the non-linearity and ill-posedness of the problem [1,2]. As a countermeasure to non-linearity, the Born Approximation (BA) [3] could be very attractive, although it is safely applicable only in the weak scattering regime. A possible way to overcome these limitations is represented by the Distorted Born Iterative Method (DBIM) [4], which relies on iterated linear approximations of the scattering equations.

In this paper, in the same spirit of DBIM, an iterative method is proposed, wherein at each iteration the problem is linearized by means of a recently introduced field approximation based on the emerging framework of the virtual scattering experiments (VE) [5-8]. The resulting *Distorted Iterated Virtual Experiments* (DIVE) scheme is intrinsically different from DBIM and it is expected to outperform this latter because of the wider range of validity of the intermediate linearizations [6]. Moreover, as countermeasure to ill-posedness, the relevant paradigm of Compressive Sensing (CS) is exploited in order to obtain nearly optimal reconstruction of extended but piecewise homogeneous scatterers.

II. STATEMENT OF THE PROBLEM AND VIRTUAL EXPERIMENTS

Let us consider the canonical 2D TM scalar problem and an unknown nonmagnetic object embedded in a nonmagnetic homogeneous medium with complex permittivity ε_b. Let the unknown object with support Ω and complex permittivity ε_s. The scatterer is probed by means of some antennas located in the far-field of Ω on a closed curve Γ and the measurements of the resulting scattered fields are taken on Γ by adopting a multiview-mulstistatic configuration.

The equations describing the relevant scattering problem are:

$$E_s(\boldsymbol{r_m},\boldsymbol{r_t}) = k^2 \int_{\Omega} G_b(\boldsymbol{r_m},\boldsymbol{r'})\,\chi(\boldsymbol{r'})E(\boldsymbol{r'},\boldsymbol{r_t})d\boldsymbol{r'} = \mathcal{A}_e[\chi E], \qquad \boldsymbol{r_t},\boldsymbol{r_m} \in \Gamma \tag{1}$$

$$E(\boldsymbol{r},\boldsymbol{r_t}) = E_i(\boldsymbol{r},\boldsymbol{r_t}) + k^2 \int_{\Omega} G_b(\boldsymbol{r},\boldsymbol{r'})\,\chi(\boldsymbol{r'})E(\boldsymbol{r'},\boldsymbol{r_t})d\boldsymbol{r'} = E_i + \mathcal{A}_i[\chi E], \qquad \boldsymbol{r} \in \Omega, \boldsymbol{r_t} \in \Gamma \tag{2}$$

where E_i, E_s and E are the incident, scattered and total field, respectively, $k = \omega\sqrt{\mu_b\varepsilon_b}$ is the wavenumber in the host medium, $G_b(\boldsymbol{r},\boldsymbol{r'})$ is the Green's function pertaining to the background and $\mathcal{A}_e$ and $\mathcal{A}_i$ are a short notation for the integral radiation operators.

In the inverse scattering problem the aim is to estimate the unknown contrast function χ from measured scattered fields E_s. Due to the properties of the involved operator in (1), the problem is ill-posed [2] and, as the total field also depends on the unknown contrast, it is also non-linear. In the adopted formulation, multiple experiments have been considered to increase as much as possible the amount of independent information [8] and improve the performances of the inversion strategy. Once collected, this amount of independent information can be re-organized in a different way by taking advantage from the linearity of the scattering phenomenon. Starting from this simple linear 'transformation' of the scattering data, one can re-arrange the original experiments into virtual experiments (VE) [5-7] without requiring additional measurements.

III. DISTORTED ITERATED VIRTUAL EXPERIMENTS

The idea of transforming the original experiments into VE opens the way to a new iterative procedure in which the nonlinear inverse problem is solved through successive linearizations, in the same spirit of DBIM. Since the VE are conveniently derived from a smart processing of the scattering data (rather than on a priori assumption), the DIVE approach reaches better and better performance with respect to DBIM. The procedure can be summarized in five steps.

1) *Initialization*: a first estimate χ^1 of the contrast function is achieved by using the VE based linear approximation introduced in [5]. Obviously,

other more favorable starting guesses can be considered with respect to the a priori information.

2) *Scenario update*: the forward scattering problem pertaining to χ^k, the background medium at the *kth* iteration, is solved in order to update the total (background) field $E_b{}^k$ and compute the anomalous field $\Delta E_s{}^k$ (that is the difference between E_s and the field scattered when $\chi = \chi^k$). Then, the Green's function $G_b{}^k$ with respect to χ^k is numerically computed by exploiting the reciprocity theorem.

3) *Convergence control*: a stopping rule is considered by defining the relative residual error (RRE) at k^{th} iteration as $RRE^k = \|\Delta E_s{}^k\|_2 / \|E_s\|_2$. If RRE^k is less than a set threshold (10^{-5}) or is larger than the RRE^{k-1}, the procedure terminates. Otherwise, the iterative procedure continues.

4) *VE update*: in order to identify and localize possible perturbation and corrections $\Delta\chi^k$ with respect to the current reference scenario χ^k, we solve in each sampling point $\boldsymbol{r_s}$ the (distorted) linear sampling method (LSM) equation [9,10]:

$$\sum_{t=1}^{N} \Delta E_s{}^k(\boldsymbol{r_t},\boldsymbol{r_m})\alpha^k(\boldsymbol{r_t}\,,\boldsymbol{r_s}) = G_b{}^k(\boldsymbol{r_m},\boldsymbol{r_s}) \qquad (3)$$

wherein $\alpha^k(\boldsymbol{r_t}\,,\boldsymbol{r_s})$ are the sought auxiliary excitations coefficients which allow to define the support indicator of the perturbation and to select on it some *pivots points* $\boldsymbol{r_p}$, in order to build the new VE [5].

5) *Contrast update via linear inversion*: the relevant data equation (1) is applied to the case of partially known scenario and recast in terms of VE as:

$$\Delta\mathcal{E}_s{}^k(\boldsymbol{r_m},\boldsymbol{r_p}) = \int_{\Omega} G_b{}^k(\boldsymbol{r_m},\boldsymbol{r}')\,\Delta\chi^k(\boldsymbol{r}')\mathcal{E}^k(\boldsymbol{r}',\boldsymbol{r_p})d\boldsymbol{r}' \qquad (4)$$

where $\Delta\mathcal{E}_s{}^k$ and $\mathcal{E}^k$ are the anomalous and total fields arising in the VE, respectively. By considering the physical meaning of (regularized) eq.(3) [9,10], the unknown total field in eq.(4) can be approximated by:

$$\mathcal{E}^k(\boldsymbol{r},\boldsymbol{r}_p) = \mathcal{E}_b{}^k(\boldsymbol{r},\boldsymbol{r}_p) + LP\{G_b{}^k(\boldsymbol{r},\boldsymbol{r}_p)\} \qquad (5)$$

where $\mathcal{E}_b{}^k$ is the virtual background field obtained by recombining $E_b{}^k$ through the coefficients α^k, and the second addendum is a low pass filtered version of $G_b{}^k$. Note that approximation (5), unlike the DBA, is a "scatterer aware" approximation as it takes into account the contribution of the anomaly through $\mathcal{E}_b{}^k$ at each step.
In order to solve the linearized distorted problem (4), we use the CS theory as regularization technique and suppose to deal with extended

targets exhibiting piecewise constant dielectric profiles, which can be conveniently represented in terms of step functions [11]. Finally, a new profile is generated by adding the reconstruction to the current reference scenario, that is $\chi^{k+1} = \chi^{k} + \Delta\chi^{k}$.

6) *Return to step 2*. The iteration continues until the stopping criterion is fulfilled.

IV. CONCLUSION

A new iterative inversion scheme is introduced, in which the inverse scattering problem is solved through successive linearizations based on the virtual experiments framework. The new proposed method is able to outperform the usual DBIM and allows to exploit the Compressive Sensing paradigms for the solution of non linear inverse problem.

Further details on the approach together with numerical examples (with numerical and experimental data) will be presented at the Conference.

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