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Too much waste, not enough rationing: The failure of stochastic, competitive markets ☆

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Abstract

There are good reasons why sellers often post prices before the realization of demand shocks. We study whether equilibrium prices chosen *ex ante* coincide with the *ex-ante* prices that maximize expected aggregate surplus. The main result is that even in the competitive limit there is a divergence. Waste is excessive and entry decisions are distorted. The problem is that for competitive firms to sell in low-demand states involves a costly sacrifice of high-state revenue.

JEL classification: D61; D81; H23

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1. Introduction

Restaurants typically set prices before knowing how many customers turn up on a particular night. Sometimes they run out of specific dishes, other times they throw food away. Airlines, theatres and other sellers of perishable goods face similar dilemmas. Finding the profit-maximizing

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inventory for an individual firm facing stochastic demand is a classic problem in operations research. Known as the news vendor problem, it seems to have first been addressed by Edgeworth (1888).[1] This paper examines whether the competitive equilibrium of a market of this sort is socially efficient. Given that, for good reasons, price must be chosen before demand uncertainty is resolved, market failure is found to be pervasive.[2] Even in risk-neutral atomistic markets, the *ex-ante* prices that maximize expected welfare need not coincide with the equilibrium prices.[3] Moreover, the number of "active" firms in a competitive equilibrium may not be welfare maximizing. As demand is never entirely certain and price adjustment is rarely instantaneous, the systematic market failure identified here is potentially widespread.

We are not the first to look at this problem, but we find a different answer. Prescott (1975) provides an analysis of competitive equilibrium when sellers must set price in advance.[4] In this model, equilibrium is efficient. Carlton (1978), in a different model of advance pricing, draws the same conclusion. In both models, consumers negatively affected by demand shocks have their willingness to pay reduced to zero. Shocks lower willingness to pay in the models analysed here, but not necessarily to zero, giving rise to the difference in the conclusions. Since these two early studies, the welfare question of whether equilibrium *ex-ante* prices are socially optimal under stochastic demand does not seem to have been addressed, though work has continued on the nature of equilibrium.

Our setup involves firms with fixed capacity selling a homogeneous, perishable good to *ex-ante* identical consumers. Consumers know prices, can only visit one seller per period, make simultaneous choices where to shop knowing their own demand, but do not observe the aggregate demand state. As a benchmark, if there is no uncertainty and demand is continuous then, when the number of firms is sufficiently large, as shown in Section 5 of Peck (2018), these assumptions yield a socially efficient, market-clearing Nash equilibrium in prices, identical to the Walrasian equilibrium.[5] We show that the consequence of introducing aggregate demand shocks is that the equilibrium does not necessarily maximize aggregate surplus subject to price and entry being chosen prior to the realization of demand and to an unchanged matching process.[6]

Even when individual sellers constitute such a low fraction of total supply that they have negligible impact on the rest of the market, so are able to sell to as many buyers as they want as long as they deliver to each of them the equilibrium expected utility, this does not imply

[1] In the operations research literature, selling price is normally fixed.

[2] Fixing prices *ex ante* may be due to menu costs, rational inattention to judging the macro state, credibly preventing consumer hold-up, avoiding adverse behavioural responses from consumers (as in the snow-shovel case of Kahneman et al., 1986).

[3] Under risk neutrality, the appropriate welfare criterion is expected surplus. Risk aversion requires weighting of gains and losses. Typically, there is a missing risk market and therefore market failure is immediate.

[4] Bryant (1980) generalizes the equilibrium.

[5] The deterministic case with discrete demand is degenerate. Suppose that there is a mass of consumers with valuation £15 and another with valuation £10. The total mass of consumers exceeds market capacity, but capacity exceeds the mass of high-value types. If the smallest unit of currency is a penny, there is a two-price equilibrium with a high price of £10.01 and a low price of £10, with fractional waste at the high price to equalise profit. This equilibrium is arbitrarily close to the efficient equilibrium. Without a smallest currency unit, there is no efficient two-price equilibrium.

In a model in which firms can offer personalized prices, Burguet and Sákovics (2017) also find a market clearing equlibrium without the need to specify rationing rules.

[6] The model is formally a case of competitive/directed search with rival matching, but is distinct from most of that literature in incorporating demand shocks. It is the randomness that is the basis for the constrained inefficiency result, which contrasts with the efficiency typically found in competitive/directed search, such as Eeckhout and Kircher (2010), Proposition 8.

that sellers are price takers.[7] As noted by Carlton (1978), a firm charging more than its rivals loses customers, but this lowers the probability of being rationed, limiting the decline in sales. Deep price cuts may be necessary to utilize capacity in low-demand states. This is privately expensive in terms of lost good-state revenue. The potential gains to consumers from better capacity utilization in low-demand states are not captured by firms, so the price tends to be above the welfare maximizing level. Only when consumers are either unaffected by shocks or else disappear altogether is there no inefficiency. There is no reason for either a social planner or profit maximizing firms to be concerned with absent consumers, so the equilibrium does maximize welfare.

At issue is not just whether price is correct given market capacity, entry incentives are also distorted in a competitive equilibrium. There are opposing forces. Rationing implies the seller does not capture the full social value of their output in high-demand states whilst when demand is low, the social value of marginal capacity is zero, but revenue is nevertheless earned by entrants. Even if price is efficient given the number of sellers, entry will not normally be efficient.

The remainder of the paper is in five sections. Section 2 and 3 presents two simple models of rationing and waste with unit demand. In the first, shocks lower everyone's willingness to pay, but not to zero. Inefficiency arises. In the second model, shocks have heterogeneous effects. If affected consumers drop out of the market completely, there is efficiency, but when the effects are less drastic, equilibrium is inefficient. Section 4 discusses some extensions of the model. Section 5 places results in the context of the literature. Section 6 draws some brief conclusions.

2. Non-drastic shocks with homogeneous impact

The key to the welfare results is how shocks impact demand. One possibility is that affected consumers are not willing to buy at any price, as analyzed in the previous literature, such as by Prescott (1975), Carlton (1978), Deneckere and Peck (1995) and Dana (1999). We designate this a drastic shock. Alternatively, adverse shocks may lower willingness to pay, but not to zero.

To illustrate the novel results possible with non-drastic shocks, consider a one-period, risk-neutral economy with a continuum of identical consumers of measure M. Each consumer demands at most one unit of a good, which perishes at the end of the period. The valuation of the good depends on the macro state, good and bad. Willingness to pay is $v \in \{v^B, v^G\}$, with $v^G > v^B > 0$. The good state occurs with probability $P(v = v^G) = x$, and the bad state with $P(v = v^B) = 1 - x$. There are n firms, each with capacity K, in terms of the measure of consumers that can be served. Production cost is zero. Every firm sets its price prior to the realization of the macro state. We will denote the exogenous capacity per capita, nK/M, by θ, and provisionally assume that $\theta < 1$, so the measure of consumers exceeds the capacity to serve them. The model will subsequently be extended to endogenize entry of sellers.

The timing is: 1) sellers simultaneously post prices; 2) consumers observe prices and their own valuation (in this section, consumers are identically affected by the state, so can perfectly infer it from their own demand); 3) consumers can visit at most one seller knowing that, if there is excess demand, it is random which of them is served.

[7] The market cannot clear in all states if price must be set *ex ante*, so the concept of Walrasian equilibrium is not directly applicable. Instead, competition is again taken as Nash equilibrium with large numbers of firms. For example, Rothschild and Stiglitz (1976) title their seminal paper "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information", although the equilibrium is not Walrasian.

2.1. Equilibrium

As in Deneckere and Peck (1995), in deciding which seller to visit consumers mix with probabilities that equalize expected utility whichever seller is chosen. The continuum assumptions mean that the measure of consumers choosing a particular seller can be treated as determinate. A single-price, pure-strategy, symmetric, consumer equilibrium is initially sought.

Denote the expected surplus of a consumer visiting a seller charging price p as $E[S(p)]$. A seller charging p, when the other sellers charge the same price, has expected (gross) profit

$$E[\pi(p,p)] = \begin{cases} xpK, & \text{if } p \in (v^B, v^G]; \\ \pi(p,p) = pK, & \text{if } p \le v^B. \end{cases} \tag{1}$$

Since $\theta < 1$, if $p \le v^B$, every firm sells its entire capacity, K, in both states, whereas if $p \in (v^B, v^G]$, sales are only made in the good state. A deviant firm, charging p_d when the others charge p, obtains expected profit $E[\pi(p_d, p)]$ and its service rate (the probability that a visiting buyer obtains the good) will normally differ from the equilibrium service rate.

As a preliminary, we note that there cannot be an equilibrium at either $p \in (v^B, v^G)$ or $p < v^B$. If all sellers charge $p \in (v^B, v^G)$, sales are only made in the good state, and not all buyers are served. If a seller deviates to a slightly higher price, the probability of being rationed must be marginally lower than at other sellers to make the new offer acceptable. That is, the measure of customers visiting the deviant, denoted by m, smoothly declines in p_d but, as there is no demand jump, m still exceeds K. The deviant continues to sell out and its expected revenue rises, making the deviation to higher prices profitable. Similar reasoning rules out all sellers charging a price $p < v^B$.

The two possible equilibria are: 1) $p = v^G$; 2) $p = v^B$.

1) Equilibrium at $p = v^G$

A candidate equilibrium is that all sellers charge $p = v^G$. In this case, each firm's expected revenue is

$$E[\pi(p,p)] = E[\pi(v^G, v^G)] = xv^G K.$$

At $p = v^G$, consumers only wish to buy in the good state, with service rate $nK/M = \theta$, and obtain expected surplus

$$E[S(p)] = E[S(v^G)] = 0.$$

The best deviation price is $p_d = v^B$, the highest price at which sales are made in both states. Sales are then K in both states and revenue $E[\pi(p_d, p)] = \pi(v^B, v^G) = v^B K$. The deviation is unprofitable if the revenue at $p_d = v^B$ is lower than that at $p = v^G$, that is

$$\pi(v^B, v^G) = v^B K \le xv^G K = E[\pi(v^G, v^G)].$$

The condition for an equilibrium at $p = v^G$ is thus

$$v^B \le xv^G. \tag{2}$$

If the willingness to pay in the bad state is lower than the threshold in (2), firms are able to set a high price that more than offsets the losses of unused capacity in the bad state.

Proposition 1. *If $v^B \leq xv^G$, there is a single-price equilibrium at $p = v^G$.*

2) Equilibrium at $p = v^B$

A second candidate equilibrium is that all sellers charge $p = v^B$. At this price, each sells out regardless of the macro state, earning revenue

$$\pi(v^B, v^B) = v^B K.$$

For consumers, expected surplus is

$$E[S(v^B)] = x\theta(v^G - v^B),$$

where the equilibrium service rate in both states is θ, but no surplus is obtained in the bad state.

The optimal deviation is not to set $p_d = v^G$ as consumers would then obtain no surplus and would strictly prefer a seller charging $p = v^B$ despite not being sure to be served. Setting a deviation price that results in good-state rationing cannot be optimal since a higher price does not lose sales. A price that just eliminates rationing is potentially an optimal deviation but this is not immediate since setting a higher price still may not eliminate all deviant sales. As the market share of the deviant is not negligible, the service rate of the other sellers may decrease enough to compensate for the deviant's higher price. It will now be shown that it is less profitable to price such that the measure of consumers attracted is $m \in (0, K)$ than if it is set to yield $m = K$. Taking into account how deviation affects the service rates of other sellers, if the deviant attracts m visitors, at non-deviant sellers the service rate is $(n-1)K/(M-m)$. There is no rationing with $m < K$ so, to equalize surplus across sellers, the deviant's price must satisfy $E[S(p_d)] = E[S(v^B)]$, or

$$v^G - p_d = \frac{(n-1)K}{M-m}(v^G - v^B), \tag{3}$$

which yields

$$p_d = \frac{v^G(M-m) - (v^G - v^B)(n-1)K}{M-m}. \tag{4}$$

The deviant's expected revenue is thus $E[\pi(p_d, v^B)] = xp_d m$. Using (4),

$$\frac{dE[\pi(p_d, v^B)]}{dm} = \frac{(1+x)v^G(M-m)^2 + (v^G - v^B)(n-1)[(1+x)M - m]K}{(M-m)^2} > 0,$$

so the optimal deviation price, denoted by $\widetilde{p}$, is such that $m = K$ and equal to

$$\widetilde{p} = \frac{v^G M - [(v^G - v^B)n + v^B]K}{M-K}.$$

Finally, it must be determined whether a deviation from v^B to $\widetilde{p}$ is profitable. Raising the price to $\widetilde{p}$, and losing bad-state sales, lowers profit if

$$E[\pi(\widetilde{p}, v^B)] = x\widetilde{p}K \leq v^B K = \pi(v^B, v^B),$$

or

$$n \geq \frac{(xv^G - v^B)M}{x(v^G - v^B)K} + \frac{(1-x)v^B}{x(v^G - v^B)}. \tag{5}$$

For an equilibrium at $p = v^B$, the number of firms must be equal or exceed the threshold in (5).[8] When $v^B \leq xv^G$, that is the condition in (2) for an equilibrium at $p = v^G$ holds, the

[8] Multiple single-price equilibria suggests the possibility of a two-price equilibrium. Suppose that n_H of the n sellers shift to a price p_H, higher than v^B, and at which the service rate is unity. So, $n - n_H$ firms continue to sell at $p = v^B$, with service rate $(n - n_H)K/(M - n_H K)$. The high price, deriving from the equal-surplus condition at p_H and v^B, is

inequality in (5) is satisfied, so there will not necessarily be an equilibrium at $p = v^B$ also. When $v^B > xv^G$, that is an equilibrium at $p = v^G$ does not exist, the condition in (5) is always satisfied.[9]

Proposition 2. *If the condition in (5) holds, there is an equilibrium at $p = v^B$. This may coexist with an equilibrium at $p = v^G$. If there is not an equilibrium at $p = v^G$, there is always an equilibrium at $p = v^B$.*

2.2. Welfare

Expected per capita welfare at price p is the expected value of each consumer's surplus plus each firm's revenue, which can be written as

$$E[W(p)] = \begin{cases} \left[xv^G + (1-x)v^B\right]\theta, & \text{if } p \le v^B; \\ xv^G\theta, & \text{if } p \in (v^B, v^G]. \end{cases} \tag{6}$$

To find the welfare optimum, note that welfare is independent of price in each of the two ranges of (6). If price exceeds v^B, a reduction to v^B, or below, has no effect on the good-state surplus, but creates value of v^B per consumer in the bad state. Given capacity, the welfare maximizing price is therefore any $p \le v^B$. In an equilibrium with $p = v^G$, price is inefficiently high, entailing pointless bad-state waste.

2.3. Entry

So far, the number of sellers has been fixed at a level that results in equilibrium rationing and waste. Entry decisions will now be considered, endogenizing whether rationing and waste emerge in equilibrium and whether market forces generate the efficient number of sellers. The game is augmented so that, at the first stage, firms can choose to sink a capacity cost. Amongst the sellers that choose to enter, the second-stage pricing game is as already analyzed. A rationing/waste equilibrium to the whole game is possible if sellers differ in their capacity costs. Specifically, in a market in which there are N potential entrants, $C(n/N)$, increasing in n/N, is the cost of the n_{th} cheapest seller. The reason for this formulation is to allow a clear analysis of the effect of market size. A larger economy has more potential sellers, but the same proportion of them with costs below a given level.[10]

$$p_H = \frac{v^B(n-n_H)K + v^G(M-nK)}{M-n_HK},$$

which is increasing in the number of firms charging it. The expected revenue at p_H is $E[\pi(p_H, v^B)] = xp_HK$, which is also increasing in the number of high-price firms, as the difference of expected return at n_H+1 and at n_H is

$$\frac{x(v^G-v^B)(M-nK)K^2}{(M-n_H)[M-(n_H+1)K]} > 0.$$

This implies that whatever the division of sellers between high and low price a switch in one direction or the other would raise the switchers' expected revenue. Even if there is an integer number of high price firms at which revenue is equalized, it would not be an equilibrium. A two-price equilibrium is therefore precluded.

[9] The requirement in (5) can be rewritten as

$$(xv^G - v^B)\left(\frac{M}{nK} - 1\right) + \frac{v^B}{n}(1-x) \le (1-x)v^B.$$

Since $M > nK$, the condition holds when $v^B > xv^G$.

[10] The analysis in Section 2 could be regarded as the case in which $C(n/N) = 0$ if $n \le N$.

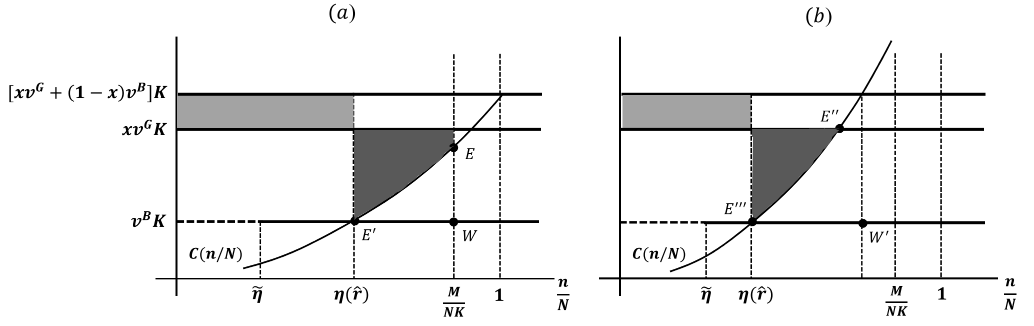

Fig. 1. Entry: (a) Low $C(n/N)$; (b) High $C(n/N)$.

Ignoring the integer constraint on n and N, when $v^B \leq xv^G$, there is an equilibrium at $p = v^G$, with $n = n_1$ entrants, if $E[\pi(v^G, v^G)] = xv^G K = C(n_1/N)$, that is, if the marginal entrant just covers its costs.[11] In addition, to be in the rationing and waste zone, we require $n_1 K < M$. Comparing the private return to the aggregate benefit of an extra entrant, we have that $E[\pi(v^G, v^G)] = xv^G K < [xv^G + (1-x)v^B]K$, so socially efficient capacity is in excess of the equilibrium n_1. Both price and entry are wrong.

There is a second equilibrium at $p = v^B$ with $n = n_2$ entrants if $\pi(v^B, v^B) = v^B K = C(n_2/N)$, and $n_2 K < M$. In addition, it must be unprofitable to deviate to a price above v^B. This is condition (5) which depends on the size of the market. In the spirit of Edgeworth (1888), market size can be represented by a scalar, $r \geq 1$, which multiplies up the number of sellers and the measure of customers. As, for example, in Vives (1988) or Yosha (1997), a large economy comprises r identical smaller economies. In an r-replica economy, the proportion of potential sellers that must enter for an equilibrium at $p = v^B$ must exceed a threshold,

$$\frac{n}{N} \geq \frac{(xv^G - v^B)M}{x(v^G - v^B)NK} + \frac{(1-x)v^B}{x(v^G - v^B)rN} \equiv \eta(r). \tag{7}$$

The larger the economy, the lower is the minimum fraction of sellers that must enter for an equilibrium at $p = v^B$ to be possible. As $r \to \infty$, the requirement tends to $n/N \geq (xv^G - v^B)M/x(v^G - v^B)NK \equiv \tilde{\eta}$. In addition to $xv^G < v^B$, for an equilibrium at $p = v^B$, and zero profit, it is necessary that the economy is sufficiently large. In such an equilibrium, as already noted, the number of sellers is inefficiently low.

Proposition 3. *A rationing/waste equilibrium with endogenous entry is never efficient, even subject to price and entry having to be chosen ex ante. A different price and/or number of sellers would yield higher expected aggregate surplus.*

Fig. 1 summarizes the entry equilibrium when $v^B \leq xv^G$. In panel (a), supply is sufficiently large that there is a no-rationing equilibrium at E, where $v^B K < C(n/N) < xv^G K$ and $n/N = M/NK$, or $nK = M$. Price is above v^B and there is waste in the bad state. This equilibrium is therefore inefficient.[12] There is also an equilibrium at E' with $p = v^B$ if $r \geq \hat{r}$. The equilibrium

[11] As gross returns do not increase in entry, the integer constraint implies that n_1 is the largest integer for which $xv^G K \geq C(n_1/N)$. This procedure follows Mankiw and Whinston (1986).

[12] Were supply sufficiently large that $C(n/N) \leq v^B K$, with $n/N = M/NK$, there would be an efficient equilibrium with price below v^B.

at E has the merit that output is efficiently priced, but there is less of it than at point E'. From a welfare perspective, the loss from an equilibrium at E is the non-consumption of output in the bad state, as in the equilibrium at E'. The expected value of the loss is shown by the light grey area to set against the expected surplus from extra good-state consumption shown by the dark grey area. It is ambiguous which equilibrium is preferable. The less elastic the cost curve, the more important it is to get price right. Panel (b) is identical to (a), except the supply curve is shifted to the left. Now there are equilibria at E'' and E'''. Both involve rationing, but only the equilibrium at E'' involves waste. Again, the two welfare areas are shown. If the light grey is greater, it is better to have the equilibrium with no waste.[13] Neither of the two equilibria maximize welfare subject to price and entry being chosen *ex ante*. That occurs at W and W' in the two configurations.

Proposition 4. *When there are two equilibria, it is ambiguous whether the one with less waste is welfare superior, but neither maximize welfare subject to ex-ante price and entry choices.*

3. Shocks with heterogeneous impact

The literature mostly assumes that consumers affected by a negative shock disappear (that is, will not consume at any price). As a result, the bad state differs from the good state only in that the market shrinks. In other words, shocks are drastic but heterogeneous. In this section, we present a model that enables a comparison of drastic and non-drastic shocks. Again, there are two macro states, good and bad, and each consumer has willingness to pay $v \in \{v^B, v^G\}$, with $v^G > v^B \geq 0$. In the good state, which occurs with probability $P(G) = x$, all consumers value the good at $v = v^G$. In the bad state, which occurs with $P(B) = 1 - x$, a fraction, $\phi \in (0, 1)$, of consumers value the good at $v = v^G$ and the rest at $v = v^B$. The number of sellers is given exogenously, with $\theta = nK/M < 1$.

The previous model of homogeneous shocks was initially solved for a fixed number of sellers irrespective of their market share. As it is well known that equilibrium is inefficient when firms have appreciable market power, of most interest is whether inefficiencies persist in atomistic markets. In larger markets, the effect of a deviation by one seller is spread over more sellers, so has a diminishing effect on their service rates. In a r-replica economy, as $r \to \infty$, it will be shown there is no effect at all. This is the classic concept of perfect competition where the actions of any individual firm has no effect on the rest of the market.

Two cases will be distinguished: in sub-section 3.1, $v^B = 0$; in sub-section 3.2, $v^B > 0$.

3.1. Drastic shocks

It is immediate that pricing is efficient. Pricing above v^G cannot be profitable, so everyone that has a positive value of the good in the bad state gets to consume it. In the good state, capacity is fully utilized and as all active consumers value the good equally, it does not matter which of them consume. This does not imply that entry is efficient. Extra capacity is only socially useful in the good state, but as competition for customers in the bad state drives the equilibrium price below its consumption value, an entrant captures less than the social value in the good state. Offsetting this, an entrant obtains a share of bad-state revenue even though the total number of

[13] It is easily confirmed with numerical examples that the two areas can be of this relative magnitude.

consumers served in this state is independent of its entry, so no extra social value is created. It therefore seems ambiguous whether the incentive to enter is excessive or insufficient.

In the drastic case, $v^B = 0$. In addition to $\theta < 1$, it is assumed that $nK/\phi M = \theta/\phi > 1$, so bad state capacity is more than enough to supply all active consumers (those with a willingness to pay $v = v^G$). Consumers only observe their own demand so must infer the probability of which state applies. The prior probability of the good state is x, the unconditional probability for each consumer of being active is $x + \phi(1-x)$ so, by Bayes' rule, a consumer assesses the probability of the good state, conditional on being active, as

$$P(G \mid active) = \frac{x}{x+\phi(1-x)} \equiv \xi.$$

The expected surplus of a customer of the p seller is

$$E[S(p)] = \xi\theta(v^G - p) + (1-\xi)(v^G - p),$$

where the service rate in the good state is θ, and in the bad state is unity.

If all sellers set price $p \le v^G$, there is rationing in the good state and unused capacity in the bad state. The expected (gross) profit of each seller is

$$E[\pi(p, p)] = xpK + \phi(1-x)p\frac{M}{n}.$$

Each seller sells out all capacity in the good state and $\phi M/n$ in the bad state (the measure of customers per seller, M/n, is naturally invariant to r-replications of the economy).

To test for an equilibrium in the rationing/waste zone at price p, consider whether a deviant can profit by setting price $p_d < p$. The deviant diverts bad-state sales from other sellers and continues to sell out in the good state. If, contrary to our assumption, buyers knew for sure that the bad state had occurred, even an infinitesimal price reduction would be enough to ensure that the deviant sells out in this state, with the demand jump making the deviation profitable. As active buyers do not know the state, but must infer probabilities from their own demand, this limits how many consumers switch in response to a lower price. There is a chance the state turns out to be good, in which case switchers will face increased rationing. As a result, there is no jump in deviant demand from a price cut, making equilibrium possible.

The deviant must attract more consumers than the p-sellers, and thus offer a lower service rate so as to equalize expected surplus. This requires that the measure of customers the deviant attracts in the good state (bad state), m (ϕm), satisfies $E[S(p_d)] = E[S(p)]$, or

$$\xi\frac{K}{m}(v^G - p_d) + (1-\xi)(v^G - p_d) = \xi\theta(v^G - p) + (1-\xi)(v^G - p). \tag{8}$$

The service rate in the bad state is 1 for all sellers. In the good state, the service rate at the deviant is K/m, while the service rate at non-deviants is given by $\lim_{r\to\infty}(rn-1)K/(rM-m) = nK/M = \theta$ (in the competitive limit, the effect of deviation on the service rate of the non-deviants can be ignored). From (8),

$$m = \frac{x(v^G - p_d)K}{\phi(1-x)(p_d - p) + x(v^G - p)\theta}, \tag{9}$$

which is decreasing in p_d. Using m from (9), the optimal deviation within the rationing/waste zone (with $m > K$ in the good state, and $\phi m < K$ in the bad state) solves

$$\max_{p_d} E[\pi(p_d, p)] = xp_dK + \phi(1-x)p_dm = x\left[1 + \frac{\phi(1-x)(v^G - p_d)}{\phi(1-x)(p_d - p) + x(v^G - p)\theta}\right]p_dK. \tag{10}$$

Differentiating (10) with respect to p_d, then setting $p = p_d$, the necessary condition for an equilibrium is

$$\left.\frac{dE[\pi(p_d,p)]}{dp_d}\right|_{p_d=p}=\frac{[\phi(1-x)+x\theta][x(v^G-p)\theta-\phi(1-x)pK}{x(v^G-p)\theta^2}=0.$$

Solving for p, the candidate equilibrium price is

$$p=\frac{xv^G\theta}{\phi(1-x)+x\theta}\equiv p_e,$$

where $p_e<v^G$. Substituting $p=p_e$ in (10),

$$E[\pi(p_d,p_e)]=xv^GK. \tag{11}$$

When the other firms charge p_e, the expected revenue of each seller is independent of its own price, as long as the deviation is in the interval that leads to rationing and waste. Outside this interval, deviations lower revenue. Reducing price from the level that just utilizes capacity in the bad state evidently lowers profits. Eliminating rationing by charging a price above v^G results in no sales, as the probability of rationing cannot be lowered below zero to compensate for the higher price. It follows that the unique rationing/waste equilibrium is p_e.

Notice that p_e rises as θ increases. With more entry, each seller has fewer bad-state customers and, therefore, gives more weight to the price that is appropriate for the good state considered alone. This means entry increases price, which just offsets the effect on return of the decline in each firm's bad-state demand.

We can also show that it is not profitable for the deviant to set a price such that it sells out also in the bad state. Indeed, using (9), the sell-out price in the bad state satisfies $\phi m=K$, which evaluated at $p=p_e$, can be rewritten as

$$\phi m=\frac{x(v^G-p_d)K}{(1-x)p_d}=K,$$

yielding $p_d=xv^G\equiv p_s^B$, and deviant's return $\pi(p_s^B,p_e)=xv^GK$. Thus, p_s^B represents the lowest possible deviation price. Since $\pi(p_s^B,p_e)=xv^GK=E[\pi(p_e,p_e)]$, the deviant has no strict incentive to choose p_s^B. It follows that p_e identifies the unique, pure-strategy rationing/waste equilibrium price.

Expected per capita welfare (including both active and inactive consumers) at p is the expected value of the (gross) surplus,

$$E[W(p)]=[x\theta+\phi(1-x)]v^G,$$

which is independent of price, and is invariant to r-replications of the economy. Therefore, socially efficiency requires $p\le v^G$. Given entry, the equilibrium price is efficient.

Turning to entry, as there is excess capacity in the bad state, the social benefit of an extra seller is equal to xv^GK. It follows that the private return to entry equals the social return.

Proposition 5. *When shocks are drastic and heterogeneous, if the equilibrium price involves both rationing and waste, it is efficient and rises with θ.*

3.2. Non-drastic shocks

With non-drastic shocks, all buyers have willingness to pay v^G in the good state, whilst in the bad state, fraction ϕ of buyers value the good at $v=v^G$ with the rest valuing it at $v=v^B>0$. So, if $p\le v^B$, all consumers seek to buy in both states but, if $p\in(v^B,v^G)$, in the bad state only ϕ of consumers visit a seller. For welfare maximization, it is no longer obvious that $p\le v^B$. Although waste is eliminated in the bad state, allocative efficiency is not achieved as, with random rationing, some consumers with $v = v^B$ are served despite others with $v = v^G$ being excluded. It will be shown that, if v^B is not too high, the equilibrium price is that found in sub-section 3.1, yet welfare is higher at price $p \leq v^B$.

To determine the welfare maximizing price two price ranges will be compared: $p \leq v^B$; and $p \in (v^B, v^G)$.

At a price $p \leq v^B$, sales are K in both states, and consumers are always rationed. If it is random who is served, average consumption value in the good state is v^G, and in the bad state is $\phi v^G + (1-\phi)v^B$. So, per capita expected welfare is

$$E[W(p)] = \{xv^G + (1-x)\,[\phi v^G + (1-\phi)v^B]\}\theta. \tag{12}$$

If, instead, $p \in (v^B, v^G)$, as in sub-section 3.1, per capita welfare is

$$E[W(p)] = [x\theta + \phi(1-x)]v^G. \tag{13}$$

Comparing (12) and (13), welfare is higher at a price $p \leq v^B$, rather than at any $p \in (v^B, v^G)$, if

$$v^B \geq \tfrac{\phi(1-\theta)v^G}{(1-\phi)\theta} \equiv \underline{v}^B. \tag{14}$$

So, $\underline{v}^B$ is the lowest bad-state valuation such that it is welfare maximizing to set a price at which capacity is fully utilized in the bad state.

Turning to the equilibrium analysis, the strategy is to find the highest bad-state valuation, denoted by $\overline{v}^B$, that preserves the equilibrium at p_e. If $\overline{v}^B > \underline{v}^B$, there is an interval in which the equilibrium price exceeds the welfare maximizing price. If $v^B = 0$, the equilibrium price is p_e, with expected revenue $E[\pi(p_e, p_e)]$. When $v^B > 0$, there is a jump in bad-state deviant demand at $p_d = v^B$. Deviant sales are then K in both states.[14] Expected revenue is $E[\pi(v^B, p_e)] = v^B K$. To preserve equilibrium at p_e, the requirement is $E[\pi(v^B, p_e)] = v^B K < xv^G K = E[\pi(p_e, p_e)]$, or $v^B \leq xv^G$, which is the same condition derived in (2). So, the highest bad-state valuation for equilibrium at p_e is

$$xv^G \equiv \overline{v}^B, \tag{15}$$

which corresponds to the sell-out price in the bad state, p_s^B, derived in sub-section 3.1.

From (14) and (15), if $\phi \in [0, \widehat{\phi}]$, where

$$\widehat{\phi} = \tfrac{x\theta}{1-(1-x)\theta},$$

then $\overline{v}^B \geq \underline{v}^B$, and the equilibrium is inefficient. When few buyers are affected by the shock, it is less attractive to deviate to v^B, expanding the zone of pricing inefficiency.

To determine the efficiency of entry when $\phi \in [0, \widehat{\phi}]$, note that at the socially efficient price there is rationing in both states. An extra seller includes the value of bad-state sales, and is equal to K times the bracketed expression in (12). In equilibrium sales are only made in the good state, and each seller's expected revenue is $E[\pi(p_e, p_e)] = xv^G K$. Hence, if heterogeneous set-up costs lead to an equilibrium with rationing and waste, entry is below the socially efficient level (although, without price regulation, an entry subsidy would lower welfare).[15]

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14 At price just above v^B, deviant customers in the good state are $m > K$. At price below v^B, all affected buyers choose the deviant in both states, so the deviant sells out in both states if $\theta/\phi > 1$, as we assume. Note that consumers with $v = v^G$ may not find it worth visiting the deviant, as the affected buyers may make rationing in both states excessive.

15 It may be optimal to have two prices but, to show that the equilibrium may not be efficient, it suffices to consider the single-price optimum.

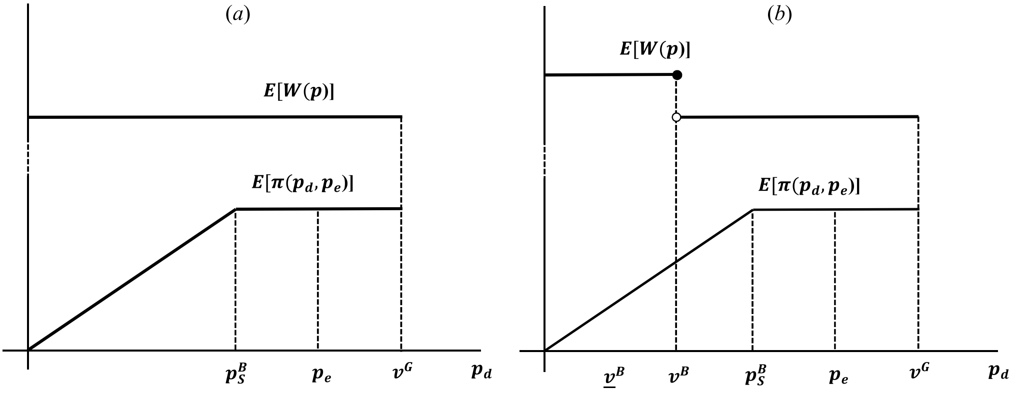

Fig. 2. Heterogeneous shocks: deviant's revenue and welfare: (a) Drastic shocks; (b) Non-drastic shocks.

Proposition 6. *When shocks are non drastic, if $\phi \in [0, \widehat{\phi}]$, the competitive price exceeds the socially efficient price. Equilibrium entry is then inefficiently low.*

Fig. 2 illustrates the nature of equilibrium price and its relation to the welfare optimum, in panel (a) for the drastic case and in panel (b) for the non-drastic case.

4. Extensions

4.1. Advance sales

The inefficiency of *ex-ante* pricing arises because adverse shocks result in available goods not being consumed despite having positive valuations. If consumers commit to buy before the state is revealed this problem is avoided. The cost to the consumer at time of use is then zero. For example, if a restaurant charges a non-returnable deposit if a table is booked a week in advance, diners will tend to arrive even if the weather is inclement. There are problems though with such marketing schemes. One is that consumers may be risk or regret averse. Another is that buyers may be subject to idiosyncratic shocks. If it is the individual who feels under the weather, it is more efficient that someone else gets the table. As we now show, under these circumstances advance selling may not be an equilibrium.[16]

Augment the homogeneous shocks model of Section 2 by adding an idiosyncratic shock. At the start of the consumption day, $(1-\alpha)M$ consumers, with $\alpha \in (0, 1)$, are randomly hit by an idiosyncratic shock (say "illness"), which reduces their valuation to zero. For the unaffected consumers, their valuation is $v = v^G$ in the good macro state and $v = v^B$ in the bad state. Propose an equilibrium with only spot sales at $p = v^G$. As before, deviating to a spot price of v^B is unprofitable if $v^B \leq xv^G$. Now, consider a deviation to advance sales at p_d, chosen to be the highest price at which consumers will buy in the competitive limit. If we denote the expected surplus with advance sales by $E[S(p, \alpha)]$, the deviation must satisfy

$$E[S(p_d, \alpha)] = \alpha[x(v^G - p_d) + (1-x)(v^B - p_d)] - (1-\alpha)p_d = 0 = E[S(v^G)],$$

[16] A related analysis of advance sales is provided by Dana (1998), who incorporates heterogeneous consumers into the Prescott (1975) model.

where the consumer loses p_d in case of illness. Thus, $p_d = \alpha[xv^G + (1-x)v^B] \equiv p_\alpha$. Revenue in the spot equilibrium at $p = v^G$ is $E[\pi(v^G, v^G)] = xv^G K$. Denoting by $\pi(p_\alpha, v^G)$ the return with advance sales, a deviation is unprofitable if

$$\pi(p_\alpha, v^G) = \alpha[xv^G + (1-x)v^B]K \leq xv^G K = E[\pi(v^G, v^G)],$$

which is satisfied if $\alpha \leq xv^G/[xv^G + (1-x)v^B]$.

4.2. Monopoly

The inefficiency of competitive equilibrium in the presence of *ex-ante* pricing and stochastic demand is due to rationing, giving even atomistic firms some price-setting power. It is therefore instructive to compare competitive and monopoly equilibria. Monopoly is understood as the item only being available from one firm, which can choose the number of sellers (shops) and the price they set. To abstract from monopsony, it is assumed that the reason for increasing cost is that some locations are more expensive than others.

In the model with homogeneous shocks of Section 2, when $v^B \leq xv^G$, a monopoly sets $p = v^G$ as, irrespective of its capacity, revenue is higher than at $p = v^B$. The extra revenue that the monopoly obtains from adding a seller is $xv^G K$. Hence, the monopoly equilibrium is identical to the high-price competitive equilibrium, including capacity choice. Under competition, there is also the possibility of a low-price equilibrium at $p = v^B$, but with less total capacity than at the high-price equilibrium at $p = v^G$, which is also the monopoly solution. As previously noted, the high-price outcome may be welfare preferred to the low-price competitive equilibrium. Monopoly may therefore be preferred to competition if the criterion is aggregate surplus. Competition may drive price so low that entry is discouraged.[17]

In the case of drastic heterogeneous shocks of sub-section 3.1, the monopoly sets price equal to the valuation of active consumers, $v = v^G$. At any lower price, sales are the same and, at any higher price, there are no sales. The monopoly price therefore exceeds the competitive price. In choosing the number of sellers, the monopoly recognizes that, in the rationing/waste regime, extra capacity is only useful in the good state, so its value is $xv^G K$, equal to the social value of extra capacity. Under competition, the lower selling price means firms do not capture their full social value in the good state, but this is exactly offset by treating as private return the bad-state diversion of customers from other sellers. The monopoly internalizes this effect, but captures full social value in the good state. Both market structures lead to full efficiency. The inefficiency arises in the non-drastic case of sub-section 3.2, where the private incentive to cut price to draw in the impaired buyers is below the social benefit. This is true even under competition, as an individual firm deviating to a lower price suffers a loss in good-state revenue. A monopoly lowering price at all faces the same trade off.[18]

4.3. Continuous demand

When every consumer has smooth, downward-sloping demand, there is a new issue. In the rationing state, a balance must be struck between how many customers should consume and

[17] When $v^B > xv^G$, $p = v^B$ under both market structures, leading to equal but insufficient industry capacity.

[18] Lowering price at a single seller is less attractive to a monopoly than a competitive seller, as it internalizes the business stealing effect.

how much should each of them buy. The Appendix analyzes the case of homogeneous shocks. In the drastic case, the competitive equilibrium is efficient but, in the non-drastic case, welfare may be higher at a price below the equilibrium level. Once again, the private incentive to cut price is below the social incentive. A lower price now reduces welfare in the good state. More is consumed per person but, as fewer people are served, the average valuation of consumption declines. Nevertheless, the price cut loses more revenue than it costs in good-state efficiency, so atomistic firms charge an excessive price.

5. Related work

Prescott (1975) provides a simple model of advance pricing in which the equilibrium is efficient. He assumes *ex-ante* identical consumers and unit demand. The fraction of active consumers varies with the state, but not their willingness to pay. This is the drastic-shock formulation. Consumers arrive at random and visit the lowest price seller with stock remaining. The equilibrium involves multiple prices. Higher price sellers do not sell out in low demand states. There is idle capacity in many states, but the private and social value of creating extra capacity coincide. Our model differs in that consumers make simultaneous not sequential choices of seller. This is not the fundamental reason for the differing efficiency results. In our first model, for example, the inefficient high-price equilibrium is unchanged with sequential arrivals. The issue is how adverse shocks affect demand. If the willingness to pay of some or all consumers is depressed but not to zero, there is inefficiency because it now matters which consumers are served in the rationing states.

Deneckere and Peck (2012), in the static section of their paper, extend the Prescott model. Shocks still determine the number of active consumers, but now there are heterogeneous positive valuations. In equilibrium, unsold stock in bad states may coexist with late arriving low but positive valuation consumers facing prices too high for them to buy. Ideally, items should only go unconsumed if nobody values them, but this does not address whether different *ex-ante* prices would increase aggregate surplus. For example, if a social planner were to set a single price of zero, there will never be unsold stocks unless their consumption value is zero. Nevertheless, as the wrong buyers may get the good, a zero price may not be efficient. The relevant question seems to be whether a social planner could set *ex-ante* prices (or find some other mechanism) to deliver higher expected welfare, which is not addressed.

Carlton (1978) analyzes competitive equilibria in the presence of stochastic demand and endogenous but predetermined price. He notes that the competitive equilibrium is not welfare maximizing when there is risk aversion or consumers are heterogeneous *ex ante*. But, when these features are absent, efficiency is achieved, even with smooth demand. Carlton's model differs from ours in various respects, such as variable firm capacity and allowing idiosyncratic shocks as well as aggregate shocks. These are not the reasons for the different conclusions under risk neutrality and homogeneous consumers. First, the drastic shocks formulation is adopted. Second, Carlton restricts deviations to capacity and price pairs that preserve the number of buyers selecting a seller, presenting an informal dynamic justification for ruling out more general price deviations (see also Carlton, 1991).[19] Our equilibrium allows sellers to choose deviations that involve the number of consumers selecting a seller to vary. In a further analysis, Carlton (1979)

[19] "When firms remain competitive by offering the given level of utility, they randomly receive their equal share of the L customer"(p. 575).
See also his eq. (4), and the discussion in Deneckere and Peck (1995).

studies a vertical production chain in a framework with *ex-ante* pricing. There is a private incentive for downstream firms to vertical integrate in whole or part, which affects the risk borne by the supplying industry, an externality leading to market failure. Horizontal externalities, the source of market failure in our analysis, are not examined.

Deneckere and Peck (1995) analyze a unit demand model with simultaneous search and drastic shocks. Capacity is a choice variable for sellers, and observed by buyers.[20] Their results concern the nature of equilibrium, but its efficiency is not analyzed. Dana (1999) extends the Prescott model of sequential search to imperfect competition but, again, the question of the efficiency of equilibrium is not addressed. Peters (1984) assumes smooth demand, but the randomness is due to the law of large numbers not holding in the mixed strategy buyer equilibrium, but welfare is not analyzed. Myatt and Wallace (2016) study a price setting industry selling differentiated goods under demand uncertainty. Their primary interest is in the causes and consequences of information acquisition. It is assumed that firms have quadratic costs, but the possibility of rationing is not considered.

6. Conclusions

The market failure analyzed here is intrinsic to competitive equilibrium under stochastic demand and *ex-ante* price setting. Our point is not that welfare falls short of what is achievable if price could be adjusted *ex post so as* always to clear the market. The claim is that if price must be set before demand is known, the *ex-ante* price and entry that maximizes welfare will not normally coincide with their equilibrium values. Rationing emerges in high-demand states, allowing even competitive atomistic firms a degree of price-setting power despite being utility takers. As a result, given industry capacity, price tends to be excessive. Rationing is diminished but waste is increased. Aggregate welfare is higher if the *ex-ante* price is below its equilibrium value. Only in the case of drastic shocks is equilibrium price efficient. There is no need to take absent consumers into account either in the welfare calculus or in determining equilibrium.

Whether or not price is efficient, entry signals are also distorted relative to the *ex-ante* optimum. Adding a seller relieves rationing in good states, so yields consumer benefits not fully captured by the entrant, giving rise to too few sellers. A new seller may also sell in excess supply states although this does not expand total consumption, a force tending to expand supply beyond the efficient level. When consumers demand multiple units there is a downside to low prices. Unrationed buyers increase purchases, reducing the number of consumers served. From an efficiency viewpoint it is better to serve more consumers with each buying less, given demand is downward sloping. Some waste may, therefore, be efficient but sellers do not directly capture these allocative gains. We find that with smooth demand and homogeneous drastic shocks, the equilibrium is efficient but when the shocks are non-drastic price is excessive. The distinction between drastic shocks which eliminate some buyers and non-drastic shocks which merely lower willingness to pay appears crucial for welfare. In most applications, the latter case seems the most reasonable representation.

Prescott (1975) analyzes a product market, the vacancy rate of hotel rooms, but the intention is to shed light on the efficiency of the labor market. Our results can be directly translated into a labor market setting. If employers post wages without knowing supply conditions, however competitive the labor market, it will not generate an *ex-ante* efficient outcome.

[20] We consider entry, but sellers do not have a capacity choice. Even with capacity variable, an equilibrium in prices is still required, so our results hold.

Appendix A

A.1. Homogeneous, drastic shocks

In the good state, all consumers are active, each with demand $1 - p$. A consumer able to buy at price $p \leq 1$, obtains surplus $S(p) = (1 + p)(1 - p)/2 - p(1 - p) = (1 - p)^2/2$. In the bad state, all consumers drop out of the market and sales are zero whatever the price.[21] Total capacity per capita is θ. We examine the competitive limiting case in which $r \to \infty$, so a deviant setting a non-equilibrium price does not affect the service rate at non-deviants.

Analysis of the drastic case is trivial. As there is no market in the bad state, it can be ignored in determining the equilibrium. All action is in the good state so, in the competitive case, equilibrium price will clear the market and by the First Welfare Theorem it is efficient.

Proposition A1. *If shocks are drastic, the equilibrium is market clearing in the good state and is efficient.*

A.2. Homogeneous, non-drastic shocks

In the case of non-drastic shocks, demand per consumer is $1 - p$ in the good state, but lower in the bad-state at $\phi - p$, with $\phi \in (0, 1)$, a parallel shift. Consumers can therefore perfectly infer the macro state from their own demand. The market clearing price in the good state is $p^G = 1 - \theta$ and in the bad state is $p^B = \phi - \theta$. For simplicity, it is assumed that, if $p = p^G$, demand in the bad state is positive, that is $\theta + \phi > 1$. If there is excess demand, some consumers are randomly selected to buy as much as they want, but others are not served at all (first come, fully served).

Proposition A2. *If shocks are non drastic and homogeneous, the equilibrium price may exceed the welfare maximizing price.*

Proof. We first consider the welfare problem. If $p \leq p^G$, bad state consumer surplus is $S(p, \phi) = (\phi + p)(\phi - p)/2 - p(\phi - p) = (\phi - p)^2/2$. In a rationing/waste regime, if $p \in [p^B, p^G]$, per capita welfare is the expected value of consumer surplus plus revenue,

$$E[W(p)] = x \tfrac{\theta}{1-p} \tfrac{(1+p)(1-p)}{2} + (1 - x) \tfrac{(\phi+p)(\phi-p)}{2}.$$

The first-order condition is $dE[W(p)]/dp = x\theta/2 - (1 - x)p = 0$, and the second-order condition is negative, so if the solution falls in the rationing/waste interval, the global welfare-maximizing price is[22]

$$p = \tfrac{x\theta}{2(1-x)} \equiv p_w.$$

It might be thought that ϕ would figure in the expression for p_w. When ϕ is high, there is less waste at a given price, suggesting that more weight should be given to eliminating rationing by charging a higher price. However, at the margin the trade-offs are independent of ϕ. As all demands are linear with the same slope, a price rises loses the same number of sales independently

[21] Efficient rationing would involve a limit to how much can be bought, as in Peck (2018). We assume it is not feasible to implement this scheme, as detecting multiple orders is difficult. Similar results to obtain with efficient rationing.

[22] If $p_w > p^G$ welfare is maximized at p^G, and if $p_w < p^B$ welfare is maximized at p^B.

of ϕ. These marginal sales are all valued at the ruling price. Hence, the welfare cost of a price rise in the bad state is independent of ϕ, and therefore so is p_w. The condition for $p_w \leq p^G$ is

$$\theta \leq \frac{2(1-x)}{2-x} \equiv \overline{\theta}.$$

We now turn to the equilibrium analysis. As in the drastic-shock case, there cannot be an equilibrium with price below the market-clearing price in the bad state. A firm deviating to an epsilon higher price would still sell out in both states, thereby increasing profits, so the equilibrium price cannot be below p^B. Additionally, there cannot be an equilibrium at a price above the market-clearing level as this is a Bertrand zone with an epsilon price deviation leading to a jump in demand. We now show that there is an equilibrium at the market clearing price in the good state, p^G, when the welfare-maximizing price is $p_w \in (p^B, p^G)$. If all firms charge p^G, the service rate is unity in both states, and each seller obtains expected revenue

$$E[\pi(p^G, p^G)] = xp^G K + (1-x)p^G(\phi - p^G)\frac{K}{\theta} = \frac{[\theta-(1-x)(1-\phi)](1-\theta)K}{\theta}. \tag{A1}$$

A deviant that charges $p_d < p^G$, when all other sellers charge p^G, offers a service rate below unity in the good state (a deviation to $p_d > p^G$ is not profitable as there will not be any sales). At price p_d, consumers distribute themselves so that $E[S(p_d)] = E[S(p^G)]$, or

$$x\frac{K}{(1-p_d)m}S(p_d) + (1-x)S(p_d,\phi) = xS(p^G) + (1-x)S(p^G,\phi), \tag{A2}$$

where the probability of being served in the good-state at price p_d is $K/(1-p_d)m$. Substituting m from (A2), the deviant's expected revenue, at p_d is

$$\begin{aligned} E[\pi(p_d, p^G)] &= xp_d K + (1-x)p_d(\phi - p_d)m = \\ &= xp_d K\left[1 - \frac{(1-x)(1-p_d)(\phi-p_d)}{(1-x)p_d^2+2\phi(1-x)(1-p_d-\theta)+(2-2x-\theta)\theta+x-1}\right]. \end{aligned}$$

The deviation is unprofitable if $E[\pi(p^G, p^G)] \geq E[\pi(p_d, p^G)]$, which holds if

$$\frac{[\phi(1-x)+\theta+x-1]\{x(2\theta-1)x+2\phi(x(1-\theta)+\theta-1)+(1-\theta)^2\}}{(1-x)[(1-\phi)x(2\theta-1)-(1-\theta)(\phi+\theta-1)]} \equiv \underline{p} < p_d < p^G.$$

At some point, p_d is sufficiently low that the deviant sells out in both states. It cannot be profitable to set price below this level. The highest price, denoted by p_s^B, at which the deviant sells out in the bad state satisfies

$$(\phi - p_s^B)m = K. \tag{A3}$$

Using m from (A3), at the bad-state sell-out price, $E[S(p_s^B)] = E[S(p^G)]$, or

$$\xi\frac{\phi-p_s^B}{1-p_s^B}S(p_s^B) + (1-\xi)S(p_s^B,\phi) = \xi S(p^G) + (1-\xi)S(p^G,\phi), \tag{A4}$$

where the probability of being served in the good state by the deviant is $(\phi - p_s^B)/(1 - p_s^B)$. There are two solutions to (A4), and the relevant one (below p^G) is

$$p_s^B = \frac{\phi(2-x)+x-\sqrt{4(1-\phi-\theta)^2+4(1-\phi)x(2\theta+\phi-1)+(1-\phi)^2x^2}}{2}.$$

At p_s^B, the deviant sells out capacity in both states, so a lower deviation is certainly not optimal. If $\underline{p} \leq p_s^B$, there is an equilibrium at p^G. The requirement is

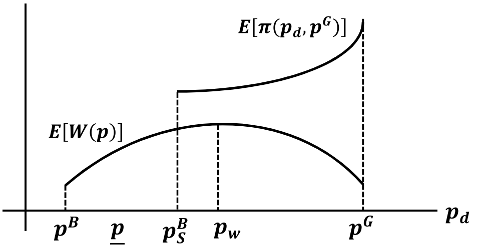

Fig. 3. Non-drastic shocks. Parameters: $x = 0.6$, $\theta = 0.5$, $\phi = 0.55$.

$$\phi \leq 1 - \frac{[(2-x)\theta - 2(1-x)]\theta^2}{(1-x)(1-\theta)[1-x-(3-2x)\theta]} \equiv \phi'.$$

It then follows that, if $\theta \leq \overline{\theta}$, $\overline{\phi} \leq \phi'$. So, when p_w is in the rationing/waste regime, $\underline{p} \leq p_s^B$, and all downward deviations from p^G are unprofitable. $\square$

Fig. 3 plots the deviant's revenue function, $E[\pi(p_d, p^G)]$, and the welfare function, $E[W(p)]$, for an example. Deviation from p^G is not profitable as the revenue is lower for each $p_d \in [p_s^B, p^G]$. The welfare maximizing price, p_w, is below p^G.

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