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On the effect of the aspect ratio on the mixed convection in a vertical cylindrical cavity with rotating inner wall

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Abstract. The thermal fluidynamic behaviour of a variety of fluids filling either vertical cylinders or vertical annuli under external conditions giving rise to natural convective motions has been thoroughly investigated, both experimentally and numerically, because of the several possible engineering applications that can be well approximated and described by these kinds of configurations. This study aims at investing through a numerical analysis the mixed convection arising inside cavities formed between two horizontal adiabatic disks and two isothermally and differentially heated vertical cylindrical walls, one of which is rotating. Previous studies reported and discussed the role played by both the Rayleigh number the Reynolds number for the case of the rotation of either the inner or the external wall and for a specified geometry of the cavity. The focus of the present study is to analyse the role played by the aspect ratio, $A=H/(R_e-R_i)$, for the case of rotating inner cylinder. It is reported that this role is particular relevant with respect to the flow structures established within the cavity and, as a consequence, also with respect to the Nusselt number.

1. Introduction

The natural convection of a fluid in an annulus between two vertical rotating concentric cylinders is a topic of major interest in many fields of applications, such as electrical machineries, thermal energy storage systems and chemical mixing equipment. In literature, some interest has been focused on the analysis of "buoyancy dominated flows", i.e. natural convective flows that are generated in static systems. Example of this are the studies of Vahl Davis and Thomos [1] and Kumar and Kalman [2], where correlations between the number of Nusselt and physical parameters have been determined. Other works deal with the dynamical contribution occurring in vertical annular cavity in which one (or both) of the walls, either of the cylinder or of the annulus, rotates. The numerical study of free convection in a rotating annulus was conducted by Williams [3]. De Vahl Davis et alii. [4] analyzed a vertical annular cavity with rotating inner wall and top and bottom surfaces. G. Urquiza et alii. [5] proposed a numerical study to evaluate the influence of the Prandtl number on the flow and the heat transfer rate on a rotating cylindrical annulus.

A specific configuration is the annular cavity between two concentric vertical cylinders filled with a fluid and closed by two horizontal disks. The heat exchange occurs between the fluid and the isothermally inner heated and outer cooled surfaces of the cylinder, whilst both the bottom and top horizontal disks are assumed to be adiabatic. Moreover, one or both of the cylindrical walls are considered to be rotating with constant angular velocity. As consequence, mixed convection arises in the system due to the contribution of natural convection, originated by the differential heating imposed at the vertical walls, and the forced convection established by the rotation of one of the cylinders.
Such a system is studied by M. Venkatachalappa [6] for different parameters as the speed rotation of both the inner and outer surfaces delimiting the vertical annular cavity, the buoyancy as expressed by the Rayleigh number, and the type of fluid as expressed by specified values of the Prandtl number (Pr=0.01, 0.1, 1). Therefore, the results reported in [6] hold for liquid metals and for gases.
With the aim of characterising the behaviour of liquids of moderate Pr, the same configuration, schematized in Fig. 1, was analyzed by the authors for a fluid with $Pr=5$ in the case of rotation of either the inner [7] or the external wall [8], for an enclosure specified by an aspect ratio $A=H/(R_e-R_i)=1$ and a radius ratio $RK=R_e/R_i=5$. In particular, the thermal and the flow fields, the profiles of the temperature and of the velocity components and the Nusselt number were described for the case of the cavity differentially heated inside and cooled outside, under a variety of values of the Rayleigh number and of the Reynolds number, within the limit of mixed convection.
In the present study the attention is focused on the relative influence of the forced component on both the flow structures and heat transfer between the vertical walls by varying the aspect ratio of the enclosure, for Pr=5.

2. Mathematical Model

The system considered in the present study is an enclosure, as shown in Fig. 1, formed by two differentially and isothermally heated vertical coaxial cylindrical walls of height H and inner and outer radius R_i and R_e, respectively, and two horizontal adiabatic disks.

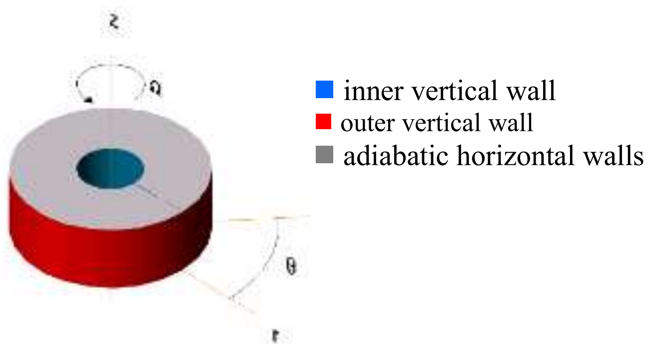

Fig. 1 Scheme of the system geometry

The reference coordinate system is cylindrical (r, θ, z) and three velocity components u, w, v, can be consequently identified along the three coordinates. The system symmetry with respect to the vertical axis ensures the flow independence with respect to the angular position, θ; therefore, the system can be modelled in 2D just with respect to r and z. Fixed coordinates are considered with the corresponding choice of boundary conditions at the walls. In line with Boussinesq's approximation, the fluid density is assumed to be constant in all terms, except for the buoyancy term that linearly depends on the temperature. The model was derived in terms of primitive variables u, v, w, p and T by expressing the mass, energy and momentum along the directions r, z and θ, as it follows:

$$\frac{1}{r}\frac{\partial ru}{\partial r}+\frac{\partial v}{\partial z}=0 \tag{1}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+v\frac{\partial T}{\partial z}=a\nabla^2 T \tag{2}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+v\frac{\partial u}{\partial z}=-\frac{1}{\rho_0}\frac{\partial p}{\partial r}+\frac{\mu}{\rho_0}\left(\nabla^2 u-\frac{u}{r^2}\right)+\frac{\rho}{\rho_0}\cdot\left(\frac{w^2}{r}\right) \tag{3}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+v\frac{\partial v}{\partial z}=-\frac{1}{\rho_0}\frac{\partial p}{\partial z}+\frac{\mu}{\rho_0}\nabla^2 v+g\beta(T-T_0) \tag{4}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+v\frac{\partial w}{\partial z}=\frac{\mu}{\rho_0}\left(\nabla^2 w-\frac{w}{r^2}\right)-\frac{\rho}{\rho_0}\cdot\left(\frac{uw}{r}\right) \tag{5}$$

The model was rewritten in non-dimensional form considering the following scaling factors:

- Vertical coordinate: H [m]
- Radial coordinate: $\Delta r/\eta$ [m]
- Reference velocity: $V_0 = \Omega \cdot R_i$ [m/s]
- Reference time: H/V_0 [s]
- Reference pressure: $2\rho_0 V_0$ [Pa].

which corresponds to the following choice of the non-dimensional system variables:

$$A=\frac{H}{\Delta R};\quad RK=\frac{R_e}{R_i};\quad \eta=RK-1;$$
$$z'=\frac{z}{H};\quad r'=\frac{r}{R_i}=\eta\frac{r}{\Delta R};\quad t'=t\frac{V_0}{H};$$
$$u'=\frac{u}{V_0};\quad w'=\frac{w}{V_0};\quad v'=\frac{v}{V_0};$$
$$p'=\frac{p+\rho_0 gH}{\rho_0 V_0^2};\quad T'=\frac{2(T-T_0)}{\Delta T};\quad \mathrm{Re}=\frac{2\Omega\Delta R^2}{\nu};$$
$$Ra=\frac{\beta\cdot g\,\Delta T\Delta R^3}{\nu a};\quad Pr=\frac{c_p\mu}{k}=\frac{\nu}{a};\quad Fr=\frac{\Omega^2\Delta R}{g}. \tag{6}$$

Under the previous choices the non-dimensional model was therefore reformulated as:

$$(\eta A)\frac{\partial ru}{\partial r}+\frac{\partial v}{\partial z}=0 \tag{7}$$

$$\frac{\partial T}{\partial t}+(\eta A)\cdot u\frac{\partial T}{\partial r}+v\frac{\partial T}{\partial z}=\frac{2}{Pr\ Re}\left[(\eta^3 A)\cdot\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)+\left(\frac{\eta}{A}\right)\frac{\partial^2 T}{\partial z^2}\right] \tag{8}$$

$$\frac{\partial u}{\partial t}+(\eta A)\cdot u\frac{\partial u}{\partial r}+v\frac{\partial u}{\partial z}=(\eta A)\cdot\left(1-2\frac{RaFr}{\mathrm{Pr}\ \mathrm{Re}^2}T\right)\cdot\left(\frac{w^2}{r}\right)+$$
$$+\frac{2}{\mathrm{Re}}\left[(\eta^3 A)\cdot\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)+\left(\frac{\eta}{A}\right)\frac{\partial^2 u}{\partial z^2}\right]-\frac{\partial p}{\partial r} \tag{9}$$

$$\frac{\partial v}{\partial t}+(\eta A)\cdot u\frac{\partial v}{\partial r}+v\frac{\partial v}{\partial z}=-(\eta A)\cdot\frac{1}{Fr}\left(1-2\frac{RaFr}{\mathrm{Pr}\ \mathrm{Re}^2}T\right)+$$
$$+\frac{2}{\mathrm{Re}}\left[(\eta^3 A)\cdot\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}\right)+\left(\frac{\eta}{A}\right)\frac{\partial^2 v}{\partial z^2}\right]-\frac{\partial p}{\partial z} \tag{10}$$

$$\frac{\partial w}{\partial t}+(\eta A)\cdot u\frac{\partial w}{\partial r}+v\frac{\partial w}{\partial z}=(\eta A)\cdot\left(1-2\frac{RaFr}{\mathrm{Pr}\ \mathrm{Re}^2}T\right)\cdot\left(\frac{uw}{r}\right)+$$
$$+\frac{2}{\mathrm{Re}}\left[(\eta^3 A)\cdot\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)+\left(\frac{\eta}{A}\right)\frac{\partial^2 w}{\partial z^2}\right] \tag{11}$$

where the prime symbol has been dropped. Moreover, though not used for calculations, for the analysis of the simulated flow field it was convenient to refer to the stream function ψ [9]:

$$u=\frac{1}{r}\frac{\partial\psi}{\partial z}\quad ;\quad v=\frac{1}{r}\frac{\partial\psi}{\partial r} \tag{12}$$

The model solution allows to evaluate the temperature profile and, on this basis, it is possible to calculate the Nusselt number at either of the two vertical walls. In the present study, reported results refer to the Nusselt number evaluated at the inner wall, defined as follows:

$$Nu=Nu_{r=0}=\int_0^H Q(r,y)\cdot dy=\int_0^H\frac{\partial T}{\partial r}\cdot dy \tag{13}$$

where *Q(r,y)* is the local heat flux in the radial direction.
A finite volume scheme, assessed and validated in previous studies [10], was adopted to solve the model Equations (7)–(11), defining a two-dimensional grid and differentiating the equations for each control volume. The pressure-velocity coupling was solved adopting SIMPLER algorithm, performing nodal variable calculation on the basis of *CDS* algorithm. The code inputs were the parameters characterizing the operating condition (*Re*, *Ra*, *Pr*, *Fr*), the cavity shape (*A* and *RK*), the grid characteristics (number of cells and their distribution along *r* and *z*) and the number of iterations. The output of the code was a set of matrices, one for each of the primitive variables plus an additional one for the stream function. Moreover, the code produced as outputs also the value at each iteration of the Nusselt number, of the primitive variables and of their residues, and of the number of iterations of the pressure-velocity algorithm. In this way it was possible to check the iteration process and to verify the simulation reliability. With this respect, errors below 10^{-4} were assumed satisfactory for all primitive variables, with zero iteration of the pressure algorithm at the end of the calculation. It is just mentioned that the model defined in the present study was satisfactorily validated through the

comparison with benchmark results obtained by Khellaf [11] and Ho and Tu [12] for a vertical cylindrical cavity formed by two coaxial cylinders with a rotating inner wall.

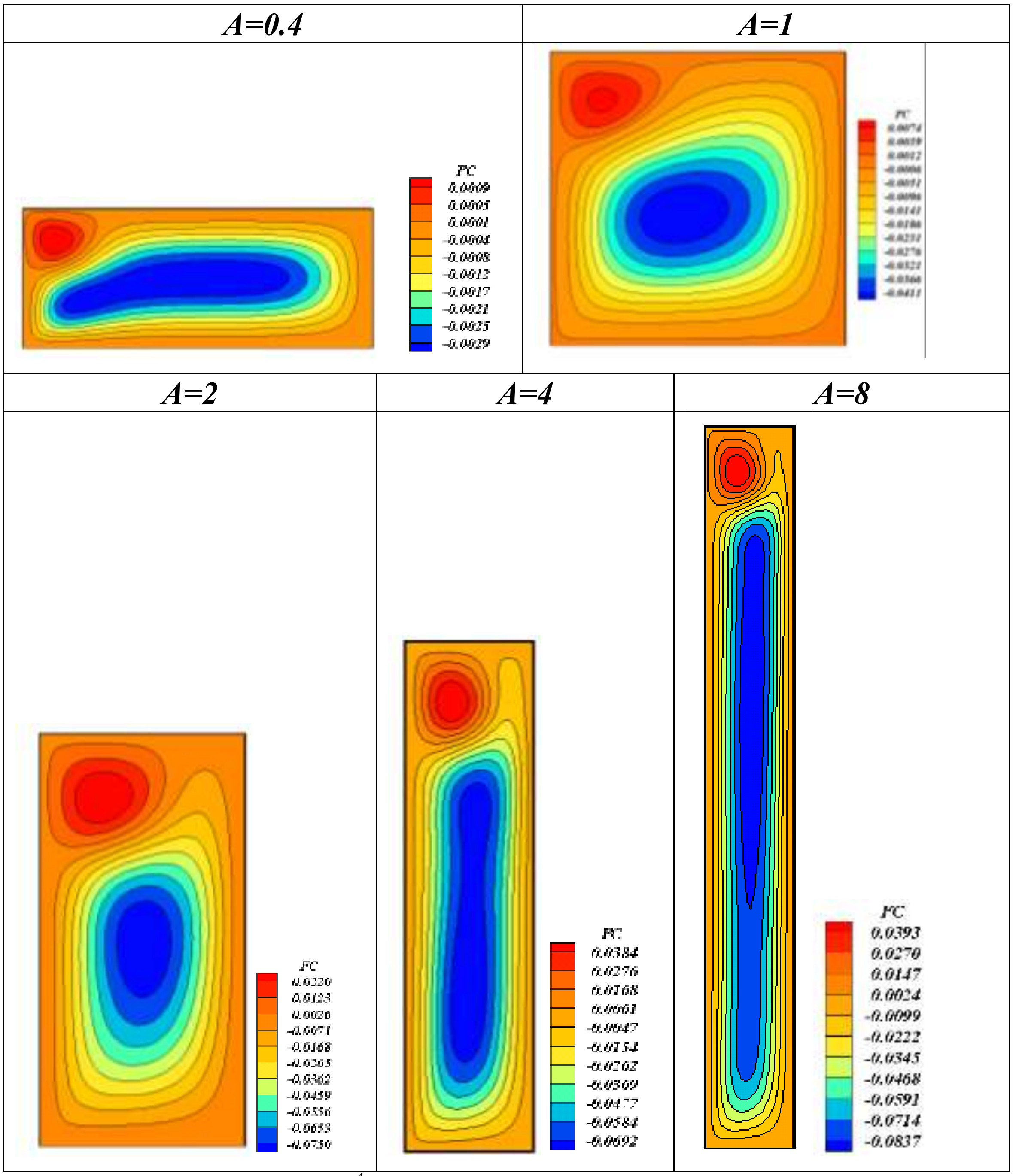

Fig. 2 Stream function for $Ra=10^4$ and A=[0.4, 1, 2, 4, 8]

3. Results

The mixed convection of a fluid of $Pr=5$ within a vertical annular enclosure with isothermally heated and cooled vertical walls was simulated for the case of rotation of the inner wall. As the interest was focused on analysing the influence of the aspect ratio, A, both the radius ratio and the Reynolds number were specified at, respectively, *RK=5* (i.e. η*=4*), *Re=100.* The flow structures reported in the following refers to aspect ratio values *A*=[0.4, 1, 2, 4, 8] and are evaluated by varying the Rayleigh number within the range $Ra \subset [10^4{:}10^5]$. It is worth observing that the calculations were performed within ranges of *Ra* and *Re* for which it is possible to neglect both viscous dissipation and the influence of density variations on centrifugal and Coriolis forces.

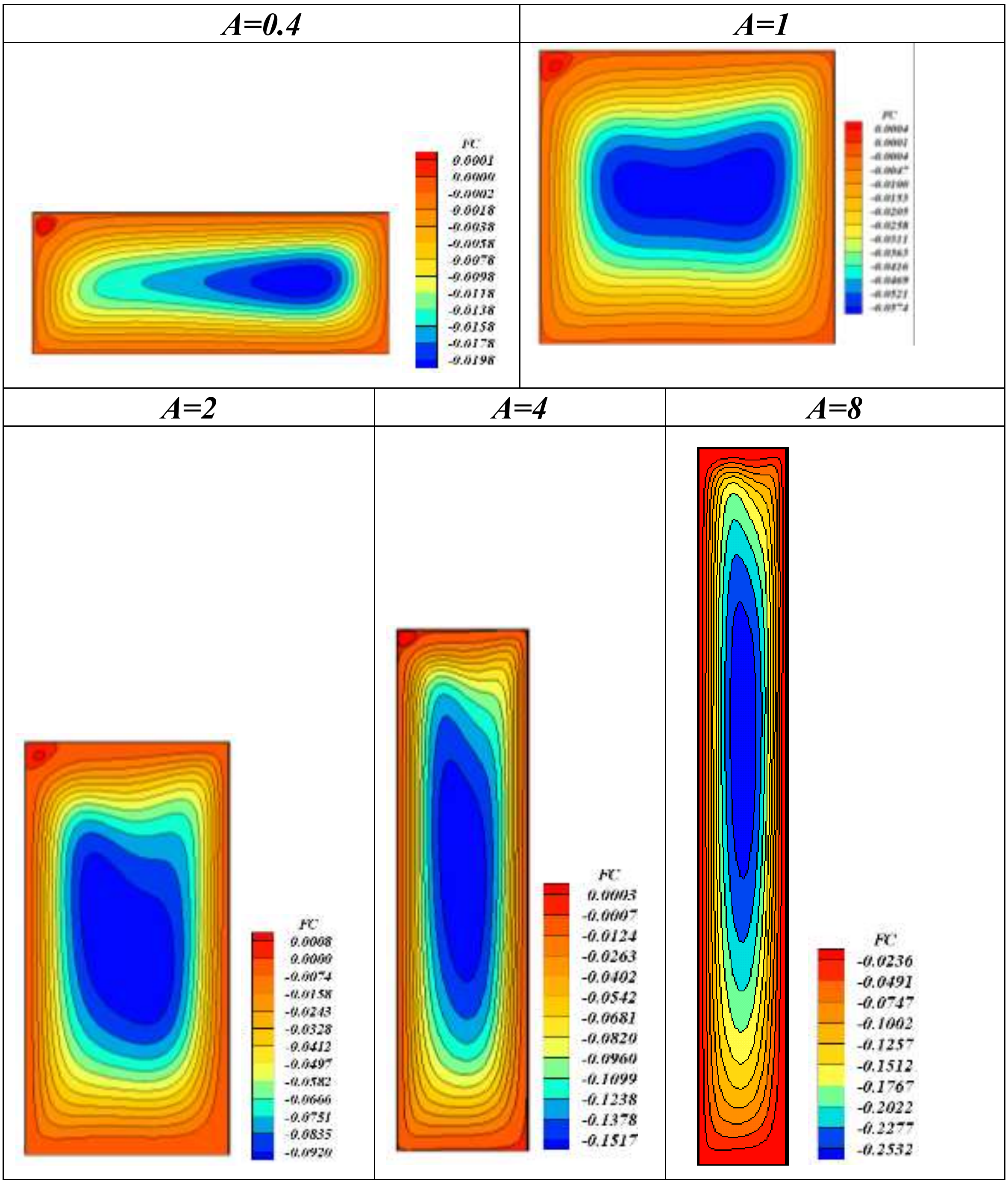

Fig. 3 Stream function for $Ra=10^5$ and A=[0.4, 1, 2, 4, 8]

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The dynamics of the fluid within the cavity is obtained by solving the numerical model. Fig. 2 and 3 report the stream functions describing the distributions of the flow structures within the right-hand side cross section of the enclosure corresponding respectively to $Ra = 10^4$ and $Ra = 10^5$ at the simulated values of the aspect ratio A.
It is possible to observe that the structure of the flow is generally bi-cellular, due to the contribution of both natural convection, caused by the radial thermal gradient, and forced convection, caused by the rotation of the inner wall. In particular, the forced convection contribution to the structure of the flow is the birth of the counter-rotating cell at the inner wall that occupies the upper part of the cavity. The natural convection, instead, is responsible of the cell that occupies the main part of the cavity.
From the analysis of Fig. 2 it is possible to observe that, considering $Ra=10^4$, the bi-cellular flow pattern structure is maintained for all of the values of A. Fig. 3 allows to observe that the growth of the Rayleigh number to $Ra=10^5$ reinforces the overall relevance of the buoyancy whilst the forced convection is dumped. This is reflected in the reduction of the size of the counter-rotating cell at the inner rotating wall that can be observed for growing aspect ratio, until A=4, and its disappearance for A=8, where the buoyancy-generates cell occupies the entire enclosure.
In order to evaluate the influence of the aspect ratio on the heat exchange, i.e. the number of Nusselt, simulations for different values of Ra and A were conducted. Fig. 4 shows the effect that the increase of A has on Nu for $Ra=10^4$ and $Ra=10^5$ and for $Re=100$. In particular, the same general dependence of Nu on A can be observed. This consists in a steep increase of Nu for $A< 1$, after which a maximum is reached around $A=1$, followed by a monotonic decrease. Moreover, the increase of Ra causes the growth of the Nu-values calculated by the numerical model with a slight decrease of the value of A corresponding to the maximum reached by Nu. Therefore, it is possible to conclude that cavities with either square or only lightly stretched cross sections show the highest values of Nu.

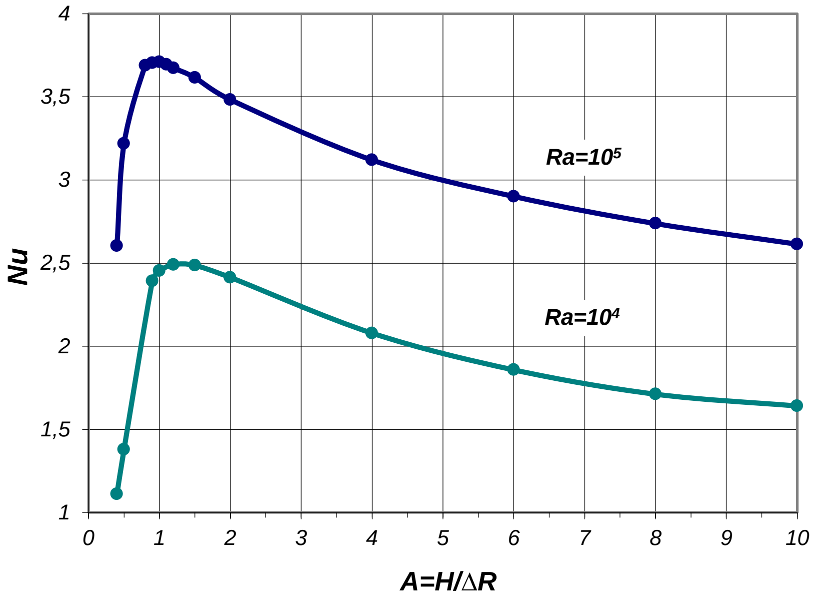

Fig. 4 Dependence of the Nusselt number on A for $Ra=10^4$ and $Ra=10^5$.

4. Conclusions

A numerical model was defined and analysed for the description of mixed convection in a vertical annular cavity differentially heated inside and cooled outside and with horizontal adiabatic disk. The case described in this study refers to influence of the aspect ratio, A, on the mixed convection arising as a consequence of the buoyancy, driven by the temperature difference between the vertical walls, and of the rotation at constant angular velocity of the inner. Results have been reported for *Re=100* and for *Ra*=10^4 and Ra=10^5 and allow to observe that a bi-cellular structure of the flow is generally observed which is lost in slender enclosures when the Rayleigh number is increased. The maximum of the Nusselt number can be observed to correspond to cross sections around the squared geometry; in fact, for such configurations it was observed that the relative importance of the force convection-driven cell is the highest, and, hence, the forced convection contribution determines the maximum increase to the Nusselt number.

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