# Toward a solution to the $R_{A A}$ and $v_{2}$ puzzle for heavy quarks 

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## A R T I C L E INFO

## Article history:

Received 17 February 2015
Received in revised form 31 May 2015
Accepted 1 June 2015
Available online 3 June 2015
Editor: J.-P. Blaizot


#### Abstract

The heavy quarks constitute a unique probe of the quark-gluon plasma properties. A puzzling relation between the nuclear modification factor $R_{A A}\left(p_{T}\right)$ and the elliptic flow $v_{2}\left(p_{T}\right)$ has been observed both at RHIC and LHC energies. Predicting correctly both observables has been a challenge to all existing models, especially for D mesons. We discuss how the temperature dependence of the heavy quark drag coefficient is responsible for a large part of such a puzzle. In particular, we have considered four different models to evaluate the temperature dependence of drag and diffusion coefficients propagating through a quark gluon plasma (QGP). All the four different models are set to reproduce the same $R_{A A}\left(p_{T}\right)$ observed in experiments at RHIC and LHC energy. We point out that for the same $R_{A A}\left(p_{T}\right)$ one can generate 2-3 times more $v_{2}$ depending on the temperature dependence of the heavy quark drag coefficient. A nondecreasing drag coefficient as $T \rightarrow T_{C}$ is a major ingredient for a simultaneous description of $R_{A A}\left(p_{T}\right)$ and $v_{2}\left(p_{T}\right)$. © 2015 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/). Funded by SCOAP3.


The ongoing nuclear collision programs at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) energies have created a medium that behaves like a nearly perfect fluid. The bulk properties of such a matter, called Quark Gluon Plasma (QGP), are governed by the light quarks and gluons [1,2]. To characterize the QGP, penetrating and well-calibrated probes are essential. In this context, the heavy quarks (HQs), mainly charm and bottom quarks, play a vital role since they do not constitute the bulk part of the matter owing to their larger mass compared to the temperature created in ultra-relativistic heavy-ion collisions (uRHICs) [3].

There are presently two main observables related to heavy quarks that have been measured at both RHIC and LHC energies. The first one is the so-called nuclear suppression factor $R_{A A}$ that is the ratio between the $p_{T}$ spectra of heavy flavored hadrons ( D and B ) produced in nucleus + nucleus collisions with respect to those produced in proton + proton collisions. More specifically at RHIC until recently it has not been possible to measure directly D and $B$ but only the leptons through their semileptonic decays. The other key observable is the elliptic flow $v_{2}=\left\langle\cos \left(2 \phi_{p}\right)\right\rangle$, a measure of the anisotropy in the angular distribution that corresponds to the anisotropic emission of particles with respect to the azimuthal angle $\phi_{p}$. Despite their large mass, experimentally measured nu-

[^0]clear suppression factor $R_{A A}$ and elliptic flow $v_{2}$ of the heavy mesons are comparable to that of light hadrons [16-19]. This is in contrast to the expectations drawn initially from the perturbative interaction of HQs with the medium which predicted an $R_{A A} \approx 0.6$ for charm quarks, $R_{A A} \approx 0.8-0.9$ for bottom quarks in the central collisions $[11,12]$ at intermediate $p_{T}$. Also the $v_{2}$ was predicted to be much smaller with respect to the light hadron ones [12].

Several theoretical efforts have been made in order to calculate the experimentally observed $R_{\mathrm{AA}}$ [16-19] and $v_{2}$ [16] for the non-photonic single electron spectra within the Fokker-Planck approach [7-10,14,20,23,25,26,34-37] and relativistic Boltzmann transport approach [15,28-33,50-52]. Furthermore, also in a pQCD framework supplemented by the hard thermal loop scheme several advances have been made to evaluate realistic Debye mass and running coupling constants $[15,25]$ and also three-body scattering effects $[10,20,21,24]$ have been implemented. It has been shown in [38] that the inclusion of both elastic and inelastic collisions within a dynamical energy loss formalism reduces the gap between the theoretical and experimental results for $R_{A A}$ as $p_{T} \geq$ $5-10 \mathrm{GeV}[39,40]$. Several other improvements have been proposed to advance the description of the data [41-43]. Interactions from AdS/CFT [57] have also been implemented $[23,27,59]$ to study the heavy flavor dynamics at RHIC and LHC. Essentially all the models show some difficulties to describe simultaneously both $R_{A A}\left(p_{T}\right)$ and $v_{2}\left(p_{T}\right)$ and such a trait is not only present at RHIC energy but also in the results coming from collisions at LHC energy [19].

In this letter we will address the impact of the temperature dependence of the interaction (drag coefficient) on both $R_{A A}$ and $v_{2}$ relation. For this we are considering four different models having different T dependent drag coefficients. For the momentum evolution of the HQ , we are using $3+1 \mathrm{D}$ Langevin dynamics. We notice that the several approaches and modelings of the HQ inmedium interaction differ significantly for the $T$ dependence of the drag coefficient they entail. One can go from drag coefficients increasing as $T^{2}$ (like in Ads/CFT) to cases with the drag coefficient decreasing with $T$. The aim of this letter is to show that, while generally a smaller $R_{A A}\left(p_{T}\right)$ corresponds to larger $v_{2}\left(p_{T}\right)$, the specific $T$ dependence of the drag can strongly modify such an amount of $v_{2}\left(p_{T}\right)$, even if the models are tuned to reproduce the same $R_{A A}\left(p_{T}\right)$ observed experimentally. Our analysis shows that it is quite unlike that a drag with $T^{2}$ dependence can generate larger elliptic flow than the one observed experimentally at both RHIC and LHC. Instead a nearly constant drag or an increasing one as $T \rightarrow T_{c}$ strongly quenches the puzzling $R_{A A}\left(p_{T}\right)-v_{2}\left(p_{T}\right)$ relation.

The standard approach to HQ dynamics in the QGP is to follow their evolution by means of a Fokker-Planck equation solved stochastically by the Langevin equations. The relativistic Langevin equations of motion for the evolution of the momentum and position of the heavy quarks can be written in the form
$d x_{i}=\frac{p_{i}}{E} d t$,
$d p_{i}=-\Gamma p_{i} d t+C_{i j} \rho_{j} \sqrt{d t}$
where $d x_{i}$ and $d p_{i}$ are the shift of the coordinate and momentum in each time step $d t$. $\Gamma$ and $C_{i j}$ are the drag force and the covariance matrix in terms of independent Gaussian-normal distributed random variables $\rho, P(\rho)=(2 \pi)^{-3 / 2} e^{-\rho^{2} / 2}$, which obey the relations $<\rho_{i} \rho_{j}>=\delta_{i j}$ and $<\rho_{i}>=0$, respectively. The covariance matrix is related to the diffusion tensor,
$C_{i j}=\sqrt{2 B_{0}} P_{i j}^{\perp}+\sqrt{2 B_{1}} P_{i j}^{\|}$,
where $P_{i j}^{\perp}=\delta_{i j}-p_{i} p_{j} / p^{2}$ and $P_{i j}^{\|}=p_{i} p_{j} / p^{2}$ are the transverse and longitudinal projector operators respectively. Under the assumption, $B_{0}=B_{1}=D$, Eq. (2) becomes $C_{i j}=\sqrt{2 D(p)} \delta_{i j}$. Such an assumption strictly valid only for $p \rightarrow 0$, is usually employed at finite $p$ in application for heavy quark dynamics in the QGP [8-10, 14,24,34].

We will discuss our results in terms of the drag coefficient $\Gamma$, but we remind that it is related to the diffusion coefficient by the fluctuation-dissipation theorem that within a Langevin approach reads $D=\Gamma E T$, for the case of the post-point Ito realization of the stochastic integral [44]. In the post-point discretization the diffusion coefficients have to be used at the momentum argument $p+d p$, where $d p$ is the increment from a pre-point Ito (Euler) time-step according to Eq. (1).

The solution of the stochastic Langevin equation needs a background medium describing the evolution of the bulk QGP matter. To describe the expansion and cooling of the bulk and its elliptic flow $v_{2}\left(p_{T}\right)$ at both RHIC and LHC, we have employed a relativistic transport code with an initial condition given by a standard Glauber model and with an evolution at fixed $\eta / s=0.16$ (similarly to viscous hydro), see Refs. [45-48] for more details.

Our objective is to demonstrate the effect of the temperature dependent interaction (drag coefficient) on the $R_{A A}$ and $v_{2}$ obtained from different models. More specifically we investigate at fixed $R_{A A}$ how the $v_{2}$ is built up under various temperature dependence of the interaction. For this purpose we consider four different modelings to calculate the drag and diffusion coefficients which are the key ingredients to solve the Langevin equation. Such
models have to be considered merely as an expedient-device to generate different $T$ dependence of the $\Gamma(T)$ but the results and conclusions deduced will be much more general because they do not depend on the way the $\Gamma(T)$ has been obtained. In this sense within a Fokker-Planck approach it is not relevant if the drag and diffusion coefficients have been evaluated considering only collisional or radiative loss.

Model-I ( $p Q C D$ ): The elastic interaction of heavy quarks with the light quarks, antiquarks and gluons in the bulk has been considered within the framework of PQCD to calculate the drag and diffusion coefficients. The scattering matrices $\mathcal{M}_{g H \mathrm{~L}}, \mathcal{M}_{q \mathrm{HQ}}$ and $\mathcal{M}_{\bar{q} H \mathrm{HQ}}$ are the well-known Combridge matrix that includes $s, t, u$ channel and their interferences terms [58]. The divergence associated with the $t$-channel diagrams due to massless intermediate particle exchange has been shielded introducing the Debye screening mass $m_{D}=\sqrt{4 \pi \alpha_{s}} T$. The temperature dependence of the coupling [55]:
$g^{-2}(T)=2 \beta_{0} \ln \left(\frac{2 \pi T}{a T_{c}}\right)+\frac{\beta_{1}}{\beta_{0}} \ln \left[\ln \left(\frac{2 \pi T}{a T_{c}}\right)\right]$
where $\beta_{0}=\left(11-2 N_{f} / 3\right) / 16 \pi^{2}, \beta_{1}=\left(102-38 N_{f} / 3\right) /\left(16 \pi^{2}\right)^{2}$ and $a=1.3 . N_{f}$ is the number of flavor and $T_{C}$ is the transition temperature.

Model-II (AdS/CFT): We have also considered the drag force from the gauge/string duality [56], namely the conjectured equivalence between conformal $\mathrm{N}=4$ SYM gauge theory and gravitational theory in anti-de Sitter space-time i.e. AdS/CFT. By matching the energy density of QCD and SYM, which leads to $T_{S Y M}=T_{Q C D} / 3^{\frac{1}{4}}$, and the string prediction for quark-antiquark potential with lattice gauge theory which gives $3.5<\lambda<8$ [57], one finds
$\Gamma_{\text {conf }}=C \frac{T_{\mathrm{QCD}}^{2}}{M_{C}}$
where $C=\frac{\pi \sqrt{\lambda}}{2 \sqrt{3}}=2.1 \pm 0.5$. The corresponding diffusion constant D can be obtained from the fluctuation-dissipation relation. Studies of heavy flavor momentum evolution within the Langevin dynamics using AdS/CFT can be found in Refs. [23,27].

Model-III (QPM): The third model recently applied to estimate the heavy flavor transport coefficients is inspired by the quasi-particle model (QPM) [62-64]. The QPM approach is a way to account for the non-perturbative dynamics by T-dependent quasi-particle masses, $m_{q}=1 / 3 g^{2} T^{2}, m_{g}=3 / 4 g^{2} T^{2}$, plus a T dependence background field known as bag constant. Such an approach is able to successfully reproduce the thermodynamics of IQCD [60] by fitting the coupling $g(T)$. To evaluate the drag and diffusion coefficients we have employed QPM tuned to the thermodynamics of the lattice QCD [61]. Such a fit leads to the following coupling [60]:
$g^{2}(T)=\frac{48 \pi^{2}}{\left[\left(11 N_{c}-2 N_{f}\right) \ln \left[\lambda\left(\frac{T}{T_{c}}-\frac{T_{s}}{T_{c}}\right)\right]^{2}\right.}$
where $\lambda=2.6$ and $T / T_{s}=0.57$.
Model-IV $\left(\alpha_{Q P M}(T), m_{q}=m_{g}=0\right)$ : To have a different set of drag and diffusion coefficients we are considering a case where the light quarks and gluons are massless but the coupling is from the QPM which is obtained from the fit to the lattice data. This case has to be mainly considered as an expedient to have a drag which decreases with $T$ as obtained for example in the T-matrix approach [3,14,37].

In the following, except for the case of AdS/CFT, we have calculated the drag coefficient numerically from the scattering matrix of the model by means of the standard definition of drag [6], see


Fig. 1. Variation of drag coefficient with respect to temperature.
also Ref. $[54,64]$ for a recent detailed description of the calculation of the transport coefficients for heavy quarks.

The variation of the drag coefficient with respect to temperature at $p=100 \mathrm{MeV}$ obtained within the four different models discussed above has been shown in Fig. 1. The behaviors remain quite similar also at high momentum but with different magnitude. These rescaled drag coefficients can reproduce almost the same $R_{A A}$ at RHIC energy. In AdS/CFT case the drag coefficient is proportional to $T^{2}$ whereas in $\alpha_{Q P M}(T), m_{q}=m_{g}=0$ case the drag coefficient decreases with T due to the strong coupling at low temperature. It may be mentioned here that the drag coefficient obtained from the T-matrix $[3,14,53]$ is almost constant or slightly decreasing with temperature.

We mention that the drag coefficient increases with temperature when the system behaves like a gas. For a molecular liquid the drag coefficient decreases with increasing temperature (except in a very few cases) because a significant part of the thermal energy goes into making the attraction between the interacting particles weaker, allowing them to move more freely and hence reducing the drag coefficient. The drag force of the partonic medium with non-perturbative effects may decrease with increasing temperature as shown in Refs. [ $3,14,53$ ] because in this case the medium interacts strongly more like a liquid.

In order to study the impact of the temperature dependence of the drag coefficient presented in the previous sections on the experimental observables, we have calculated the nuclear suppression factor, $R_{A A}$, using our initial charm and bottom quark distributions at initial time $t=\tau_{i}$ and final time $t=\tau_{f}$ at the freeze-out temperature as $R_{A A}(p)=\frac{f\left(p, \tau_{f}\right)}{f\left(p, \tau_{i}\right)}$. Along with $R_{A A}$ we evaluate the anisotropic momentum distribution induced by the spatial anisotropy of the bulk medium and defined as
$v_{2}=\left\langle\frac{p_{x}^{2}-p_{y}^{2}}{p_{x}^{2}+p_{y}^{2}}\right\rangle$,
which measures the momentum space anisotropy.
We have performed simulation of $A u+A u$ collisions at $\sqrt{s}=$ 200 AGeV for the minimum bias using a $3+1 \mathrm{D}$ transport approach [45, 46, 49]. The initial conditions for the bulk evolution in the coordinate space are given by the Glauber model condition, while in the momentum space we use a Boltzmann-Juttner distribution function up to a transverse momentum $p_{T}=2 \mathrm{GeV}$ and at larger momenta mini-jet distributions as calculated within PQCD at NLO order [22]. At RHIC energy, $A u+A u$ at $\sqrt{s}=200$, the maximum initial temperature of the fireball in the center is $T_{i}=340 \mathrm{MeV}$ and the initial time for the fireball simulations is $\tau_{i}=0.6 \mathrm{fm} / \mathrm{c}$ (according to the criteria $\tau_{i} \cdot T_{i} \sim 1$ ). The heavy quarks in momentum


Fig. 2. Comparison of the nuclear suppression factor, $R_{A A}$, as a function of $p_{T}$, obtained within the Langevin (LV) evolution for the four different cases, with the experimental data at RHIC energy.


Fig. 3. Comparison of the elliptic flow, $v_{2}$, as a function of $p_{T}$, obtained within the Langevin (LV) evolution for the four different cases, with the experimental data at RHIC energy.
space are distributed in accordance with the charm distribution in pp collisions that have been taken from Refs. [4,5] where in the coordinate space they are distributed according to $N_{\text {coll }}$.

The solution of the Langevin equation has been convoluted with the fragmentation functions of the heavy quarks at the quarkhadron transition temperature $T_{c}$ to obtain the momentum distribution of the D and B mesons. For the fragmentation, we use the Peterson fragmentation function:
$f(z) \propto \frac{1}{\left[z\left[1-\frac{1}{z}-\frac{\epsilon_{c}}{1-z}\right]^{2}\right]}$
where $\epsilon_{c}=0.04$ for charm quarks and $\epsilon_{c}=0.005$ for bottom quark.

In Fig. 2 we have plotted $R_{A A}$ as a function of $p_{T}$ for the four different cases obtained within the Langevin dynamics at RHIC energy. As we mentioned, we try to reproduce the same $R_{A A}$ in all the cases by rescaling the drag and diffusion coefficients. We remind that RHIC data and calculations refer to the single electrons from the semileptonic decay of D and B mesons. The $v_{2}$ for the same $R_{A A}$ has been displayed in Fig. 3 for all cases as a function of $p_{T}$. Our main striking point is that even if the $R_{A A}$ is very similar for all the four different cases, the $v_{2}$ built up is quite different depending on the temperature dependence of the drag coefficients (see Fig. 1). This is because the $R_{A A}$ is more sensitive to the early stage of the evolution whereas the $v_{2}$ is more sen-


Fig. 4. Comparison of the nuclear suppression factor $R_{A A}$ vs $v_{2}$, obtained within the Langevin (LV) evolution for the four different cases, with the experimental data at RHIC energy at $p_{T}=1.3 \mathrm{GeV}$.
sitive to the later stage of the evolution (near $T_{c}$ ). Some studies in this direction have been done also in the light flavor sector as shown in Refs. [66-68] and very recently related to the presence of magnetic monopoles [69]. The larger drag coefficient is at low temperature the larger is the $v_{2}$ even for the same $R_{A A}$. For example in the region of the peak for $v_{2}\left(p_{T}\right)$ we see a difference of about a factor 2.5 going from a $T^{2}$ dependence, like AdS/CFT to an inverse T dependence as it can occur in a liquid. This last case or at least a nearly constant drag appears to be very much favored by the comparison with the data.

This study suggests that the correct temperature dependence of drag coefficient has a crucial role for a simultaneous reproduction of $R_{A A}$ and $v_{2}$. The reason for such a relation between the two observables is that a small $R_{A A}$ (strong suppression) can be generated very quickly at the beginning of the QGP lifetime, i.e. at high T. However such a strong interaction will not be accompanied by a build-up of $v_{2}$ because the bulk medium has not yet developed a sizeable part of its elliptic flow. On the contrary to generate a large $v_{2}$ one needs there to be a strong interaction with the medium at later stages of the QGP lifetime in order to match the build-up of both $R_{A A}$ and $v_{2}$. The experimental data seem to clearly suggest that the drag of the medium cannot decrease with large power of T otherwise the interaction will be relatively weak just when a strong interaction would make possible the build of the anisotropy in momentum space. It can be here mentioned that the drag coefficient is almost constant with respect to temperature in the T-matrix case $[3,14,37,53]$. However also a QPM can be considered quite close to the data given that we have not included the coalescence mechanism that would shift the $v_{2}\left(p_{T}\right)$ in all the cases considered by about a $20-25 \%$ upward. In Fig. 4 we have introduced a new plot $R_{A A}$ vs $v_{2}$ at a given momentum ( $p_{T}=1.3 \mathrm{GeV}$ ) to promote the importance of simultaneous reproduction of $R_{A A}$ and $v_{2}$. Fig. 4 highlights how the $v_{2}\left(p_{T}\right)$ built up can differ up to a factor of around 2.5 (in the region of peak), for the same $R_{A A}\left(p_{T}\right)$, depending on the temperature dependence of the drag coefficient.

We have also extended our calculation to study $R_{A A}$ and $v_{2}$ at LHC performing simulations of $P b+P b$ at $\sqrt{s}=2.76$ ATeV energy. In this case the initial maximum temperature in the center of the fireball is $T_{0}=510 \mathrm{MeV}$ and the initial time for the simulations is $\tau_{0} \sim 1 / T_{0}=0.3 \mathrm{fm} / \mathrm{c}$. In Fig. 5 we show the $R_{A A}$ as a function of $p_{T}$ for the four different cases obtained within the Langevin dynamics at LHC energy. As we mentioned, we reproduce similar $R_{A A}$ in all the cases by rescaling the drag and diffusion coefficients. The elliptic flow $v_{2}$ for the same $R_{A A}$ has been plotted in Fig. 6 for


Fig. 5. Comparison of the nuclear suppression factor, $R_{A A}$, as a function of $p_{T}$, obtained within the Langevin (LV) evolution for the four different cases, with the experimental data at LHC energy.


Fig. 6. Comparison of the elliptic flow, $v_{2}$, as a function of $p_{T}$, obtained within the Langevin (LV) evolution for the four different cases, with the experimental data at LHC energy.
all cases as a function of $p_{T}$. Similarly to the RHIC case we get a similar trend for the $R_{A A}$ vs $v_{2}$ depending on the $T$ dependence drag coefficients.

However, as pointed out in Ref. [33], for charm quarks, which have a moderate $M / T$ ratio, a significant deviation with respect to the Brownian Langevin dynamics can be expected. In this case the full solution of the Boltzmann integral i.e. without the assumption of small collisional exchanged momenta, leads in general to a large $v_{2}\left(p_{T}\right)$. Such an effect depends on the anisotropy of the microscopic scattering and cannot be studied in terms of only the drag coefficient. It is however an effect that in general can be expected to be of the order of about $20 \%$ and does not modify the systematic studied here. A further effect that is involved in the study of HQ observable is related to the hadronization process. If the possibility of the coalescence process [13] is included, there is a further enhancement of the $v_{2}\left(p_{T}\right)$ of about a $20-25 \%$ [ $\left.9,14,70\right]$. Also the hadronic rescattering may play a role in enhancing the $v_{2}\left(p_{T}\right)$ without modifying the $R_{A A}\left(p_{T}\right)$ [65]. This however would generate a similar shift for all the cases discussed hence not affecting the discussed pattern entailed by $\Gamma(T)$. The impact of Boltzmann dynamics and hadronization by coalescence are larger at LHC and can be led to a better agreement with the data for the case $\alpha_{Q P M}(T)$ and QPM but does not modify the impact of the T-dependence of the drag coefficient discussed in this letter.

The results shown have been obtained evaluating the drag $\Gamma$ from the respective models and then the diffusion coefficient $D$ from FDT. Several other options are possible like evaluating the diffusion from the scattering matrix and the drag from the FDT or employing both drag and diffusion from the scattering matrix. We have seen that while these different options may lead to some differences, once they are tuned to $R_{A A}$, the differences in the elliptic flows stay within a $10 \%$ and in particular our main result on the impact of the T dependence of the drag is not affected by it.

In summary, we have evaluated the drag and diffusion coefficients of the heavy quarks within four different models. With these transport coefficients and heavy quark initial distributions we have solved the Langevin equation. The solution of Langevin equation has been used to evaluate the nuclear suppression factor, $R_{A A}$, and elliptic flow, $v_{2}$. The results have been compared with the experimental data both at RHIC and LHC energies. Our primary intent is to highlight how the temperature dependence of the interaction (drag coefficient) provides an essential ingredient for the simultaneous reproduction of the nuclear suppression factor, $R_{A A}$, and elliptic flow, $v_{2}$ which is a current challenge almost for all the existing model. Our work shows that the reproduction of the data on $R_{A A}\left(p_{T}\right)$ only cannot be used to determine the drag coefficient $\Gamma(T)$ of heavy quarks. We find that the different T-dependences of the drag coefficients in the literature can lead to differences in $v_{2}$ by 2-3 times even if the $R_{A A}$ is very similar. Our study suggests the correct temperature dependence of the drag coefficient cannot be a large power of $T$, like $T^{2}$ as in PQCD or AdS/CFT. We remind that $\Gamma(T)$ nearly constant or weakly decreasing with $T$ would be more typical of a liquid and not of a gas.

## Acknowledgement

We acknowledge the support by the ERC StG under the QGPDyn Grant No. 259684.

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