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MATHEMATISCHE NACHRICHTEN

Universal mappings for certain classes of operators and polynomials between Banach spaces

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Abstract

A well-known result of J. Lindenstrauss and A. Pełczyński (1968) gives the existence of a universal non-weakly compact operator between Banach spaces. We show the existence of universal non-Rosenthal, non-limited, and non-Grothendieck operators. We also prove that there does not exist a universal non-Dunford–Pettis operator, but there is a universal class of non-Dunford–Pettis operators. Moreover, we show that, for several classes of polynomials between Banach spaces, including the non-weakly compact polynomials, there does not exist a universal polynomial.

KEYWORDS

Ideals of homogeneous polynomials, surjective operator ideals, universal operator, universal polynomial

MSC (2010)

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1 | INTRODUCTION

J. Lindenstrauss and A. Pełczyński [30, Theorem 8.1] proved in 1968 that the *sum* operator $\sigma : \ell_1 \to \ell_\infty$ defined by

$$\sigma(x) := \left(\sum_{i=1}^{n} x_i \right)_{n=1}^{\infty} \qquad \text{for } x = (x_n)_{n=1}^{\infty} \in \ell_1$$

is universal for the class of non-weakly compact operators, that is, an operator $T \in \mathcal{L}(X, Y)$ is non-weakly compact if and only if there exist operators $A \in \mathcal{L}(\ell_1, X)$ and $B \in \mathcal{L}(Y, \ell_\infty)$ such that the following diagram commutes:

$$\begin{tikzcd} X \arrow[r, "T"] & Y \arrow[d, "B"] \\ \ell_1 \arrow[u, "A"] \arrow[r, "\sigma"'] & \ell_\infty \end{tikzcd}$$

W. B. Johnson [27] showed in 1971 that the formal identity operator $\ell_1 \to \ell_\infty$ is universal for the class of non-compact operators. These results are useful in order to prove that a given operator is non-weakly compact (respectively, non-compact). In 1997, M. Girardi and W. B. Johnson [21] proved that there does not exist a universal non-completely continuous operator, but there is a class $\mathcal{C}$ of universal non-completely continuous operators, that is, for every non-completely continuous operator T, there is some member of $\mathcal{C}$ that factors through T.

Here we prove the existence of universal operators for the classes of non-Rosenthal, non-Grothendieck, and non-limited operators and the existence of a universal class of non-Dunford–Pettis operators (see below the definitions of all such classes of operators). It seems quite natural and interesting to wonder about the existence of universal polynomials between Banach spaces. The notion of *ideal of k-homogeneous polynomials* is well known and has been widely studied in the literature (see, for instance, [12, §3] or the more recent paper [33]). We adapt to the polynomial setting a definition from [21]: given an integer $k \geq 1$, suppose that $\mathcal{Q}$ is a class of k-homogeneous (continuous) polynomials between Banach spaces so that a polynomial P is in $\mathcal{Q}$ whenever there exist (linear bounded) operators A, B so that $B \circ P \circ A$ is in $\mathcal{Q}$. The natural examples of such classes are the polynomials that do not belong to a given ideal of k-homogeneous polynomials. A polynomial P_0 of such a class $\mathcal{Q}$ is said to be *universal for* $\mathcal{Q}$ provided for each P in $\mathcal{Q}$, P_0 factors through P, that is, there exist operators A and B so that $B \circ P \circ A = P_0$.

The problem of the existence of universal polynomials seems to be very different with respect to the linear case and the lack of linearity introduces a degree of difficulty. In [13] we have proved that there are neither a universal non-compact polynomial nor a universal non-unconditionally converging polynomial. In the present paper we investigate the existence of a universal non-weakly compact polynomial between Banach spaces. We prove that the answer is again negative. Moreover, we show the nonexistence of universal non-Rosenthal, non-Asplund, non-limited, non-Grothendieck, and non-Dunford–Pettis polynomials. The techniques used here are in most cases different from those of [13].

Throughout, X, Y, E, and F denote Banach spaces, X^* is the dual space of X, B_X stands for its closed unit ball and S_X for its unit sphere. The closed unit ball B_{X^*} of the dual space will always be endowed with the weak-star topology. By $\mathbb{N}$ we represent the set of all natural numbers, and by $\mathbb{K}$ the scalar field (real or complex). We use the notation $\mathcal{L}(X, Y)$ for the space of all (linear bounded) operators from X into Y endowed with the operator norm. By I_X we denote the identity map on X. An operator $h \in \mathcal{L}(X, Y)$ is an *embedding* if $h(X)$ is isomorphic to X. For an embedding we use the arrow $\hookrightarrow$. The operator $k_X : X \hookrightarrow X^{**}$ is the canonical embedding of X into its bidual X^{**}.

In what follows, the notation (e_n^*) will be used for the canonical unit vector basis of ℓ_1 while (e_n) will be the canonical unit vector basis in c_0 or in ℓ_p with $p > 1$. By $\widehat{\otimes}^k_{\pi_s,s} X$ we denote the completion of the symmetric k-fold tensor product of X endowed with the symmetric projective tensor norm [20, 2.2]. Given $k \in \mathbb{N}$, we represent by $\mathcal{P}\left({}^k X, Y\right)$ the space of all k-homogeneous (continuous) polynomials from X into Y endowed with the supremum norm. For the general theory of polynomials on Banach spaces, we refer the reader to [16] and [32].

For a polynomial $P \in \mathcal{P}\left({}^k X, Y\right)$, its *linearization*

$$\overline{P} : \widehat{\otimes}^k_{\pi_s,s} X \longrightarrow Y$$

is the operator given by

$$\overline{P}\left(\sum_{j=1}^{n} \lambda_j x_j \otimes \overset{(k)}{\cdots} \otimes x_j\right) = \sum_{j=1}^{n} \lambda_j P(x_j)$$

for all $x_j \in X$ and $\lambda_j \in \mathbb{K}$ $(1 \leq j \leq n)$.

Every polynomial $P \in \mathcal{P}\left({}^k X, Y\right)$ has an extension to a polynomial

$$\widetilde{P} \in \mathcal{P}\left({}^k X^{**}, Y^{**}\right)$$

called the *Aron–Berner extension of* P. The origin of the Aron–Berner extension goes back to [2]. A survey of its properties may be seen in [40].

Recall that an operator ideal $\mathcal{U}$ is said to be *surjective* [34, 4.7.9] if, given $T \in \mathcal{L}(X, Y)$ and a surjective operator $q : G \to X$, we have that $T \in \mathcal{U}$ whenever $Tq \in \mathcal{U}$. We say that $\mathcal{U}$ is *closed* [34, 4.2.4] if, for all X and Y, the space

$$\mathcal{U}(X, Y) := \{T \in \mathcal{L}(X, Y) : T \in \mathcal{U}\}$$

is closed in $\mathcal{L}(X, Y)$.

A list of surjective operator ideals may be seen in [24]; here we consider some of them.

We recall some definitions and results from [13].

Definition 1.1. *Let $\mathcal{U}$ be a closed surjective operator ideal. As in [38] (see also [24, page 472]), we denote by $\mathcal{C}_{\mathcal{U}}(X)$ the collection of all sets $A \subset X$ so that $A \subseteq T(B_Z)$ for some Banach space Z and some operator $T \in \mathcal{U}(Z, X)$.*

Given a closed surjective operator ideal $\mathcal{U}$, let

$$\mathcal{P}_{\mathcal{U}}\left({}^k X, Y\right) := \left\{P \in \mathcal{P}\left({}^k X, Y\right) : P(B_X) \in \mathcal{C}_{\mathcal{U}}(Y)\right\}.$$

The space $\mathcal{P}_{\mathcal{U}}\left({}^kX,Y\right)$ will be endowed with the supremum norm. This construction is used in [4] for general operator ideals $\mathcal{U}$.

The following result can be found in [13, Proposition 2.4]. See also the proof of [39, Proposition 1] which works for $\mathcal{P}_{\mathcal{U}}$ with an immediate argument.

Proposition 1.2. *If $\mathcal{U}$ is a closed surjective operator ideal, the space $\mathcal{P}_{\mathcal{U}}\left({}^kX,Y\right)$ is closed in $\mathcal{P}\left({}^kX,Y\right)$.*

Proposition 1.3. *Given Banach spaces X and Y, and $k \in \mathbb{N}$, the space $\mathcal{P}\left({}^kX,Y\right)$ is isomorphic to a complemented subspace of $\mathcal{P}\left({}^{k+1}X,Y\right)$, that is, there are operators j and π*

$$\mathcal{P}\left({}^kX,Y\right) \xrightarrow{j} \mathcal{P}\left({}^{k+1}X,Y\right) \xrightarrow{\pi} \mathcal{P}\left({}^kX,Y\right)$$

such that $\pi \circ j = I_{\mathcal{P}({}^kX,Y)}$. Moreover, if $\mathcal{U}$ is a closed surjective operator ideal, restricting j and π to the spaces $\mathcal{P}_{\mathcal{U}}$, we have

$$\mathcal{P}_{\mathcal{U}}\left({}^kX,Y\right) \xrightarrow{j} \mathcal{P}_{\mathcal{U}}\left({}^{k+1}X,Y\right) \xrightarrow{\pi} \mathcal{P}_{\mathcal{U}}\left({}^kX,Y\right),$$

that is, j and π take polynomials in $\mathcal{P}_{\mathcal{U}}$ into polynomials in $\mathcal{P}_{\mathcal{U}}$.

The proof of the above result is contained in [5, Proposition 5.3] (see also [8, Proposition 5] and [13, Propositions 2.5]) and implies that, given $\psi \in S_{X^*}$, a polynomial Q belongs to $\mathcal{P}_{\mathcal{U}}\left({}^kX,Y\right)$ if and only if the polynomial $j(Q) := \psi Q$ belongs to $\mathcal{P}_{\mathcal{U}}\left({}^{k+1}X,Y\right)$, where ψQ is pointwise multiplication of ψ and Q.

For some of the results in this paper, Figure 1 may be helpful. The diagram in the figure is adapted from code of [19] which is based on [9].

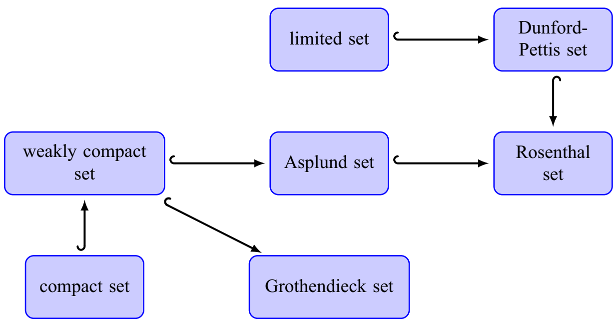

FIGURE 1 Some classes of sets in a Banach space

2 | THE WEAKLY COMPACT AND RELATED CASES

If $\mathcal{W}$ denotes the (surjective) ideal of weakly compact operators, $\mathcal{P}_{\mathcal{W}}\left({}^kX,Y\right)$ is the space of all k-homogeneous weakly compact polynomials from X into Y, that is, the space of all polynomials $P \in \mathcal{P}\left({}^kX,Y\right)$ such that $P(B_X)$ is relatively weakly compact in Y. We show that there does not exist a universal non-weakly compact polynomial, and extend this result to related classes.

Proposition 2.1. *Let*

$$P := \phi_1 \cdots \phi_{k-1} T \in \mathcal{P}\left({}^kX,Y\right) \setminus \mathcal{P}_{\mathcal{W}}\left({}^kX,Y\right),$$

where $\phi_1, \dots, \phi_{k-1} \in X^*$, not necessarily pairwise different, and $T \in \mathcal{L}(X,Y)$. Then, the Aron–Berner extension $\widetilde{P}$ of P is not Y-valued.

Proof. *Since $P \notin \mathcal{P}_{\mathcal{W}}\left({}^kX,Y\right)$, there is a net $(x_\alpha) \subset B_X$ such that $(P(x_\alpha))_\alpha$ does not have any weakly convergent subnet [29, Chapter 5, Theorem 2]. In particular, for every subnet (x_β) of (x_α), we must have $\phi_j(x_\beta) \nrightarrow 0$ $(1 \le j \le k-1)$; otherwise, since T is bounded, we would have*

$$P(x_\beta) = \phi_1(x_\beta) \cdots \phi_{k-1}(x_\beta) T(x_\beta) \longrightarrow 0,$$

a contradiction.

Using the weak**-compactness of* $B_{X^{**}}$ *and passing to a subnet if necessary, we may assume that* (x_α) *is weak***-convergent to some* $x^{**} \in B_{X^{**}}$ *[29, Chapter 5, Theorem 2]. In particular,*

$$\langle \phi_j, x^{**} \rangle = \lim_\alpha \phi_j(x_\alpha) \neq 0 \quad \textit{for } \ 1 \leq j \leq k-1 \,.$$

Since the second adjoint T^{**} *of* T *is weak***-to-weak** *continuous, we have*

$$T(x_\alpha) \xrightarrow{\textit{weak}^*} T^{**}(x^{**}) \,.$$

If $T^{**}(x^{**}) \in Y$*, the net* $(T(x_\alpha))_\alpha$ *would be weakly convergent, and the net*

$$(P(x_\alpha))_\alpha = \big(\phi_1(x_\alpha) \cdots \phi_{k-1}(x_\alpha) T(x_\alpha)\big)_\alpha$$

would also be weakly convergent, a contradiction.

Therefore,

$$\widetilde{P}(x^{**}) = \langle \phi_1, x^{**} \rangle \cdots \langle \phi_{k-1}, x^{**} \rangle T^{**}(x^{**}) \in Y^{**} \backslash Y$$

and the proof is finished. □

Remark 2.2. *Note that there are non-weakly compact polynomials in* $\mathcal{P}\big({}^k X, Y\big)$ *with* Y*-valued Aron–Berner extension. A typical example is the polynomial* $Q \in \mathcal{P}\big({}^k \ell_2, \ell_1\big)$ *used in the proof of the following Theorem 2.3.*

Theorem 2.3. *For* $k > 1$*, there does not exist a universal non-weakly compact* k*-homogeneous polynomial.*

Proof. *Suppose* $P \in \mathcal{P}\big({}^k E, F\big)$ *is universal non-weakly compact. Let* $Q \in \mathcal{P}\big({}^k \ell_2, \ell_1\big)$ *be the polynomial given by* $Q(\eta) := \big(\eta_n^k\big)_{n=1}^\infty$ *for* $\eta = (\eta_n)_{n=1}^\infty \in \ell_2$*. Since* $Q \notin \mathcal{P}_{\mathcal{W}}\big({}^k \ell_2, \ell_1\big)$*, we may factor* P *in the form*

$$\begin{tikzcd} \ell_2 \arrow[r, "Q"] & \ell_1 \arrow[d, "B"] \\ E \arrow[u, "A"] \arrow[r, "P"] & F \end{tikzcd}$$

Since $\widetilde{Q} = Q$*, the Aron–Berner extension of* $B \circ Q \circ A$ *is* F*-valued. Therefore,* $\widetilde{P}(E^{**}) \subseteq F$*.*

Choose an operator $S \in \mathcal{L}(X, Y) \backslash \mathcal{W}(X, Y)$ *and* $\psi_1, \ldots, \psi_{k-1} \in S_{X^*}$*. By iterating the proof of Proposition 1.3 (see [13, proofs of Proposition 2.2 and 2.5]), we have*

$$\psi_1 \cdots \psi_{k-1} S \in \mathcal{P}\big({}^k X, Y\big) \backslash \mathcal{P}_{\mathcal{W}}\big({}^k X, Y\big) \,.$$

Hence, there are $U \in \mathcal{L}(E, X)$ *and* $V \in \mathcal{L}(Y, F)$ *such that* P *factors in the form*

$$\begin{tikzcd} X \arrow[r, "\psi_1 \cdots \psi_{k-1} S"] & Y \arrow[d, "V"] \\ E \arrow[u, "U"] \arrow[r, "P"] & F \end{tikzcd}$$

Easily, we have

$$P = V \circ (\psi_1 \cdots \psi_{k-1} S) \circ U = (\psi_1 \circ U) \cdots (\psi_{k-1} \circ U) T \notin \mathcal{P}_{\mathcal{W}}\big({}^k E, F\big) \,,$$

where $T := V \circ S \circ U \in \mathcal{L}(E, F)$*.*

By Proposition 2.1, $\widetilde{P}(E^{**}) \not\subset F$*, a contradiction.* □

Recall that a subset A of a Banach space is said to be *Rosenthal* if every sequence in A has a weak Cauchy subsequence. An operator $T \in \mathcal{L}(X, Y)$ is said to be *Rosenthal* if $T(B_X)$ is a Rosenthal set. We denote by $\mathcal{R}$ the ideal of Rosenthal operators.

Proposition 2.4. *Every embedding of ℓ_1 into ℓ_∞ is a universal non-Rosenthal operator.*

Proof. *Let $h : \ell_1 \hookrightarrow \ell_\infty$ be an embedding. Let $T : X \to Y$ be a non-Rosenthal operator. Let $(x_n) \subset X$ be a sequence such that $(T(x_n))$ does not admit a weak Cauchy subsequence. By Rosenthal's ℓ_1-theorem, there is a subsequence that we still denote by $(T(x_n))$ equivalent to the ℓ_1-basis. Define $A \in \mathcal{L}(\ell_1, X)$ by $A(e_k^*) := x_k$. Let $B_0 : \overline{\operatorname{span}}\{T(x_n) : n \in \mathbb{N}\} \to \ell_1$ be the isomorphism such that $B_0(T(x_n)) = e_n^*$. Let $B_1 := h \circ B_0$. By the injectivity of ℓ_∞, B_1 admits an extension to an operator $B \in \mathcal{L}(Y, \ell_\infty)$. Then, the following diagram commutes:*

$$\begin{tikzcd}
X \arrow[r, "T"] & Y \arrow[d, "B"] \\
\ell_1 \arrow[u, "A"] \arrow[r, hook, "h"'] & \ell_\infty
\end{tikzcd}$$

and the proof is finished. □

Corollary 2.5. *Let $\mathcal{U}$ be a closed surjective operator ideal such that $\mathcal{W} \subseteq \mathcal{U} \subseteq \mathcal{R}$. Then, for $k > 1$, there does not exist a universal non-$\mathcal{P}_{\mathcal{U}}$ k-homogeneous polynomial.*

Proof. *It is enough to note that the polynomial Q used in the proof of Theorem 2.3 is non-Rosenthal and, therefore, it is not in $P_{\mathcal{U}}$ and that Proposition 2.1 also holds for polynomials of the form*

$$\psi_1 \cdots \psi_{k-1} S \in \mathcal{P}\left({}^k X, Y\right) \setminus \mathcal{P}_{\mathcal{R}}\left({}^k X, Y\right).$$ □

An operator $T \in \mathcal{L}(X, Y)$ is said to be *Asplund* if it factors through a Banach space each of whose separable subspaces has a separable dual [37].

It is proved in [37, Theorem 2.13] that the Haar operator $H : \ell_1 \to \ell_\infty(\Delta, \mu)$, where μ is the Haar measure on the Cantor set Δ, is a universal non-Asplund operator. The ideal of Asplund operators (see also [35]) satisfies the hypothesis of Corollary 2.5. Hence, we have:

Corollary 2.6. *For $k > 1$, there does not exist a universal non-Asplund k-homogeneous polynomial.*

Consider the (surjective) ideal $\mathcal{GR}$ of Grothendieck operators. Recall that $T \in \mathcal{L}(X, Y)$ is a *Grothendieck operator* [17] if every w^*-null sequence $(y_n^*) \subset Y^*$ is mapped by the adjoint T^* into a weakly null sequence $(T^*(y_n^*))$ in X^*. A subset $K \subset X$ is called a *Grothendieck set* if, for all $T \in \mathcal{L}(X, c_0)$, the set $T(K)$ is relatively weakly compact in c_0. Hence, $T \in \mathcal{GR}(X, Y)$ if and only if, for every bounded subset $A \subset X$, $T(A)$ is a Grothendieck set in Y [17, Section 1].

We shall prove that there is a universal non-Grothendieck operator but there does not exist a universal non-Grothendieck polynomial.

Theorem 2.7. *The operator $T_0 \in \mathcal{L}(\ell_1, c_0)$ given by $T_0(e_n^*) := e_1 + e_2 + \cdots + e_n$ for every $n \in \mathbb{N}$ is universal for the class of non-Grothendieck operators.*

Proof. *Let $T \in \mathcal{L}(X, Y) \setminus \mathcal{GR}(X, Y)$. Then, there is $S \in \mathcal{L}(Y, c_0)$ such that $S \circ T$ is non-weakly compact [17, Lemma 1.3]. Therefore, we can find a sequence $(x_n) \subset B_X$ such that $(S(T(x_n)))$ contains no weakly convergent subsequence. By [26, Theorem I.1.10], we can assume that $(S(T(x_n)))$ is a basic weak Cauchy sequence. Let $(y_n^*) \subset \ell_1$ be the coefficient functionals of $(S(T(x_n)))$. Every $e \in \overline{\operatorname{span}}\{S(T(x_n))\}$ has a unique representation of the form $e = \sum_{n=1}^{\infty} y_n^*(e) S(T(x_n))$. As in the proof of [30, Theorem 8.1], the sequence $\left(\sum_{i=1}^{n} y_i^*(e)\right)_{n=1}^{\infty}$ is convergent. Define*

$$U : \overline{\operatorname{span}}\{S(T(x_n))\} \longrightarrow c_0 \quad \textit{by} \quad U(e) := \left(\sum_{n=1}^{\infty} y_n^*(e), \sum_{n=2}^{\infty} y_n^*(e), \ldots\right) \in c_0 .$$

Clearly, U is bounded. By the separable injectivity of c_0 [6, Theorem 2.3], U has an extension $\widetilde{U} \in \mathcal{L}(c_0, c_0)$. Let $A \in \mathcal{L}(\ell_1, X)$ be given by $A(e_n^*) := x_n$. Then, for every $m \in \mathbb{N}$, we have

$$\begin{aligned}
\widetilde{U}\circ S\circ T\circ A\left(e_m^*\right) &= \widetilde{U}\circ S\circ T(x_m)\\
&= \left(\sum_{n=1}^{\infty} y_n^*(S(T(x_m))), \sum_{n=2}^{\infty} y_n^*(S(T(x_m))), \cdots\right)\\
&= \left(1,\overset{(m)}{\ldots},1,0,0,\ldots\right) = e_1+e_2+\cdots+e_m = T_0(e_m^*),
\end{aligned}$$

so the diagram

$$\begin{tikzcd}
X \arrow[r, "T"] & Y \arrow[r, "S"] & c_0 \arrow[d, "\widetilde{U}"] \\
\ell_1 \arrow[u, "A"] \arrow[rr, "T_0"'] & & c_0
\end{tikzcd}$$

commutes and this finishes the proof. □

Theorem 2.8. *For $k > 1$, there does not exist a universal non-Grothendieck k-homogeneous polynomial.*

Proof. *Since the unit vector basis of ℓ_1 is not a Grothendieck set in ℓ_1, the polynomial Q given in the proof of Theorem 2.3 is non-Grothendieck. Therefore, by the proof of Theorem 2.3, if there were a universal non-Grothendieck k-homogeneous polynomial $P_0 \in \mathcal{P}\left({}^kE, F\right)$, its Aron–Berner extension would be F-valued. On the other hand, if $P := \phi_1 \cdots \phi_{k-1}T \in \mathcal{P}\left({}^kX, Y\right) \setminus \mathcal{P}_{\mathcal{GR}}\left({}^kX, Y\right)$, P is also non-weakly compact. By Proposition 2.1, $\widetilde{P}(X^{**}) \not\subset Y$, and an easy adaptation of the proof of Theorem 2.3 yields a contradiction.* □

One of the referees has kindly pointed out the interest to study the existence of a universal non-Radon–Nikodým operator. Recall that an operator $T \in \mathcal{L}(E, F)$ is called *Radon–Nikodým* if for every probability measure space (Ω, μ) and every operator $S \in \mathcal{L}(L_1(\Omega, \mu), E)$, the operator $T \circ S$ is *representable,* that is, there is $g \in L_\infty(\mu, E)$ such that

$$T \circ S(f) = \int_\Omega f\, g \,\mathrm{d}\mu \qquad (f \in L_1(\Omega, \mu))$$

(see [34, 24.2], [15, Definition III.1.3]).

We feel that the search for a universal non-Radon–Nikodým operator would require some effort that we leave for a (hopefully) forthcoming paper. This solution would possibly come from a careful adaptation of ideas of [21].

Let (I_n) be a sequence of subintervals of $[0, 1]$ such that each point of $[0, 1]$ belongs to infinitely many I_n and $\mu(I_n) \to 0$ where μ is Lebesgue measure on $[0, 1]$ [11]. Define $U \in \mathcal{L}(L_1[0, 1], c_0)$ by

$$U(f) := \left(\int_{I_n} f \,\mathrm{d}\mu\right)_{n=1}^{\infty}.$$

Then U is non-Radon–Nikodým but is near Radon–Nikodým (also called strongly regular) [28]. The operator U might be a candidate for universality for the class of non-Radon–Nikodým operators.

3 | OTHER CLASSES OF OPERATORS AND POLYNOMIALS

In this section we consider other surjective operator ideals, namely the ideal of limited operators and the ideal of Dunford–Pettis operators. A bounded subset $K \subset X$ is *limited* (respectively, *Dunford–Pettis*) if

$$\lim_n \sup_{x\in K} \left|x_n^*(x)\right| = 0$$

for every weak*-null (respectively, weakly null) sequence $(x_n^*) \subset X^*$.

A bounded subset $K \subset X^*$ is called an L-*set* if

$$\lim_n \sup_{x^* \in K} \left| x^*(x_n) \right| = 0$$

for every weakly null sequence $(x_n) \subset X$. It is easily seen that every relatively compact subset of X is limited, every limited set is Dunford–Pettis, and every Dunford–Pettis set in a dual space is an L-set (this last assertion is obvious from the definitions and is stated in the proof of [18, Corollary 1]), but the converse assertions are false in general.

An operator $T \in \mathcal{L}(X, Y)$ is *limited* (respectively, *Dunford–Pettis*) if $T(B_X)$ is limited (respectively, Dunford–Pettis). We prove the existence of universal non-limited operators and the nonexistence of a universal non-Dunford–Pettis operator. However, we show that there is a class of universal non-Dunford–Pettis operators. We also study the related polynomial cases.

Proposition 3.1. *The natural inclusion $i : \ell_1 \to c_0$ is a universal non-limited operator.*

Proof. *Let $T \in \mathcal{L}(X, Y)$ be a non-limited operator. Then, there are a bounded sequence $(x_n) \subset X$, a weak*-null sequence $(y_n^*) \subset Y^*$, and $\delta > 0$ such that*

$$\left| \left\langle T(x_n), y_n^* \right\rangle \right| > \delta \qquad (n \in \mathbb{N}) .$$

The sequence $\left(T^*\left(y_n^*\right)\right)$ is weak-null but it is not norm null. Hence, by [25, Lemma 3.1.19], there are a subsequence $\left(T^*\left(y_{n_k}^*\right)\right)$ and a bounded sequence $(z_n) \subset X$ such that*

$$\left\langle z_n, T^*\left(y_{n_k}^*\right) \right\rangle = \delta_{n,k} .$$

Define $A \in \mathcal{L}(\ell_1, X)$ by $A\left(e_n^*\right) := z_n$ and $B \in \mathcal{L}(Y, c_0)$ by

$$B(y) := \left(y_{n_k}^*(y) \right)_{k=1}^{\infty} .$$

We show that the following diagram commutes:

$$\begin{tikzcd} X \arrow[r, "T"] & Y \arrow[d, "B"] \\ \ell_1 \arrow[u, "A"] \arrow[r, "i"'] & c_0 \end{tikzcd}$$

Indeed,

$$B\left(T\left(A\left(e_m^*\right)\right)\right) = B(T(z_m)) = \left(\left\langle T(z_m), y_{n_k}^* \right\rangle\right)_{k=1}^{\infty} = \left(\left\langle z_m, T^*\left(y_{n_k}^*\right)\right\rangle\right)_{k=1}^{\infty} = (\delta_{m,k})_{k=1}^{\infty} = e_m = i(e_m^*),$$

so $B \circ T \circ A = i$. □

We say that a polynomial $P \in \mathcal{P}\left({}^k X, Y\right)$ is *limited* if $P(B_X)$ is a limited set in Y. We shall prove that there does not exist a universal non-limited polynomial.

Lemma 3.2. *If there is a universal non-limited k-homogeneous polynomial, then there is a universal non-limited k-homogeneous polynomial defined on ℓ_1.*

Proof. *If $P_0 \in \mathcal{P}\left({}^k E, F\right)$ is a universal non-limited polynomial, there is a sequence $(x_n) \subset B_E$ such that $(P(x_n))$ is not limited. Define $A \in \mathcal{L}(\ell_1, E)$ by $A\left(e_n^*\right) = x_n$ for all $n \in \mathbb{N}$. Then $P_0 \circ A$ is a universal non-limited polynomial on ℓ_1.* □

Proposition 3.3. *If $P_0 \in \mathcal{P}\left({}^k \ell_1, F\right)$ is a universal non-limited polynomial then we can take $F = c_0$ and $P_0 = \xi^{k-1} i$ where $i : \ell_1 \to c_0$ is the natural inclusion and $\xi \in \ell_\infty$.*

Proof. *The proof is as in [13, Proposition 2.7]. We sketch it for the reader's convenience. Since $P_0 \in \mathcal{P}\left({}^k \ell_1, F\right)$ is non-limited, Proposition 1.3 implies that there is a non-limited operator $T \in \mathcal{L}(\ell_1, F)$. Choose $0 \neq \eta \in \ell_\infty$. Define $P \in \mathcal{P}\left({}^k \ell_1, F\right)$*

by $P(x^*) := \langle x^*, \eta\rangle^{k-1} T(x^*)$ *for all* $x^* \in \ell_1$*. By Proposition 1.3,* P *is non-limited. Therefore,* P_0 *factors through* $\eta^{k-1}T \in \mathcal{P}\left({}^k\ell_1, F\right)$ *in the form* $P_0 = B \circ \left(\eta^{k-1}T\right) \circ A$*. Letting* $S := B \circ T \circ A$ *and* $\psi := \eta \circ A \in \ell_\infty$*, we obtain* $P_0 = \psi^{k-1}S$*. Again by Proposition 1.3,* i *factors through* S*. Letting* $\xi := \psi \circ U \in \ell_\infty$*, we obtain that* $\xi^{k-1}i$ *factors through* $\psi^{k-1}S = P_0$*, and* $\xi^{k-1}i$ *is universal for the class of non-limited polynomials.* □

Theorem 3.4. *For* $k > 1$*, there does not exist a universal non-limited k-homogeneous polynomial.*

Proof. *Suppose that there is a universal non-limited k-homogeneous polynomial* P_0*. By Proposition 3.3 it may be chosen of the form* $P_0 = \xi^{k-1}i \in \mathcal{P}\left({}^k\ell_1, c_0\right)$*. As in [13, Theorem 2.8], define* $P \in \mathcal{P}\left({}^k\ell_1, c_0\right)$ *by*

$$P(\phi) := \left(\phi_1^{k-2}\phi_n^2\right)_{n=1}^{\infty} \qquad (\phi = (\phi_n)_{n=1}^{\infty} \in \ell_1).$$

Note that P *is non-limited. Indeed, for* $r > 1$*,*

$$P\left(e_1^* + e_r^*\right) = \left(\left\langle e_1, e_1^* + e_r^*\right\rangle^{k-2}\left\langle e_n, e_1^* + e_r^*\right\rangle^2\right)_{n=1}^{\infty} = \left(\left\langle e_n, e_1^* + e_r^*\right\rangle^2\right)_{n=1}^{\infty} = e_1 + e_r$$

and

$$\sup_r \left|\left\langle P\left(e_1^* + e_r^*\right), e_n^*\right\rangle\right| = \sup_r \left|\left\langle e_1 + e_r, e_n^*\right\rangle\right| = 1 \quad \text{for all } r \in \mathbb{N} \quad (r > 1).$$

Since $\left(e_n^*\right)$ *is weak*-null in* ℓ_1 *and* $e_1^* + e_r^* \in 2B_{\ell_1}$*, we obtain that* $P\left(2B_{\ell_1}\right) = 2^k P\left(B_{\ell_1}\right)$ *is non-limited, so* P *is non-limited. By the assumption, there are operators* $A \in \mathcal{L}(\ell_1, \ell_1)$ *and* $B \in \mathcal{L}(c_0, c_0)$ *such that the following diagram commutes:*

$$\begin{tikzcd} \ell_1 \arrow[r, "P"] & c_0 \arrow[d, "B"] \\ \ell_1 \arrow[u, "A"] \arrow[r, "\xi^{k-1}i"'] & c_0 \end{tikzcd}$$

The same proof of [13, Theorem 2.8], which is omitted because it is rather technical, leads to a contradiction which finishes the proof. □

We now consider the class of Dunford–Pettis operators. We shall prove that there is neither a universal non-Dunford–Pettis operator nor a universal non-Dunford–Pettis polynomial. Recall that an operator $T \in \mathcal{L}(X, Y)$ is *completely continuous* if it takes weakly null sequences of X into norm null sequences in Y.

Proposition 3.5. *([22, Proposition 3.2]) An operator* $T \in \mathcal{L}(X, Y)$ *is completely continuous if and only if* $T^*\left(B_{Y^*}\right)$ *is an L-set.*

Theorem 3.6. *There does not exist a universal non-Dunford–Pettis operator.*

Proof. *Suppose that* $T_0 \in \mathcal{L}(E, F)$ *is a universal non-Dunford–Pettis operator. Let* $T \in \mathcal{L}(X, Y)$ *be a non-completely continuous operator. We follow the first step of the proof of [21, Theorem 5] to construct a "better" non-completely continuous operator* $J \in \mathcal{L}(Z, \ell_\infty)$ *where* Z *will be reflexive. We can find a weakly null sequence* $(x_n) \subset X$ *with* $(T(x_n))$ *bounded away from zero. By passing to a subsequence if necessary, we can assume that* $(T(x_n))$ *is a basic sequence in* Y*. By [14, Corollary 7], there exists a reflexive Banach space* Z *with an unconditional basis* (z_n) *such that the operator* $S : Z \to X$ *which takes* z_n *into* x_n *is continuous. Let* $U \in \mathcal{L}(Y, \ell_\infty)$ *be given by* $U(y) := (\langle y, y_n^*\rangle)_{n=1}^{\infty}$ *where* $\left(y_n^*\right)$ *is a bounded sequence in* Y^* *such that* $\left\{T(x_n), y_n^*\right\}$ *is a biorthogonal system. The operator* $J := U \circ T \circ S \in \mathcal{L}(Z, \ell_\infty)$ *takes* z_n *into the nth unit vector* e_n *of* ℓ_∞*. The reflexivity of* Z *implies that* J *is non-completely continuous.*

By Proposition 3.5, $J^* \in \mathcal{L}\left(\ell_\infty^*, Z^*\right)$ *is non-Dunford–Pettis, since every Dunford–Pettis set in a dual space is an L-set (this is clear from the definitions and is stated in the proof of [18, Corollary 1]). Since* T_0 *is universal non-Dunford–Pettis, there are* $A \in \mathcal{L}\left(E, \ell_\infty^*\right)$ *and* $B \in \mathcal{L}(Z^*, F)$ *such that the following diagram commutes:*

$$\begin{tikzcd} \ell_\infty^* \arrow[r, "J^*"] & Z^* \arrow[d, "B"] \\ E \arrow[u, "A"] \arrow[r, "T_0"'] & F \end{tikzcd}$$

Taking adjoints:

$$\begin{tikzcd}
Z \arrow[r, "J^{**}=J"] & \ell_\infty \arrow[d, "A^* \circ k_{\ell_\infty}"] \\
F^* \arrow[u, "B^*"] \arrow[r, "T_0^*"'] & E^*
\end{tikzcd}$$

so the following diagram commutes:

$$\begin{tikzcd}
Z \arrow[r, "S"] \arrow[rrr, bend left, "J"] & X \arrow[r, "T"] & Y \arrow[r, "U"] \arrow[dr, "A^* \circ k_{\ell_\infty} \circ U"'] & \ell_\infty \arrow[d, "A^* \circ k_{\ell_\infty}"] \\
F^* \arrow[u, "B^*"] \arrow[ur, "S \circ B^*"'] \arrow[rrr, "T_0^*"] & & & E^*
\end{tikzcd}$$

Since T_0 is non-Dunford–Pettis, T_0^* is non-completely continuous [23, Proposition 2.1]. The above diagram shows that T_0^* is a universal non-completely continuous operator which is in contradiction with [21, Theorem 5]. This completes the proof. □

The following two results draw heavily on [21]. The first one yields the existence of a class of universal non-Dunford–Pettis operators. The second one gives more information when the range space is super-reflexive.

Theorem 3.7. *Let $\mathcal{C}$ be the class of all operators from ℓ_1 into a reflexive sequence space with a normalized unconditional basis (u_n) which take the unit vector basis of ℓ_1 into (u_n). Then $\mathcal{C}$ is universal for the class of non-Dunford–Pettis operators.*

Proof. *Let $T \in \mathcal{L}(X, Y)$ be a non-Dunford–Pettis operator. Then, T^* is non-completely continuous [23, Proposition 2.1]. So there are a weakly null sequence $(y_n^*) \subset Y^*$ and $\delta > 0$ such that*

$$\left\| T^*\left(y_n^*\right) \right\| > \delta \qquad (n \in \mathbb{N}).$$

We can assume that $\left(T^*\left(y_n^*\right)\right)$ is basic. By [25, Lemma 3.1.19], we can find a bounded sequence (x_j) in X such that $\left\langle x_j, T^*\left(y_n^*\right)\right\rangle = \delta_{j,n}$. As in the beginning of the proof of Theorem 3.6, there are a reflexive Banach space G with an unconditional basis (g_n), and an operator $A \in \mathcal{L}(G, Y^*)$ such that $A(g_n) = y_n^*$. Define $B \in \mathcal{L}(X^*, \ell_\infty)$ by $B(x^*) := \left(\left\langle x_j, x^*\right\rangle\right)_{j=1}^\infty \in \ell_\infty$. Then

$$B \circ T^* \circ A(g_n) = B \circ T^*\left(y_n^*\right) = \left(\left\langle x_j, T^*\left(y_n^*\right)\right\rangle\right)_{j=1}^\infty = (\delta_{j,n})_{j=1}^\infty = e_n .$$

Letting $J \in \mathcal{L}(G, \ell_\infty)$ be defined by $J(g_n) = e_n$, the diagram

$$\begin{tikzcd}
Y^* \arrow[r, "T^*"] & X^* \arrow[d, "B"] \\
G \arrow[u, "A"] \arrow[r, "J"'] & \ell_\infty
\end{tikzcd}$$

is commutative. The sequence $\left(g_n^*\right)$ of the coefficient functionals of (g_n) is an unconditional basis of G^* [31, Proposition 1.b.1, Theorem 1.b.5, and page 19]. Define the operators $S \in \mathcal{L}(\ell_1, G^*)$, $U \in \mathcal{L}(\ell_1, X)$, and $V \in \mathcal{L}(Y, G^*)$ by

$$S(e_n^*) := g_n^* \qquad (n \in \mathbb{N}),$$

$$U(e_n^*) := x_n \qquad (n \in \mathbb{N}),$$

$$V := A^* \circ k_Y .$$

Then,

$$\begin{aligned}\left\langle g_n, V\circ T\circ U\left(e_m^*\right)\right\rangle &= \langle g_n, V\circ T(x_m)\rangle = \langle g_n, A^*\circ k_Y\circ T(x_m)\rangle \\ &= \langle A(g_n), k_Y\circ T(x_m)\rangle = \langle T(x_m), A(g_n)\rangle \\ &= \left\langle T(x_m), y_n^*\right\rangle = \delta_{n,m} = \left\langle g_n, g_m^*\right\rangle = \left\langle g_n, S\left(e_m^*\right)\right\rangle .\end{aligned}$$

Hence, the following diagram

$$\begin{tikzcd} X \arrow[r, "T"] & Y \arrow[d, "V"] \\ \ell_1 \arrow[u, "U"] \arrow[r, "S"'] & G^* \end{tikzcd}$$

commutes and the proof is complete since $S\in \mathcal{C}$. ☐

Theorem 3.8. *Let $\mathcal{C}$ be the collection of all natural inclusions $i_q : \ell_1 \to \ell_q$ $(1 < q < \infty)$. Then $\mathcal{C}$ is universal for the class of non-Dunford–Pettis operators with range in a super-reflexive space.*

Proof. *Let $T\in\mathcal{L}(X,Y)$ be a non-Dunford–Pettis operator, with Y a super-reflexive space. Then T^* is non-completely continuous, so we can find a weakly null sequence $\left(y_n^*\right)$ in Y^* such that $\left(T^*\left(y_n^*\right)\right)$ is bounded away from zero. We can assume that $\left(y_n^*\right)$ is normalized and basic. Again by [25, Lemma 3.1.19], passing to a subsequence if necessary, we can find a bounded sequence (x_j) in X such that $\left\langle x_j, T^*\left(y_n^*\right)\right\rangle = \delta_{j,n}$. Since Y^* is super-reflexive [7, 4, I, §3, Corollary 8], there are $1 < p < \infty$ and $A\in\mathcal{L}\left(\ell_p, Y^*\right)$ such that $A(e_n) = y_n^*$ for every $n\in\mathbb{N}$ [7, 4, II, Theorem 1]. Let q be the conjugate index of p. Let $V := A^*\circ k_Y$ and let $U\in\mathcal{L}(\ell_1, X)$ be given by $U\left(e_n^*\right) = x_n$. Then we have*

$$\begin{aligned}\left\langle e_n, V\circ T\circ U\left(e_m^*\right)\right\rangle &= \langle e_n, A^*\circ k_Y\circ T(x_m)\rangle = \langle A(e_n), k_Y\circ T(x_m)\rangle \\ &= \left\langle y_n^*, k_Y\circ T(x_m)\right\rangle = \left\langle T(x_m), y_n^*\right\rangle = \delta_{m,n} = \left\langle e_n, i_q\left(e_m^*\right)\right\rangle ,\end{aligned}$$

and the diagram

$$\begin{tikzcd} X \arrow[r, "T"] & Y \arrow[d, "V"] \\ \ell_1 \arrow[u, "U"] \arrow[r, "i_q"'] & \ell_q \end{tikzcd}$$

is commutative, which concludes the proof. ☐

We say that a polynomial $P\in\mathcal{P}\left({}^kE, F\right)$ is *Dunford–Pettis* if $P\left(B_E\right)$ is a Dunford–Pettis set in F. The proof of the following lemma is as in Lemma 3.2.

Lemma 3.9. *If there is a universal non-Dunford–Pettis polynomial, then there is also a universal non-Dunford–Pettis polynomial defined on ℓ_1.*

We can now prove the following

Theorem 3.10. *If $k > 1$, there does not exist a universal non-Dunford–Pettis k-homogeneous polynomial.*

Proof. *Suppose that $P_0\in\mathcal{P}\left({}^k\ell_1, F\right)$ is a universal non-Dunford–Pettis polynomial. Let $T\in\mathcal{L}(X,Y)$ be a non-Dunford–Pettis operator. For every index $i\in\{1,\dots,m-1\}$, let*

$$j_i : \widehat{\otimes}^i_{\pi_s,s} X \hookrightarrow \widehat{\otimes}^{i+1}_{\pi_s,s} X \qquad \text{and} \qquad \pi_i : \widehat{\otimes}^{i+1}_{\pi_s,s} X \longrightarrow \widehat{\otimes}^{i}_{\pi_s,s} X$$

be the operators introduced in [8] such that $\pi_i \circ j_i$ is the identity map on $\widehat{\otimes}^i_{\pi_s,s}X$. The operator $T \circ \pi_1 \circ \cdots \circ \pi_{k-1} : \widehat{\otimes}^k_{\pi_s,s}X \to Y$ cannot be Dunford–Pettis since $T = T \circ \pi_1 \circ \cdots \circ \pi_{k-1} \circ j_{k-1} \circ \cdots \circ j_1$ is not so. Let $\delta : X \to \widehat{\otimes}^k_{\pi_s,s}X$ be the canonical polynomial given by $\delta(x) := x \otimes \overset{(k)}{\cdots} \otimes x$ for $x \in X$, and let

$$P := T \circ \pi_1 \circ \cdots \circ \pi_{k-1} \circ \delta \in \mathcal{P}\left({}^k X, Y\right) .$$

Its linearization

$$\overline{P} = T \circ \pi_1 \circ \cdots \circ \pi_{k-1} : \widehat{\otimes}^k_{\pi_s,s}X \longrightarrow Y$$

is non-Dunford–Pettis, so the polynomial P is non-Dunford–Pettis either [3, Theorem 4.5]. Therefore, there are operators $A \in \mathcal{L}(\ell_1, X)$ and $B \in \mathcal{L}(Y, F)$ such that the following diagram commutes

$$\begin{tikzcd} X \arrow[r, "P"] & Y \arrow[d, "B"] \\ \ell_1 \arrow[u, "A"] \arrow[r, "P_0"'] & F \end{tikzcd}$$

Taking linearizations, we obtain the following commutative diagram

$$\begin{tikzcd} \widehat{\otimes}^k_{\pi_s,s}X \arrow[r, "\overline{P}"] & Y \arrow[d, "B"] \\ \widehat{\otimes}^k_{\pi_s,s}\ell_1 \arrow[u, "\otimes^k A"] \arrow[r, "\overline{P_0}"'] & F \end{tikzcd}$$

Let $i : \ell_1 \to \widehat{\otimes}^k_{\pi_s,s}\ell_1$ be an onto isomorphism (for the nonsymmetric tensor product, see [36, Exercise 2.6]; the symmetric case may be viewed as an application of the Pelczyński decomposition technique [1, Theorem 2.2.3]). Then the following diagram is commutative:

$$\begin{tikzcd} \widehat{\otimes}^k_{\pi_s,s}X \arrow[rr, "\pi_1 \circ \cdots \circ \pi_{k-1}"] & & X \arrow[r, "T"] & Y \arrow[d, "B"] \\ \ell_1 \arrow[u, "(\otimes^k A) \circ i"] \arrow[r, "i"'] & \widehat{\otimes}^k_{\pi_s,s}\ell_1 \arrow[ul, "\otimes^k A"'] \arrow[rr, "\overline{P_0}"'] & & F \end{tikzcd}$$

Setting $A_1 := \pi_1 \circ \cdots \circ \pi_{k-1} \circ \left(\otimes^k A\right) \circ i \in \mathcal{L}(\ell_1, X)$, we obtain $\overline{P_0} \circ i = B \circ T \circ A_1$, so $\overline{P_0} \circ i$ is a universal non-Dunford–Pettis operator, in contradiction with Theorem 3.6. $\square$

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