Journal of Physics G: Nuclear and Particle Physics

LETTER

Spin-dipole nuclear matrix element for the double beta decay of ^{76}Ge by the (^{3}He, *t*) charge-exchange reaction

To cite this article: H Akimune *et al* 2020 *J. Phys. G: Nucl. Part. Phys.* **47** 05LT01

Letters

IOP Publishing Journal of Physics G: Nuclear and Particle Physics

J. Phys. G: Nucl. Part. Phys. **47** (2020) 05LT01 (9pp) https://doi.org/10.1088/1361-6471/ab7a87

Letter

Spin-dipole nuclear matrix element for the double beta decay of ^{76}Ge by the (^{3}He, *t*) charge-exchange reaction

H Akimune[1], H Ejiri[2,13], F Hattori[1], C Agodi[3], M Alanssari[4], F Cappuzzello[3,5], D Carbone[3], M Cavallaro[3], G Coló[6], F Diel[7], C A Douma[8], D Frekers[4], H Fujita[9], Y Fujita[9], M Fujiwara[2], G Gey[2], M N Harakeh[8], K Hatanaka[2], K Heguri[1], M Holl[4], A Inoue[2], N Kalantar-Nayestanaki[8], Y F Niu[10], P Puppe[4], P C Ries[11], A Tamii[2], V Werner[11] and K Zuber[12]

[1] Konan University, 8 Chome-9-1 Okamoto, Higashinada, Kobe, Hygo, Japan
[2] RCNP, Osaka University, Ibaraki, Osaka, 567-0047, Japan
[3] INFN Laboratori Nazionali del Sud, via S. Sofia 62, I-95125 Catania, Italy
[4] Institut fűr Kernphysik, Universita˝t Műnster, D-48341 Műnster, Germany
[5] Dipartimento di Fisica e Astronomia 'Ettore Majorana', Catania University, via S. Sofia 64, I-95125 Catania, Italy
[6] Dipartimento di Fisica, Universitá degli Studi di Milano, Via Celoria 16, I-20133 Milano, Italy
[7] Universität zu Kőln, Kőln University, Zülpicher Str. 77, D-50937 Köln, Germany
[8] KVI-CART, University of Groningen, Zernikelaan 25, 9747 AA Groningen, The Netherlands
[9] Department of Physics, Osaka University, Toyonaka, Osaka 560, Japan
[10] Nuclear Science and Technology, Lanzhou University, Lanzhou 730000, People's Republic of China
[11] Institut fűr Kernphysik, TU Darmstadt, Schlossgartenstr. 9, D-64289 Darmstadt, Germany
[12] IKTP, TU Dresden, D-01069 Dresden, Germany

E-mail: ejiri@rcnp.osaka-u.ac.jp

Received 13 December 2019, revised 21 February 2020
Accepted for publication 26 February 2020
Published 26 March 2020

CrossMark

Abstract

Nuclear matrix elements (NMEs) for double beta decays (DBDs) are crucial for studying the neutrino mass and other neutrino properties beyond the standard electro-weak model by measuring neutrino-less DBDs. The spin-dipole (SD) $J^{\pi} = 2^{-}$ NME is one of the major components associated with the DBD NME. The SD NME for ^{76}Ge was derived for the first time by using the

[13] Author to whom any correspondence should be addressed.

^{74,76}Ge (^{3}He, t) at RCNP Osaka. The obtained SD NME for the ^{76}Ge $\rightarrow$ ^{76}As ground-state transition is $|M^-_{\mathrm{EXP}}(\mathrm{SD})| = 1.5 \times 10^{-3}$ in natural units. This is smaller by a coefficient around $k \approx 0.2$ with respect to the quasi-particle model NME $|M^-_{\mathrm{QP}}(\mathrm{SD})|$. The impact of the reduced (quenched) SD NME on DBD neutrino studies is discussed.

Keywords: double beta decay of ^{76}Ge, charge-exchange reaction, spin-dipole matrix element, quenching of axial-vector coupling g_A

(Some figures may appear in colour only in the online journal)

Neutrino-less double beta decay (DBD) is a very sensitive and realistic probe for studying neutrino properties such as the Majorana nature, the absolute mass scale and the mass hierarchy, the lepton-sector CP phase and other properties beyond the standard electro-weak model. The nuclear matrix element (NME) is a key element for extracting the effective neutrino mass and other properties of particle-physics interest beyond the standard model from the experimental DBD rate, if the decay is observed. It is even crucial for the design of the DBD detector since the DBD-isotope mass required to measure the rare DBD depends very much on the NME, as we discuss later. These subjects are discussed in [1–5]. DBD NMEs are discussed in [6–13], and single-β NMEs are in [1–3, 14, 15].

This letter presents the first experimental study of a major component—the spin-dipole (SD) single-β NME M^-(SD)—for a key DWB nucleus ^{76}Ge [3, 7, 8]. It reports the ^{76}Ge $\rightarrow$ ^{76}As ground-state transition M^-(SD) value determined from the (^{3}He, t) charge-exchange reaction (CER) at the medium energy of $E(^3\mathrm{He}) = 420$ MeV, where the spin-isospin central interaction to excite the present SD state gets dominant over the isospin central interaction and other non-central/tensor interactions [2, 3]. Here the SD NME is one of the major components associated with the DBD NME because the orbital angular-momentum transfer $\Delta L \approx 1$ (i.e. dipole) just corresponds to the momentum transfer of $q \approx 60\,\mathrm{MeV}/c$ associated with the neutrino propagator in the DBD nucleus and the spin-flip SD NME itself is large in this mass region. ^{76}Ge is one of the key DBD nuclei in DBD experiments because Ge detectors with the ^{76}Ge isotopes are used as DBD detectors with high energy-resolution, which is very important to separate very rare (10^{-34} per sec. or less) low-energy DBD signals from other background ones [7, 9].

It is extremely difficult to calculate DBD NMEs accurately since they are very sensitive to nucleonic and non-nucleonic correlations in nuclei and to nuclear medium effects, some of which are effectively incorporated in the effective axial-vector coupling g_A^{eff}. Consequently, the calculated DBD NMEs strongly depend on nuclear models and nuclear parameters, being scattered within an order of magnitude [3, 9]. Thus experimental inputs are quite important in helping DBD NME evaluation [2, 3, 7, 9].

The neutrino-less DBD is written as ${}^A_Z X \leftrightarrow {}^{A}_{Z-2}X$ with A being the mass number and Z being the atomic number. We consider the light Majorana-mass process, which is of great interest because the non-zero neutrino-less DBD rate is expected from the neutrino-oscillation experiments [7, 9]. The process is expressed schematically as ${}^A_Z X \leftrightarrow {}^{A}_{Z-1}X \leftrightarrow {}^{A}_{Z-2}X$, where the light virtual-neutrino with the medium momentum of $q \approx 20$–$200\,\mathrm{MeV}/c$ is exchanged in the intermediate nucleus ${}^{A}_{Z-1}X$ [3, 9].

The neutrino-less DBD NME is associated with the τ^- - and τ^+ - side single-β NMEs, $M^-(\alpha)$ for ${}^A_Z X \rightarrow {}^{A}_{Z-1}X$ and $M^+(\alpha)$ for ${}^{A}_{Z-1}X \leftarrow {}^{A}_{Z-2}X$, with τ^- and τ^+ being the isospin-lowering and isospin-raising operators and α being the transition mode. The single-β NMEs associated with the DBD NME are $M^{\pm}$ (J^{π}) with J^{π} being the spin-parity of the intermediate state. The states with $J^{\pi} = 0^{\pm}$, $1^{\pm}$, $2^{\pm}$ and so on up to $J \approx 6$ are involved [3, 4]. Among them, $M^{\pm}(2^{-})$ plays an important role for the neutrino-less DBD NME, while $M^{\pm}(1^{+})$ does for the two-neutrino DBD NME. The 2^{-} transition, which is mainly due to the spin dipole (SD) operator discussed later, is denoted as SD, while the 1^{+} one due to the Gamow–Teller (GT) is denoted as GT.

So far, the single-β $M^{\pm}$(GT) for DBD nuclei have been studied by using measured single-$\beta^{\pm}$ and EC rates (conventionally *ft* values) for some DBD nuclei with the ground state 1^{+}. CERs on DBD nuclei have been used to measure $M^{\pm}$(GT) for the low-lying GT(1^{+}) states in DBD nuclei, as given in [3]. Recently muon CERs have been shown to be a useful tool to study the $M^{+}(J^{\pi})$ in wide energy and momentum ranges [16–19]. Experimental studies of the DBD NMEs by using lepton, photon and nuclear probes are given in recent reviews [2, 3, 20]. The quenching of the axial-vector coupling g_A for the large momentum transfer, which is relevant to the neutrino-less DBD, is discussed in [21]. The GT response for ^{116}Sn has been studied recently [22]. Double CERs are interesting to study DBD NMEs [23, 24].

The CER cross section for the α mode excitation by the α mode interaction is expressed in terms of the α mode nuclear response $B(\alpha)$ as [2, 3, 14, 15]

$$\frac{\mathrm{d}\sigma(\alpha)}{\mathrm{d}\Omega} = C_{\alpha} B(\alpha), \tag{1}$$

$$B(\alpha) = (2J_i + 1)^{-1} |M(\alpha)|^2, \tag{2}$$

$$C_{\alpha} = K(\alpha, \omega) F(\alpha, q, \omega) J(\alpha, \omega)^2, \tag{3}$$

where $K(\alpha, \omega)$ and $J(\alpha, \omega)$ are the kinematic factor and the volume integral of the α mode ^{3}He–n interaction, respectively, for the momentum q and energy ω transfers. In the present case of the even–even DBD nucleus with the initial state spin $J_i = 0$, one gets $B(\alpha) = |M(\alpha)|^2$. The kinematic q, ω-dependence of $F(\alpha, q, \omega)$ is the calculated kinematic distribution by the distort wave Born approximation. The CER responses for low-lying states with $J^{\pi} = 0^{+}$, 1^{+} and 2^{-} in DBD nuclei are discussed in [2, 3, 25, 29]. The present CER given in equation (1) is based on the dominant single-step direct reaction to excite the SD state by the spin isospin interaction as discussed before [2, 3, 25].

The high energy-resolution (^{3}He, *t*) CERs for GT (α = GT) transitions have been applied extensively for decades at RCNP to study τ^{-}-side GT responses in DBD and neighboring nuclei, where the GT responses (B(GT)) for the ground states in the mass region are known from the measured EC/β^{+} rates. Thus, the coefficient C_{GT} to relate the GT CER cross-section to the GT response is known experimentally, as discussed in the reviews [2, 3, 13]. Note that the C_{α} coefficient reflects the kinematic factor for the incident and out-going (external) particles, while the $B(\alpha)$ response does the nuclear (internal) structure of the target nucleus to be studied.

The SD NMEs $M^{\pm}$(SD) for DBD nuclei, however, have not been known experimentally. The ground states in the intermediate nuclei for the DBD nuclei, except ^{76}Ge, are not the SD (2^{-}) state, thus no EC/β^{+} data are available. In case of ^{76}Ge, the M^{+}(SD) is known from the β^{-}-decay rate, but the M^{-}(SD) is not known because the EC rate is too small to be measured accurately.

Recently, SD states in DBD nuclei have been shown to be well excited by the (^{3}He, *t*) CERs at RCNP, and the SD cross-sections are compared with the FSQP (Fermi surface quasi particle) SD responses [25], but the SD NMEs are not derived since the coefficient C_{SD} to relate the CER cross section to the SD response is not known experimentally.

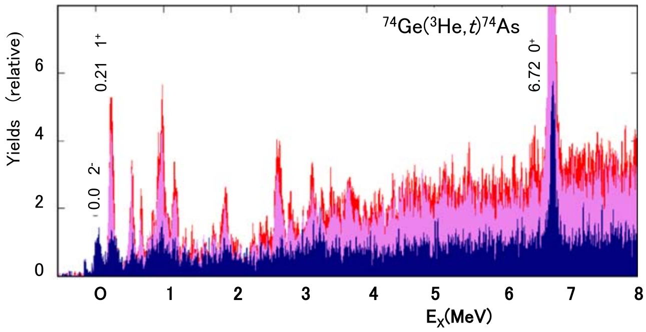

Figure 1. The energy spectra of the ^{74}Ge (^{3}He, t)^{74}As reaction is plotted against the excitation energy of ^{74}Ge. The spectra at $\theta \approx 0°–1°$ (red), $\theta \approx 1°–2°$ (pink) and $\theta \approx 2°–3°$ (blue) are overlaid. The GT and F states with $\Delta L = 0$ show a peak (red) at the forward angles of $\theta \approx 0°–1°$, while the SD ground state with $\Delta L = 1$ shows a large yield (blue) at larger angles of $\theta \approx 2°–3°$. The other peaks are mostly transitions to GT (1^+) states, which are well populated at forward angles in the present CER.

So, in the present work, we select ^{76}Ge, which has been extensively studied in DBD experiments by using high energy-resolution Ge detectors [26, 27], and we study experimentally the CER cross section for the ground SD state in ^{74}Ge, where the SD response B (SD) is known from the SD EC rate ($f_1 t$ value), in order to derive the coefficient $C_{\rm SD}$. Then, using this $C_{\rm SD}$ in ^{74}Ge for ^{76}Ge and the cross section measured in the previous experiment for the SD state in ^{76}Ge [25, 28, 29], we get the response B(SD) for ^{76}Ge.

The (^{3}He, t) CER on ^{74}Ge was studied at RCNP. The experimental details are the same as those for the previous experiment [28]. The incident 420 MeV ^{3}He^{++} beam was provided by the RCNP ring cyclotron combined with the AVF injector cyclotron. Note that the present medium-energy CER preferentially excites simple quasi-particle axial-vector states as the present SD state via the isospin spin interaction, and the excitation of such complicated (high level-density) states as excited by low-energy projectiles is very weak. The ^{3}He beam was transported to the target via the WS beam line. The out-going triton (t) from the (^{3}He, t) CER on ^{74}Ge was momentum-analyzed by using the high energy-resolution spectrometer GRAND RAIDEN [30], and was detected by the focal-plane detectors consisting of a set of multi-wire drift chamber for the t-track reconstruction and the plastic counter for the particle identification. The target used is a thin 0.25 mg cm^{-2} Ge foil enriched to 94% in ^{74}Ge. The RCNP high-energy-resolution system made it possible to realize the energy resolution of around 70 keV, including the contribution from the target, in FWHM to separate the 2^- ground-state transition from the 1^+ and other excited-state transitions at around 200 keV [31].

The measured energy spectrum for ^{74}Ge is shown in figure 1. The 2^- (SD) ground state and the 0^+ 6.72 MeV state (isobaric analogue state, IAS) are clearly excited. The energy spectrum for the CER on ^{76}Ge with the 1.4 mg cm^{-2} ^{76}Ge target was extracted from the previous data on ^{76}Ge in [28] as shown in figure 2, where the SD ground-state and the 8.31 MeV IAS transitions are well observed. The observed angular distribution for the ^{74}Ge SD state shows a typical distribution characteristic of the orbital angular-momentum transfer $\Delta L = 1$ in accordance with the calculated distribution as shown in figure 3 and with the angular distribution for the ^{76}Ge SD state.

Here, we consider two modes, α = SD for the 2^- ground state and α = F for the 0^+ IAS. The IAS state is strongly excited in the CER, and is conventionally used as a reference state since the response is given by the sum-rule limit of B(F) $= N - $ Z, N and Z being the

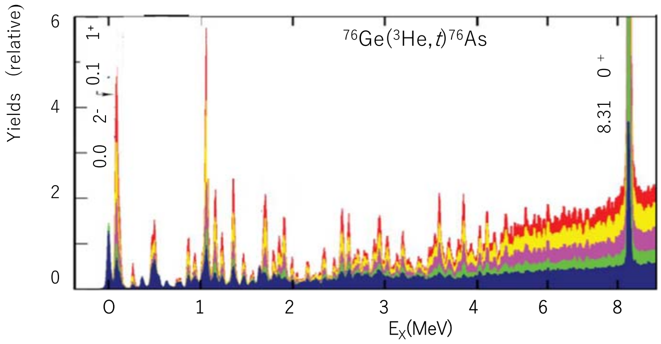

Figure 2. The energy spectra of the ^{76}Ge(^{3}He, t)^{76}As reaction is plotted against the excitation energy of ^{76}As. The spectra at $\theta \approx 0°$–$0.5°$ (red), $\theta \approx 0.5°$–$1°$ (yellow), $\theta \approx 1°$–$1.5°$ (violet), $\theta \approx 1.5°$–$2°$ (green) and $\theta \approx 2°$–$2.5°$ (blue) are overlaid. The GT and F(IAS) states with $\Delta L = 0$ show a peak (red) at the forward angles of $\theta \approx 0°$–$1°$, while the SD ground state with $\Delta L = 1$ shows a large yield (blue) at larger angles of $\theta \approx 2°$–$3°$. The other peaks are mostly transitions to GT(1^+) states as in ^{74}Ge CER in figure 1 [28].

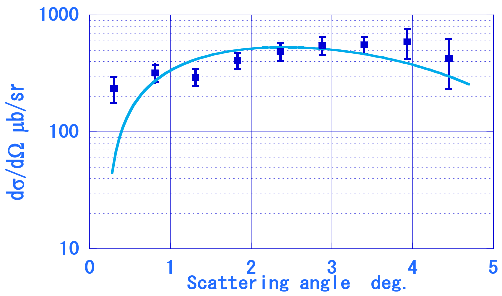

Figure 3. Measured (squares) and calculated (distorted wave Born approximation: solid line) angular distributions for the SD state in the ^{74}Ge (^{3}He, t)^{74}As. The solid line (mainly $\Delta L = 1$) includes a small component of $\Delta L = 3$ at the large angles [28].

neutron and proton numbers of the target nucleus. This is based on the good isospin symmetry in the nucleus [2, 32, 33]. We note here that the SD ground-state is an isolated particle-bound state and has no quasi-free scattering background, and IAS is a strongly excited τ^- giant resonance (GR) and there is no F-type quasi-free background. There are a tail of the GT GR and a GT quasi-free scattering underneath the huge IAS peak. They are rather smooth as a function of the excitation energy and the sum of them is around 6% of the IAS peak height. They are corrected for by using a smooth BG line. There are some (3–7 per MeV) faint GT peaks with the intensity of the order of 1% of the IAS one. One of them could happen to be hidden in the 70 keV IAS peak. Including all of them, the total error of a few % due to the uncertainty of the background subtraction is included in the final error.

Then, using the F cross section and the F response as references, the SD cross section and the SD response are expressed as

$$\frac{\mathrm{d}\sigma(\mathrm{SD})}{\mathrm{d}\sigma(\mathrm{F})} = R(\mathrm{SD/F})\frac{B(\mathrm{SD})}{B(\mathrm{F})}, \quad R(\mathrm{SD/F}) = \frac{C_{\mathrm{GT}}}{C_{\mathrm{F}}}, \tag{4}$$

where dσ(SD) and dσ(F) are, respectively, the differential cross-section for the SD transition at $\theta \approx 2.3°$ and that for the F(IAS) transition at $\theta \approx 0°$. Note the SD ($\Delta L = 1$) and F ($\Delta L = 0$) cross sections have maximum, respectively, at around $2.3°$ ($q \approx 0.3\,\mathrm{fm}^{-1}$) and $0°$ ($q \approx 0\,\mathrm{fm}^{-1}$) in their angular ($q$) distributions for both ^{74}Ge and ^{76}Ge, as shown in figures 3 and figure 3 in [28], and thus dσ(SD) and dσ(F) are rather stable, being insensitive to the momentum transfer (angle), at $\theta \approx 2.3°$ and $0°$, respectively.

$R(\mathrm{SD}/\mathrm{F})$ is the ratio of the SD to F coefficients as given in equation (4). This is a kind of the phase-space ratio, depending on the kinematic condition, and thus one can assume the same for both the ^{74}Ge and ^{76}Ge CERs, as explained below, while the cross-section ratios for the ^{74}Ge and ^{76}Ge CERs reflect, respectively, the nuclear response ratios for the ^{74}Ge and ^{76}Ge CERs. The ratio $R(\mathrm{SD}/\mathrm{F})$ is expressed by a product of the three SD to F ratios of $R_{\mathrm{K}}(\mathrm{SD}/\mathrm{F})$ for the kinematic factors of $K(\alpha, \omega)$, $R_{\mathrm{D}}(\mathrm{SD}/\mathrm{F})$ for the distortion factors of $F(\alpha, q, \omega)$ and $R_{\mathrm{VI}}(\mathrm{SD}/\mathrm{F})$ for the volume integral squares of $J(\alpha, \omega)^2$ (see equation (3)). The kinematic factor depends on the energy and the momentum. The incident ^{3}He energy is 420 MeV and the outgoing t energies are around 418 MeV and 411 MeV for the SD and F states in both the ^{74}Ge and ^{76}Ge CERs. The momentum transfers for both the ^{74}Ge and ^{76}Ge CERs are almost (99%) the same. Therefore, the kinematic conditions for the CERs on both nuclei are nearly the same. So the ratio $R_K(\mathrm{SD}/\mathrm{F})$ for $K(\alpha, \omega)$ is nearly the same within 1%. The distortion factor for ^{76}Ge is smaller by 2% than that for ^{74}Ge, but the ratio $R_{\mathrm{D}}(\mathrm{SD}/\mathrm{F})$ remains nearly the same within 0.5%. The volume integral does not depend on the mass number. The present SD and IAS transitions from the ground state (0^+) to the ground state (2^-) and the IAS are similar quasi-neutron $\to$ quasi-proton transitions for both the ^{74}Ge and ^{76}Ge CERs [25, 34]. Thus $R_{\mathrm{VI}}(\mathrm{SD}/\mathrm{F})$ for the volume integral squares is nearly the same for both the ^{76}Ge and ^{74}Ge CERs within 2%. Accordingly, the ratios $R(\mathrm{SD}/\mathrm{F})$ for both the ^{76}Ge and ^{74}Ge CERs remain the same within a few%, which is included in the final error. Actually, the coefficient C_{GT}, which is a kind of the GT unit cross section, stands for the common proportionality coefficient associated with the GT transition and the value for C_{GT} has been determined experimentally by referring to the coefficient in the neighboring nuclei as used in the present SD case [2, 3, 35, 36].

The measured SD to F cross section ratio is $\mathrm{d}\sigma(\mathrm{SD})/\mathrm{d}\sigma(\mathrm{F}) = 0.0442 \pm 0.0031$ for the $^{74}\mathrm{Ge} \to {}^{74}\mathrm{As}$ ground-state transition. The large error (7%) is mainly a statistical one since the systematic errors cancel out in the ratio. The SD to F response ratio is derived by using the B(SD) $= 2.82 \pm 0.34$ in units of 10^{-6} (n.u. natural unit)^2 [34] from the EC rate [31] and the B(F) $= N - Z = 10$ as $B(\mathrm{SD})/B(\mathrm{F}) = 0.282 \pm 0.034$ in units of 10^{-6} (n.u.)^2. Then, the ratio is $R(\mathrm{SD}/\mathrm{F}) = 0.157 \pm 0.022$ in units of 10^{-6} (n.u.)^2 for ^{74}Ge.

The cross-section ratio is derived as $\mathrm{d}\sigma(\mathrm{SD})/\mathrm{d}\sigma(\mathrm{F}) = 0.0295 \pm 0.0015$ for ^{76}Ge from the CER data on ^{76}Ge in [28]. The SD and F(IAS) transitions are shown in the (^{3}He, t) energy spectrum for ^{76}Ge in figure 2. By using this cross-section ratio and the coefficient ratio of R(SD/F) for ^{74}Ge one gets the response ratio $B(\mathrm{SD})/B(\mathrm{F}) = 0.188 \pm 0.039$ in units of 10^{-6} (n.u.)^2 for ^{76}Ge. Here, the SD response includes the systematic error of around 15% associated with contributions from non-central (tensor) interactions and spin-octupole (SO) transitions to the major SD cross-sections for ^{74,76}Ge at $\theta \approx 2.3°$. Then, using the F response of $B(\mathrm{F}) = N - Z = 12$ for ^{76}Ge, the SD response is derived as $B(\mathrm{SD}) = 2.26 \pm 0.46$ in units of 10^{-6} (n.u.)^2, where the final error is mainly the systematic one. Then, the SD NME for ^{76}Ge is obtained as $|M^{-}_{\mathrm{EXP}}(\mathrm{SD})| = 1.50 \pm 0.15$ in units of 10^{-3} n.u.. Note that we used the F cross section at $\theta \approx 0°$ where the momentum transfer is around $q \approx 0.085\,\mathrm{fm}^{-1}$, and that the cross section extrapolated to $q = 0\,\mathrm{fm}^{-1}$ is larger by 7% for both ^{74}Ge and ^{76}Ge. This effect cancels out in the present ratio of the C_{F} coefficients for both nuclei. The obtained SD NME for ^{76}Ge is similar to the NME of 1.68 in units of 10^{-3} n.u. for ^{74}Ge [34], but smaller than the

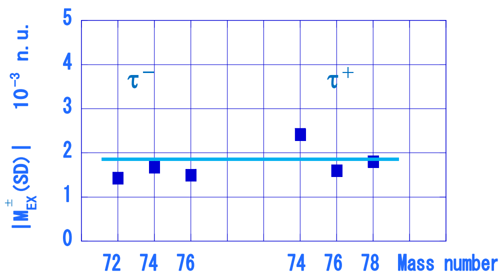

Figure 4. Experimental SD NMEs (squares) for the Ge ground-state to ground-state transitions, $|M^{\pm}_{\mathrm{EXP}}(\mathrm{SD})|$, in units of 10^{-3} n.u. in the mass region of $A = 72$–78. τ^- and τ^+ are for the τ^--and τ^+-side NMEs. The errors of the experimental NMEs are within the size of the squares. The solid line is an average value.

value evaluated on the basis of the FSQP [25]. The present SD NME for ^{76}Ge derived from the present CER, together with the ones for the neighboring nuclei derived from EC/$\beta^{\pm}$ decay data [34], are all around $|M^{\pm}_{\mathrm{EXP}}(\mathrm{SD})| \approx 1.9 \pm 0.5$ in units of 10^{-3} n.u., as shown in figure 4.

Now we briefly discuss the obtained SD NME. Since the main component of the present SD transition is the spin-stretched quasi-particle transition of $(l + 1/2)_n \leftrightarrow ((l - 1) - 1/2)_p$ with $l = 4$, the transition operator in the present case of $q \approx 0.3$ fm^{-1} is given by the first-order SD one in the reaction interaction NME, as in the first-order lepton-sector weak-interaction (i.e. β-decay) NME [1, 2, 14, 15]. It is expressed as

$$T(\mathrm{SD}) = g\tau^{\pm}[i^1\sigma \times rY_1]_2, \tag{5}$$

where τ and σ are the isospin and spin operators, respectively, and g is the interaction constant. Note that the nuclear radius r (inverse of momentum q) is included in M(SD), following the convention of β–γ NMEs, and thus the momentum q is included in $K(\mathrm{SD}, \omega)$ [3, 25]. Here, we follow the definition of the SD NME obtained from the SD f_1t as given in equation (3) in [34], which differs from that in [14].

The SD NME for the quasi-particle transition is expressed as

$$M^{\pm}(\mathrm{SD}) = k^{\pm}M^{\pm}_{\mathrm{QP}}(\mathrm{SD}), \tag{6}$$

where $M^{\pm}_{\mathrm{QP}}(\mathrm{SD})$ is the SD NME in the quasi-particle model and $k^{\pm}$ is the re-normalization coefficient. The quasi-particle NMEs scatter around $M^{\pm}_{\mathrm{QP}}(\mathrm{SD}) = 10$–$15$ in units of 10^{-3} n.u. [34]. The experimental SD NMEs are indeed much reduced by the coefficient of $k^{\pm} \approx 0.2$ with respect to the single quasi-particle NMEs [1, 2, 34]. The reduction of the SD NMEs in Ge nuclei is in accordance with the reductions for the axial-vector β–γ NMEs in medium heavy nuclei [1–3, 14, 15, 34, 37]. The reduction is due to nucleonic multipole and $\tau\sigma$ interactions, non-nucleonic (mesons, isobars) interactions and other interactions, which are not in the simple quasi-particle model. Among them, the strong nucleonic $\tau\sigma$ interaction gives rise to the SD giant resonance at the high excitation region and plays a crucial role for the reduction of the axial-vector NMEs for low-lying states, as shown by using the $\tau\sigma$ multipole interaction and QRPA (quasi-particle random-phase approximation) in [38, 39] and in the reviews [1–3, 15].

The nucleonic $\tau\sigma$ interaction is somehow incorporated in the p–n (proton–neutron) QRPA model [34]. Then, the NME is given as

$$M^{\pm}(\alpha) = k^{\pm}_{\rm NM} M^{\pm}_{\rm QR}(\alpha), \tag{7}$$

where $M^{\pm}_{\rm QR}(\alpha)$ is the pnQRPA model NME and $k^{\pm}_{\rm NM}$ stands for the re-normalization (quenching) coefficient due to non-nucleonic interactions and nuclear medium effects, which are not explicitly included in the pnQRPA model. The coefficient $k^{\pm}_{\rm NM}$ is conventionally expressed as $g_A^{\rm eff}/g_A$ where $g_A^{\rm eff}$ is the effective axial-weak coupling and $g_A = 1.27$ is the coupling for a free nucleon in unit of the vector coupling g_V [1, 3]. The pnQRPA NME for the ^{74}Ge is $M^{-}_{\rm QR}(\rm SD) = 4.87$ in units of 10^{-3} n.u. [34], and the coefficient is $k^{\pm}_{\rm NM} \approx 0.35$. This shows a severe re-normalization (reduction) effect for the SD NME. It is interesting to evaluate the SD NME for ^{76}Ge by using the pnQRPA.

The neutrino-less DBD NME is expressed by [3, 20]

$$M^{0\nu} = [M^{0\nu}(\rm GT) + M^{0\nu}(\rm T)] + \left(\frac{g_V}{g_A}\right)^2 M^{0\nu}(\rm F), \tag{8}$$

where $M^{0\nu}(\rm GT)$, $M^{0\nu}(\rm T)$ and $M^{0\nu}(\rm F)$ are the axial-vector, tensor and vector DBD-NMEs, respectively, and g_V/g_A is the vector coupling in unit of the axial-vector coupling of $g_A = 1.27$. Here g_A^2 for the GT and T NMEs is included in the phase-space factor. The $M^{0\nu}(\rm GT)$ involves the axial-vector NMEs with 1^+, 2^-, 3^+ and others via the neutrino potential for the virtual Majorana neutrino [3].

The axial-vector and tensor DBD-NMEs are considered to be reduced with respect to the quasi-particle and pnQRPA DBD-NMEs by the re-normalization (quenching) coefficients of $k^2 \approx 0.04$ and $k^2_{\rm NM} \approx 0.2$ with respect to the simple quasi-particle and pnQRPA DBD-NMEs, respectively [3, 20, 40]. Then the DBD NME $M^{0\nu}$ may be reduced by the re-normalization (quenching) coefficient of around 0.3 with respect to the pnQRPA DBD NME, depending on the relative weight of the vector NME in equation (8). The DBD isotope (detector) mass required for a given neutrino-mass sensitivity is inversely proportional to $(M^{0\nu})^4$ [3, 7]. Then, the detector mass gets two orders of magnitude more than in case of the pnQRPA DBD-NME without the quenching [3, 20].

The two-neutrino $\beta\beta$ NMEs with the low momentum of $q \leqslant a$ few MeV$/c$ consists of $M^{-}(\rm GT) \times M^{+}(\rm GT)/\Delta$ with Δ being the energy denominator [4, 7, 8]. It is reduced by the coefficient $k \approx k^- \times k^+ \approx 0.04$, with $k^{\pm} \approx 0.2$ being the reduction coefficients for the $\tau^{\pm}$ GT NMEs, with respect to the quasi-particle model NME [3, 41, 42].

On the other hand, in case of the neutrino-less DBD, the major components associated with the DBD-NME are the τ^--side and τ^+-side 2^- NMEs at the medium momentum of $q = 30$–100 MeV$/c$ via the neutrino potential. Theoretical calculations on the neutrino-less DBD-NME by using, for example, such pnQRPA model with the re-normalization coefficient that reproduces the experimental τ^--and τ^+-SD NMEs for ^{76}Ge are interesting. They will be presented elsewhere.

Acknowledgments

The present experiment E425 was supported by RCNP Osaka University. The authors thank RCNP and the accelerator group for the support. The authors, H A and H E, thank Prof. J Suhonen for very valuable and intensive discussions on SD NMEs, pnQRPA and DBDs, and Prof. R Zegers for very productive and interesting discussions on CERs. The authors from INFN thank ERC for the Grant Agreement No. 714 625 and the author, V W, for the DFG contract SFB 1245.

ORCID iDs

H Akimune https://orcid.org/0000-0001-5237-5718
H Ejiri https://orcid.org/0000-0003-0568-3528
G Coló https://orcid.org/0000-0003-0819-1633
K Zuber https://orcid.org/0000-0001-8689-4495

References

[1] Ejiri H 1978 *Phys. Rep.* **38** 85
[2] Ejiri H 2000 *Phys. Rep.* **338** 265
[3] Ejiri H, Suhonen J and Zuber K 2019 *Phys. Rep.* **797** 1
[4] Suhonen J and Civitarese O 1998 *Phys. Rep.* **300** 123
[5] Faessler A and Šimkovic F 1998 *J. Phys. G. Nucl. Part. Phys.* **24** 2139
[6] Zuber K 1998 *Phys. Rep.* **305** 295
[7] Ejiri H 2005 *J. Phys. Soc. Japan.* **74** 2101
[8] Avignone F, Elliott S and Engel J 2008 *Rev. Mod. Phys.* **80** 481
[9] Vergados J, Ejiri H and Šimkovic F 2012 *Rep. Prog. Phys.* **75** 106301
[10] Suhonen J and Civitarese O 2012 *J. Phys. G: Nucl. Part. Phys.* **39** 124005
[11] Vergados J, Ejiri H and Šimkovic F 2016 *Int. J. Mod. Phys.* E **25** 1630007
[12] Engel J and Menéndez J 2017 *Rep. Prog. Phys.* **60** 046301
[13] Frekers D and Alanssari M 2019 *Eur. Phys. J.* A **54** 177
[14] Bohr A and Mottelson B 1969 *Nuclear Structure* 1 (Amsterdam: Benjamin Inc.)
[15] Ejiri H and de Voigt M J A 1989 *Gamma Ray and Electron Spectroscopy in Nuclear Physics* (Oxford: Clarendon)
[16] Kortelainen M and Suhonen J 2002 *Eur. Phys. Lett.* **58** 666
[17] Hashim I H, Ejiri H, Shima T, Takahisa K, Sato A, Kuno Y, Ninomiya K, Kawamura N and Miyatake Y 2018 *Phys. Rev.* C **97** 014617
[18] Zinatulina D *et al* 2019 *Phys. Rev.* C **99** 024327
[19] Jokiniemi L, Suhonen J, Ejiri H and Hashim I H 2019 *Phys. Lett.* B **794** 143
[20] Ejiri H 2019 *Frontiers Phys.* **7** 30
[21] Ejiri H 2019 *J. Phys. G: Nucl. Part. Phys.* **46** 125202
[22] Douma C A *et al* 2020 *Eur. Phys. J.* A **56** 51
[23] Cappuzzello F *et al* 2015 *Eur. Phys. J.* A **51** 145
[24] Cappuzzello F *et al* 2018 *Eur. Phys. J.* A **54** 72
[25] Ejiri H and Frekers D 2016 *J. Phys. G Nucl. Part. Phys.* **43** 11LT01
[26] Agostini M *et al* 2018 *Phys. Rev. Lett.* **120** 132503
[27] Aalseth C *et al* 2018 *Phys. Rev. Lett.* **120** 132502
[28] Thies J H *et al* 2012 *Phys. Rev.* C **86** 014304
[29] Frekers D, Alanssari M, Ejiri H, Holl H, Poves A and Suhonen J 2017 *Phys. Rev.* C **95** 034619
[30] Fujiwara M *et al* 1999 *Nucl. Instrum. Meth. Res.* A **422** 484
[31] Singh B and Farlam A R 2006 *Nucl. Data Sheets* **107** 1923
[32] Taddeucci T N *et al* 1981 *Phys. Rev.* C **25** 1094
[33] Taddeucci T N *et al* 1987 *Nucl. Phys.* A **469** 125
[34] Ejiri H, Soukouti N and Suhonen J 2014 *Phys. Lett.* B **729** 27
[35] Zegers R G *et al* 2007 *Phys. Rev. Lett.* **99** 202501
[36] Frekers D, Puppe P, Thies J H and Ejiri H 2013 *Nucl. Phys.* A **916** 219
[37] Ejiri H and Suhonen J 2015 *J. Phys. G: Nucl. Part. Phys.* **42** 055201
[38] Ejiri H, Fujita J and Ikeda K 1968 *Phys. Rev.* **176** 1277
[39] Ejiri H 1971 *Nucl. Phys.* A **166** 594
[40] Suhonen J 2017 *Phys. Rev.* C **96** 05501
[41] Ejiri H 2009 *J. Phys. Soc. Japan.* **78** 074201
[42] Ejiri H 2017 *J. Phys. G: Nucl. Part. Phys.* **44** 115201