PAPER • OPEN ACCESS

Search for second order response of nuclei to isospin probes and their connection to double beta decay

To cite this article: F. Cappuzzello and for the NUMEN Collaboration 2020 *J. Phys.: Conf. Ser.* **1610** 012003

Search for second order response of nuclei to isospin probes and their connection to double beta decay

F. Cappuzzello[1,2,3]

for the NUMEN Collaboration

[1]Istituto Nazionale di Fisica Nucleare, Laboratori Nazionali del Sud, Catania, Italy
[2]Dipartimento di Fisica e Astronomia "Ettore Majorana", Università di Catania, Italy
[3]Centro Siciliano di Fisica Nucleare e Struttura della Materia, Italy

cappuzzello@lns.infn.it

Abstract. In order to get quantitative information on neutrino absolute mass scale from the possible measurement of the $0\nu\beta\beta$ decay half-lives, the knowledge of the Nuclear Matrix Elements (NME) involved in such transitions is mandatory. NMEs are not observables and can only be accessed by theory. However, the many-body nature of the nuclear state involved in the decay, makes this task possible only at approximate level. In this perspective, several experimental approaches have been proposed in the years in order to provide useful information to better constrain the theory. Here a short overview of the role of charge exchange reactions in this scenario is given, with particular emphasis on second order processes, known as Double Charge Exchange (DCE) reactions.

1 Introduction

Neutrinoless double beta decay ($0\nu\beta\beta$) is potentially the best resource to probe the Majorana nature of neutrino and to extract its effective mass from decay rate measurements. Moreover, if observed, $0\nu\beta\beta$ decay will show that the total lepton number is not conserved. Presently, this physics case is significantly driving research "beyond the Standard Model" and could offer the way toward a Grand Unified Theory of fundamental interactions and contribute to unveil the source of matter-antimatter asymmetry observed in the Universe.

Since the $\beta\beta$ decay process involves transitions in atomic nuclei, nuclear structure issues must be also accounted for in order to describe it. In particular, the $0\nu\beta\beta$ decay rate $[T_{1/2}]^{-1}$ can be factorized as a phase-space factor $G_{0\nu}$, the Nuclear Matrix Element (NME) $M_{0\nu}$ and a term $f(m_i, U_{ei}, \xi_i)$ containing a combination of the masses m_i, the mixing coefficients U_{ei} of the neutrino species and the Majorana phases ξ_i. Thus, if the NMEs are established with sufficient precision, the $f(m_i, U_{ei}, \xi_i)$ function, containing physics beyond the standard model, can be extracted from $0\nu\beta\beta$ decay rate measurements or bounds [1].

The evaluation of the NMEs is presently based on model calculations from different methods, e.g. Quasi-particle Random Phase Approximation (QRPA), large scale shell-model, Interacting Boson Model (IBM), Energy Density Functional (EDF), ab-initio [2-6]. All of these approaches propose different truncation schemes of the still unsolved full nuclear many-body problem into a solvable one, limited to a model space. The purpose is to include, as much as possible, the relevant degrees of freedom which allow a complete description of the problem. However, this condition cannot be easily checked without a comparison with experimental data. Indirect hints of the reliability of model calculations could come from their relative convergence to common values, even if this condition would not exclude that common unverified assumptions are still present in all models. High precision experimental information from Single Charge Exchange (SCE), transfer reactions and Electron Capture (EC) are also used to constrain the calculations [7-11]. However, the ambiguities in the models are still too large to provide accurate values of the NMEs. Discrepancy factors larger than two are presently reported in literature [12]. In addition, some assumptions, common to the different competing calculations, could cause unknown overall systematic uncertainties [13]. A pertinent example is about the use of quenched coupling constants within the nuclei, especially for the axial-vector weak interaction, which is strongly debated nowadays [14-22].

In this scenario, the experimental study of other nuclear transitions where the nuclear charge is changed by two units leaving the mass number unvaried, in analogy to the $\beta\beta$-decay, could provide important information.

Recently, the use of modern high resolution and large acceptance spectrometers has been proven to be effective in order to face the main experimental challenges and to extract quantitative information from DCE reactions. The measurement of DCE high-resolution energy spectra and accurate absolute cross sections at very forward angles is crucial to identify the transitions of interest [23,24]. The concurrent measurement of the other relevant reaction channels allows to isolate the direct DCE mechanism from the competing multi-nucleon transfer processes.

Based on these results, the NUMEN (*NUclear Matrix Elements for Neutrinoless double beta decay*) [25] project was recently proposed, with the aim to investigate the nuclear response to DCE reactions for all the isotopes explored by present and future studies of $0\nu\beta\beta$ decay [26,27].

2 A View of Heavy Ion Single Charge Exchange Reactions

Single Charge Exchange (SCE) reactions are common tools for investigation of nuclear states. In a SCE reaction induced by a projectile a on a target A, a proton (neutron) of the target is converted into a neutron (proton), $\Delta Z_A = \pm 1$, $\Delta N_A = \mp 1$, keeping the mass number A unchanged, with opposite transition simultaneously occurring in the projectile, $\Delta Z_a = \mp 1$, $\Delta N_a = \pm 1$. In the isospin representation, SCE reactions probe the isovector excitations generated, at two-body level, by $\tau_{a\pm}\tau_{A\mp}$ combination of the isospin rising and lowering operators acting on a nucleon in the projectile a and the target A, respectively. The monopole component $\Delta L = 0$ has attracted special interest, since the associated $\sigma\tau$ operator is analogous to the Gamow-Teller (GT) one acting in the spin transferring β-decay. In the years, a wealth of studies of SCE reactions has been reported. Excellent reviews of the early activities can be found in key articles by F. Osterfeld [28] for the theoretical aspects and by W. P. Alford and B.M. Spicer [29] for a survey of the experimental explorations. Also important is the paper by T. N. Taddeucci et al. [30], which proposed a useful factorization of the CE cross section. Here we focus the attention on heavy-ion induced SCE, mainly discussed in a recent review article from H. Lenske et al. [31].

SCE reactions are manifestations of the strong interaction, mediated by the exchange of mesons with isovector nature, the lightest of which are the pions π. At momentum transfer sensibly smaller than the π mass, the meson form factors do not influence appreciably the SCE dynamics and a simpler description in terms of smoothly energy dependent coupling factors is possible. This is similar to the weak interaction, where constant coupling factors g_v and g_a scale the isospin and spin-isospin operators. In this way, the analogy between τ (Fermi) and $\sigma\tau$ (GT) operators of the strong and weak interactions becomes closer. In this perspective, SCE reactions offer the opportunity to complement β-decay studies of the nuclear response to isovector probes. The best example is the study of isovector monopole nuclear response, ($\Delta J^{\pi}=1^+$, $\Delta L=0$; $\Delta\sigma=1$; $\Delta\tau=1$) for the GT, which is intrinsically limited to a reduced energy window accessible by β-decay, but not for SCE reactions. Since the $\sigma\tau$ is not a symmetry for nuclear systems, the associated GT strength is broadly spread over many states as a function of the excitation energy around the Gamow-Teller Resonance (GTR) [32,33]. The exact GT distribution is a characterizing property of each nucleus, reflecting in a detailed way its peculiar many-body nature. For that reason, the exploration of GT strength has soon gained a central relevance in the development of nuclear physics. A relevant finding is that only part of the strength (from about 50% to 70%) predicted by the model-independent sum rule for GT [34], [28] is found in the experiments [35], at least in the region of the GTR or even up to about 50 MeV excitation energy. Beyond this limit, it is hard to extract the monopole strength from the experiments with the necessary accuracy. Recent results from Douma et al. have analyzed in detail the high energy region in the excitation energy spectra of Sb isotopes, showing that a careful treatment of the quasi-free component in the reaction cross section for the (^{3}He,t) could mitigate this discrepancy [36].

Another old puzzle, maybe connected to the latter, is that the GT strengths extracted from measured cross sections of isolated transitions are systematically smaller than predicted with different nuclear structure models and a quenching factor of about 0.7 is typically needed to reproduce the data. In this scenario a recent study with state-of-art ab-initio approach has demonstrated that a consistent treatment of two-body currents is indeed necessary to almost completely get rid of this discrepancy [37].

An important aspect of SCE reactions, when used for investigation of GT modes in nuclei, is that the momentum transfer should be kept as small as possible in order to filter out $\Delta L \neq 0$ components in the collision or easily distinguished in the data analysis. This also ensures that the tensor components of the isovector nucleon-nucleon interaction (ΔJ^π=1$^+$, ΔL=2; $\Delta\sigma$=1; $\Delta\tau$=1) have a small impact on the observed ΔJ^π=1$^+$ strength. Such condition is best matched when the incident energy is typically above 100 MeV/u and the scattering angle is close to zero degree. Following this strategy the measured cross sections for (n,p) and (p,n) reactions at energies above 100 MeV were found proportional to known β^+ and β^- strengths, respectively, even if the achieved experimental resolution does not allow to separate all the GT states in the energy spectra, somewhat reducing the sensitivity of these experimental tools. Complementary results have been achieved by SCE reactions induced by heavier projectiles, such as the (d,^{2}He), (t,^{3}He), (^{7}Li,^{7}Be) (^{12}C,^{12}N) (^{18}O,^{18}F) for the β^+-like target transitions, or the (^{3}He,t), (^{12}C,^{12}B) for the β^--like class.

From the experimental side, state-of-art results have been obtained by the (^{3}He,t) reaction performed at 140 MeV/u at the Grand Raiden magnetic spectrometer of RCNP in Osaka [38-40] due to the zero degree mode available for the spectrometer and the high energy resolution (FWHM typically ~ 25 keV). A remarkable proportionality (better than 5%) between measured cross sections and known β^- strengths have been reported as a general finding, at least for not suppressed states, for a large number of states in many targets. As a consequence, the RCNP facility has become the ideal place for high resolution GT studies. For the β^+ transitions remarkable results have been obtained by the (d,^{2}He) at KVI and RIKEN laboratories [41-46]. Experimentally, the high efficient detection of the two protons decaying from ^{2}He has allowed to get an overall energy resolution of about 100 keV in the missing mass spectra. About 100 MeV/u bombarding energy was chosen, as discussed above, and the center-of-mass detection angle for the ^{2}He system was around zero degrees. Again a close proportionality between nuclear matrix elements extracted from SCE cross sections and those extracted from β^+ and EC studies was found.

An interesting application of high-resolution (^{3}He,t) and (d,^{2}He) reactions is to map the GT response of specific nuclei, which are intermediate systems in known two-neutrino double beta decays ($2\nu\beta\beta$). The GT response in the intermediate system is separately explored from the parent and the daughter side. Among the many 1$^+$ states populated in the two reactions, it is possible to infer what states give relevant contribution to the $2\nu\beta\beta$, as those which are significantly populated in both SCE processes. A drawback of this technique is that only the transition probabilities to individual 1$^+$ states are extracted from the experiments for each step, while in the $2\nu\beta\beta$ the amplitudes are needed with the proper phase, since they add coherently. A simple case is obtained when a single 1$^+$ state is found to be dominant in the intermediate state, since in this case no coherent sum is needed. Approximate schemes have also been proposed for 1$^+$ transitions close to the Fermi level [47]. Recently the (^{3}He,t) reaction has been used to map also the 2$^-$ state distribution, which opens a new interesting way to provide accurate information for $0\nu\beta\beta$ NME [48].

When moving to heavier projectiles, a typical problem is to take their complex many-body nature into account in the SCE cross section analyses [49]. The projectile-target potential needs to be described with high accuracy both in the entrance (Initial State Interaction, ISI) and the outgoing (Final State Interaction, FSI) channel. In this case, the quasi-elastic SCE reactions are localized in the nuclear surfaces of the colliding systems, due to the strong absorption of the incoming waves in the inner part of heavy nuclei. This aspect of the heavy-ion reaction mechanism is crucial, since it allows to convert the full many-body reaction problem into a much simpler one, where direct reactions as SCE can be treated as small perturbations of the direct elastic scattering, which is described by an appropriate nucleus-nucleus average optical potential. Modern techniques to build ISI and FSI potentials by double folding integrals of the nucleon-nucleon interaction with the densities of the colliding systems have proven to be accurate enough for this purpose [50-54], especially when elastic scattering data of the projectile-target system are available at the same energy of the SCE reaction cross sections. In this way, the SCE reaction matrix elements can be directly extracted from the experimental cross sections and connected to the nuclear response to two-body operators, as those discussed above for the GT case and even for higher multipolarities. However, other quasi-elastic mechanisms in the projectile-target collision are in principle allowed. For example, multi-nucleon transfer reactions, where the colliding partners exchange nucleons, could have a non-negligible contribution to SCE channel. In particular, the transfer of a proton/neutron from the projectile to the target (stripping process) followed by the transfer to the projectile of a neutron/proton from the target (pick-up process) is a two-step mechanism which feeds the same outgoing channel as the direct one-step SCE reaction induced by two-body nucleon-nucleon interaction. The two-step mechanism is sensitive to the nucleon-nucleus mean field potential and cannot probe the nucleon-nucleon interactions at the origin of F and GT response of nuclei. This is an unwanted complication that should be taken into account, especially in heavy-ion induced SCE reactions, and possibly minimized by an appropriate choice of the experimental conditions [55-57]. From the theory point of view this problem has been extensively debated in the past with major advances achieved thanks to the development of microscopic approaches for the data analysis. An updated view of the status of the field is found in ref. [31]. As a general finding, the two-step mechanisms tend to be small at incident energies far above the Coulomb barrier. This has been reported in (^{12}C,^{12}B) [58], (^{12}C,^{12}N), (^{13}C,^{13}N) [59] and in (^{7}Li,^{7}Be) reactions [60-69] explored at different energies from 5 to 70 MeV/u and on different targets. In references [65], [70] it was shown that quantitative information on GT matrix elements can be extracted from (^{7}Li,^{7}Be$_{gs}$(3/2^{-})) and (^{7}Li,^{7}Be$_{0.43}$ MeV(1/2^{-})) measured cross sections for isolated transitions. The results, obtained at about 8 MeV/u bombarding energy for light neutron rich nuclei as ^{11}Be, ^{12}B, ^{15}C and ^{19}O, indicate that a good accuracy (better than 10%) is achieved, providing that a fully consistent microscopic approach is used for the ISI, FSI and the reaction form factors.

An interesting aspect of heavy-ion induced SCE reactions is that a significant amount of linear momentum is available during the collision and it is transferred to the final asymptotic state, even at forward angles. This feature is normally considered a drawback of heavy-ion induced SCE reactions, as the typical focus is in studying the L=0 modes, namely the GT one. However, this property is interesting since neither β-decay nor many of the light ions induced CE reactions can effectively probe the nuclear response to the higher multipoles of the isospin (F-like) and spin-isospin (GT-like) operators. Nowadays much interest is given to this aspect of nuclear response for its implications in $0\nu\beta\beta$ decay matrix elements [71,72] where high order multipoles are considered to give a major contribution [73]. Thus, the exploration of heavy-ion induced SCE reactions has recently regained favor, with the consequent need to develop suitable experimental techniques and advanced theoretical analysis for a detailed description of the data [49].

3 Heavy-Ion induced Double Charge Exchange Reactions

A Double Charge Exchange (DCE) reaction is a process induced by a projectile a on a target A, in which two protons (neutrons) of the target are converted in two neutrons (protons), $\Delta Z_A = \pm 2$, $\Delta N_A = \mp 2$, being the mass number A unchanged, with opposite transition simultaneously occurring in the projectile,

$\Delta Z_a = \mp 2,\ \Delta N_a = \pm 2$. In the isospin representation, DCE reactions probe the double isovector excitations generated, at four-body level, by $\tau_{a\pm}\tau_{a\pm}\tau_{A\mp}\tau_{A\mp}$ combination of the isospin rising and lowering operators acting on two nucleons in the projectile *a* and the target *A*, respectively. If we limit to only the target excitations, DCE transitions can also occur as result of (π^+,π^-) or (π^-,π^+) reactions or $\beta\beta$-decays, the latter allowed only for positive Q-value.
In analogy with the SCE reactions, DCE probe nuclear response to the isospin degree of freedom, despite here the second order effects are selected. It is useful to recall the main features of known nuclear processes connected to second order isospin operators. $2\nu\beta\beta$-decays, induced by the heavy gauge bosons of the weak interaction, are sensitive to the nuclear response to a sequence of two GT operators acting independently and probing the low momentum component of nuclear wave functions. $0\nu\beta\beta$-decays, which are also induced by the weak interaction, are connected to the nuclear response to two-body isospin operators in a broad range of momenta distributed around 0.5 fm^{-1} and consequently in a wide range of multipolarities [73]. As a consequence, despite $2\nu\beta\beta$-decays and $0\nu\beta\beta$-decays connect the same nuclear states and are both generated by the weak interaction, they are sensitive in the momentum space to quite different regions of the involved wave functions. In practice, the link between the two phenomena is not strong enough for a safe extrapolation of $0\nu\beta\beta$ NMEs from $2\nu\beta\beta$ NMEs.
Pion-induced DCE reactions require the isospin components of the strong interaction acting twice. At a nucleonic level, two independent nucleons interact sequentially with the π fields. In the first step, the charged incident pion is converted to a neutral one $n(\pi^+,\pi^0)p$; in the second step the neutral pion is converted to a charged one as follows $n(\pi^0,\pi^-)p$. A similar sequence occurs for DCE induced by negative pions according to the following reaction chain $p(\pi^-,\pi^0)n$ followed by $p(\pi^0,\pi^+)n$.
For pionic DCE reaction, Johnson and collaborators have emphasized the important role of the $\Delta_{33}(1232)$ resonance [74]. Auerbach et al. have investigated in depth the nuclear structure aspects entering into the description of (π+, π−) reactions [75,76], pointing out that nucleon–nucleon correlations are playing a central role. The (π^+, π^-) process can be explained by a two-nucleon mechanism through the formation and decay of intermediate $\Delta_{33}(1232)$ resonances. Due to the spin-less nature of pions spin-isospin nuclear responses are not directly accessed and thus are difficult to observe. Extensive studies of (π^+, π^-) were performed in the 80's [77-79] leading to the observation of second order collective excitations as the Double Isobaric Analogue State (DIAS) or the Isobaric Analogue State built on the top of the Giant Dipole Resonance (GDR-IAS). Instead, no Double Gamow-Teller (DGT) was observed, maybe due to the above mentioned weak sensitivity to spin modes for this probe. Recently the peculiar feature of (π^+, π^-) process to be driven by nucleon-nucleon correlations have attracted the interest of Lenske et al. [31] in the view that, these kinds correlation-driven processes are not specific for pion-induced DCE but will occur in similar form also in other hadronic reactions. They can be observed, however, only on systems which support rank-2 isotensor processes, thus excluding nucleons. If, in addition, we require that the final ejectile should be in a particle-stable state, then heavy-ion DCE reactions with ^{12}C,^{18}O,^{20}Ne nuclei or heavier are the natural choice.
An interesting aspect of DCE reactions induced by nuclear collisions is that no light projectiles can be practically used. The lightest projectiles allowed are tritons or ^3He, and even in these cases the (t,3p) or the (^3He,3n) reactions are very challenging from the experimental point of view and, to our knowledge, never explored. Also moving to heavier projectiles the experiments appear rather demanding. First pioneering explorations of the heavy-ion induced DCE are the (^{18}O,^{18}Ne), (^{18}O,^{18}C) and (^{14}C,^{14}O) reactions, which were performed at Berkeley, NSCL-MSU, IPN-Orsay, ANU-Pelletron, Los Alamos laboratories [80-84] at energies above the Coulomb barrier. The main purpose was to determine the mass of neutron rich isotopes by reaction Q-value measurements. However, these experiments were not conclusive for deeper spectroscopic investigations, mainly due to the poor statistical significance of the few DCE observed events; thus no other experiments were proposed. Also the theory, which started to study the DCE reaction mechanism [85,86], soon followed the trend and the field was abandoned for a long time.

Recently, major interest has raised along DCE studies, also for their possible connection to *ββ*-decays. New reactions have been considered, such as the (^{8}He,^{8}Be) [87], the (^{11}B,^{11}Li) [88] or the (^{12}C,^{12}Be) [89], explored at RIKEN and RCNP at energies between 80 and 200 MeV/u. The (^{8}He,^{8}Be) was used to search for the tetra-neutron (4n) system by the ^{4}He(^{8}He,^{8}Be)4n at 186 MeV/u [87]. The (^{11}B,^{11}Li) and the (^{12}C,^{12}Be) were investigated with the main goal to find the DGT resonance and provide quantitative information about the DGT sum-rule, important for modern nuclear structure theories [90]. Another new DCE reaction, the (^{20}Ne,^{20}O) have been introduced by the NUMEN project [25] [91], with the aim to probe $\beta^{-}\beta^{-}$-like nuclear response. In addition, important results have been recently achieved by the renewed use of the (^{18}O,^{18}Ne) reaction in upgraded experimental conditions [23,24]. In reference [23] the ^{40}Ca(^{18}O,^{18}Ne)^{40}Ar was studied at 15 MeV/u at the MAGNEX facility of the INFN-LNS [92-94] showing that high mass, angular and energy resolution energy spectra and accurate absolute cross sections are at our reach, even at very forward angles. In addition, a schematic analysis of the reaction cross sections demonstrated that relevant quantitative information on DCE matrix elements can be extracted from the data.

In analogy to the case of heavy-ion induced SCE reactions, an important issue for the DCE is to quantify the contribution coming from multi-nucleon transfer reactions. In this case the effects start from the 4th order in the nucleon-nucleon potential since two protons (neutrons) should be stripped from the projectile and two neutrons (protons) picked-up from the target. In Ref. [23] it was shown that, under the experimental conditions set for the experiment at INFN-LNS, the contribution of multi-nucleon transfer was negligible (less than 1%). Similar results are found in the preliminary analysis of the other explored cases. Consequently, the leading DCE reaction mechanism is connected to nucleon-nucleon isovector interaction. This acts between two neutrons (protons) in the projectile and two protons (neutrons) in the target for the (^{18}O, ^{18}Ne) and the (^{20}Ne, ^{20}O) reactions, respectively. A useful way to consider the DCE process is by means of the exchange of two charged π or ρ mesons between the involved nucleons. An interesting question is whether the two mesons are exchanged independently of each other in analogy to $2\nu\beta\beta$-decays [95] or in a correlated way, as in the $0\nu\beta\beta$-decays [31], [96]. This last question is quite interesting for the connection of DCE to $0\nu\beta\beta$-decays. In addition, this aspect is also important from the point of view of nuclear reaction theory, since it could indicate a new way to access nucleon-nucleon short-range correlations [31].

4 DCE reactions and 0νββ decays

The availability for the first time of valuable data on DCE reactions raises the question how they can be profitably used toward the experimental access to $0\nu\beta\beta$ decay NMEs. In ref. [23] it has been pointed out that, although the DCE and $0\nu\beta\beta$ decay processes are mediated by different interactions, there are a number of important similarities among them:

- Parent/daughter states of the $0\nu\beta\beta$ decay are the same as those of the target/residual nuclei in the DCE;
- Short-range Fermi, Gamow-Teller and rank-2 tensor components are present in both the transition operators, with relative weight depending on incident energy in DCE. Performing the DCE experiments at different bombarding energies could give sensitivity to the individual contribution of each component;
- A large linear momentum (~100 MeV/c) is available in the virtual intermediate channel in both processes [12]. This is a distinctive similarity since other processes such as single β decay, $2\nu\beta\beta$ decay, light-ion induced SCE cannot probe this feature [97]. An interesting development is the recently proposed μ-capture experiments at RCNP [98], [99];
- The two processes are non-local and are characterized by two vertices localized in a pair of valence nucleons;
- Both processes take place in the same nuclear medium. In medium effects are expected to be present in both cases, so DCE data could give a valuable constraint on the

Journal of Physics: Conference Series **1610** (2020) 012003 doi:10.1088/1742-6596/1610/1/012003

theoretical determination of quenching phenomena on *0νββ*. One should mention for example that in single *β* decay, 2ν*ββ* decay [5] and SCE reactions [29], the limited model space used in the calculations and the contribution of non-nucleonic degrees of freedom and other correlations require a renormalization of the coupling constants in the spin-isospin channel. However, an accurate description of quenching has not yet been fully established and other aspects of the problem can give important contributions [14] [22];

- An off-shell propagation through virtual intermediate channels is present in the two cases. The virtual states do not represent the asymptotic channels of the reaction and their energies can be different from those (measurable) at stationary conditions [100].

The descriptions of NMEs for DCE and 0νββ decay present the same degree of complexity, with the advantage for DCE to be "accessible" in laboratory. In Refs. [101] and [102] such analogy have been investigated and a good linear correlation between double GT transitions to the ground state of the final nucleus and 0νββ decay NMEs is reported for pf-shell nuclei. However, a simple relation between DCE cross sections and ββ-decay half-lives is not trivial and needs to be explored.

References

[1] H. Ejiri, J. Suhonen, K. Zuber, Physics Reports 797, 1 (2019).
[2] E. Caurier, J.Menendez, F. Nowacki, A. Poves, Phys. Rev. Lett. 100 (2008) 052503.
[3] J. Suhonen and M. Kortelainen, Int. Journ. of Mod. Phys. E 17, 1 (2008).
[4] N. L. Vaquero, T.R. Rodriguez, J. L.Egido, Phys. Rev. Lett. 111 (2013) 142501.
[5] J. Barea, J. Kotila, F. Iachello, Phys. Rev. C 87 (2013) 014315.
[6] A. A. Kwiatkowski et al., Phys. Rev. C 89, 045502 (2014).
[7] H. Akimune et al., Phys. Lett. B 394, 23 (1997).
[8] J. P. Schiffer et al., Phys. Rev. Lett. 100 112501 (2008).
[9] D. Frekers, Prog. Part. Nucl. Phys. 64, 281 (2010).
[10] C. J. Guess et al., Phys. Rev. C 83, 064318 (2011).
[11] S. J. Freeman and J. P. Schiffer, J. Phys. G: Nucl. Part. Phys. 39, 124004 (2012).
[12] J. Barea, J. Kotila and F. Iachello, Phys. Rev. Lett. 109, 042501 (2012).
[13] Report to the Nuclear Science Advisory Committee, Neutrinoless Double Beta Decay, 2014.
[14] J. T. Suhonen, Frontiers in Physics 5 (2017) 55.
[15] J. Fujita and K. Ikeda, Nucl. Phys. 67, 145 (1965).
[16] D. H. Wilkinson, Nucl. Phys. A 225, 365 (1974).
[17] J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 91, 034304 (2015).
[18] J. Suhonen and O. Civitarese, Phys. Lett. B 725, 153 (2013).
[19] A. Faessler, G.L. Fogli, E. Lisi, V. Rodin, A.M. Rotunno, and F. Šimkovic, J. Phys. G: Nucl. Part. Phys. 35, 075104 (2008).
[20] R. G. H. Robertson, Modern Phys. Lett. A 28, 1350021 (2013).
[21] S. Dell'Oro, S. Marcocci, and F. Vissani, Phys. Rev. D 90, 033005 (2014).
[22] J. Menendez, D. Gazit, and A. Schwenk, Phys. Rev. Lett. 107, 062501 (2011).
[23] F. Cappuzzello et al., Eur. Phys. J. A 51: 145 (2015).
[24] H. Matsubara, et al., Few-Body Systems 54, 1433 (2013).
[25] F. Cappuzzello et al., Eur. Phys. J. A 54, 72 (2018).
[26] A. Spatafora et al., Phys. Rev. C 100 (2019) 034620.

[27] S. Calabrese et al., Acta Phys. Pol. 49 (2018) 275.
[28] F. Osterfeld, Review of Modern Physics 64, 491 (1992).
[29] W. P. Alford and B.M. Spicer, Advances in Nucl. Phys. 24, 1 (1998).
[30] T. N. Taddeucci et al. Nucl. Phys. A469 125 (1987).
[31] H. Lenske et al., Prog. Part. Nucl. Phys. 109 (2019) 103716.
[32] K. Ikeda, S. Fujii, and J. I. Fujita, Phys. Lett. 3, 271 (1963).
[33] D. E. Bainum et al., Phys. Rev. Lett. 44, 1751 (1980).
[34] M. C. Vetterli, O. Hausser, R. Abegg, et al. Phys. Rev C 40 (1989) 559.
[35] S. D. Bloom, C. D. Goodman, S. M. Grimes, and R. F. Hausman, Jr., Phys. Lett. B 107, 336 (1981).
[36] C.A. Douma et al., Eur. Phys. J. A 56 (2020) 51.
[37] F. Cappuzzello et al., Nature Communications 6, 6743 (2015).
[38] M. Fujiwara et al., Nucl. Instr. and Meth. A 422 (1999) 484.
[39] Y. Fujita et al., Nucl. Instrum. Methods Phys. Res. B 126, 274 (1997).
[40] H. Fujita et al., Nucl. Instrum. Methods Phys. Res. A 484, 17 (2002).
[41] H. Okamura et al., Phys. Lett. B345 (1995) 1.
[42] S. Rakers et al., Nucl. Instr. and Meth. A 481, 253 (2002).
[43] H. Ohnuma et al., Phys. Rev. C 47 (1992) 648.
[44] H. Dohmann et al., Phys. Rev. C 78 (2008) 041602(R).
[45] E.-W. Grewe et al., Phys. Rev. C 78, 044301 (2008).
[46] D. Frekers, Progress in Particle and Nuclear Physics 57, 217 (2006).
[47] H. Ejiri, J. Phys. Soc. Jpn. 81, 033201 (2012).
[48] L Jokiniemi et al. Phy. Rev. C 98 (2018) 024608.
[49] H. Lenske et al., Phys. Rev. C 98, 044620 (2018).
[50] M. Cavallaro et al., Phys. Rev. C 88, 054601 (2013).
[51] M. Cavallaro et al., Phys. Rev. Lett. 118 (2017) 012701.
[52] D. Carbone et al., Phys. Rev. C 95 (2017) 034603.
[53] M. J. Ermamatov et al., Phys. Rev. C 94 (2016) 024610.
[54] M. J. Ermamatov et al., Phys. Rev. C 96 (2017) 044603.
[55] M.J. Ermamatov et al., Phys. Rev. C 94 (2016) 024610.
[56] B. Paes et al., Phys. Rev. C 96 (2017) 044612.
[57] E. N. Cardozo et al., Phys. Rev. C 97 (2018) 064611.
[58] H. G. Bohlen et al., Nucl. Phys. A 488 (1988) 89c.
[59] W. von Oertzen Nucl. Phys. A 482 (1988) 357c.
[60] S. Nakayama et al., Phys. Lett. B 246, 342 (1990).
[61] S. Nakayama et al., Phys. Rev. C 60 (1999) 047303.
[62] T. Annakkage et al., Nucl. Phys. A 648 (1999) 3.
[63] J. Cook et al., Phys. Rev. C 30, 1538 (1984).
[64] N. M. Clarke and J. Cook. Nucl. Phys. A 458 (1986) 137.
[65] F. Cappuzzello et al., Nucl. Phys. A 739, 30 (2004).
[66] F. Cappuzzello et al., Phys. Lett. B 516, 21 (2001).

[67] A. Etchegoyen et al., Phys. Rev. C 38, 2124 (1988).
[68] C. Nociforo et al., Eur. Phys. J. A SP 27, 283 (2006).
[69] F. Cappuzzello et al., Europhys. Lett. 65 (2004) 766.
[70] M. Cavallaro, Nuovo Cimento C 34, n. 5 (2011).
[71] H. Ejiri, N.Soukouti, J.Suhonen, Physics Letters B 729 (2014) 27.
[72] L. Jokiniemi, J. Suhonen, and H. Ejiri 2016, 8417598 (2016).
[73] J. Suhonen and O. Civitarese, Physics Reports 300, 123 (1998).
[74] M.B. Johnson, E.R. Siciliano, H. Toki, A. Wirzba, Phys. Rev. Lett. 52 (1984) 593–596.
[75] N. Auerbach, W.R. Gibbs, E. Piasetzky, Phys. Rev. Lett. 59 (1987) 1076–1079.
[76] N. Auerbach, W.R. Gibbs, J.N. Ginocchio, W.B. Kaufmann, Phys. Rev. C 38 (1988) 1277–1296.
[77] A. Fazely and L. C. Liu, Phys. Rev. Lett. 57, 968 (1986).
[78] S. Mordechai, et al., Phys. Rev. Lett. 61, 531 (1988).
[79] N. Auerback et al., Phys. Rev. Lett. 59 (1987) 1076.
[80] J. Cerny, et al., Proc. 3° Int. Conf. on Nuclei Far from Stability, Cargese, 1976.
[81] J. Blomgren, et al., Phys. Lett. B 362, 34 (1995).
[82] L. K. Fifield et al., Nucl. Phys. A 385 (1982) 505.
[83] F. Naulin, et al., Phys. Rev. C 25, 1074 (1982).
[84] D. M. Drake, et al., Phys. Rev. Lett. 45, 1765 (1980).
[85] D. R. Bes, O. Dragun, E.E. Maqueda, Nucl. Phys. A 405, 313 (1983).
[86] C.H. Dasso and A. Vitturi, Phys. Rev. C 34, 743 (1986).
[87] K. Kisamori et al., Phys. Rev. Lett. 116, 052501 (2016).
[88] K. Takahisa et al., arXiv:1703.08264 (2017).
[89] M. Takaki et al., RIKEN Accel. Prog. Rep. 47 (2014).
[90] H. Sagawa, T. Uesaka, Phys. Rev. C 94, 064325 (2016).
[91] M. Cavallaro et al., Proceedings of Science, PoS(BORMIO2017) 015 (2017).
[92] F. Cappuzzello et al. Eur. Phys. J. A 52: 167 (2016).
[93] M. Cavallaro et al., Results in Phys. 13 (2019) 102191.
[94] M. Cavallaro et al., Nucl. Instr. and Meth. B 463 (2020) 334.
[95] J.I. Bellone et al., arXiv:1912.03043.
[96] E. Santopinto et al., Phys. Rev. C 98 061601(R) (2018).
[97] P. Puppe et al., Phys. Rev. C 84 051305 (2011).
[98] L. Jokiniemi et al., Phys. Lett.B 794 (2019) 143.
[99] H. Ejiri et al., J. Phys. Soc. Jpn. 82, 044202 (2013).
[100] L. Mandelstam and Ig. Tamm, J. Phys. USSR 9 (1945) 249.
[101] J. Menéndez, N. Shimizu and K. Yako, arXiv:1712.08691v1.
[102] N. Shimizu, J. Menéndez and K. Yako, arXiv:1709:01088.