Article

Cognitive Profiling of Nodes in 6G through Multiplex Social Network and Evolutionary Collective Dynamics

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Abstract: Complex systems are fully described by the connectedness of their elements studying how these develop a collective behavior, interacting with each other following their inner features, and the structure and dynamics of the entire system. The forthcoming 6G will attempt to rewrite the communication networks' perspective, focusing on a radical revolution in the way entities and technologies are conceived, integrated and used. This will lead to innovative approaches with the aim of providing new directions to deal with future network challenges posed by the upcoming 6G, thus the complex systems could become an enabling set of tools and methods to design a self-organized, resilient and cognitive network, suitable for many application fields, such as digital health or smart city living scenarios. Here, we propose a complex profiling approach of heterogeneous nodes belonging to the network with the goal of including the multiplex social network as a mathematical representation that enables us to consider multiple types of interactions, the collective dynamics of diffusion and competition, through social contagion and evolutionary game theory, and the mesoscale organization in communities to drive learning and cognition. Through a framework, we detail the step by step modeling approach and show and discuss our findings, applying it to a real dataset, by demonstrating how the proposed model allows us to detect deeply complex knowable roles of nodes.

Keywords: 6G networks; complex networks; multiplex social network; epidemic spreading; evolutionary game theory; cognitive networks; community detection

1. Introduction

The ongoing process of telecommunications evolution towards 6G, along with the growing number of mobile users and the demand of bandwidth-intensive services and high data-rate applications, are creating a large volume of traffic, making a complex dare which needs to be addressed with innovative approaches [1,2]. The worldwide research activity is focused on defining the next-generation 6G systems in order to take into consideration a confluence of trends as densification, higher rate, massive antenna and emerging trends that include innovations in terms of services and devices, complexity and artificial intelligence (AI), computing and sensing [3]. Ranging from simple sensors to sophisticated devices, and including different network scenarios as cellular, vehicular, Wi-Fi, Internet of Things (IoT) and Internet of Everything (IoE) systems which connect millions of people and billions of machines, we expected that the sixth generation systems will meet the demands for a fully connected and intelligent network. Taking into consideration the forthcoming 6th generation (6G) communication networks, it will address the constraints and the performance requirements of the applications and innovative services which need highly increasing resources, introducing new approaches and technologies as well as revolutionary network features [2,4].

The designing and modeling processes of 6G network need new paradigms to move the system from closed hierarchical structures towards open and distributed networks, including self-organization, self-adaptation and optimization of interactions and functions

of nodes. Thus, it can be achieved through the introduction of a complex systems approach. The traditional attributes applied to measure and characterize communication networks such as interference, coverage, throughput, robustness and costs are not able to describe dynamics and crucial aspects of future wireless and mobile networks. That is why there is growing interest in studying communication networks from a complex systems perspective, taking into account methodologies and tools built to analyze emerging behaviors, and cooperative and collaborative dynamics, among the elementary units of the system [2,5,6]. The upcoming 6G has the ambition to introduce heterogeneous interconnected elements which can dynamically interact with each other as well as with their environment in an unpredictable and unplanned way. Human beings, their devices and, consequently, their behaviors are active elements of the networks, becoming a sort of heterogeneous and aggregated things, with a crucial role in technical systems and in the design of the networking functions, representing the interacting part of a complex socio-technical ecosystem [7,8]. From the mobile users and their devices (from wearable to hand-held) or the heterogeneous smart devices that form the IoT networks to the plethora of entities of each sub-network that constitute a 6G environment, these can be represented by nodes of a multiplex social network, as shown in Figure 1.
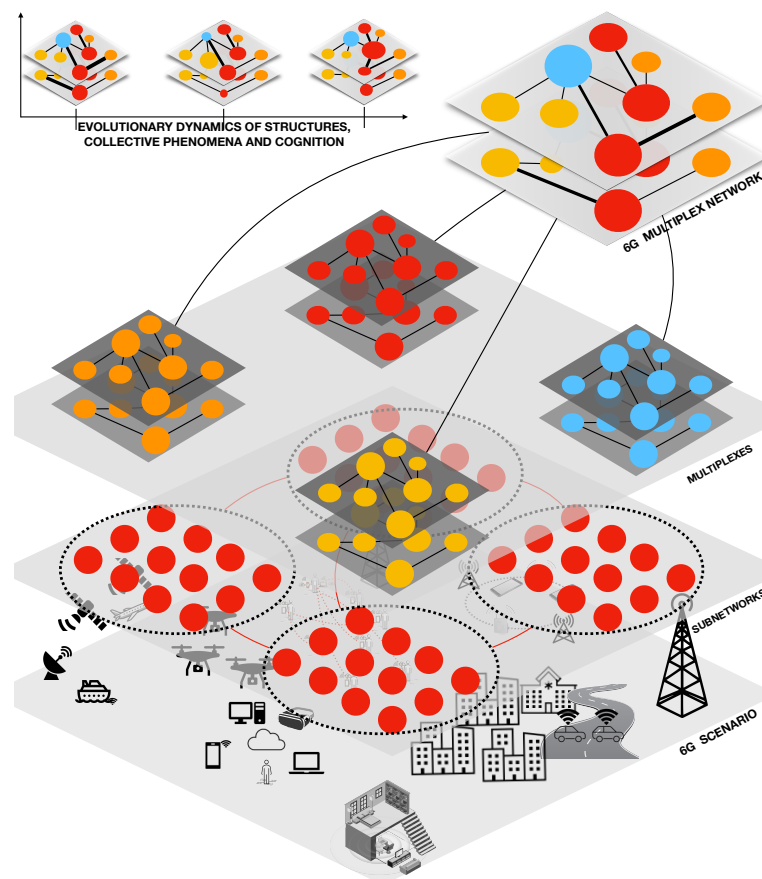

Figure 1. Multiplex representation mined from 6G subsystems. This figure shows the schematic representation through the multiplex network of a 6G scenario, including different types of sub-networks, various interactions and the evolutionary dynamics of collective phenomena.

The multiplex dimension offers a key change in perspective for the structural analysis, and it represents the suitable representation to study the emergence of complex properties of the network [9,10]. The 6G network is intrinsically suitable to be modeled as the multidimensional relational systems of different sub-networks, represented by various graphs that embed interacting elements in different ways [2].

The designing, modeling and monitoring of the behavior of these systems will be challenges to address. Since we are rapidly moving from a closed to an open and distributed system, and to a completely dynamic topology that will be characterized by a vast heterogeneity, this requires an intelligence at the node level. The node, in a fully user-centric network architecture, plays a key role in content diffusion, learning, computation and organization, and consequently, a new characterization is becoming necessary. For this reason, we propose a node profiling process based on a complex network approach, which embeds, step by step, the knowledge extracted from the introduction of the multiplex dimension, the analysis of the dynamics of collective phenomena as diffusion and competition, applying the epidemics spreading modeling and the evolutionary game theory, from the mesoscale hierarchical organization of the network in communities. We detail the profiling technique through the introduction of metrics and parameters that enables the analysis of three different aspects, structural, collective dynamical and community-based. The paper is organized as follows: in Section 2 we briefly introduce background and methods. In Section 3, we detail the modeling approach to the profiling, in terms of structural, collective and community analysis. After, we show and discuss our findings in Section 4 to conclude in Section 5.

2. Materials and Methods

2.1. Multiplex Social Network and Mesoscale Structure

In order to show the hidden dynamics of complex systems it is useful to rely on multi-dimensional network representation, which is able to unveil interesting structural properties and interdependence. A single graph representation, where elementary units are mapped into network nodes and their interactions into links, can result in a loss of knowledge about the structural complexity and connectivity [9,11]. Representing all the links in an equivalent way can lead to a loss in details and information, and so, to an incomplete description about phenomena of real-world networks. Multilayer networks embody a generalization of traditional network theory, representing a novel framework to study graphs, in which nodes have different layers of connectivity [12]. These networks are able to properly describe the complex connectivity, by distinguishing in the different layers the several channels of interactions, which can be different for relevance, context and meaning [9,10,13]. The concept of multilayer networks extends to other mathematical objects, such as multiplex networks, that describe a large number of real networks, including biological [14], social and technological systems, social networks and relationships [15], epidemic and social contagions [16], air transportation networks and brain computing [5,17–20].

The multiplexity constitutes an additional dimension of analysis which makes the full characterization of behavior in complex systems possible, unveiling crucial structural properties [21]. Furthermore, an even deeper analysis of complexity in connections can be done characterizing interactions by a given cost, a distance or a weight, reflecting their intensity in the different layers, like in the case of weighted multiplex networks [13].

Moreover, a multiplex network has a rich structure that is often reflected in its community [22]. In the literature, there are several algorithms proposed to detect communities in complex networks [23]. Since there is a tendency of nodes of the networked systems to cluster together in groups, triggering a non-trivial structure, it is interesting to analyze the mesoscale structure [22]. The aim is to unveil a hidden organization of nodes and its interplay with the structure of multiplex network, nodes profile, nature of links, and layers existence.

2.2. Collective Dynamics of Diffusion and Competition

The multiplex networks lead us to explore in what measure the heterogeneous nodes and their ties in the different layers have an impact on emerging phenomena like cascading failures, super-diffusion, spreading and epidemic dynamics [15,17–21,24]. The diffusion dynamics can be modeled as a spreading process of diseases [5,25], social contagion

of behaviors in networks [15] or misinformation and emotions spreading [26]. These processes, based on classical epidemiological models [25], find applications in many fields, from smart cities to healthcare [6,27], characterizing different collective phenomena that spread inter-personally through the social networks [16,28]. The nature of social ties has a pivotal role in shaping the structure, the heterogeneity of nodes and the diffusion in the network. There is an upsurge of social and communication technologies and an increasing in the need to embody what is traced about the activity of networked individuals. It is interesting to investigate the effect of the diffusion processes that co-evolve in the network, based on content exposure, event dynamics, collective attention and awareness, shifting the dynamics of social behaviors, also impacting on the nature of nodes and ties [5,28]. To study the evolutionary dynamics of competition, we use the Evolutionary Game Theory (EGT), which is an extension of the traditional Game Theory, and it was applied to the study of genetics, adaptation, and frequency of gene's appearance [29]. The focus is the dynamic of changing strategies, and games are a way to investigate which behavior emerges from the interaction among individuals and if it is able to persist. The purpose of these theories is to use mathematical models to describe and predict what will happen, through games, in real situations.

Games are defined by a set of N players who can choose among a set $A_i = a_{i1}, a_{i2}, ..., a_{in}$ of available strategies; with each strategy, players receive a payoff, which acts as a feedback and scores the preference [30]. EGT is useful to study the collective dynamics of cooperation and competition, through the so-called social dilemmas. In these dilemmas, there are two players and two possible strategies: cooperation and defection. They model conflict situations in which the most profitable strategy for individuals does not correspond to the most convenient for the entire social community. Thus, the players and the society would benefit more from mutual cooperation, receiving a higher benefit than in the case of mutual defection. Each of them is described by its payoff matrix representing the interactions' rules [14].

Players will both receive a reward R in correspondence of mutual cooperation and a punishment P in the case of mutual defection. A defector will get the temptation payoff T when playing against a cooperator, while the cooperator obtains the so-called sucker payoff S. Here, we consider different social dilemmas as the Prisoner's Dilemma (PD), Snowdrift Game (SD), Stag-Hunt Game (SH), and Harmony Game (HG) [31,32], for the investigation of the evolutionary dynamics of the cooperation.

3. Results

3.1. Scenario

We propose a nodes' profiling technique based on a modeling framework, as we presented in Figure 2. The scheme put in evidence the introduction of the multiplex dimension to detect a profiling in structural terms, also enabling the analysis of the diffusion and the competition dynamics with the aim at analyzing the collective contribution to the characterization of nodes. Finally, the clustering techniques for the community detection enable us to track down an added value to the nodes' role linked to a mesoscopic point of view.
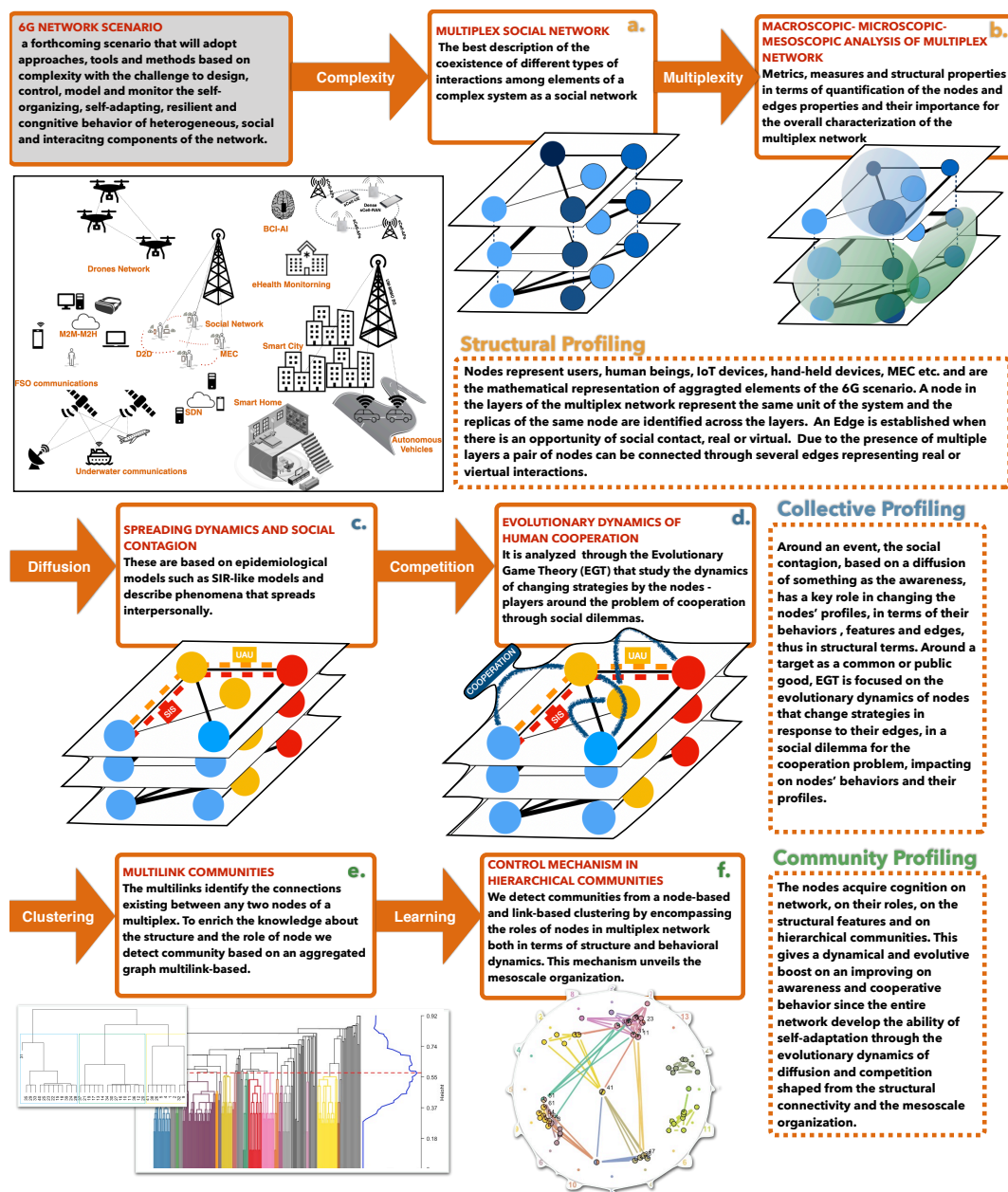
Figure 2. Modeling Approach: a technical scheme of nodes' profiling in a 6G network scenario. The figure graphically describes the various steps and the key aspects to profile the elements that constitute the 6G scenario into a complex network heterogeneous environment of cognitive elements. The approach starts from the application of the complexity and the multiplex representation and analysis to extract structural features as detailed, respectively, in (**a**,**b**). The scheme shows the collective profiling based on the analysis of diffusion and competition, representing two co-evolving and interdependent dynamics, as, respectively, detailed in (**c**,**d**). The community profiling, as in (**e**,**f**), is characterized by the detection of communities based on node and link hierarchical clustering obtained through a multilink graph.

3.2. Structural Profiling

A complex network involves basic units connected to each other with multiple kinds of relationships among them. Embedding the interactions in different layers in accordance to their types, we can consider the multiplex social network representation and analysis [5,9,10,15]. Following the scheme presented in Figure 2, we consider a multiplex social network M, assuming that it can represent a 6G sub-network and it is referred to different kinds of interactions among nodes. A node represents an aggregation of different basic units of the network (as wearable or hand-held devices, IoT systems, etc.). The multiplex network consists of a population of N nodes and $l = l_1, l_2, \dots, l_M$ layers.

Here, we consider $M = 2$ layers, where the layer l_1 is referred to a real contact network, extracted from a real case [33], while the second one is referred to a virtual layer following a theoretical scheme as, for example, a Scale-Free network [34] or a Small-World network [35]. In the literature, the multiplex network is defined as a set of graphs G, with $G = (V, E)$, with vertices V and edges E. The set of nodes N is the same for each layer, whereas the set of links E changes according to the layer [9]. Each network G_l is described by the adjacency matrix, denoted by A^l with elements a_{ij}^l, where $a_{ij}^l = 1$, if the node i and the node j are connected through a link on a layer l, otherwise $a_{ij}^l = 0$. Alternatively, in the weighted case, $a_{ij}^l = w_{ij}^l > 0$, if there is a link between i and j with a weight w_{ij}^l, otherwise $a_{ij}^l = 0$, with w_{ij} a real positive number, namely the weight of the link between i and j. In this case, a weighted multiplex network is completely specified by the weighted adjacency matrix W^l, with elements w_{ij}^l [13]. With k_i^l, we indicate the degree of the node i in a given layer l, that considers the total number of connections. Moreover, other basic features that we include in the profiling are represented by the edge overlap $o_{ij}^{[l_1,l_2]}$, that take into consideration the fact that a pair of nodes can be connected by ties in different layers, and the overlapping degree o_i [9]. In the case of weighted multiplex networks, the latter are, respectively, expressed as weighted overlap o_{ij}^w and weighted overlapping degree o_i^w [9,13]. The weights, which can be distributed heterogeneously, are strongly correlated with the structure [13], shedding light on the relevance of the links in a layer of the multiplex network representation. For that reason and with the aim at embodying social aspects into structural profiling, we apply the definition of the weights of the links as function of some key metrics, as follows:

$$w_{ij}^l = h_{ij}^l |aw_i - aw_j| \tag{1}$$

where h_{ij}^l indicates the tendency to interact with similar nodes (as equal devices) or with that ones that are in similar conditions (capacity, resources, etc.) [5,9,13]. Furthermore, we include the gap value, between i and j, of the node awareness aw. In a fixed time T, in which we observe the network and its activity, we define aw as the awareness level of a node i, that represents acquired knowledge on the sub-network to which it belongs, that is about activity about task requests, as a result of a complex discovering process. For that reason, we estimate the awareness of a node i as function of the participation coefficient P_i, and the att_i attention level, as follows:

$$aw_i|_{t=T} = aw_i|_{t=0} + \sum_T att_{i|t-e} + P_i \tag{2}$$

The multiplex participation coefficient P_i ponders the awareness since it measures the heterogeneity of the number of neighbors across the layers of each node at the edge of the weighted link, measuring the probability of acquiring more knowledge as a result of different interactions distributed across the layers of the multiplex network [5,9]. The P_i value is added to incorporate the richness of the knowledge extracted from the introduced multiplex dimension. Furthermore, the awareness can be computed as the result of the monitoring of the attention level, traced through the analysis of the user-generated data [28,36]. Data can be produced by devices in terms of willingness to cooperate for a set of tasks in IoT cognitive systems or it can represent the activity of nodes produced in social media during collective phenomena of interest, impacting on the widespread participation of networked users, on the behaviors of each node and its interactions and decisions in the dynamical evolution of the concerned phenomena [5]. We show the multiplex network M in Figure 3.

A complete list of parameters and defined metrics with its fair meaning, referring to the structural profiling, is summarized in Table 1.
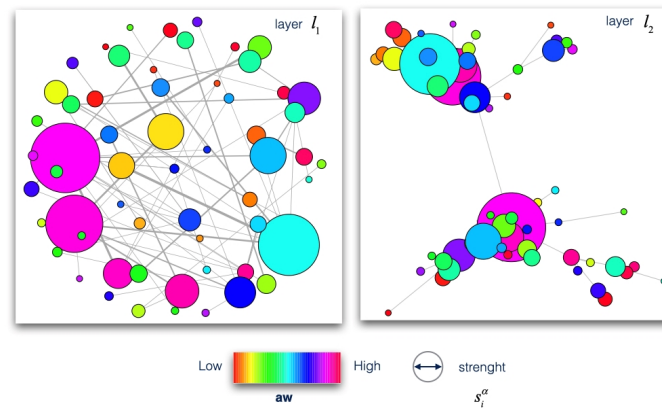
Figure 3. Multiplex Network Representation. The figure describes both layers of the multiplex network M. The layer l_1 is obtained from a real contact case, as explained in Section 3.2, while the layer l_2, follows a theoretical scheme. Here, l_2 is represented as a scale-free network. We shed light on the strength of each node (as the nodes' size), and the awareness of each node (as the nodes' color).

Table 1. Structural Profiling Parameters and Defined Metrics. Here, we summarize the general formalism of the multiplex networks nodes' parameters and some defined statistical estimators, used to draw up the structural profiling of nodes in a 6G scenario represented and analyzed through the proposed framework and the mentioned above mathematical methods and tools, based on complex systems and networks.

Structural Profiling	Formalism	Description
Parameters [9,13]	N	Population of Nodes of the Multiplex M
	M	Numbers of Layers of the Multiplex M
	A^l	Adjacency Matrix of the Layer l
	a_{ij}	Elements of the Matrix A^l
	k_i^l	Degree of Node i in the Layer l
	o_i	Overlapping Degree of Node i
	W^l	Weighted Adjacency Matrix of the layer l
	o_{ij}^w	Weighted Overlap of edge $i - j$
	o_i^w	Weighted Overlapping Degree of node i
	s_i^l	Strength of node i in the layer l
	Y_i^l	Inverse participation ratio of node i in the layer l
	P_i	Participation Coefficient of the node i
	H_i	Entropy of a node i
Defined Metrics	w_{ij}^l	Weights of interactions between $i - j$
	h_{ij}	Homophily of a pair of nodes $i - j$
	aw_i	Awareness of a node i
	$att_{i|T-e}$	Attention of a node i during T around e

3.3. Collective Profiling

Multiplex networks representation, as showed in Figure 3, constitutes the most suitable network structure to understand and investigate on collective dynamical processes and their complex interdependence [37]. In this second step, we consider key features extracted from the analysis of parameters and statistical estimators of two interdependent and co-evolving spreading processes and the evolutionary dynamics of the cooperation among nodes. Firstly, we study the spreading of two interdependent processes in the multiplex network M, modeled as disease spreading process [15] and thought of as "composed-SIR" models, that is as an extension of the classical SIR epidemic model [38]. One process is referred to a specific phenomenon as epidemic [5], collective attention [28] or a social contagion [16], while the second one is related to the diffusion of the awareness on what is

transmitted through the first one [15,39]. Heterogeneity and awareness, for each node, are included in order to describe the impact of the diffusion and competition into the structural connectivity and the evolutionary dynamics. Each node has a different awareness aw, representing the acquired knowledge from an interest or participation on a collective phenomena. As a consequence, each node, will be heterogeneously prone to join a diffusion process, since the awareness acquired has an influence on the behaviors of nodes. Firstly, we analyze two co-evolving spreading processes in the multiplex network M. The first one is based on the content shared or a phenomena of interest. Taking into consideration the nature of the phenomena or content, we can diagrammatically express the first process, in terms of reaction-diffusion equations, following two different hypothesis as indicated below as the $S^h I S^h$ or $S^h I R$ spreading processes:

$$S^h I S \Rightarrow S^h \xrightarrow{\beta_i^\alpha} I \xrightarrow{\mu} S^h \tag{3}$$

$$S^h I R \Rightarrow S^h \xrightarrow{\beta_i^\alpha} I \xrightarrow{\mu} R \tag{4}$$

The $S^h I S^h$ can occurs when a node, based on its interactions, becomes prone to participate to the diffusion, and after that it returns in the condition to the starting pools of nodes. Differently, in the $S^h I R$ the last transition is replaced by the step which occurs when a node have acquired a permanent condition and it is not available to participate again in the diffusion. Both models are characterize by the states S^h, defined as "heterogeneous susceptible state", where a node is predisposed to be involved in the spreading process [15]. The state I indicates the condition in which a node is involved or infected, while R represents the recovered state. The probability of call off the diffusion is equal to μ, while the β_i^α is the diffusion rate for each node i at each layer α in the multiplex network M. The latter represents the probability that a node i in the layer α is predisposed or susceptible to be involved in the diffusion process. We assume that, the involvement for a node in the network means that a node i is in the state of informed or infected. The heterogeneous diffusion rate depends on the weighted structural connectivity through the measures of inverse participation ratio Y_i^α [13] and the rate of awareness λ_i^α, defined in Equation (8).

$$\beta_i^\alpha = \frac{1}{1+\lambda_i^\alpha} \cdot \frac{1}{Y_i^\alpha} \tag{5}$$

Similarly, the second spreading process, focused on awareness diffusion, is diagrammatically expressed in terms of reaction–diffusion equation also in two possible different cases, as follows:

$$U^h A U^h \Rightarrow U \xrightarrow{\lambda_i} A \xrightarrow{\delta} U^h \tag{6}$$

$$U^h A F \Rightarrow U \xrightarrow{\lambda_i} A \xrightarrow{\delta} F \tag{7}$$

with λ_i^α and δ, defined as transition rates. We distinguish between the rate of awareness λ_i^α for each node i at each layer α of the multiplex M and the rate of awareness after being aware. The U^h state expresses a condition in which a node is heterogeneously unaware, while in A, it is aware about the phenomena that spreads inter-personally in the network. In the second case, the state F is introduced as the "faded state", in which a node decreases the attention and the interest to improve its knowledge about the phenomena of interest. The rate of awareness is expressed as follows:

$$\lambda_i^\alpha = \frac{s_i^\alpha}{1+s_i^\alpha} \cdot \lambda \tag{8}$$

where s_i^α is the strength of node i in the layer α of the multiplex network M [13].

Starting from the assignment of a state probability for each node i in M, to be in one of the initial states as (SU)-(SA)-(IA), and choosing the suitable spreading models for the co-evolution process, as indicated in Figure 4 and expressed in Equations (3) and (4), we add, in the profiling process, some features extracted from the analysis of the co-evolution in

the multiplex network considered. To obtain the contagion threshold, we need to apply the MMCA method, investigating the steady state of the system, indicated as β_c. The Dynamic Microscopic Markov Chain Approach (MMCA) enables us to explore the dynamics of the co-evolution on a weighted multiplex network. Following the mathematical approach, as expressed in [5], we get the density of involved nodes ρ_i:

$$\rho^I = \frac{1}{N} \sum_{i=1}^{N} p_i^{IA} \tag{9}$$

where p_i^{IA} are the probabilities of being involved and aware. On this matter, it is important to underline that the threshold model depends on the complex dynamical interplay between social contagion and the awareness spreading in M and the values of β_i^α that change in accordance with the awareness state λ_i^α, the network structure of M, and the double heterogeneity expressed in S state and U state. Consequently, the *aw* awareness of a node as expressed in Equation (2) also depends on the β and λ value in terms of attention which represents the social-aware factor that varies in function of the two rates. Both processes co-evolve modeling the sharing or participation in terms of content, interests, attention and awareness, and in terms of active or passive involvement [6,16,39]. Furthermore, we inspect the cooperative behaviors and its dynamics among nodes, which include social factors in their aggregated nature, by introducing the game theoretical approach [14]. We introduce different dilemmas as the iterated forms of Prisoner's Dilemma game (PD), Snowdrift game (SD), Stag-Hunt game (SH) and Harmony game (HG) [31]. In the analysis of the interdependent collective dynamics, we run and compare the dilemmas for a number of rounds such that a dynamical steady state is reached, in order to describe not only the problem of cooperation, but also its evolution. Nodes are treated as players that, in each elementary round of the game can decide to change or maintain its strategy, playing the game with all its neighbors in both layers of the multiplex M, in line with the Fermi function $W(S_x \rightarrow S_y)$, expressed as follows:

$$W(S_i \rightarrow S_j) = c_i \eta_i \frac{1}{1 + exp[\frac{P_i - P_j}{\delta_{ij} K}]} \tag{10}$$
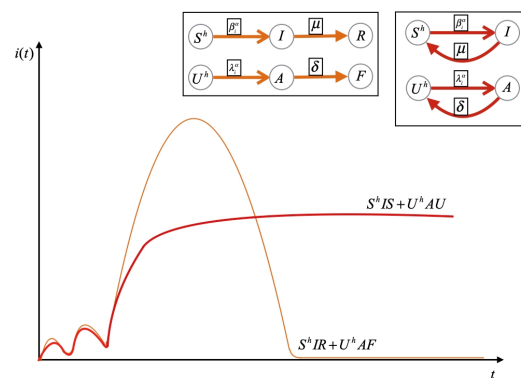

Figure 4. Density of Involved Nodes $i(t)$ versus time t, on the two processes in co-evolution. The outbreak of spreading in time is subjected to statistical variability. After a common exponential growth, we shed light on how the density of involved ("involved" means informed or infected) nodes $i(t)$ change in two different cases (respectively, orange and red) of the co-evolving processes that can converge to zero ($S_h IR - U_h AF$) or to a constant number ($S_h IS - U_h AU$).

Firstly, with this function we take into account the payoff difference $P_i - P_j$, the homophily measure δ_{xy}, a communicability measure η_x and a noise factor K to evaluate the probability that a player i on the layer l_1 decides to adopts the strategy S_j of node j playing on the layer l_2 [14]. Accounting for that, at time step t, each node can occupy one state of

the co-evolving spreading processes, and considering that, $q_i(t)$ represents the probability of node i not being involved and with $r_i(t)$ the probability of unaware node i staying unaware at the same time step t, we include in the Fermi function the dependency from the c_i factor, defined as follows:

$$c_i = \frac{1 - q_i}{1 - r_i} \tag{11}$$

This includes the joint impact of the co-evolving spreading processes and the evolutionary dynamics of the cooperation among nodes in the multiplex network M, assessed in the temporal window T. We summarize parameters and defined metrics of the collective dynamics aspects in Table 2.

Table 2. Collective Profiling Parameters and Defined Metrics. Here, we summarize the general formalism linked to the analysis of the collective dynamics of diffusion and competition that involve the networks nodes in the multiplex network M. We list the key parameters and some defined statistical estimators used to compose the collective profiling of nodes in a 6G scenario represented and analyzed through the proposed framework and the mentioned above mathematical tools, based on complex systems and networks.

Collective Profiling	Formalism	Description
Parameters [5]	S^h	Heterogeneous Susceptible State
	I	Involved State
	R	Recovered State
	U^h	Heterogeneous Unaware State
	A	Aware State
	F	Faded State
	μ	Probability to transit from state I to state R
	δ	Fading Rate
	ρ_I	Density of Infected Node
Defined Metrics	β_i^α	Heterogeneous involvement rate
	λ_i^α	Heterogeneous awareness rate
	$W(S_i \rightarrow S_j)$	Fermi function
	c_i	Impact factor of the co-evolving processes

3.4. Community Profiling

The whole multiplex network M can be also described via its multilink [22] with the aim at describing a mesoscopic dimension. Every pair of nodes in M is connected by a multilink $\overrightarrow{m}_{ij} = (m_{ij}^{[l_1]}, m_{ij}^{[l_2]})$ with l_1 and l_2, layers of the multiplex M, with $m_{ij}^l = a_{ij}^l$ indicating the set of all links connecting these nodes in the different layers [22]. If $\overrightarrow{m} = 0$ there are no interactions between the two nodes in the multiplex M. This measure shows the basic motif [9] that contributes to the unveiling of correlations between structure and function of a network [12]. We construct a weighted aggregated network $\widehat{G}^w$, based on the multilink $\overrightarrow{m}_{ij}$, with the adjacency matrix $A_{ij} = \theta(\sum a_{ij}^{[l]})$, with $\theta(x)$ a step function [22]. For detecting community, we introduce the multidegree $k_i^{\overrightarrow{m}}$ for each node i of the population, showing how many multilinks $\overrightarrow{m}$ are incident on the node i [13]. In line with the previous steps of profiling, we propose two community detection methods introducing the mesoscopic analysis of the network. The resulting hierarchy embeds both the richness of the structure and the collective dynamics in the cognitive profiling since each node belongs to a community with a specific role that dynamically tunes in response to the changes in behaviors and social factors. Thus, we combine the different interdependent steps of profiling and we apply two hierarchical clustering technique to the multiplex network, respectively, a node-based and a link-based technique. The node-based hierarchical clustering analyzes a set of dissimilarities through the application of an agglomeration method. We define a dissimilarity structure based on the distance between each pair of nodes in function of the

number of cooperations n_{ci}, the social dilemma, the between-ness centrality c_b, and the multidegree $k_i^{\overrightarrow{m}}$ [9,22]. We construct a dendrogram via hierarchical clustering method containing information about the structure and based on the dissimilarity matrix. More specifically, we evaluate a defined score function for each pair of nodes to estimate the distance between them, as indicated below:

$$hc_{ij|\widehat{G}} = \frac{c_b(i)}{n_{c_i} + k_i^{\overrightarrow{m}}} - \frac{c_b(j)}{n_{c_j} + k_j^{\overrightarrow{m}}}. \tag{12}$$

With this approach, each node belongs to a community with a role in the hierarchical organization of the multiplex network M. Similarly, by constructing the aggregated graph multilink-based $\widehat{G}$, we furthermore apply a hierarchical clustering by grouping instead the links in different communities showing a hidden mesoscale structural organization, highlighting how nodes can belong, at the same time, to different link-based communities. We summarize parameters and defined metrics of the community profiling aspects in Table 3. We detail the output of these methodologies in the next section.

Table 3. Community Profiling Parameters and Defined Metrics. Here, we summarize the general formalism linked to the mesoscale analysis applied to the multiplex network M. We list the key parameters and some defined metrics used to compose the community profiling of nodes in a 6G scenario represented and analyzed through the proposed framework and the mentioned above mathematical tools, based on complex systems and networks.

Community Profiling	Formalism	Description
Parameters [9,13]	$\overrightarrow{m}_{ij}$	Multilink between i, j
	$\widehat{G}^w$	Weighted Aggregated Network
	A_{ij}	Elements of $\widehat{G}^w$
	$k_i^{\overrightarrow{m}}$	Multidegree of node i
	n_{ci}	Number of cooperations
	c_b	Between-ness Centrality
Defined Metrics	$hc_{ij|\widehat{G}}$	Score Function for dissimilarity structure

4. Discussion

Simulations have been performed considering a weighted multiplex network M as explained in Section 3. To build the model, do computation and obtain our results, we used the programming language R and the IDE RStudio. The figures were generated thanks to the package Plotly and Linkcomm [40–43]. We consider a population $N = 61$ in the two layers l_1, l_2 representing two distinct kinds of weighted interactions and connectivity between nodes. In the first layer l_1, the interactions are based on the graph extracted from a real contact network [33], while in l_2, we consider a theoretical scheme for the same population of nodes N. In Figure 5, we can observe how the structural heterogeneity in the multiplex representation of the networks leads to highlight several key aspects for the structural roles of nodes. For a heterogeneous weighted multiplex network, we study how certain metrics are distributed across different layers and in the whole structure. We have computed the degree k_i^l, the strength s_i^l and the overlapping degree o_i as indicated in the two heath-map plots. Both plots display metrics for the multiplex M composed by the first layer based on real contact network while through the second layer we can compare the case of a scale-free network (in the top-left plot) with a small-world network (in top-right plot). For the sake of clarity, we indicate the first resulting multiplex M as $RC - SF$, and the second one as $RC - SW$. As shown in Figure 5, our findings exhibit two different levels of heterogeneity. We can notice how in the $RC - SW$ case, there is a small heterogeneity due to a high clustering and modularity, meaning that there are groups of nodes more highly connected than the rest of the network with an abundance of high degree nodes

that act as hubs of the network. Instead, in the $RC-SF$ case, there are few hubs in the network since it exhibits a highly heterogeneous and high degree correlations. In the other two panels below in Figure 5, the curves correspond to the distribution of the participation coefficient P_i, in the range $[0,1]$, considering this variation in function of the Z-score of the overlapping degree, $z(o_i)$. We extract profiling features structurally classifying the nodes of the multiplex in three classes, focused, mixed and multiplex nodes, putting in evidence the fitness based on the two cases $RC-SF$ and $RC-SW$, the awareness distribution of aw_i and the homophily distribution of the values h_{ij}, in accordance with the modeling approach. Our findings highlight an increasing of the P_i value in both cases ($RC-SF$–$RC-SW$), and a higher density of regular multiplex nodes. Decreasing the homophily (from the left plot to the right plot below), the findings exhibit a decreasing of P_i for the $RC-SW$ case that has a more homogeneous distribution with a higher density of regular mixed nodes. Differently, in the same condition, in the $RC-SF$, the findings show an increasing of $z(o_i)$ and a higher density in the multiplex hubs, since, although there is a higher value of homophily, the structural heterogeneity of the topology produces a more heterogeneous distribution of nodes' roles.
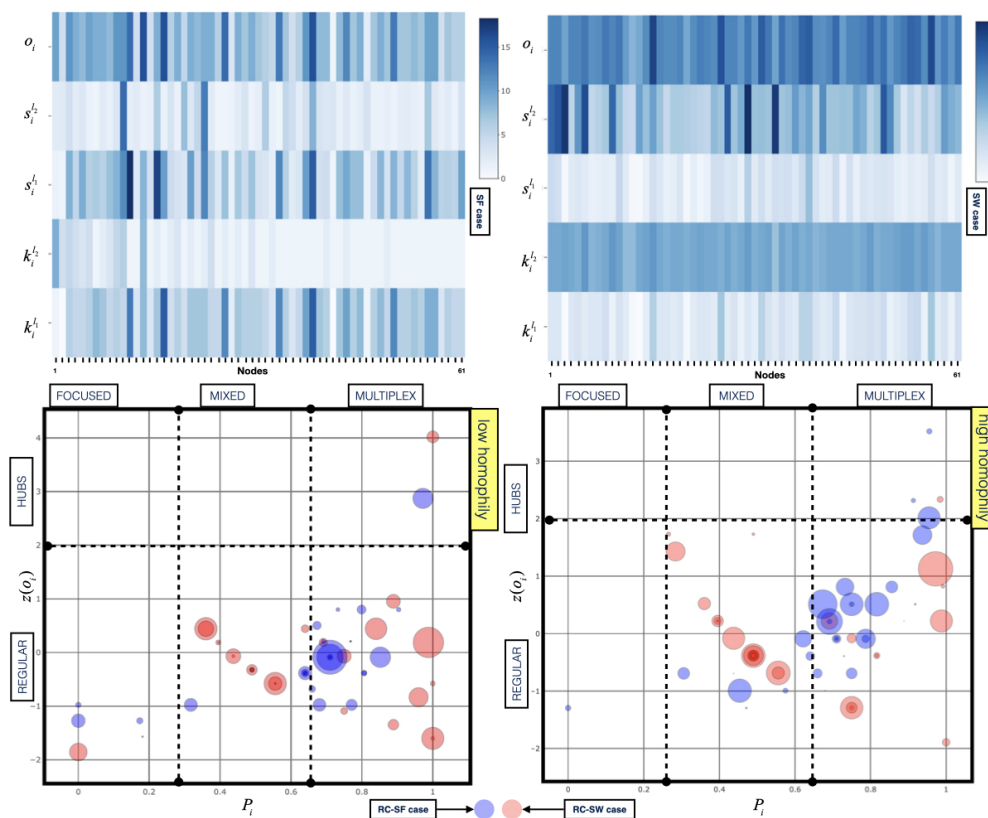

Figure 5. Roles of Nodes from Structural Analysis. We show four panels to shed light on the roles of each node in M as a result of the structural profiling. At the top, the two heat-maps show with a color-code the value of degree k_i^α, strength s_i^α and overlapping degree o_i for each node, from the largest (darkest) to the smallest (brightest) in both cases of scale-free (SF) and small-world networks (SW) as assumption for the graph of the second layer of the multiplex that describes a social-based interaction graph, jointly with the first layer based on a real contact network (RC). Below, the other two panels show two cartography obtained by plotting, for each node, the multiplex participation coefficient P_i versus the Z-score of the total overlapping degree o_i in the cases of a theoretical distribution with high standard deviation δ for low homophily and low standard deviation δ for high homophily, referring to the distribution of similarity features among nodes. In both plots, comparing the cases $RC-SF$ and $RC-SW$, we identify specific roles of nodes put in evidence by the plots, in terms of focused, mixed, multiplex, regular or hubs nodes.

Figure 6 points out the profiling features embedding a measure of how a node of the multiplex structure is involved in the collective dynamics. We show the findings of

diffusion and competition dynamics, by displaying the trend of infection rate β_i in function of the awareness rate λ_i jointly with our modeling approach presented in Section 3.
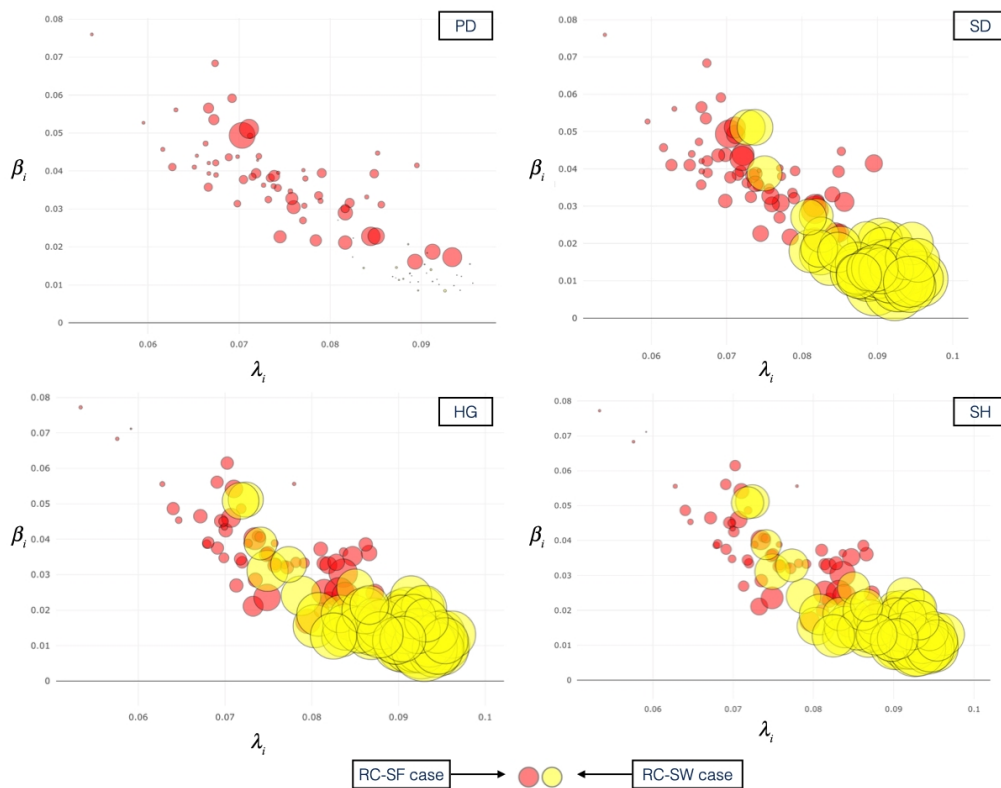

Figure 6. Roles of Nodes from Collective Dynamics Analysis. We show four plots to shed light on the roles of each node of the population considered for the multiplex network M as a result of the collective profiling. The plots display, in the plane λ-β, the trend of involved rate in function of the awareness rate, comparing both cases of scale-free (SF) and small-world (SW) networks as assumption for the graph of the second layer of the multiplex that describe a social-based interaction graph, jointly with the first layer based on a real contact network (RC), showing in red color the $RC - SF$ case, while in yellow the $RC - SW$ case. In accordance with the model and the interdependence between the evolutionary dynamics linked to the co-evolving spreading processes and the evolutionary game theory, we set and compare the four social dilemmas (PD-SD-HG-SH) displaying the trend of the spreading dynamics in relation to the number of cooperation n_c for each node (as dot size variation) of the multiplex network M.

The plots show the resulting trend, in the plane λ-β, of the spreading dynamics in conjunction with the cooperative behavior of nodes in the multiplex M, taking into consideration the two cases $RC - SF$ and $RC - SW$, and the four social dilemmas PD-SD-HG-SH. Since the diffusion dynamics impact on the evolutionary dynamics of cooperation, we compare the modeling of the conflict situations with the different dilemmas. We find out how the increasing of the awareness rate λ_i, and the consequently decreasing of infections rate β_i produce an impact on the collective cooperation dynamics. Namely, in case of social dilemmas in which the cooperation dominates the defection as SD, HG and SH, a more homogeneous number of cooperations for each node is more evident in the case of $RC - SW$. Differently, in the plot referred to the PD game, in case of $RC - SF$ we detect a more heterogeneous distribution of cooperative behavior. In addition, in the PD, the increasing of λ_i results in a decreasing of β_i, in both cases $RC - SF$ and $RC - SW$, while in the other social dilemmas cases, this impact is stronger in $RC - SW$ then in $RC - SF$. What is more, in the $RC - SF$ case and SD-HG-SH, the decreasing of β_i with the increasing of λ_i is up to a specific threshold, resulting in a change in strategy, in accordance with the Fermi function as expressed in Equation (10), shedding light on the increased size of nodes in line with the increasing average in number of cooperation for each node.

In Figure 7, we show four panels linked to the community profiling detailed in the Section 3.4. By defining the dissimilarity between nodes based on Equation (12) we apply a hierarchical clustering of nodes as showed in the (a), (b) plots of Figure 7, that, respectively, exhibits the dendrogram and the graph based colored partitions of the six different communities. The detected groups of nodes based on a complex score function that take into consideration the structural and collective dynamical properties, show a community connectivity lower than the multiplex one. This unveils how the multiple interactions in the multiplex structure of each node represent a key point in a hierarchical characterization of the nodes' roles. In the plot (c) and (d) (produced through the R package "linkcomm" [43]) of Figure 7, we find out results linked to the multilink and the link-based hierarchical clustering to reveal the richness of the network at the mesoscale level. We show in (c) the cluster dendrogram and the sixteen communities detected, and in (d) a graph visualization of the multilink-based aggregated graph $\widehat{G}$. This methodology sheds light on in what measure the nodes of the multiplex structure, independently of its layer activity, can belong to different communities, revealing the hidden information extracted from the mesoscale structure organized in communities. The findings demonstrate that since the nodes belongs to multilinks communities, on average they have a high community activity, suggesting that the network can be expanded in many layers of different interactions. Moreover, the plot (d) of Figure 7 also underlines the brokering function of certain nodes among whose belonging to many communities (dots with multiple colors), playing a pivotal role in the cognitive collective dynamics.
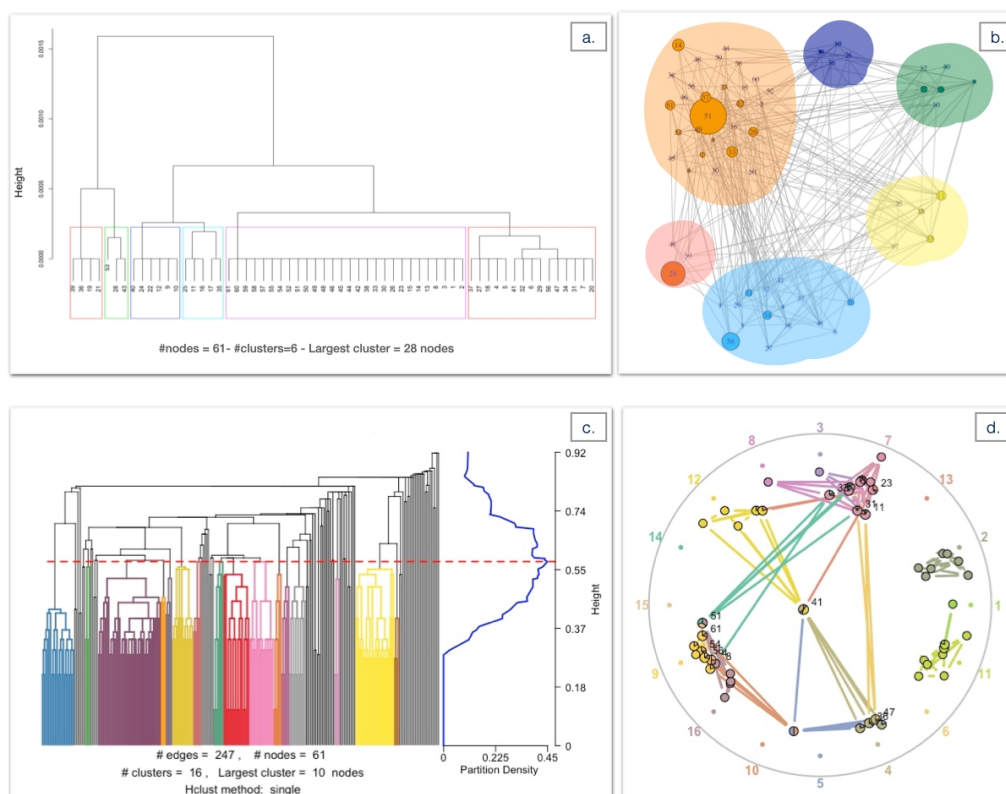

Figure 7. Roles of Nodes from Community Analysis. In the (**a**,**b**) panel, we display the findings of the hierarchical clustering of nodes based on the score function $hc_{ij|\widehat{G}}$. In (**a**), we show the cluster dendrogram, with each colored square shedding lights on different node communities, determined by cutting the dendrogram at a specific height. In panel (**b**), we display the visualization of nodes communities using graph highlighting the six communities of the weighted aggregated graphs $\widehat{G}$, whose interaction are referred to existing multilinks between a pair of nodes, and it is extracted from the multiplex network M. Moreover, we underline the multidegree values $k_i^{\overrightarrow{m}}$ for each node in accordance with their size. In the panel (**c**,**d**), we exhibit the findings of a link-based clustering of the graph $\widehat{G}$ of the multiplex M, to detect the links communities. In (**c**), we show the cluster dendrogram highlighting the sixteen clusters detected. In (**d**), we illustrate the visualization of links communities using graph. Panel (**d**) shows a Spencer circle layout with numbers around the circumference of the circle that are referred to community ID.

5. Conclusions

Over the last few years, the analysis and modeling of communication networks as complex dynamical systems has attracted high interest, especially in the interdisciplinary research. The 6G network is intrinsically suitable to be modeled as a multidimensional relational system of different sub-networks, represented by various graphs which embed elements that interact in different ways. These elements represent connected heterogeneous things that can share resources, archives, and tasks to provide services. These ecosystems involve devices with constrained resources and computational capabilities and call for novel algorithms and a new characterization for dynamically managing lightweight and simple services as a microservice. It represents a task for a physical edge element of the network, that in a virtual space is connected with other elements, seen as resources able to perform different tasks with the aim at providing an application that is in line with the consumer's requirements. Heterogeneous things and devices (as sensors or hand-held devices), virtualized in edge cloud represent data and computational resources for tasks. In the proposed work, the elements are physical, virtual and concatenated resources. The interest mainly falls on the fact that those systems can be effectively described as complex networks, leveraging on the complex systems theory to engineer, design and model a 6G network. The complex approach for 6G systems will envision distributed artificial intelligence, to implement a fully user-centric network architecture. Introducing complexity through the proposed profiling approach, in 6G network, allows nodes to acquire cognition ability, as the results of processes able to disentangle grades of knowledge on their connectivity and behaviors on multiple different channels by deepening the investigation of structural and social aspects of the network, the collective dynamics of diffusion and competition, and the learning of belonging to various communities. In this paper, we propose a node profiling process that, step-by-step, allows defining of different aspects extracted from a complex network analysis, to shape a profile which embeds macroscopic, microscopic, mesoscopic, dynamical and learning properties. Our proposed profiling framework describes a set of inter-operable abstract classes referred to processes which constitute a cognitive level for nodes and community of nodes in network. We detail metrics and parameters, theory and analytical tools to study the coexistence of various type of interaction among nodes, the multiplex dimension of the network and the interplay of collective dynamics and the mesoscale organization.

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