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Rationalization of indecisive choice behavior by pluralist ballots[☆]

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ABSTRACT

We describe a bounded rationality approach for indecisive choice behavior, in which all, some or none of the items in a menu may be selected. Choice behavior is s-pluralist if there is a population of voters – encoded by arbitrary binary relations – such an item is selected from a menu if and only if it is endorsed by a share of voters larger than s. We prove that all forms of pluralism are equivalent to Axiom α. We also examine special forms of pluralism, in which the share of voters is either minimum (liberal) or more than one half (democratic).

1. Introduction

Many models of multi-self choice have been designed with the purpose of explaining context-sensitive behavior, in which the decision maker (DM) is observed to select exactly one item from each menu: see, among several contributions of the topic, Ambrus and Rozen (2014), Fudenberg and Levine (2006), Manzini and Mariotti (2007), May (1954), and Silva-Leander and Seth (2017). In a variety of manners, these models accommodate choices of DMs whose behavior violates Chernoff (1954)'s property of 'contraction consistency'. This property – which, following Sen (1971), will be hereafter referred to as *Axiom α* – says that if an item is chosen from a menu, then it must be also selected from any submenu containing it. The specialized literature often regards Axiom α as a compelling property of choice consistency, which ought to be preserved to label a behavior as rational. Ambrus and Rozen (2014) even use the number of its violations to measure the degree of irrationality of a behavior.

This paper does the opposite job for the most general expression of selection, here called an 'extensive choice'. Our observable is a possibly indecisive choice behavior, in which the DM is allowed to select all, some or none of the items in each menu. Then, within a multi-self formulation of the selection problem, we describe a bounded rationality model that explains *exactly* all extensive choices satisfying Axiom α.

For comparison, consider the multi-self model called *rationalization by multiple rationales* (*RMR*), designed by Kalai et al. (2002) for single-valued choices. The authors argue that violations of Axiom α can be explained if every choice set has an epistemic value, and the DM applies a suitable rationale (linear order) for the specific choice problem posed by that menu.[1] The choice problem triggers the application of one (any) of the available linear orders, but there is no explicit structural link between menus and rationales.[2]

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1 Compare to Sen (1993), who provides arguments for 'rational' ways to contradict Axiom α.

2 For a context-sensitive version of the RMR model, where the justifying rationale is directly linked to the menu, see Giarlotta et al. (2022b).

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Suppose, on the other hand, that the DM is allowed to select more than one item from a menu. Then it may be the case that she finds reasons different from classical rationality principles to test the suitability of each selection. The next example illustrates this point.

Example 1. *Mrs. Watson's Movie Selection (Eliaz and Ok, 2006, Example 1).* Mrs. Watson has to choose a movie to rent for her children, Alice and Tom. The set of available movies is $X = \{x, y, z\}$. Children's preferences over X are represented by the following rankings: $y \succ_A z \succ_A x$ for Alice, and $x \succ_T y \succ_T z$ for Tom. Then Mrs. Watson's choice c is as follows (set delimiters are omitted):

$$c(xyz) = xy\,, \quad c(xy) = xy\,, \quad c(xz) = xz\,, \quad c(yz) = y\,,$$

where a decision like $c(xz) = xz$ means that she will flip a coin to decide which movie to rent between x and z. Although Mrs. Watson's approach contradicts a classical tenet of rationality, namely WARP, Eliaz and Ok (2006) illustrate how plausible her way to settle conflicts may be considered.

Alternatively – and more in the flavor of this paper – Mrs. Watson's selections might be explained by the following mental process. In any choice problem (say, the selection from the menu $\{x, z\}$), she flips a coin between the eligible movies that are *best for at least one of the children*. Then her apparently conflicting choices become a natural consequence of her attitude. Indeed, she explains her choice from the menu $\{x, z\}$ by the fact that both Tom's favorite (x) and Alice's favorite (z) cannot be disregarded. For a similar reason, z is rejected from $\{x, y, z\}$, because it is never the best in any of the two available rankings. Mrs. Watson's selections are all justified by this mechanism: the flip of a coin between the best alternatives for at least one of the two rationales, namely Alice's and Tom's preferences.

The latter argument is subtly different from the spirit of the RMR model in a critical feature: Mrs. Watson is (voluntarily) bound to the opinions of her children. When the rationales are fixed, her opinion is determined by them. The rationales are not a mere consultative tool, instead they enforce actions. To emphasize the role of the rationales in the process of rationalization, we shall refer to them as 'voters', and the selections they enforce will be called 'ballots'. The model that stems from this ballot procedure allows us to draw a neat demarcation line between 'rational' and 'non-rational' extensive choice behavior. The next example illustrates this point.

Example 2. *A Non-liberal Choice.* Suppose that a DM chooses from the grand set $X = \{x, y, z\}$ as follows: $c(xz) = x$, and $c(A) = A$ for any other menu. According to the most basic presentation of our model, a *liberal* justification of this choice behavior is a finite population V of voters over X such that the options selected from each menu are exactly those chosen by *at least one* voter in V. Here we represent a *voter* by an 'unrestricted' binary relation $\gg$ on X, that is, an arbitrary subset of $X \times X$, which possibly displays loops or cycles.[3] In other words, a liberal justification of c consists of a finite family $V = \{\gg_i : i \in I\}$ of unrestricted binary relations over X such that, for each menu $A \subseteq X$ and candidate $x \in A$,

$$x \in c(A) \quad \iff \quad x \in \max(A, \gg_i) \text{ for some } i \in I.$$

(As usual, $\max(A, \gg_i)$ is the subset of non-dominated items of A, namely $x \in A$ such that $y \gg_i x$ holds for no $y \in A$.) Note that $\gg_i$ canonically induces an extensive choice υ_i over X, defined by $\upsilon_i(A) = \max(A, \gg_i)$ for all $A \subseteq X$: we call υ_i the *ballot* over X derived from $\gg_i$. Therefore, a liberal justification for c is equivalently given by a family $\mathcal{V} = \{\upsilon_i : i \in I\}$ of ballots over X such that for each $A \subseteq X$ and $x \in A$,

$$x \in c(A) \quad \iff \quad x \in \upsilon_i(A) \text{ for some } i \in I.$$

It turns out that such a family (of voters, or, equivalently, ballots) does not exist, and so c cannot be explained by the liberal paradigm. We conclude that this behavior must be labeled *non-rational* by the simplest instance of our approach.

A direct proof of the non-liberality of the choice described in Example 2 is not immediate, although the number of alternatives is tiny. A simpler proof relies on the normative implications of our model: liberality is equivalent to the satisfaction of Axiom α (Aizerman and Aleskerov, 1995), but Axiom α is violated (z is selected from the grand menu X, but it is not chosen from its submenu $\{x, z\}$).

The liberal model described above may appear excessively permissive, insofar as a single ballot (equivalently, voter) suffices to ensure the election of a candidate. In contrast, consider a 'pluralist' version of our approach, in which an option is selected from a menu if and only if it is chosen *by more than a fixed share* of the ballots/voters. Formally, there is a finite collection $\mathcal{V} = \{\upsilon_i : i \in I\}$ of ballots over X and a share $s \in [0, 1)$ such that, for each $A \subseteq X$ and $x \in A$,

$$x \in c(A) \quad \iff \quad x \in \upsilon_i(A) \text{ for more than } s \cdot |I| \text{ ballots.}$$

When the share is 'large enough' (e.g., $s = 0.5$), such a justification has more solid grounds than a liberal one, because it provides a more robust support for a decision.

Rather surprisingly, the normative implication of the general version of our model is the same regardless of the value of the share: in fact, here we show that an extensive choice is s-pluralist (for any s) if and only if Axiom α holds (Theorem 1). Our findings provide a generalization of the characterization given by Aizerman and Aleskerov (1995), whereby liberalism is extended to all types of pluralism.

A natural question arises: *Can we distinguish a specific pluralist paradigm from a different one?* To answer this question in the positive, we only need to focus on two instances of pluralism, namely: (i) 'liberal' pluralism, where a single endorsement suffices for the selection of an item; and (ii) 'democratic' pluralism, where the endorsement of a strict majority is necessary and sufficient for selection. Then we prove that we can differentiate liberal from democratic behavior by looking at the minimum number of voters that provide a rationalization. To that end, here we first determine a tight upper bound to the minimum number of voters necessary for a liberal justification (Theorem 2) and then derive the asymptotic behavior of this number (Corollary 2).

This paper is organized as follows. Section 2 collects preliminaries about choice rationalization, and introduces ballots and voters. In Section 3, we define and characterize rationalization by pluralist ballots, and then dwell on the minimum number of voters necessary for a justification. Section 4 concludes the paper.

2. Rationalization, ballots, and voters

Let X be a finite grand set of $n \geqslant 2$ alternatives (also called items or candidates). As usual, 2^X denotes the family of all subsets of X, and each $A \in 2^X$ is a *menu*. An *extensive choice* over X is any map $c : 2^X \to 2^X$ satisfying the following property:

(Contractiveness) $c(A) \subseteq A$ for each $A \in 2^X$.

If, in addition, c satisfies

(Decisiveness) $c(A) \neq \varnothing$ when $A \neq \varnothing$,

[3] The notation suggests that $x \gg y$ is interpreted as x 'dominates' y. The arbitrariness of $\gg$ allows for situations of the type '$x \gg x$' or '$x \gg y \gg x$'.

then it is simply called a *choice* (correspondence). In particular, we shall use the expression *'choice function'* for a single-valued choice, when a unique item is selected from each menu. To simplify notation, we often omit set delimiters, writing $c(x) = x$ in place of $c(\{x\}) = \{x\}$, $c(xyz) = xy$ in place of $c(\{x, y, z\}) = \{x, y\}$, etc.

Historically, choices have been an object of careful study, whereas extensive choices have usually been disregarded. A notable exception is given by Gerasimou (2018), who uses extensive choices in his inspection of choice-theoretic explanations for deferral-permitting models and their revealed preference analysis.[4] The author explains that imposing decisiveness blurs the distinction between 'choice' and 'decision': in fact, the former term is a restricted version of the latter, which allows for a *status quo* outside option or deferral. In a very recent work, Costa-Gomes et al. (2002) argue again in favor of considering non-decisive behavior: in fact, they report on experiments suggesting that choice models rejecting decisiveness may offer a powerful lens to study revealed preferences. Following this stream of analysis and evidence, our main results explicitly avoid the assumption of decisiveness, and concern the utmost general form of behavior, that is, extensive choices.

A binary relation $\succ$ over X is an arbitrary subset of $X \times X$. In particular, $\succ$ is *asymmetric* if $x \succ y$ implies $\neg(y \succ x)$ for all $x, y \in X$; in this case, $\succ$ is also *irreflexive*, that is, $x \succ x$ holds for no $x \in X$. The classical notion of rationality in *revealed preference theory* is encoded by the existence of an asymmetric relation that explains choice behavior by maximization (Samuelson, 1938):

Definition 1. An extensive choice c over X is *rationalizable* if there is an asymmetric relation $\succ$ over X such that $c(A) = \max(A, \succ)$ for all $A \in 2^X$, where $\max(A, \succ)$ is the set $\{x \in A : y \succ x \text{ for no } y \in A\}$.

As usual, we interpret $y \succ x$ as 'item y dominates item x', and so $\max(A, \succ)$ collects all non-dominated items in A. Note that if c is a rationalizable choice, then the asymmetric relation $\succ$ that rationalizes c must be unique; moreover, $\succ$ is also *acyclic*, i.e., no *cycle* $x_1 \succ x_2 \succ \ldots \succ x_k \succ x_1$ of $k \geqslant 3$ items can exist.

The notion of rationality presented in Definition 1 does not fit well the very concept encoded by an extensive choice, because it may fail to explain indecisive choice behavior. Indeed, if c is a rationalizable extensive choice, then all menus A of size one or two *must* have a nonempty choice set $c(A)$. Therefore, a rational but possibly indecisive DM is allowed not to decide only for menus with at least three items, but she is *forced* to select at least one item from all menus of size two and one. In this regard, Costa-Gomes et al. (2002) report on experimental findings in psychology with a story from Shafir et al. (1993) (see also Gerasimou, 2018): Thomas Schelling "*was presented with two attractive encyclopedias and, finding it difficult to choose between the two, ended up buying neither [...] Had only one encyclopedia been available he would have happily bought it.*". However this behavior has been contradicted in experiments with one and two alternatives (Tversky and Shafir, 1992, Section 4).

To further argue against a forced selection on menus of size two or one, consider the following examples. In a political ballot, if there are only two candidates x and y belonging to extreme wings, and I have very moderate political views, then I shall decide not to vote at all (that is, $c(xy)$ is empty). By the same token, if I am allergic to chocolate, and a restaurant only offers a chocolate cake x as house dessert, then I will avoid taking dessert (that is, $c(x)$ is empty). All in all, the existence of pairs of items that pairwise eliminate each other, as well as the possibility of having self-excluding alternatives, should be explicitly allowed, because both cases are compatible with abstract rationality as well as experimental findings.

The next definition relaxes the classical notion of (asymmetric) rationalizability allowing the rationalizing preference be arbitrary:

Definition 2. An extensive choice c over X is a *ballot* if there is an arbitrary binary relation $\gg$ over X, called a *voter*, such that

$$c(A) = \max(A, \gg) = \{x \in A : y \gg x \text{ for no } y \in A\}$$

for each $A \in 2^X$. We denote a ballot over X by v, w, etc. (instead of c), and identify it with any of the voters that explain it by maximization.[5]

The existence of a loop $x \gg x$ says that item $x \in X$ is 'intrinsically bad' in the voter's eyes, and so will never be selected. The item eliminates itself as in the case of a person allergic to chocolate: the fact that x contains chocolate deems x ineligible, and this is independent of the structure of X and any possible binary comparison.

A situation of the type $x \gg y \gg x$ may mean that either (i) there are scenarios for which a voter considers x strictly better than y, and different scenarios in which the converse happens, or (ii) the two items are too difficult to compare. Such a situation triggers indecisiveness between x and y, and the voter ends up choosing none of the two distinct items, as in the case of a political ballot or the two encyclopedias.

The difference between the two notions of rational behavior, respectively encoded by Definitions 1 and 2, only materializes for indecisive DMs:

Lemma 1.

(i) *For a choice, being rationalizable and being a ballot are equivalent conditions.*
(ii) *For an extensive choice, being rationalizable implies being a ballot. The converse implication is false.*

Proof. Straightforward. □

It is well-known that the rationalizability of a (decisive) choice behavior is characterized by the satisfaction of two classical properties of consistency:

Axiom α**:** for all $A, B \subseteq X$ and $x \in X$, if $x \in A \subseteq B$ and $x \in c(B)$, then $x \in c(A)$;

Axiom γ**:** for all $A, B \subseteq X$ and $x \in X$, if $x \in c(A)$ and $x \in c(B)$, then $x \in c(A \cup B)$.

Axiom α is due to Chernoff (1954). It says that any item selected from a menu is also chosen from any smaller menu containing it: that is why this property is often called *standard contraction consistency*. Its role in the abstract theories of rational individual choice and social choice is central, sometimes in the form of *independence of irrelevant alternatives* (Arrow, 1950). The normative appeal of Axiom α is rather strong, as discussed in Eliaz and Ok (2006, Remark 1) and Heller (2012, Section 2.2). In fact, Nehring (1997, p. 407) goes even further, calling Axiom α "*the mother of all choice consistency conditions*".

4 See also Gerasimou (2016).

5 A ballot may be rationalized by more than one voter. For instance, the extensive choice c such that $c(A) = \varnothing$ for all $A \in 2^X$ is rationalized by any binary relation over X in which all items are self-excluding. Uniqueness of the rationalization of a ballot may be achieved by imposing further conditions; however, here we do not deal with this issue, because it is out of the scope of the paper.

Axiom γ, often referred to as *standard expansion consistency*, was introduced by Sen (1971). It says that if an item is chosen in two menus, then it is also selected from the larger menu obtained as their union.

Theorem (*Sen, 1971*). *A choice is rationalizable if and only if both Axiom α and Axiom γ hold.*

This characterization still holds for extensive choices, provided that we employ the relaxed notion of rationality described in Definition 2:

Proposition 1 (*Aizerman and Aleskerov, 1995, Theorem 2.5*). *An extensive choice is a ballot if and only if Axioms α and γ hold.*

For a proof of Proposition 1, see Aleskerov and Monjardet (2002, Theorem 2.8).

3. Rationalization by ballots

Here we introduce the main notion of the paper (Section 3.1), justify it (Section 3.2), and characterize it (Section 3.3). We also examine two special cases of the pluralist paradigm (Section 3.4).

3.1. Preliminaries

Definition 3. Let s be any real number such that $0 \leqslant s < 1$, here called a *share*. An extensive choice $c : 2^X \to 2^X$ is *s-pluralist* if there is a finite family $\mathcal{V} = \{v_1, \ldots, v_k\}$ of (not necessarily distinct) ballots over X such that for any $A \in 2^X$ and $x \in X$,

$$x \in c(A) \quad \Longleftrightarrow \quad \frac{\left|\{i : x \in v_i(A)\}\right|}{k} > s\,. \tag{1}$$

We say that c is *rationalizable by ballots* if it is s-pluralist for some share $s \in [0, 1)$.

Equivalently, c is s-pluralist when there is a family of (not necessarily distinct) voters such that a candidate is selected from a menu if and only if it is endorsed by a share of voters strictly larger than s. It will always be understood that voters/ballots in a given family need not be pairwise distinct.

The model of rationalization by ballots described in Definition 3 aims to build a bridge between choice theory and voting theory. In fact, a selection process is explained by means of a population of voters, who make a *collective* decision by appealing to a *socially acceptable* paradigm, here encoded by a threshold.

Next, we state some simple properties of the model described in Definition 3.

Lemma 2. *Any extensive choice that is rationalizable by ballots satisfies Axiom α.*

Proof. This follows from a routine application of Definition 3 and the fact that ballots satisfy Axiom α (Proposition 1). □

The 0-pluralist case is also called *liberal* (see Section 3.4). It has a very simple representation, which owes to the fact that, when $s = 0$, the equivalence (1) becomes: $x \in c(A)$ if and only if $x \in v_i(A)$ for some i.

Lemma 3. *An extensive choice c is 0-pluralist if and only if there exists a family $\mathcal{V} = \{v_1, \ldots, v_k\}$ of ballots such that $c = \bigcup_{i=1}^k v_i$ (i.e., $c(A) = \bigcup_{i=1}^k v_i(A)$ for any menu A).*

Proof. Let c be an extensive choice over X. By (1), c is 0-pluralist if and only if there is a family $\mathcal{V} = \{v_1, \ldots, v_k\}$ of k ballots over X such that for any $A \in 2^X$,

$$c(A) = \left\{x \in A : \frac{\left|\{i : x \in v_i(A)\}\right|}{k} > 0\right\}$$

$$= \left\{x \in A : x \in v_i(A) \text{ for some } i\right\} = \bigcup_{i=1}^k v_i(A).$$

This proves the claim. □

Combining Lemma 3 with Theorems 2.5(a) and 5.4 in Aizerman and Aleskerov (1995), we readily derive the following fact:

Corollary 1. *Any extensive choice that satisfies Axiom α is 0-pluralist.*

Proof. By Theorem 5.4 in Aizerman and Aleskerov (1995), if c is an extensive choice that satisfies Axiom α, then there is a finite family $\{v_i\}_{i=1}^q$ of extensive choices, which satisfy Axioms α and γ, and are such that $c = \bigcup_{i=1}^q v_i$. Therefore by Proposition 1 above, $\{v_i\}_{i=1}^q$ are ballots. Now the claim readily follows from Lemma 3. □

3.2. Interpretation and relation with literature

To start, observe that the standard maximization model of rationality is a basic instance of rationalization by ballots. In fact, if an extensive choice c is a ballot, then for each $s \in [0, 1)$ the family $\mathcal{V} = \{c\}$ is such that

$$x \in c(A) \quad \Longleftrightarrow \quad \frac{\left|\{i : x \in v_i(A)\}\right|}{1} > s$$

for any $A \in 2^X$ and $x \in X$.

Note also that the 0-pluralist (liberal) expression of choice correspondences is similar to *weakly pseudo-rationalizable* choices studied by Stewart (2020). Here the author investigates choices for which there is a finite collection of *decisive* ballots $\{v_i\}_{i=1}^q$ such that $c = \bigcup_{i=1}^q v_i$. Axioms α and *weak* γ (as defined in Stewart, 2020) characterize his model. Our Lemma 3 assures that the liberal version of choices rationalized by ballots is a relaxed form of weakly pseudo-rationalizable choices, since the ballots studied here are not necessarily decisive. For this reason, weak γ can be violated in 0-pluralist (extensive) choices.

Next, consider the *rationalization by multiple rationales (RMR)* described by Kalai et al. (2002). Formally, an *RMR* of a choice function c over X is a set $\mathscr{L}$ of linear orders[6] over X such that, for all nonempty $A \subseteq X$, the item $c(A)$ is $\succ$-maximal for some $\succ$ in $\mathscr{L}$. Any choice function over n items has an RMR with n linear orders.[7] The liberal version of rationalization by ballots is reminiscent of the RMR model, but deviates from it for at least two reasons: (1) the range of our analysis is much wider, because our observables are extensive choices (instead of choice functions); (2) our model is testable (see Theorem 1 below).

Alternative interpretations of a rationalization by ballots are related to other bounded rationality approaches. Multi-self decision making – see, e.g., May (1954) and Manzini and Mariotti (2007) – provides interpersonal or intrapersonal frameworks for aggregation of preferences. The essential idea of this paradigm is that in order to derive choices over alternatives, a DM resorts to a family $\mathcal{V}$ of fictitious selves (also called ‘motivations’ or ‘priorities’). In this paper we employ the suggestive term ‘ballots’ instead, and do not impose an interpersonal interpretation. We also assume that each justifying ballot can be generally indecisive (including singletons and doubletons). Even more important, we consider the general case of extensive choices, instead of the very restricted scenario of choice functions. From this point of view, an extensive choice is rationalizable by ballots whenever there

[6] A *linear order* is an asymmetric, transitive and complete binary relation.

[7] Proposition 1 in Kalai et al. (2002) shows that $n - 1$ linear orders suffice.

is a family of (possibly fictitious or unobservable) ballots with the property that the items selected from each menu are exactly those endorsed by more than a fixed share of ballots.

Yet another interpretation of our model is related to *multiple criteria decision making*. In this scenario, alternatives are characterized by attributes, and each attribute is explained by a rationale. When the DM only attends to selected characteristics, she makes a choice by the attributes that stem from the maximization of the corresponding rationales. In our model, choices are justified differently: a DM decides to accept an alternative from a menu when it is choosable for a fixed share of attributes. For this reason its predictions are compatible with conflict-inducing multi-attribute choices mentioned by Tversky and Shafir (1992). The authors argue that pairs of alternatives that dominate each other in salient characteristics explain paradoxical experimental findings. In the language of our model, we can consider the case of two attributes with associated rationales $\gg_1$ and $\gg_2$ such that $x \gg_1 y$ and $y \gg_2 x$. Here rationalization by ballots recommends $c(x) = x$, $c(y) = y$, and as a result we observe $c(xy) = \emptyset$.

3.3. Characterization

The proof of the characterization of our model is lengthy. In order to have a more structured sequence of arguments, first we present a restricted version of it:

Proposition 2. *The following statements are equivalent for an extensive choice c:*

(i) *c satisfies Axiom α;*
(ii) *c is s-pluralist for some rational share $s \in [0, 1)$;*
(iii) *c is s-pluralist for each rational share $s \in [0, 1)$.*

Proof. Since (iii) $\Longrightarrow$ (ii) is obvious, and (ii) $\Longrightarrow$ (i) is Lemma 2 (note that s is not required to be a rational number), we only show (i) $\Longrightarrow$ (iii).
(i) $\Longrightarrow$ (iii): Suppose $c : 2^X \to 2^X$ satisfies Axiom α, and let $s \in \mathbb{Q} \cap [0, 1)$. We prove the claim in three steps:

1. c is 0-pluralist;
2. c is t-pluralist for some $t \in (s, 1) \cap \mathbb{Q}$;
3. c is s-pluralist.

Step 1: Apply Corollary 1.[8] Let $\mathcal{V} = \{v_1, \ldots, v_n\}$ be a finite family of ballots witnessing the fact that c is 0-pluralist.

Step 2: We build on the family $\mathcal{V}$ of ballots given at Step 1. Since the sequence $\left\{\frac{k^2+k-1}{k^2+nk}\right\}_{k=1}^{\infty}$ converges to 1, and $\frac{k^2+k-1}{k^2+nk} < 1$ for any $k \geqslant 1$, there exists $m \in \mathbb{N}$ such that $s < \frac{m^2+m-1}{m^2+nm} < 1$. Set $t := \frac{m^2+m-1}{m^2+nm} \in \mathbb{Q} \in (0, 1)$. We claim that c is t-pluralist. To prove the claim, we construct a finite family $\mathcal{W}$ of ballots witnessing that c is t-pluralist. Let $\mathcal{V}'$ be a family of *neutral* ballots having size $|\mathcal{V}'| = m^2$, that is, for each $v' \in \mathcal{V}'$, the equality $v'(A) = A$ holds for all menus $A \subseteq X$.[9] Furthermore, let $\mathcal{V}''$ be the family of ballots obtained by replicating m times each ballot in the original family $\mathcal{V}$. Set $\mathcal{W} := \mathcal{V}' \cup \mathcal{V}''$, hence $\mathcal{W}$ is made of $m^2 + mn$ ballots. To check that $\mathcal{W}$ is a t-pluralist representation of c, let $a \in A \subseteq X$. The forward implication in (1) holds, because if $a \in c(A)$, then

$$\frac{\left|\{w \in \mathcal{W} : a \in w(A)\}\right|}{|W|} = \frac{\left|\{v' \in \mathcal{V}' : a \in v'(A)\}\right| + m \cdot \left|\{v \in \mathcal{V} : a \in v(A)\}\right|}{|\mathcal{W}|} \geqslant \frac{m^2 + m}{m^2 + mn} > t.$$

For the converse, suppose $\frac{|\{w \in \mathcal{W} : a \in w(A)\}|}{|\mathcal{W}|} > t$. Then there is $v \in \mathcal{V}$ such that $a \in v(A)$, since otherwise $\frac{|\{w \in \mathcal{W} : a \in w(A)\}|}{|\mathcal{W}|} = \frac{m^2}{m^2+mn} < t$. Since $\mathcal{V}$ witnesses that c is 0-pluralist, we obtain $a \in c(A)$. This proves that (1) holds.

Step 3: We suitably modify the family $\mathcal{W}$ defined at Step 2. For brevity, denote by $p := m^2 + mn$ the size of the family $\mathcal{W}$. First we argue that we can assume that the number $p' := p(\frac{t}{s} - 1)$ is a positive integer. Indeed, $\frac{t}{s} - 1$ is a positive rational number, hence a sufficiently large replication of $\mathcal{W}$ satisfies $p' \in \mathbb{N}$. (Observe that all replications of $\mathcal{W}$ are obviously t-pluralist justifications of c.) Now consider a family $\mathcal{W}'$ of p' *hypercritical* ballots, that is, ballots such that the selection from each menu is always empty. Finally, set $\mathcal{Z} := \mathcal{W} \cup \mathcal{W}'$. Note that $|\mathcal{Z}| = p + p' = p \cdot \frac{t}{s}$.
We check that $\mathcal{Z}$ is an s-pluralist representation of c. Indeed, for each $a \in X$, we have

$$\frac{\left|\{z \in \mathcal{Z} : a \in z(A)\}\right|}{\left|\mathcal{Z}\right|} = \frac{\left|\{w \in \mathcal{W} : a \in w(A)\}\right|}{p \cdot \frac{t}{s}} = \frac{\left|\{w \in \mathcal{W} : a \in w(A)\}\right|}{\left|\mathcal{W}\right|} \cdot \frac{s}{t}.$$

It follows that

$$\frac{\left|\{z \in \mathcal{Z} : a \in z(A)\}\right|}{\left|\mathcal{Z}\right|} > s \iff \frac{\left|\{w \in \mathcal{W} : a \in w(A)\}\right|}{\left|\mathcal{W}\right|} > t.$$

This completes the proof. □

Now we are ready to prove the main result of this paper:

Theorem 1. *The following statements are equivalent for an extensive c:*

(i) *c satisfies Axiom α;*
(ii) *c is s-pluralist for some share $s \in [0, 1)$;*
(iii) *c is s-pluralist for each share $s \in [0, 1)$.*

Proof. We have already observed that the proof of (ii) $\Longrightarrow$ (i) in Proposition 2 goes through without requiring the share s be rational. Thus, it suffices to show that (i) $\Longrightarrow$ (iii) holds.

To that end, suppose $c : 2^X \to 2^X$ satisfies Axiom α, and let $s \in (0, 1)$. Select $t \in \mathbb{Q}$ such that $s < t < 1$. By Proposition 2, c is t-pluralist: let $\mathcal{V}$ be a family of p ballots over X witnessing this fact. By replicating the society as many times as needed, we can assume without loss of generality that $p \geqslant 2$, $pt \in \mathbb{N}$, and $t < \frac{p-1}{p}$.

Consider the sequence $\left\{t - \frac{tk}{p+k}\right\}_{k=0}^{\infty}$, which converges to 0 as k diverges. We claim that the inequality

$$t - \frac{t(k-1)}{p + (k-1)} < t - \frac{tk - 1}{p + k} \tag{2}$$

holds for all $k \in \mathbb{N}$. Since $t < \frac{p-1}{p}$, the claim holds for $k = 0$. Routine computations show that (2) is equivalent to $p(t-1) + 1 - k < 0$, which is true for $k \geqslant 1$. This proves (2). Let q be the minimum positive integer such that $t - \frac{tq}{p+q} \leqslant s$. Since

$$t - \frac{tq}{p+q} \leqslant s < t - \frac{t(q-1)}{p+q-1} < t - \frac{tq-1}{p+q},$$

there is $\delta \in [0, 1)$ such that $s = t - \frac{tq-\delta}{p+q} = \frac{\left(t+\frac{\delta}{p}\right)p}{p+q}$. To complete the proof, we show that c is s-pluralist in two steps.

[8] See Aizerman and Aleskerov (1995), Theorem 2.5(a) and Theorem 5.4.

[9] Since $\mathcal{V}'$ is a family (and not a set, because elements can be repeated), the size $|\mathcal{V}'|$ of $\mathcal{V}'$ is the cardinality of the index set.

Step 1: *c is $\left(t+\frac{\delta}{p}\right)$-pluralist.* By assumption, for any $A \in 2^X$, we have

$$a \in c(A) \iff \left|\{v \in \mathcal{V} : a \in v(A)\}\right| > t|\mathcal{V}|.$$

Furthermore,

$$\left|\{v \in \mathcal{V} : a \in v(A)\}\right| \geqslant t|\mathcal{V}|+1 > t|\mathcal{V}|+\delta \geqslant t|\mathcal{V}|,$$

since $\left|\{v \in \mathcal{V} : a \in v(A)\}\right|$ and $t|\mathcal{V}|$ are integers. Thus the claim follows from

$$a \in c(A) \iff \left|\{v \in \mathcal{V} : a \in v(A)\}\right| > t|\mathcal{V}|+\delta = tp+\delta.$$

Step 2: *c is s-pluralist.* Let $\mathcal{V}'$ be a family of q hypercritical ballots (empty choices) over X. Set $\mathcal{W} := \mathcal{V} \cup \mathcal{V}'$, hence $|\mathcal{W}| = p+q$. For each $A \in 2^X$, we have

$$\begin{aligned} a \in c(A) &\iff \frac{\left|\{v \in \mathcal{V} : a \in v(A)\}\right|}{p} > t+\frac{\delta}{p} \\ &\iff \frac{\left|\{w \in \mathscr{W} : a \in w(A)\}\right|}{p} > t+\frac{\delta}{p} \\ &\iff \frac{\left|\{w \in \mathscr{W} : a \in w(A)\}\right|}{|\mathscr{W}|} > \frac{(t+\frac{\delta}{p})p}{p+q} = s. \end{aligned}$$

Thus the family $\mathcal{W}$ provides a s-pluralist justification of c, as claimed. $\square$

3.4. Two paradigms of pluralistic justification

Theorem 1 asserts that all s-pluralist justifications are witnessed by the same behavioral property, hence either all of them are valid explanations of a given choice behavior, or none is. Here we show that this equivalence vanishes as soon as we consider a very simple feature of a pluralist rationalization, namely the minimum number of ballots that is required for a justification by ballots.[10] To achieve this goal we just need to single out the following two benchmark forms of pluralism:

Definition 4. Let $c: 2^X \to 2^X$ be an extensive choice. A *liberal (or 0-pluralist) representation of c of size k* is any family $\mathcal{V} = \{v_1, \dots, v_k\}$ of $k \geqslant 1$ ballots over X such that

$$x \in c(A) \iff x \in v_i(A) \text{ for some } v_i \in \mathcal{V} \tag{3}$$

for any $A \in 2^X$ and $x \in X$. The *liberal number* $\mathsf{lib}(c)$ of c is the minimum number of ballots in a liberal representation of c if there is one, and infinite otherwise, that is,

$$\mathsf{lib}(c) := \begin{cases} \text{size of a smallest liberal} & \\ \quad \text{representation of } c & \text{if } c \text{ Axiom } \alpha \text{ holds,} \\ \infty & \text{otherwise.} \end{cases}$$

Similarly, a *democratic (or 0.5-pluralist) representation of c of size k* is any family $\mathcal{V} = \{v_1, \dots, v_k\}$ of $k \geqslant 1$ ballots over X such that

$$x \in c(A) \iff \left|\{i : x \in v_i(A)\}\right| > \frac{k}{2} \tag{4}$$

for any $A \in 2^X$ and $x \in X$. The *democratic number* $\mathsf{dem}(c)$ of c is

$$\mathsf{dem}(c) := \begin{cases} \text{size of a smallest democratic} & \\ \quad \text{representation of } c & \text{if Axiom } \alpha \text{ holds,} \\ \infty & \text{otherwise.} \end{cases}$$

[10] This analysis is reminiscent of the *index of rationality* of a given choice function, defined as the minimum number of linear orders in an RMR for it (Kalai et al., 2002).

Thus $\mathsf{lib}(c) < \infty$ if and only if $\mathsf{dem}(c) < \infty$ if and only if Axiom α holds for c. While the two paradigms are indistinguishable for a rationalizable extensive choice c (because $\mathsf{lib}(c) = \mathsf{dem}(c) = 1$), they behave differently for cases of non-rationalizability.

Example 3. Let c be the choice over $X = \{x, y, z\}$ defined by $c(xy) = xy$, $c(xz) = x$, $c(yz) = y$, and $c(xyz) = x$. Note that c satisfies Axiom α but not Axiom γ, and so it fails to be rationalizable. A simple computation shows that $\mathsf{lib}(c) = 2$ and $\mathsf{dem}(c) = 3$.

Example 4. Let c be the choice over $X = \{0, 1, 2, 3, 4, 5\}$ defined by

$$c(A) = \begin{cases} A \setminus \{0\} & \text{if } 0 \in A \text{ and } |A| \geqslant 4 \\ A & \text{otherwise.} \end{cases}$$

One can show that $\mathsf{dem}(c) \leqslant 5$ and $\mathsf{lib}(c) \geqslant 10$. Thus the liberal and democratic paradigms are *very* far apart in this case. This can be interpreted as saying that a situation in which a government only disregards a single item when the latter appears in a large menu (otherwise being fully neutral) is better suited for a democratic model rather than a liberal one.

The following result provides some upper bounds for the democratic (resp. liberal) number in terms of the liberal (resp. democratic) number:

Lemma 4. *For any extensive choice c, we have:*

(i) $\mathsf{dem}(c) \leqslant 2\,\mathsf{lib}(c)$;
(ii) $\mathsf{lib}(c) \leqslant 2^{\mathsf{dem}(c)-1}$.

Proof. . Let c be an extensive choice over X. If Axiom α fails for c, then $\mathsf{dem}(c) = \mathsf{lib}(c) = \infty$, hence (i) and (ii) are verified. Next, suppose c satisfies Axiom α. By Theorem 1, c is both democratic and liberal, and so $\mathsf{dem}(c)$ and $\mathsf{lib}(c)$ are finite. In what follows, we prove (i) and (ii) by deriving a representation of a certain type (resp. democratic, liberal) from one of the other type (resp. liberal, democratic).

(i): This inequality follows from the fact that for any liberal representation of c, we can always create a democratic representation of c by doubling the number of ballots, where all new ballots are neutral (i.e., voters choose everything).

(ii): Let $\mathcal{V}$ be a democratic representation of c such that $|\mathcal{V}| = \mathsf{dem}(c)$. Below we construct a family

$$\mathcal{W}_{\mathcal{V}} = \left\{ w_{\mathcal{U}} : \mathcal{U} \subseteq \mathcal{V} \text{ and } |\mathcal{U}| > \tfrac{|\mathcal{V}|}{2} \right\}$$

of ballots over X that liberally represents c. The ballot $w_{\mathcal{U}} \in \mathcal{W}_{\mathcal{V}}$ is such that

$$a \in w_{\mathcal{U}}(A) \iff a \in u(A) \text{ for all } u \in \mathcal{U} \tag{5}$$

for each $A \in 2^X$ and $a \in X$. To prove that $\mathcal{W}_{\mathcal{V}}$ liberally represents c, we show that

$$a \in c(A) \iff a \in w_{\mathcal{U}}(A) \text{ for some } w_{\mathcal{U}} \in \mathscr{W}_{\mathcal{V}} \tag{6}$$

for each $A \in 2^X$ and $a \in X$. If $a \in c(A)$, then, since $\mathcal{V}$ is a democratic representation of c, there is a subfamily $\mathcal{U} \subseteq \mathcal{V}$ such that $|\mathcal{U}| > |\mathcal{V}|/2$ and $a \in u(A)$ for all $u \in \mathcal{U}$. Now (5) yields $a \in w_{\mathcal{U}}(A)$, which proves the forward implication in (6). We prove the reverse implication in (6) by contrapositive. Suppose $a \notin c(A)$. If $a \in w_{\mathcal{U}}(A)$ for some $\mathcal{U} \subseteq \mathcal{V}$ such that $|\mathcal{U}| > |\mathcal{V}|/2$, then (5) gives $a \in u(A)$ for all $u \in \mathcal{U}$, which contradicts the fact that $\mathcal{V}$ is a democratic representation of c. Thus, there is no $\mathcal{U} \subseteq \mathcal{V}$, with $|\mathcal{U}| > |\mathcal{V}|/2$, such that $a \in w_{\mathcal{U}}(A)$. Thus $\mathcal{W}_{\mathcal{V}}$ liberally represents c. Now the bound (ii) follows easily. Indeed, letting $n := |\mathcal{V}| = \mathsf{dem}(c)$, we have:

$$|\mathcal{W}_{\mathcal{V}}| \;=\; \sum_{k=\lfloor \frac{n}{2} \rfloor+1}^{n} \binom{n}{k} \;\leqslant\; \frac{2^n}{2} \;=\; 2^{\mathsf{dem}(c)-1},$$

and so $\mathsf{lib}(c) \leqslant |\mathcal{W}_{\mathcal{V}}| \leqslant 2^{\mathsf{dem}(c)-1}$, as claimed. □

Lemma 4 partially answers the following question: *If we have a liberal (resp. democratic) representation of a possible indecisive choice behavior, how large should the population of voters be in order to obtain a democratic (resp. liberal) justification for the same behavior?* In this respect, the proof of Lemma 4 is instructive, because it explicitly shows how to canonically construct a liberal (resp. democratic) representation from a democratic (resp. liberal) one.

To conclude our analysis of these special paradigms of pluralistic justifications, we establish the asymptotic behavior of the liberal number of choices satisfying Axiom α.

Theorem 2. *For any extensive choice c over n elements satisfying Axiom α,*

$$\mathsf{lib}(c) \;\leqslant\; \binom{n-1}{\lfloor \frac{n-1}{2} \rfloor}. \tag{7}$$

Moreover, the upper bound (7) is tight.

Proof (Sketch)**.** Let c be an extensive choice over a set X of size $n \geqslant 2$ such that Axiom α holds. For each $x \in X$, let $\mathscr{A}_x$ be the collection of the $\subseteq$-maximal sets $A \subseteq X$ such that $x \in c(A)$.

Claim: *There is a liberal representation $\mathcal{V}$ of c such that $|\mathcal{V}| \leqslant \max_{x \in X} |\mathscr{A}_x|$.*

Proof of Claim. Let $x_1, \ldots, x_n$ be the distinct items in X that are selected in at least a menu. By Axiom α, it holds that $c(x_i) = x_i$ for each i. Moreover, let

$$A_{x_i,1}, \;\ldots, A_{x_i,k_i} \tag{8}$$

be the distinct members of $\mathcal{A}_{x_i}$ for each $i = 1, \ldots, n$ (hence, $k_i \geq 1$), and set

$$\overline{k} := \max_{1 \leq i \leq n} k_i.$$

To make the lists (8) all of the same length $\overline{k}$, we pad those having length strictly less than $\overline{k}$ by repeating their last item as many times as necessary; formally, let

$$A_{x_i,j} := A_{x_i,k_i} \quad \text{for } i = 1, \ldots, n \quad \text{and} \quad j = k_i + 1, \ldots, \overline{k}.$$

For every $j = 1, \ldots, \overline{k}$, let $\gg_j$ be the binary relation over X defined by

$$\gg_j \;:=\; \bigcup_{i=1}^{n} \big\{(y, x_i) : y \in X \setminus A_{x_i,j}\big\} \cup \bigcup_{x \in X \setminus \{x_1,\ldots,x_n\}} \big\{(x, x)\big\},$$

and let v_j be the ballot canonically obtained from $\gg_j$ by maximization. It is not difficult to show that $\mathcal{V} := \{v_1, \ldots, v_{\overline{k}}\}$ is a liberal representation of c. Since

$$|\mathcal{V}| \leq \overline{k} = \max_{1 \leq i \leq n} k_i = \max_{x \in X} |\mathscr{A}_x|,$$

the Claim is fully proved. □

We now complete the proof of (7) showing $|\mathcal{V}| \leqslant \binom{n-1}{\lfloor \frac{n-1}{2} \rfloor}$. For $x \in X$, let

$$\mathscr{A}'_x := \{A \setminus \{x\} : A \in \mathscr{A}_x\},$$

so that $|\mathscr{A}'_x| = |\mathscr{A}_x|$ holds. Plainly, each $\mathscr{A}'_x$ is a Sperner family over $X \setminus \{x\}$, as its members are mutually $\subseteq$-incomparable. Hence, by Sperner's theorem (Sperner, 1928; Lubell, 1966), we have $|\mathscr{A}'_x| \leqslant \binom{n-1}{\lfloor \frac{n-1}{2} \rfloor}$. Now the Claim yields $|\mathcal{V}| \leqslant \binom{n-1}{\lfloor \frac{n-1}{2} \rfloor}$, as wanted.

Finally, we prove tightness. It suffices to invoke the following combinatorial result (its long proof is omitted, but is available upon request):

Lemma 5. *Let $X_n = \{0, 1, \ldots, n\}$ be a nonempty set. For each $1 \leqslant k \leqslant n+1$, let $c_{n,k}$ be a choice over X_n satisfying the following conditions for all $A \in 2^X$:*

(a) $A \setminus c_{n,k}(A) \subseteq \{0\}$,
(b) $A \setminus c_{n,k}(A) = \{0\} \iff 0 \in A$ *and* $|A| > k$.

Then $c_{n,k}$ satisfies Axiom α. Furthermore,

$$\mathsf{lib}(c_{n,k}) = \binom{n}{k-1} \qquad \textit{and}$$

$$\begin{cases} \mathsf{dem}(c_{n,k}) \leqslant 2k-1 & \textit{if } \frac{n}{2} < k \leqslant n+1 \\ \mathsf{dem}(c_{n,k}) \leqslant 2(n-k) & \textit{if } 1 \leqslant k \leqslant \frac{n}{2}. \end{cases}$$

The choice $c := c_{n-1,\lfloor \frac{n-1}{2} \rfloor+1}$ over X_{n-1} is such that $|X_{n-1}| = n$ and $\mathsf{lib}(c) = \binom{n-1}{\lfloor \frac{n-1}{2} \rfloor}$. □

Using Stirling's approximation, we obtain

Corollary 2. *For any extensive choice c over n elements satisfying Axiom α,*

$$\mathsf{lib}(c) \;=\; \mathcal{O}\left(\frac{2^n}{\sqrt{n}}\right).$$

4. Final remarks

Several models show that effective theories of individual choice can be founded on tenets that relax the classical Weak Axiom of Revealed Preference. We identify a liberal maximization process that can be tested solely by Axiom α. A less radical justification of the same class of choices places them in a pluralist scenario with respect to an arbitrary share. Our procedure continues the strand of literature on rationalizability of choice with multi-self models that began with May (1954).

Alternatively, Manzini and Mariotti (2007) explain some empirically important 'boundedly rational' patterns of single-valued choice by a *rational shortlist method (RSM)*.[11] In a RSM, a first rationale (an asymmetric binary relation) gives a selection of alternatives (a shortlist); then a second rationale determines the unique selection. Note that the 'default route' example in Manzini and Mariotti (2007, Section I.B) is neither RSM nor s-pluralist.

Rationalization by multiple rationales (Kalai et al., 2002) does not use rationales in a sequential order. Our model resorts to a multiplicity of rationales, and it does not use them sequentially either. As in an RMR approach, one rationale justifies the choice of an item in the liberal (i.e., 0-pluralist) model. Unlike that model, however, neither of the rationales is irrelevant, and we allow for two framework effects, because both the menu and the item may affect the mental argument – the particular *self* – that acts for justification. Furthermore, the introduction of a new voter (a bounding rationale) modifies the choice.

Computational complexity remains unexplored. Antecedents with respect to the RMR model include Apesteguía and Ballester (2010) and Demuynck (2011), who show that the problems of rationalizing choices by a minimum number – or by a fixed

11 Many choice models of bounded rationality resort to some kind of sequential procedure to explain an observed behavior: see Giarlotta et al. (2022a) for a uniform treatment of all these models.

number – of rationales are NP-complete. Also the problem posed by sequential choice by k rationales (Manzini and Mariotti, 2007) is NP-complete when $k \geq 3$ (Demuynck, 2011, Theorem 3).

Declaration of competing interest

All authors have no declaration of interest to make.

Data availability

No data was used for the research described in the article.

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