

**Quasigluon lifetime and confinement from first principles**

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The mass and the lifetime of a gluon are evaluated from first principles at a finite temperature across the deconfinement transition of pure SU(3) Yang-Mills theory, by a direct calculation of the pole of the propagator in the complex plane, using the finite temperature extension of a massive expansion in the Landau gauge. Even at  $T = 0$ , the quasigluon lifetime is finite, and the gluon is canceled from the asymptotic states, yielding a microscopic proof of confinement from first principles. Above the transition, the damping rate is a linear increasing function of temperature as predicted by standard perturbation theory.

DOI: [10.1103/PhysRevD.96.114020](https://doi.org/10.1103/PhysRevD.96.114020)**I. INTRODUCTION**

Quarks and gluons are believed to be confined because no free-particle asymptotic states have ever been observed. However, despite the success of QCD, no formal proof of confinement has been derived from first principles yet. The analytical results of standard perturbation theory break down in the infrared (IR), where most of our knowledge of non-Abelian theories relies on numerical nonperturbative methods.

In the last decades, important achievements have been reached by the simulation of huge lattices and by improved truncation schemes of Dyson-Schwinger equations (DSE). In the pure gauge sector, the gluon propagator has attracted a lot of interest because of the direct dynamical information that could be extracted by its detailed knowledge. Unfortunately, lattice simulations and numerical solutions of DSE have provided a very accurate description of the propagator in the Euclidean space, but few direct information on the analytic properties that determine the physical dynamical behavior of the gluon. Actually, the analytic continuation of a limited set of data points to Minkowski space is an ill-defined problem. While some evidence for positivity violation has emerged, the numerical attempts only give qualitative results at best [1]. On the other hand, for the study of the hot matter created in heavy ion collisions, quasiparticle models are quite successful and make use of temperature-dependent phenomenological masses and widths for the quasigluons [2–5].

In this paper, for the first time, the real and imaginary part of the gluon mass are evaluated from first principles across the deconfinement transition of pure SU(3) Yang-Mills theory, by a direct calculation of the pole of the gluon propagator in the complex plane as a function of the temperature. The result is achieved by a finite-temperature extension of a massive expansion [6–9] that was shown to provide very accurate analytical expressions for the propagator and the self energy in the IR. Even in the limit  $T \rightarrow 0$ , the imaginary part of the mass saturates at a finite value

$\gamma \approx 0.48$  GeV, yielding a very short finite lifetime  $\tau = 1/\gamma$  that eliminates the gluon from the asymptotic states. Thus, the gluon is confined, and the quasigluon can only exist as a short-lived intermediate state at the origin of a gluon-jet event [10].

From a formal point of view, the massive expansion of Ref. [7] is obtained by a perturbation theory expanding around a massive zeroth order free propagator in a Landau gauge. As first pointed out in Ref. [11] and fully discussed in Ref. [12], the expansion can be derived by a variational argument as an expansion around the best vacuum that minimizes the Gaussian effective potential [13–24]. A massive vacuum for the gluon is shown to be energetically favored, but the actual mass scale cannot be derived by that method because of the lack of any scale in Yang-Mills theory. Moreover, the expansion can be further optimized by a best choice of the subtraction point and can be classified as a special case of renormalization-scheme (RS) optimized perturbation theory (OPT) [25].

The massive expansion has many merits. It is based on the Faddeev-Popov gauge-fixed Yang-Mills Lagrangian in the Landau gauge, namely the same Lagrangian used in most of the lattice simulations. There are no Landau poles in the IR nor diverging mass terms. No spurious parameters or mass counterterms are required, yielding an analytical calculational method from first principles. Minkowski space is the native environment of the expansion, and Wick rotation is only used as a standard and rigorous tool for the actual evaluation of the elementary integrals by dimensional regularization. At a one loop and  $T = 0$ , the self energy  $\Sigma(p^2)$  and the propagator  $\Delta(p^2)$  can be evaluated analytically, providing analytic functions that can be easily continued to the Euclidean space, where the agreement with the lattice data is impressive [9].

The method is very predictive since the adimensional ratio  $\sigma(p^2) = \Sigma(p^2)/p^2$  is determined up to an additive constant which, as usual, depends on the renormalization scheme and should be absorbed by a change of the wave function renormalization constant. In other words, the

derivative of the function  $\sigma$  is fully determined yielding a universal prediction for the derivatives of the inverse dressing functions, in perfect agreement with the lattice data [9], without having to fix any parameter.

Back to the propagator, once the mass scale is fixed by a comparison with the lattice or with the phenomenology, the only free parameter is the subtraction point. Its change should be absorbed by a change of the wave function renormalization constant but it is not, because of the one-loop approximation. The residual dependence of the propagator on the subtraction point, i.e., on the additive constant of the adimensional ratio  $\sigma$ , can be further optimized by RD-OPT. Surprisingly, an optimal choice of the additive constant exists that makes the neglected higher order terms vanish, at least in the Euclidean space where a comparison with the lattice data can be made. Strictly speaking, that only tells us that higher order terms can be written as  $\Sigma \approx \text{const} \times p^2$  and can be absorbed by a change of the subtraction point. Thus, the optimized one-loop approximation provides reliable analytical functions for the gluon and ghost propagator and can be easily continued and studied in the whole complex plane [8].

An important prediction of the calculation is the existence of complex conjugate poles in the gluon propagator. Their existence was conjectured but not proven before. While usual dispersion relations do not hold in the presence of complex poles [26], no violation of causality and unitarity emerges by a careful analysis, as fully discussed by Stingl more than twenty years ago [10]. The imaginary part of the mass leads to short-lived intermediate states that cannot be present among the asymptotic states. Thus, the existence of complex poles is a direct microscopic proof of confinement. Moreover, even if the propagator is a gauge dependent function, its poles are gauge-invariant physical observables [27], and their dependence on temperature would be of primary importance for a microscopic description of the deconfinement transition.

The extension to finite temperature is straightforward and only requires the evaluation of the thermal parts of the graphs that are retained in the expansion. Explicit expressions have been derived in the very different approach of Ref. [28] that shares some of the massive one-loop graphs. Some new crossed graphs are required in the present massive expansion and can be obtained by a simple derivative. All thermal parts are finite but require a numerical one-dimensional integration. While the details of the explicit calculation will be published elsewhere, in this paper, the trajectory of the poles of the gluon propagator is studied in the complex plane, as a function of temperature, in the long wavelength limit. A crossover is found from a low temperature intrinsically confined gluon to a high temperature thermal quasiparticle.

The nonmonotonic behavior of the mass and the linear increase at high temperature are in qualitative agreement with the predictions of phenomenological quasiparticle

models [2–5] and of high temperature perturbative calculations [29], giving us more confidence in the genuine physical nature of the poles even at  $T = 0$ .

The paper is organized as follows: In Sec. II, some features of the massive expansion at  $T = 0$  are clarified and discussed together with the physical meaning of the poles and their relevance for a microscopic description of the deconfinement transition; in Sec. III, the extension to a finite temperature is described, and the trajectory of the poles is studied in the complex plane, as a function of the temperature, in the long wavelength limit where they give the mass of the quasigluon; in Sec. IV, a general qualitative discussion of deconfinement is given at the light of the present findings.

## II. COMPLEX POLES AND CONFINEMENT

At  $T = 0$ , the gluon propagator of pure SU(3) Yang-Mills theory can be evaluated by the optimized one-loop massive expansion of Ref. [7], and the explicit analytical result can be continued to the whole complex plane as discussed in Ref. [8]. We refer to those papers for the details of the calculation.

The massive expansion is obtained by just adding a mass term  $m_0^2$  to the quadratic part of the standard Faddeev-Popov Lagrangian in a Landau gauge and subtracting the same mass term in the interaction by a counterterm  $\delta\Gamma = m_0^2$ . Thus, the total Lagrangian is unchanged, and its exact study would lead to the same results of lattice simulations.

The self energy is expanded by the standard perturbation theory retaining only two-point graphs with no more than three vertices and no more than one loop, as shown in Fig. 1. The internal gluon lines in the graphs are given by the massive zeroth order propagator

$$\Delta_m(p^2) = [-p^2 + m_0^2]^{-1}, \quad (1)$$

and the mass  $m_0$  is the only energy scale in the calculation. It can only be fixed by comparison with the phenomenology or lattice data. The counterterm cancels all spurious mass divergences, and the expansion can be renormalized by a standard wave function renormalization in a dimensional regularization scheme. At the one loop, the graphs can be evaluated analytically, and explicit expressions were reported by Tissier and Wschebor [30,31] for most of the graphs in Fig. 1. The crossed graphs in Fig. 1, containing

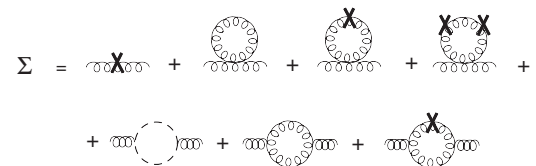


FIG. 1. Two-point graphs with no more than three vertices and no more than one loop. The cross is the counterterm  $\delta\Gamma = m_0^2$ .

one or more insertions of the counterterm, are obtained by a simple derivative of the other graphs, and explicit expressions can be found in Ref. [7]. We observe that at the tree level, the first graph in Fig. 1 cancels the mass shift  $m_0^2$  in the propagator, and the renormalized dressed gluon propagator  $\Delta(p)$  can be written as

$$\Delta(p) = \frac{Z}{-p^2 - \Sigma_L(p)} = \frac{J(p)}{-p^2}, \quad (2)$$

where  $Z$  is the wave function renormalization constant,  $J(p)$  is the dressing function, and  $\Sigma_L$  is the sum of all self-energy graphs containing loops. Thus, the dynamical generation of mass arises from loops, and no mass would be predicted in QED by the same method.

As usual, at the one loop, we can write  $Z$  as the product of a finite renormalization constant  $z$  times a diverging factor  $1 + \alpha\delta Z$ , where  $\alpha$  is some coupling here taken as  $\alpha = 3N\alpha_s/(4\pi)$ , where  $\alpha_s = g^2/(4\pi)$  is the strong coupling. The dressing function reads

$$zJ(p)^{-1} = 1 + \alpha[F(p^2/m_0^2) - \delta Z], \quad (3)$$

where the adimensional function  $F(s)$  is just the self energy divided by  $p^2$

$$F(p^2/m_0^2) = \frac{\Sigma_L(p^2)}{\alpha p^2}. \quad (4)$$

Its finite part is an explicit analytical expression that does not depend on any parameter and is evaluated in Ref. [7] by the sum of the finite parts of the graphs in Fig. 1. The divergent part is canceled by the divergent part of  $\delta Z$ , yielding a finite result. However, the finite part of  $\delta Z$  depends on the subtraction point of the renormalization scheme, and its arbitrary choice should be compensated in Eq. (3) by a change of the finite multiplicative renormalization factor  $z$  that is arbitrary anyway. Thus, the function  $F(s)$  is defined up to an additive (finite) renormalization constant. Moreover, we can divide by  $\alpha$  and absorb the coupling in the arbitrary factor  $z$  yielding

$$zJ(p)^{-1} = F(p^2/m_0^2) + F_0, \quad (5)$$

where the new constant  $F_0$  is the sum of all the constant terms.

As anticipated in the Introduction, the method is very predictive since the derivative of the inverse dressing function is given by the derivative of the function  $F$  and acquires a universal form that does not depend on any parameter and has been found in perfect agreement with the lattice data in the IR [9]. Integrating back, the dressing function does depend on the integration constant  $F_0$ , which is related to the arbitrary choice of the subtraction point. The residual dependence of the propagator on the choice of

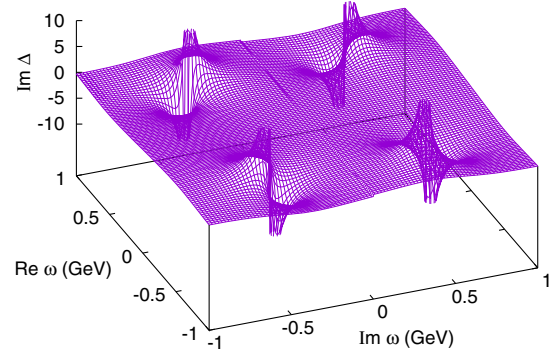


FIG. 2. Imaginary part of the one-loop gluon propagator  $\Delta(p^2)$  evaluated by Eq. (5) for  $F_0 = -1.05$  and  $m_0 = 0.73$  GeV in the complex plane  $\omega = \sqrt{p^2}$ . A very small discontinuous structure can be seen on the real axis [26].

$F_0$  can be optimized in the Euclidean space by a comparison with the lattice data. As shown in Refs. [7,9], an impressive agreement is obtained by taking  $F_0 = -1.05$  at the mass scale  $m_0 = 0.73$  GeV. Even if the optimal values might change as a function of temperature, we will take those values as fixed in the present paper. Their eventual variation would lead to a variational improvement of the method at a finite temperature.

Denoting by  $\omega$  the physical energy in Minkowski space and by  $\mathbf{k}$  the three momentum, so that  $p^2 = \omega^2 - \mathbf{k}^2$ , we can study the propagator in the complex  $\omega$  plane and in the long-wavelength limit, where  $\omega = \sqrt{p^2}$ . By a direct calculation through Eq. (5), the dressing function has two pairs of opposite complex conjugate poles at  $\omega = \pm(m \pm i\gamma)$ , where the real part  $m = 0.63$  GeV and the imaginary part  $\gamma = 0.48$  GeV. A plot of the imaginary part of the gluon propagator  $\Delta$  is shown in Fig. 2.

As discussed in Ref. [26], the gluon propagator is very well approximated by the sum of the principal parts

$$\Delta_R(\omega) = \sum_{\pm} R_{\pm} \left[ \frac{1}{\omega - (m \pm i\gamma)} - \frac{1}{\omega + (m \pm i\gamma)} \right], \quad (6)$$

where  $R_{\pm}$  are complex conjugate residues. The difference  $\Delta - \Delta_R$  contains the very small discontinuous structure that can be observed on the real axis. That structure has no poles and is not relevant for the asymptotic states, so that we can safely take  $\Delta \approx \Delta_R$  in the following discussion. A rational propagator like  $\Delta_R$  in Eq. (6) was conjectured in the past and predicted by phenomenological models like the refined version [32–34] of the Gribov-Zwanziger model [35]. As shown by Stingl [10], the existence of complex conjugate poles would not violate unitarity or causality because the quasigluon would be canceled from the asymptotic states. Actually, the existence of complex masses can be seen as a microscopic proof of confinement. Following the argument of Stingl [10] and taking  $\Delta \approx \Delta_R$ , we can Fourier transform the propagator in Minkowski space and write

$$\Delta(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \Delta_R(\omega) e^{i\omega t} = -2e^{-\gamma|t|} |R| \sin(m|t| + \phi), \quad (7)$$

where  $R_{\pm} = |R|e^{\pm i\phi}$ . Even at  $T = 0$ , the elementary excitations of the vacuum are short-lived quasiglons with an intrinsic lifetime  $\tau_0 = 1/\gamma$ . In other words, all S-matrix elements involving one or more external gluons are zero and cannot give rise to any unitarity or causality problem. During its short life, for  $|t| \ll \tau_0$ , the quasigluon behaves like an eigenstate with energy  $m$ .

On the other hand, a thermal theory does not require the existence of asymptotic states, and the quasiglons contribute to the free energy and to other thermodynamic quantities. In that sense, they can appear as the elementary degrees of freedom of a hot plasma above the deconfinement transition temperature  $T_c$ . That motivates an extension to the finite temperature of the massive expansion.

We must mention that no complex poles were found by the numerical solution of DSE in Minkowski space [36]. However, that calculation could be sensitive to the special ansatz that is used for the truncation of the infinite set of integral equations. For instance, the existence of a peak on the real axis was claimed in Ref. [36] and replaced by a smooth function in Ref. [37] by the same authors. Moreover, in any numerical calculation in the complex plane, the choice of the correct Riemann sheet might not be a simple task [38]. Thus, the present extension to a finite temperature might also be useful for establishing the genuine physical nature of the complex poles, since the results of standard perturbation theory should be approached when the temperature is high enough above  $T_c$ .

Another independent test for the reliability of the formalism would arise from a direct check of the gauge invariance of the poles, which is predicted by general arguments [27] and formally shown in other schemes like the Gribov-Zwanziger framework [39]. While the present work is in the Landau gauge, the formalism could be easily extended to a generic linear covariant gauge, along the lines discussed in Ref. [12]. When fixing the gauge by a covariant term  $(\partial A)^2/(2\alpha)$ , with  $\alpha \neq 0$ , the added mass  $m_0$  in the quadratic part of the Lagrangian should be replaced by a pure *transverse* mass term, which is canceled by a *transverse* mass counterterm  $\delta\Gamma$  in the interaction. Since the total Lagrangian is still unchanged, Becchi-Rouet-Stora-Tyutin invariance is not broken and the exact sum of the expansion yields a vanishing longitudinal polarization. Thus, the dressed longitudinal propagator is known exactly and is equal to the free one,  $\Delta_L = -\alpha/p^2$ , which is massless because no longitudinal mass was inserted in the quadratic part of the Lagrangian. The transverse propagator and its poles can be evaluated as before, by the massive expansion, using the zeroth order propagator which has a massive transverse part, still given by Eq. (1), and a massless (exact) longitudinal part.

However, since Becchi-Rouet-Stora-Tyutin is broken in the quadratic part of the Lagrangian, we do not expect that the poles would be exactly invariant at any finite order of the approximation. Thus, their gauge dependence would measure the accuracy of the approximation at any order.

### III. POLE TRAJECTORY AT FINITE $T$

The extension of the massive expansion to a finite temperature is straightforward but tedious. All graphs in Fig. 1 acquire a thermal part, and explicit expressions were reported in Ref. [28] for most of the required one-loop graphs. The new crossed graphs (including one or more insertions of the counterterm  $\delta\Gamma = m_0^2$ ) can be obtained by a simple derivative with respect to  $m_0^2$ . All thermal parts are finite but require a numerical one-dimensional integration over the internal three-vector modulus  $q$ . Explicit expressions will be published elsewhere.

The analytic continuation of integral functions is not trivial if singular points are integrated along the integration path. As discussed in Ref. [38], we must check that the integration over  $q$  on the real axis does not meet any singular point of the logarithmic functions. Otherwise, a modified path must be chosen before the analytic continuation can be undertaken. By inspection of the explicit expressions [28], branch cuts might be present, originating at the singular branch point of the logarithmic function

$$L_{\beta}(z_{\alpha}) = \log \left[ \frac{z_{\alpha}^2 + \omega_{+\beta}^2}{z_{\alpha}^2 + \omega_{-\beta}^2} \right], \quad (8)$$

where  $z_{\alpha} = i\omega \pm i\sqrt{q^2 + \alpha^2}$  and  $\omega_{\pm\beta}^2 = (q \pm k)^2 + \beta^2$ . Here,  $\alpha$  and  $\beta$  are masses equal to 0 or  $m_0$ , while  $k$  is the external three-vector modulus. Assuming the existence of a branch point at  $q = q_0$  on the real axis, it must satisfy

$$\pm 2q_0 k = \alpha^2 - \beta^2 - k^2 + \omega^2 \pm 2\omega\sqrt{q_0^2 + \alpha^2}, \quad (9)$$

where the  $\pm$  signs are independent of each other. Taking  $\omega = x + iy$  with  $y > 0$ , the imaginary part of Eq. (9) gives  $x = \mp\sqrt{q_0^2 + \alpha^2}$  and substituting back in the real part, we obtain  $\omega_{\pm\beta}^2 + y^2 = 0$  which is never satisfied unless  $y = \beta = 0$ . Thus, if  $\omega$  is not real, the branch point  $q_0$  cannot be real, and the integral over  $q$  on the real axis gives an analytic function of  $\omega$ . That condition is fulfilled around the poles, where  $y \approx \gamma > 0$ . We can safely continue analytically the numerical thermal integrals from the Euclidean space ( $x = 0, y > 0$ ) to the whole upper half plane. Moreover, in the large wavelength limit  $k \rightarrow 0$ , the logarithmic function can be written as  $L_{\beta}(z_{\alpha}) \approx \log [1 + \mathcal{O}(k)]$ , and the argument of the log never vanishes. Thus, in that limit, there are no branch points at all and the numerical thermal integrals over  $q$  can be safely continued to the whole complex  $\omega$  plane.

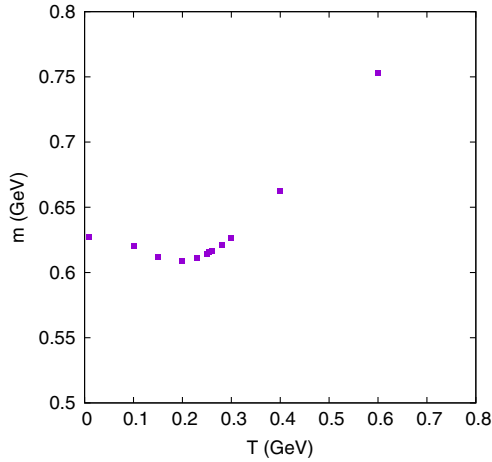


FIG. 3. The quasigluon mass  $m$ , i.e., the real part of the pole location in the complex  $\omega$ -plane for  $k \rightarrow 0$ , is shown as a function of temperature for  $F_0 = -1.05$  and  $m_0 = 0.73$  GeV.

The method could be used for a study of the full dispersion relations as functions of temperature and three-vector  $k$ , by following the location of the poles in the longitudinal and transverse projections of the propagator. However, in the present paper, we will content ourselves with the long wavelength limit  $k \rightarrow 0$ , where the longitudinal and transversal quasiparticles must have the same complex masses because there are no privileged directions. We checked that the poles of the longitudinal and transverse projections coalesce in that limit.

In principle, the additive renormalization constant  $F_0$  and the mass scale  $m_0$  should be optimized as functions of the temperature. Here, they are fixed at their optimal value at  $T = 0$ , neglecting their change at a finite temperature. Thus, the method might be improved by some variational argument.

In Figs. 3 and 4, the real part  $m$  and the imaginary part  $\gamma$  of the pole position in the complex  $\omega$  plane are displayed as functions of the temperature.

As shown in Fig. 3, the quasigluon mass  $m$  is not monotonic. It decreases below 200 MeV, reaches a minimum at  $T \approx 200$  MeV, and then increases approaching a linear behavior above 400 MeV. A nonmonotonic behavior was observed on the lattice for the longitudinal inverse propagator  $1/\Delta(0)$ , which defines a Debye mass [40,41]. However, the two definitions of mass are quite different. While  $\Delta(0)$  is a mass scale that depends on renormalization, gauge choice, and polarization, the real part of the pole  $m$  is the dynamical mass of the quasigluon, according to Eq. (7). It does not depend on the polarization and is expected to be gauge invariant [27]. We checked that the correct qualitative behavior is observed for the inverse propagator at  $\omega = 0$  in the long wavelength limit  $k \rightarrow 0$ , where we find a nonmonotonic longitudinal projection (Debye mass) and a monotonically increasing transverse projection (magnetic mass), as already shown in Ref. [28]

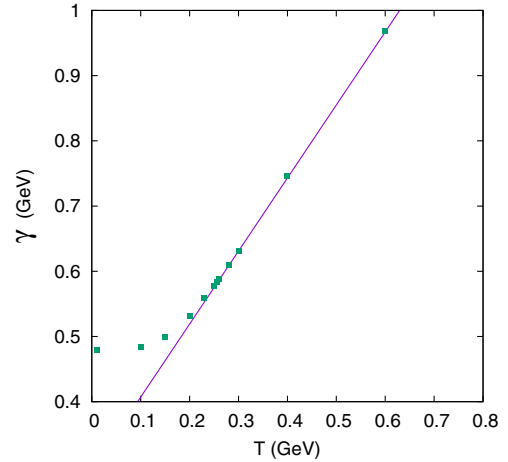


FIG. 4. The quasigluon damping rate  $\gamma = 1/\tau$ , i.e., the imaginary part of the pole location in the complex  $\omega$  plane for  $k \rightarrow 0$ , is shown as a function of temperature for  $F_0 = -1.05$  and  $m_0 = 0.73$  GeV. The straight line is the linear function  $\gamma = \gamma_0 + bT$  with  $\gamma_0 = 0.295$  GeV and  $b = 1.12$ .

by a massive expansion. We observe that different results are obtained reversing the order of the two limits  $\omega \rightarrow 0$  and  $k \rightarrow 0$ . For a finite  $\omega$ , the longitudinal and transverse projection must coincide in the limit  $k \rightarrow 0$ , because there are no privileged directions. While for a finite  $k$ , the limit  $\omega \rightarrow 0$  gives different definitions for the transverse and longitudinal propagators even when  $k$  is very small, yielding different limits for the Debye mass and the magnetic mass.

A more direct comparison can be made with phenomenological models that usually predict a nonmonotonic quasigluon mass around the deconfinement transition [2,5]. A minimum is found at the same temperature  $T \approx 200$  MeV in Ref. [2], while it is pushed above  $1.5T_c$  in Ref. [5]. No discontinuity is found for the mass in those models, even if that conclusion is in part the consequence of the details of the models that sometimes use a divergent ansatz for the mass from the beginning [3]. A finite and continuous mass across the transition has been explained in Ref. [4] by the coupling to the Polyakov loop. On that point, no reliable conclusion can be reached by the present calculation since we expect that even a first-order transition might be rounded by the one-loop approximation. However, Fig. 3 shows a clear crossover: above 300 MeV, the mass approaches the linear increasing behavior expected by perturbation theory [29]; decreasing the temperature below 200 MeV, the mass *increases* again because of the dynamical mass generation, leading to a strong deviation from the perturbative behavior, with a residual intrinsic mass  $m \approx 630$  MeV at  $T = 0$ .

According to Eq. (7), the imaginary part of the pole is the quasigluon damping rate  $\tau^{-1} = \gamma$  and is shown in Fig. 4 as a function of temperature. Again, we observe a remarkable crossover with a linear behavior above 300 MeV and a strong deviation from that behavior below 200 MeV,

where the lifetime  $\tau(T)$  saturates at the residual intrinsic value  $\tau_0^{-1} = \tau(0)^{-1} \approx 480$  MeV. As shown in the figure, the linear behavior is approached very quickly above the transition, and the quasigluon becomes an ordinary thermal quasiparticle with a damping rate that is very well approximated by the linear expression

$$\tau^{-1} = \gamma_0 + bT, \quad (10)$$

where  $\gamma_0 = 0.295$  GeV and  $b = 1.12$ . Extrapolating to a high temperature, we are tempted to compare the effective coupling  $b$  with the coefficient expected by perturbation theory  $\gamma/T = a\alpha_s/2$ , where the value  $a \approx 6.6$  was evaluated by resummation of hard thermal loops [29]. The comparison would give a reasonable  $\alpha_s = 2b/a = 0.34$ , which is the actual coupling at 2 GeV. We must mention that the standard perturbation theory fails to predict that coefficient unless the hard thermal loops are resummed in a consistent way [29]. In the present massive expansion, the existence of a mass scale  $m_0$  inside the loops should mitigate the problem, and the hard thermal loops are not expected to be relevant unless  $T \gg m_0$ . Thus, in the range of the temperature of Fig. 4, where  $m_0 = 0.73$  GeV, the effect of hard thermal loops should be negligible.

Below 200 MeV, where ordinary perturbation theory breaks down, the damping rate  $\gamma$  deviates from the linear behavior and saturates, because of the existence of a quasigluon intrinsic finite lifetime  $\tau_0$  that does not arise from thermal effects. The quasigluon remains short-lived even when  $T \rightarrow 0$  and acquires a very different behavior than ordinary thermal long-lasting quasiparticles. Thus, the crossover describes a transition from an intrinsically confined quasigluon to ordinary quasiparticle behavior. It is remarkable that, albeit continuous, the transition takes place in the narrow range of temperature between 200 and 300 MeV, that compares well with the critical temperature  $T_c \approx 270$  MeV observed in the lattice [42].

#### IV. DISCUSSION

The massive expansion of Ref. [7] provides an analytical approach to QCD in the IR by perturbation theory. Based on the original Faddeev-Popov gauge-fixed Yang-Mills Lagrangian in a Landau gauge, the optimized expansion is in very good agreement with the lattice in the Euclidean space [9] and allows for a straightforward analytic continuation to Minkowski space [8]. Thus, the prediction of a finite lifetime for the quasigluon can be seen as a microscopic proof of confinement. At a finite temperature, the pole trajectory describes a crossover from an intrinsically confined quasigluon for  $T < 200$  MeV to an ordinary thermal quasiparticle for  $T > 300$  MeV. In the high-temperature phase, the standard linear behavior is recovered, strengthening the reliability of the method.

A physical description emerges, where the quasigluons are intrinsically damped in the confined phase, with a short

lifetime  $\tau_0$  that does not arise from thermal effects. Since the lifetime is even shorter in the deconfined phase, with  $\gamma(T) \approx m(T)$ , one could even question what the word ‘‘deconfinement’’ really means. Moreover, even the usual notion of a quasiparticle can be questioned when the distance of the pole from the real axis is so large that no relevant resonance structure can be observed in the imaginary part of the propagator on the real axis, as shown in Fig. 2. However, even when  $\gamma$  loses its meaning as a width of a broad resonance, according to Eq. (7), it retains its meaning as a damping rate  $\tau^{-1}$  of the quasigluon short-lived intermediate state.

A parallel can be made with the scaling theory of localization and with the crossover from weak to strong disorder that is observed in condensed matter. Assuming that in a disordered sample of size  $L$  the electrons are localized at the Fermi energy, any effect on the conductivity can only be observed if the localization length  $\xi$  of the states is shorter than  $L$ , while no phenomenological effect can be seen if  $\xi \gg L$  since the states appear as extended at the scale  $L$ . At a finite  $T$ , the electron coherence can only be probed at the dephasing scattering scale  $L(T) \sim 1/T^n$ , which is mainly due to inelastic scattering. Thus, at a high temperature, when  $L(T) < \xi$ , the effects of disorder are weak and, even if the scattering length is shorter, the electrons can be described by ordinary perturbation theory because the intrinsic localization length  $\xi$  is larger than the effective sample size  $L(T)$ . On the other hand, in the low temperature limit,  $L(T)$  gets very large and when  $L(T) > \xi$ , the electrons appear as strongly localized, with large deviations from the standard picture of thermal quasiparticles.

In heavy ion collisions, the time scale of the process is very large compared to the gluon lifetime  $\tau$ , so that the intermediate quasigluon states can only generate gluon-jet events. However, at the high temperature reached during the process, the quasiparticles can only be probed during their very short thermal lifetime  $\tau_{th}(T) \sim 1/T$  to be compared with the intrinsic lifetime  $\tau_0$  at  $T = 0$ . Thus, in the high temperature limit, when  $\tau(T) < \tau_0$ , no phenomenological evidence of confinement can appear in the thermodynamic behavior of the hot plasma. The quasigluon looks like deconfined even if its lifetime is shorter. While in the low temperature limit, the thermal lifetime  $\tau_{th}(T)$  gets very large, so that when  $\tau_{th}(T) > \tau_0$ , the short intrinsic lifetime of the quasigluon emerges and the gluon looks like confined, with large deviations from the predictions of standard perturbation theory.

Even if the crossover is found at the correct range of temperatures, without adjusting any free parameter, the present approach fails to predict the sharp first-order transition that is expected in pure SU(3) Yang-Mills theory [42]. In principle, there is no evidence that the mass and lifetime must be discontinuous at the transition. They have never been measured by lattice simulations and might not

be the correct order parameter. A continuous mass was found in Ref. [4] by assuming a coupling to the Polyakov loop. However, it is likely that any sharp change would be rounded by the present one-loop calculation.

It would be interesting to explore the effects of a further variational optimization of the mass parameter  $m_0$  as a function of the temperature, along the lines that have been recently discussed in Ref. [12]. A temperature-dependent

mass parameter was successfully employed in the phenomenological approach of Ref. [28] and could make some difference at the transition point. However, while the mass term was added to the Lagrangian in that work and used as a fit parameter, here the mass would arise from first principles by a variational approach to the exact Yang-Mills theory, yielding a very predictive tool for the study of QCD in the IR.

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- [1] D. Dudal, O. Oliveira, and P.J. Silva, *Phys. Rev. D* **89**, 014010 (2014).
- [2] M. Nahrgang, J. Aichelin, P.B. Gossiaux, and K. Werner, *Phys. Rev. C* **93**, 044909 (2016).
- [3] P. Castorina, V. Greco, D. Jaccarino, and D. Zappalà, *Eur. Phys. J. C* **71**, 1826 (2011).
- [4] M. Ruggieri, P. Alba, P. Castorina, S. Plumari, C. Ratti, and V. Greco, *Phys. Rev. D* **86**, 054007 (2012).
- [5] S. Plumari, W.M. Alberico, V. Greco, and C. Ratti, *Phys. Rev. D* **84**, 094004 (2011).
- [6] F. Siringo, arXiv:1509.05891.
- [7] F. Siringo, *Nucl. Phys.* **B907**, 572 (2016).
- [8] F. Siringo, *Phys. Rev. D* **94**, 114036 (2016).
- [9] F. Siringo, *EPJ Web Conf.* **137**, 13016 (2017).
- [10] M. Stingl, *Z. Phys. A* **353**, 423 (1996).
- [11] F. Siringo, in *Correlations in Condensed Matter under Extreme Conditions*, edited by G.G.N. Angilella and A. La Magna (Springer International Publishing AG, New York, 2017); F. Siringo, arXiv:1701.00286.
- [12] G. Comitini and F. Siringo, arXiv:1707.06935.
- [13] P.M. Stevenson, *Phys. Rev. D* **32**, 1389 (1985).
- [14] F. Siringo, *Phys. Rev. D* **62**, 116009 (2000).
- [15] F. Siringo, *Europhys. Lett.* **59**, 820 (2002).
- [16] F. Siringo and L. Marotta, *Int. J. Mod. Phys. A* **25**, 5865 (2010).
- [17] F. Siringo and L. Marotta, *Phys. Rev. D* **78**, 016003 (2008).
- [18] F. Siringo and L. Marotta, *Phys. Rev. D* **74**, 115001 (2006).
- [19] F. Siringo, *Phys. Rev. D* **86**, 076016 (2012).
- [20] I. Stancu and P.M. Stevenson, *Phys. Rev. D* **42**, 2710 (1990).
- [21] I. Stancu, *Phys. Rev. D* **43**, 1283 (1991).
- [22] M. Camarda, G. G. N. Angilella, R. Pucci, and F. Siringo, *Eur. Phys. J. B* **33**, 273 (2003).
- [23] L. Marotta, M. Camarda, G. G. N. Angilella, and F. Siringo, *Phys. Rev. B* **73**, 104517 (2006).
- [24] L. Marotta and F. Siringo, *Mod. Phys. Lett. B* **26**, 1250130 (2012).
- [25] P.M. Stevenson, *Nucl. Phys.* **B868**, 38 (2013).
- [26] F. Siringo, *EPJ Web Conf.* **137**, 13017 (2017).
- [27] R. Kobes, G. Kunstatter, and A. Rebhan, *Phys. Rev. Lett.* **64**, 2992 (1990).
- [28] U. Reinosa, J. Serreau, M. Tissier, and N. Wschebor, *Phys. Rev. D* **89**, 105016 (2014).
- [29] E. Braaten and R. D. Pisarski, *Phys. Rev. D* **42**, 2156 (1990).
- [30] M. Tissier and N. Wschebor, *Phys. Rev. D* **82**, 101701(R) (2010).
- [31] M. Tissier and N. Wschebor, *Phys. Rev. D* **84**, 045018 (2011).
- [32] D. Dudal, J. A. Gracey, S. P. Sorella, N. Vandersickel, and H. Verschelde, *Phys. Rev. D* **78**, 065047 (2008).
- [33] D. Dudal, S. P. Sorella, N. Vandersickel, and H. Verschelde, *Phys. Rev. D* **77**, 071501 (2008).
- [34] D. Dudal, S. P. Sorella, and N. Vandersickel, *Phys. Rev. D* **84**, 065039 (2011).
- [35] D. Zwanziger, *Nucl. Phys.* **B323**, 513 (1989).
- [36] S. Strauss, C. S. Fischer, and C. Kellermann, *Phys. Rev. Lett.* **109**, 252001 (2012).
- [37] H. Sanchis-Alepuz, C. S. Fischer, C. Kellermann, and L. von Smekal, *Phys. Rev. D* **92**, 034001 (2015).
- [38] J.-P. Blaizot, A. Ipp, and A. Rebhan, *Ann. Phys. (Amsterdam)* **321**, 2128 (2006).
- [39] M. A. L. Capri, D. Dudal, A. D. Pereira, D. Fiorentini, M. S. Guimaraes, B. W. Mintz, L. F. Palhares, and S. P. Sorella, *Phys. Rev. D* **95**, 045011 (2017).
- [40] P. J. Silva, O. Oliveira, P. Bicudo, and N. Cardoso, *Phys. Rev. D* **89**, 074503 (2014).
- [41] A. Maas, J.M. Pawłowski, L. von Smekal, and D. Spielmann, *Phys. Rev. D* **85**, 034037 (2012).
- [42] B. Lucini, M. Teper, and U. Wenger, *J. High Energy Phys.* **01** (2004) 061.