

Reply to “*Presence of a fundamental acceleration scale in galaxies*” and “*A common Milgromian acceleration scale in nature*”

Davi C. Rodrigues^{1,2}, Valerio Marra^{1,2}, Antonino Del Popolo^{3,4,5} & Zahra Davari⁶

¹Center for Astrophysics and Cosmology, CCE, Federal University of Esp rito Santo, 29075-910, Vit ria, ES, Brazil.

²Department of Physics, CCE, Federal University of Esp rito Santo, 29075-910, Vit ria, ES, Brazil.

³Dipartimento di Fisica e Astronomia, Universit  di Catania, Viale Andrea Doria 6, 95125 Catania, Italy.

⁴Institute of Modern Physics, Chinese Academy of Sciences, POB 31, Lanzhou 730000, People’s Republic of China.

⁵INFN sezione di Catania, Via S. Sofia 64, 95123 Catania, Italy.

⁶Department of Physics, Bu Ali Sina University, Hamedan, Iran.

The correspondences by McGaugh *et al*¹ and Kroupa *et al*² question our results³ (hereafter R18). In essence, they state that our results are in conflict with Li *et al*⁴ (hereafter L18) and criticize the priors that we used in our analysis. We show that L18 has no implication for our results and that our priors are adequate for our analysis.

L18 shows that the RAR⁵ can be inferred from individual fits of galaxies, and they find no evidence for a variable acceleration scale a_0 . This does not contradict the fact that R18 does find strong evidence against a fundamental a_0 . The test performed by R18 is more robust and sensitive to this question. Indeed, the test by L18 is based on a visual comparison of the cumulative distribution functions (CDF) of χ^2_ν (reduced χ^2). The idea is to see how the overall fitting performance is improved when a_0 becomes a free parameter. However, this test is not suitable to infer the compatibility among the a_0 values from different galaxies, and it is not correct to classify this approach as a Bayesian statistical analysis, as done in the McGaugh *et al* correspondence. The definitions of χ^2 and χ^2_ν used in L18 are not standard and doing a simple variation on their definition of χ^2_ν suggests a different conclusion. Interestingly, although we do not support this CDF test, their approach applied to the main sample used in R18 favours our priors (see Figure 1). No criticism on our results can be done from this CDF analysis.

In R18 we use Bayesian inference to test the compatibility between the acceleration scales a_0 inferred from individual galaxies. To this end, the unknown acceleration scale a_0 must enter as a free parameter with a flat prior and the constraint $a_0 > 0$. We find unjustified the choice of a Gaussian prior on a_0 , as advocated in part of the analysis of L18: indeed, neither the RAR or MOND predict a fundamental a_0 value nor its dispersion. Consequently, here we will not discuss this case further. Stellar mass-to-light ratios (Υ_*) and galaxy distances (D) enter in our analysis as nuisance parameters, which may capture unaccounted for systematics. Whenever there is doubt

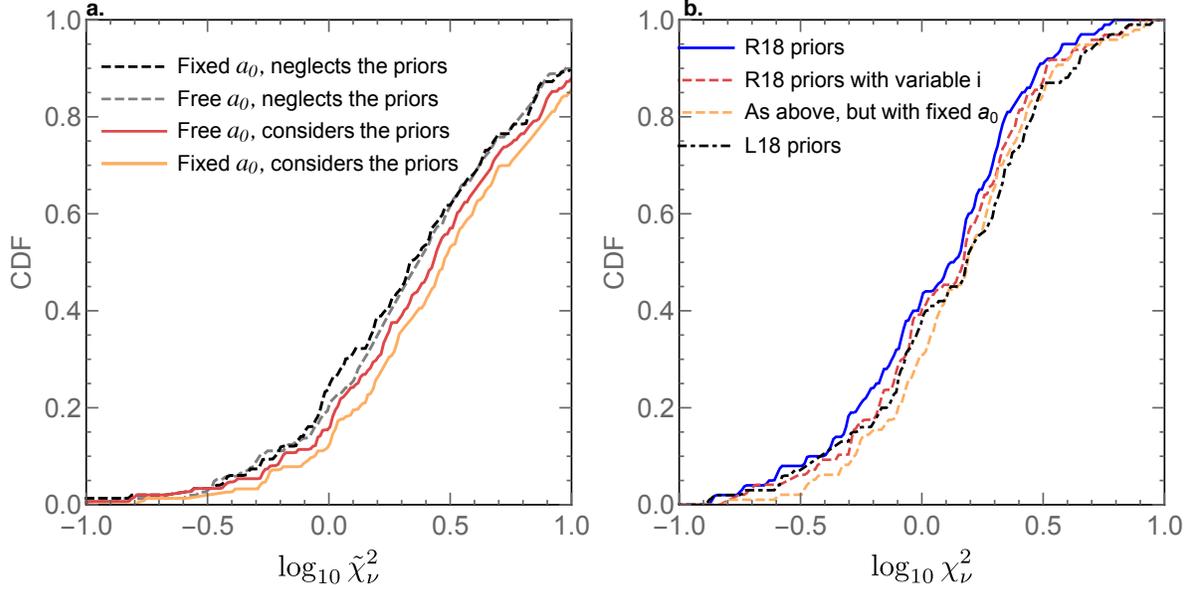


Figure 1: Analyses of the χ^2 CDF. a. The dashed curves reproduce the results shown in L18. The $\tilde{\chi}^2$ is not the usual one, it is based on the accelerations, instead of the circular velocities⁴. L18 evaluates the $\tilde{\chi}^2$ at the parameter values that maximize the full posterior P (with priors) but does not minimize $\tilde{\chi}^2$. Consequently, $\tilde{\chi}^2$ is an effective quantity which includes no information on the priors (although they were used in the minimization process). The same plot shows the L18 results when using another effective quantity: the minimum of $-2 \ln P$ divided by the number of degrees of freedom (solid lines). For the latter approach, the CDF of the free a_0 case has an advantage. **b.** The 100 galaxies that constitute the main analysis of R18 are considered (we recall that galaxies with poor MONDian fits were rejected³, thus the CDF achieves its maximum faster). The standard χ^2 function (the one with respect to velocities) is used here, with: *i*) the same priors of R18, *ii*) the priors of R18 together with inclination variations with the same Gaussian prior of L18, *iii*) as the previous case, but with fixed $a_0 = 1.2 \times 10^{-13} \text{km/s}^2$, and *iv*) all the priors of L18 with free a_0 . This plot follows the L18 convention when computing χ^2 : the priors enter only in the maximization of the posterior, not in χ^2 . R18 priors are favoured, but this CDF analysis is qualitative and not robust. No criticism on our results can be done based on this analysis.

about the probability distribution of a parameter, flat priors are better than Gaussian priors as they are the most conservative option⁶. In our analysis, the effect of nuisance parameters is to enlarge the credible intervals of a_0 . Indeed, as we show in Figure 2, by fixing all the nuisance parameters at their reference values⁵ and repeating our analysis with free a_0 , the dispersion of the modes of the posteriors of a_0 is essentially the same, but the credible intervals are much tighter.

We were generous with the Υ_* constraints: flat priors in a 3σ interval⁷ centered on the disk and bulge values that most favour the fundamental a_0 interpretation⁵. For D we used a 20% flat prior on the SPARC reference values for all the galaxies, without entering on the merits of each type of distance estimation. The chosen tolerance is larger than the median value of the relative uncertainties (14%). Considering Kroupa *et al* comments, we add that flat priors provide larger credible intervals than using the corresponding Gaussian priors. Also, if a galaxy cannot be well fitted with this constraint, it is removed from our sample due to our quality cuts³. That said, it is worthwhile to evaluate the dependence of our results on the galaxies with the largest relative uncertainties, this is shown in Figure 2. By removing either galaxies with distance uncertainties larger than 20% or those measured from the Hubble flow, a fundamental acceleration scale is still rejected at more than 10σ (see also R18 for further details).

One can use an unlimited number of nuisance parameters to describe a galaxy, we selected the most relevant ones and found a result with high level of statistical significance (for all our quality cuts, it was always beyond 10σ , and for our main analysis, we find $\sim 50\sigma$). No reasonable inclination variation can wash away those 50σ , since by eliminating all the galaxies more susceptible to inclination changes our conclusion is the same, see Figure 2. At last, we also recall that we analysed 18 galaxies from the THINGS sample, and we found similar results.

On the histograms presented in the McGaugh *et al* correspondence. Our results³ are not based on best fits. A histogram on these quantities can argue in favour of the emergent nature of the RAR but it does not directly test the compatibility between the acceleration scales. We also add that the analysis of McGaugh *et al* does not consider the same sample of data that we used to infer our conclusions, since our quality cuts were not considered. We use their suggestion of eliminating galaxies with large distance uncertainties in Figure 2.

On the quality cuts of Kroupa *et al* and inclinations fits. After a sequence of cuts on the sample from which the RAR⁵ is derived, 81% of the RAR sample is removed and still 17% of the remaining galaxies are incompatible with MOND, according to their criteria. *A posteriori* adjustments on particular features of each galaxy are evoked such that compatibility with MOND, in some sense, is achieved. We do not find that this is a good argument in favour of MOND. If there is reason to suspect of observational data issues, all the galaxies need to be reevaluated, not

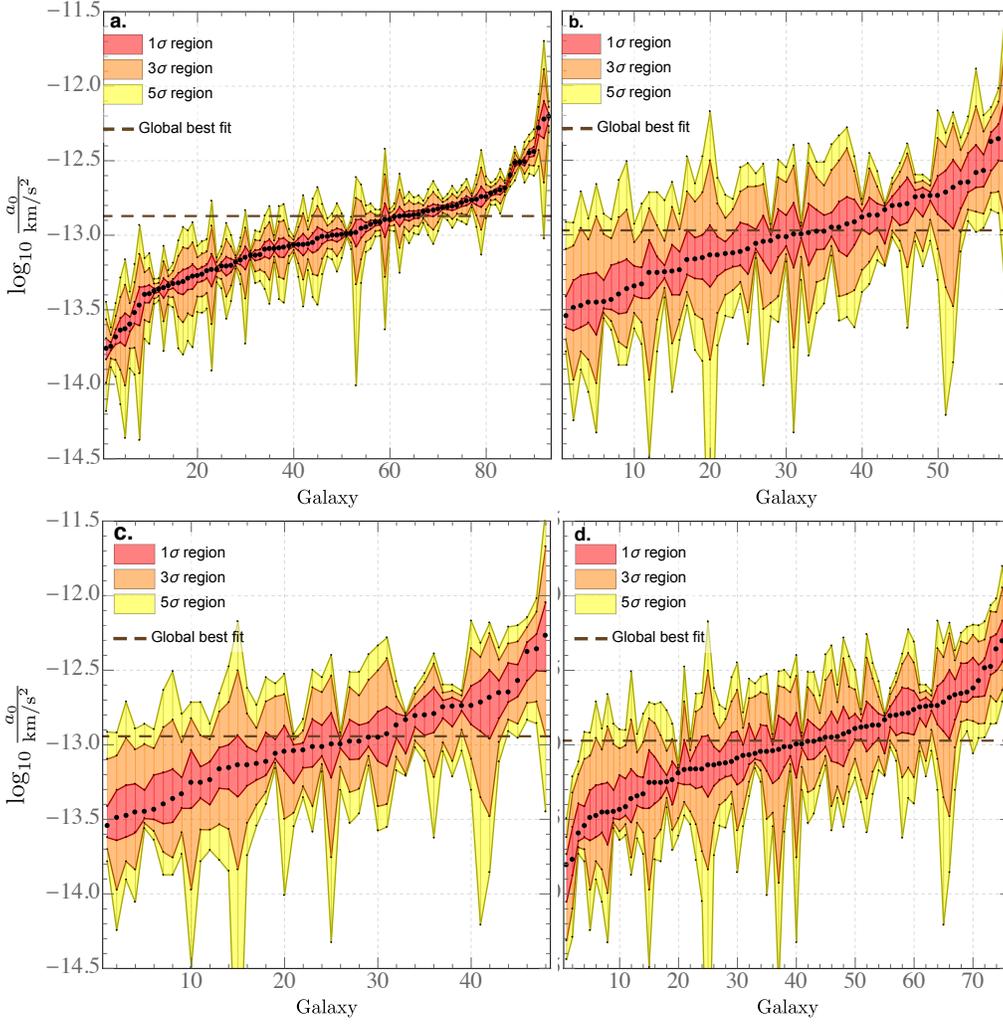


Figure 2: **Posterior probability distributions of a_0 .** The black dots are the modes of the a_0 posteriors (maximum probability after marginalizing on the other parameters), the brown dashed line is the global best fit for a_0 , the red, orange and yellow regions show the 1σ , 3σ and 5σ credible intervals³. **a.** Similar to Figure 1 of R18, but without the nuisance parameters: 93 galaxies pass the main quality cuts, a fundamental acceleration is rejected at very high confidence, and the modes of the a_0 posteriors cover a similar interval in spite of the change on the priors. **b.** As in Figure 1 of R18, but with an additional quality cut: galaxies with relative error on D larger than 20% are excluded. This leaves 59 galaxies and a fundamental acceleration is rejected at 26σ . **c.** Similar to the previous case, but excluding all the galaxies with distance inferred from the Hubble flow. This leaves 48 galaxies and a fundamental acceleration is rejected at 22σ . **d.** R18 always excludes galaxies with inclination (i) less than 30° , but one can consider a stronger quality cut, such that galaxies with $i - 2\sigma_i \leq 40^\circ$ are excluded, where σ_i is the SPARC observational error. This leaves 76 galaxies and a fundamental acceleration is rejected at 41σ .

only those with problems with MOND. Among their cuts, the strongest one is the Hubble flow one, which we used in Figure 2. At last, the plot with best fits on inclinations does not address the issue of compatibility between acceleration scales, nor any probability has been quantified.

Acknowledgements DCR and VM thank CNPq and FAPES (Brazil) for partial financial support. ADP was supported by the Chinese Academy of Sciences and by the President’s international fellowship initiative, grant no. 2017 VMA0044. ZD thanks the ministry of science, research and technology of Iran for financial support.

References

1. McGaugh, S. S., Li, P., Lelli, F. & Schombert, J. M. Presence of a fundamental acceleration scale in galaxies. *Nat. Astron.* (2018). <https://doi.org/10.1038/s41550-018-0615-9>.
2. Kroupa, P. *et al.* A common milgromian acceleration scale in nature. *Nat. Astron.* (2018). <https://doi.org/10.1038/s41550-018-0622-x>.
3. Rodrigues, D. C., Marra, V., Del Popolo, A. & Davari, Z. Absence of a fundamental acceleration scale in galaxies. *Nat. Astron.* **2**, 668–672 (2018). 1806.06803.
4. Li, P., Lelli, F., McGaugh, S. & Schormbert, J. Fitting the radial acceleration relation to individual SPARC galaxies. *Astron. Astrophys.* **615**, A3 (2018). 1803.00022.
5. McGaugh, S., Lelli, F. & Schombert, J. Radial Acceleration Relation in Rotationally Supported Galaxies. *Phys. Rev. Lett.* **117**, 201101 (2016). 1609.05917.
6. Gregory, P. C. *Bayesian logical data analysis for the physical sciences* (Cambridge U., 2010).
7. Meidt, S. E. *et al.* Reconstructing the stellar mass distributions of galaxies using S⁴G IRAC 3.6 and 4.5 μm images: II. The conversion from light to mass. *Astrophys. J.* **788**, 144 (2014). 1402.5210.