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Hybrid circuits to model and control fusion plasma instabilities

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Abstract: Controlling nonlinear phenomena is essential in many real-world applications. This paper explores the possibility to design novel nonlinear control strategies for nuclear fusion devices. In particular, a nonlinear phenomena relevant for nuclear fusion research is analyzed and two nonlinear control approaches, namely thermostat control and pellet injection, are opportunely designed.

Keywords: Nonlinear circuits, chaos control, nuclear fusion.

1. INTRODUCTION

The idea of controlling nonlinear phenomena has been deeply explored in the last decades [Vaidyanathan (2016)]. Nonlinearity, in fact, plays a key role in modeling physical systems, especially those in which the spatial dynamics is important as the temporal dynamics. The nonlinear nature of such kind of dynamical systems determines the complexity of the emergent behavior, inducing the condition for which chaos and nonlinear oscillations may occur.

The onset of a complex behavior is often considered as a drawback, so that the need of designing control strategies to mitigate chaotic dynamics comes out. The control of nonlinear systems became a fundamental topic in control theory during the last decades, giving rise to a plethora of different strategies and approaches aimed at suppressing chaos from a given dynamical system, among which we mention robust adaptive control [Buscarino et al. (2017)], fuzzy logic control [Johnston (1994)], sliding mode control [Edwards et al. (1998)], Lyapunov based approaches [Freeman et al. (2008)]. These methods are fundamental for real-world applications offering practical solutions which can be implemented by means of simple devices.

On the other hand, the conditions for which chaos does not emerge and nonlinear oscillations occur can be associated to the unwanted state of a system. In this paper we will focus on the dynamical behavior of a simple low dimensional model for nuclear fusion plasma related phenomena. In this case, the chaotic behavior is associated to the *normal* operating condition of the physical system, while its suppression is linked to the onset of plasma instabilities. Hence, for such a system an *antichaos* control strategy must be outlined, i.e. a control action which induces the appearance of chaotic behavior.

Nuclear fusion represents an innovative technique for energy production and a promising option for the future. However, up to now, the possibility to obtain a self-sustained fusion reaction is far from being realized. Different reactors, with different operating modes are still investigated with the aim of reaching conditions for self-sustained fusion.

One of the most studied nuclear fusion device is the Tokamak [Apicella et al. (2018)], a reactor with a toroidal chamber in which high magnetic fields confine the hydrogen isotopes to the plasma state at very high temperature and pressure conditions. Even though operating under high confinement regime (H-mode) [Lituadon et al. (2017)] implies higher energy confinement time, nonlinear unstable phenomena are likely to occur such as Edge localized modes (ELMs) and sawtooth. These kinds of instabilities may lead to severe damages to the vessel components threatening the overall Tokamak performance. As a consequence, modeling such nonlinear dynamics is fundamental for defining suitable control strategies.

In this contribution, the possibility to control these events is investigated and two different nonlinear control techniques are designed and applied to the model presented in [Constantinescu et al. (2011)] in order to mitigate ELMs occurrence, i.e. in order to avoid the dynamical regime standing for ELMs occurrence. In particular, we adopted an approach based on the realization of a circuit analogue of the dynamical system, i.e. a simple electronic circuit obeying to the same dynamical equations of the considered model. This approach allows us to meet a twofold goal:
i.) the possibility to test different control strategies without the need of running actual fusion experiments and implementing the devices providing the contol action on the reactor;
ii.) the possibility to take explicitly into account the non-ideality of the phenomenon, since a circuit implementation is always subjected to unavoidable component tolerances and nonidealities. This latter point is definitely different from considering a more detailed model: while including an higher level of representation in the model may lead to severe increase of the complexity of the model itself, implementing a real device allows to test the robustness of the proposed control techniques with respect to different

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sources of uncertainty, maintaining a reasonable level of complexity. The use of digital simulator, instead, allows only to test the correctness of the model design, and not its robustness.

The paper is organized as follows: in Section 2 the design and implementation of the circuit analogue of the considered model are presented; Section 3 discusses two different control strategies able to allow the emergence of a chaotic behavior; finally, Section 4 draws the conclusions of the paper.

2. ANALYSIS AND DESIGN OF A PLASMA LOWER DIMENSIONAL MODEL

The dynamical information carried out by plasma variables, especially during instabilities, is fundamental to control the occurrence of spatiotemporal phenomena affecting plasma performance and its confinement time.
The model in [Constantinescu et al. (2011)] is based on two plasma macroscopic quantities: the magnetic field and the plasma pressure. It simulates the interplay between a relaxation dynamics, related to the instabilities phenomena (magnetohydrodynamic (MHD) force balance) and the energy conservation, that is the driving part of the system which determines typical changes in profiles or profile gradients. Therefore complex plasma phenomena can be modeled by using a dynamical system accounting for the interplay between the instabilities dynamics and the power conservation.

2.1 The model

The normalized dimensionless model reads as:

$$\begin{aligned} \dot{x} &= (z-1)y - \delta x \\ \dot{y} &= x \\ \dot{z} &= \eta(h - z - y^2 z) \end{aligned} \tag{1}$$

where $x = \dot{\xi}$ with ξ equal to the magnetic field, $y = k\xi$, $z = p$ is the pressure gradient and and δ, η, and h are the three bifurcation parameters that determines the different dynamical behavior of the system. In particular, η is the heat diffusion coefficient, δ corresponds to the dissipation/relaxation of the instability related to the ELMs burst, and h represents the normalized input power of the system, that determines the ELMs burst.

Plasma can undergo instability processes only above a critical input power threshold, i.e. $h > 1$. In the following, we will set $h = 1.5$, so that the necessary conditions to observe dynamical regimes characteristic of plasma instabilities are met. The effective onset of the plasma instability can be observed by varying the other two independent parameters in (1), namely η and δ. Following [Constantinescu et al. (2011)], the ELMs regime corresponds to sawtooth-like periodic oscillations, while the chaotic behavior is associated to the reduction of the edge pressure gradient, thus mitigating the onset of the instability [Evans et al. (2006)].

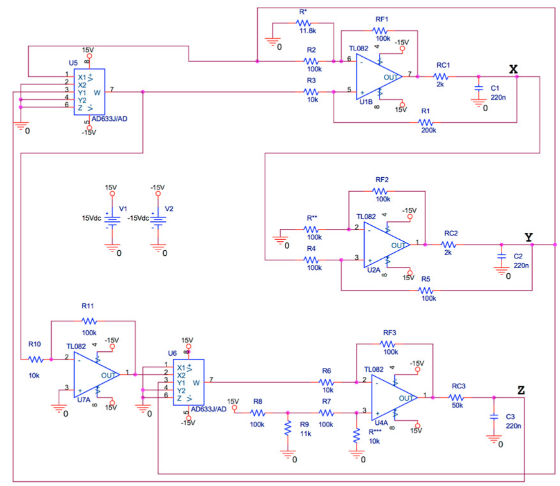


Fig. 1. Electric scheme of the circuit.

2.2 Circuit implementation

A circuit implementation of the nonlinear model 1 was designed and realized, by following the state variable approach [Buscarino et al. (2014)].
The physical realization of the ideal model constitutes a powerful instrument to reproduce the plasma dynamical behaviour, by using a very simple circuit built with low cost electronic components.
The equations of the overall circuit are the following:

$$\begin{aligned} \dot{x} &= \frac{1}{R_{C1}C_1}\left[\left(\frac{R_{F1}}{R_1}-1\right)x - \frac{R_{F1}}{R_2}y + \frac{R_{F1}}{R_3}V^*\right] \\ \dot{y} &= \frac{1}{R_{C2}C_2}\left[\frac{R_{F2}}{R_4}x + \left(\frac{R_{F2}}{R_5}-1\right)y\right] \\ \dot{z} &= \frac{1}{R_{C3}C_3}\left[\frac{R_{F3}}{R_7}V - z - \frac{R_{F3}}{R_6}V^{**}\right] \end{aligned} \tag{2}$$

where $V^* = \frac{yz}{10}$, $V = 15V$, $V^{**} = \frac{y^2 z}{10}$.

In figure 1 is shown the electrical scheme of the circuit.

The active integrators are implemented by using three operational amplifier, instead the nonlinearity is realized with analog multipliers. The circuit parameters are determined in order to match 1 with 2. Specifically the chosen values are the following:

$R_1 = 200k\Omega$, $R_2 = 100k\Omega$, $R_3 = 10k\Omega$, $R^* = 11.8k\Omega$
$R_4 = 100k\Omega$, $R_5 = 100k\Omega$, $R^{**} = 100k\Omega$, $R_7 = 100k\Omega$,
$R^{***} = R_6 = 10k\Omega$, $R_7 = 100k\Omega$, $R_8 = 100k\Omega$,
$R_9 = 11k\Omega$, $R_{10} = 10k\Omega$, $R_{11} = 100k\Omega$,
$R_{F1} = R_{F2} = R_{F3} = 100k\Omega$, $R_{C1} = R_{C2} = 2k\Omega$,
$C_1 = C_2 = C_3 = 220nF$.

The R_{C3} resistor is a potentiometer with nominal value equal to $100k\Omega$. By changing this value it is possible to obtain different values of η.

Figure 2 represents the physical implementation of the circuit, obtained by using off-the-shelf components.

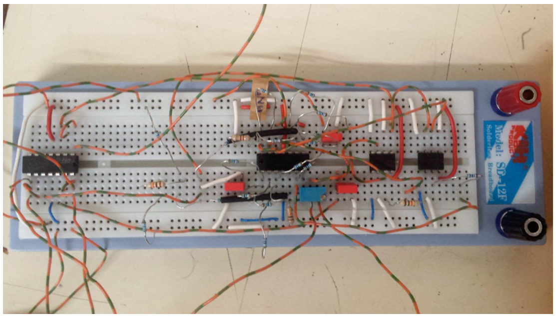
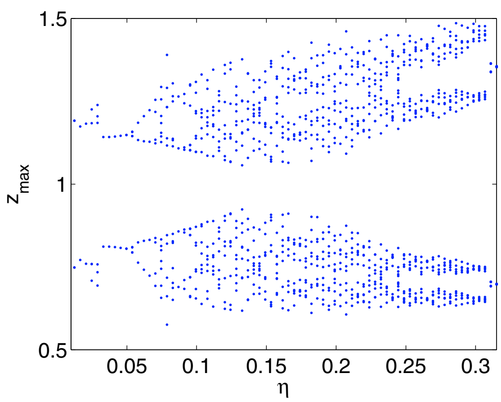
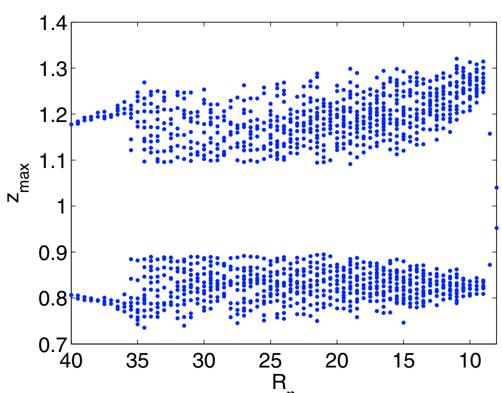


Fig. 2. Implementation of the nonlinear circuit.



(a)



(b)

Fig. 3. (a) Simulated bifurcation diagram of z variable respect to the bifurcation parameter η. (b)Experimental bifurcation diagram of z variable respect to the bifurcation parameter η.

In the Fig. 3 the numerical (3(a)) and the experimental bifurcation diagrams (3(b)) with respect the bifurcation parameter η are shown. The two diagrams show the local maxima and minima of the variable z.

3. CHAOS CONTROL

The aim of this section is to show two different approaches to control the dynamical behavior of the system, and, in particular, to suppress ELMs regime. Motivated by the physical meaning of the considered model, we focused on control strategies which involves the injection of an external driving signal. During real nuclear experiments, in fact, instabilities are mitigated by injecting pellet at given rates in the vacuum chamber.

Pellet injection can be, thus, considered as an open-loop control strategy implemented by the application of an external pulse signal [Buscarino et al. (2017)],[Constantinescu et al. (2011)]. In this case, the control parameters are the amplitude, the frequency and the width of the pulses.

Introducing a feedback which regulates the amplitude and the frequency of the external control signal, we get a situation similar to the so-called thermostat control, illustrated by [Nosé (1984)], [Hoover (1985)]. It represents a closed-loop control approach, in which a feedback is added in system 1 by means of a further dynamical variable which mediates between the behavior of a state variable and the control action. From a physical point of view, we can consider that a magnetic perturbation acts on the system, such that δ in (1) is varied through the thermostat control.

A similar control strategy has already been applied in Constantinescu et al. (2013), where an external magnetic perturbation has been applied as a pulse to the system in (1) in an open-loop scheme. In this work, a closed-loop arrangement is proposed by exploiting the thermostat control

3.1 Pellet injection by using Arduino® board

On the basis of the work presented in [Constantinescu et al. (2011)], an external perturbation is applied on the first equation of the model 1:

$$\begin{aligned} \dot{x} &= -\{1-[z+P(t)]\}y - \delta x \\ \dot{y} &= x \\ \dot{z} &= \eta(h - z - y^2 z) \end{aligned} \tag{3}$$

The perturbation has the following expression:

$$P(t) = a \cdot exp\{-[t - t_{period}\ int(t/t_{period}) - \delta_p]^2/b\} \tag{4}$$

where a is the amplitude, t_{period} symbolizes the period, $delta_p$ represents the shift, b is the width and the function int represents the integer part of the argument.
The parameters value of the system are chosen in order to work in the ELMs region: h=1.5, $\delta = 0.5$, $\eta = 0.009$.
The perturbation parameters have the following values: a=5, $t_{period} = 50$, b=1, and δ_p=2.
The perturbation is obtained by using the microcontroller board Arduino®Uno. The circuit implementation is modified in order to match the first equation of the model 3 in the following way:

$$\dot{x} = \frac{1}{R_{C1}C_1}\left[\left(\frac{R_{F1}}{R_1}-1\right)x - \frac{R_{F1}}{R_2}y + \frac{R_{F1}}{R_3}V^* + \frac{R_{F1}}{R_P}\frac{P(t)y}{10}\right] \tag{5}$$

with $R_P = 111.4k\Omega$, and $R^* = 9.6k\Omega$.

Based on the results obtained it is possible to affirm that the plasma stability is changed by the perturbation, that increases the ELMs frequency. In fact, the dynamical evolution of the plasma pressure gradient radically changes when the external perturbation is applied passing from the ELM regime to the chaotic region as shown in Fig.(4).

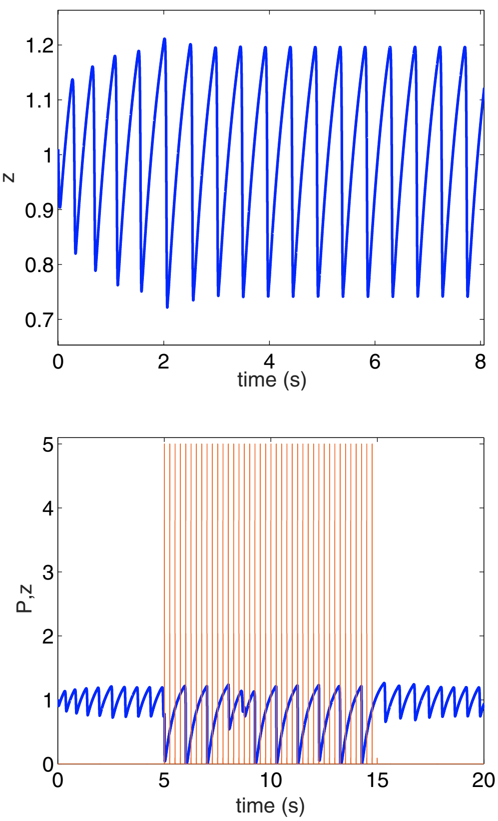


Fig. 4. (a)Dynamical evolution of z variable before (a) and after pellet injection (b) when $\eta = 0.5$ and t_{period}=50.

It is worthy to notice that for low t_{period} the system behavior is chaotic, thus revealing the effectiveness of pellet injection in suppressing ELMs.

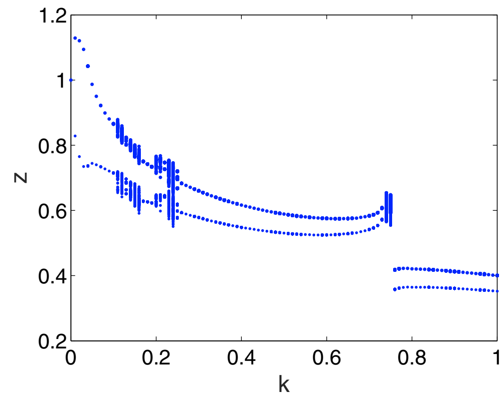
3.2 Thermostat control

The system of equations that modelize the system behaviour is shown in 6:

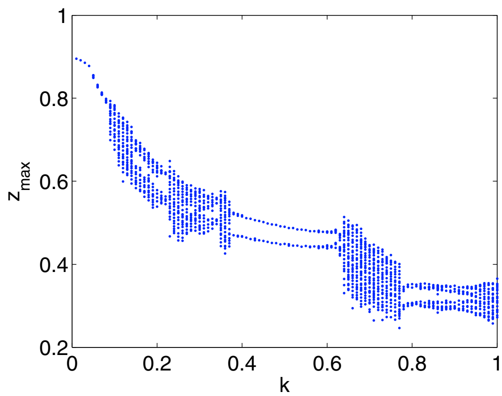
$$\begin{aligned}\dot{x} &= (z-1)y - \delta x - ux \\ \dot{y} &= x \\ \dot{z} &= \eta(h - z - y^2 z) \\ \dot{u} &= x^2 - k\end{aligned} \tag{6}$$

This method uses the same structure of the Nosé-Hoover oscillator, see Sprott et al. (2017), in order to obtain the thermostat control.
The system parameters are chosen to work in the ELMs region with the following values: $\delta = 0.5$, $\eta = 0.009$, and h=*1.5*. The control parameter k is changed in the range $[0, 1]$. The circuit implementation 2 is modified in order to add a fourth equation representing the new state variable and introduce this contribution in the first equation:

$$\begin{aligned}\dot{x} &= \frac{1}{R_{C1}C_1}\left[\left(\frac{R_{F_1}}{R_1} - 1\right)x - \frac{R_{F_1}}{R_2}y + \frac{R_{F_1}}{R_3}V^* - \frac{R_{F_1}}{R_C}V_C\right] \\ \dot{u} &= \frac{1}{R_{C4}C_4}\left[-\frac{R_{F_4}}{R_{10}}V_k + \frac{R_{F_4}}{R_{11}}V^{***} + \left(\frac{R_{F_4}}{R_{12}} - 1\right)u\right]\end{aligned} \tag{7}$$



(a)



(b)

Fig. 5. (a) Simulated bifurcation diagram of z variable respect to the control parameter k. (b) Experimental bifurcation diagram of z variable respect to the control parameter k.

where: $V_C = \frac{ux}{10}$, $V^{***} = \frac{x^2}{10}$, $R_C = 10k\Omega$, $R^* = 67k\Omega$, $R_{F4} = R_{10} = R_{12} = 100k\Omega$, $R_{11} = 10k\Omega$, $R^{****} = 11k\Omega$, $C_1 = C_2 = C_4 = 10nF$, $C_3 = 470nF$, $RC_3 = 4k\Omega$ (the components values not specified are the same of the first implementation shown in 2).

Changing the value of the control parameter k we pass from ELMs region to the chaotic region and the average value of z is reduced.
In figure 5 are shown the ideal bifurcation diagram (5(a)) and the experimental bifurcation diagram (5(b)):

4. CONCLUSION

In this paper we introduced an experimental setup based on an analog circuit implementation of a model relevant to plasma fusion behavior. In particular, the model is able to mimic the behavior of magnetic variables in the vacuum chamber of a fusion reactor during an experiment, catching the dynamical properties of plasma instabilities. This unwanted behavior is represented by stable limit cycle in the phase portrait of the model, while to drive away from instability the system a chaotic behavior is needed, so that an antichaos strategy can be adopted.

We proposed two control strategies with the aim of inducing the appearance of chaos and implemented both by means of simple analog/digital devices. The experimental results reported in this paper allow us to assess the validity of the approach. The possibility to design suitable control strategy is clearly obtained, providing a new tool to test them without the need of running a fusion experiment.

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