



## INFLUENCE OF LINK MODELS IN THE ASSESMENT OF THE SEISMIC RESPONSE OF MULTI-STOREY EBFS

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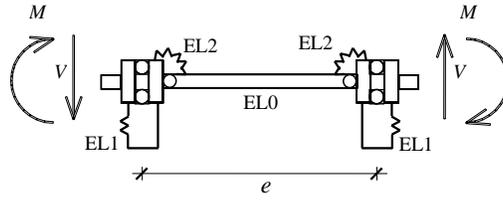
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**ABSTRACT:** The cyclic response of links in eccentrically braced structures has often been modelled as elastic plastic with kinematic hardening or as elastic perfectly plastic. These models were based on the Euler-Bernoulli beam with concentrated flexural plastic hinges and considered ending springs to simulate the inelastic shear and flexural response of the link. Recently, the authors have proposed a simple but refined link model in which the response of the hinges is dictated by the uniaxial material model proposed by Zona and Dall'Asta for buckling restrained braces. This model takes into account both kinematic and isotropic hardening. Previous investigation has proved that the cyclic response provided by the proposed link model matches satisfactorily that obtained by experimental tests. This new model is considered here to assess the effectiveness of some common and simpler models in which the effect of the isotropic hardening is not taken into account explicitly. The comparison is made on multi-storey structures subjected to artificial accelerograms. The structures are characterized by different number of storeys and link lengths and are designed according to Eurocode 8. The investigation considers both global and local response parameters, namely maximum top-storey displacement, maximum transient and residual drifts and plastic rotations of links.

### 1. Description of the models

Four different models are considered in this paper. All of them consist of elements connected in series (Fig. 1). The central element (EL0) has the same length and moment of inertia of the link and simulates the flexural elastic behaviour of the link (the shear stiffness of this element is infinite). The two elements at each end of the link (EL1 and EL2) are zero length and connect the beam segments outside the link to the central element EL0. The first of these two elements (EL1) simulates the elastic and inelastic shear behaviour of half a link, while the second (EL2) simulates the inelastic flexural behaviour of the ending



**Fig. 1 – Elements of the link model**

part of the link (the elastic stiffness of this element is assumed equal to infinity). The nodes of EL1 are allowed to have only relative vertical displacements; those of EL2 may have only relative rotations. The models differ because of the inelastic response of the elements EL1 and EL2.

### 1.1. Model 1: isotropic and kinematic hardening

The first model (M1) has recently been proposed in Bosco et al. (2015) and considers isotropic hardening explicitly. The response of the two elements EL1 and EL2 is defined by means of the uniaxial material model proposed by Zona and Dall'Asta (2012) for buckling restrained braces. This uniaxial material model considers a simple rheological scheme where a spring “0” is connected in series with a friction slider in parallel with a spring “1”. The stiffness of the spring “0”, named  $k_0$ , is equal to the initial elastic stiffness of the element while the stiffness of the spring “1”, named  $k_1$ , influences the post-yield behaviour of the element. The hysteretic response is described by the nonlinear relationship between force and deformation in the friction element. The full description of this model requires that values be given to the stiffnesses  $k_0$  and  $k_1$ , to the initial yield force  $F_{y0}$  and to the maximum yield force for the fully saturated isotropic hardening condition  $F_{y\max}$ . In addition, the values of two positive non-dimensional constants have to be specified. The first constant  $\beta$  controls the rate of the isotropic hardening; the second,  $\alpha$ , controls the trend of the transition from the elastic to the plastic response.

The elastic stiffness  $k_{L1,0}$  of element EL1 is calculated by means of the equation

$$k_{L1,0} = \frac{GA}{\chi e/2} \quad (1)$$

where  $A$  is the area of the link cross-section,  $G$  is the tangent modulus of elasticity,  $e$  is the link length and  $\chi$  is the shear coefficient. This latter parameter is calculated here as

$$\chi = \frac{b(d-t_f)^5}{240A \cdot i^4} \left[ 15 \frac{\eta^2}{\xi} + 10\eta + 5\eta \frac{b^2}{(d-t_f)^2} + 2\xi \right] \quad (2)$$

where  $i$  is the radius of gyration of the cross-section,  $\xi = t_w/b$  and  $\eta = 2t_f/(d-t_f)$ ,  $b$  is the width of the flange,  $d$  is the depth of the section,  $t_f$  is the thickness of the flange,  $t_w$  is the thickness of the web.

The initial yield force  $V_y$  of the same element is equal to the plastic shear force  $V_p$  of the link specimen, calculated by means of the relationship

$$V_p = 0.6 f_{yw} t_w (d - 2t_f) \quad (3)$$

where  $f_{yw}$  is the tensile yield strength. Based on several numerical tests on links, the following values of the parameters above have been suggested (Bosco et al., 2015) to simulate the cyclic response of links in which stiffeners are disposed as required by modern seismic codes

$$k_{L1,1} = 0.442 \% k_{L1,0} \quad V_{y,\max} = 1.308 V_y \quad (4a)$$

$$\alpha_{L1} = 0.485 \quad \beta_{L1} = 0.113 \quad (4b)$$

The ultimate shear force of short links (i.e. the shear force corresponding to a plastic rotation angle equal to 0.08 rad) corresponding to the fully saturated isotropic hardening is

$$V_u = 1.308 \cdot V_p + \frac{0.08 \cdot e/2 \cdot k_{L1,0}}{k_{L1,0}/k_{L1,1} - 1} \quad (5)$$

Element EL2 is characterised by a very high elastic stiffness  $k_{L2,0}$  because the elastic flexural deformability of the link is simulated by element EL0. The bending moment at yield  $M_y$  is assumed equal to the plastic moment  $M_p$  of the entire cross-section

$$M_p = f_{yf} b t_f (d - t_f) + f_{yw} \frac{t_w}{4} (d - 2t_f)^2 \quad (6)$$

where  $f_{yf}$  is the tensile yield strength of the flanges. The suggested values of the post-elastic stiffness  $k_{L2,1}$ , that of the fully saturated bending moment  $M_{y,max}$ , and those of the parameters  $\alpha_{L2}$  and  $\beta_{L2}$  are

$$k_{L2,1} = 0.795\% \frac{6EI}{e} \quad M_{y,max} = 1.212 M_y \quad (7a)$$

$$\alpha_{L2} = 0.025 \quad \beta_{L2} = 0.054 \quad (7b)$$

Owing to this, the ultimate bending moment of long links (i.e. the bending moment corresponding to a plastic rotation angle equal to 0.02 rad) is

$$M_u = 1.212 \cdot M_p + 0.02 \cdot k_{L2,1} \quad (8)$$

## 1.2. Models 2 and 3: kinematic hardening

In the second model (M2) and in the third model (M3) the behaviour of elements EL1 is elastic-plastic with kinematic hardening (Fig. 2a, 2b). The effect of the isotropic hardening is not considered explicitly in these models. In particular, in model M2 an equivalent kinematic strain hardening is used to include the effects of both isotropic and kinematic hardening while in model M3 the effect of the isotropic hardening is included assuming that the yield shear force and the yield bending moments are higher than the plastic values provided by Eqs. (3) and (6).

Specifically, in model M2, the elastic stiffness and the plastic shear force of element EL1 are equal to those assigned to model M1. The post elastic stiffness is such that the shear force corresponding to a plastic rotation angle equal to 0.08 rad is equal to that provided by model M1 at the same plastic rotation angle, i.e.

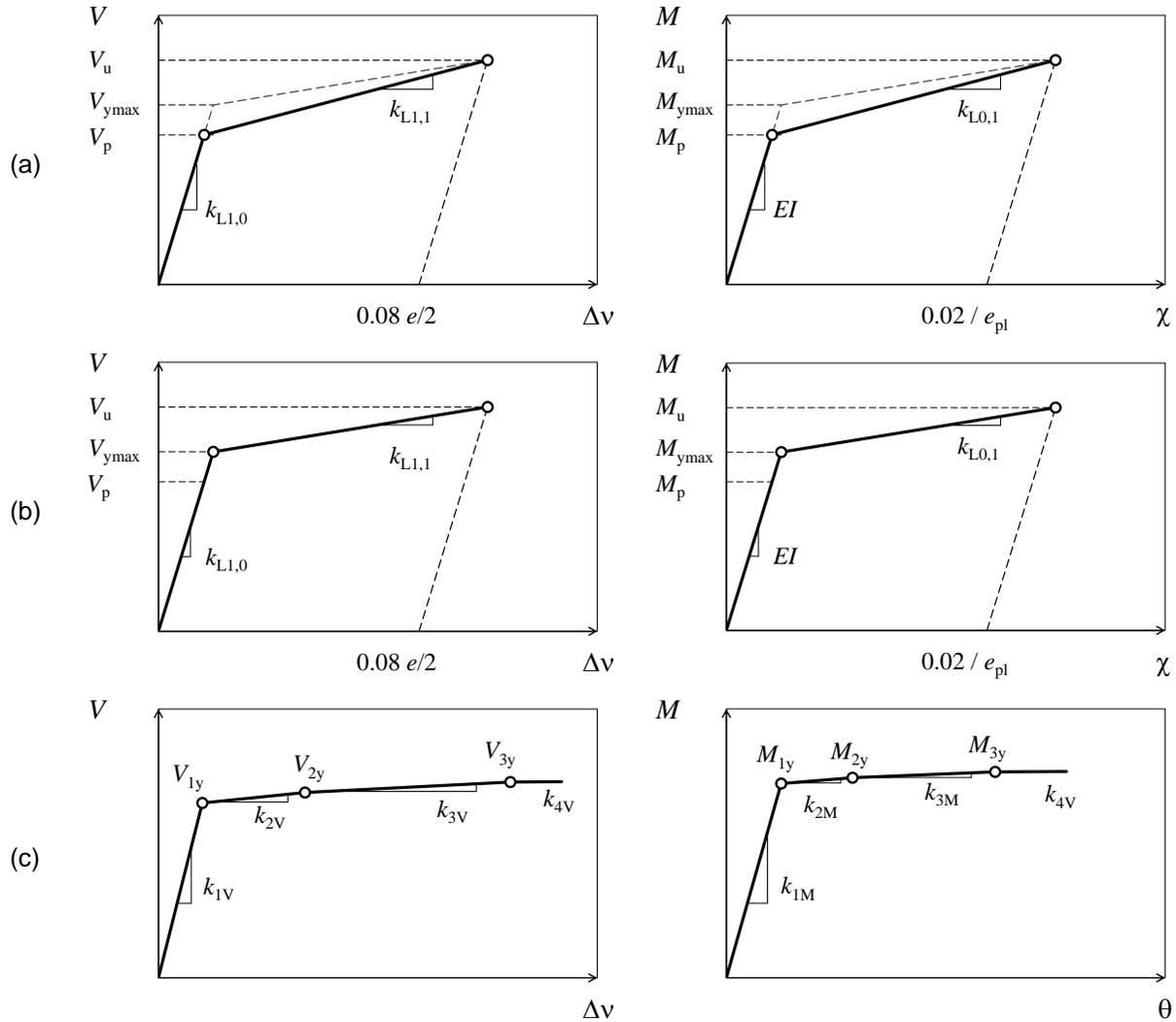
$$k_{L1,1} = \frac{V_u - V_p}{0.08 \cdot e/2 \cdot k_{L1,0} + (V_u - V_p)} \cdot k_{L1,0} \quad (9)$$

Elements EL2 are not included in the model because element EL0 is a beam with hinge element. The plastic hinges of element EL0 are characterised by a length  $e_{pl}$  equal to  $1/100 e$  and by an elastic-plastic with kinematic hardening moment-curvature relationship. This moment-curvature relationship is characterised by a plastic bending moment equal to  $M_p$ , an elastic flexural stiffness equal to  $EI$  and a post elastic stiffness equal to

$$k_{L0,1} = \frac{M_u - M_p}{0.02 \cdot EI/e_{pl} + (M_u - M_p)} \cdot EI \quad (10)$$

The flexural stiffness in the equation above is such the bending moment corresponding to a plastic rotation angle equal to 0.02 rad is equal to that provided by model M1 at the same plastic rotation angle.

Instead, in model M3, the yield shear force and the yield bending moments are equal to the values corresponding to the fully saturated isotropic hardening condition, i.e.  $V_y = V_{y,max}$  and  $M_y = M_{y,max}$ . In



**Fig. 2 – Response of element EL1 and plastic hinge of element EL2 according to model: (a) M2, (b) M3, (c) M4**

addition, the post yield stiffness  $k_{L1,1}$  of element EL1 is equal to the corresponding stiffness of model M1 while the post yield stiffness of the plastic hinge of element EL0 is

$$k_{L0,1} = \frac{M_u - M_{y,max}}{0.02 \cdot EI / e_{pl} + (M_u - M_p)} \cdot EI \quad (11)$$

### 1.3. Model 4: Ramadan and Ghobarah

The last model considered in the paper is that proposed by Ramadan and Ghobarah (1995). According to this model, elements EL1 consist of three translational springs while elements EL2 consist of three rotational springs. In each element, the three springs operate in parallel in order to achieve a multi-linear behaviour (Fig. 2c). The individual resistances and the stiffness of the shear and flexural sub-hinges are adjusted so that the combined values governing the multi-linear shear force–deformation and bending moment–rotation relationships are

$$V_{1y} = 0.90 V_p \quad V_{2y} = 1.06 V_{1y} \quad V_{3y} = 1.12 V_{1y} \quad (12a)$$

$$k_{2V} = 0.030 k_{1V} \quad k_{3V} = 0.015 k_{1V} \quad k_{4V} = 0.002 k_{1V} \quad (12b)$$

$$M_{1y} = M_p \quad M_{2y} = 1.03 M_p \quad M_{3y} = 1.06 M_p \quad (13a)$$

$$k_{2M} = 0.030 k_{1M} \quad k_{3M} = 0.015 k_{1M} \quad k_{4M} = 0.002 k_{1M} \quad (13b)$$

In the equations above, the plastic resistance  $V_p$  and  $M_p$  are calculated by Eqs. (3) and (6), the elastic stiffness  $k_{1V}$  by Eq. (1) and  $k_{1M}$  is equal to  $6EI/e$ . The shear spring follows an isotropic hardening behaviour according to the equation

$$V_{yx} = V_y [1 + 0.8 \exp(-10 \mu_c)] \quad (14)$$

where  $\mu_c$  is the accumulated deformation in the shear spring. Element EL0 is assumed to be rigid because the elastic flexural stiffness is considered by means of the rotational spring.

## 2. Design of multi-storey structures

The eccentrically braced frames investigated in this paper were designed in previous research (Bosco et al., 2014) and constitute simplified models of the structure of an apartment building having squared plan (24m x 24m) and geometric and mass properties equal at all floors (Fig. 3). The structure of this building is defined by the intersection of two groups of four three-span plane frames oriented along two orthogonal directions and located symmetrically to the geometric centre. Frames on the perimeter of the building are designed to resist the entire horizontal force and are endowed with eccentric braces disposed in the central span according to the split K-braced configuration. The interstorey height is equal to 3.3 m. To nullify the interaction between deck and links, two beam members are introduced at each level of the eccentrically braced frame instead of the traditional single section. Some geometric properties of these structures are varied within wide ranges in order to obtain systems with different dynamic and mechanical properties. In particular, the number of storeys  $n_s$  is varied from 4 to 12 (in step of 4) and the link length  $e$  is equal to either 0.1 to 0.3 times the length  $L$  of the braced span. Vertical dead and live loads are assumed constant on every floor level and are defined by characteristic values ( $G_k$  and  $Q_k$ ) equal to 4.4 and 2.0 kN/m<sup>2</sup>, respectively. All the structures stand on soft soil (soil C according to Eurocode 8) and are designed assuming a peak ground acceleration equal to 0.35 g and a behaviour factor equal to 5. The design internal forces on members are determined by either the modal response spectrum analysis (MRSA) or the lateral force method of analysis (LFMA). These two different methods of analysis are considered to investigate the effects of the link modelling on structures characterised by a more uniform distribution of plastic deformation of links (systems designed by MRSA) or by plastic deformations concentrated in a few stories (tall systems designed by LFMA). The non-dissipative members are designed according to the capacity design principles as per Eurocode 8. All the members that do not belong to the braced frames are designed to sustain gravity load only. In the following, the frames are

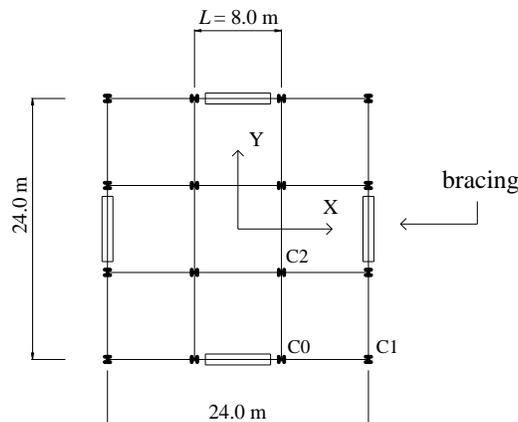


Fig. 3 – Plan of the building

identified by a label obtained by adding the number of storeys (04, 08 or 12), the link length (10 or 30 for systems with  $e/L$  equal to 0.10 or 0.30, respectively) and the design method of analysis (M for MRSA or S for LFMA)

### 3. Numerical analyses and response parameters

In this section, the response of the multi-storey structures is determined by incremental nonlinear dynamic analysis. The numerical analyses are performed by the program OpenSEES. The numerical model includes both the braced frames and the gravity columns, which are continuous for the entire height of the building and pinned at the base. Beam segments outside links, braces and columns are modelled by means of elastic elements. Links are modelled by each of the considered models.

The Rayleigh formulation is used to introduce damping. Mass and stiffness proportional damping coefficients are defined so that the first and the third modes of vibration of the structure are characterised by an equivalent viscous damping factor equal to 0.05. The stiffness proportional damping coefficient is applied to the initial stiffness matrix of the elements. In accordance with Ricles and Popov (1987) no stiffness proportional damping is considered for the zero length elements of the link. The seismic input consists of 10 artificially generated accelerograms (Amara et al., 2014).  $P-\Delta$  effects are included in the numerical analyses. The peak ground acceleration is scaled in step of 0.04g. For each considered value of peak ground acceleration, it is verified that seismic demands of dissipative and non-dissipative elements do not exceed the corresponding capacities in terms of deformations and strengths, respectively. Thus, the peak ground acceleration is scaled up to the value  $a_{gu}$  that leads to the first achievement of the ultimate rotation capacity  $\varphi_u$  of links or to yielding or buckling of non-dissipative members. Specifically, yielding and buckling resistances for combined values of axial force and bending moments of non-dissipative members are calculated according to Eurocode 3 assuming the partial safety factors  $\gamma_{M0}$  and  $\gamma_{M1}$  are equal to 1 while the plastic rotation capacity  $\varphi_u$  of links is calculated according to Eurocode 8 as

$$\varphi_u = \begin{cases} 0.08 \text{ rad} & \text{if } eV_p/M_p \leq 1.6 \\ 0.02 \text{ rad} & \text{if } eV_p/M_p \geq 3.0 \\ \text{linear interpolation} & \text{if } 1.6 < eV_p/M_p < 3.0 \end{cases} \quad (15)$$

For each accelerogram scaled to  $a_{gu}$ , the heightwise distribution of the link normalised plastic rotation and that of the residual drift angle is investigated. Specifically, the link normalised plastic rotation is calculated as the ratio of the maximum plastic rotation  $\varphi$  required at the link of the  $i$ -th storey to the corresponding rotation capacity  $\varphi_u$ ; the residual drift angle  $\Delta_{res}$  is calculated as the ratio of the residual drift to the interstorey height.

Finally, the values of  $a_{gu}$ ,  $\varphi/\varphi_u$  and  $\Delta_{res}$  predicted by the considered models are averaged over the number of the considered accelerograms and compared.

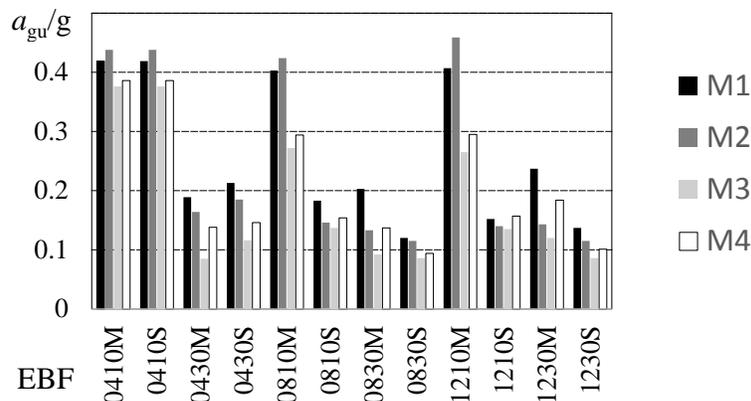
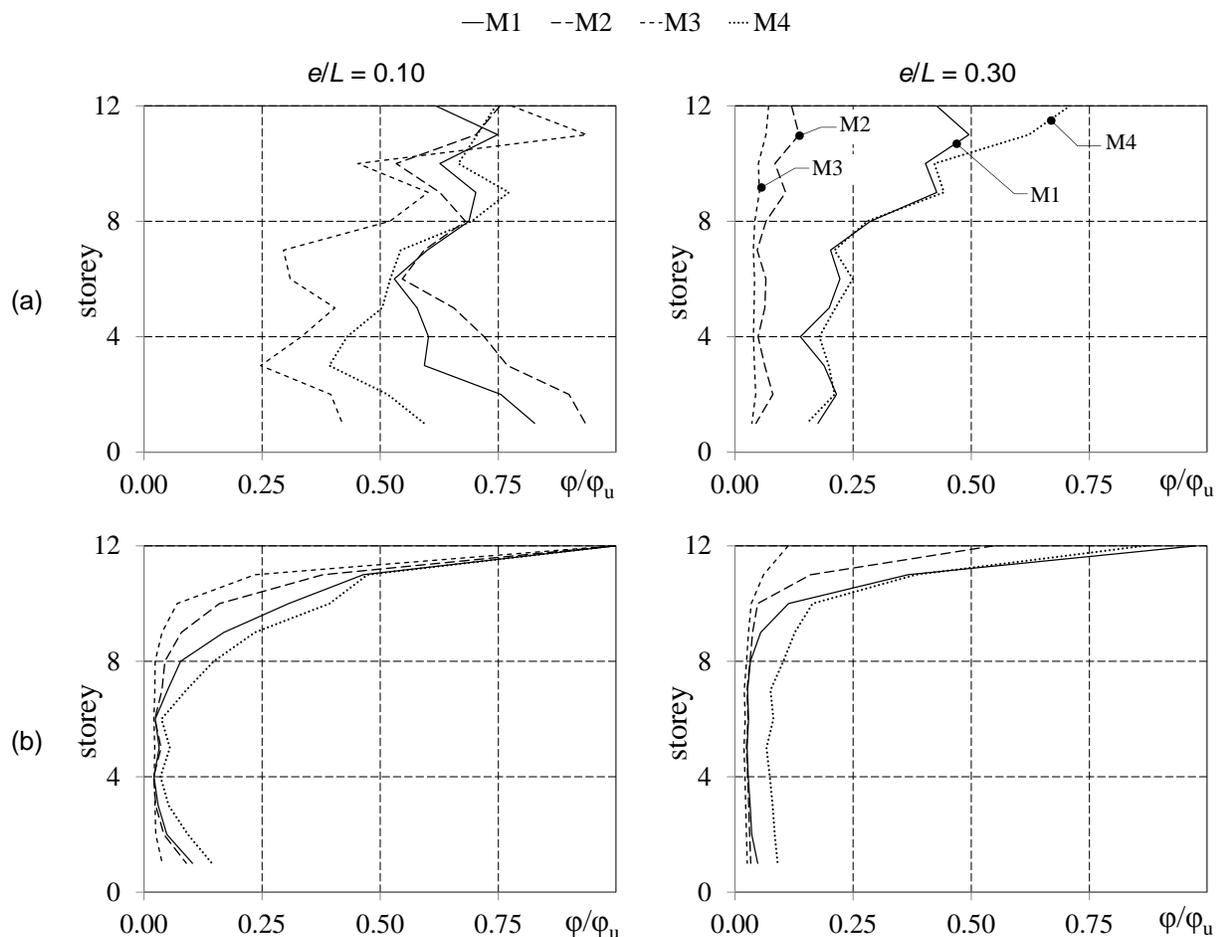


Fig. 4 – Comparison between the predicted ultimate peak ground accelerations

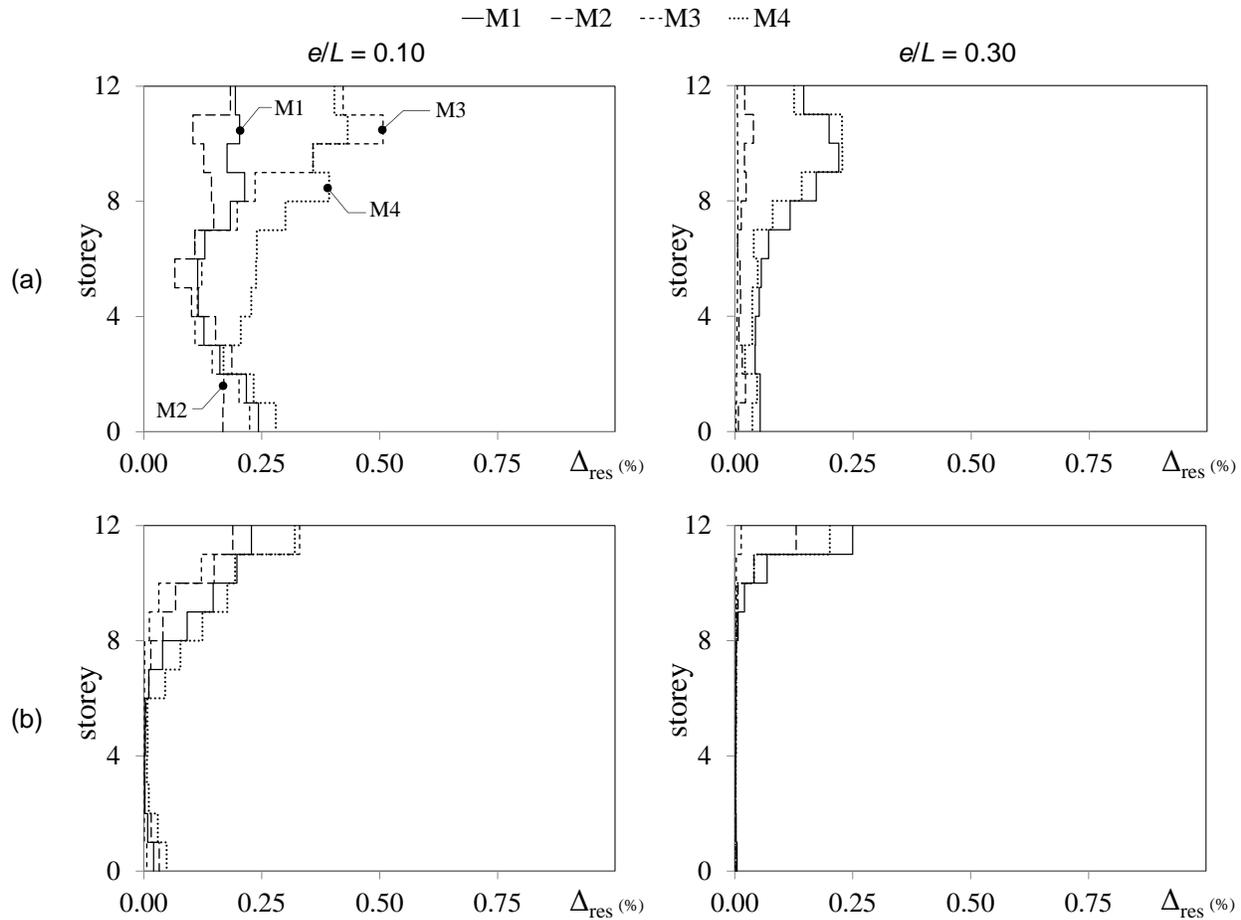
## 4. Comparison of the models

### 4.1. Ultimate peak ground acceleration

The average values of  $a_{gu}$  obtained for the considered models are compared in Fig. 4. In the figure, different colours are used to represent results obtained by the considered models. The figure shows that the models provide values of  $a_{gu}$  very close to each other when applied to predict the response of low-storey systems with short links (0410M, 0410S) or high storey systems designed by LFMA (0810S, 1210S, 0830S, 1230S). The greater differences between the values of  $a_{gu}$  are obtained when considering systems designed by MRSA, especially in the case of long links (0830M, 1230M). Out of the considered models, model M2 (dark grey bars) is that which gives the values of  $a_{gu}$  closest to those provided by model M1 (black bars), although it significantly underestimates the ultimate peak ground acceleration in the case of systems with long links designed by MRSA. Model M3 (light grey bars) is that which provides the lowest values of  $a_{gu}$ . In order to investigate deeper on the above-mentioned differences, it is worth noting that in systems with short links the value  $a_{gu}$  is always related to the first achievement of the ultimate rotation capacity of links; instead, in systems with long links, some yielding of the beam segments outside links may occur. In particular, for the considered structures with long links, models M2 and M3 always predict the achievement of the yielding resistance of the beam segment outside the link while, according to models M1 and M4, the achievement of the ultimate plastic rotation of the link is recorded only for some accelerograms. Thus, the differences on the predicted values of  $a_{gu}$  are partially



**Fig. 5 – Comparison between the predicted normalized plastic rotations of links: systems designed by (a) model response spectrum analysis, (b) lateral force method of analysis**



**Fig. 6 – Comparison between the predicted residual storey drift angles: systems designed by (a) model response spectrum analysis, (b) lateral force method of analysis**

due to the different failure mechanism and are related to the different value of the bending moment that, according to the considered models, are transmitted by the link for assigned values of plastic deformation.

#### 4.2. Link normalised plastic rotation

Fig. 5 shows the comparison between the link normalised plastic rotations predicted by the models for the considered 12-storey structures. The solid line is used to represent the results predicted by model M1, different types of hatches are used for the other models. For the structure with short links ( $e/L = 0.10$ ) designed by MRSA (Fig. 5a), the normalised plastic rotation of links is significant at all the storeys. However, the values predicted by the models are scattered especially at the lower storeys. As an example, at the third storey, the percentage difference between the normalised plastic rotation provided by models M2, M3 and M4 and that given by model M1 is 30%, -58% and -33%, respectively. Model M4 seems to predict normalised plastic rotations closest to those obtained by model M1. In the corresponding systems designed by LFMA (Fig. 5b), the normalised plastic rotation is very high only at the upper storey while the plastic behaviour of the links of the other storeys is limited. For this reason, all the models provide a similar distribution of the considered response parameter. In the systems with long links designed by either MRSA or LFMA ( $e/L = 0.30$ ), the plastic rotations predicted by model M2 and M3 are very low because they are obtained for ultimate peak ground accelerations that are significantly smaller than those corresponding to the models M1 and M4. Again, a good agreement is recorded between the

predictions obtained by these two models. Even if not shown in any figure, these results are confirmed when considering 4- and 8-storey frames.

### 4.3. Residual drift angles

Fig. 6 shows the comparison between storey drift angles predicted by the 4 considered models for the same structures analysed in the previous sub-section. When short links are considered, the model M2 is that providing the results closest to those of the model M1. However, the percentage differences between the predicted residual drift angles are significantly greater than those obtained for the link normalised plastic rotation. Models M3 and M4 provide larger residual drifts especially in the upper storeys. When long links are considered, models M1 and M4 give similar results, while models M2 and M3 give residual drifts close to zero. This result is not surprising owing to the low values of the plastic rotations of links corresponding to the ultimate peak ground accelerations predicted by these models.

## 5. Conclusions

This paper investigates the effectiveness of some simplified models of steel link beams commonly used in the seismic assessment of eccentrically braced frames. The benchmark is represented by the seismic response obtained by means of a simple but refined link model (M1 model) that has recently been proposed by the writers and takes into account both kinematic and isotropic hardening. The comparison is carried out on the seismic response at collapse of 12 eccentrically braced frames designed as per Eurocode 8. The seismic response at collapse of the analysed frames is determined by incremental nonlinear dynamic analysis. The frames differ because of the number of storeys, the geometrical link length and the design method of analysis. The seismic response is expressed in terms of ultimate peak ground acceleration, link normalised plastic rotations and residual storey drift angles.

The results of incremental nonlinear dynamic analyses show that the simplified model in which the effect of isotropic hardening is represented by an equivalent (increased) kinematic hardening (model M2) gives the values of ultimate peak ground acceleration closest to those provided by reference model for systems with short links. Instead, the simplified model in which the effect of isotropic hardening is represented by increasing the value of the yielding shear force and bending moments (model M3) underestimates significantly the same response parameter. Intermediate results are provided by the model proposed by Ramadan and Ghobarah (model M4).

The major differences between the responses predicted by the analysed models are recorded for systems with long links. Indeed, in this case, models M2 and M3 predict the yielding of the beam segment outside link while this behaviour is recorded only for some accelerograms when the other two considered models are adopted.

The percentage difference between the predicted normalised plastic rotations provided by models is not negligible for structures in which links have large plastic rotations at all the storeys. The greatest differences between the models are obtained when residual drifts are considered.

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